

## Simultaneous Maximization of Spatial and Temporal Autocorrelation in Spatio-temporal Data

Allan Aasbjerg Nielsen, *Technical University of Denmark*

Based on Switzer and Green's maximum autocorrelation factor (MAF) analysis for multivariate image data [Min/Max Autocorrelation Factors for Multivariate Spatial Imagery (1984), Technical Report 6, Department of Statistics, Stanford University] an extension for spatio-temporal data is suggested. Traditional MAF analysis generates new orthogonal variates  $a^T X$  from the original multivariate image data  $X$  by maximizing spatial autocorrelation. This is done by solving the generalized eigenproblem represented by the Rayleigh coefficient  $\lambda = a^T \Sigma_{\Delta} a / a^T \Sigma a$  where  $\Sigma$  is the dispersion of  $X$  and  $\Sigma_{\Delta}$  is the dispersion of the difference between  $X$  and  $X$  spatially shifted. Hence, the new variates are obtained from the conjugate eigenvectors  $a$  and the autocorrelations obtained are  $1 - \lambda/2$ , i.e., high autocorrelations are associated with small eigenvalues and vice versa. Often  $\Sigma_{\Delta}$  is calculated by means of a pool of a horizontal and a vertical shift. If the data are not spatial but temporal the spatial shift is replaced by a temporal shift causing the temporal autocorrelation to be maximized. Such a temporal MAF analysis is equivalent to Molgedey and Schuster's method for calculating independent components of temporal data [Separation of a Mixture of Independent Signals Using Time Delayed Correlations, *Physical Review Letters* 72(23) (1994), 3634–3637]. If the data are both spatial and temporal it is suggested here to calculate  $\Sigma_{\Delta}$  by pooling both spatial and temporal shifts. Results from such a simultaneous maximization of spatial and temporal autocorrelation on two years of global monthly mean sea surface temperature and sea surface height anomaly data are compared with results from 1) maximization of temporal autocorrelation alone, 2) maximization of spatial autocorrelation alone, and 3) so-called empirical orthogonal function (EOF) analysis which is traditionally used in oceanography and which is equivalent to principal component analysis [Preisendorfer, *Principal Component Analysis in Meteorology and Oceanography* (1988), Elsevier].

[Allan Aasbjerg Nielsen, Informatics and Mathematical Modelling, Informatics and Mathematical Modelling Building 321, DK-2800 Kongens Lyngby, Denmark; [aa@imm.dtu.dk](mailto:aa@imm.dtu.dk), [www.imm.dtu.dk/~aa](http://www.imm.dtu.dk/~aa)]