Evaluation of the Wishart Test Statistics for Polarimetric SAR Data

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Abstract—**A test statistic for equality of two covariance matrices following the complex Wishart distribution has previously been used in new algorithms for change detection, edge detection and segmentation in polarimetric SAR images. Previously, the results for change detection and edge detection have been quantitatively evaluated. This paper deals with the evaluation of segmentation. A segmentation performance measure originally developed for single-channel SAR images has been extended to polarimetric SAR images, and used to evaluate segmentation for a merge-using-moment algorithm for polarimetric SAR data.**

Keywords: Polarimetry; SAR; Wishart; segmentation; evaluation

I. INTRODUCTION

Edge detection and segmentation are important examples of low-level operators, providing the basic information for higher-level algorithms. It is very important that such low-level operators are adapted to the image statistics to provide optimal results. The probability density function (pdf) for homogeneous areas in single-channel SAR images is readily described by the Gamma pdf. Using the Gamma pdf for homogeneous areas in single-channels SAR images, the ratio detector was developed in the middle eighties [1][2].

Also, segmentation algorithms for SAR images have been suggested in the literature. Most of these apply to single-channel SAR images but multi-channel algorithms have also been described. One approach for segmentation is based on edge detection and region growing as described in [3-5]. Another approach is to perform segment merging in an initially over-segmented image using an appropriate test statistic. This approach has for instance been used for single channel SAR images in the MUM (Merging Using Moments) segmentation algorithm described in [5][6].

An appropriate representation of the polarimetric SAR data is the covariance matrix, i.e. a 3x3 Hermitian, positive definite matrix which follows a complex Wishart pdf. Based on this pdf a test statistic for equality of two such matrices and an associated asymptotic probability for obtaining a smaller value of the test statistic were developed in [7][8]. Previously, algorithms for change detection [7][8], edge detection [9][10] and segmentation [11] have been developed using the test statistic [12][13]. The advantage of the test statistic is that it uses the full polarimetric information in a uniform way, instead of handling each element of the covariance matrix individually followed by a heuristic combination of the results.

The results reported in [7-13] clearly show that the Wishart test statistic uses the full polarimetric information. An important aspect of algorithm evaluation is quantitative evaluation of the performance for different polarimetric modes, i.e. multipolarisation SAR (i.e. all off-diagonal elements are zero), azimuthally symmetric mode (i.e. all co- and cross-elements are zero), and fully polarimetric mode.

Quantitative evaluation of the test statistic applied to change detection and edge detection has already been performed in [8] and [10], respectively. In [8], the test statistic and the associated asymptotic probability for obtaining a smaller value of the test statistic were computed for a number of different areas for two different acquisitions of the Danish airborne, polarimetric EMISAR [14] to assess the potential of the test statistic for change detection. The potential of the test statistic for edge detection was evaluated in [10] using both EMISAR data and simulated polarimetric SAR data. In both cases the azimuthally symmetric mode performed better than the multipolarisation (or diagonal) mode and the fully polarimetric mode. The difference to the diagonal mode is due to the additional information in the off-diagonal elements. For the fully polarimetric mode it is probably due to a larger noise contribution for the off-diagonal elements

The present paper will concentrate on the evaluation of the segmentation results previously reported in [11]. The paper is structured as follows: In Section II the performance measure used to evaluate the results are presented, followed by the results in Section III and the conclusions in Section IV.

II. PERFORMANCE MEASURES

Quantitative segmentation performance measures were suggested by Caves et al. [15] for single-channel SAR data. In this section, these measures are briefly outlined, and hereafter an extension to polarimetric SAR data is proposed.

A. Single-channel SAR

A ratio image, formed by dividing the original image with its segmentation, is suggested in [15] as a performance measure. A ratio image, where each pixel is divided by its true backscatter coefficient, σ^0, would consist of pure speckle with

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mean 1, and hence residual structures in the ratio image would indicate regions where the segmentation has failed [15].

A global measure suggested in [15] is the average log-likelihood, L, per pixel of the intensity image I given a specified σ^0 image (i.e. the segment image), μ, defined as

$$L = \frac{1}{n}\sum_{i=1}^{n} \ln(f(I_i \mid \mu_i)) \tag{1}$$

where $f(I_i|\mu_i)$ is the pdf of the intensity I_i given the mean μ_i, and n is the total number of pixels in the image. A more accurate segmentation will generate a more accurate mean image, and hence a higher likelihood. Assuming the intensities to be Gamma distributed in homogeneous areas, it is shown in [15] that the only contribution from the segments to the L is from the socalled normalised log measure, D, given by

$$D = \sum_{k=1}^{m} \frac{n_k}{n} \overline{\ln r_k} = \overline{\ln r} \tag{2}$$

where m is the number of segments, n_k is the number of pixels in segment k, r_k is the ratio for segment k, r is the ratio for the entire image, and $\bar{x}$ means spatial average.

B. Polarimetric SAR

For a polarimetric SAR, data are readily described by the covariance matrix. If the p x p sample covariance matrix, **C**, has N number of looks, we define the Hermitian matrix $\mathbf{z} = N\,\mathbf{C}$. $\mathbf{z}$ follows a complex Wishart pdf $W_C(p,N,\mathbf{\Sigma})$ having mean covariance matrix $\mathbf{\Sigma}$ [16], i.e.

$$p(\mathbf{z} \mid p, N, \mathbf{\Sigma}) = \frac{|\mathbf{z}|^{N-p}}{\Gamma_p(n)|\Sigma|^N} e^{-tr(\Sigma^{-1}\mathbf{z})} \tag{3}$$

where | | and tr() denote the determinant and the trace, respectively, and

$$\Gamma_p(n) = \pi^{\frac{1}{2}p(p-1)} \sum_{j=1}^{p} \Gamma(N - j + 1) \tag{4}$$

where Γ() is the Gamma function.

The average log-likelihood of the covariance image, **z**, given a specified **Σ** image (i.e. the segment image) is given by

$$L_{Wishart} = \frac{1}{n}\sum_{i=1}^{n} \ln(p(\mathbf{z}_i \mid p, N, \mathbf{\Sigma}_i)) \tag{5}$$

where $\mathbf{\Sigma}_i$ means the average covariance matrix within the segment corresponding to pixel i. Using (3) we have

$$\begin{aligned} L_{Wishart} = &-\ln \Gamma_p(N) - \frac{1}{n}\sum_{i=1}^{n} tr(\mathbf{\Sigma}_i^{-1}\mathbf{z}_i) \\ &- p\overline{\ln|\mathbf{z}|} + N(\overline{\ln|\mathbf{z}|} - \overline{\ln|\mathbf{\Sigma}|}) \end{aligned} \tag{6}$$

The first and third terms do not depend on the segmentation results. The second term is for a single segment, k, given by

$$\begin{aligned} \frac{1}{n_k}\sum_{\forall i \in Segk} tr(\mathbf{\Sigma}_i^{-1}\mathbf{z}_i) &= \frac{1}{n_k} tr(\sum_{\forall i \in Segk} \mathbf{\Sigma}_i^{-1}\mathbf{z}_i) \\ &= \frac{1}{n_k} tr(\mathbf{\Sigma}_k^{-1} \sum_{\forall i \in Segk} \mathbf{z}_i) \\ &= \frac{1}{n_k} tr(\mathbf{\Sigma}_k^{-1} n_k \mathbf{\Sigma}_k) = tr(\mathbf{I}) = p \end{aligned} \tag{7}$$

The second term in (6) for the entire image is consequently equal to p. Hence, only the fourth term in (6) is dependent on the segmentation. It can be written as

$$D_{Wishart} = \sum_{k=1}^{m} \frac{n_k}{n} D_{Wishart,k} \tag{8}$$

with

$$\begin{aligned} D_{Wishart,k} &= \overline{\ln|\mathbf{z}_k|} - \overline{\ln|\mathbf{\Sigma}_k|} \\ &= \frac{1}{n_k}\sum_{\forall i \in Segk} \ln|\mathbf{z}_i| - \frac{1}{n_k}\sum_{\forall i \in Segk} \ln|\mathbf{\Sigma}_i| \\ &= \frac{1}{n_k}\sum_{\forall i \in Segk} \ln\frac{|\mathbf{z}_i|}{|\mathbf{\Sigma}_i|} \end{aligned} \tag{9}$$

and hence combining (8) and (9)

$$D_{Wishart} = \frac{1}{n}\sum_{i=1}^{n} \ln\frac{|\mathbf{z}_i|}{|\mathbf{\Sigma}_i|} \tag{10}$$

Comparing (2) and (10) we can see that the ratio between the determinant in the covariance image and in the corresponding segmented image plays the same role for the polarimetric SAR as the ratio between intensities does for the single-channel SAR. The log-ratios will both have zero mean when the segmentation describes well the average structure in the image, and different from zero otherwise.

III. RESULTS

The log-ratio defined in (10) has been applied to the segmented images described in [11], where a MUM algorithm based on the Wishart pdf was used. The segmentation algorithm was applied to the L-band EMISAR image shown in Figs 1and 2, where the backscatter coefficients are shown in Fig. 1 and the phase difference between HH and VV is shown in Fig. 2. The average log-ratio for the entire image for the three modes described earlier was found to be –0.85 for the diagonal mode, -0.75 for the azimuthally symmetric mode, and –1,00 for the fully polarimetric mode. Hence, the best result is obtained for the azimuthally symmetric mode, and the worst result for the fully polarimetric mode.

In Figs. 3 and 4 are shown the log-ratio for the azimuthally symmetric mode and the diagonal mode, respectively. Here values close to zero are shown as black, and other values are coloured. Smaller structures such as buildings and hedges are seen to produce large ratios in both cases, showing that the segmentation is not optimum for such structures. A significant difference is seen for the fields labelled A and B in Fig. 1. For these fields the ratio is much larger in the diagonal mode. Due to the small difference for the backscatter coefficients for these fields and some of the neighbouring fields (cf. Fig. 1), the MUM algorithm does not produce separate segments for these fields using the diagonal mode [11]. This is, however, the case for the azimuthally symmetric mode due to the separation of the fields using the phase difference as seen in Fig. 2

IV. CONCLUSIONS

A segmentation performance measure originally developed for single-channel SAR images was extended to polarimetric SAR data. It was then tested on segmented EMISAR L-band polarimetric data, and showed that the performance of the azimuthally symmetric mode was better than both the diagonal and the fully polarimetric modes.

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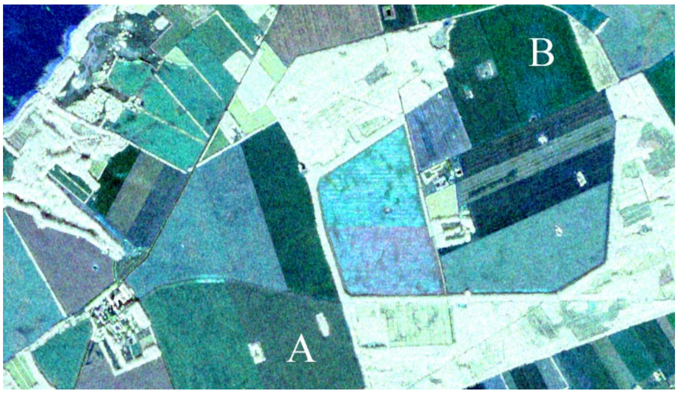

Figure 1. EMISAR L-band image (HH:green, VV: blue, HV: red)

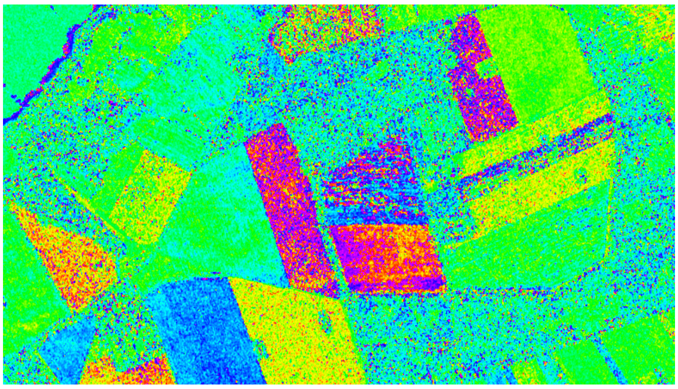
Figure 2. Phase difference between HH and VV for Fig.1 image

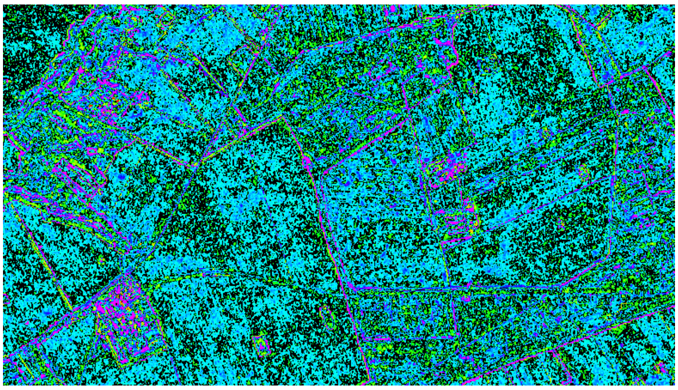
Figure 3. Log ratio for azimuthally symmetric mode

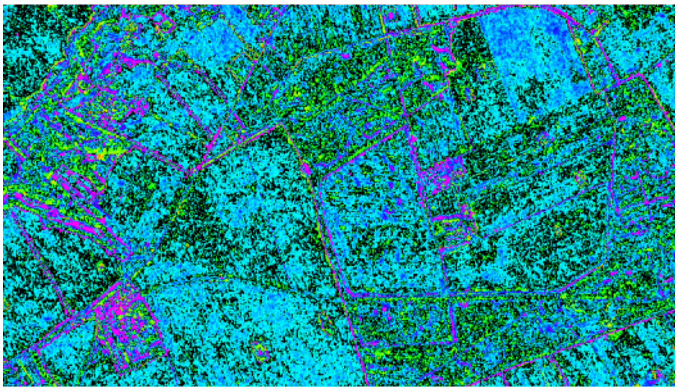
Figure 4. Log ratio for diagonal mode