From Shapes to Manifolds

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Overview

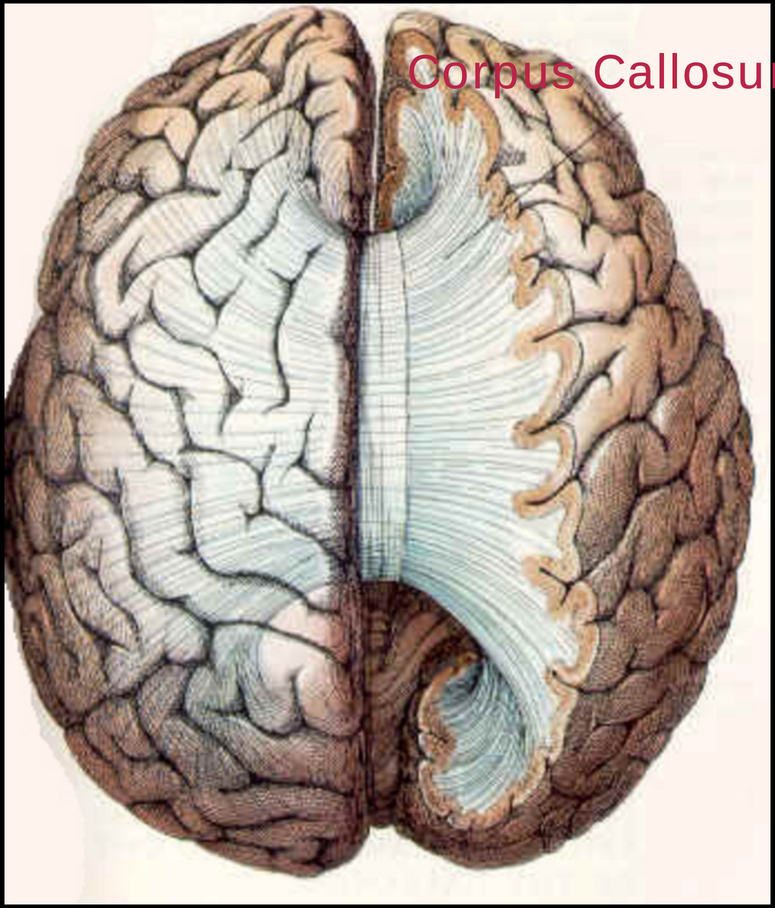
- An introduction to the topics of the summer school
- Conceptual understanding of the themes
- Shapes and shape representations
- Putting shapes into space(s)
- Shapes on manifolds
- Manifold navigation

A typical scenario

- I know a man!
- He can see things that others can not see!
- He is an expert!
- Doctor X believes that he can "see" on a hand X-ray if the patient is in risk of arthritis!
- Specifically Doctor X is sure that the *shape of the joints* is an estimator for arthritis!

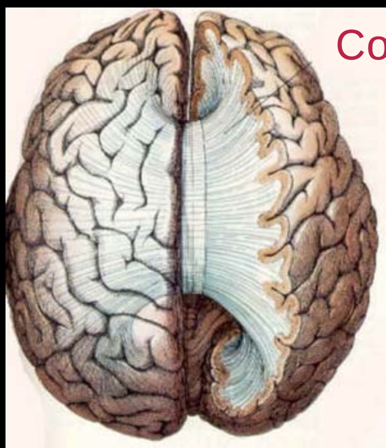
Can we verify that?

Scenario II

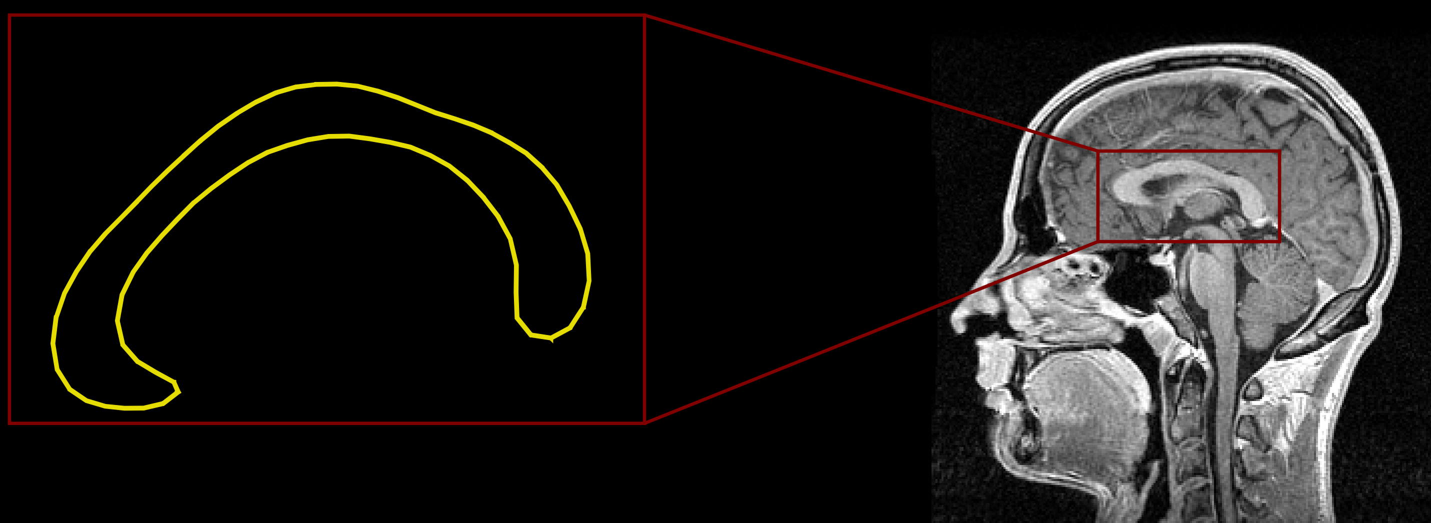


- MR images have been captured of a large group of people
- Cognitive abilities measured as well
- Is there a correlation between *how the brain looks* and how we behave?
- Does the shape of corpus callosum tell us something?

Scenario II



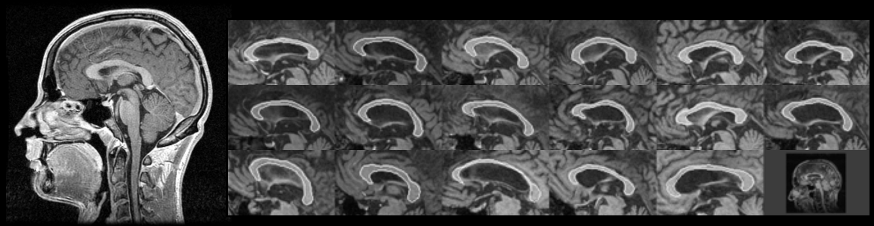
- We can get the MR slice with the corpus callosum from all the patients



Scenario III

- An experienced hearing aid fitter has seen a lot of ears!
- Some hearing aid users are very difficult to fit. Why?
- Large variation in the shape of ears
- Ear canals change shape when people chews
- Is it possible to learn about the shape and use it?

A project starts



600 MR scans and behavioural data

1000 historical X-rays

A boxful of something that look like ear canals

- The situation is now that we got a lot of nice data
- We would like to learn these secrets that the experts believe the shape contains

Where do I start?

What did the supervisor say?

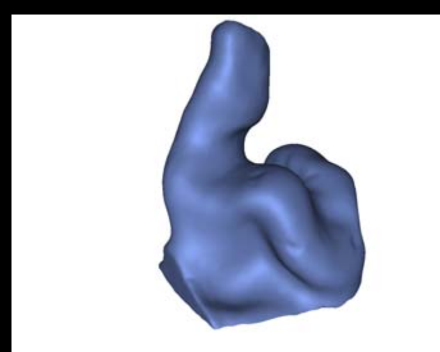
- Make a shape model
- Use manifold learning
- The rest of this presentation is for those of you that left your supervisor in confusion!

A boxful of something that look like ear canals

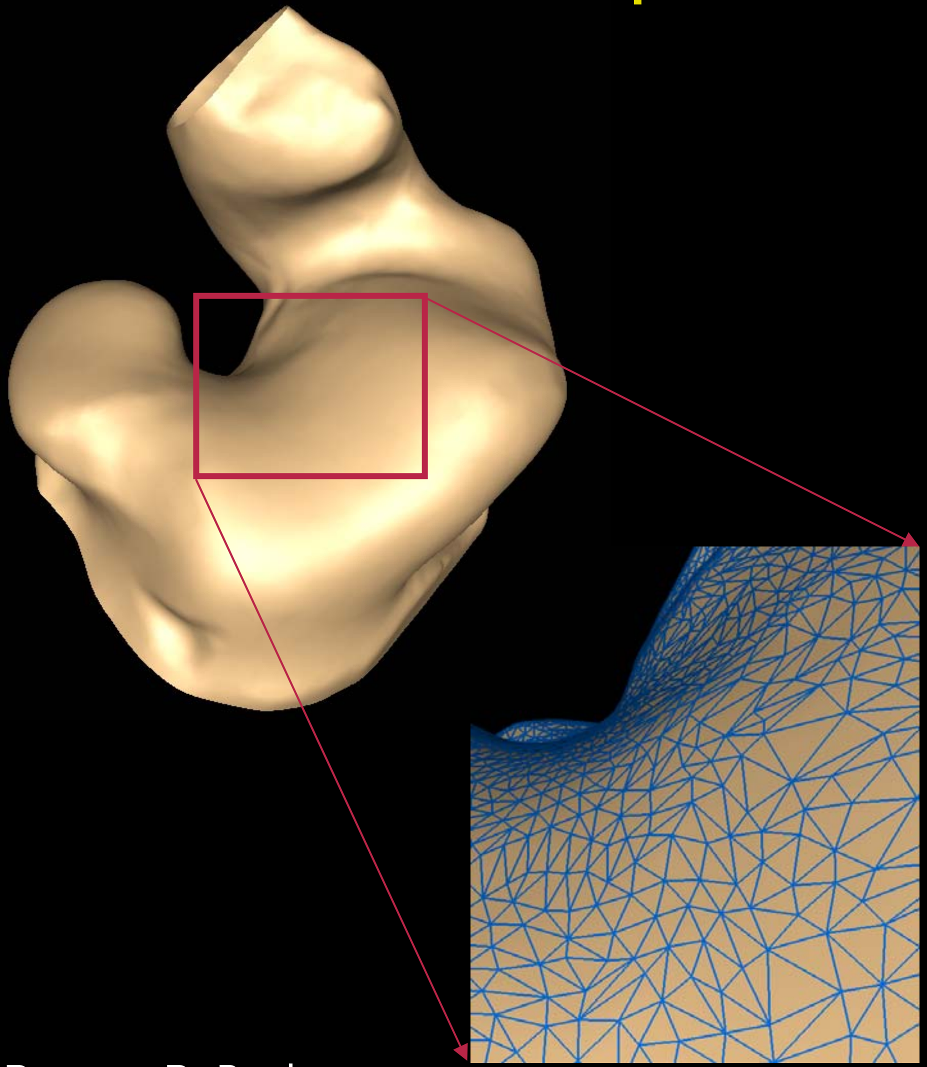
Ear Impression

Digitalisation

- We need a digital representation of our objects
- We want the geometry of the ear canal – not the volumetric properties
- Laser scanning
- Accurate surface description of the ear impression

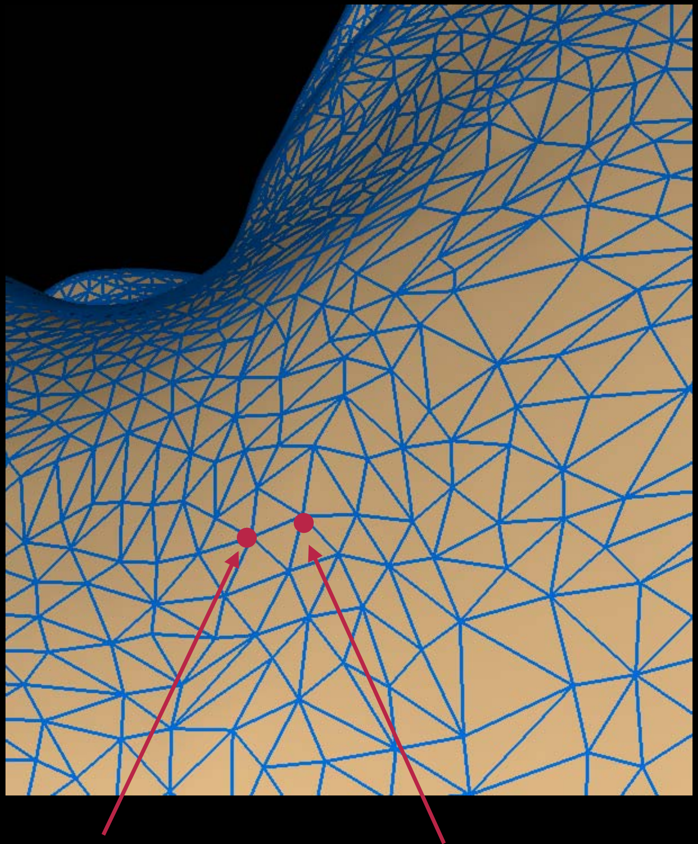


Initial shape representation



- The ear shape is represented as a 2D surface embedded in 3D space
- In our case it is a triangulated surface
- Connected points
- The shape of the ear canal is defined by the positions of the mesh points

Initial shape representation II



i=17

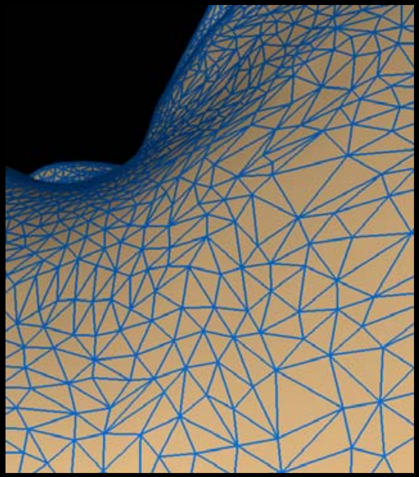
(x,y,z) = (10,9,1)

i=8192

(x,y,z) = (11,10,0)

- Each point is defined by an id number (i) and a coordinate (x,y,z)
  - Total number of point N=11.400 so: 1 <= i <=11.400
- In 2D shapes points can be ordered
  - Not in 3D!
  - Neighbour points can have very different ids
  - Ids typically assigned by the capture device

Putting a shape in space

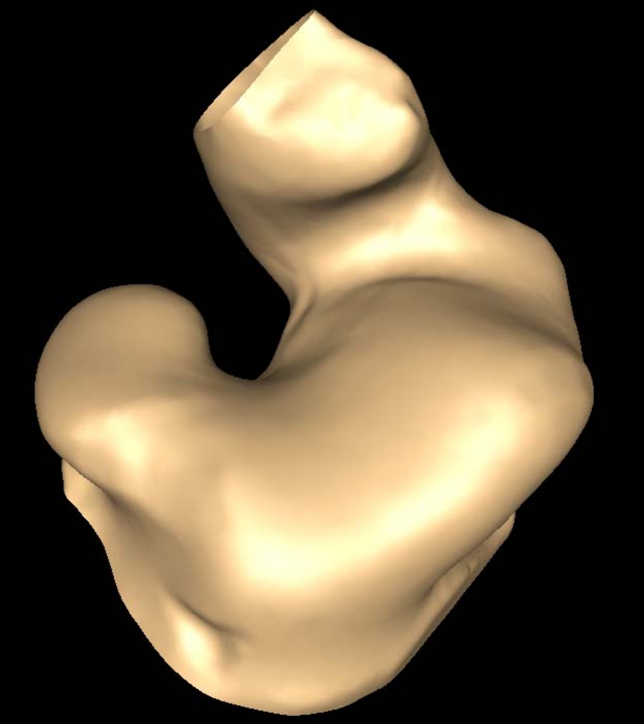


$$1 : (x_1, y_1, z_1)$$
$$2 : (x_2, y_2, z_2)$$
$$\vdots$$
$$N : (x_N, y_N, z_N)$$

- The shape is represented as an array of (x,y,z) coordinates
- Trick number one!
  All coordinates are put into one vector!
- 11.440 points
  - Vector with 34.320 elements!

$$\mathbf{x} = [x_1, y_1, z_1, \ldots, x_N, y_N, z_N]^T$$

Putting a shape into space II

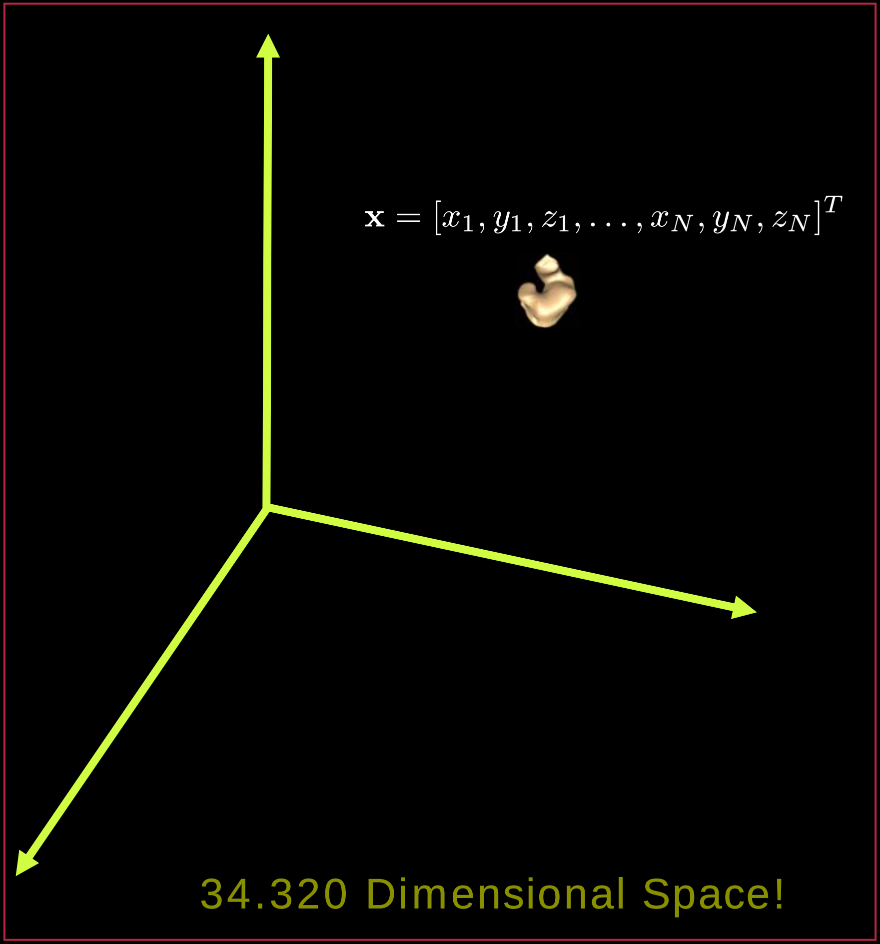


$$\mathbf{x} = [x_1, y_1, z_1, \ldots, x_N, y_N, z_N]^T$$

- On ear canal is now described using one vector
- A vector can also be seen as a point in space!

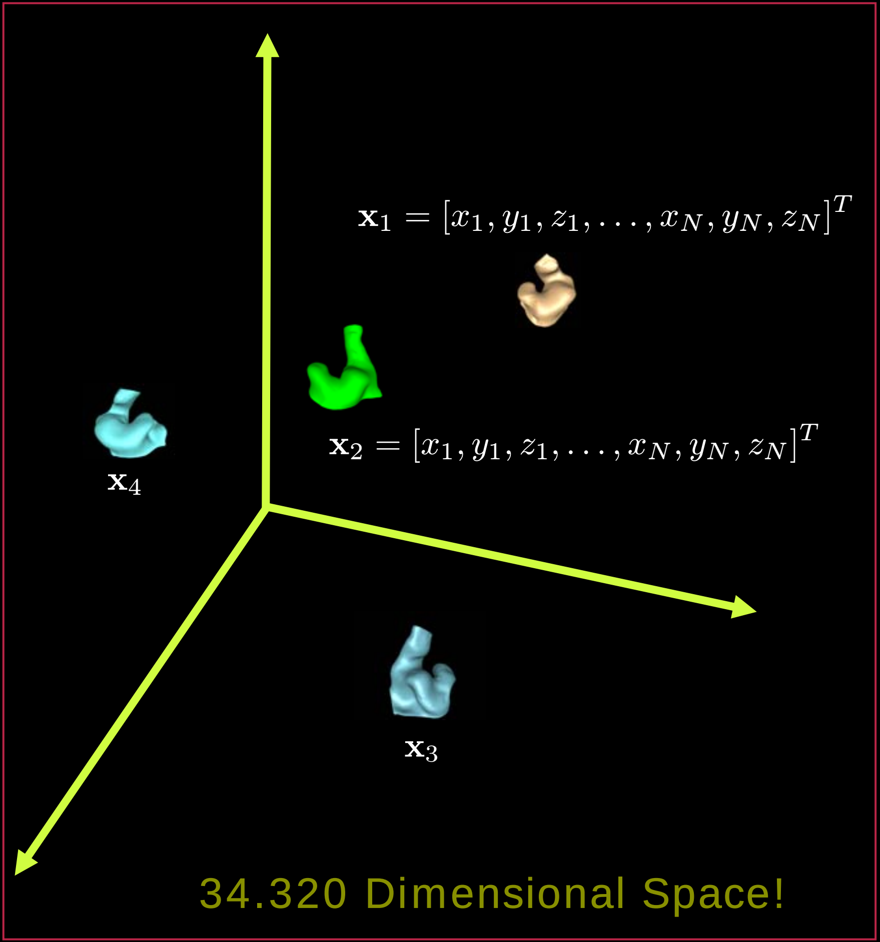
Trick number two!

Coordinates in space



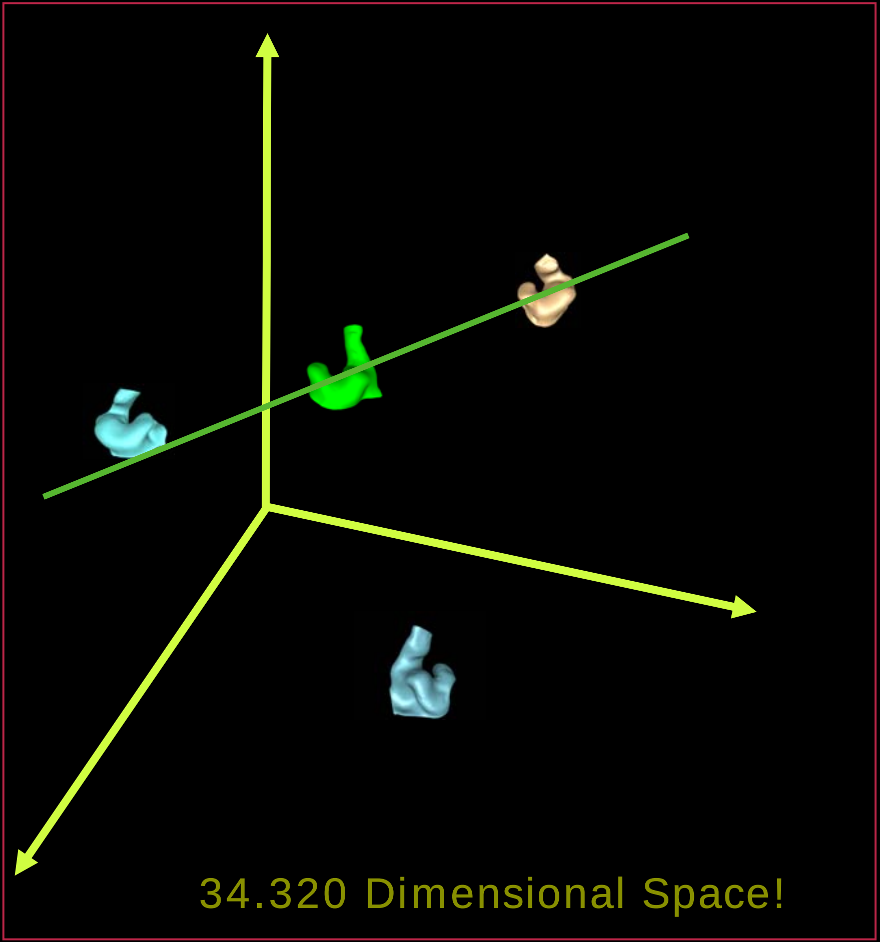
- On ear canal is now described using one vector
- A vector can also be seen as a coordinate in space!
- Not 2D space, not 3D space, not 4D space...
- 34.320 Dimensional Space!
- An ear canal has a position in this space!

Ears in Space



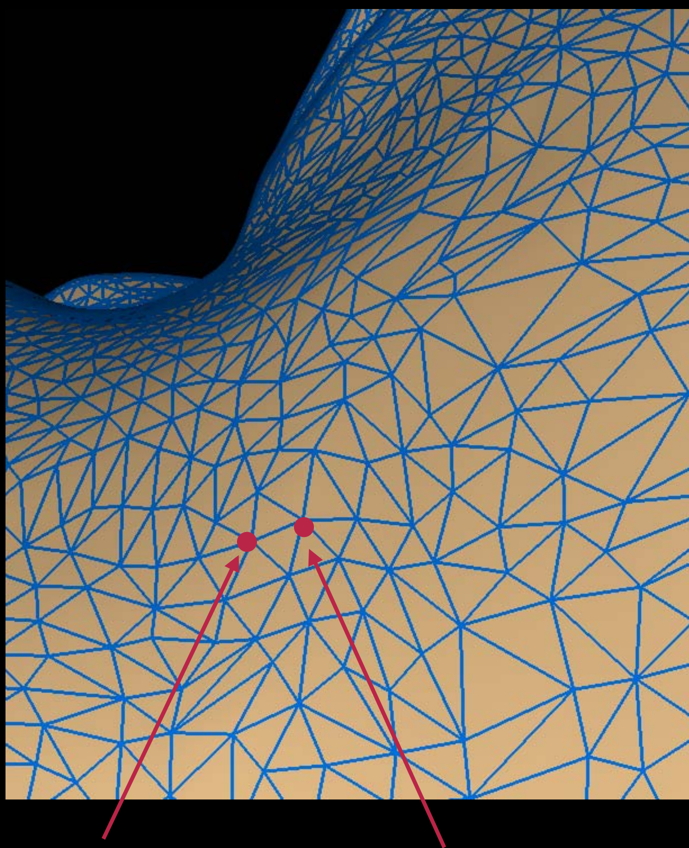
- An ear canal has a position in space!
- Another ear canal appears
  - in the same space
  - different position = different shape
- All ear canals have a place in this space!

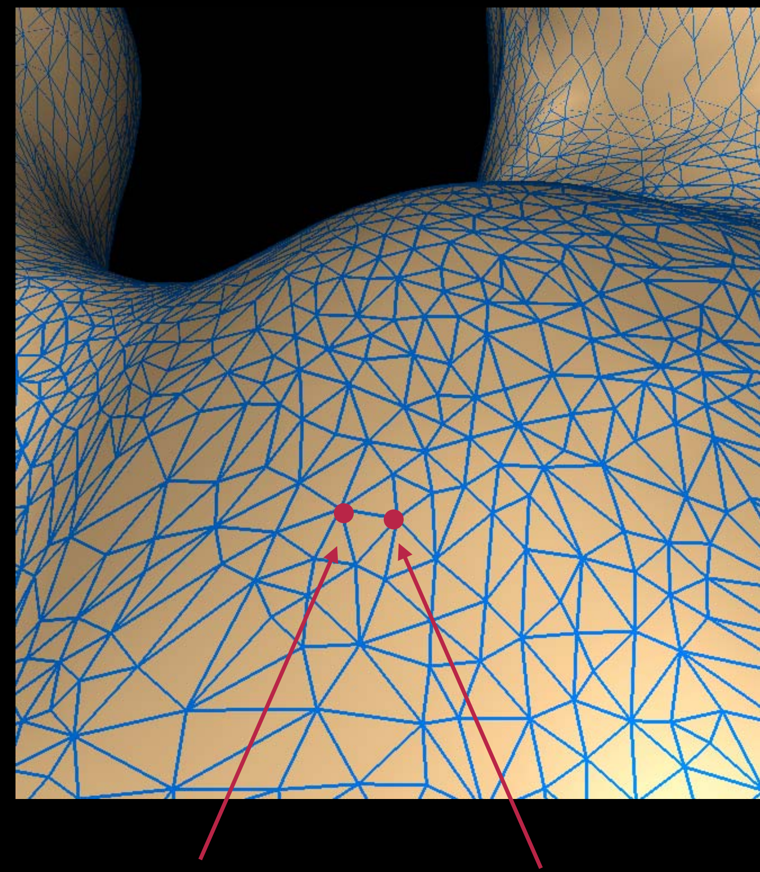
Ready for Manifold learning?



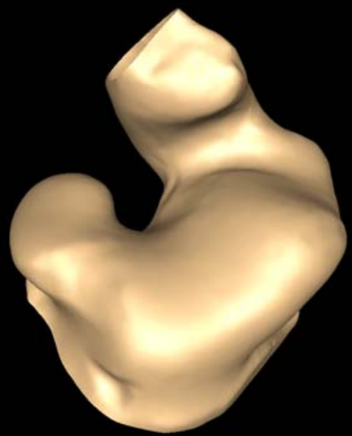
- In principle we are ready for manifold learning
- I just "forgot" to mention some details
- I will get back though

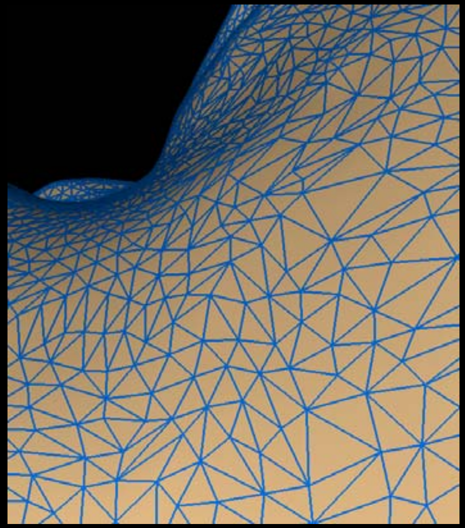
Two shapes

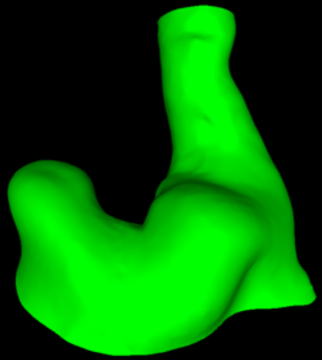


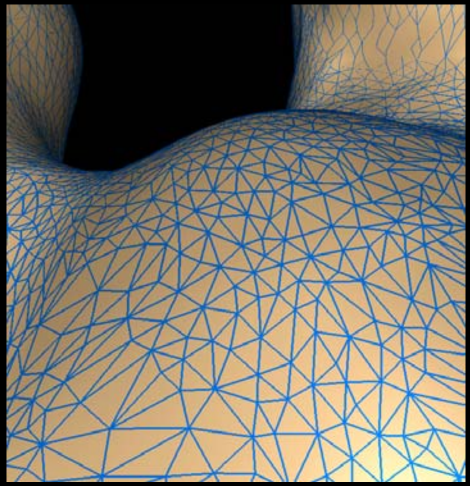


Two shapes





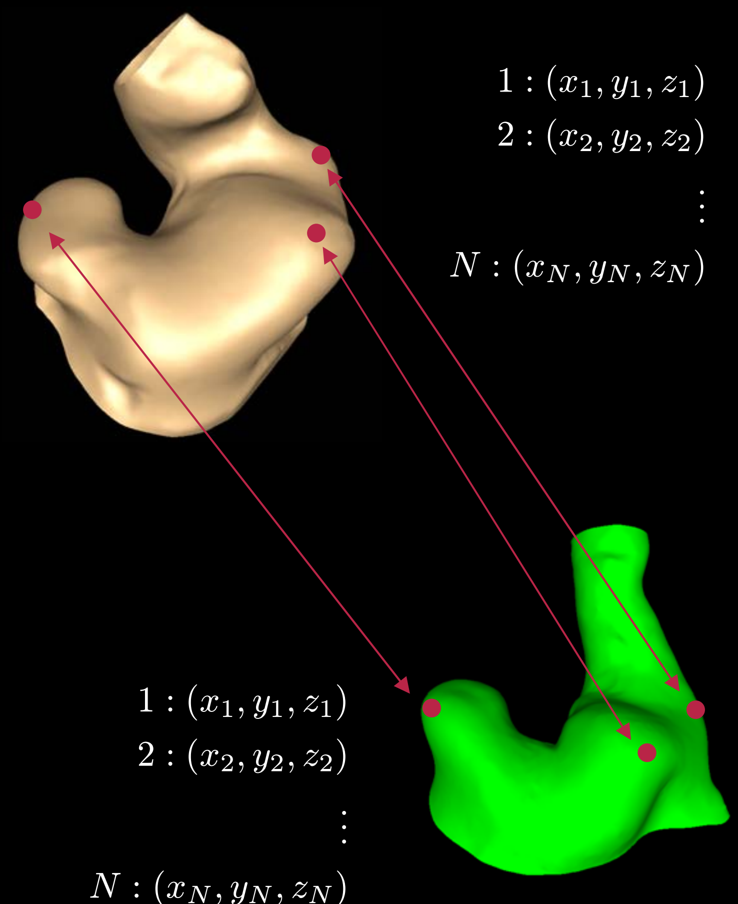




- Two similar shapes acquired with the same scanner
- Completely different mesh layout
- Not the same number of points
- No correspondence
- Their *vector representations* are not in the same space

We need point correspondence!

Point Correspondence

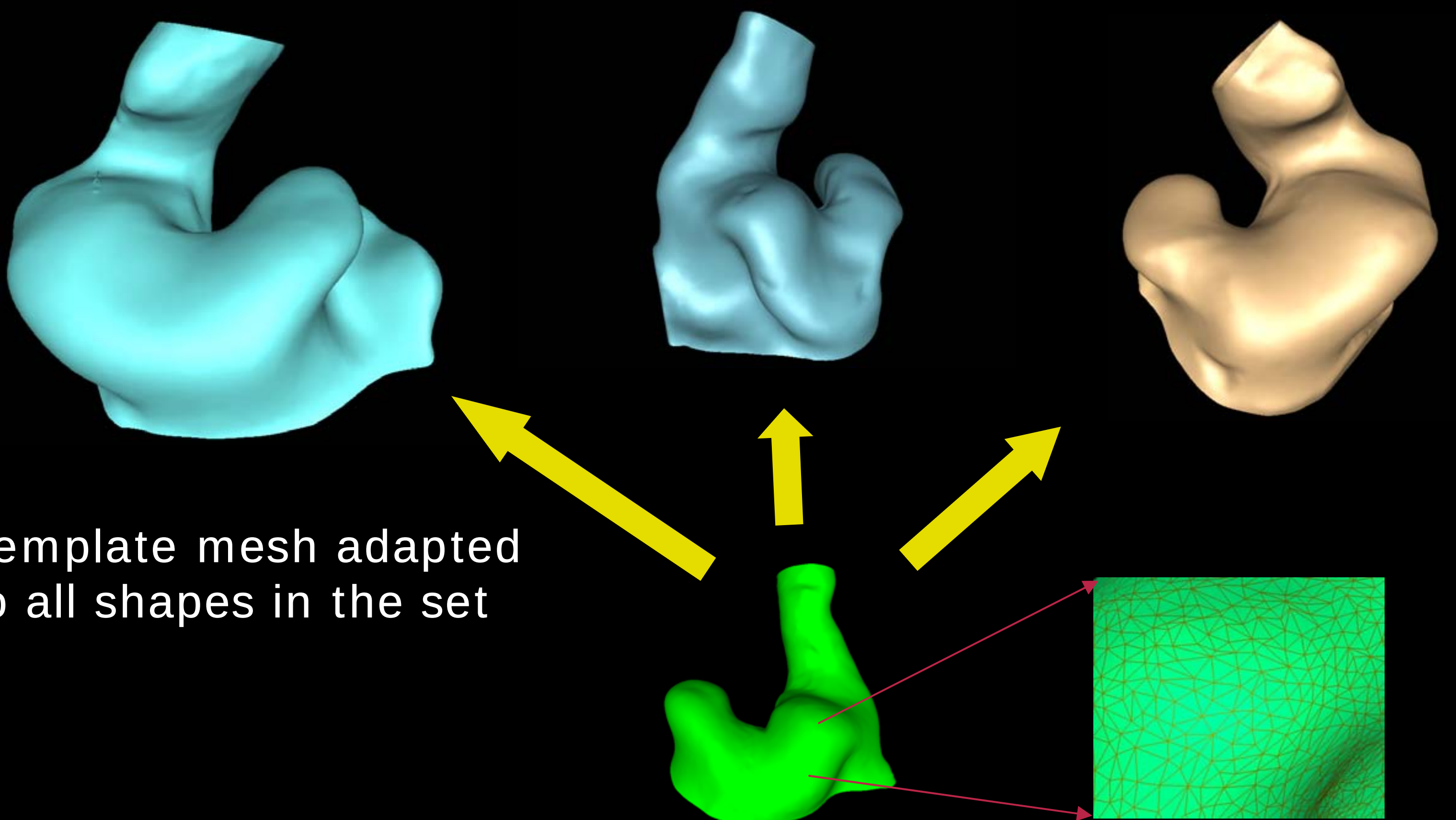


- A point with a given id is placed on the same identifiable area
  - Anatomical
  - Geometrical (curvature)
  - (texture)

  on all shapes

All shapes should have the same number of points and triangles.

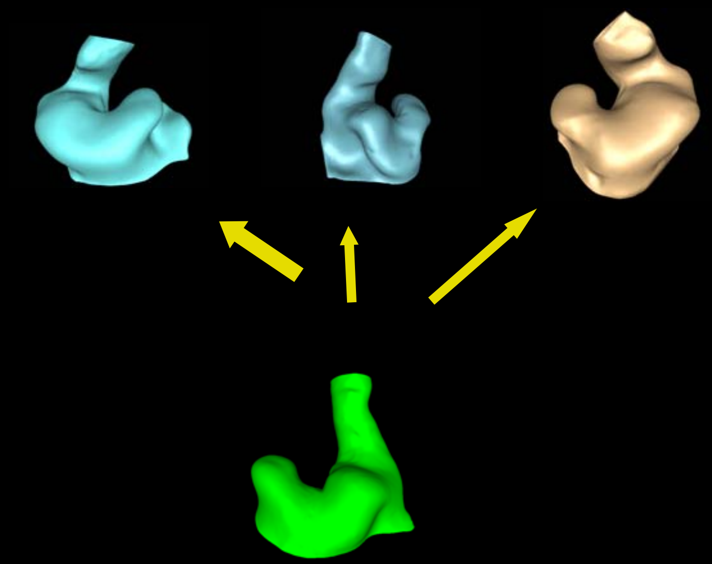
Creating Correspondence



Template mesh adapted
to all shapes in the set

Template shape with a *template mesh*

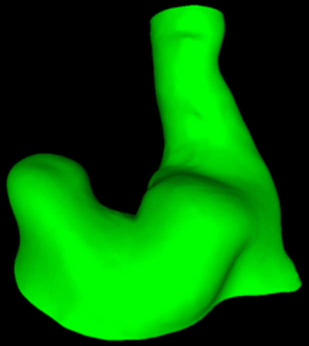
One to many or many to many?



- One to many by far the most common approach
- Alternative is to use a global approach
  - Landmarks placed and moved on all shapes simultaneously
  - Somewhat beyond the scope here

Correspondence as registration

Target shape

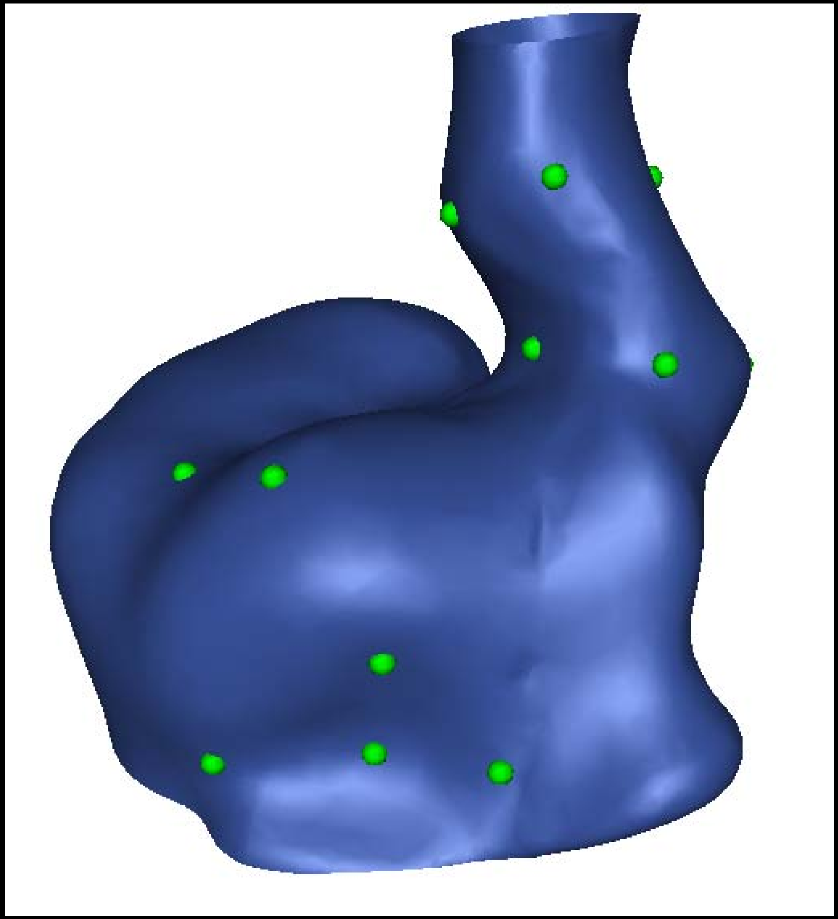


Template shape

- The template shape is *registered* to all the shapes in the shape set one by one
- Atlas mapping
- *Image registration* is a huge field!

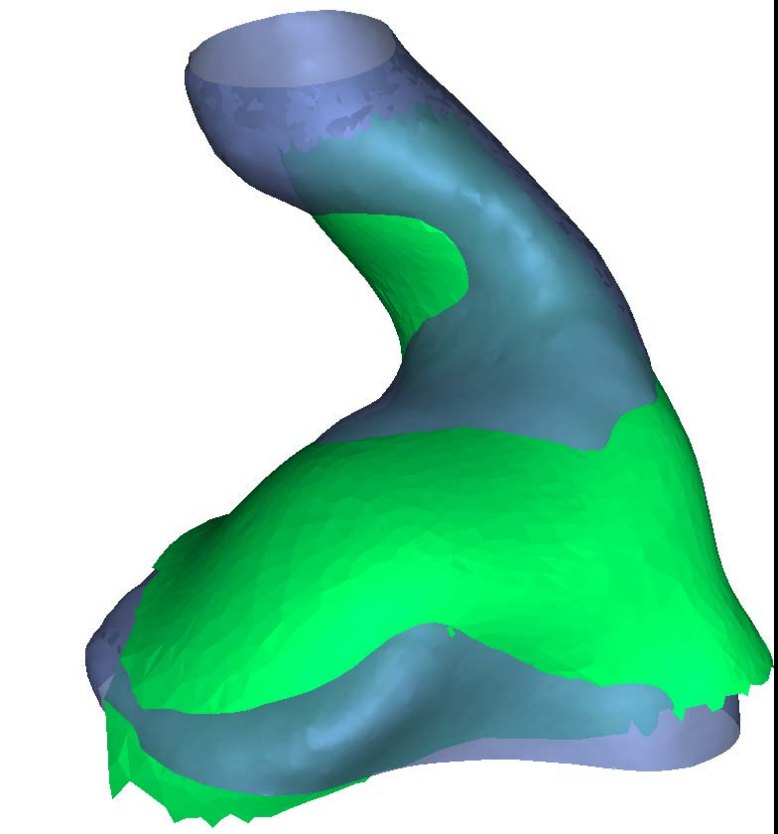
The presented method is just one out of many possible

Manual annotation



- An expert placed 18 anatomical landmarks on each ear
- Also on the template ear

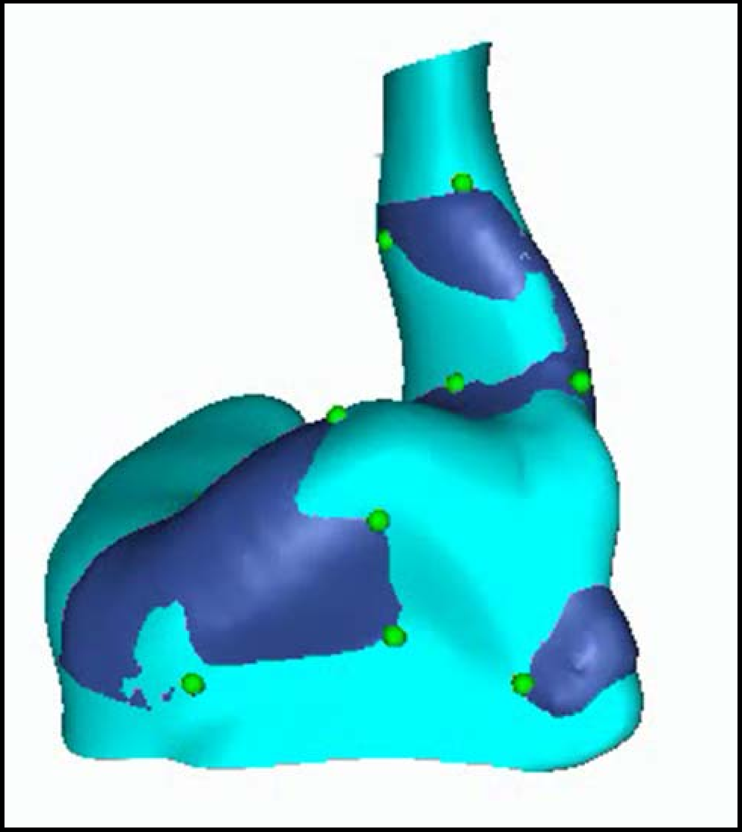
Initial correspondence – rigid alignment



Template and Target

- Rigid alignment: Translation and rotation
- The template shape is translated and rotated so it fits the target shape
- Transformation minimises distances between template and target landmarks
- Even simpler than the popular Iterative Closest Point (ICP) method
  - But more robust

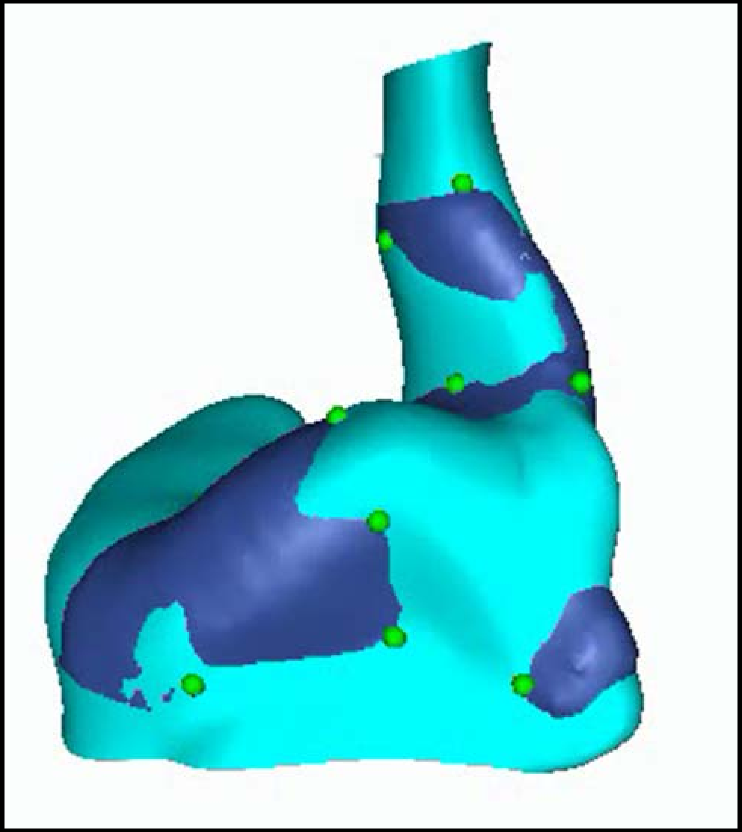
Non-rigid registration



Template and Target

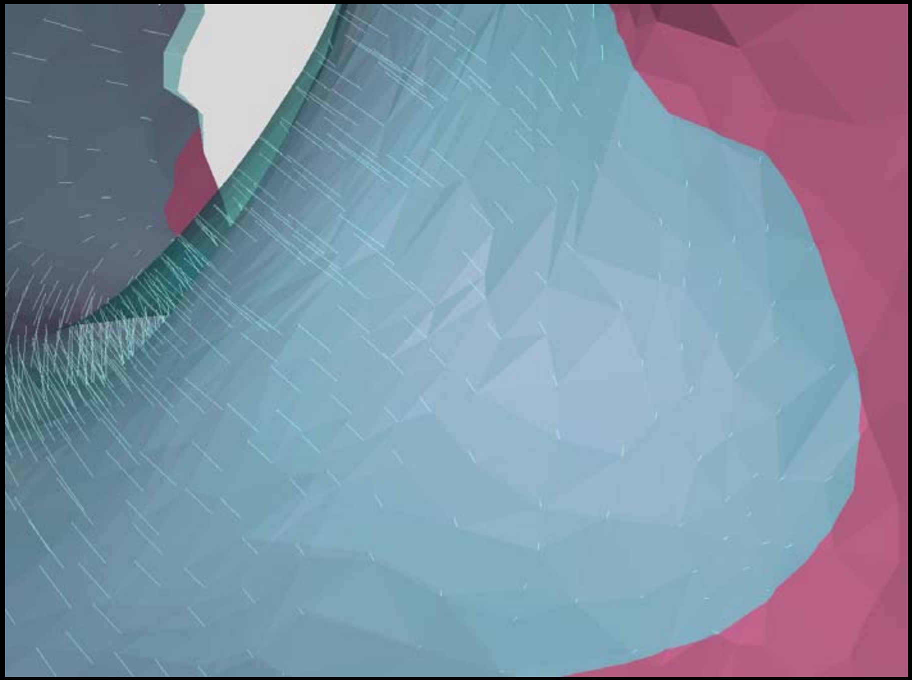
- Rigid alignment is not enough – shapes too different
- Spline based method used
- Template is deformed so the landmarks fit exactly the landmarks of the target

Creating the correspondence



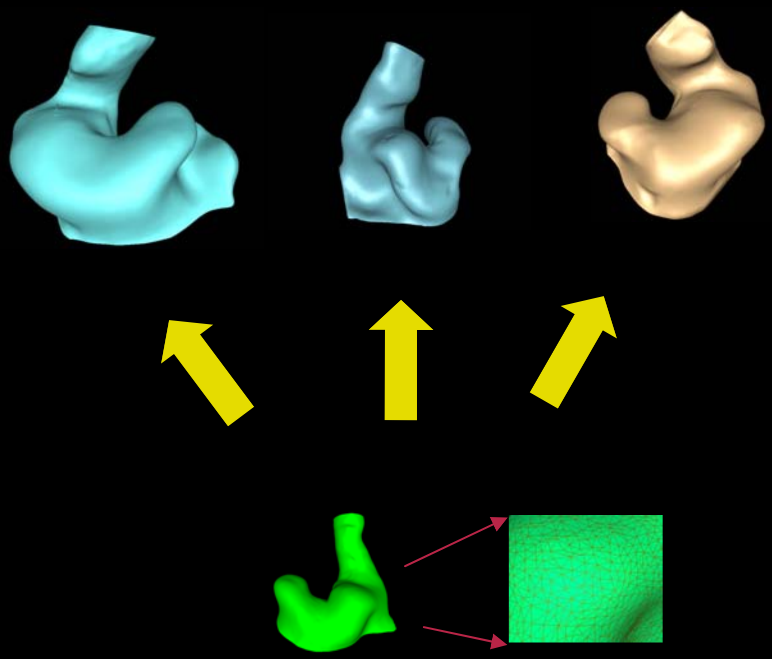
- The two surfaces are now very close
- The points from the template should now be *copied* to the target

Creating the correspondence



- The two surfaces are now very close
- The points from the template should now be *copied* to the target
- For each point on the template find the closest point on the target
- A new mesh is created to represent the target
  - The projected points
  - Mesh structure from the template

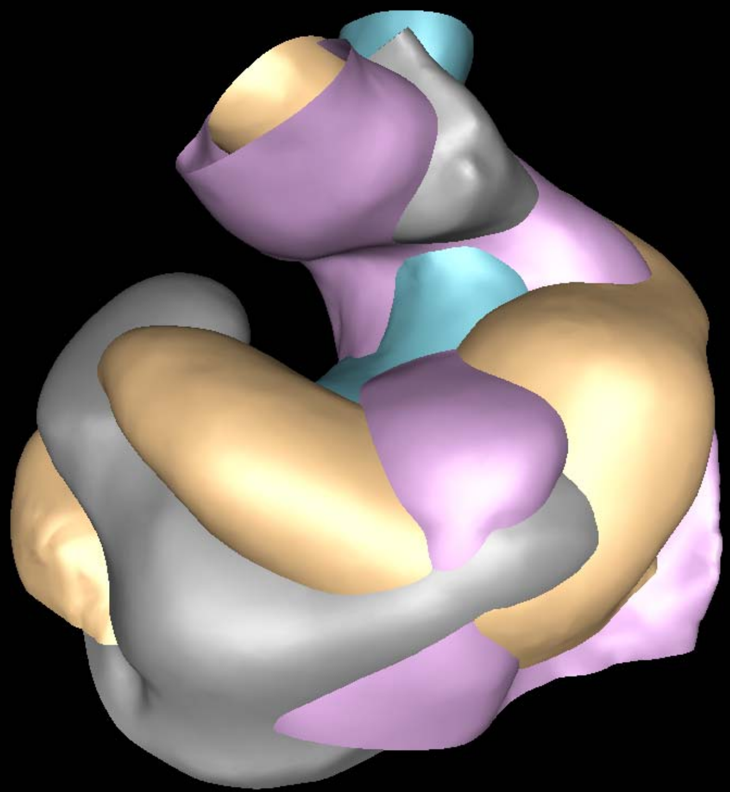
We have correspondence



$$\mathbf{x} = [x_1, y_1, z_1, \ldots, x_N, y_N, z_N]^T$$

- Different shapes
  - Same representation
  - Same number of points
  - Points are placed at the same anatomical places

Procrustes

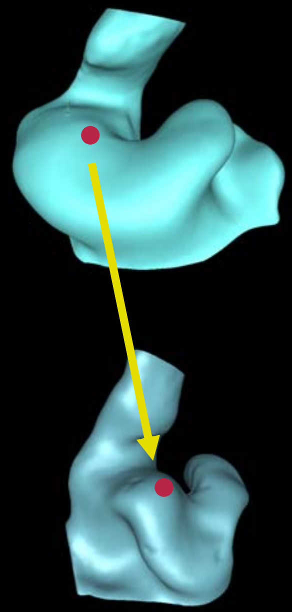


- We have correspondence
- Shapes are not aligned in a common coordinate system
- Solution: Procrustes
- In 3D it is an iterative method
- Aligns all shapes to a common average shape

Standard algorithm. Implemented in many frameworks (vtk f.ex.)

We have correspondence

- We can start to analyse shape differences
- One point moved corresponds to three elements changed in the shape vector

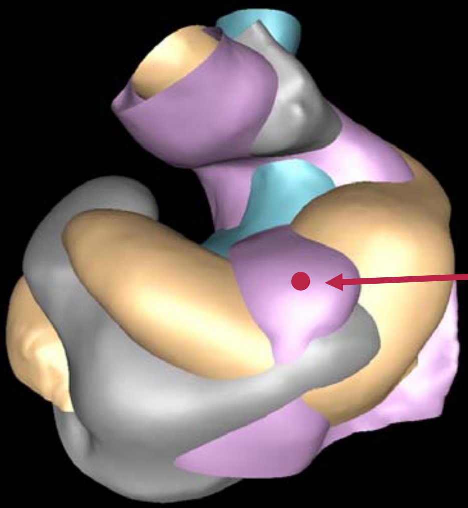


$$\mathbf{x}_1 = [x_1, y_1, z_1, \ldots, x_{5100}, y_{5100}, z_{5100}, \ldots, x_N, y_N, z_N]^T$$

$$\mathbf{x}_2 = [x_1, y_1, z_1, \ldots, x_{5100}, y_{5100}, z_{5100}, \ldots, x_N, y_N, z_N]^T$$

Point statistics

- We can look at the "movement" of the point over the set of shapes
- M shapes
- Statistics: Average point + standard deviation of the point movement



$$\mathbf{x}_1 = [x_1, y_1, z_1, \ldots, x_{5100}, y_{5100}, z_{5100}, \ldots, x_N, y_N, z_N]^T$$

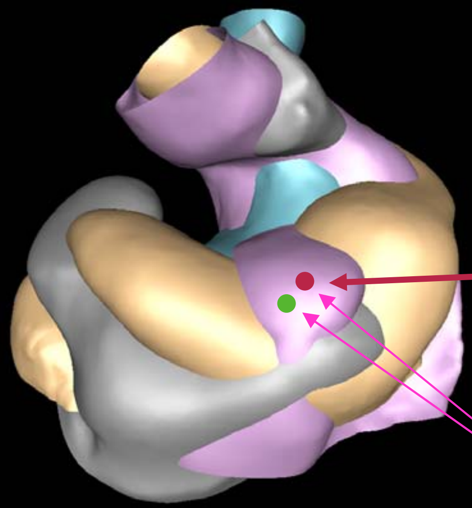
$$\mathbf{x}_2 = [x_1, y_1, z_1, \ldots, x_{5100}, y_{5100}, z_{5100}, \ldots, x_N, y_N, z_N]^T$$

$$\vdots$$

$$\mathbf{x}_M = [x_1, y_1, z_1, \ldots, x_{5100}, y_{5100}, z_{5100}, \ldots, x_N, y_N, z_N]^T$$

Point statistics II

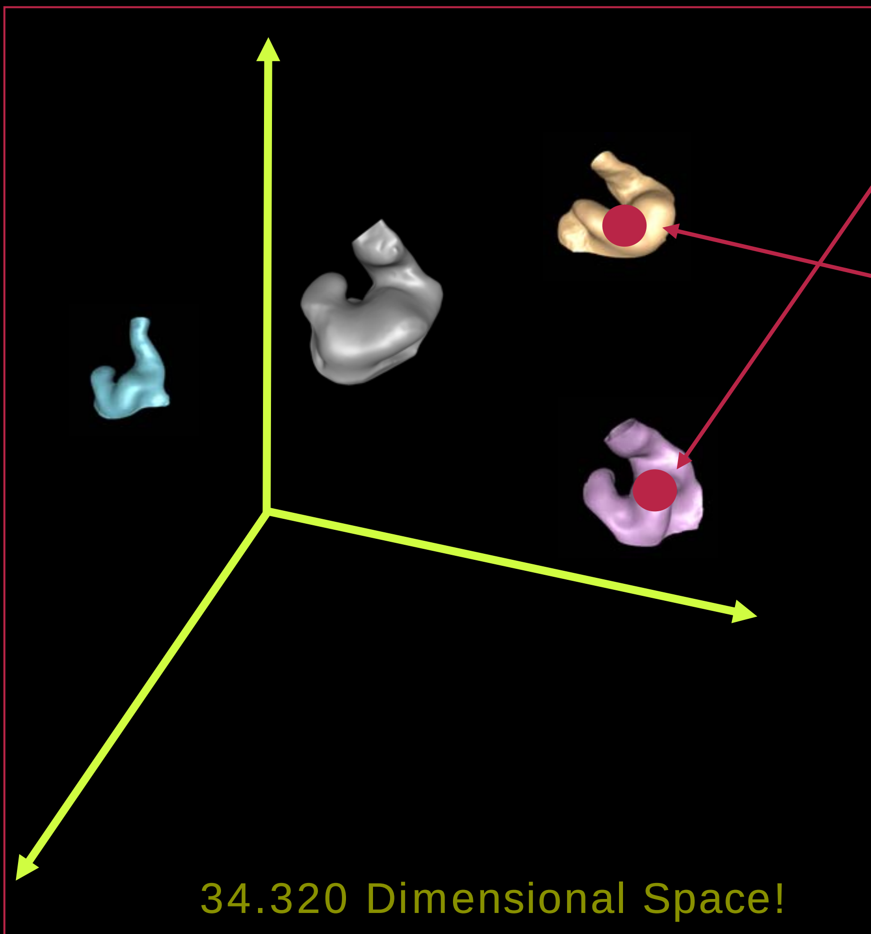
- Statistics on more points
- Movement of neighbour point highly correlated



$$\mathbf{x}_1 = [x_1, y_1, z_1, \ldots, x_{5100}, y_{5100}, z_{5100}, \ldots, x_N, y_N, z_N]^T$$
$$\mathbf{x}_2 = [x_1, y_1, z_1, \ldots, x_{5100}, y_{5100}, z_{5100}, \ldots, x_N, y_N, z_N]^T$$
$$\vdots$$
$$\mathbf{x}_M = [x_1, y_1, z_1, \ldots, x_{5100}, y_{5100}, z_{5100}, \ldots, x_N, y_N, z_N]^T$$

High correlation

Back in Space

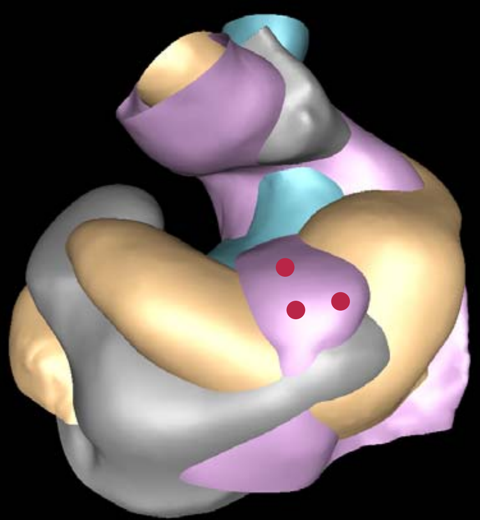


We make a shape:

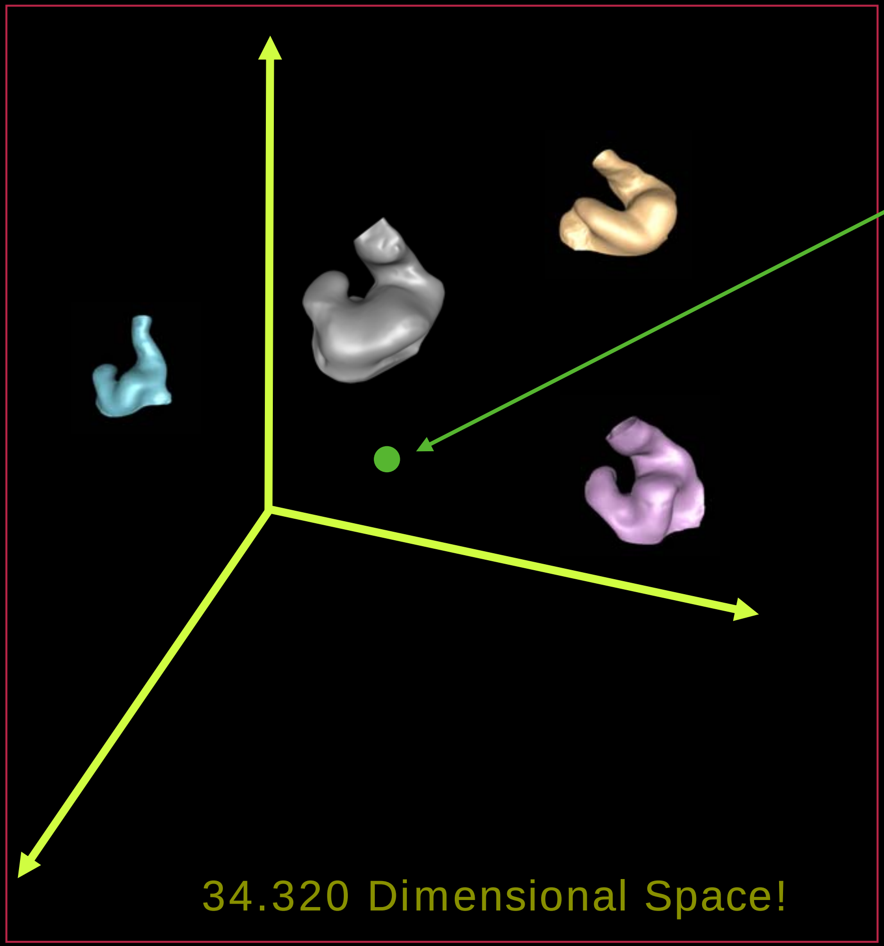
$$\mathbf{x} = [x_1, y_1, z_1, \ldots, x_N, y_N, z_N]^T$$

Copy positions from 

Copy positions from 



Shape Synthesis



We make an empty shape:

$$\mathbf{x} = [x_1, y_1, z_1, \ldots, x_N, y_N, z_N]^T = \mathbf{0}$$

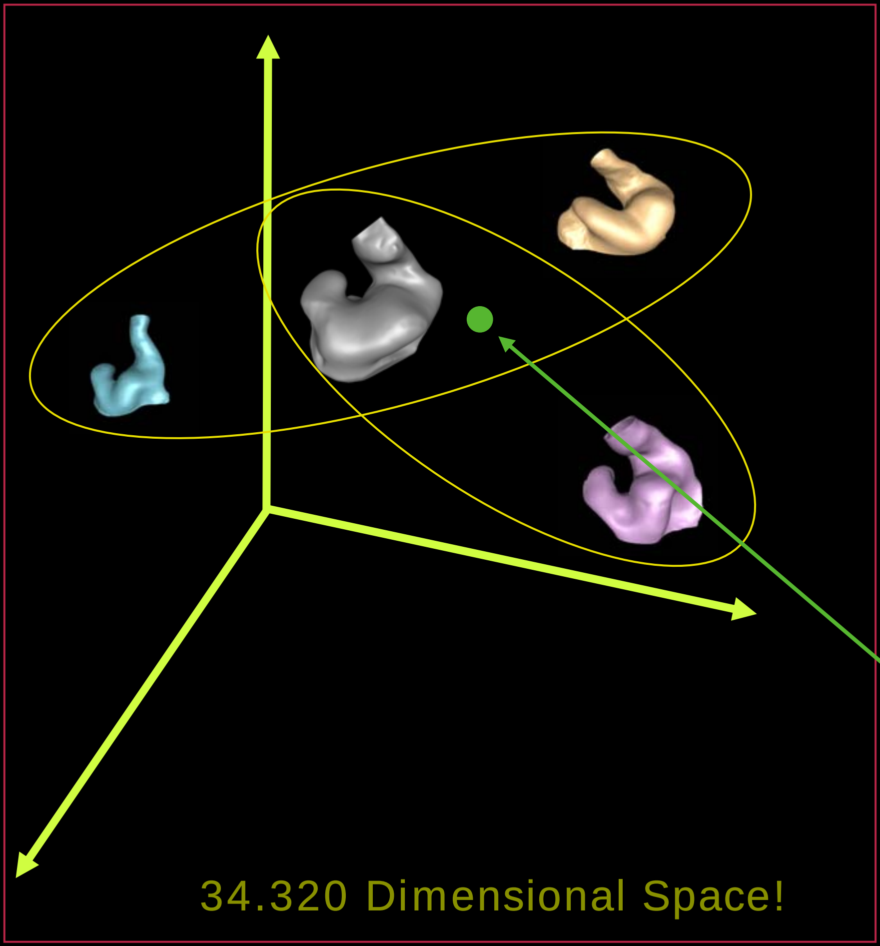
Pick a random point in space

Copy coordinates into the shape

$$\mathbf{x} = [x_1, y_1, z_1, \ldots, x_N, y_N, z_N]^T$$

Copy positions to 

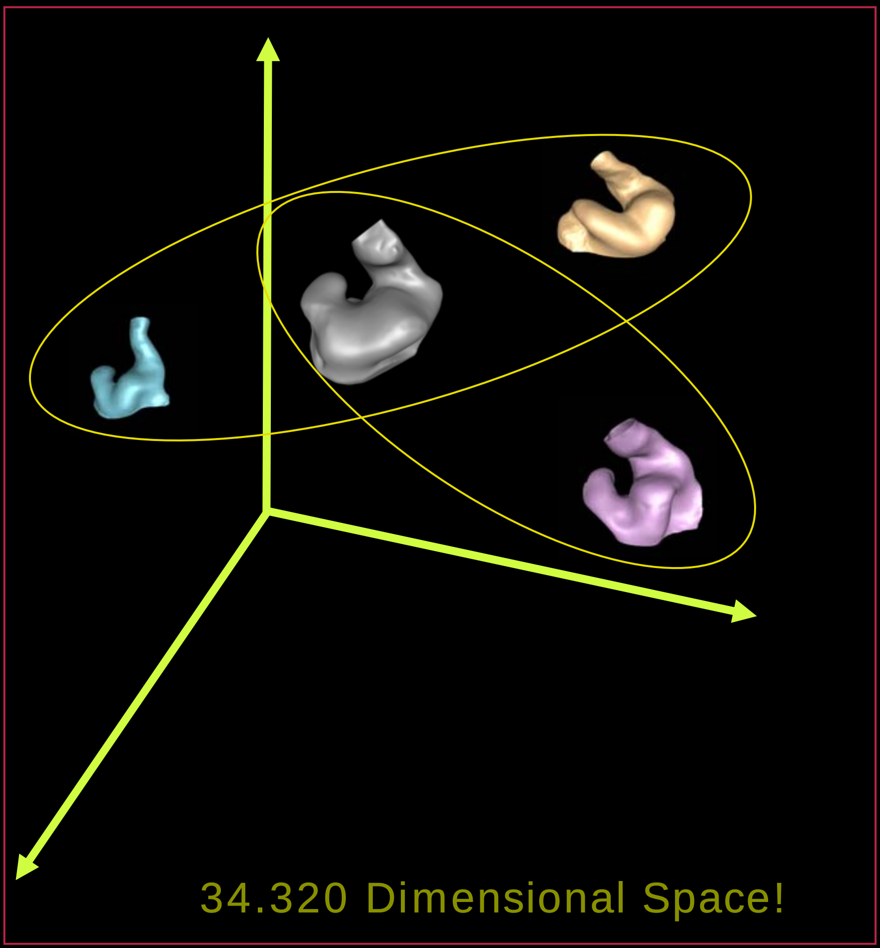
Shape Synthesis



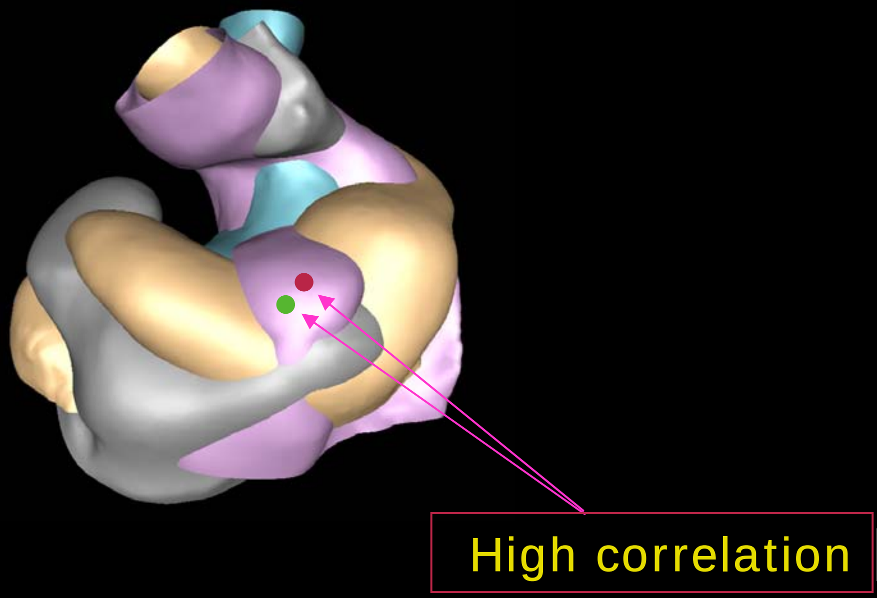
- How to synthesise plausible ears?
- We need to *map the space* that ears occupy
- In other words we need to learn about the manifold the ears are placed on
- Then we can synthesise new ears on that manifold

$$\mathbf{x} = [x_1, y_1, z_1, \ldots, x_N, y_N, z_N]^T = \mathbf{0}$$

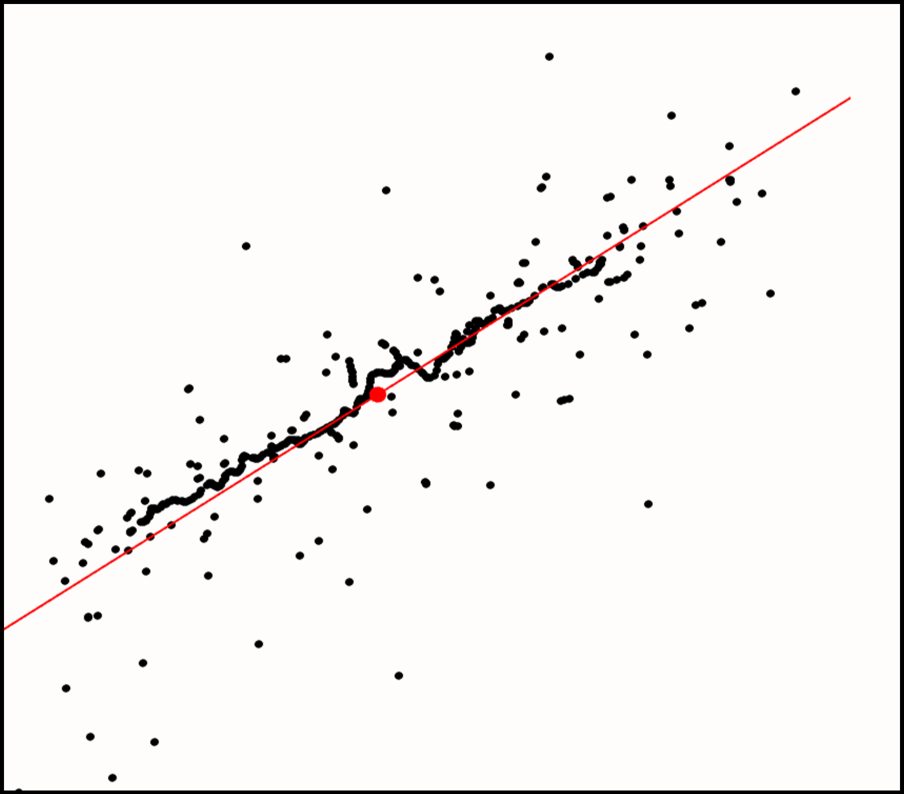
Shape manifold



- Why do we believe that the ears are placed on a manifold?
- Why not randomly around in space?

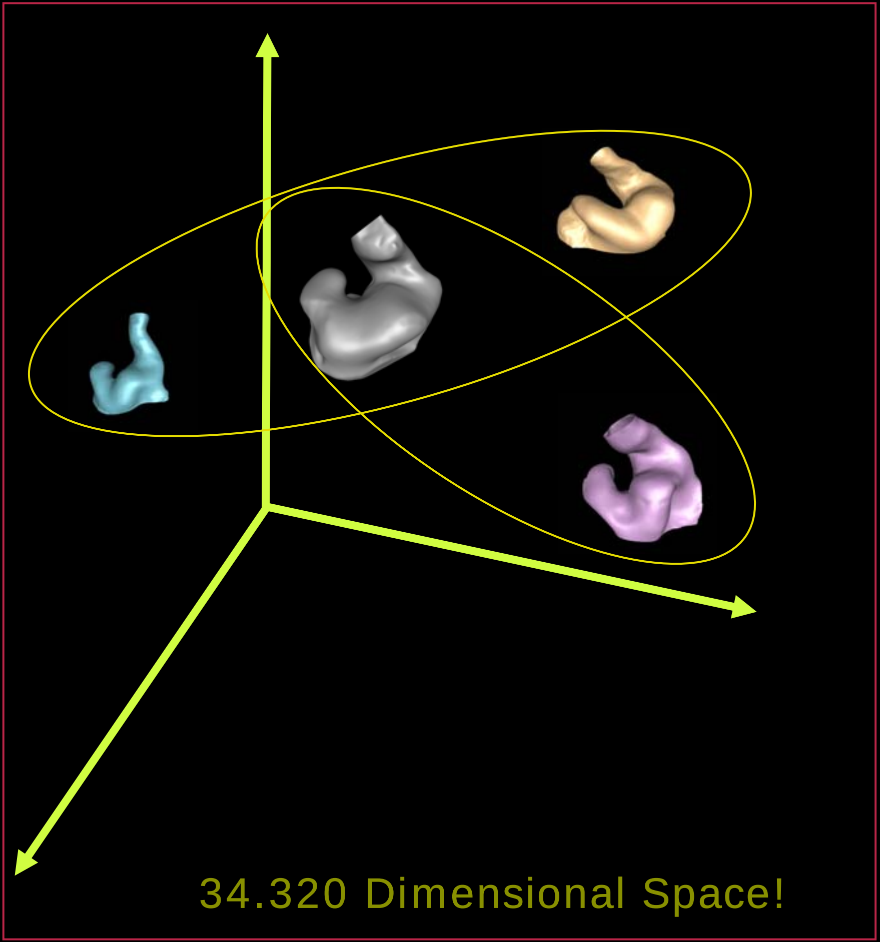


Dimensionality Reduction



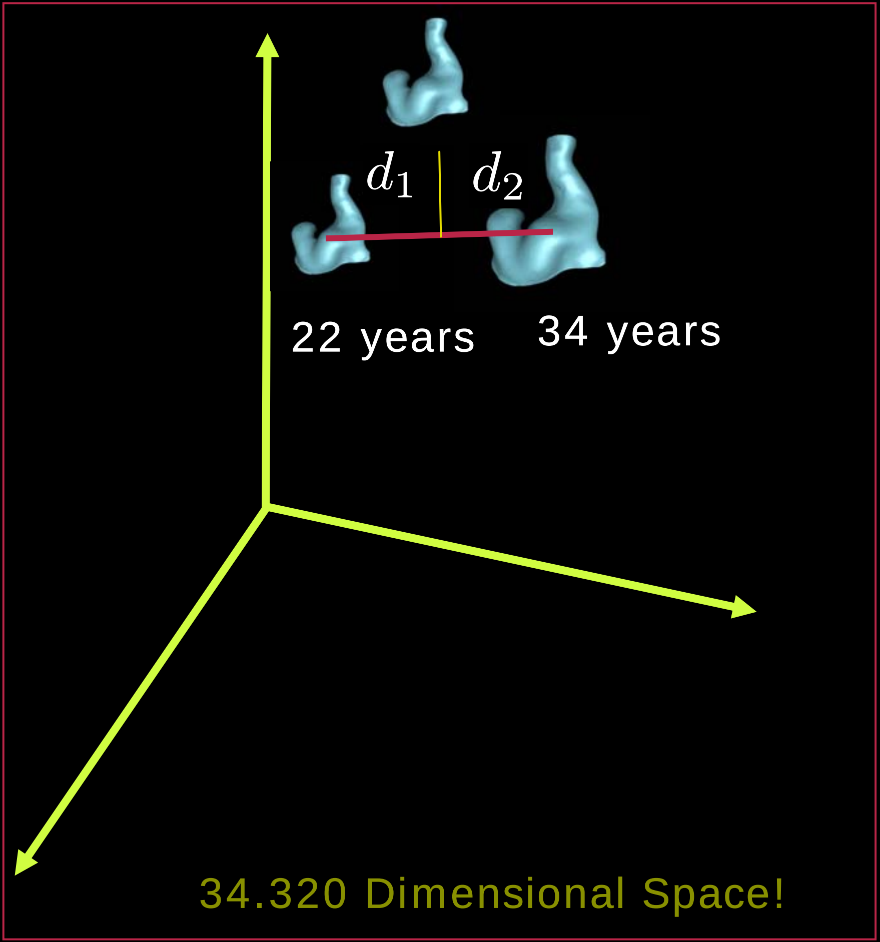
- Highly correlated variables can typically be explained by fewer parameters

Manifold learning



- How do we describe the manifold?
- How de we reduce the dimensions without loosing important information?

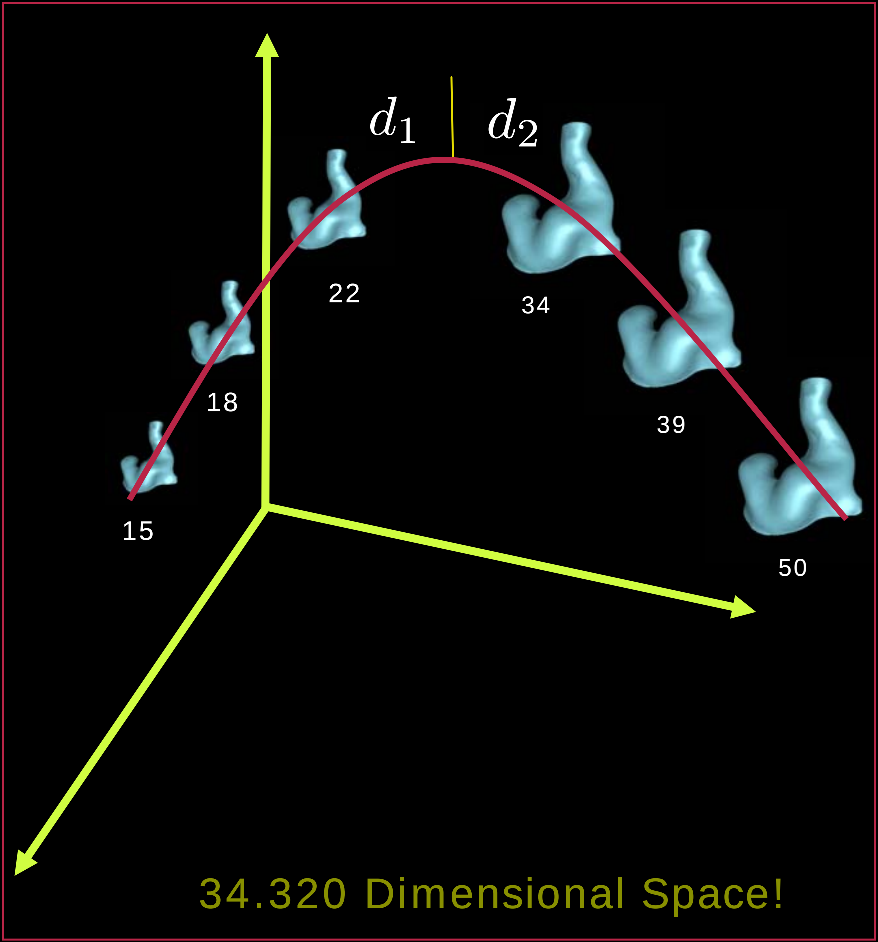
An (artificial) example



- Growth modelling
- Ear shape acquired at age 22 and age 34
- How did it look at age 28?
- Find the point halfway between the two
  - Synthesise the shape
- Euclidean distances

$$d_1 = d_2$$

An (artificial) example II

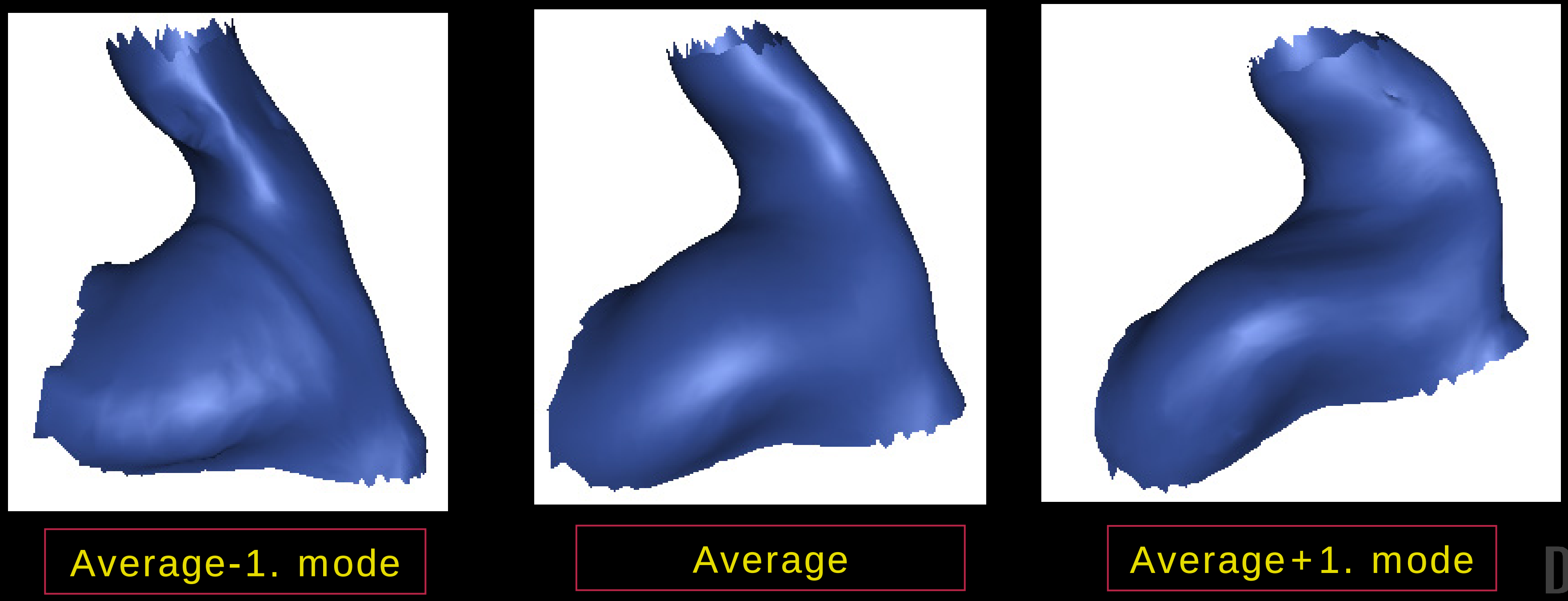


- Ear shape acquired at other times as well
- Still ok to use the Euclidean distance to estimate the shape at age 28?
- Estimate the growth manifold
- Calculate distances on the manifold
- Non-Euclidean metric

$$d_1 = d_2$$

Some results

- Main variation of the shape of the ear canal
- Found using principal component analysis
- First mode of variation
- 7 modes explain 95% of the total variation



Conclusion

- Shape models can be cast into a manifold learning framework
- Shape models can be build in many ways
- Creating correspondence is a huge topic
- Manual annotation was used
  - Many fully automated exist

Questions?