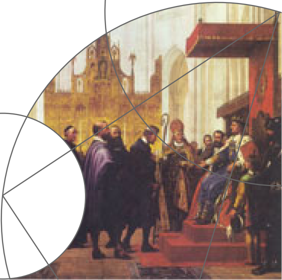
Optimal Net Surface Segmentation

Application to Airway Walls in CT Images

Jens Petersen



Introduction

Wu and Chen's optimal net surface problems[1]:

- Globally optimal solution (given the graph discretization).
- Multiple surfaces in multiple dimensions.
- Surface cost functions and geometric constraints.
- Polynomial time solution using maximum flow algorithms.

[1] X. Wu, D. Z. Chen, LNCS, 2002, vol. 2380, pp. 1029-1042

Introduction - Comparison with Graph Cut[2]

Advantages:

- Can optimally deal with more than two labels.
- Initial knowledge of surface orientation and position can be used to develop shape priors.

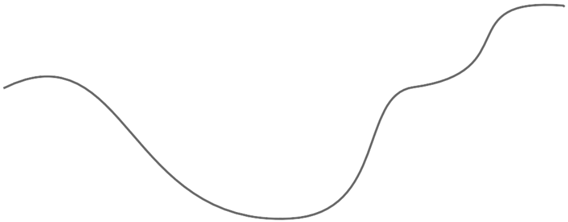
Disadvantages:

- The sought surface(s) must be terrain-like within some initially known transformation.

[2] Y. Boykov et al, 2001, PAMI, vol. 23, No. 11, pp. 1222-1239

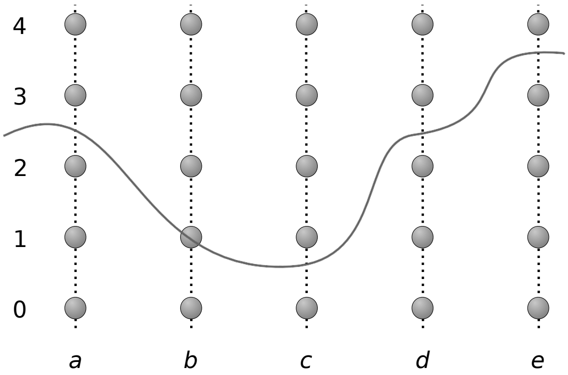
Optimal Net Surface Problem

Goal: Find some terrain-like surface.



Optimal Net Surface Problem

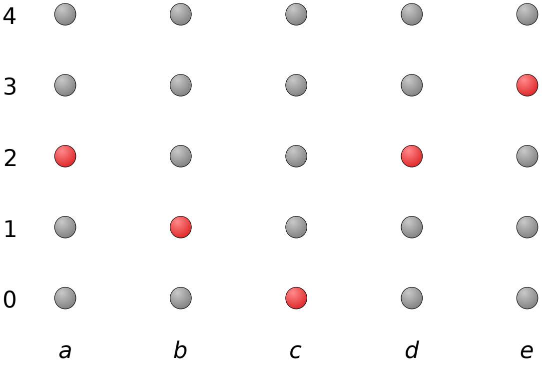
Graph vertices belong to a disjoint set of columns V_B.



Optimal Net Surface Problem

A net surface N in G is a subset of V, such that:

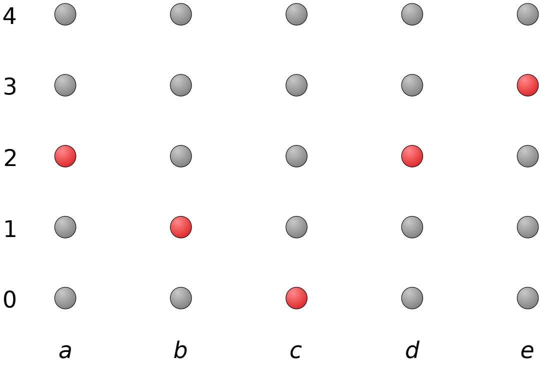
- Each vertex in N belongs to exactly one column in V_B
- There exists exactly one vertex in each column belonging to N.



Optimal Net Surface Problem

The optimal net surface problem:

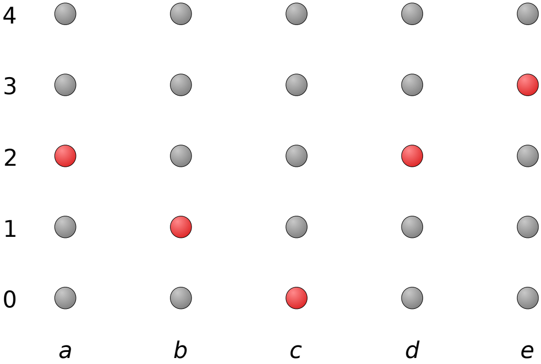
$$\min_{N}\left(\sum_{i_k \in N} w(i_k) + \sum_{\{i_k, j_l\} \in N} f_{i,j}(|k-l|)\right)$$



Optimal Net Surface Problem

Vertex cost function (positive).

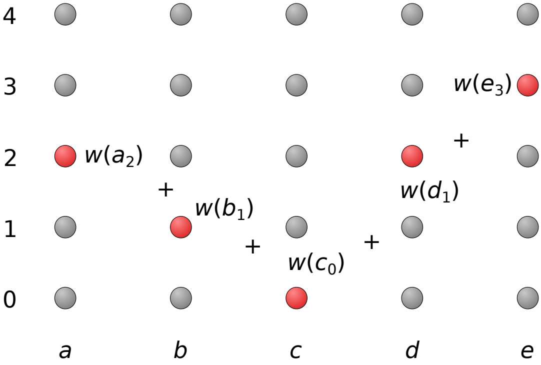
$$\min_{N}\left(\sum_{i_k\in N} w(i_k) + \sum_{\{i_k, j_l\}\in N} f_{i,j}(|k-l|)\right)$$



Optimal Net Surface Problem

Cost of all vertices in the surface.

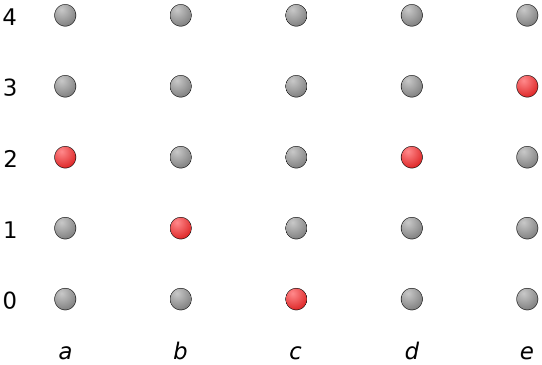
$$\min_{N} \left(\sum_{i_k \in N} w(i_k) + \sum_{\{i_k, j_l\} \in N} f_{i,j}(|k-l|) \right)$$



Optimal Net Surface Problem

Edge cost function (convex, non-decreasing).

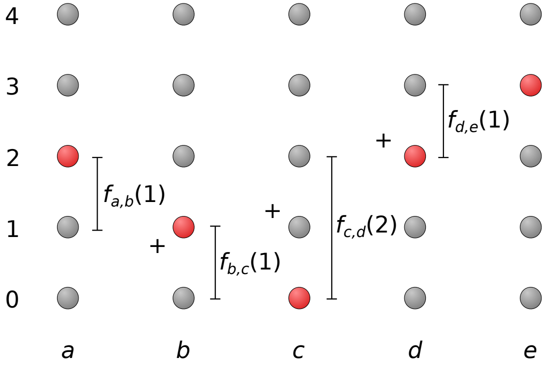
$$\min_N \left(\sum_{i_k \in N} w(i_k) + \sum_{\{i_k, j_l\} \in N} f_{i,j}(|k-l|) \right)$$



Optimal Net Surface Problem

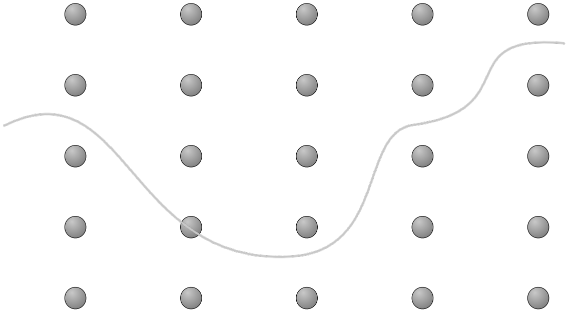
Cost of all vertex pairs in the surface.

$$\min_N \left(\sum_{i_k \in N} w(i_k) + \sum_{\{i_k, j_l\} \in N} f_{i,j}(|k-l|) \right)$$



Optimal Net Surface Problem

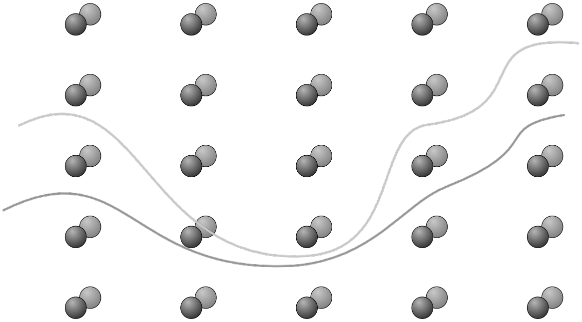
Multiple dependent surfaces, how?



Optimal Net Surface Problem

Multiple dependent surfaces, how?

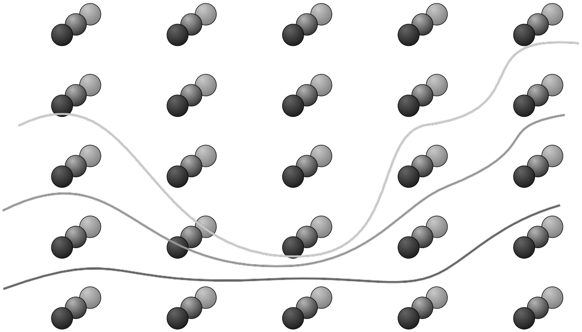
- Add a sub-graph of columns for each surface.



Optimal Net Surface Problem

Multiple dependent surfaces, how?

- Add a sub-graph of columns for each surface.
- Inter-surface constraints easily added.

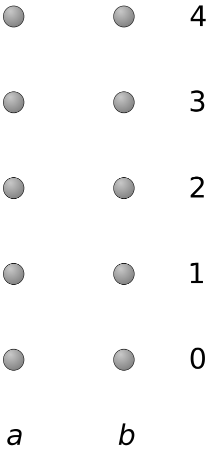


Optimal Net Surface - Minimum Cut Graph

Goal:

- Optimal surface(s) given by top-most vertices in minimum cut source set.

How?



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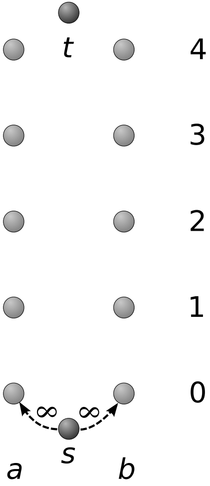
- Add source and sink nodes.



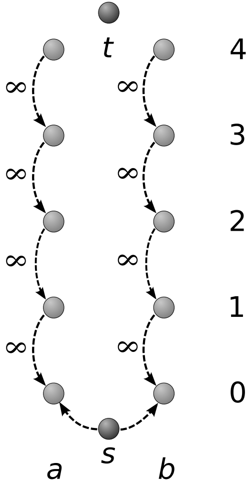
Optimal Net Surface - Minimum Cut Graph

Goal:

- Optimal surface(s) given by top-most vertices in minimum cut source set.

How?

- Add source and sink nodes.
- Force solution to be a net surface.



Optimal Net Surface - Minimum Cut Graph

Goal:

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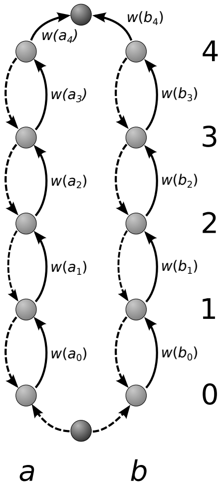
How?

- Add source and sink nodes.
- Force solution to be a net surface.
- Implement vertex cost function.



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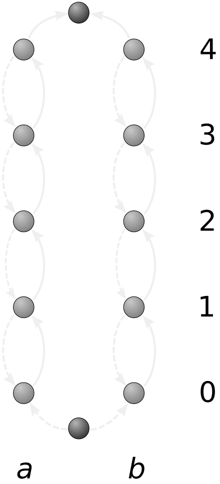

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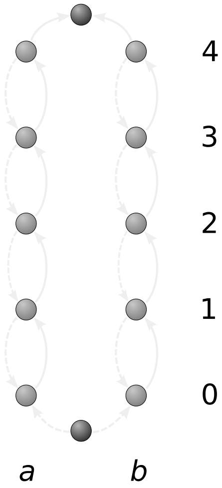
- Implement edge cost function.

$$g_{i,j}(x) = \begin{cases} 0 & \text{if } x < 0 \\ f_{i,j}(1) & \text{if } x = 0 \\ \begin{array}{l} f_{i,j}(x+1) - 2f_{i,j}(x) + \\ f_{i,j}(x-1) \end{array} & \text{if } x > 0 \end{cases}$$

Assume WLOG: $f_{i,j}(0) = 0$ and note: $g_{i,j}(x) \geq 0$.



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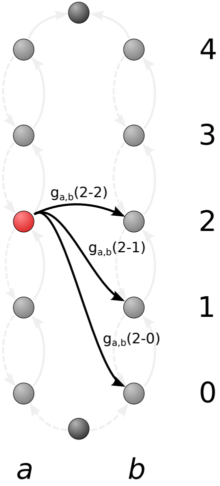

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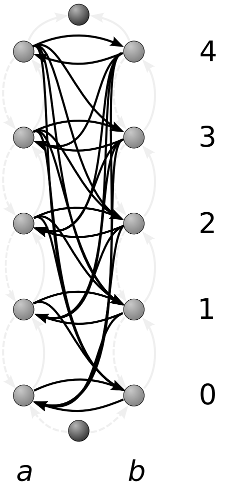

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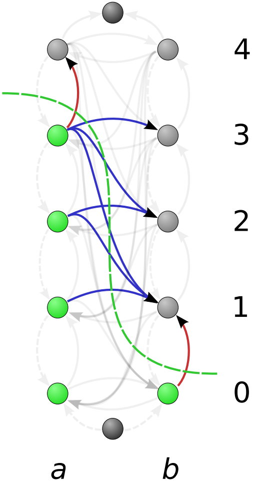
Example cut:

$$w(a_3) + w(b_0) + 3g_{a,b}(0) + 2g_{a,b}(1) + g_{a,b}(2)$$



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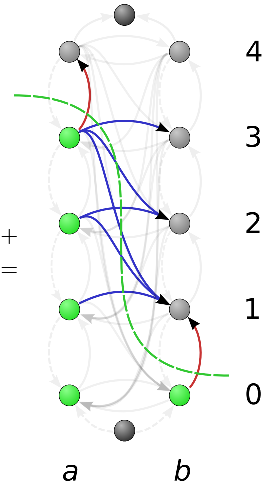
$$w(a_3) + w(b_0) + 3f_{a,b}(1) + 2(f_{a,b}(2) - 2f_{a,b}(1) + f_{a,b}(0)) + f_{a,b}(3) - 2f_{a,b}(2) + f_{a,b}(1) \quad + \quad =$$



Optimal Net Surface - Minimum Cut Graph

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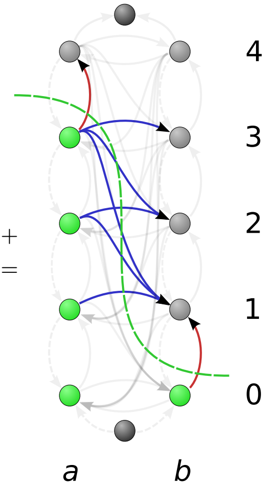
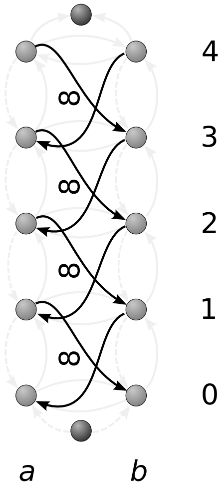


Optimal Net Surface - Minimum Cut Graph

Infinite cost edges → hard constraints.

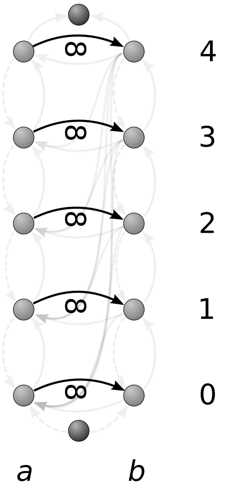
- Change at most one index.



Optimal Net Surface - Minimum Cut Graph

Infinite cost edges $\rightarrow$ hard constraints.

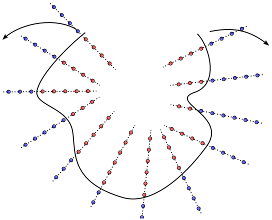
- Force solution in one column to be above or same level as other.



Graph Space - How?

Easy if the sought surface (or contour):

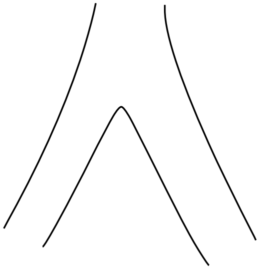
- Is terrain-like.
- Can be easily 'unfolded': tubular, star-shaped, etc.



However most surfaces are not like that.

Graph Space - Different Approaches

Approach of Liu et al.[3]:



- Initial rough segmentation.

[3] Liu et al., First International Workshop on Pulmonary Image Analysis, 2008, pp. 103-111

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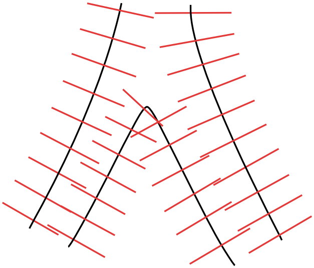

- Initial rough segmentation.
- Columns at surface points in normal direction.

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[3] Liu et al., First International Workshop on Pulmonary Image Analysis, 2008, pp. 103-111

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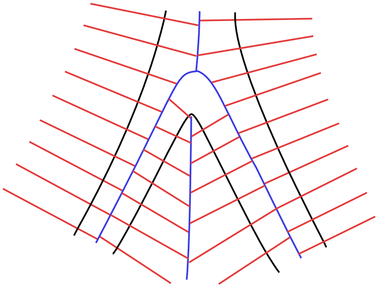

- Initial rough segmentation.
- Columns at surface points in normal direction.
- Length: distance to the inner and outer medial axes.

[3]Liu et al., First International Workshop on Pulmonary Image Analysis, 2008, pp. 103-111

Graph Space - Different Approaches

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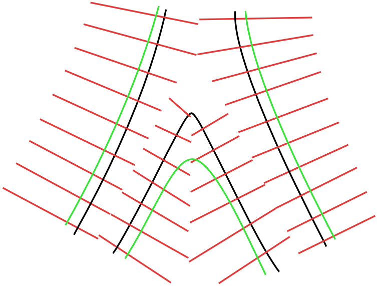
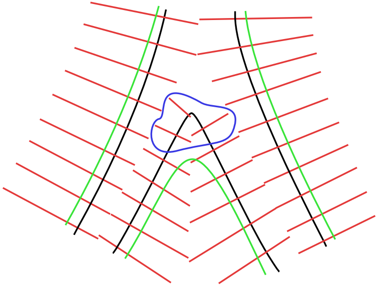


Columns can be too short.

[3] Liu et al., First International Workshop on Pulmonary Image Analysis, 2008, pp. 103-111

Graph Space - Different Approaches

Approach of Liu et al.[3]:



Poor results in regions with high curvature!

[3] Liu et al., First International Workshop on Pulmonary Image Analysis, 2008, pp. 103-111

Graph Space - Flow Lines

Our approach[4]:

- Combine regularization of the rough initial segmentation with computation of columns.
- Columns constructed as greatest ascent/descent flow lines from surface points in the smoothed initial segmentation.

[4] Petersen et al., LNCS, 2011, vol. 6801, pp. 49-60

Graph Space - Flow Lines

Our approach[4]:

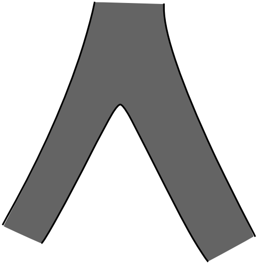
- Combine regularization of the rough initial segmentation with computation of columns.
- Columns constructed as greatest ascent/descent flow lines from surface points in the smoothed initial segmentation.

Advantages:

- Well suited for surfaces with high curvature.
- Noise and small errors can be removed by increasing regularization.
- Surfaces do not self-intersect.

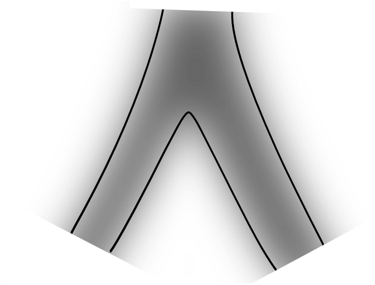
[4] Petersen et al., LNCS, 2011, vol. 6801, pp. 49-60

Graph Space - Flow Lines



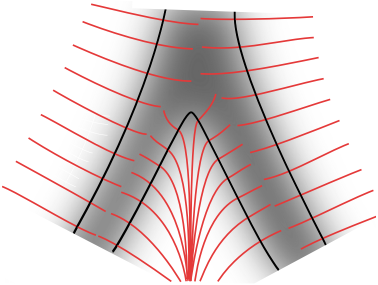
1. Compute rough initial segmentation obtaining a binary image.

Graph Space - Flow Lines



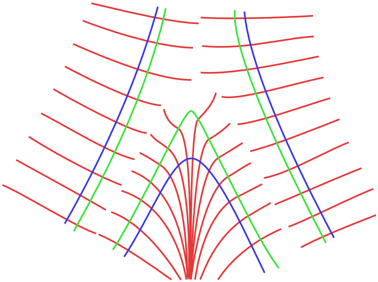
1. Compute rough initial segmentation obtaining a binary image.
2. Convolve binary image with some regularization filter.

Graph Space - Flow Lines



1. Compute rough initial segmentation obtaining a binary image.
2. Convolve binary image with some regularization filter.
3. Trace flow lines from surface points of the initial segmentation inward and outward.

Graph Space - Flow Lines



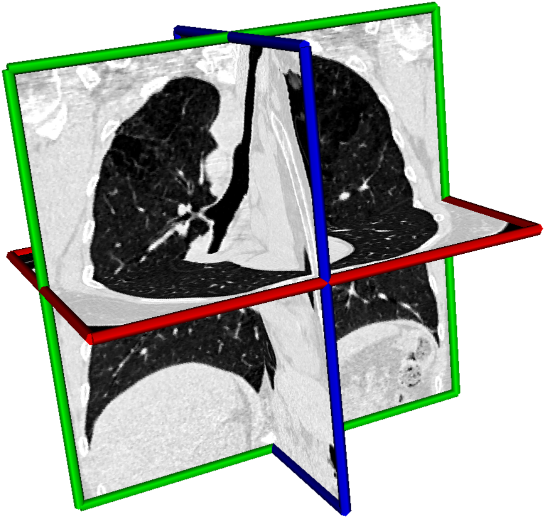
1. Compute rough initial segmentation obtaining a binary image.
2. Convolve binary image with some regularization filter.
3. Trace flow lines from surface points of the initial segmentation inward and outward.
4. Find surfaces using an optimal net surface method with columns following the flow lines.

Application to Airway Walls in CT Images

Why do we want to segment airway walls in CT?

- Relevant in connection with Chronic Obstructive Pulmonary Disease (COPD).
    - Airway lumen narrowing.
    - Airway wall thickening.
- Two surface problem $\rightarrow$ inner and outer wall surface.

Application to Airway Walls in CT Images



Application to Airway Walls in CT Images

Initial segmentation Lo et al.[5]:

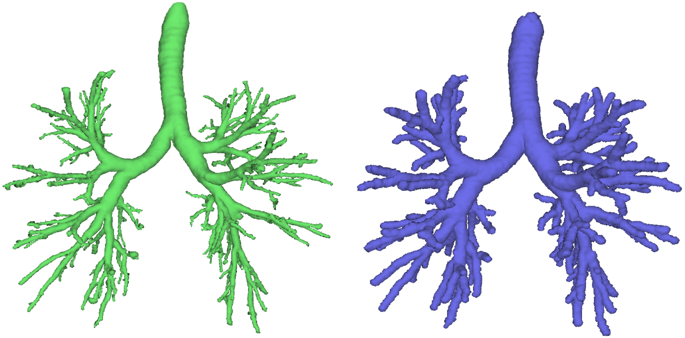
- Segmentation of the lumen surface.

Cost functions:

- Vertex cost: weightings of the intensity derivatives.
- Edge cost:
  - Linear non-smoothness penalty for each surface.
  - Linear surface separation penalty.

[5] Lo et al., LNCS, 2009, vol. 5762, pp. 51-58

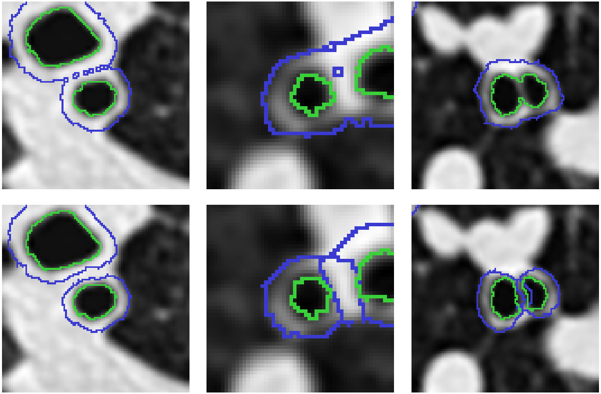
Results - Visualizations



(a) Inner Surface (b) Outer Surface

Results - Cross-sections near bifurcations

Flow line columns with Gaussian regularization



Medial axes + normal direction columns

Results - Manual Annotations

319 Manually annotated cross-sectional slices.

	Normal direction	Flow line
Dice coefficient	0.87 ± 0.09	$\mathbf{0.89} \pm \mathbf{0.06}$
Contour distance (mm)	0.11 ± 0.13	$\mathbf{0.09} \pm \mathbf{0.10}$

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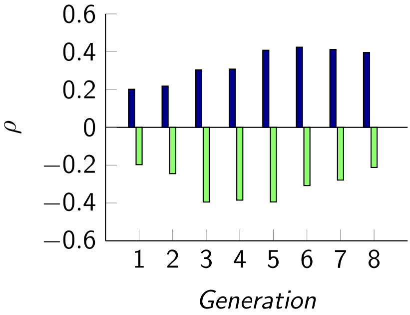
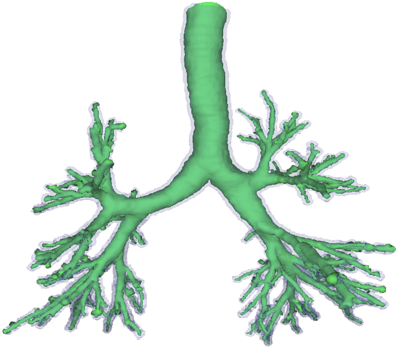
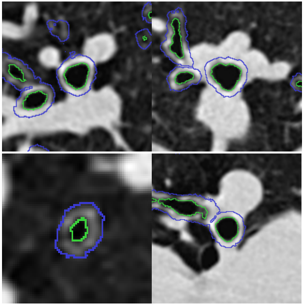
Results - Correlation with Lung Function

- 480 low dose (120 kV and 40 mAs), $0.78mm \times 0.78mm \times 1mm$
- Measures: lumen volume (blue) and wall area percentage (green).
- Spearman correlation with FEV1 (% predicted).



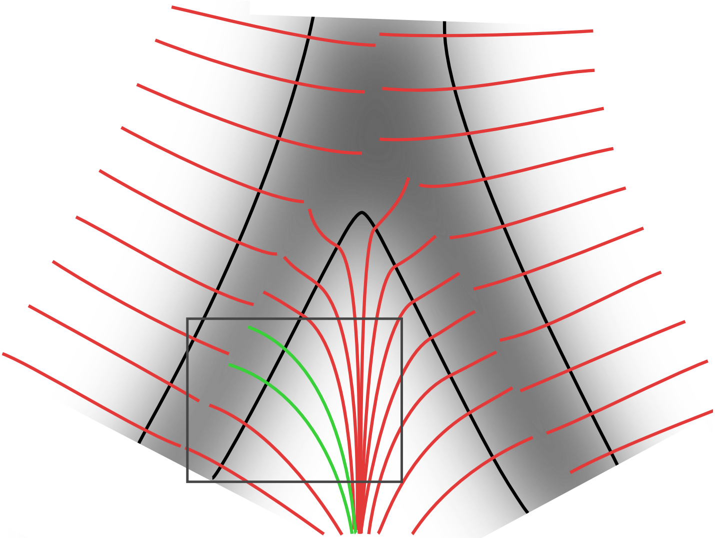
Questions?





Thank you!

Extra Slides: Graph in More Detail



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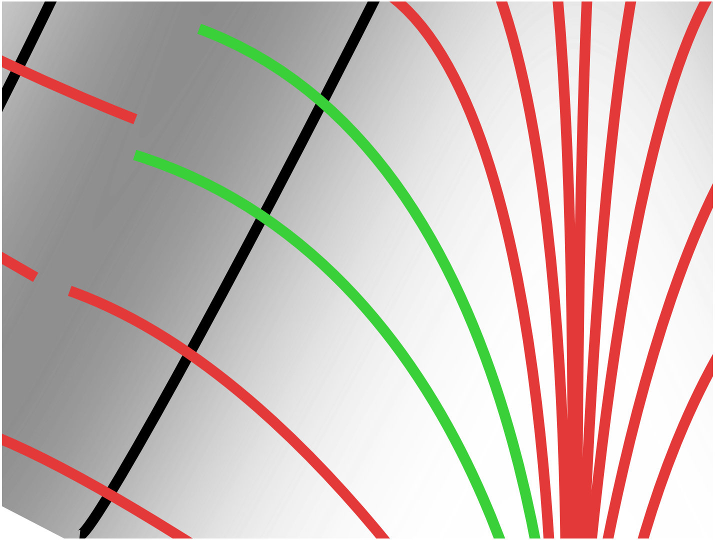

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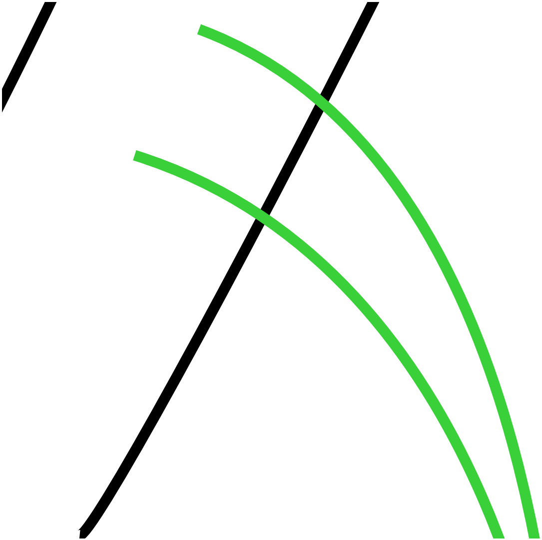

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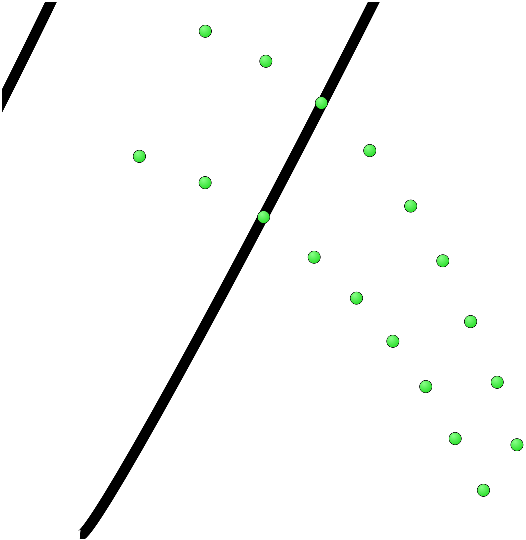

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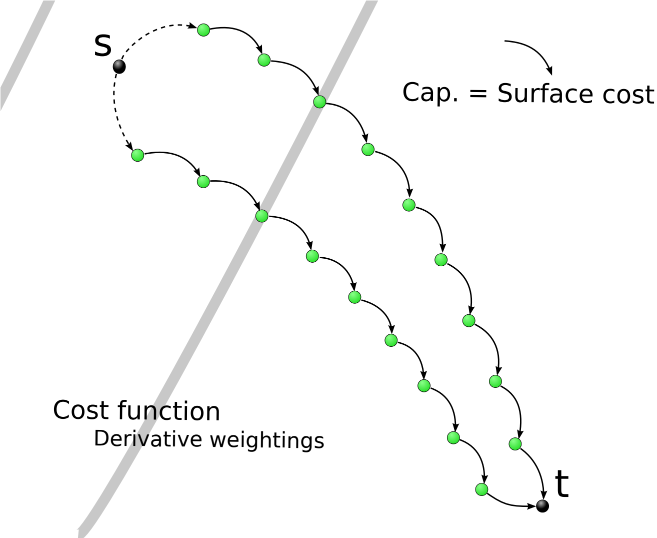

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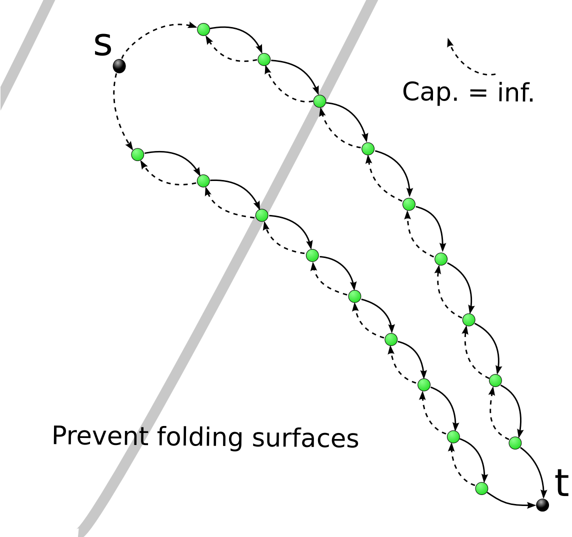

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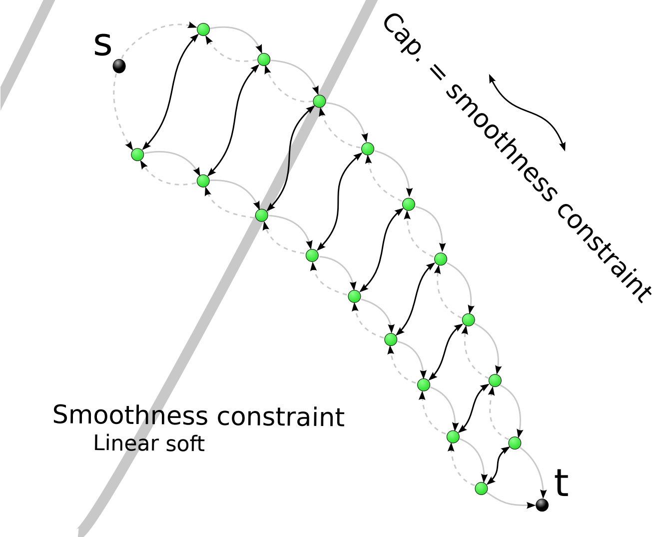

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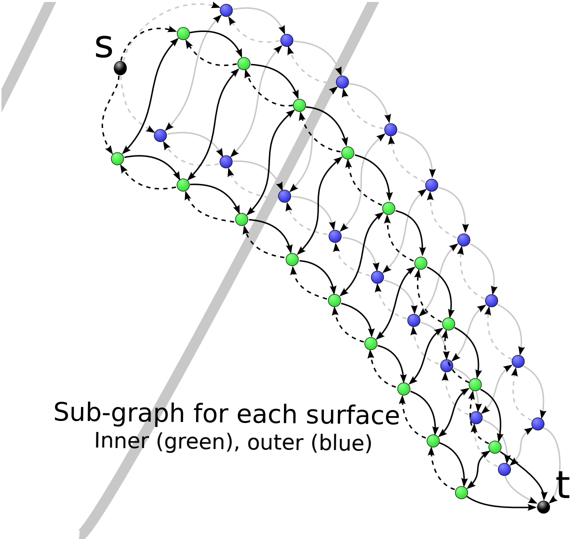

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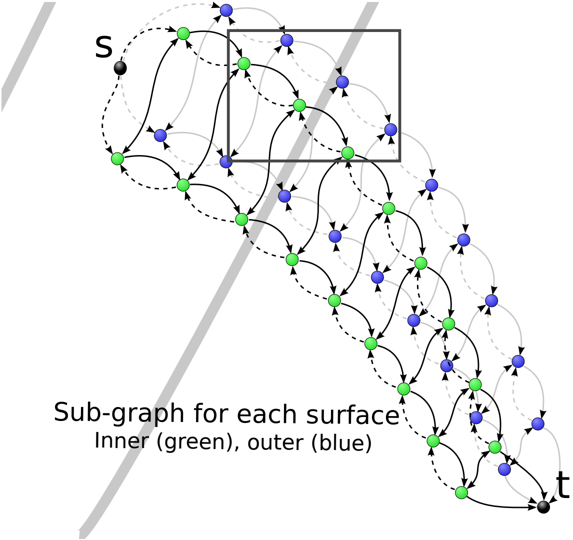

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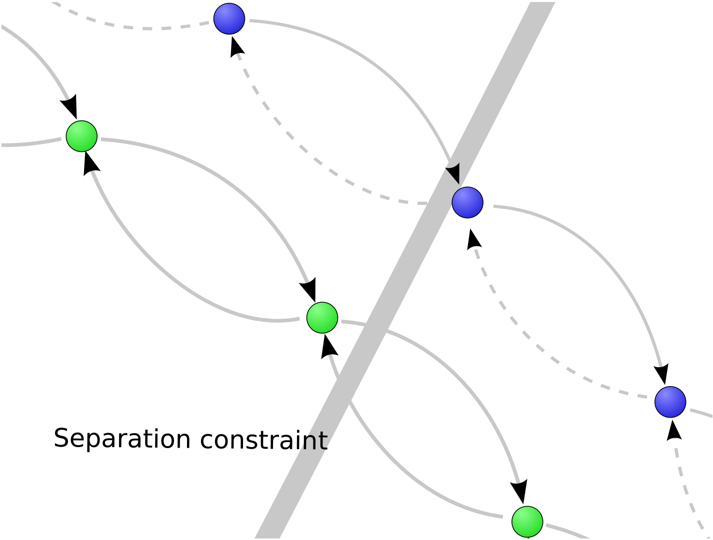

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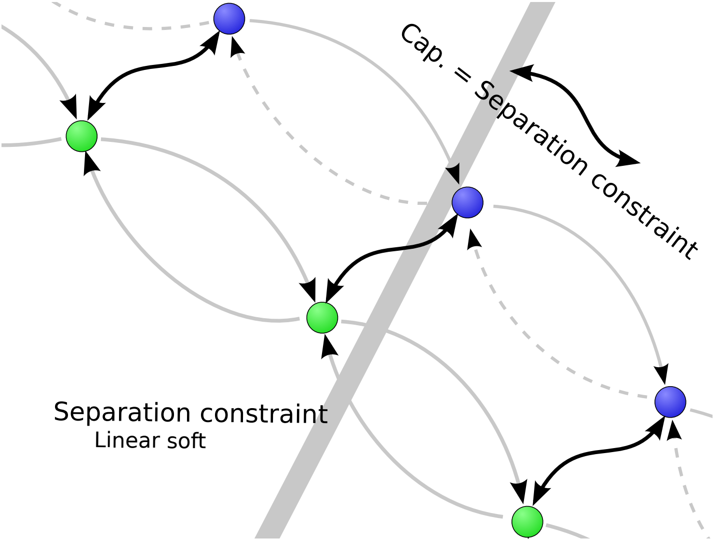

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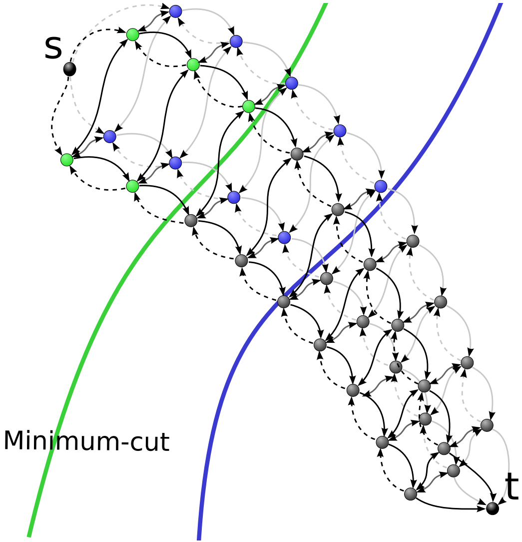

Extra Slides: Training

Data:

- 329 Cross-sectional images from 8 subjects.
- Randomly extracted perpendicular to and centered on airway centerline.
- Manually annotated with lumen and complete airway area.

Training:

- Inner and outer surface smoothness constraints.
- Inner and outer cost function derivative weightings.
- Separation constraint.

Method:

- Similar to coordinate search.
- Criteria: Dice coefficient.