

Local Branching – A brand new method

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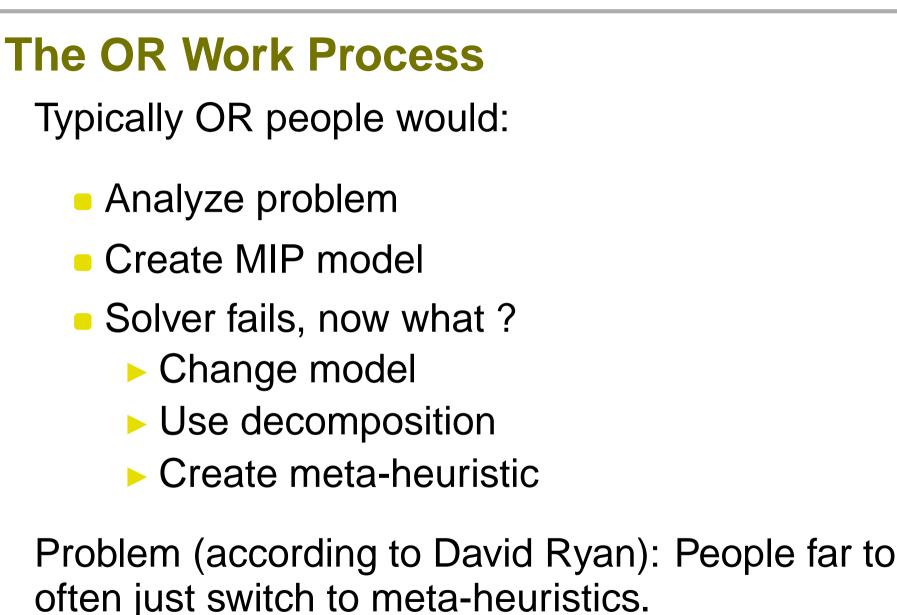
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Outline

- Heuristics for Mixed Integer Programs
 - Rounding
 - Local branching
 - Relaxation induced neighbourhood search
 - Guided dives
 - Experimental results







Meta-heuristics and MIP

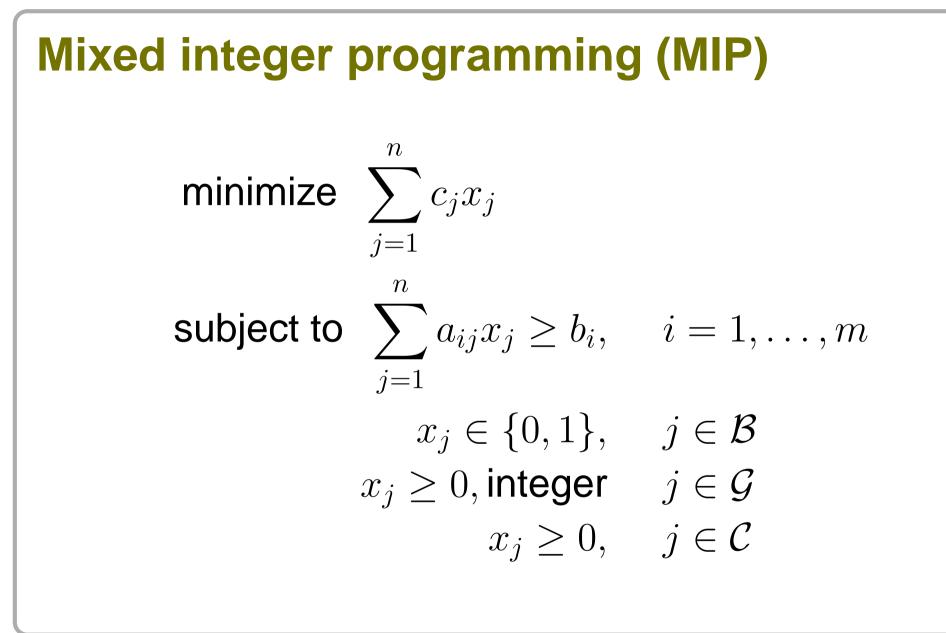
There are two reasons why switching to meta-heuristics are problematic:

- Some work (the MIP model) is discarded ...
- Handling constraints in meta-heuristics is hard: Set Partitioning, Metsa Öy ...

Alternatively we can use MIP-heuristics: Rounding, RINS or Local Branching. These are very new (2003), but not very complicated, techniques ...











Local branching

Given a MIP model:

- Construct a feasible solution (may be hard ...)
- Add a "locality" constraint, which limits the search area around the current feasible solution $\bar{\mathbf{x}}$...
- Enter the main loop
- Find the new best around $\bar{\mathbf{x}}$
- Switch to the new solution
- Continue until no better solutions can be found

What's a locality constraint ???

Let $\overline{\mathcal{B}} := \{j \in \mathcal{B} : \overline{x}_j = 1\}$ be the binary support of $\overline{\mathbf{x}}$. Hamming distance between $\overline{\mathbf{x}}$ and \mathbf{x} :

$$\Delta(\mathbf{x}, \bar{\mathbf{x}}) := \sum_{j \in \mathcal{B}} |x_j - \bar{x}_j| = \sum_{j \in \overline{\mathcal{B}}} (1 - x_j) + \sum_{j \in \mathcal{B} \setminus \overline{\mathcal{B}}} x_j$$

 $k\text{-}\mathsf{OPT}$ neighbourhood of $\bar{\mathbf{x}}$: Set of solutions \mathbf{x} for which

 $\Delta(\mathbf{x}, \bar{\mathbf{x}}) \le k$





Local branching: Algorithm

How do we search a k-OPT neighboorhood? Just add constraint $\Delta(\mathbf{x}, \overline{\mathbf{x}}) \leq k$ to the MIP, and solve the reduced MIP to optimality!





Local branching: Algorithm II

- When k is small, the new MIP is usually easy to solve.
- May use a truncation scheme, i.e., a time- or node-limit for the solution of the reduced MIP.
- Tactical branching: Having explored the neighbourhood $\Delta(\mathbf{x}, \overline{\mathbf{x}}) \leq k$, we may add the constraint $\Delta(\mathbf{x}, \overline{\mathbf{x}}) \geq k + 1$ to the MIP.





Experimental results

Local branching final comments:

- Again: An algorithm which can solve problems
 NON of the metaheuristics can solve ...
- Relatively easy to implement in e.g. GAMS ...

