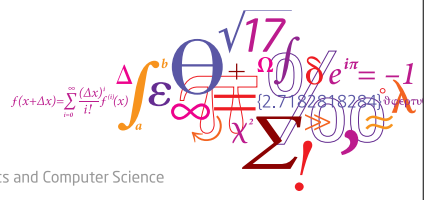
02465: Introduction to reinforcement learning and control

The finite-horizon decision problem

Tue Herlau

DTU Compute, Technical University of Denmark (DTU)



DTU Compute
Department of Applied Mathematics and Computer Science

Lecture Schedule

Dynamical programming

1. **The finite-horizon decision problem**
   7 February
2. Dynamical Programming
   14 February
3. DP reformulations and introduction to Control
   21 February

Control

4. Discretization and PID control
   28 February
5. Direct methods and control by optimization
   7 March
6. Linear-quadratic problems in control
   14 March
7. Linearization and iterative LQR
   21 March

Reinforcement learning

8. Exploration and Bandits
   28 March
9. Bellmans equations and exact planning
   4 April
10. Monte-carlo methods and TD learning
    11 April
11. Model-Free Control with tabular and linear methods
    25 April
12. Eligibility traces
    2 May
13. Deep-Q learning
    9 May

Syllabus: `https://02465material.pages.compute.dtu.dk/02465public`
Help improve lecture by giving feedback on DTU learn

Reading material:

- [Her25, Chapter 4] Introduction

Learning Objectives

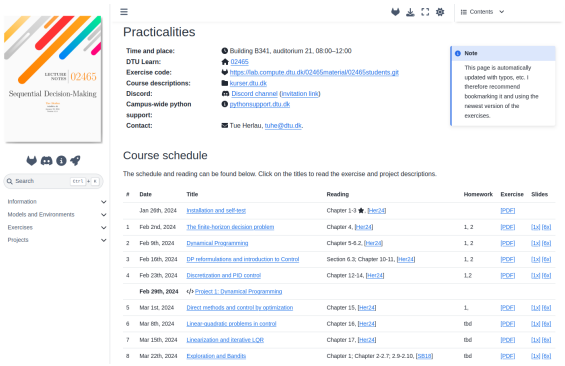
- Introduction and key definitions
- Python and object-oriented programming

cweb

Course practicalities

Course webpage

`02465material.pages.compute.dtu.dk/02465public/index.html`



Course practicalities

Where and what

DTU Learn Announcements, assignment hand-ins, quizzes

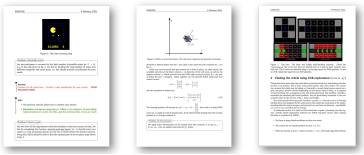
Course homepage Exercises, projects, slides, documentation, installation, etc. `https://02465material.pages.compute.dtu.dk/02465public`

Off-hours QA Discord. See link on homepage.

- Exercises
  - Building B341, auditorium 21
  - Building B341, IT-015
  - Building B341, IT-019
- Ask **project-related question** online so that everyone has the same information (i.e. not in class)

Course practicalities

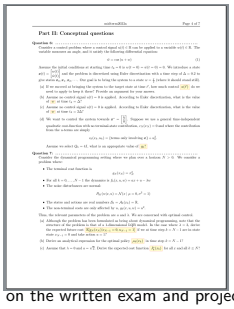
Project work



- Groups of 1, 2 or 3 students

  Part 1 Dynamical programming **(available now)**
  Part 2 Control
  Part 3 Reinforcement Learning
- The projects are subject to DTUs rules of collaboration/Code of Conduct
  - This includes the individual programming in Part 3.

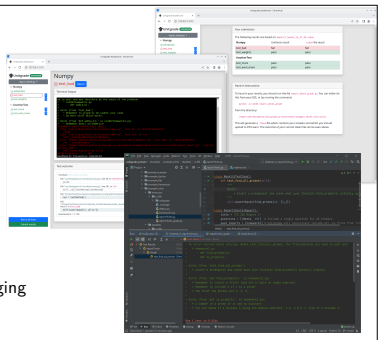
Exam



- The 4-hour written exam will contain:
  - Multiple-choice questions
  - Written-answer questions
  - Programming questions
- Your evaluation is an overall assessment based on the written exam and project work
  - The project work is 20%.

N.b. the exam is planned to be in English and not Danish. You can request that I change the language to Danish. I don't think this is to anyones advantage since many terms don't have a good Danish equivalent, however, it is up to you. If you wish that the exam is translated please contact me before week 6 of the course.

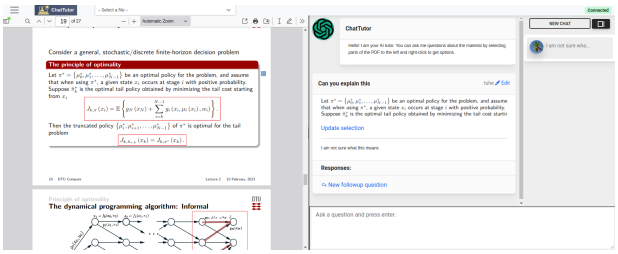
Creating handins



See 'Lecture 0' on the homepage

- Tests are available locally for debugging
- A grade script generate a handin
- Example usage:
  - `python -m irlc.project0.fruit_project_grade`
  - Hand in your code/scores by uploading the `.token` file

ChatTutor



- ChatTutor allows you to ask questions to **both** TAs and an AI (ChatGPT)
- The platform will collect the data you put in (i.e., same as any other webpage!)
- Sign-up link: `https://chattutor.dk/s/112/bylink/jWOVynKVW6/as/ST/`

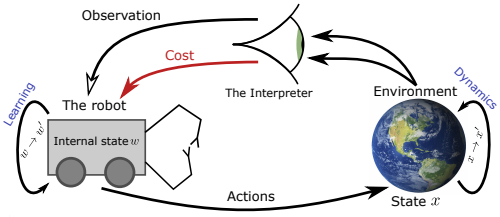
Welcome!

Types of machine learning

Supervised learning Learn a function $f(x_i) \mapsto \hat{y}_i$ to minimize a **loss**

Unsupervised learning Learn a **structure** to **summarize data**

Sequential decision making

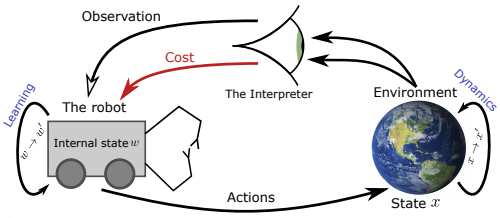


Make decisions, one after another, to bring about a desired outcome

- Observe the world
- Take action
- Obtain cost

Minimize total cost

`lecture_01_pacman.py`

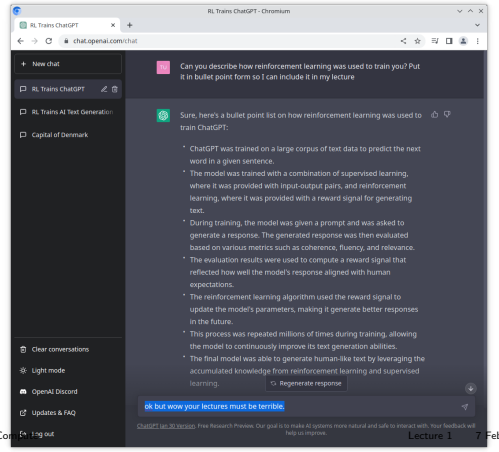


- Time is really important (sequential data)
- Must optimize behavior of dynamical systems using information that becomes progressively available as the systems evolve
- Future cost and state of the system will depend on current actions and state

Alpha-Go (2016)

- Self-learning Go supercomputer
- Defeated world champion Lee Sedol in 2016
- Notable mentions: Atari/Dota/Starcraft II learners

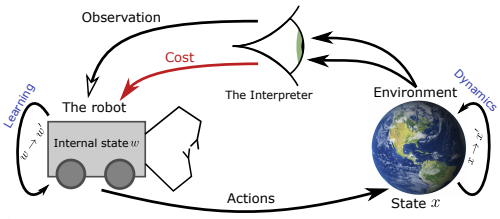
ChatGPT (2022)



How to address this problem

- Establish vocabulary
- Build a mathematical model
- Use the model to solve problems

The decision problem



State The configuration of the environment x

Action The robots output-signal

Cost/reward A number. Depends on state x and action u

Example: Atari

States RAM memory state

Observations Pixel-based snapshots $H \times W \times 3$

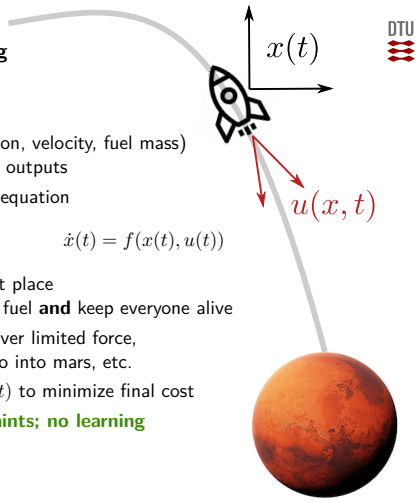
Actions Discrete joystick actions

Dynamics Discrete, stochastic (what the emulator does)

Cost High-score

Don't know dynamics; must learn from scratch

Example: Mars landing



Time Continuous

State/Actions $x(t)$: (Position, velocity, fuel mass)
$u(t)$: thruster outputs

Dynamics A differential equation

$$\dot{x}(t) = f(x(t), u(t))$$

Cost Land the right place
and use little fuel **and** keep everyone alive

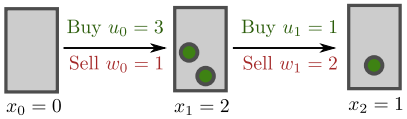
Constraints Thrusters deliver limited force,
ship cannot go into mars, etc.

Objective Determine $u(t)$ to minimize final cost

Really important constraints; no learning

`lecture_01_car_random.py`

Inventory control



- We order a quantity of an item at period $k = 0, \ldots, N-1$ so as to meet a stochastic demand

  x_k stock at the beginning of the kth period,
  $u_k \geq 0$ stock ordered at the beginning of the kth period.
  $w_k \geq 0$ Demand during the k'th period

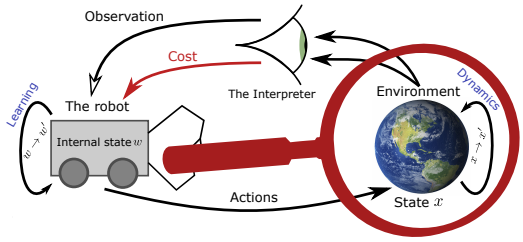
- Dynamics: $x_{k+1} = \min(\max(x_k + u_k - w_k, 0), 2)$
- Cost to minimize:

$$\underbrace{u_k}_{\text{cost-to-order items}} + \underbrace{(x_k + u_k - w_k)^2}_{\text{Satisfy demand + limit inventory size}}$$

- Select actions $u_0, \ldots, u_{N-1}$ to minimize cost

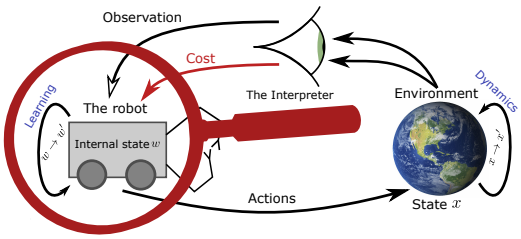
We want proven optimal rule for ordering

The environment



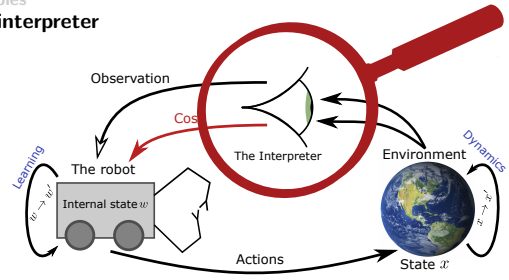
- Nature can be stochastic or deterministic
- The problem can be continuous-time or discrete-time
- We can know the dynamics or not

The agent



Policy How the robot chooses actions at given times/states

The interpreter



Reward The **immediate** evaluation of current step

Agents goal Maximize **cumulative** reward

Reward Hypothesis

Every desired behavior of the agent can be described by the maximization of expected cumulative reward

Making sense of these distinctions

- Why so many things in one course?
  - Study-line requirement
  - A single problem, and a single solution + tricks
  - A better overview (right tool for the job)
- Today, we will look at the problem

Basic control setup: Environment dynamics

Finite time Problem starts at time 0 and terminates at time N. Indexed as $k = 0, 1, \ldots, N$.

State space The states x_k belong to the **state space** $x_k \in \mathcal{S}_k$

Control The available controls u_k belong to the **action space** $\mathcal{A}_k(x_k)$, which may depend on x_k

Dynamics

$$x_{k+1} = f_k(x_k, u_k, w_k), \quad k = 0, 1, \ldots, N-1$$

Disturbance/noise A random quantity w_k with distribution

$$w_k \sim P_k(W_k | x_k, u_k)$$

Cost and control

Agent observe x_k, agent choose u_k, environment generates w_k

Cost At each stage k we obtain cost

$$g_k(x_k, u_k, w_k), \quad k = 0, \ldots, N-1 \quad \text{and} \quad g_N(x_k) \text{ for } k = N.$$

Action choice Chosen as $u_k = \mu_k(x_k)$ using a function $\mu_k : \mathcal{S}_k \to \mathcal{A}_k(x_k)$

$$\mu_k(x_k) = \{\text{Action to take in state } x_k \text{ in period } k\}$$

Policy The collection $\pi = \{\mu_0, \mu_1, \ldots, \mu_{N-1}\}$

Rollout of policy Given x_0, select $u_k = \mu_k(x_k)$ to obtain a **trajectory** $x_0, u_0, x_1, \ldots, x_N$ and **accumulated cost**

$$\text{Cost-of-rollout} = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, \mu_k(x_k), w_k)$$

Expected return (approximate) Generate T rollouts according to π

$$J_\pi(x_0) \approx \frac{1}{T} \sum_{i=1}^{T} \{\text{Cost-of-rollout } i\}$$

Quiz 1: Discuss and answer on DTU Learn

How do you feel about this argument? Justify your answer:

Decision-making is about determining the appropriate sequence of actions $u_0, \ldots, u_{N-1}$.
Once executed, we get a total cost. Let's say that on average this is $c(\mathbf{u})$.
Thus, decision-making is ultimately an optimization problem: Find the sequence that on average minimize the cost:

$$u_0, \ldots, u_{N-1} = \arg\min_{\mathbf{u}} c(\mathbf{u}).$$

a. It is computationally too complicated to solve such an optimization problem

b. It is infeasible to derive or learn the function $c(\mathbf{u})$

c. Actually nothing is wrong: It is just not a theoretically interesting/fruitful way to approach decision-making

d. Something else is wrong with the argument

e. Don't know

Pre-semester quiz


```python
# chapter1/lecture1_code.py
class MyClass:
    def __init__(self, a):
        self.my_variable = a

    def some_function(self):
        print("The variable I got was", self.my_variable)

class MyOtherClass(MyClass):
    def __init__(self, a, b):
        super().__init__(a)
        print("I also got", b)
```

This is new -- I have not used class inheritance before. The code is mysterious.

I have seen code like this before, but it is not something I have used. I think I can pick it up.

I have written code that inherit from other classes (i.e., something like the second class). I am not an expert, but it is not something that worries me

This is easy. I have written code like this before and can reason about what it does.

Initiatives

What I have done:

- Re-structured the project work
- Simplification of exercises + videos
- Course notes on Python + online documentation
- This lecture
- Changed exam format
- Course co-responsible for the new mandatory programming course (02002/3) in 2023

What I hope you will do:

- Decide to learn this – you can!
- Set aside some time in the first block
- Don't give up:
  - Programming was not taught correctly – 100% valid criticism
  - You need to learn new programming techniques through your career

Pacman game loop (without objects)

```python
# chapter1/lecture1_code.py
walls = np.ndarray( ) # Initialize a walls-variable
food = np.ndarray( )
pacman_x = 4
pacman_y = 6

for k in range(10):
    # Use the walls and pacman_x, pacman_y to figure out what actions are available.
    available_actions = ... # compute using the walls-variable
    # Do some sort of planning (search?) by using the walls, pacman_x, pacman_y.
    # select the best possible action
    # Compute the outcome of the action:
    pacman_x = pacman_x + action_x
    pacman_y = pacman_y + action_y
    # Compute the reward
    # Let the agent learn based on the outcome and reward
```

(about 500 lines total)

Same with two agents and two environments

```python
# chapter1/lecture1_code.py
for k in range(10):
    if environment_type == 2:
        available_actions = ... # compute using the walls-variable
    else:
        available_actions = ... # This environment may differ
    if agent_type == 1: # Agent plan it's actions
        pass    # do planning of first type
    elif agent_type == 2:
        pass    # do planning of the second type
    if environment_type == 1: # Compute the outcome of the action:
        pacman_x = pacman_x + action_x
        pacman_y = pacman_y + action_y
        # Compute the cost-function
    else:
        pass # Updates relevant for second environment
        # Compute the cost function
    if agent_type == 2:  # Allow the agent to learn based on cost
        pass # Learning for the second agent
    else:
        pass # Learning method for the first agent
```


Using objects

```python
# chapter1/lecture1_code.py
env = InventoryEnvironment() # Create an instance of the inventory environment
agent = RandomAgent(env)     # Create an instance of a random-action agent
train(env, agent)            # Train the agent
```

Training-function:

```python
# chapter1/lecture1_code.py
def train(env, agent):
    s = env.reset()       # Reset and get first state, x_0
    for k in range(10):
        a = agent.pi(s) # The policy computes the action
        sp, r, done = env.step(a) # Environment computes next state, reward
        agent.train(s, a, sp, r, done)      # Let the agent train
```

(this is a very rough sketch. Well get to the real training function soon)

The simplest class

The smallest and friendliest `class`

```
>>> class BasicClass:  # Classnames are usually upper-case
...     pass           # `pass` is a special keyword which does nothing
...
```

Each class **instance** function like it's own little box of variables:

```
>>> a = BasicClass()   # Create an instance of the class
>>> a.name = "My first class"  # You can write data to the class like this
>>> b = BasicClass()   # Another instance. a and b are not related and can store different data:
>>> b.name = "Another class"
>>>
>>> print("Class a:", a.name)
Class a: My first class
>>> print("Class b:", b.name)
Class b: Another class
```


A class with a function

```
>>> class BasicDog:
...     name = "Unnamed dog"  # Each dog-instance will have the property name
...     def read_nametag(self):
...         # This is a class-function. Note we must pass it `self` as a first argument,
...         # instance of the class itself (i.e. the current object). This is how we can
...         print("This dog is named", self.name, "please give me treats!")
...
>>> dog = BasicDog()
>>> dog.name
'Unnamed dog'
```

`self` refers to the class instance

```
>>> dog.read_nametag()  # Invoke the read_nametag() function. Note we don't pass the ob
This dog is named Pluto please give me treats!
```


`def __init__` function is called when the class is created

```
>>> class BetterBasicDog:
...     def __init__(self, name):
...         self.name = name
...         self.age = 0
...         print(f"The __init__() function has been called with name='{name}'")
...     def birthday(self):
...         self.age = self.age + 1
...         print("Hurray for", self.name, "you are now", self.age, "years old")
...
```

Arguments can be passed along like this

```
>>> d1 = BetterBasicDog("Pluto")        # the __init__ function is now called
The __init__() function has been called with name='Pluto'
>>> d2 = BetterBasicDog(name="Lassie")  # Also support named arguments
The __init__() function has been called with name='Lassie'
```

Functions can change the `state` of the class

```
>>> d1.birthday()
Hurray for Pluto you are now 1 years old
>>> d1.birthday()
Hurray for Pluto you are now 2 years old
```


Quiz 2: What is the outcome of this code?

```
>>> class BetterBasicDog:
...     def __init__(self, name):
...         self.name = name
...         self.age = 0
...         print(f"The __init__() function has been called with name='{name}'")
...     def birthday(self):
...         self.age = self.age + 1
...         print("Hurray for", self.name, "you are now", self.age, "years old")
...
>>> d1 = BetterBasicDog("Pluto")
The __init__() function has been called with name='Pluto'
```

```python
# chapterOpythonC/quiz.py
d1 = BetterBasicDog("Pluto")
d1.birthday()
d1.age = 5
d1.name = "Lassie"
d1.birthday()
```

a. Ignore changes and prints out `"Hurray for Pluto you are now 1 years old"`

b. Accept changes and prints out `"Hurray for Lassie you are now 6 years old"`

c. It gives an error – it is not possible to set the age.

d. It uses `name` but ignores `age`, so we get:
`"Hurray for Lassie you are now 1 years old"`

e. Don't know.

The parrot

```
>>> class Parrot:
...     def __init__(self):
...         self.words = ["Squack!"]
...     def learn(self, word):
...         self.words.append(word)
...     def speak(self):
...         return random.choice(self.words)  # Return a random word
...     def vocabulary(self):
...         return self.words
...
```

```
>>> parrot = Parrot()
>>> words = ["sugar", "sleep well", "(parrot noises)", "*honk*"]
>>> for word in words:
...     parrot.learn(word)
...
>>> for _ in range(3):  # Say three words
...     parrot.speak()
...
'Squack!'
'Squack!'
'sleep well'
>>> print("Vocabulary", parrot.vocabulary())
Vocabulary ['Squack!', 'sugar', 'sleep well', '(parrot noises)', '*honk*']
```


Inheritance

```
>>> class Parrot:
...     def __init__(self):
...         self.words = ["Squack!"]
...     def learn(self, word):
...         self.words.append(word)
...     def speak(self):
...         return random.choice(self.words)  # Return a random word
...     def vocabulary(self):
...         return self.words
...
```

`ForgetfulParrot`: Is like the regular `Parrot`, except the learn-function

```
>>> class ForgetfulParrot(Parrot):
...     # The Parot class is used as a template.
...     # All functions in the Parot-class are therefore 'imported' as default, including 'self.words'
...     def learn(self, word):  # This function overwrite the 'actual' learn function in the Parot class
...         self.words = [word]  # This parrot only know a single word
...
```

Inheritance: The functions are *"copy-pasted"* into the `ForgetfulParrot`

```
>>> old_parrot = ForgetfulParrot()
>>> old_parrot.learn("damn remote")
>>> old_parrot.learn("Jeopardy")
>>> print("Vocabulary", old_parrot.vocabulary())
Vocabulary ['Jeopardy']
```


Inheritance continued

More **inheritance**: Make a squeak before and after every word:

```
>>> class Parrot:
...     def __init__(self):
...         self.words = ["Squack!"]
...     def learn(self, word):
...         self.words.append(word)
...     def speak(self):
...         return random.choice(self.words)  # Return a random word
...     def vocabulary(self):
...         return self.words
...
```

Where is the bug?

```
>>> class BadSqueekyParrot(Parrot):
...     def __init__(self, squeek="Quck!"):
...         self.squeek = squeek
...     def speak(self):
...         return f"{self.squeek} {random.choice(self.words)} {self.squeek}"
...
>>> squeeky = BadSqueekyParrot(squeek="Kvak-Kvak")
>>> squeeky.learn("Good night!")
Traceback (most recent call last):
  File "<console>", line 1, in <module>
  File "<console>", line 5, in learn
AttributeError: 'BadSqueekyParrot' object has no attribute 'words'
```


Use `super()` to access functions in the parent class

```
>>> class SqueekyParrot(Parrot):
...     def __init__(self, squeek="Quck!"):
...         super().__init__()  # Call the 'Parot' class __init__ method to set up the words-variable.
...         self.squeek = squeek # save the squeek variable
...     def speak(self):
...         word = super().speak() # Use the speak() function defined in the Parrot class.
...         return f"{self.squeek} {word} {self.squeek}"
...
>>> squeeky = SqueekyParrot(squeek="Kvak-Kvak")
>>> squeeky.learn("Good night!")
>>> squeeky.learn("Tell that damn bird to shut it's beak")
>>> squeeky.learn("Sugar!")
>>> squeeky.speak()
'Kvak-Kvak Good night! Kvak-Kvak'
>>> squeeky.speak()
'Kvak-Kvak Sugar! Kvak-Kvak'
```

Why classes in this course?

Consistency When we inherit from `Parrot`, we **know** the functions should be called `speak`, `learn` (and not `talk`, `practice`)

- `Env`: (`reset`, `step`, `action_space` and a few other)
- `Agent`: (`pi`, `train`)

Functionality Inheritance allows us to re-use code

- In control theory, we will use inheritance to add simulation-functionality to all models

The inventory environment

```
# inventory_environment.py
class InventoryEnvironment(Env):
    def __init__(self, N=2):
        self.N = N                              # planning horizon
        self.action_space      = Discrete(3)    # Possible actions {0, 1, 2}
        self.observation_space = Discrete(3)    # Possible observations {0, 1, 2}

    def reset(self):
        self.s = 0                              # reset initial state x0=0
        self.k = 0                              # reset time step k=0
        return self.s, {}                       # Return the state we reset to (and an

    def step(self, a):
        w = np.random.choice(3, p=(.1, .7, .2))    # Generate random disturbance
        s_next = max(0, min(2, self.s-w+a))           # next state; x_{k+1} =  f_k(x_k,
        reward = -(a + (self.s + a - w)**2)           # reward = -cost      = -g_k(x_k,
        terminated = self.k == self.N-1               # Have we terminated? (i.e. is k=
        self.s = s_next                               # update environment state
        self.k += 1                                   # update current time step
        return s_next, reward, terminated, False, {}  # return transition information
```

Recall $x_{k+1} = x_k - w_k + a_k$ (clipped at 0 and 2) and e.g. $P(w = 0) = \frac{1}{10}$

The Agent:

```
# inventory_environment.py
class RandomAgent(Agent):
    def pi(self, s, k, info=None):
        """ Return action to take in state s at time step k """
        return np.random.choice(3) # Return a random action
```

- The policy $\mu_k(x_k)$ corresponding to `pi(x, k, info)`
- A training function which is given x_k, u_k and x_{k+1} plus obtained reward plus additional information
- In each exercise session, you will write at least one agent
- Look at the `Agent`-class
- `truncated=False`; `info` is 'extra information' (see documentation)

The `train`-function

The train-function computes an episode as follows:

```python
# inventory_environment.py
def simplified_train(env: Env, agent: Agent) -> float:
    s, _ = env.reset()
    J = 0  # Accumulated reward for this rollout
    for k in range(1000):
        a = agent.pi(s, k)
        sp, r, terminated, truncated, metadata = env.step(a)
        agent.train(s, a, sp, r, terminated)
        s = sp
        J += r
        if terminated or truncated:
            break
    return J
```

Above computes the sum-of-reward for one episode:

```python
    # inventory_environment.py
    env = InventoryEnvironment()
    agent = RandomAgent(env)
    stats, _ = train(env,agent,num_episodes=1,verbose=False)  # Perform one rollout.
    print("Accumulated reward of first episode", stats[0]['Accumulated Reward'])
```


Approximate value function

Approximate

$$J_\pi(x_0) = \mathbb{E}\left[g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, \mu_k(x_k), w_k)\right] \tag{1}$$

As average over 1000 trajectories

```python
    # inventory_environment.py
    stats, _ = train(env, agent, num_episodes=1000,verbose=False)  # do 1000 rollouts
    avg_reward = np.mean([stat['Accumulated Reward'] for stat in stats])
    print("[RandomAgent class] Average cost of random policy J_pi_random(0)=", -avg_reward)
```


Quiz 3: Bobs friend

Bob has $x_0 = 20$ kroner. He can either:

- Action $u = 0$: Put them in the bank at a 10% interest, thereby ending up with 22 kroner.
- Action $u = 1$: Lend them to a friend.
  - With probability $\frac{1}{4}$ he looses everything ($x_1 = 0$)
  - With probability $\frac{3}{4}$ his friend gives him 12 kroner (aka one beer) as a thank you, and thus he will have $x_1 = 20 + 12 = 32$ kroner total.

Bobs goal is to decide whether to put his money in the bank, or lend them to his friend. Which one of the following statements are correct:

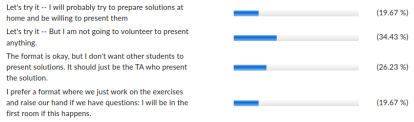
a. The state spaces are $\mathcal{S}_k = \{1, 2, \ldots, 32\}$.

b. The dynamics is $f_0(x_0, u_0, w_0) = 1.1x_0 + \frac{3}{4}(x_0 + 12u_0)$.

c. The action space is $\mathcal{A}_0(x_0) = \{0, 1\}$

d. It is not possible to determine an optimal policy since we don't know what Bobs friend will do.

Exercises



- IT015: Passive exercises; installation problems
- Aud.21 + IT019: Interactive exercises.
  Try to prepare and present homework exercises.

1 Bobs financially challenged friend

☞ Bob has $x_0 = 20$ kroner. He can either:

- Action $u = 0$: Put them in the bank at a 10% interest, thereby ending up with 22 kroner.
- Action $u = 1$: Lend them to a friend.
  - With probability $\frac{1}{4}$ he looses everything
  - With probability $\frac{3}{4}$ his friend gives him 12 kroner (aka one beer) as a thank you, and thus he will have $20 + 12 = 32$ kroner total.

Tue Herlau.
Sequential decision making.
(Freely available online), 2025.