COORDINATED VEHICLE ROUTING WITH UNCERTAIN DEMAND[1]

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ABSTRACT

Numerical optimization methods have been developed and applied successfully to many deterministic variants of the so-called vehicle routing problem (VRP). Unfortunately, existing numerical methodologies are not as effective for planning and design problems when uncertainty is a significant issue. In view of this, this presentation will show how approximation models for large-scale uncertain VRP's can complement conventional optimization methods and allow for the exploration of a broader set of design and operating strategies than is currently possible. The presentation will consider vehicle routing problems where vehicles have a finite capacity and demand is uncertain, focusing on strategies that coordinate the actions of all vehicles in the fleet in real time as information becomes available.

When uncertainty exists, systems should be designed with degrees of flexibility that allow for efficient control in real time. In the case of "single-period" vehicle routing problems, we should determine two things: (i) the system configuration, including the fleet size and composition and an initial set of vehicle routes, and (ii) a dynamic control plan (algorithm) which specifies how vehicle routes are modified in real time as information becomes available. Uncertainty should be considered when designing both the system configuration and its control algorithm. Furthermore, configuration decisions should be made with both the flow of information and the control method in mind. For the capacitated

VRP with uncertain demand, the desirability and feasibility of specific designs will depend on how and when lot size information becomes available and the degree of control that a dispatcher can exert over en-route vehicles.

Researchers have attempted to obtain optimal designs minimizing expected operating costs for problems in which customer lot size information becomes known only after the arrival of a vehicle. Unfortunately, all the solutions proposed to date are based either on configurations that are unlikely to be feasible in practice, such as single-vehicle fleets, or on feasible operating plans that are too restrictive to be appealing in practice. A possible alternative system design that may be more practical and efficient would allow tour failures to be consolidated into secondary “sweeper” routes. The approach here would be to plan initial routes as if the vehicle capacity were smaller ($q^- < q$) to ensure that few primary tours would fail, and then to serve the overflow customers with a set of secondary tours where vehicles are allowed to cooperate. Unfortunately, although this configuration is simple to describe, it is already too difficult to optimize exactly. More promising designs where vehicles would be allowed to cooperate during the primary tours are even more difficult to treat exactly.

The presentation will show how a system in which vehicles are allowed to cooperate during the primary phase can be designed and operated by minimizing and approximate "logistic cost function" of key design parameters. The effectiveness of the proposed strategies is compared against (a) current strategies in which there is little or no coordination, and (b) against deterministic strategies for equivalent problems without uncertainty. It is shown that the introduction of coordination in proper ways lowers the operation cost from the best levels that can be achieved without coordination (a) to levels close to (b).

BASIS OF THE PRESENTATION

Erera, A. L. (2000) “Design of vehicle routing systems for uncertain environments (title approximate)”, PhD thesis, Department of Industrial Engineering and Operations Research, University of California, Berkeley, CA (September-October).

Daganzo, C.F. and Erera, A. (1999) "On planning and design of logistics systems for uncertain environments" in *New Trends in Distribution Logistics*, (M.G. Speranza and P. Stahly, editors), Lecture Notes in Economics and Mathematical Systems, vol 480, Springer-Verlag.

EXAMPLES OF SPACE-CONSTRAINED VRP’s WITH UNCERTAINTY

Bertsimas, D. J. “A vehicle routing problem with stochastic demand,” *Opns. Res.* 40, 574-585 (1992).

Gendreau, M., G. Laporte and R. Seguin, “An exact algorithm for the vehicle routing problem with stochastic demands and customers”, *Trans. Sci.* 29, 143-155 (1995).

EXAMPLES OF CONTINUUM APPROXIMATION VRP’s

Daganzo C. F. , (1984a) “The length of tours in zones of different shapes”, *Trans. Res.* 18B, 135-146 and (1984b) “The distance traveled to visit N points with a maximum of C stops per vehicle: an analytic model and an application,” *Trans. Sci.* 18, 331-350.

Daganzo, C.F. and Newell, G. F. (1986) "Configuration of physical distribution networks", *Networks* 16(2), 113-132.

Newell, G. F., (1986) “Design of multiple vehicle delivery tours---III: Valuable goods”, *Trans. Res.* 20B, 377--390.

Newell, G. F. and C.F. Daganzo, (1986) “Design of multiple vehicle delivery tours--I: A ring-radial network”, *Trans. Res.* 20B, 345—364; and “Design of multiple vehicle delivery tours--II: Other metrics”, *Trans. Res.* 20B, 365-376.

Robuste, F., C.F. Daganzo and R. Souleyrette (1990) “Implementing vehicle routing models”, *Trans. Res.* 24B, 263-286.

REVIEWS AND OTHER BACKGROUND

Laporte, G. (1992) “The vehicle routing problem: an overview of exact and approximate algorithms”, *Euro. J. Opnl. Res.* 59, 345--358.

Fisher, M. “Vehicle Routing”, in *Network Routing*, (M. Ball, T. Magnanti, C. Monma and G. Nemhauser, editors), Handbooks in Operations Research and the Management Sciences, Vol. 8, pp. 1-33, Elsevier Science, Amsterdam, The Netherlands (1995).

Langevin, A., Mbaraga, P. and Campbell, J.F. "Continuous approximation models in freight distribution: an overview", *Trans. Res.* 30B, 163-88 (1996).

Daganzo, C. F. *Logistics Systems Analysis*, Springer-Verlag, New York, N.Y. (1999).

OUTLINE

- **DEFINITONS AND BACKGROOUND**

- **MODELING APPROACH: COORDINATED STRATEGIES**

  Static

  Dynamic

- **DYNAMIC CPACITY SHARING BY A VEHICLE FLEET**

  Formulation

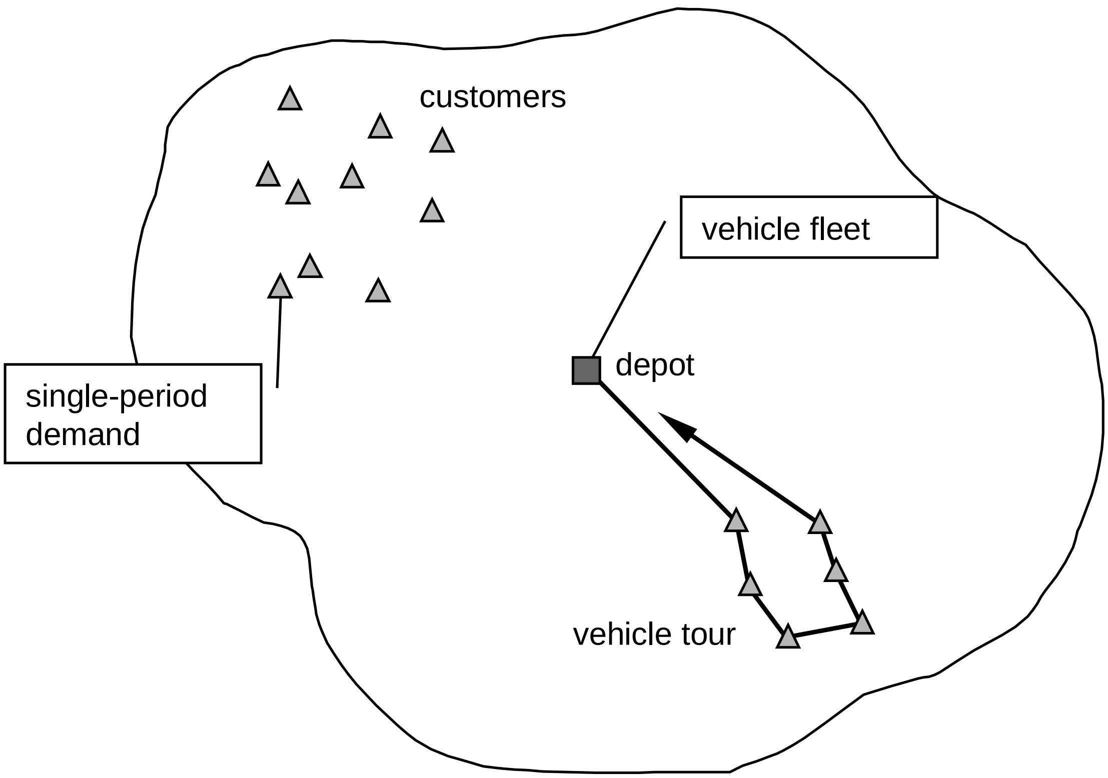
  Modeling

  Optimization

  Proof of concept

- **CONCLUSIONS**

Single-period vehicle routing



Decisions

– depot location, fleet composition, operating strategy

Possible system characteristics

– space-constraints: vehicle size
– time-constraints: time-windows, deadlines
– uncertainty
  - demand-side (locations, lot sizes, service times)
  - supply-side (travel times)

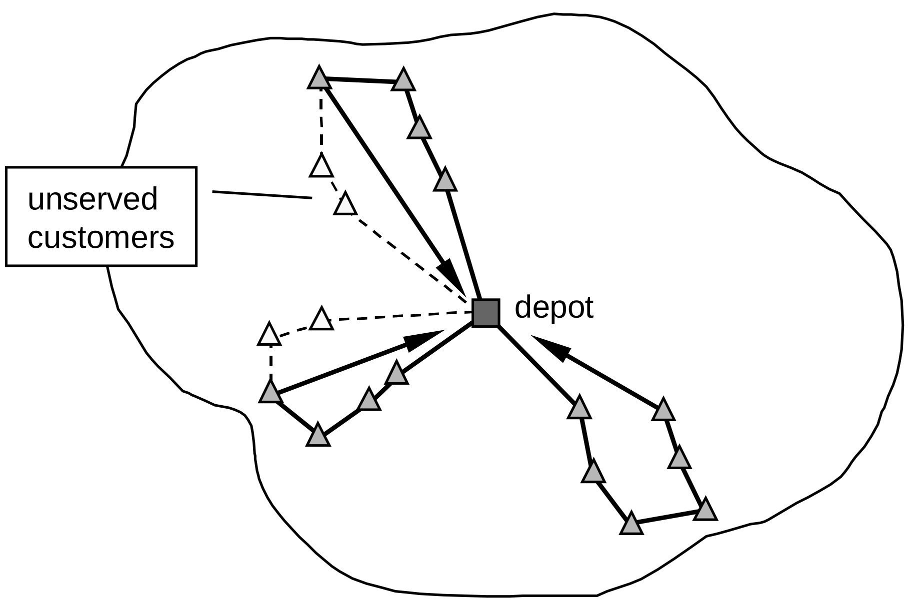
Vehicle routing and uncertainty

Deterministic vehicle routing problem

- *NP*-hard problem for minimum total distance (cost)
  - solution: set of vehicle tours
- Extensive literature: bounds, asymptotic behavior, heuristics, exact (IP) methods

Complications from demand uncertainty

- more difficult: planned tours may fail!



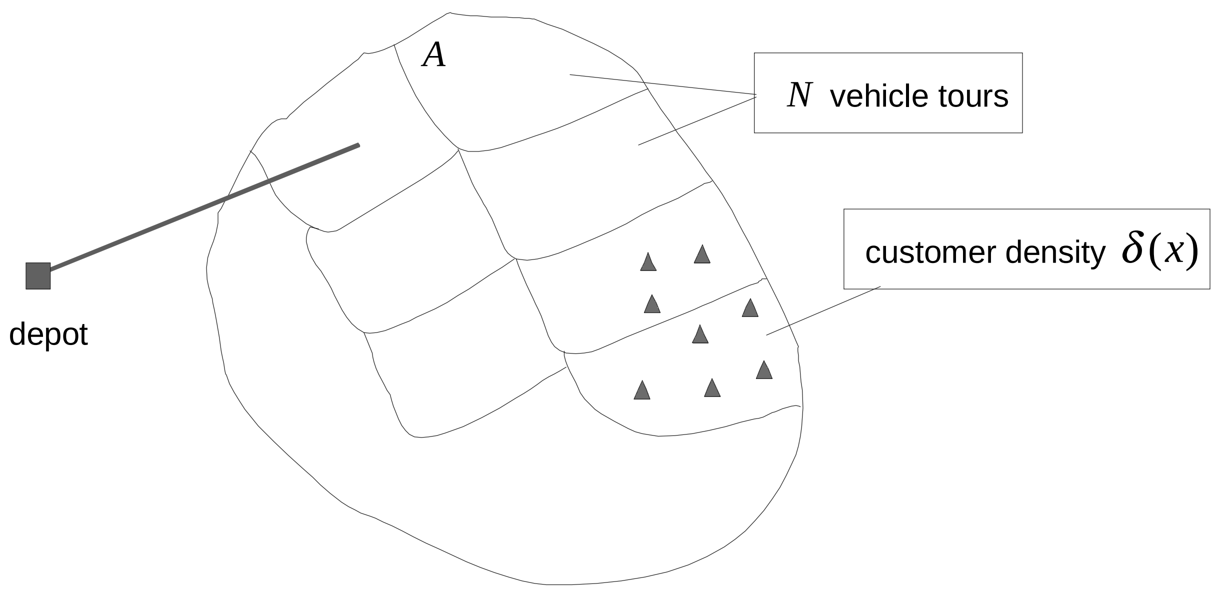
Large-scale approximations

Exploit scale under uncertainty

- continuous approx. of discrete locations, demands
- large-number laws, central limit theorem

Deterministic vehicle routing problem

- A large-number approximation for total distance
- Daganzo and Newell (1984, 1986)



$$D_T \approx 2\bar{\rho} + k_f \delta^{1/2} A$$

Space-constrained vehicle routing with uncertain demand

VRP$_{SC}$(UD)

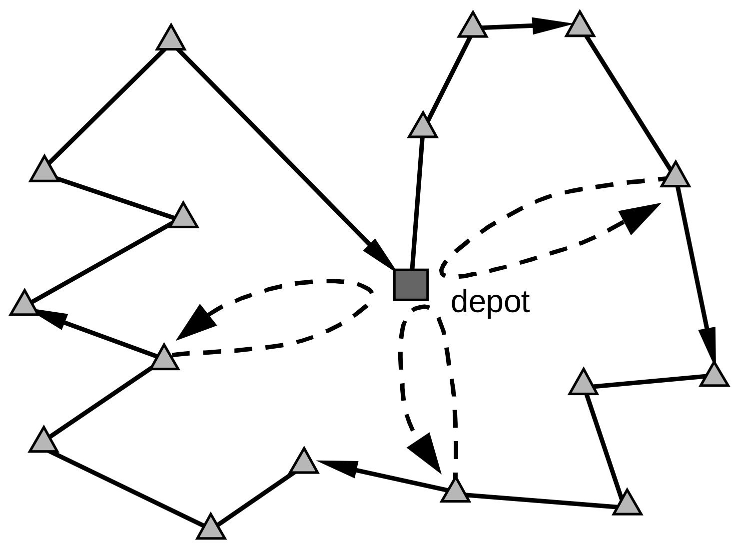
Given:	*Depot, fleet of vehicles with space capacities, customer demands random variables with known distributions, point-to-point travel costs*
Find:	Minimum expected cost operating strategy: - all customer demand satisfied - no vehicle exceeds capacity

Operating strategies

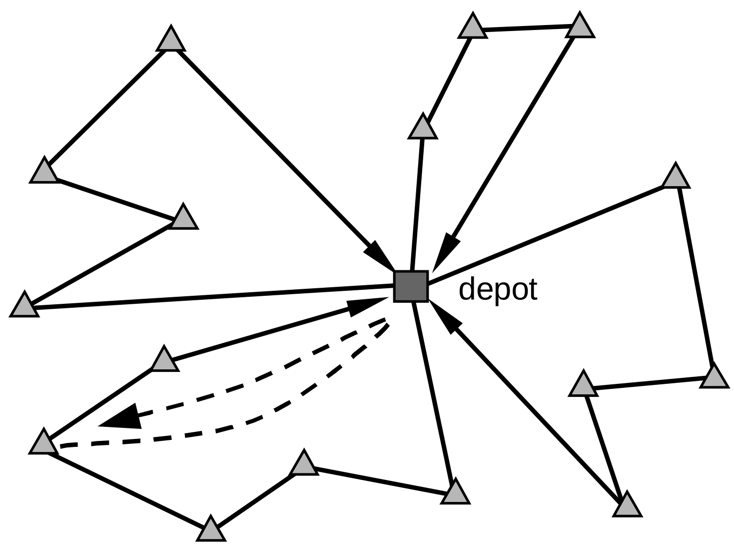
	Strategy	*References*
Uncoordinated	**Single vehicle-tour**	Dror *et al* (1989); Bastian and Rinooy Kan (1992); Bertsimas (1992)
	Multiple vehicle-tours; single-zone sweeper tours	Gendreau, Laporte, Seguin (1995,1996)
Static coordination	**Multiple vehicle-tours; multiple-zone sweeper tours**	Daganzo and Erera (1999)
Dynamic coordination	**Multiple vehicle-tours; vehicle reassignments**	**Today**

VRP_{SC}(UD): uncoordinated strategies

Single vehicle-tour

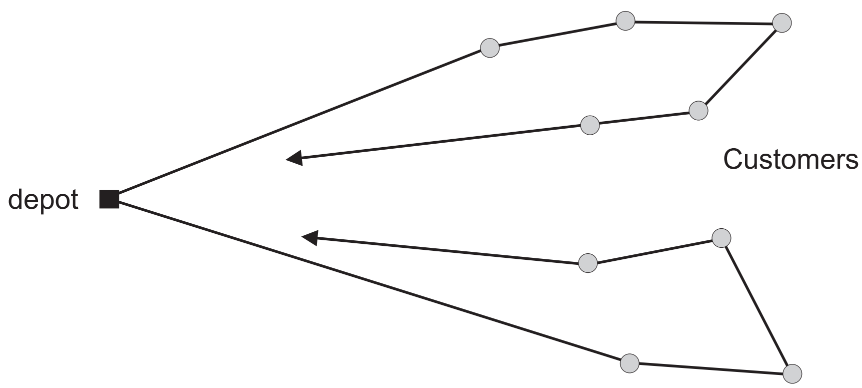


Multiple vehicle-tours

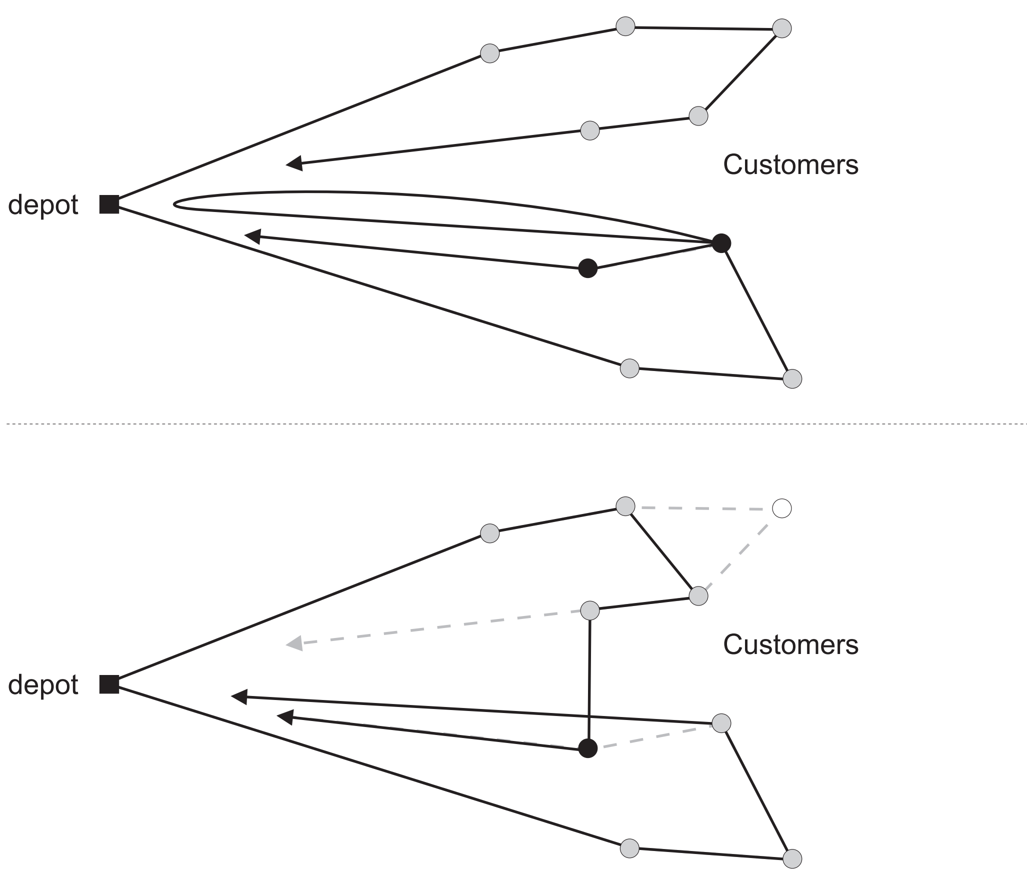


Importance of operating strategy

Single-period deterministic VRP

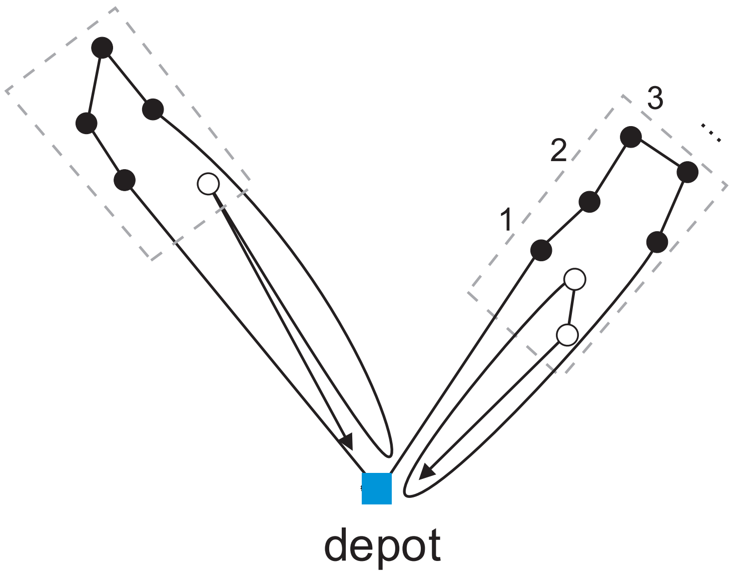


Single-period stochastic VRP



VRP_{SC}(UD): detailed models

Uncoordinated operations



Expected tour cost calculation tractable

- Example: Recursion in Bertsimas (1992)
- $O(K^2 n^2)$ per tour; K discrete demand levels

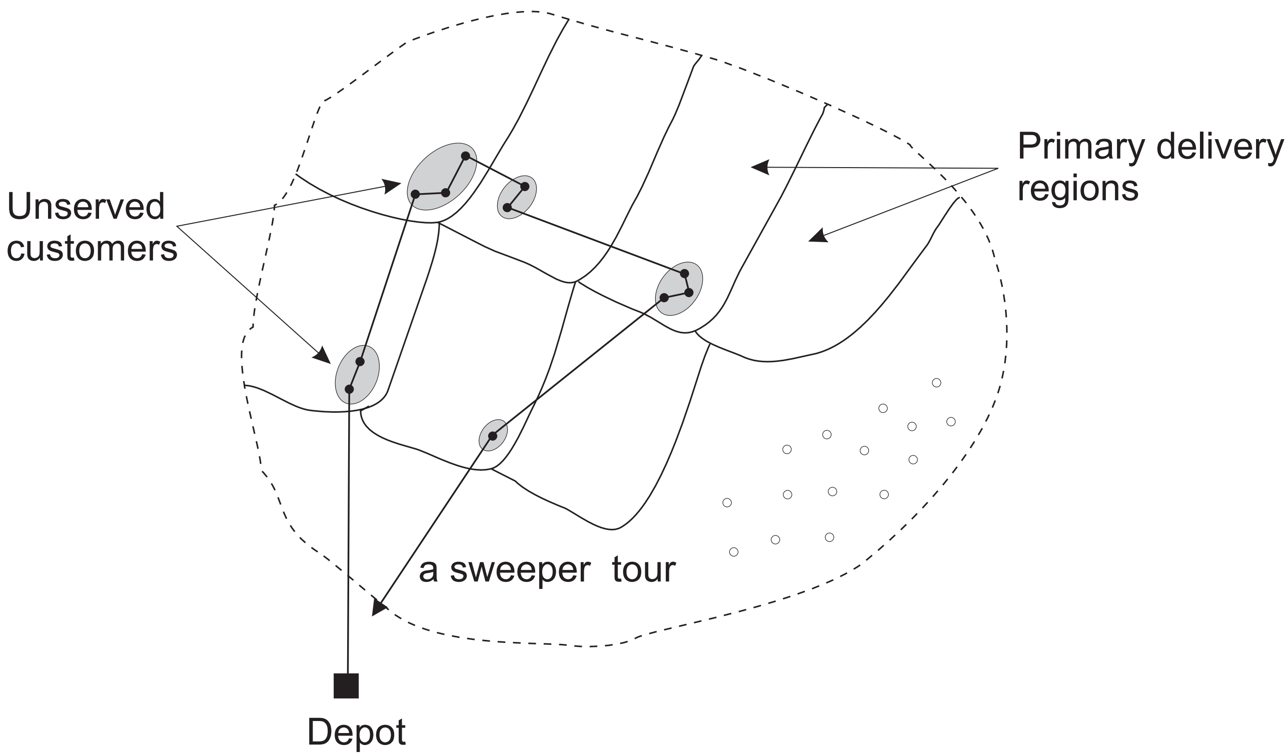
$$E[L] = \sum_{i=0}^{n} d(i, i+1) + \sum_{i=1}^{n} [\delta_i s(i,i) + \gamma_i s(i, i+1)$$

$$s(i, j) = s(i,0) + s(0, j) - s(i, j)$$

VRP$_{SC}$(UD): approximation model

Daganzo and Erera (1999)

- static coordination; multiple-zone sweep strategy
- consolidation of overflows



Modeling approach

- large-scale problem focus
- obtain approximate *logistics cost function* (LCF)
- optimization and testing

Proposed modeling approach

(1) Formulation

(2) Cost modeling

- approximate *logistics cost function* (LCF)

(3) Optimization

(4) Implementation w/ details and testing

Capacity-sharing strategies

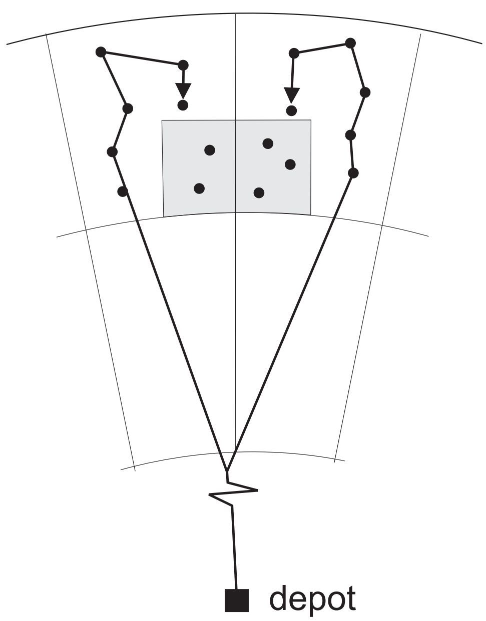
Partially-planned

(1) Operate tours to predetermined customers

(2) Non-full vehicles dynamically assigned
to unserved customers

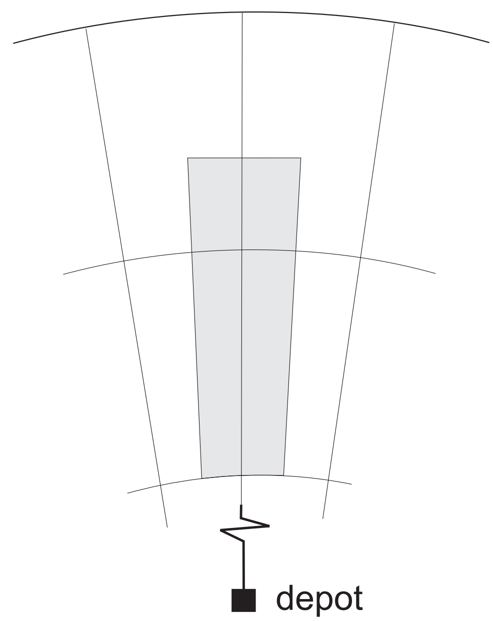
Local sharing strategies

2-vehicle capacity sharing



(a)

4-vehicle capacity sharing

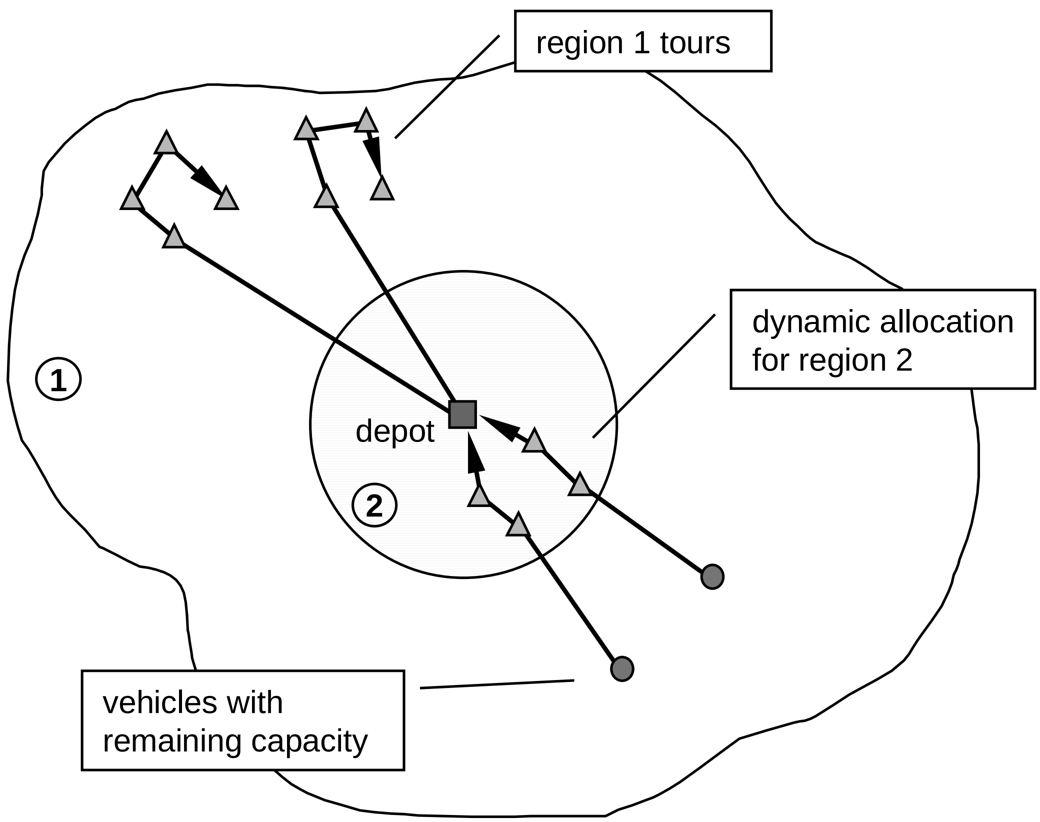


(b)

N-vehicle capacity-sharing strategy

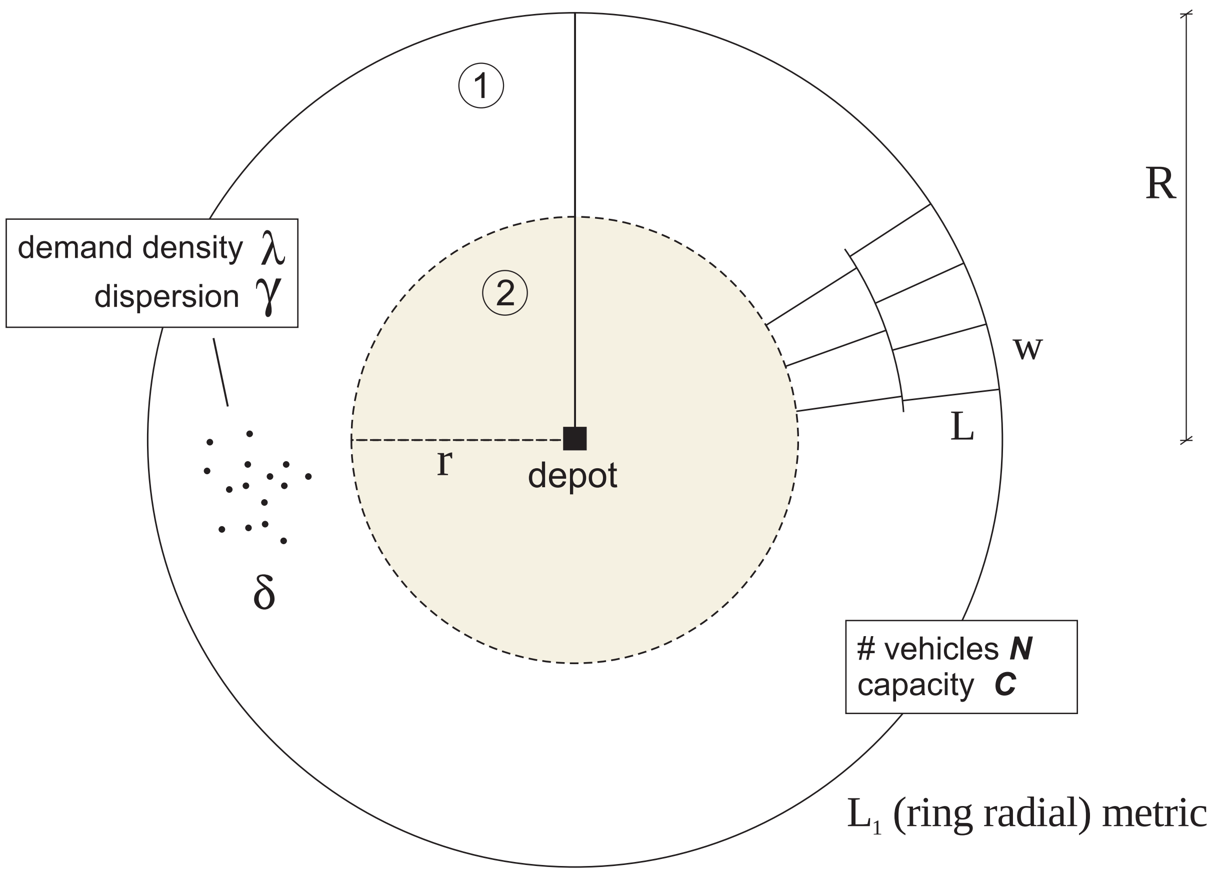
Region 1: Preplanned tours

Region 2: Dynamically-assigned tours



Formulation

Idealized service region



Design decisions

- number of vehicles, N
- region 2 radius, r
- shape of region 1 zones (w, L)
- strategies for:
  - Sweeping region 1 excess demand
  - Allocating vehicles to region 2 customers

Formulation: operating strategy

(1) Line-haul travel to region 1 zone

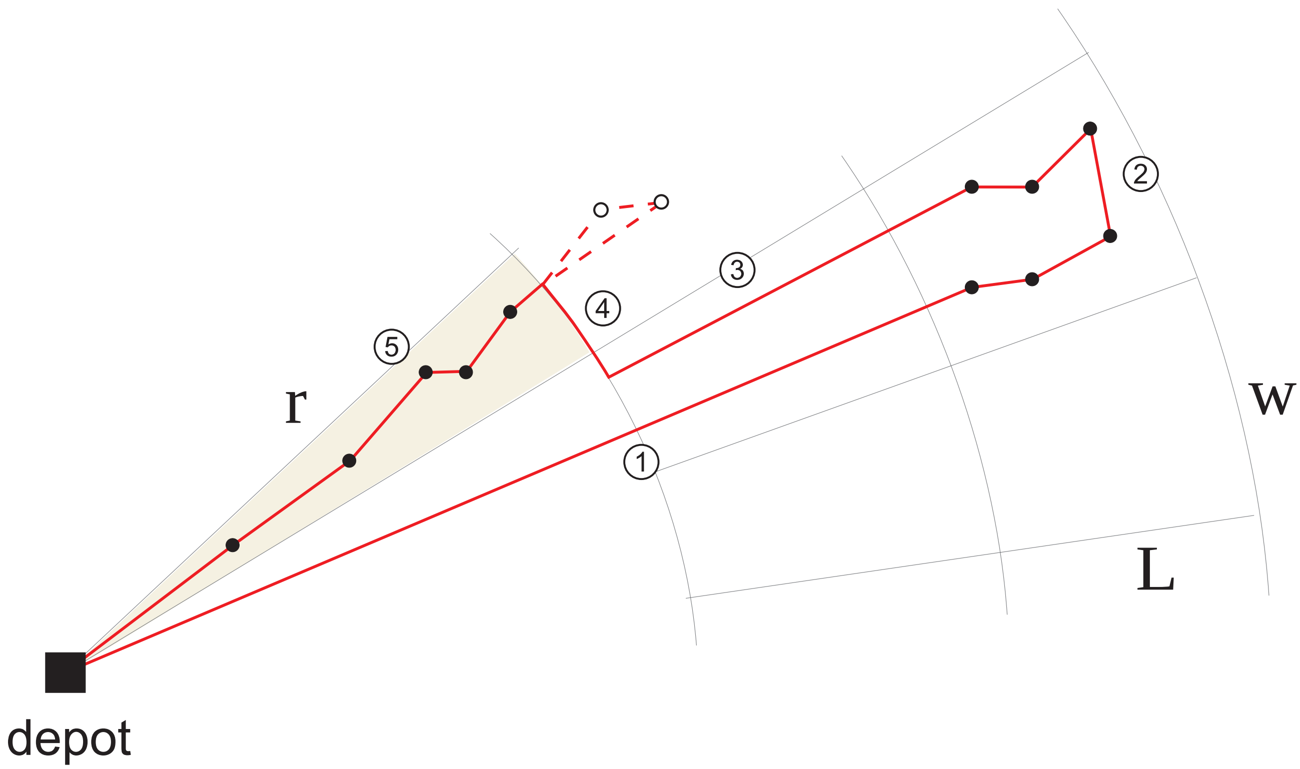
(2) Local travel between region 1 customers

(3) Line-haul return to region 2 perimeter

(4) Reposition along region 2 perimeter

(4b) Serve set of unserved region 1 customers

(5) Serve pie-shaped region 2 zone enroute to depot

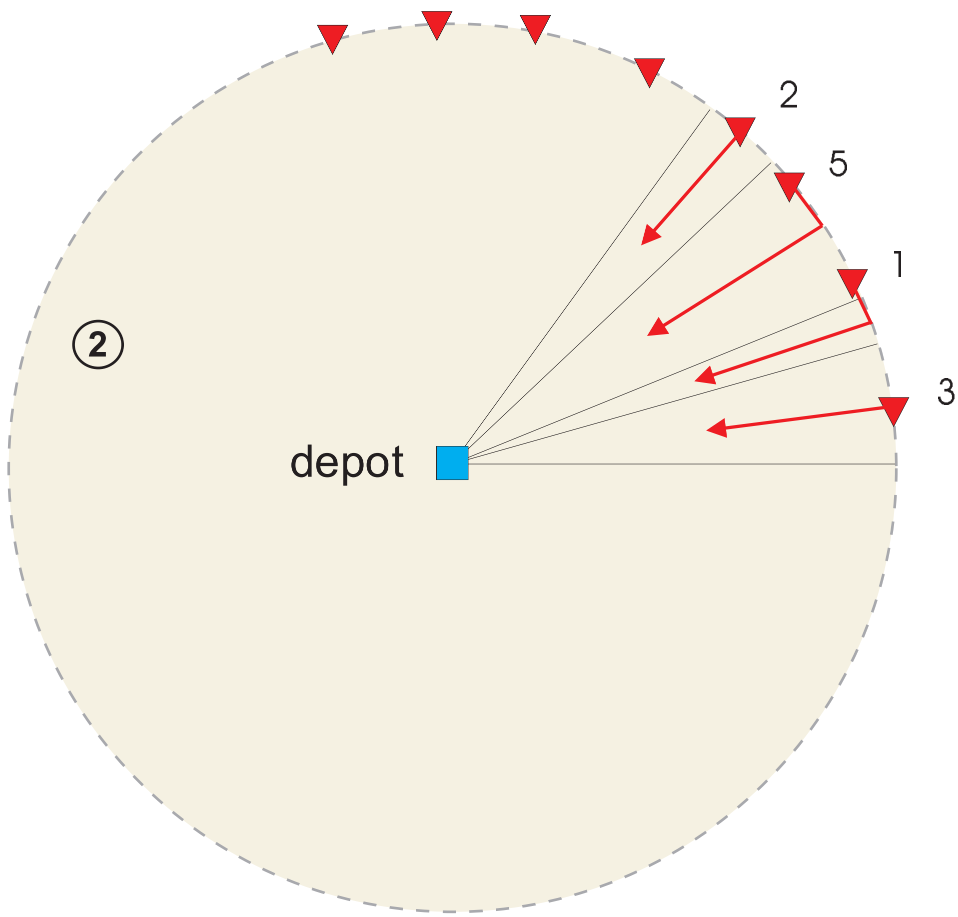


Region 2 overflow customers: depot-based sweeper tours

Region 2 dynamic assignment

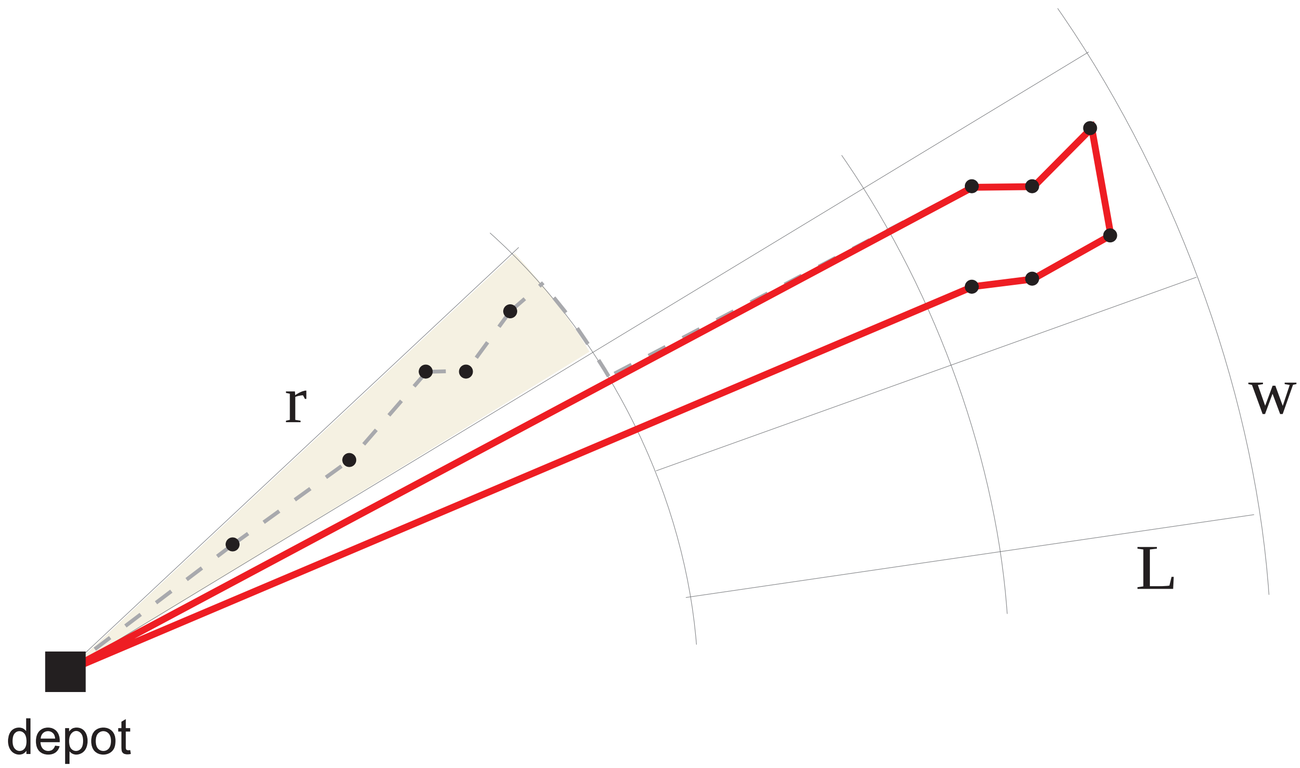
(1) Capacity proportional

(2) Minimal repositioning distance



Modeling

Total line-haul and region 1 local distance



Assumptions

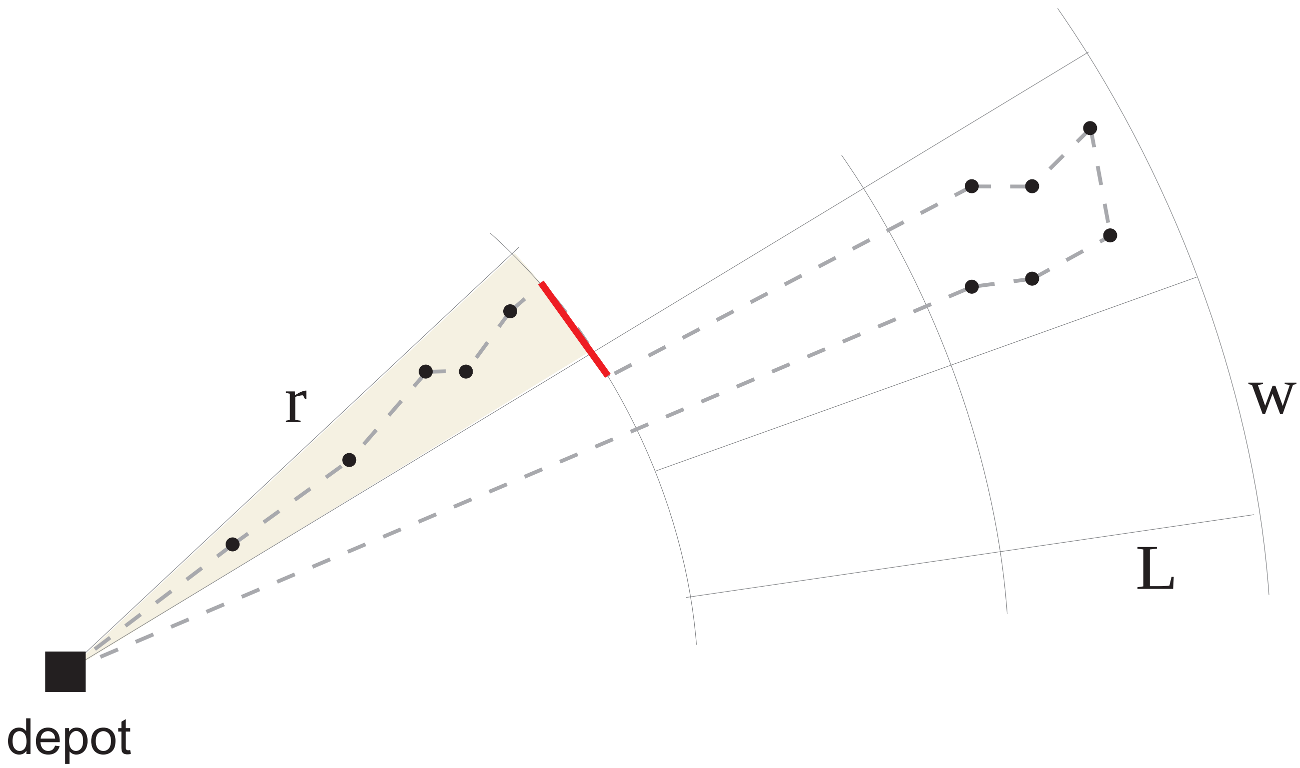
- Equal-sized zones ⇒ $A = \pi\,(R^2 - r^2)/N$
- Near-optimal dimensions (Daganzo (1984))

Expected distance

$$2N\left(\frac{2(R^3 - r^3)}{3(R^2 - r^2)}\right) + k_f \delta^{1/2} \pi\,(R^2 - r^2)$$

Modeling

Repositioning distance



Assumptions

- region 1 tour demand ~ Normal R.V.
- redistribute remaining capacity uniformly

Expected distance

$$\Phi\left(\frac{\lambda A - C}{(\gamma\lambda A)^{1/2}}\right) N(2\pi r)\sqrt{\frac{\pi\gamma\lambda A}{32N(C-\lambda A)^2}}$$

Modeling: repositioning distance

Cumulative excess capacity: diffusion process

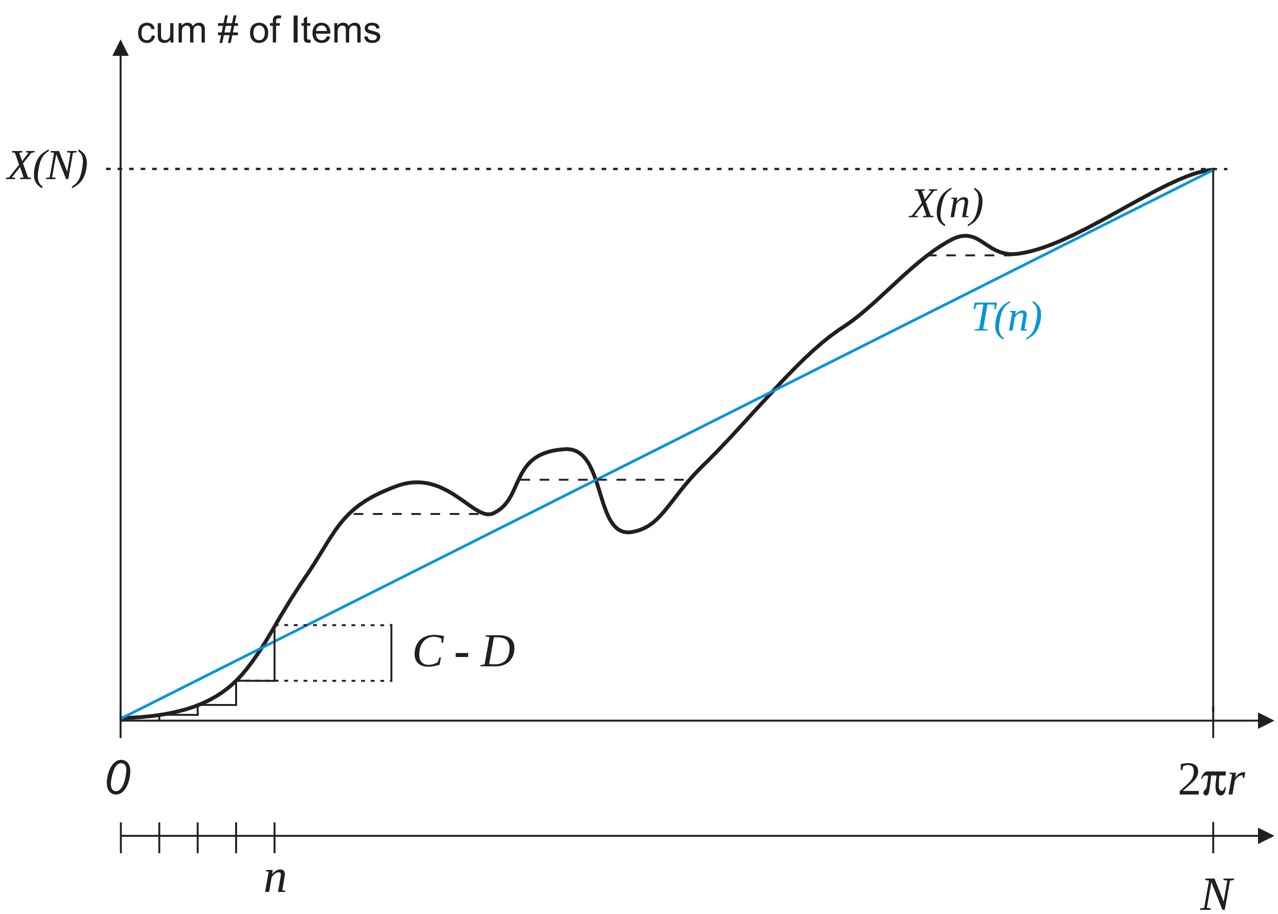
- $X(n) = (C - D)n$
- $X(n) \sim \eta(\,(C - \lambda A)n,\ \gamma\lambda A n)$ by CLT

Target curve, given $X(N)$

- $T(n) = (n/N)\,X(N)$

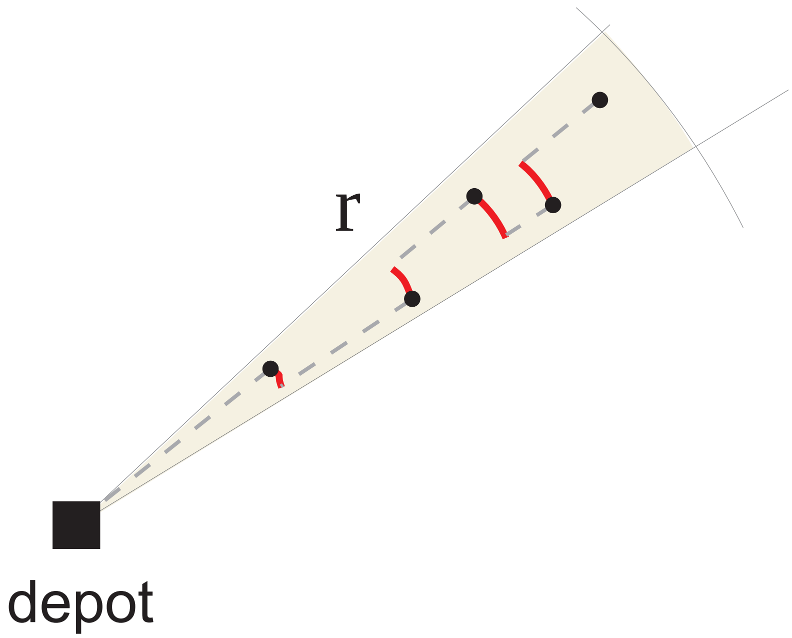
Expected reposition distance per vehicle

- E[area between curves]/E[$X(N)$]



Modeling

Region 2 local distance



Assumptions

- width of vehicle pie zone proportional to capacity
- upper bound on remaining capacity variance used

Expected distance

$$N\left(\frac{\delta r^3}{9}\right)\left(\frac{2\pi}{N}\right)^2$$

Modeling

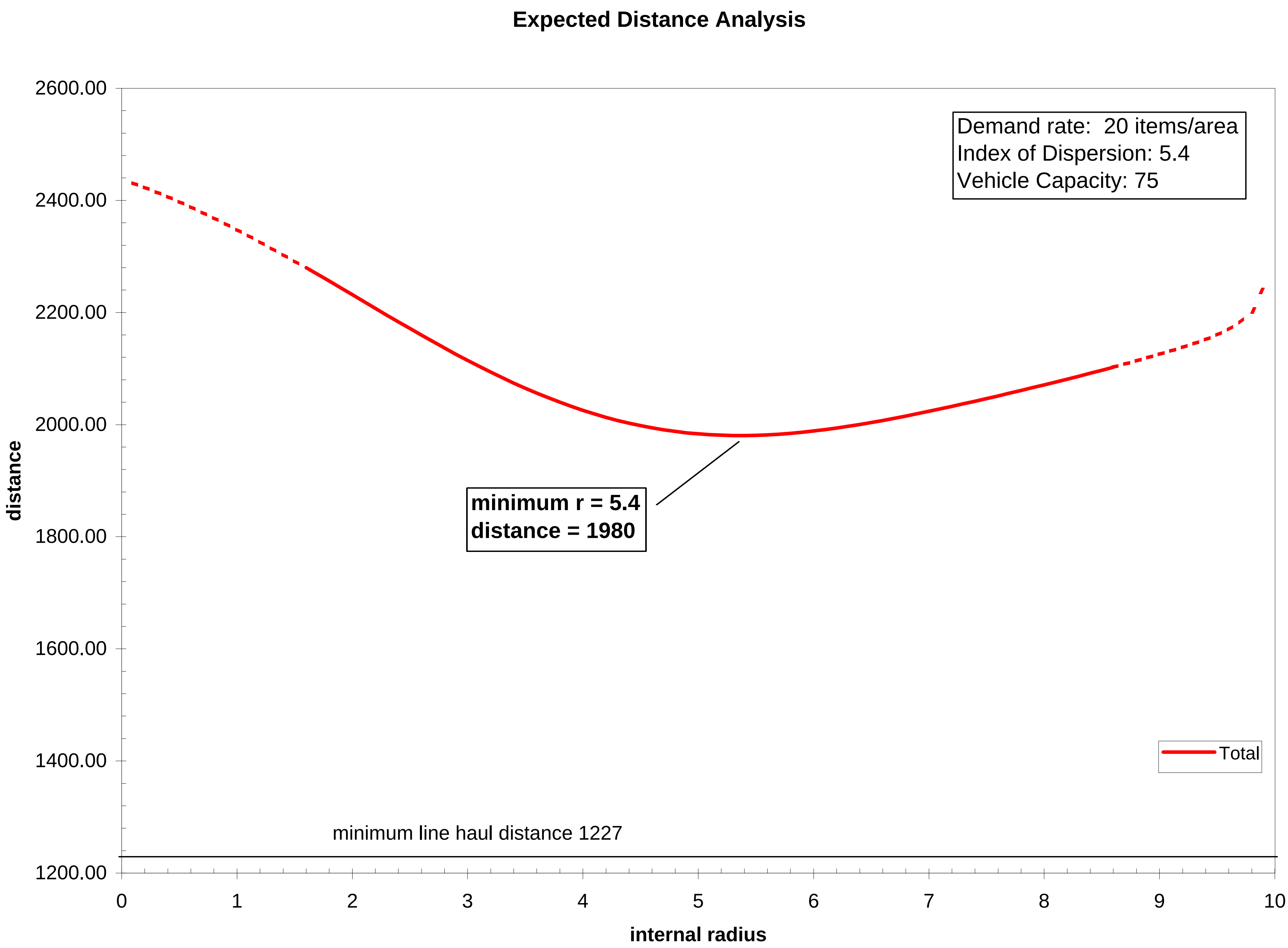
Region 1 overflow service distance

- assignment distance: included in repositioning
- lateral distance: included in region 1 local distance

$\Rightarrow$ model expected radial distance

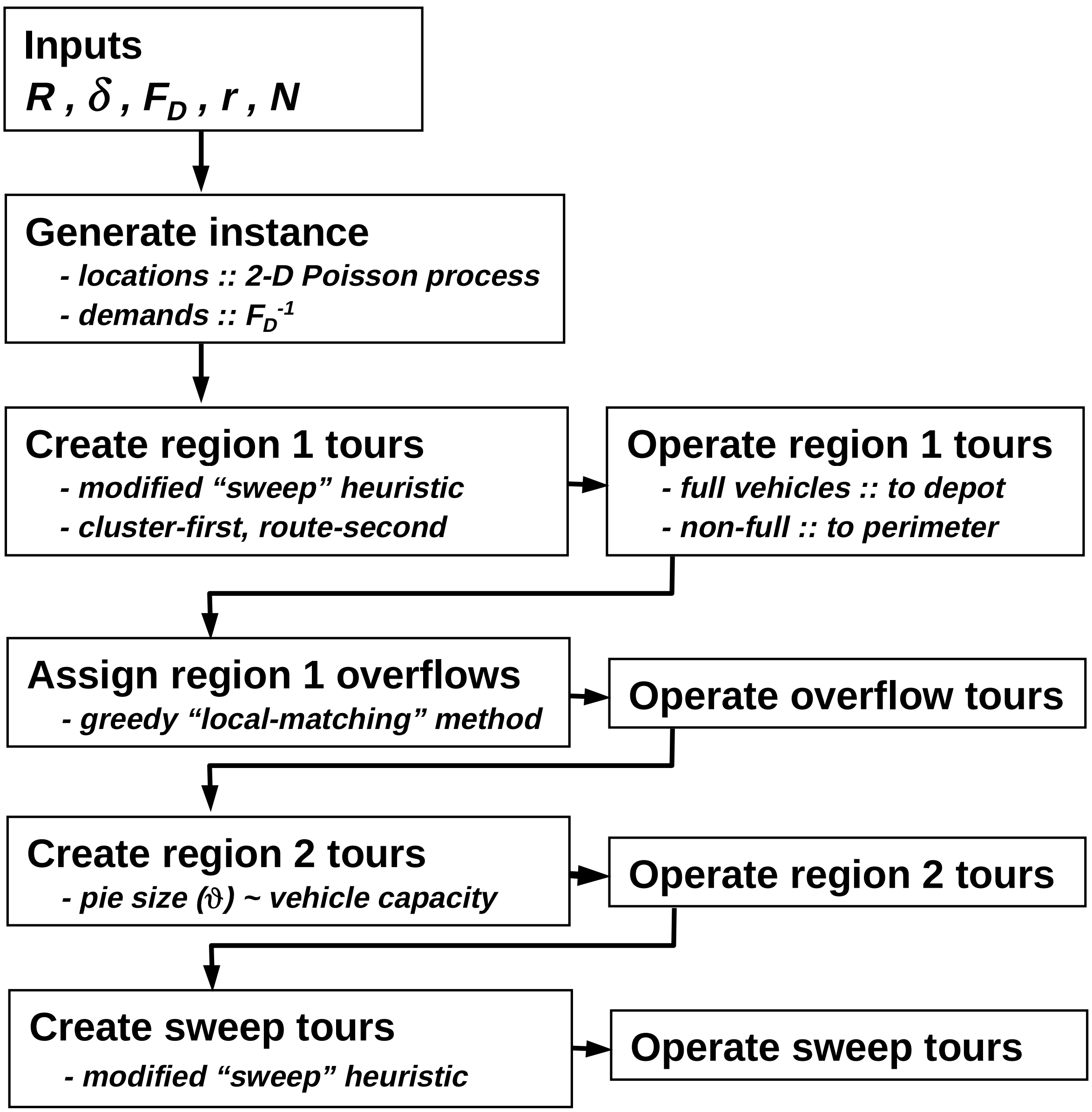
$$2N\left(\frac{2(R^3-r^3)}{3(R^2-r^2)}-r\right)\Phi\left(\frac{C-\lambda A}{(\gamma\lambda A)^{1/2}}\right)G\left(\frac{C-\lambda A}{(\gamma\lambda A)^{1/2}}\right)$$

- ***G*** : expected number of vehicles with remaining capacity needed to serve an overflow zone

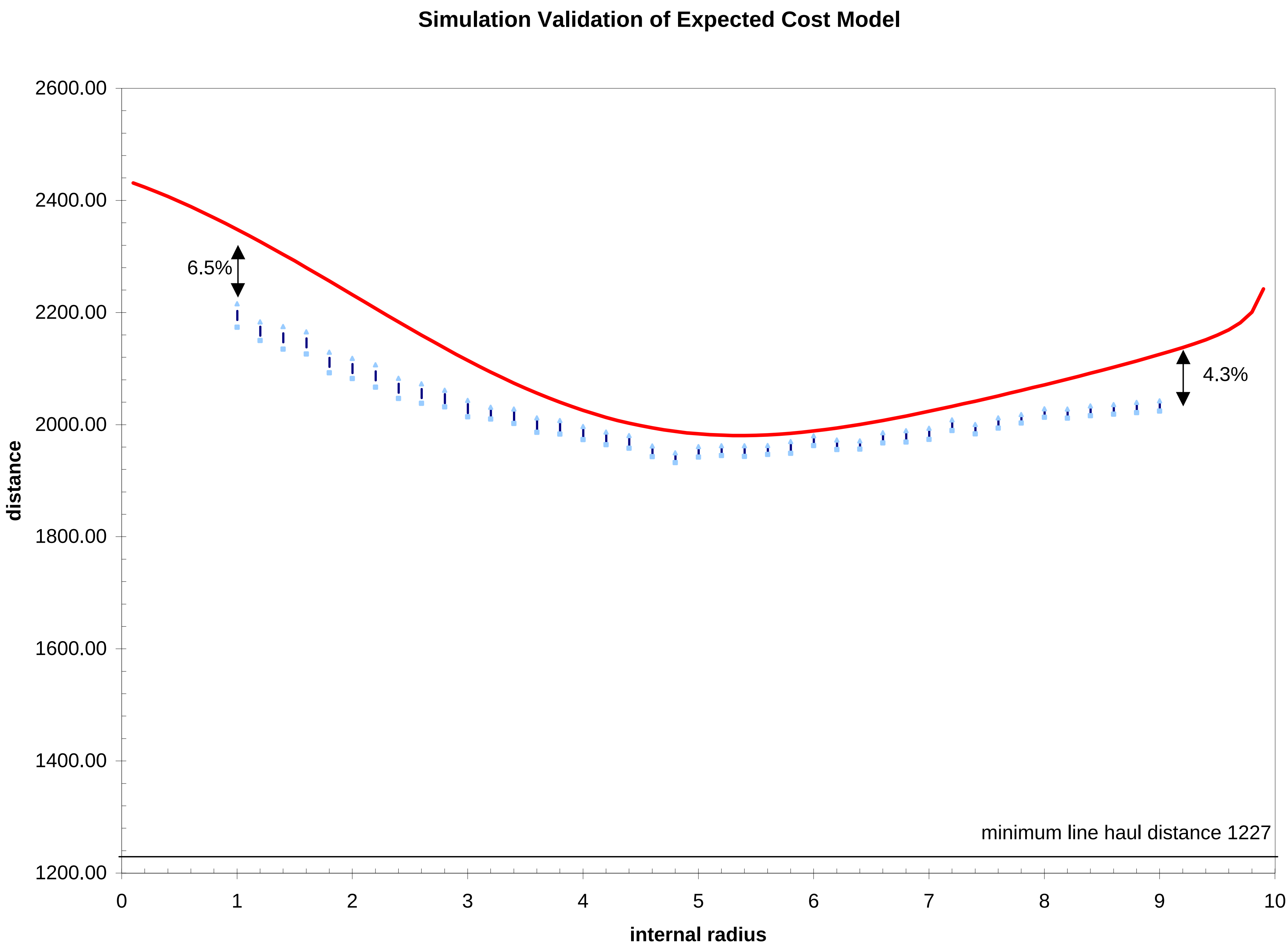


Expected Distance Analysis
Demand rate: 20 items/area
Index of Dispersion: 5.4
Vehicle Capacity: 75
minimum r = 5.4
distance = 1980
Total
minimum line haul distance 1227
distance
internal radius
2600.00
2400.00
2200.00
2000.00
1800.00
1600.00
1400.00
1200.00
0
1
2
3
4
5
6
7
8
9
10

Simulation



Inputs
R , δ , F_D , r , N
Generate instance
- locations :: 2-D Poisson process
- demands :: F_D^-1
Create region 1 tours
- modified "sweep" heuristic
- cluster-first, route-second
Operate region 1 tours
- full vehicles :: to depot
- non-full :: to perimeter
Assign region 1 overflows
- greedy "local-matching" method
Operate overflow tours
Create region 2 tours
- pie size (ϑ) ~ vehicle capacity
Operate region 2 tours
Create sweep tours
- modified "sweep" heuristic
Operate sweep tours


Simulation Validation of Expected Cost Model
2600.00
2400.00
2200.00
2000.00
1800.00
1600.00
1400.00
1200.00
distance
6.5%
4.3%
minimum line haul distance 1227
0
1
2
3
4
5
6
7
8
9
10
internal radius

Comparisons: an example

N-vehicle coordinated strategy (radius 5.4)

- predicted OC: 1980; simulated OC: 1953
- # vehicles: 92
- # customers missed on first tour: 1.3%

Uncoordinated single-zone strategy *lower bound*

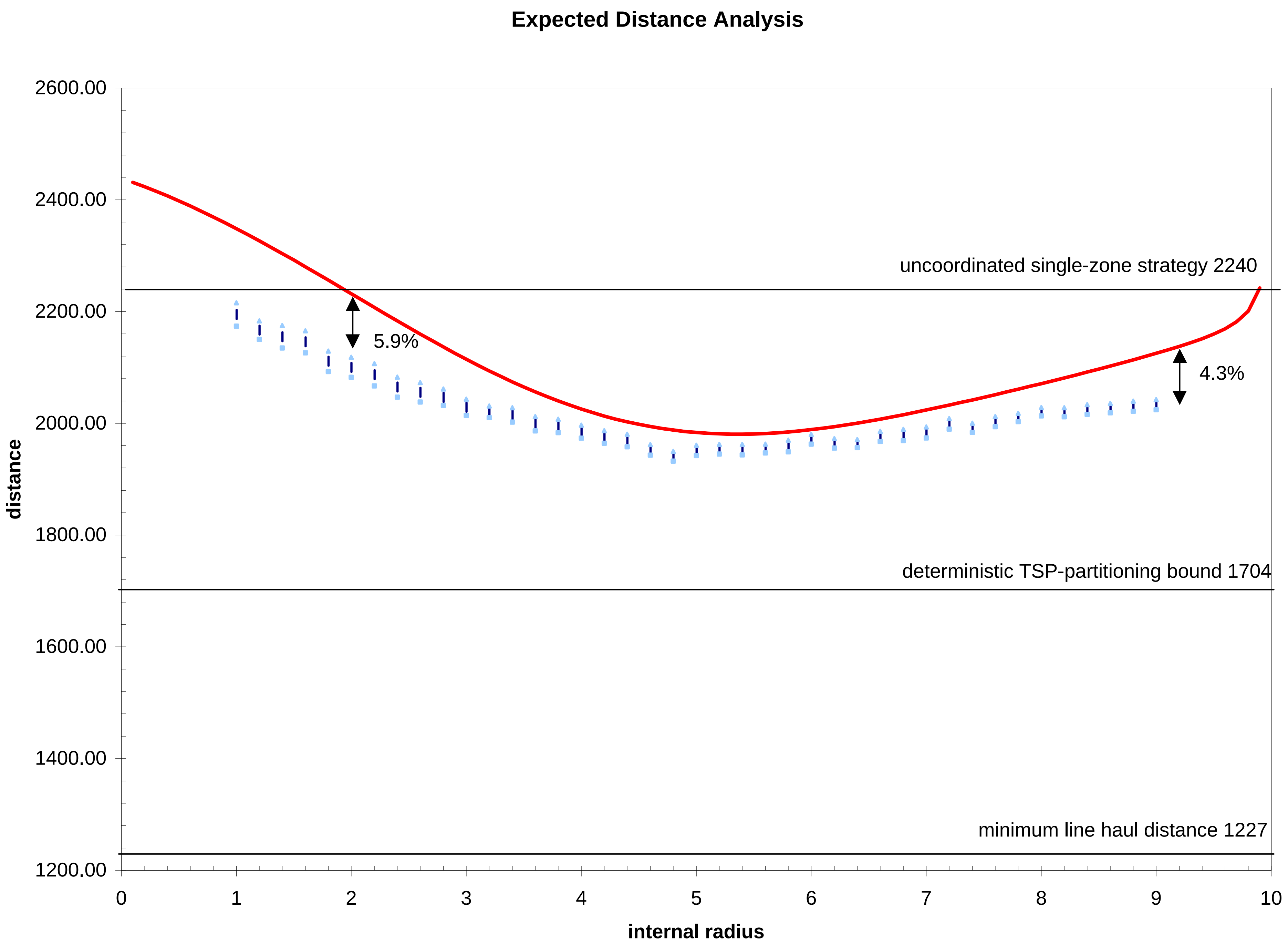
- predicted OC: 2240
- # vehicles: 120
- # customers missed on first tour: 1.3%

Savings

~ 25% fewer vehicles, 7% less distance

Comparison with deterministic bound

- TSP-tour partitioning bound OC: 1704
- Cost of uncertainty
  - *N*-vehicle strategy: 16%
  - uncoordinated strategy: 31%



Expected Distance Analysis
2600.00
2400.00
2200.00
2000.00
1800.00
1600.00
1400.00
1200.00
distance
uncoordinated single-zone strategy 2240
5.9%
4.3%
deterministic TSP-partitioning bound 1704
minimum line haul distance 1227
0
1
2
3
4
5
6
7
8
9
10
internal radius

Conclusions

Assessment

- New coordinated strategies for VRP_{SC}(UD)
- Large-scale approximations
- Preliminary proof-of-concept and validation
- Results
    - strategies improve status quo
    - large-scale approximation methods promising

Extensions

- Coordinated strategies for time-constrained vehicle routing
    - deadline problem
    - application: overnight package delivery collections
- Improved control