IMM - DTU

Seminar course: Diffusions and SDE's
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UHT/uht

Exercise 3: Numerical stochastic integration

Question 1: Write a program that takes as input to functions f and g of time, given as numeric tables over the time interval [0,T], and computes

$$\int_0^T f(t)\ dg(t)$$

in two ways: By evaluating f at the left end point of each sub-interval, and at the right end point. Verify that

$$\int_0^{2\pi} \cos t\ d\sin t$$

has the same value, whether we use the one end point or the other.

Question 2: Write a program that simulates N realizations of Brownian motion over the time interval $[0, T]$, dividing the interval into M equal-length time intervals. Verify the program by plotting a few sample paths, and by checking the statistics of the end points.

Optional: Using the simulation, compare the distribution of

$$\sup\{B_s : 0 \leq s \leq T\}$$

with that of $|B_T|$.

Question 3: Simulate a number of realizations of

$$\int_0^T B_s^2\ dB_s \text{ and } \int_0^T B_s\ ds$$

Verify the result of exercise 3.2. Guess the corresponding result for using the right end point.

Question 4: For the same integral

$$\int_0^T B_s^2\ dB_s$$

verify the Itô isometry, for example by plotting the empirical mean square of integral against the empirical mean

$$\int_0^T f_s^2\ ds$$

where $f_s = B_s^2$.