

A photograph of a room, likely a waiting area or a small office. In the center, a woman is sitting on a chair with a young child on her lap. To the left, there is a whiteboard mounted on the wall, and above it, a string of colorful flags. Below the whiteboard, there is a blue cabinet with a silver handle. On the right side, a person in a red shirt is partially visible, leaning forward. The floor is dark grey, and there is a red mat in the foreground.

Click link below to view video

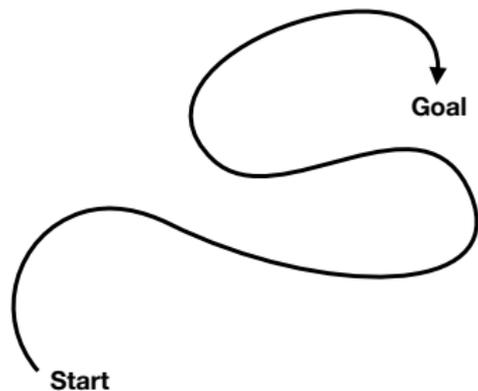
http://www2.compute.dtu.dk/~tobo/children_cabinet_cropped.mp4

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Epistemic planning =
automated *planning* + Theory of Mind reasoning

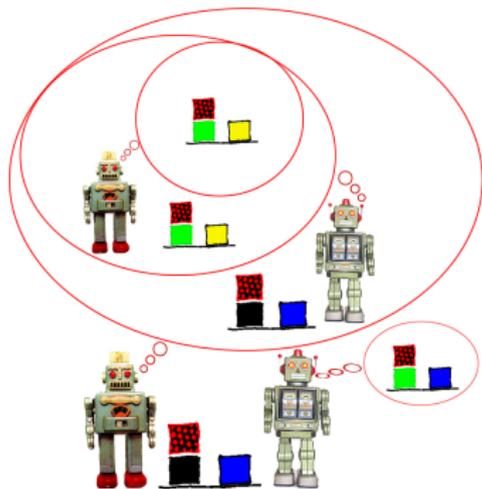
Aim: To compute plans that can take the mental states of other agents into account.

Essentially: (Decentralised) **multi-agent planning** in environments with (potentially higher-order) **information asymmetry**.



Automated planning

+



Logical reasoning about the
mental states of other agents

Syntactic vs semantic, explicit vs implicit

When moving from standard propositional states to states including a Theory of Mind, there are two distinct paths one might take.

- **Syntactic approach:** States are (sets of) formulas (e.g. formulas of S5 epistemic logic)
- **Semantic approach:** States are semantic models (e.g. epistemic models = Kripke models).

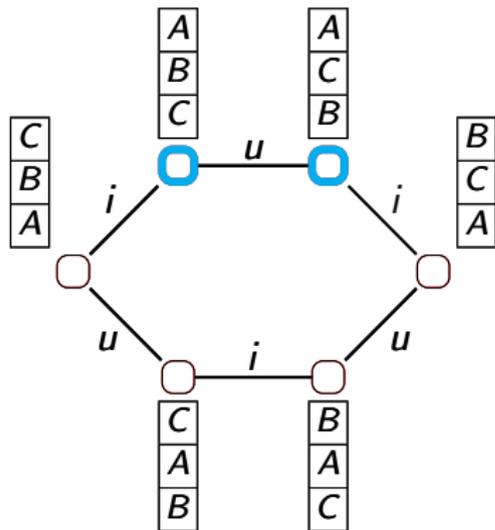
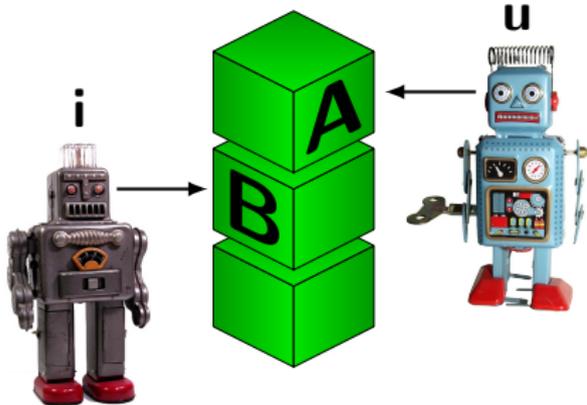
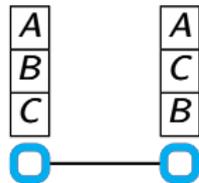
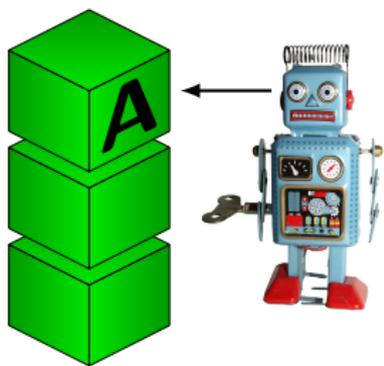
Note: For propositional planning under full observability, the approaches are trivially equivalent.

Furthermore, for the semantic approach, there is a choice between:

- **Explicit approach:** Full state space is assumed given, and solution concept is defined directly in terms of this. E.g. logics like ATEL and CSL. [van der Hoek and Wooldridge, 2002, Jamroga and Aagotnes, 2007]
- **Implicit approach:** State space is induced by initial state and action library (as in classical STRIPS/PDDL planning).

DEL-based epistemic planning is *implicit* and *semantic*.

[Bolander and Andersen, 2011]



Epistemic states: Multi-pointed epistemic models of multi-agent S5. Nodes are worlds. **Designated worlds:** ○ (those considered possible by planning agent).

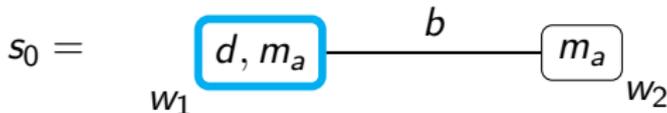
The coordinated attack problem in dynamic epistemic logic (DEL)

Two generals (agents), a and b . They want to coordinate an attack, and only win if they attack simultaneously.

d : “general a will attack at dawn”.

m_i : the messenger is at general i (for $i = a, b$).

Initial **epistemic state**:

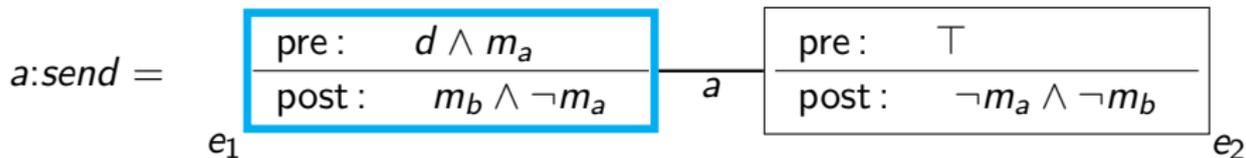


Nodes are **worlds**, edges are **indistinguishability edges** (reflexive loops not shown).

The coordinated attack problem in dynamic epistemic logic (DEL)

Recall: d means “ a attacks at dawn”; m_i means messenger is at general i .

Available **epistemic actions** (aka **action models** aka **event models**):

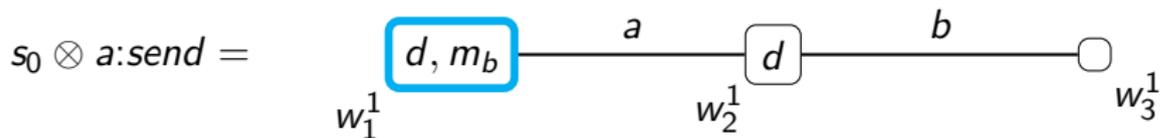
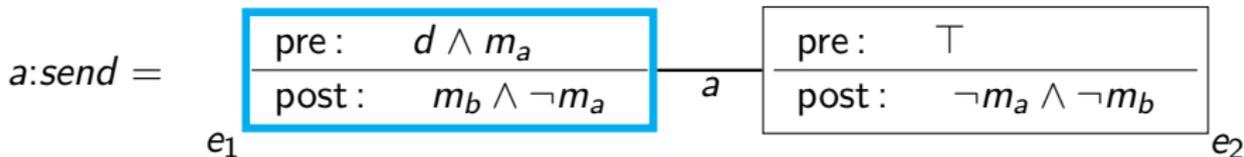
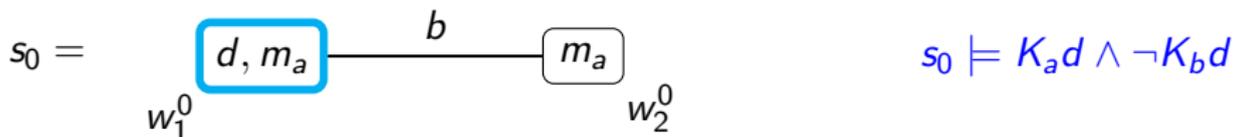


And symmetrically an epistemic action $b:send$. We read $i:\alpha$ as “agent i does α ”.

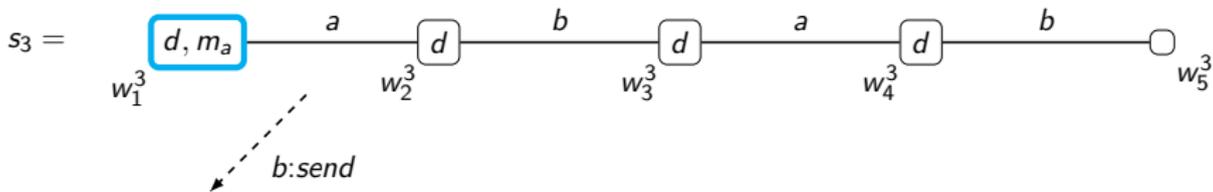
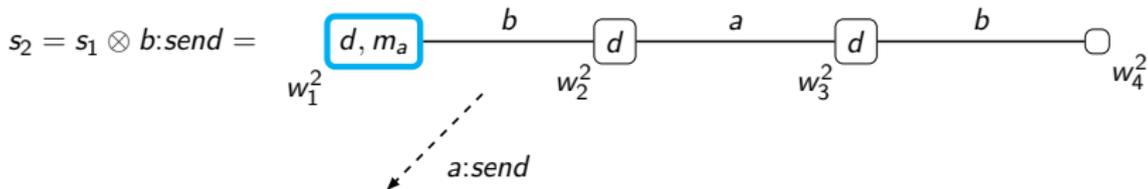
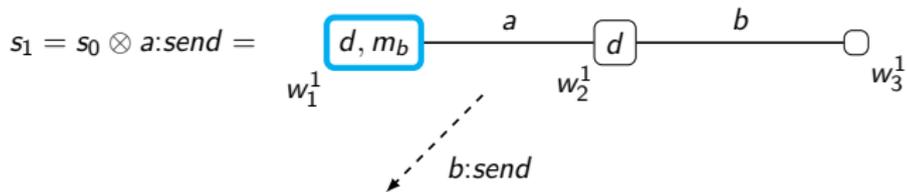
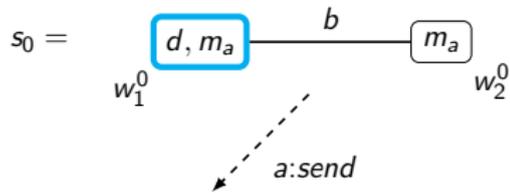
Nodes are **events**, and each event has a **precondition** and a **postcondition** (effect). The precondition is an epistemic formula and the postcondition is a conjunction of literals.

[Baltag et al., 1998, van Ditmarsch and Kooi, 2008]

The product update in dynamic epistemic logic



$s_0 \otimes a:send \models K_a d \wedge K_b d \wedge \neg K_a K_b d$



Epistemic planning tasks

Definition. An **epistemic planning task** (or simply a **planning task**) $T = (s_0, A, \gamma)$ consists of an epistemic state s_0 called the **initial state**; a finite set of epistemic actions A ; and a **goal formula** γ of the epistemic language.

Definition. A (sequential) **solution** to a planning task $T = (s_0, A, \gamma)$ is a sequence of actions $\alpha_1, \alpha_2, \dots, \alpha_n$ from A such that for all $1 \leq i \leq n$, α_i is applicable in $s_0 \otimes \alpha_1 \otimes \dots \otimes \alpha_{i-1}$ and

$$s_0 \otimes \alpha_1 \otimes \alpha_2 \otimes \dots \otimes \alpha_n \models \gamma.$$

Example. Let s_0 be the initial state of the coordinated attack problem. Let $A = \{a:send, b:send\}$. Then the following are planning tasks:

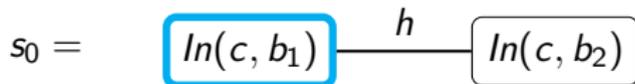
1. $T = (s_0, A, Cd)$, where C denotes common knowledge. It has no solution.
2. $T = (s_0, A, E^n d)$, where E denotes “everybody knows” and $n \geq 1$. It has a solution of length n .

[Bolander et al., 2020]

Epistemic planning example: Get the cube

- **Objects:** $\mathcal{O} = \{b_1, b_2, c\}$, two boxes b_1 and b_2 , and a cube c .
- **Agents:** $\mathcal{A} = \{h, a\}$, a human h and a robot r . The robot is the planning agent.
- **Atomic propositions:** $In(x, y)$ means x is in y , where $x, y \in \mathcal{O} \cup \mathcal{A}$ (when $y \in \mathcal{A}$, it means y is holding x).

Initial epistemic state:



The goal is for the human to hold the red cube, $In(r, h)$.

Actions specialised for the case of $\mathcal{O} = \{b_1, b_2, c\}$.

Agent i (semi-privately) **peeks** into box x :

$$i:\text{peek}(x) = \boxed{\text{pre: } \text{In}(c, x)} \xrightarrow{\mathcal{A} - \{i\}} \boxed{\text{pre: } \neg\text{In}(c, x)}$$

Agent i (publicly) **picks up** object x from y :

$$i:\text{pickup}(x, y) = \boxed{\frac{\text{pre: } \text{In}(x, y)}{\text{post: } \text{In}(x, i) \wedge \neg\text{In}(x, y)}}$$

Agent i (publicly) **puts** object x in y :

$$i:\text{putdown}(x, y) = \boxed{\frac{\text{pre: } \text{In}(x, i)}{\text{post: } \text{In}(x, y) \wedge \neg\text{In}(x, i)}}$$

Agent i (publicly) **announces** that formula φ is true:

$$i:\text{ann}(\varphi) = \boxed{\text{pre: } \varphi}$$

Get the cube: Planning task and solutions

The planning task T has the actions of the previous slide and initial state s_0 and goal γ given by:

$$s_0 = \boxed{\text{In}(c, b_1)} \xrightarrow{h} \text{In}(c, b_2) \qquad \gamma = \text{In}(r, h)$$

Solution to T , by robot R :

$$\begin{array}{l} s_0 = \boxed{\text{In}(c, b_1)} \xrightarrow{h} \text{In}(c, b_2) \\ \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \downarrow r:\text{pickup}(c, b_1) \\ s_1 = s_0 \otimes r:\text{pickup}(c, b_1) = \boxed{\text{In}(c, r)} \\ \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \downarrow r:\text{putdown}(c, h) \\ s_2 = s_1 \otimes r:\text{putdown}(c, h) = \boxed{\text{In}(c, h)} \end{array}$$

Applicability, perspective shifts, implicit coordination

Seemingly simpler solution: $h:pickup(c, b_1)$. But intuitively, this shouldn't work, since the human doesn't know the cube is in box 1...

Applicability: An action α is **applicable** in a state s if for each designated world w of s there is a designated event e of α with $w \models pre(e)$.

Perspective shift: The **perspective shift** of state s to agent i , denoted s^i , is achieved by closing under the indistinguishability relation of i . We call s^i the **perspective** of agent i on state s .

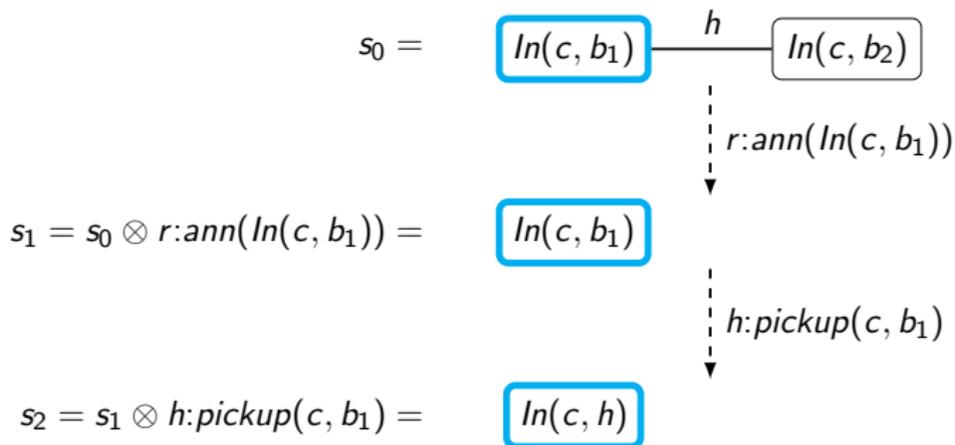


Example. $h:pickup(c, b_1)$ is not applicable in s_0 from h 's perspective.

Implicitly coordinated solution to planning task: Each action has to be applicable from the perspective of the acting agent; and the product update $s \otimes i:\alpha$ is replaced by $s^i \otimes i:\alpha$.

Get the cube: Implicit coordination

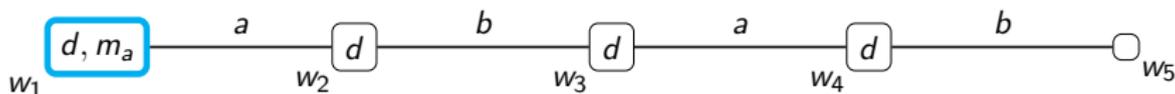
Joint solution to T , by robot R , implicitly coordinated:



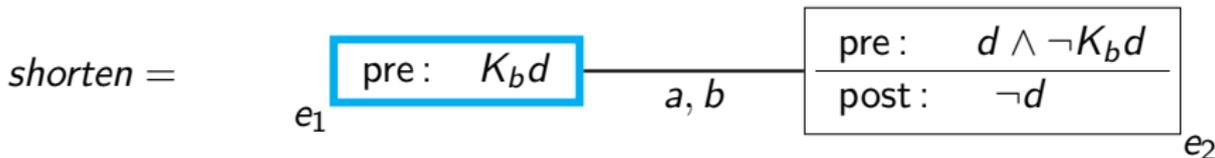
If purely epistemic actions (announcements) have a lower cost than ontic actions (moving things around), the solution above is the only optimal one.

Undecidability: lengthening and shortening chains

Consider a chain produced by the coordinated attack problem:



Using preconditions of modal depth 1 we can shorten the chain by 1:



We can now both lengthen (by *send*) and shorten chains (by *shorten*), and this allows us to encode two-counter machines \Rightarrow undecidability of the plan existence problem!

Undecidability holds even with preconditions of modal depth 1, and for purely epistemic planning (no postconditions) even for modal depth 2.

[Bolander and Andersen, 2011, Charrier et al., 2016, Bolander et al., 2020]

Some of the current challenges in epistemic planning

- **Undecidability issues:** open complexity problems.
[Bolander et al., 2020]
- **State size explosion problems:** find compact state representations.
[Charrier and Schwarzentruher, 2017, van Benthem et al., 2018]
- **The belief-revision problem in DEL:** How to recover from false beliefs without an underlying epistemic relation. Relates to the state size explosion problem.
- **Heuristics for epistemic planning:** to reduce all of the above mentioned complexity and scalability issues
- **Languages:** syntactic languages for describing actions.
[Baral et al., 2012, Baral et al., 2013]

This, and much more, is discussed in the “Epistemic Planning” special issue of AIJ currently being finalised.

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