Approximation of linear-quadratic regulators for delay differential equations with distributed time delays

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Abstract: Time delays are ubiquitous in both industrial and natural processes. It takes time to transport content from one reactor to another, animal population dynamics are affected by gestation periods, and the wake of particles moving in a viscous fluid creates an aftereffect (the Basset force). Furthermore, many time delays are not just one single value. Instead, the system contains a large collection or *distribution* of time delays. The flow velocity might vary across the pipe cross-section due to friction with the wall (e.g., Hagen-Poiseuille flow), and for each radius, there will be a different time delay. The gestation period is also not the same for all animals, incubation times in epidemics vary from person to person, and the wake force on particles moving in viscous fluids does not only affect the particle at one single point in time. Consequently, the time delays are *distributed*. Systems with such delays are less studied and the controller design is significantly more complicated than for systems with absolute delays. Therefore, in this work, we propose to approximate the involved distribution (also called a memory function or kernel) in such a way that the resulting approximate delay differential equations can be transformed analytically to a set of ordinary differential equations, for which linear-quadratic regulators can readily be designed. Using numerical examples, we will compare different memory kernel approximations and investigate the importance of their accuracy on the closed-loop performance of the resulting controller.

Keywords: Linear-quadratic regulator, delay differential equations, distributed time delays, ODE approximation, memory function approximation.