Optimal control of flexible consumers' energy demand through dynamic prices

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Objective

Aggregator companies **trade energy and energy flexibility on behalf of their customers**, e.g., power-to-X plants, other energy-intensive industry, or private consumers. In this work, we consider customers who are unwilling to let the aggregator dictate their energy demand, usually due to operation-critical process constraints, and instead, the aggregator will **incentivize** the customers to **shift their demand through dynamic energy prices**. Specifically, we want to determine both the dynamic prices and the amount of energy to supply.

Demand response model

The demand response model is essentially a stochastic **battery model** where the state of charge, x, increases or decreases based on the difference between the supplied and consumed energy, Δz, and the energy capacity, C:

$$\mathrm{d}x(t) = \frac{\Delta z(t)}{C}\,\mathrm{d}t + \sigma_x x(t)(1 - x(t))\,\mathrm{d}w(t)$$

The total energy supplied to the consumer, z, and the difference Δz are given by

$$z(t) = B(t) + \Delta z(t), \qquad \Delta z(t) = \Delta z^{\max}(t)\,\delta(t),$$

where the maximum possible difference is a piecewise linear function of the consumed energy, B:

$$\Delta z^{\max}(t) = \begin{cases} \alpha(1 - B(t)), & \delta(t) \geq 0, \\ \alpha B(t), & \delta(t) < 0 \end{cases}$$

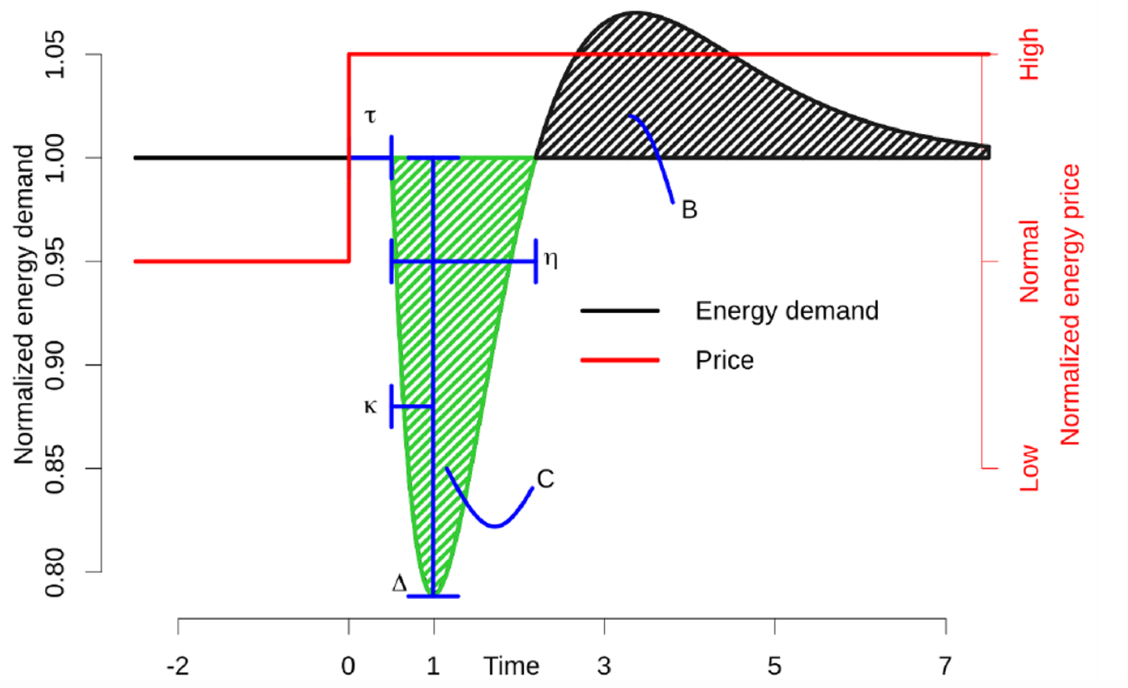
The actual proportion of the maximum possible supplied energy is given by a sigmoid function,

$$\delta(t) = \frac{2}{1 + e^{-k(\varphi(x(t)) + \psi(p(t)))}} - 1,$$

where, for simplicity, we assume linear dependences on the state of charge, x, and the price, p:

$$\varphi(t) = 1 - 2x(t),$$
$$\psi(t) = 1 - 2p(t)$$

Sketch of the demand-response model



Optimal control problem (det.)

We determine the optimal sequence of prices, p_k, and the total amount of energy to supply (and buy), $\bar{z}_k$,

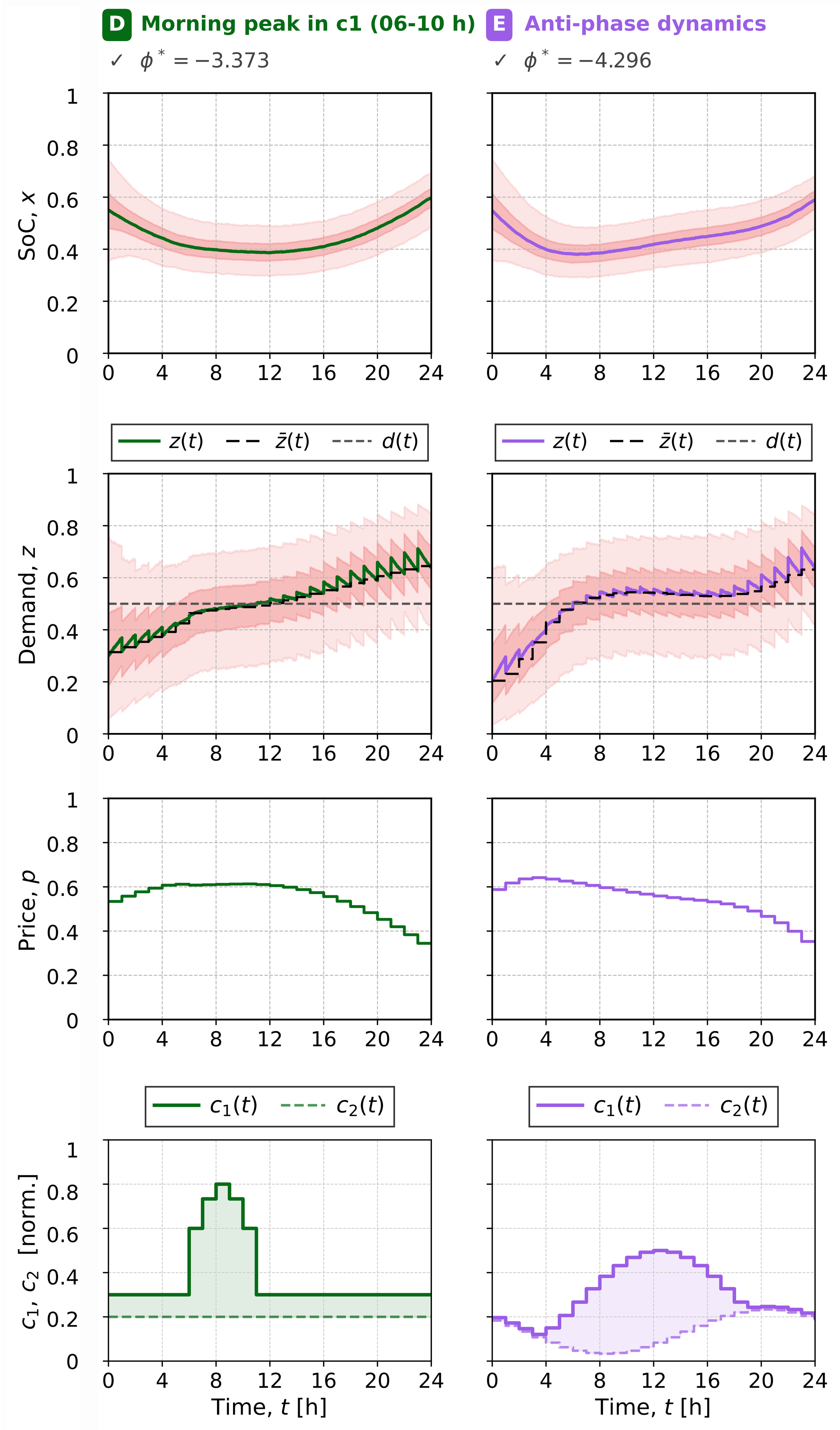
$$\min_{\{p_k, \bar{z}_k\}_{k=0}^{N-1}} \quad \phi = \int_{t_0}^{t_f} \Big[c_1(t)|z(t) - \bar{z}(t)| - p(t)\,z(t) + c_2(t)\,\bar{z}(t) \Big]\,\mathrm{d}t,$$

subject to

$$\begin{aligned}
x(t_0) &= x_0, \\
\dot{x}(t) &= f(x(t), p(t), d(t)), \quad t \in [t_0, t_f], \\
z(t) &= h(x(t), p(t), d(t)), \quad t \in [t_0, t_f], \\
p(t) &= p_k, \quad t \in [t_k, t_{k+1}[, \quad k = 0, \dots, N-1, \\
\bar{z}(t) &= \bar{z}_k, \quad t \in [t_k, t_{k+1}[, \quad k = 0, \dots, N-1, \\
p_{\min} &\leq p_k \leq p_{\max}, \quad k = 0, \dots, N-1, \\
\bar{z}_{\min} &\leq \bar{z}_k \leq \bar{z}_{\max}, \quad k = 0, \dots, N-1, \\
\frac{1}{N}\sum_{k=0}^{N-1} p_k &\leq \bar{p}_{\max}
\end{aligned}$$

Deterministic optimal control

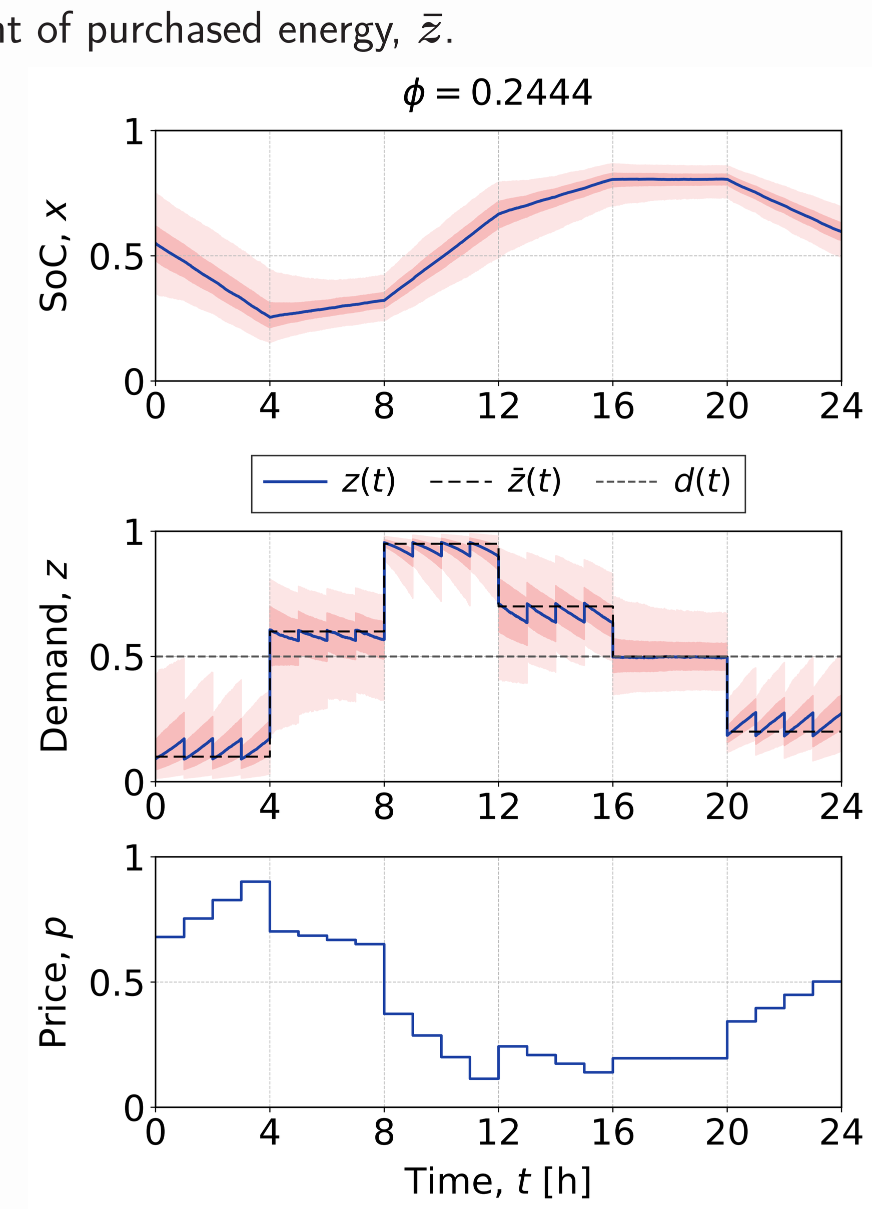
We investigate the solution for different combinations of the intra- and interday energy prices (for the aggregator), c_1 and c_2. Essentially, when c_1 and c_2 are further apart, the distance between the supplied energy, z, and the purchased energy, $\bar{z}$, becomes smaller. In all cases, it's optimal for the aggregator to sell as much energy as possible to the customers.



Although the optimal control problem involves a deterministic demand response model, we test the solution using Monte Carlo simulation of the stochastic demand response model.

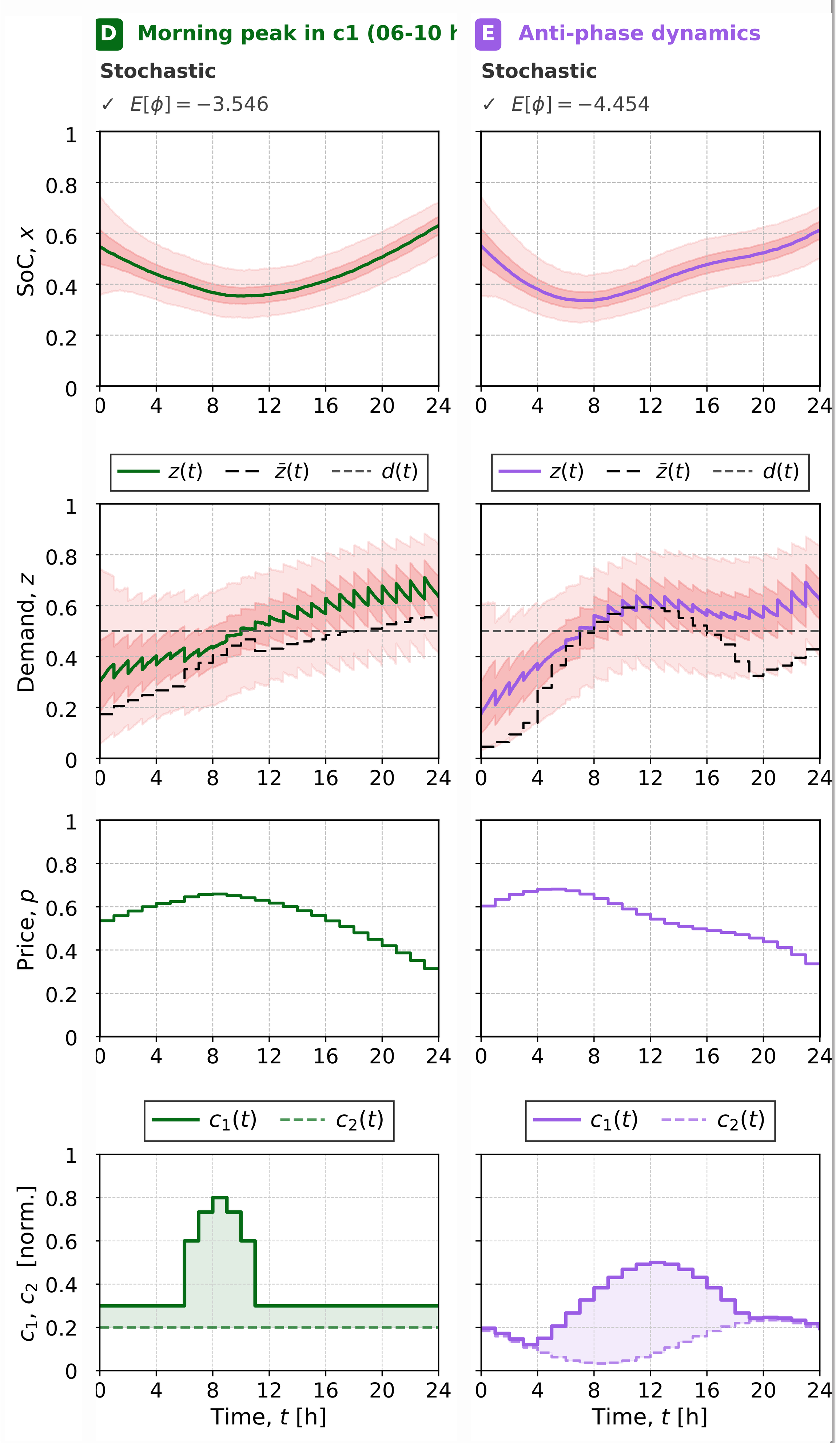
Previous work (only price)

Previous work only determined the price signal for a given amount of purchased energy, $\bar{z}$.



Stochastic optimal control

We use the stochastic demand response model instead and replace the objective function by its expected value (computed through a coarse Monte Carlo simulation). The main difference is that the distribution of z is shifted upwards relative to $\bar{z}$ because the risk then is to sell more energy to the customers.



What if we bid into markets?

Currently, we assume that the aggregator can buy any amount of energy they'd like, $\bar{z}$. In order to **place optimal bids**, the present work will be extended:

- Model of a bid of size $\hat{z}$ at price c_2

  $$\Pr(\bar{z} = \hat{z}) = \gamma(\hat{z}, c_2),$$
  $$\Pr(\bar{z} = 0) = 1 - \gamma(\hat{z}, c_2)$$

  The probability γ is assumed to be known, and it is a monotonically decreasing function of the bid size, $\hat{z}$, and a monotonically increasing function of the bid price, c_2.
- The trajectory of probable outcomes will *branch* in each control interval
- Should we consider other statistics than the mean in the objective function, e.g., mean-variance, value-at-risk (VaR), or conditional value-at-risk (CVaR)?

References

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- R.T. Rockafellar, S. Uryasev, 2000. Optimization of Conditional Value-at-risk. Journal of Risk 2(3), pp. 21–41.