A bilevel approach to model-based control of supply temperature and velocity in district heating grids

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Objective

We combine **model predictive control** with **real-time optimization** to exploit the linearity of district heating models.

Model

The model is based on an **energy balance** for the water in the pipe in the district heating grid, and it accounts for **convection** and **heat loss** to the surroundings:

$$\partial_t u(t,z) = -\partial_z N_h(t,z) + q(t,z), \quad z \in [z_0, z_f]$$

Enthalpy flux

$$N_h(t,z) = H(T(t,z), P, N(t)), \quad N(t) = \rho v(t)$$

Heat loss

$$q(t,z) = -k(T(t,z) - T_a(t))$$

Boundary conditions

Boundary condition at **heating plant**:

$$T(t, z_0) = T_s(t)$$

T_s is a manipulated input variable. The flow velocity at the heating plant is the other manipulated input.

Boundary condition at **splitting pipes**:

$$v_i(t) = \frac{A_1}{A_2 + A_3} v_1(t), \qquad i = 1, 2$$

The temperatures are unchanged: $T_i(t, z_{i,0}) = T_1(t, z_{1,f})$

Boundary condition at **merging pipes**:

$$v_3(t) = \frac{A_1}{A_3} v_1(t) + \frac{A_2}{A_3} v_2(t),$$
$$T_3(t, z_{3,0}) = \frac{A_1 v_1(t)}{A_3 v_3(t)} T_1(t, z_{1,f}) + \frac{A_2 v_2(t)}{A_3 v_3(t)} T_2(t, z_{2,f})$$

Spatial discretization

We use a **finite volume** method to discretize the energy balance spatially:

$$\dot{T}_i(t) = -\frac{v(t)}{\Delta z}(T_i(t) - T_{i-1}(t)) - \frac{k}{\rho c_P}(T_i(t) - T_a(t))$$

The ambient temperature, T_a, is a disturbance variable

The discretized model is in the form of a **linear parameter-varying** (LPV) system:

$$\dot{x}(t) = A_c(t) x(t) + B_c(t) u(t) + E_c(t) d(t),$$
$$y(t_k) = C_c(t) x(t_k) + e(t_k)$$

Disturbance model (controller)

The controller uses a model of the disturbance variables:

$$d(t) = \hat{d}(t) + s(t)$$

The *forecasted disturbance*, $\hat{d}$, is assumed to be given, and s is a random variable that represents uncertainty. It is modeled by the stochastic differential equation

$$\mathrm{d}s(t) = -\kappa s(t)\mathrm{d}t + \sigma_s \mathrm{d}w(t)$$

Augmented system:

$$\mathrm{d}\begin{bmatrix} x(t) \\ s(t) \end{bmatrix} = \left(\begin{bmatrix} A_c & E_c \\ 0 & -\kappa I \end{bmatrix} \begin{bmatrix} x(t) \\ s(t) \end{bmatrix} + \begin{bmatrix} B_c \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} E_c \\ 0 \end{bmatrix} \hat{d}(t) \right) \mathrm{d}t + \begin{bmatrix} 0 \\ \sigma_s \end{bmatrix} \mathrm{d}w(t)$$

Temporal discretization

We derive an **analytical discrete-time system**:

$$X_{k+1} = A_k X_k + B_k u_k + E_k \hat{d}_k + w_k,$$
$$y_k = C_k X_k + e_k$$

The state X contains both x and the disturbance, d, and the system matrices are computed by

$$\begin{bmatrix} A & B & E \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} = \exp\left(\begin{bmatrix} A_a & B_a & E_a \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Delta t \right),$$
$$R_w = V A^T,$$
$$\begin{bmatrix} A & V \\ 0 & A^{-T} \end{bmatrix} = \exp\left(\begin{bmatrix} A_a & \sigma\sigma^T \\ 0 & -A_a^T \end{bmatrix} \Delta t \right)$$

where the system matrices A_a, B_a, E_a, and σ are from the *augmented* system.

State estimation

We use a **stationary Kalman filter** to estimate the state:

$$\hat{X}_k = (I - K_k C_k)(A_{k-1}\hat{X}_{k-1} + B_{k-1}u_{k-1} + E_{k-1}\hat{d}_{k-1}) + K_k y_k$$

$\hat{X}_k$ is the filtered estimate, y_k is the measurement, and K_k is the stationary Kalman gain:

$$K_k = W(P_k) C_k^T (C_k W(P_k) C_k^T + R_e)^{-1}$$

The stationary covariance, P_k, is the solution to the discrete algebraic Riccati equation (DARE)

$$P_k = W(P_k) - W(P_k) C_k^T (C_k W(P_k) C_k^T + R_e)^{-1} C_k W(P_k)$$

The auxiliary function W is

$$W(P_k) = A_k P_k A_k^T + R_{w,k}$$

Optimal control

The **model predictive control** (MPC) algorithm is based on the optimal control problem

$$\min_{\{u_{j|k}\}_{j=0}^{N-1}} \quad \phi = \frac{1}{2}\sum_{j=0}^{N-1} \left(\mathbb{E}\left[\|y_{j+1|k} - \bar{y}_{j+1|k}\|_{Q_y}^2\right] + \|u_{j|k}\|_{Q_u}^2 + \|\Delta u_{j|k}\|_{Q_{\Delta u}}^2 \right)$$

subject to

$$X_{0|k} = \hat{X}_k,$$
$$X_{j+1|k} = A_{j|k} X_{j|k} + B_{j|k} u_{j|k} + E_{j|k} \hat{d}_{j|k} + w_j,$$
$$y_{j|k} = C_{j|k} X_{j|k} + e_j$$

Real-time optimization

We use **real-time optimization** (RTO) to determine the current and future consumer temperature setpoints, $\bar{y}_{j|k}$, and flow velocities, $v_{j|k}$:

$$\min_{\bar{y}_{j|k}, v_{j|k}} \quad \phi = (1-\beta)\mathbb{E}[u_{j|k}] + \beta v_{j|k},$$

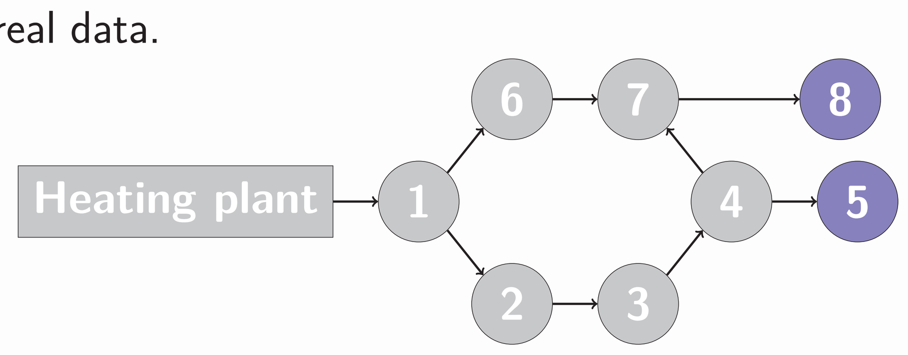
subject to

$$y_{j|k} \sim N(\mu_{j|k}^y, P_{j|k}^y),$$
$$u_{j|k} \sim N(\mu_{j|k}^u, P_{j|k}^u),$$
$$\Pr(y_{i,j|k}^{\min} \le y_{i,j|k} \le y_{i,j|k}^{\max}) \ge 1 - \alpha,$$
$$\Pr(u_i^{\min} \le u_{i,j|k} \le u_i^{\max}) \ge 1 - \alpha,$$
$$v_i^{\min} \le v_{i,j|k} \le v_i^{\max}$$

The means, μ^y and μ^u, and covariances, P^y and P^u, are obtained from the stationary closed-loop system for the MPC. They are used in the chance constraints to ensure that **enough energy is supplied to the consumers** (based on a forecast)

Test case

We use the AROMA grid to test the proposed controller. The objective is to determine the **supply temperature and velocity** in order to meet the demands of consumers (e.g., towns) in node 5 and 8. Their heat demands are based on forecasts from real data.



We use the following simple control law for comparison:

$$T_{s,k} = \begin{cases} 65^\circ\mathrm{C}, & T_{w,k} \ge 17^\circ\mathrm{C}, \\ -2.4458 T_{w,k} + 103.57^\circ\mathrm{C}, & T_{w,k} < 17^\circ\mathrm{C} \end{cases}$$

$T_{w,k}$ [°C] is the measured air temperature associated with the heat demands, and the velocity is always 2.95 km/h.

Results

Both controllers always deliver sufficiently hot water to both consumers, but in many periods, **the proposed approach is able to significantly reduce the supply temperature** without compromising the security of supply, which results in a 3.8% reduction of the economic objective. Finally, the Kalman filter is able to accurately estimate the ambient temperature.

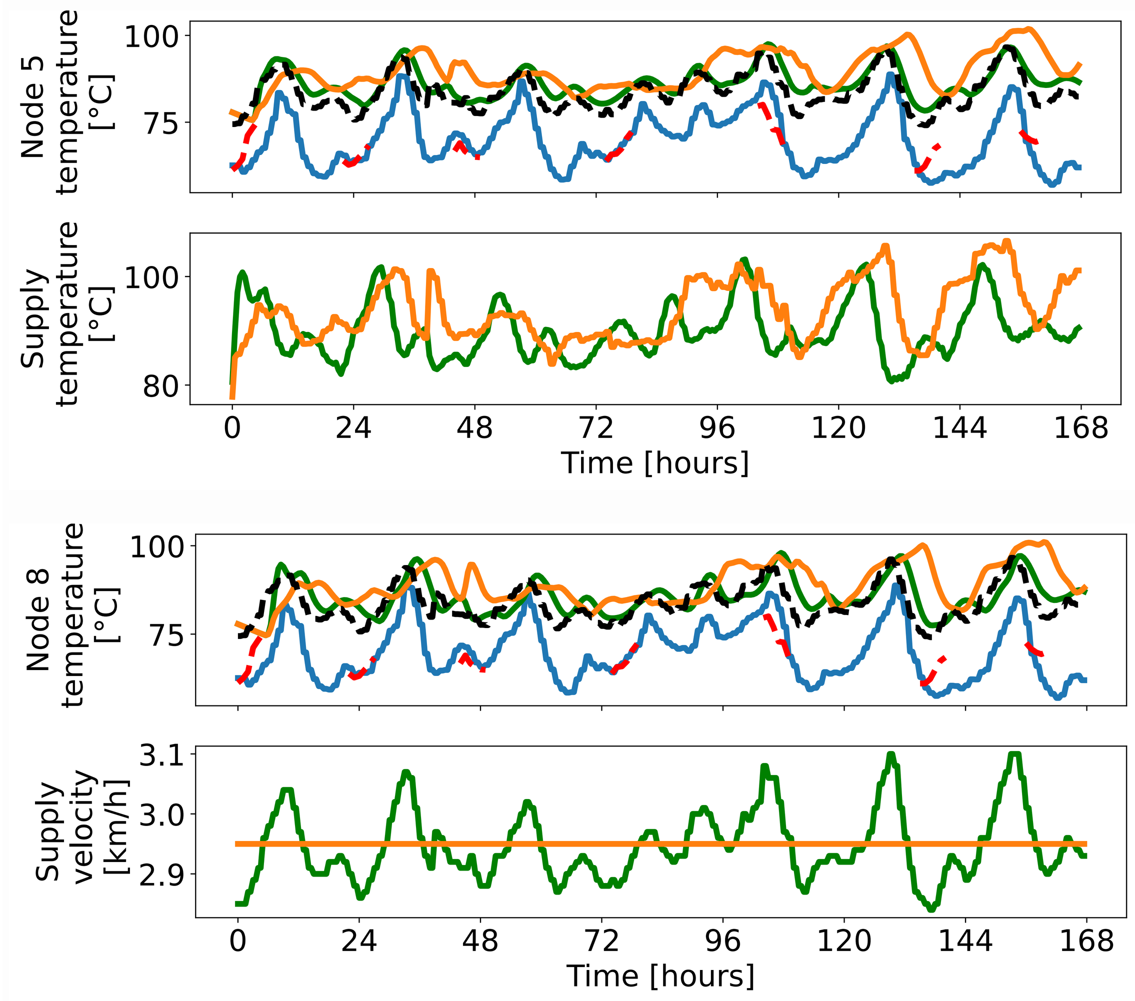


Figure 1: Top and third from the top: The temperatures at node 5 and 8 for the proposed approach (green) and the rule-based control law (orange), the forecast-based lower bound, $y_{k|k}^{\min}$, (black), the mean forecasts (red), and the true required temperature (blue). Second from the top and bottom: The supply temperature and velocity for the proposed approach and the rule-based control law.

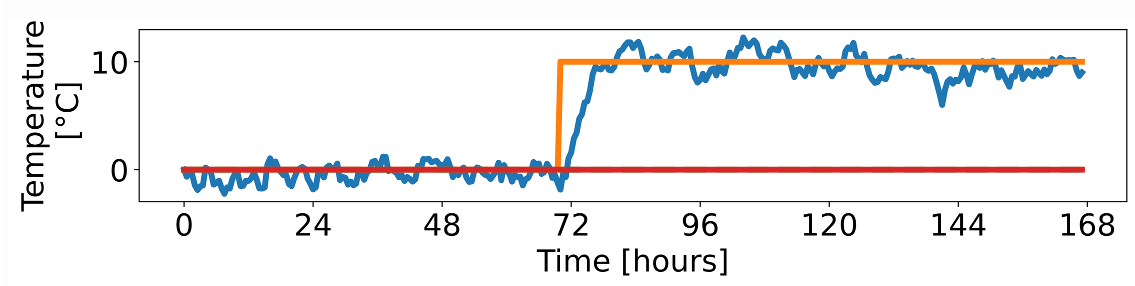


Figure 2: Disturbance. The true, d_k, (orange) and estimated, $\hat{d}_k$, (red) values, and the error estimate, $\hat{s}_k$, (blue).

References

Poulsen, M.H., Rønlev-Knudsen, T., Møller, J.K., Madsen, H., Ritschel, T.K.S., 2026. *Model Predictive Control and Robust Real-time Optimization for Flexible Operation of District Heating Systems*. **In:** Proceedings of the 23rd IFAC World Congress, August 23-28, 2026. *Accepted.*