Dynamic price signals for indirect control of flexible energy resources

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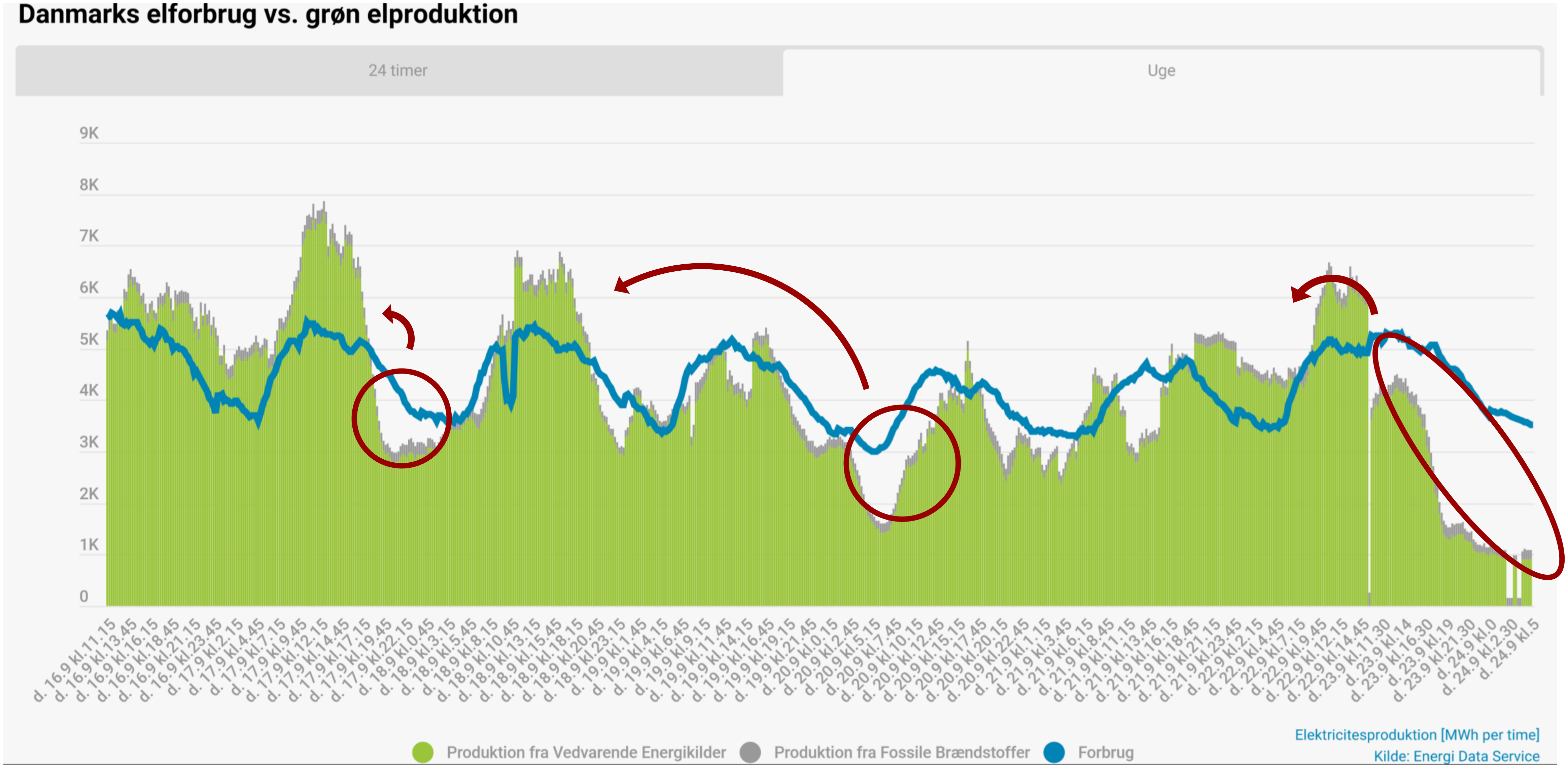
September 24, 2025

Motivation



https://ing.dk/artikel/live-danmarks-elforbrug-og-elproduktion

Motivation

Energy storage systems

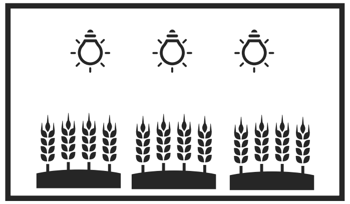
Increase renewable + curtail

Power-to-X

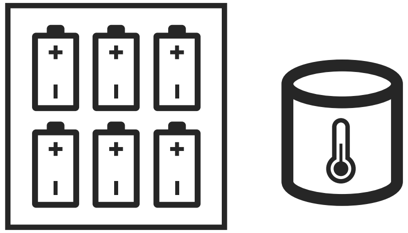
Demand-side flexibility

Types of flexible assets

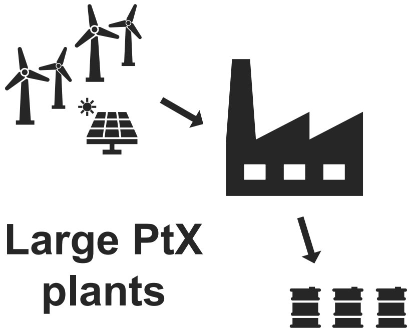
Large and intensive



Greenhouses

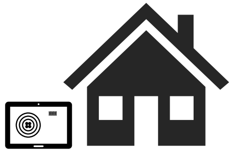


Energy storage facilities

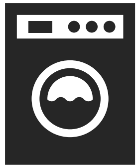


Large PtX plants

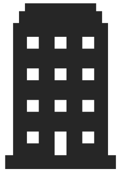
Small but widespread



Heat pumps



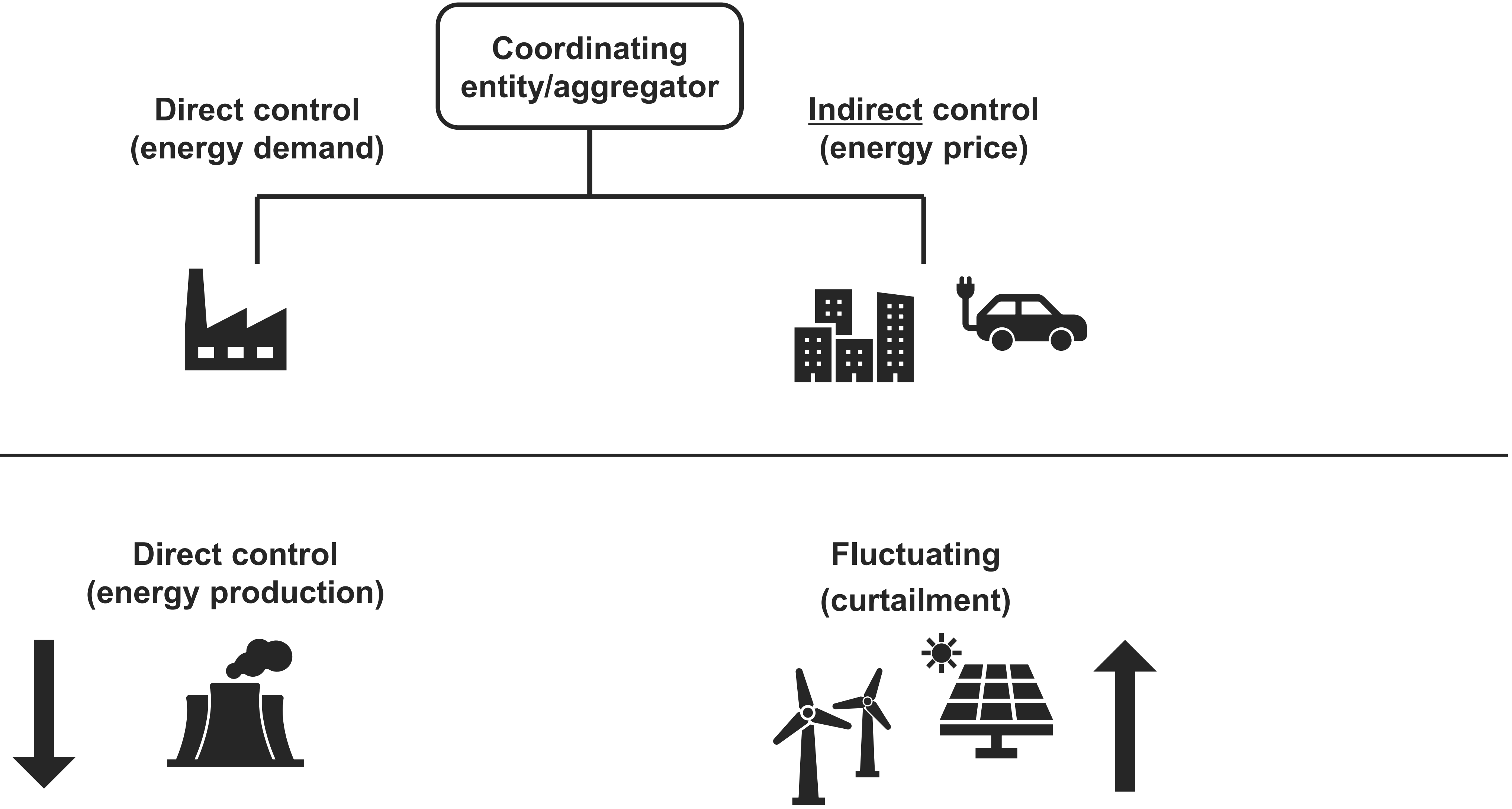
Smart appliances



Households

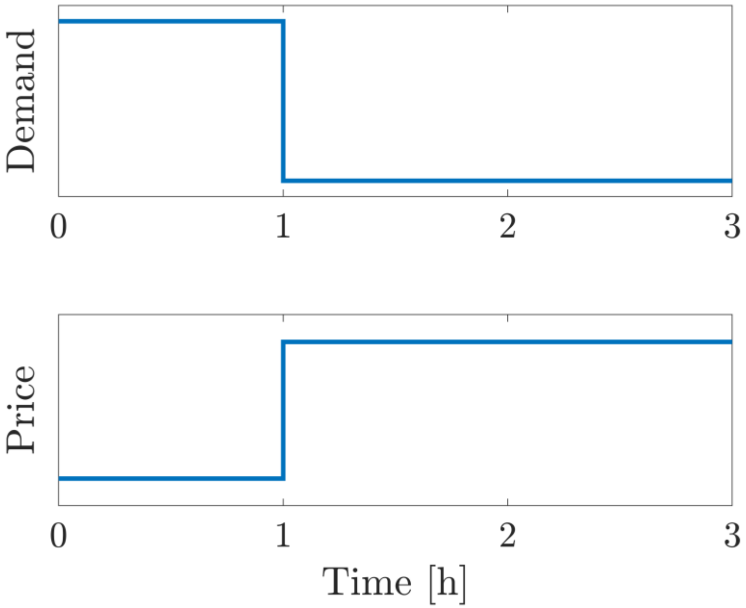
Electric vehicle charging

Direct vs. indirect control (incentivization)

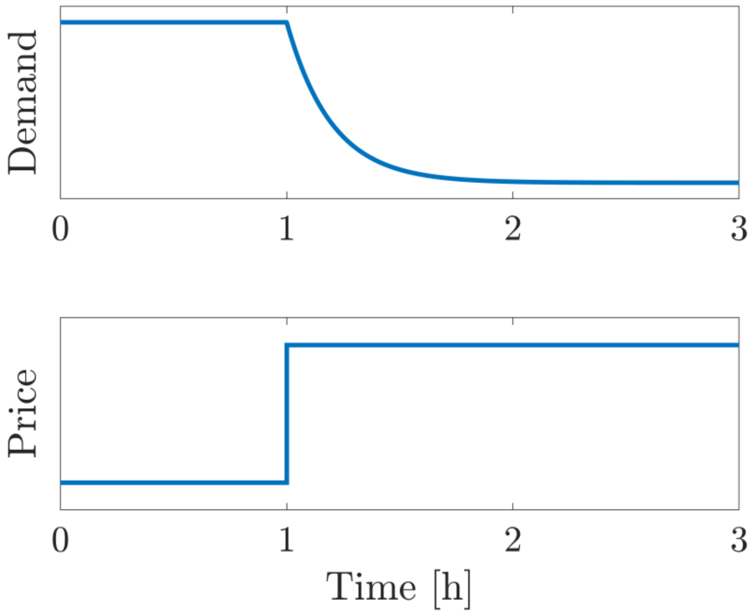


Types of flexibility (price-sensitivity)

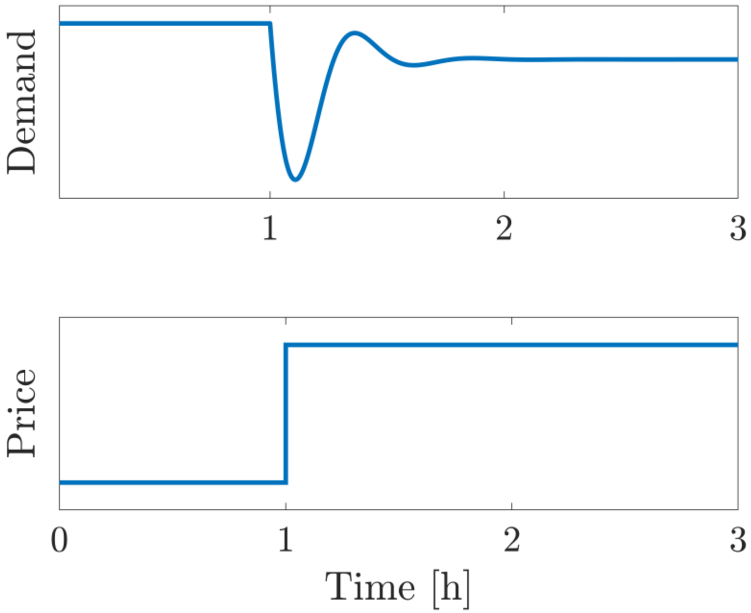
Flexible and fast



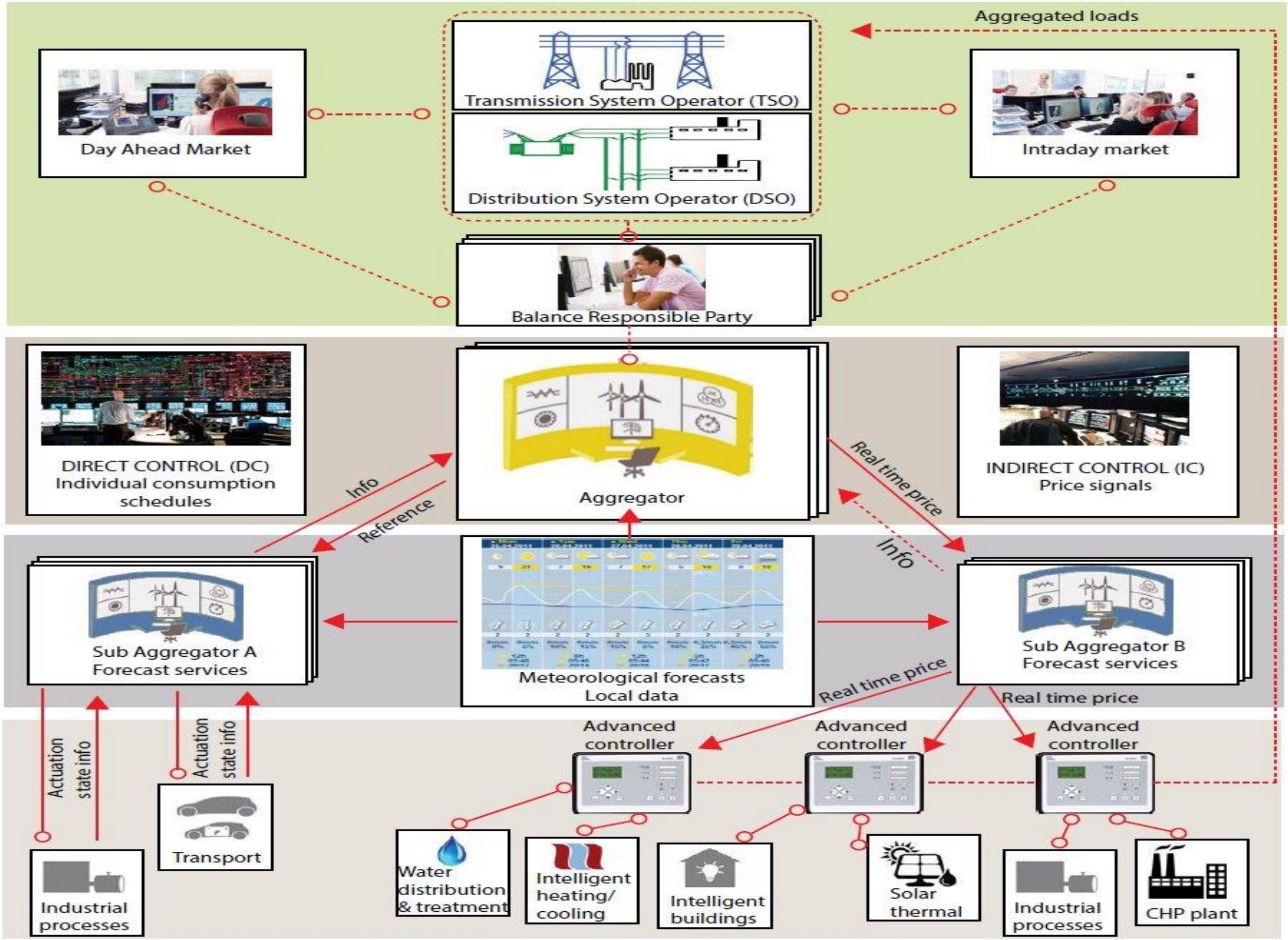
Flexible w. ramping time



Complex



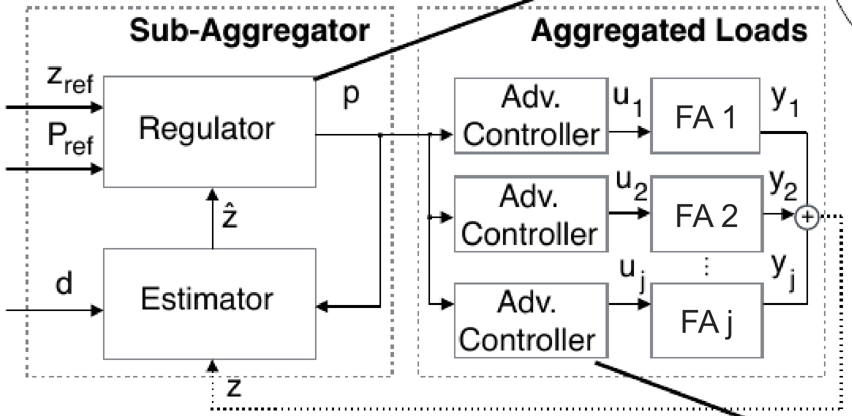
Role of aggregators



Interoperability through indirect control

Price signal generator

$$\min_{p} \quad E\left[\sum_{k=0}^{N} w_{j,k}||\hat{z}_k - z_{ref,k}|| + \mu||p_k - p_{ref,k}||\right]$$
$$\text{s.t.} \quad \hat{z}_{k+1} = f(p_k)$$



Price-sensitive control system

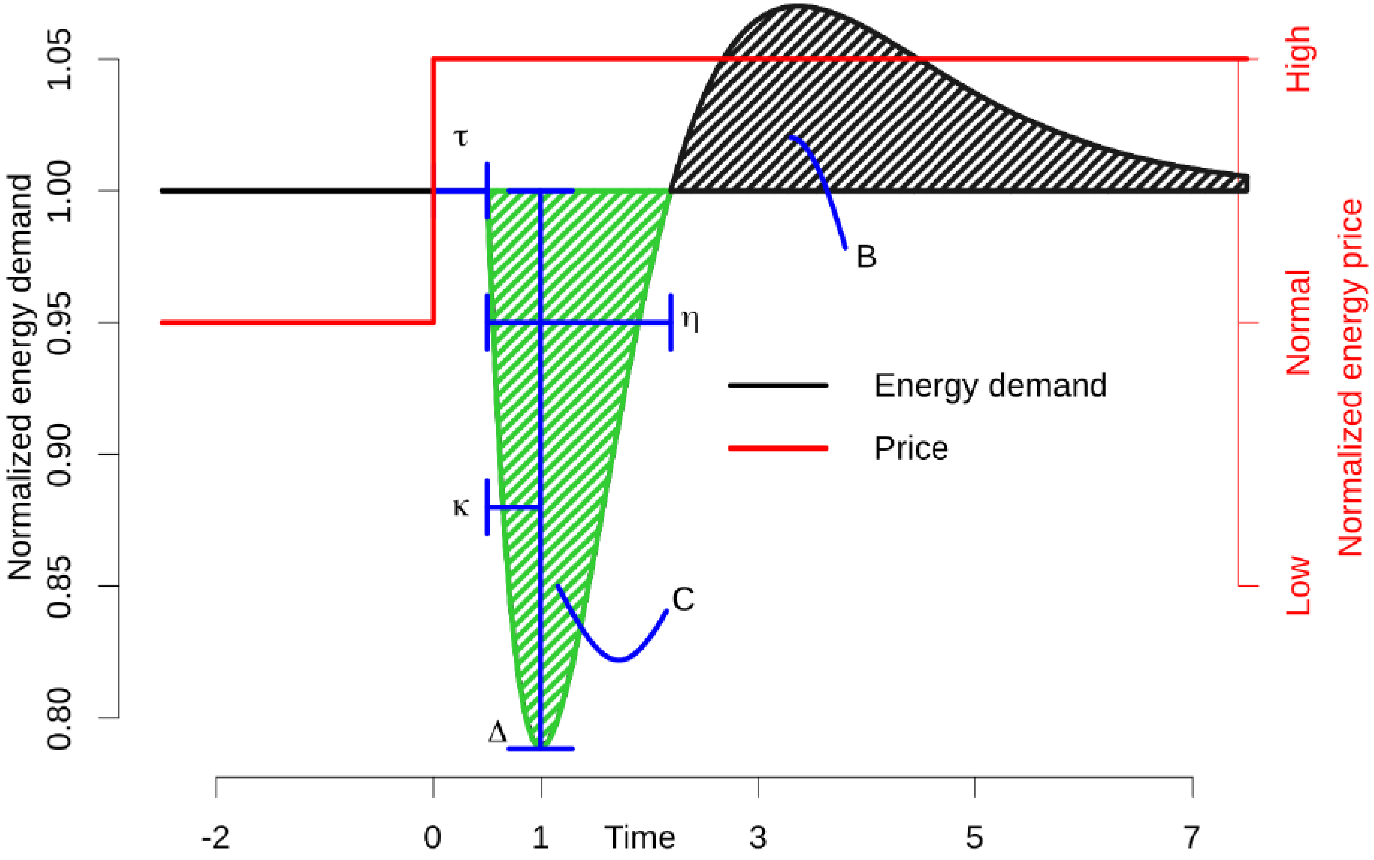
$$\min_{u} \quad E\left[\sum_{k=0}^{N}\sum_{j=1}^{J} \phi_j(x_{j,k}, u_{j,k}, p_k)\right]$$
$$\begin{aligned} \text{s.t.} \quad x_{k+1} &= Ax_k + Bu_k + Ed_k, \\ y_k &= Cx_k, \\ y_k^{min} \leq y_k &\leq y_k^{max}, \\ u_k^{min} \leq u_k &\leq u_k^{max} \end{aligned}$$

We adopt a control-based approach where the **price** becomes the driver to **manipulate** the behaviour of a certain pool flexible prosumers.

Indirect control through <u>flexibility functions</u>

Flexibility function: Predict flexible demand as function of

- **Price**
- **"State of charge"** (real or fictive)



Fundamentals of flexibility functions

State of charge

$$\dot{X}(t) = \Delta D(t)/C \tag{1}$$

Relationship between 1) state of charge and price and 2) demand

$$\Delta D(t) = \varphi(X(t), p(t)) \tag{2}$$

Properties of "relative demand function"

$$\varphi(0,0) = \alpha(1-B), \qquad \varphi(1,1) = -\alpha B, \tag{3a}$$
$$\varphi(0,p) \geq 0, \qquad \varphi(1,p) \leq 0 \tag{3b}$$

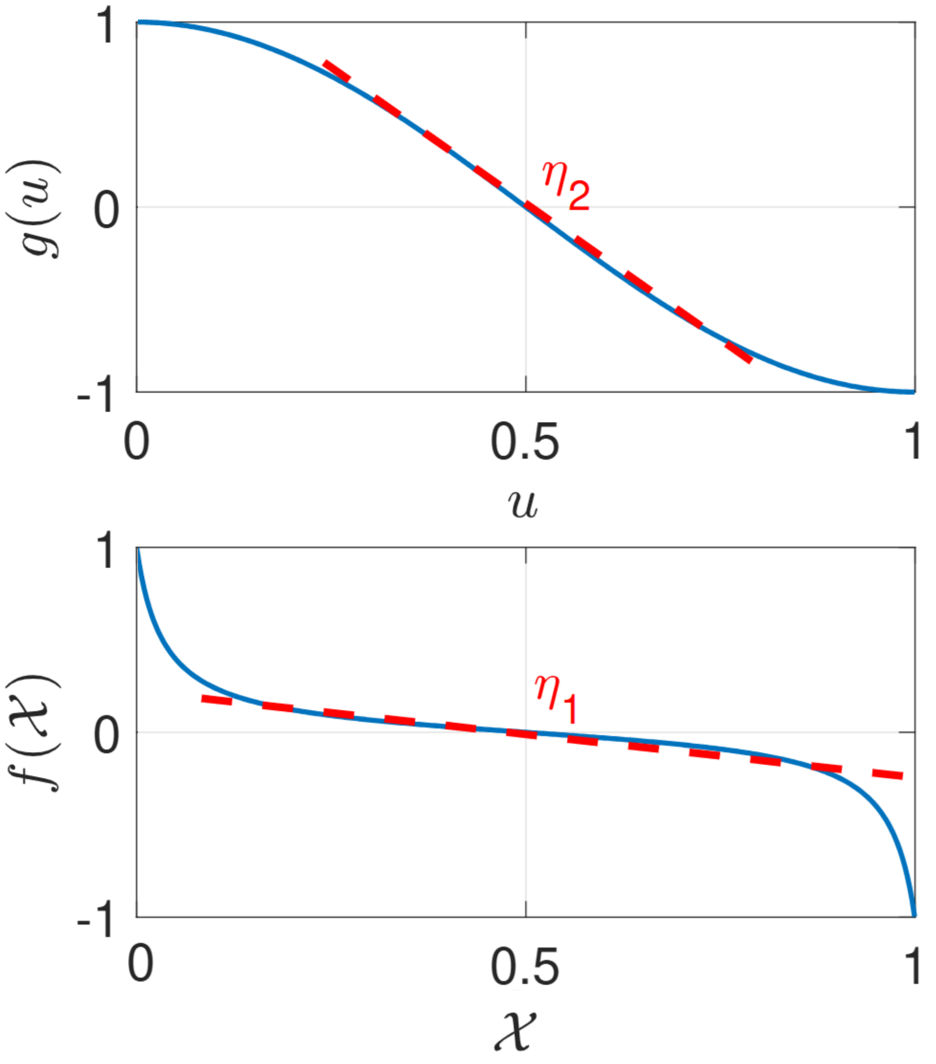
Monotonicity (monotonically decreasing in each argument)

$$\frac{\partial \varphi}{\partial X}(X,p) < 0 \quad \text{or} \quad \varphi(X,p) < \varphi(Z,p), \quad Z < X, \tag{4a}$$
$$\frac{\partial \varphi}{\partial p}(X,p) < 0 \quad \text{or} \quad \varphi(X,p) < \varphi(X,q), \quad p < q \tag{4b}$$

Demand

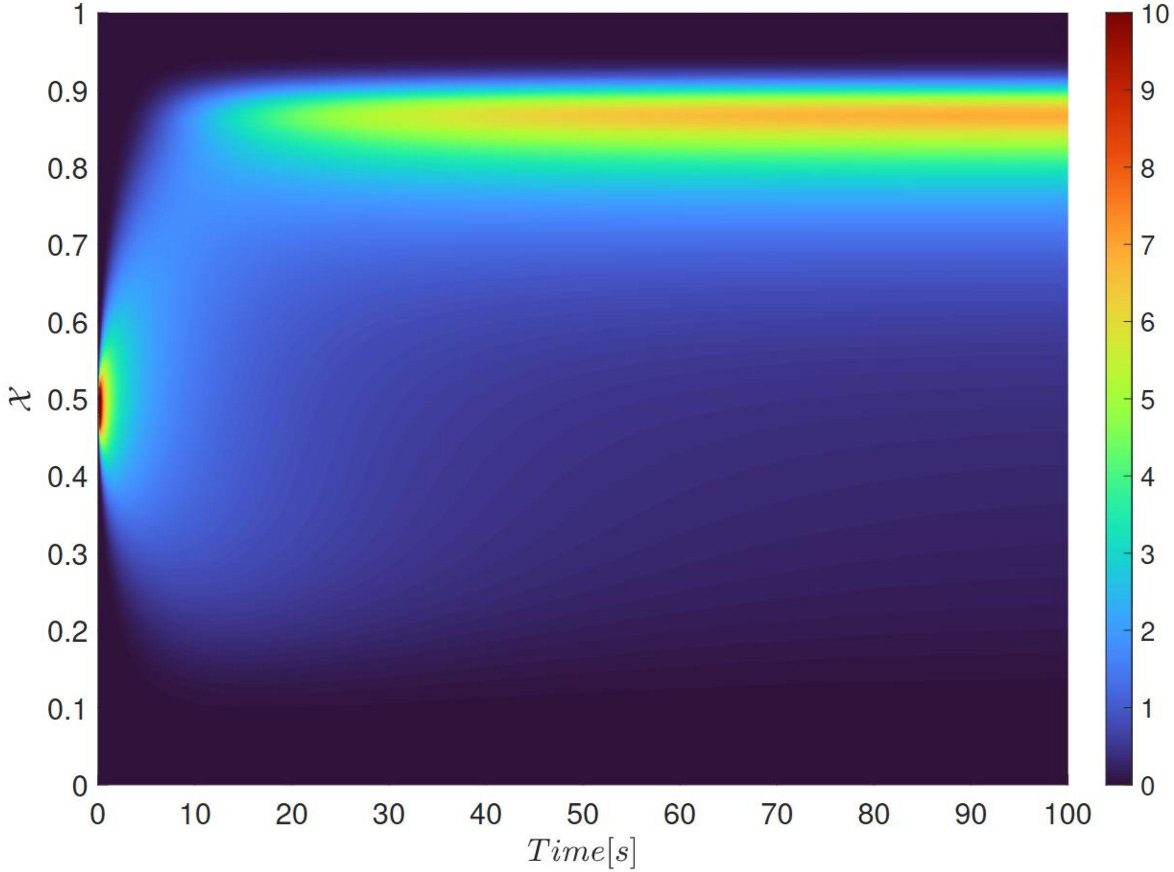
$$D(t) = B(t) + \Delta D(t) \tag{5}$$

Sensitivity to price and state of charge

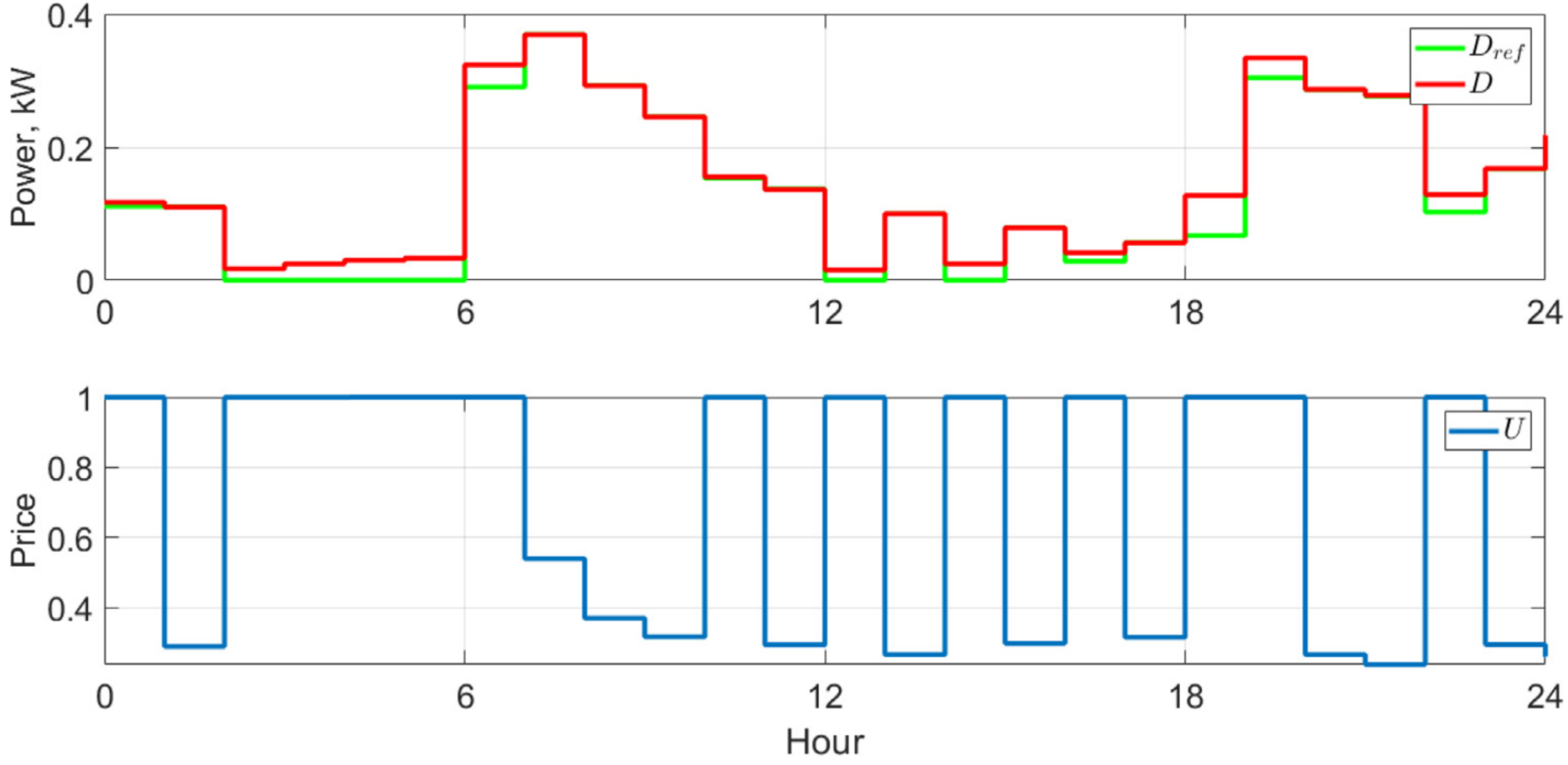


Account for uncertainty

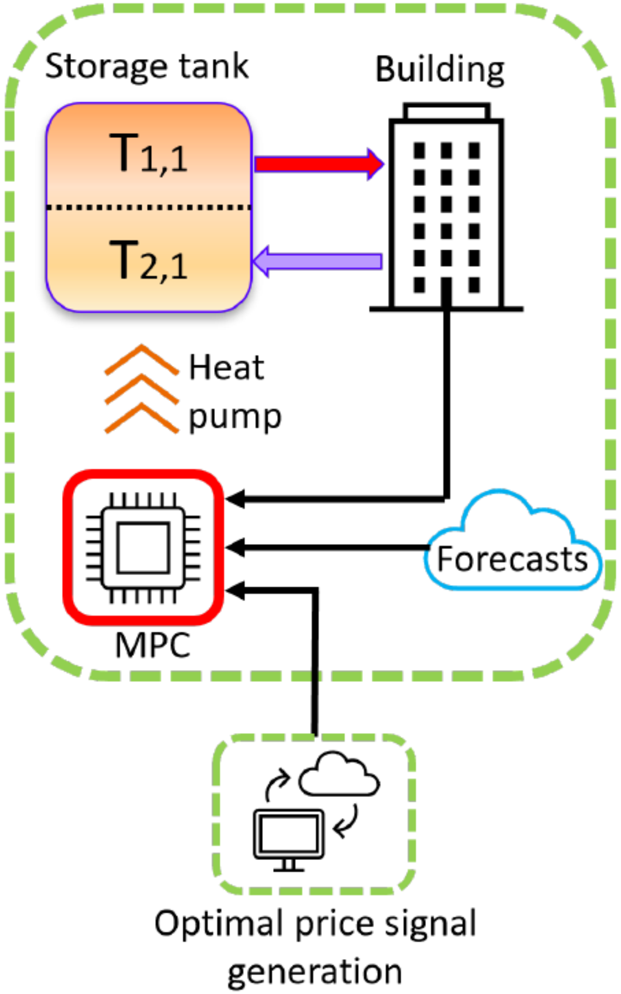
$$dx_t = \frac{\Delta D_t}{C}\,dt + \sigma_x x_t(1 - x_t)\,dW_t$$

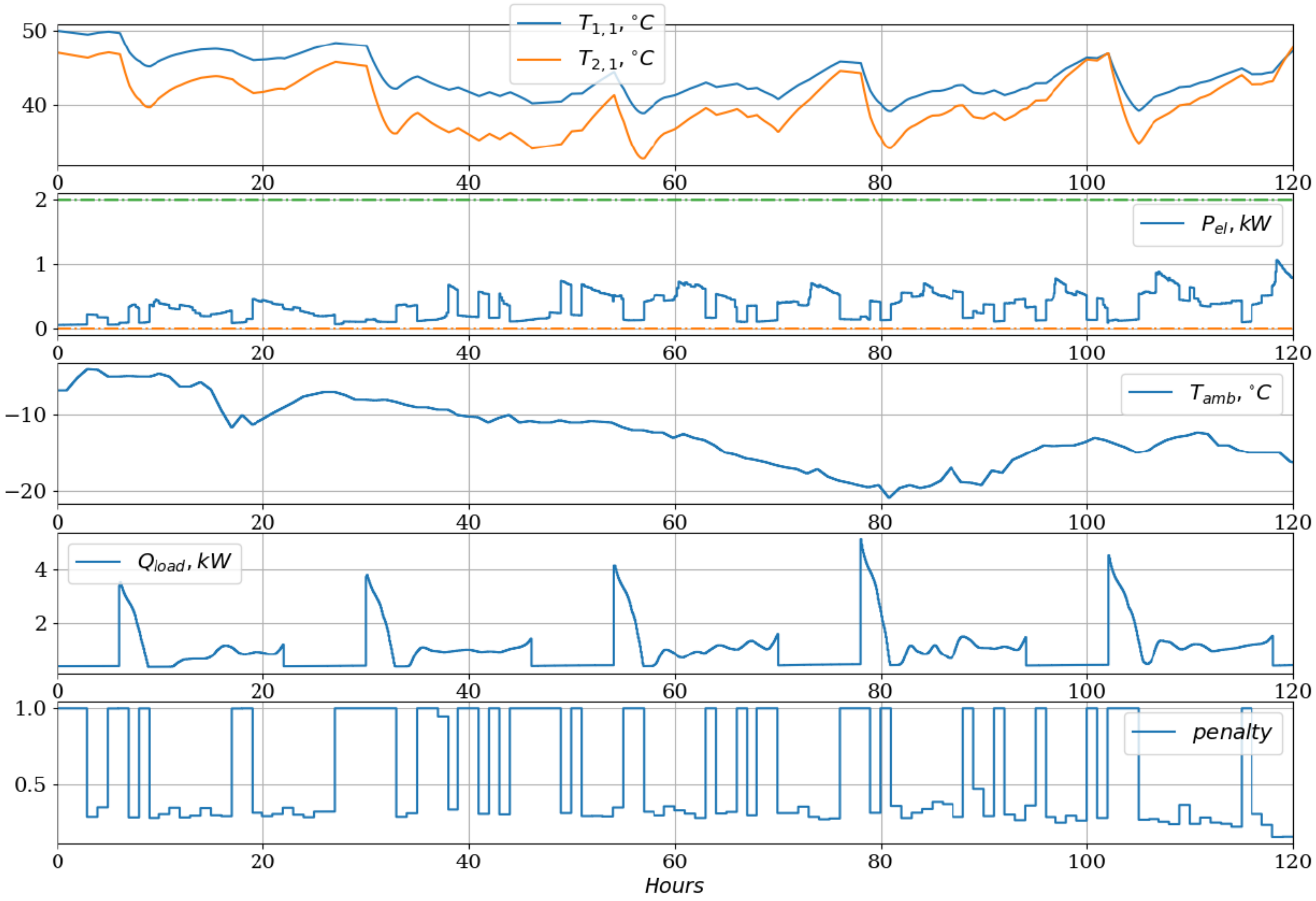


Example (day ahead prices)



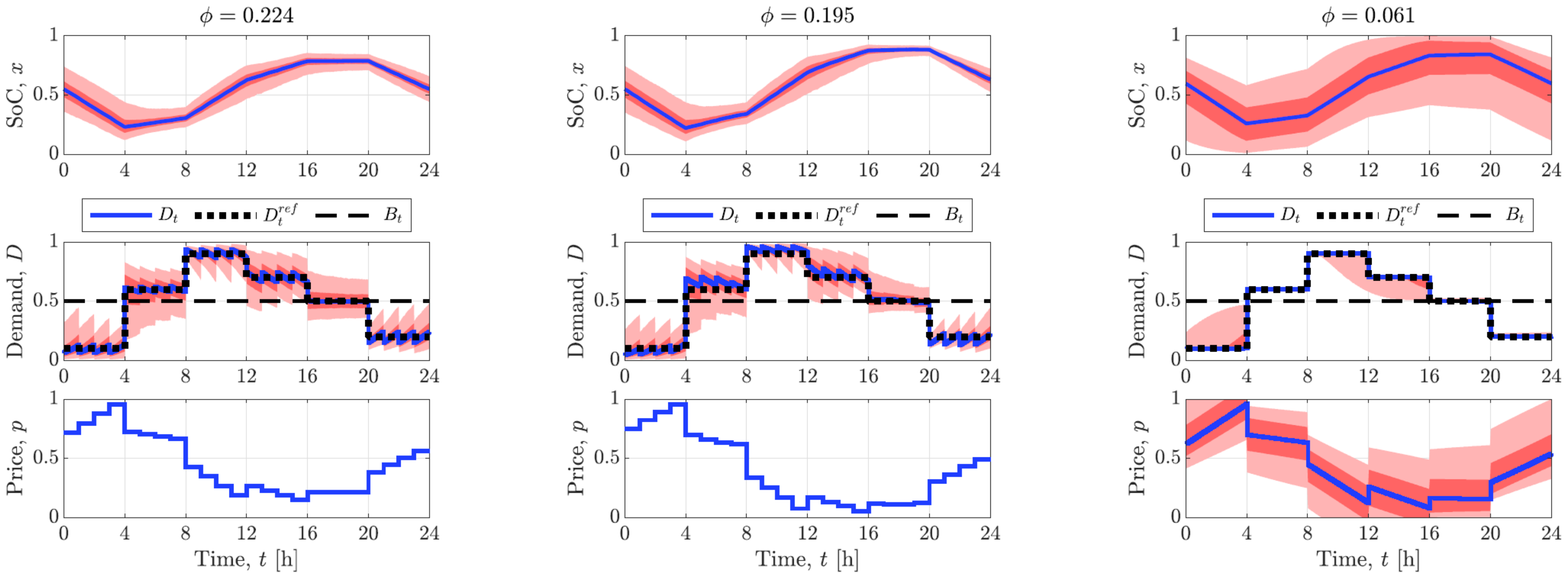
Example: Price-sensitive flexible asset





Comparison of price signal generation strategies

Day-ahead (no uncertainty) | **Day-ahead (stochastic)** | **Feedback control**



(a) Deterministic open-loop solution, hourly price update

(b) Stochastic open-loop solution, hourly price update

(c) Stochastic closed-loop solution, continuous-time price update

Fig. 2: Optimal price signals and the resulting trajectories of x_t and D_t with process noise $\sigma_x = 0.2$. Blue line: Median. Dark red area: 50% confidence interval. Light red area: 95% confidence interval.

Summary

Demand-side flexibility
of complex assets/plants

Aggregators generate
dynamic prices

Demand response model:
Flexibility function

Price signal generation:
Optimal/feedback control