## **Deblurring Images**

## Matrices, Spectra and Filtering

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 $\triangleright$  Page 7, Challenge 3. The denominator of the right-hand side should be  $\|\mathbf{A}_c \mathbf{X} \mathbf{A}_r^T\|_F$ , i.e., the norm of the blurred noise-free image (and not  $\|\mathbf{B}\|_F$ ).

▶ Page 12, Challenge 4. The displayed equation should be:

$$\mathbf{X}_k = \left(\mathbf{A}_{\mathrm{c}}
ight)_k^\dagger \mathbf{B} \left(\left(\mathbf{A}_{\mathrm{r}}
ight)_k^\dagger
ight)^T.$$

▶ Page 27. The displayed equation for the Moffat blur should be:

$$p_{ij} = \left(1 + \left(\frac{(i-k)}{s_1}\right)^2 + \left(\frac{(j-\ell)}{s_2}\right)^2\right)^{-\beta}.$$

 $\triangleright$  Page 32, Challenge 8. The displayed equation for  $\mathbf{B}_{\mathrm{ext}}$  should be:

$$\mathbf{B}_{\mathrm{ext}} = \mathrm{conv2}(\mathbf{X}_{\mathrm{ext}}, P, 'same').$$

 $\triangleright$  Page 37, the equation in the middle of the page for  $b_{22}$  must be

$$b_{22} = p_{33} \cdot x_{11} + p_{32} \cdot x_{12} + p_{31} \cdot x_{13}$$

$$+ p_{23} \cdot x_{21} + p_{22} \cdot x_{22} + p_{21} \cdot x_{23}$$

$$+ p_{13} \cdot x_{31} + p_{12} \cdot x_{32} + p_{11} \cdot x_{33}.$$

▶ Page 50, lines 7 and 8 from the bottom should be:

where **S** is an  $m \times n$  array containing the singular values of **A**, arranged such that  $\text{vec}(\mathbf{S})$  holds the diagonal elements of  $\Sigma_r \otimes \Sigma_c$ . Thus,  $\mathbf{X}_{\text{na\"{i}ve}}$  could be computed in MATLAB as

 $\triangleright$  Page 50, bottom line: Put a period after "values", and then replace " $\mathbf{S} = \text{vec}(\text{diag}(\mathbf{\Sigma}_r \otimes \mathbf{\Sigma}_c))$ " with

(MATLAB's diag function extracts the diagonal elements of a matrix and puts them into a vector; use help diag to get more information.)

▶ Page 54, Challenge 9. The text should be:

**CHALLENGE 9.** For the image iograyBorder.tif, use the PSF for Gaussian blur (with  $s_1 = s_2$ ) or Moffat blur (with  $s_1 = s_2$  and  $\beta = 3$ ) together with the bordering approach from Section 4.6 to generate noisy images for different noise levels. Then use the fast algorithm in VIP 11 to compute the naïve solution. For each noise level, how large can you make the parameters  $s_1 = s_2$  in the PSF before the errors start to dominate the reconstruction? What happens if you perform the same tests using the fast algorithm in VIP 10?

 $\triangleright$  Page 68, Figure 5.12. The title for the top rightmost figure should be  $|\mathbf{V}_{c}^{T}\mathbf{X}\mathbf{V}_{r}|$ . Same page, line 9 from bottom,  $\mathbf{V}_{c}\mathbf{X}\mathbf{V}_{r}^{T}$  should be  $\mathbf{V}_{c}^{T}\mathbf{X}\mathbf{V}_{r}$ .

▶ Page 73, line 3 below the Pointer. The Taylor expansion should read

$$(1+\epsilon)^{-1} = 1 - \epsilon + \epsilon^2 + O(\epsilon^3).$$

Hence, the two following expressions for  $\phi_i$  should read

$$\phi_i = \frac{\sigma_i^2}{\sigma_i^2 + \alpha^2} = \frac{1}{1 + \alpha^2 / \sigma_i^2} = 1 - \frac{\alpha^2}{\sigma_i^2} + \frac{\alpha^4}{\sigma_i^4} + \dots$$

and

$$\phi_i = \frac{\sigma_i^2}{\sigma_i^2 + \alpha^2} = \frac{\sigma_i^2}{\alpha^2} \frac{1}{1 + \sigma_i^2/\alpha^2} = \frac{\sigma_i^2}{\alpha^2} \left( 1 - \frac{\sigma_i^2}{\alpha^2} + \frac{\sigma_i^4}{\alpha^4} + \cdots \right).$$

▶ Page 77, Challenge 13. Change the last sentence to:

Try various choices of the Tikhonov parameter  $\alpha$  and the SVD truncation parameter k (using tol from p. 75) and determine which choices give the clearest solution image.

 $\triangleright$  Page 85, Pointer on top of page. Throughout this pointer, n should be N.

▷ HNO Functions. Lines 57–58 in psfGauss (not listed in the book) should be:

x = -fix(n/2):ceil(n/2)-1;y = -fix(m/2):ceil(m/2)-1;