EIT for beginners

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July 2014

Electrical impedance tomography

Let σ be a positive function on a domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$ and consider the elliptic PDE

$$\nabla \cdot \sigma \nabla u = \sum_{k=1}^{n} \frac{\partial}{\partial x_k} \left(\sigma \frac{\partial u}{\partial x_k} \right) = 0$$

For example σ is (possibly now complex) electrical conductivity and u potential.
For given σ there is a unique solution once the Dirichlet data $u|_{\partial\Omega}$ is specified. Similarly one can specify Neumann data $\sigma \partial u / \partial \mathbf{n}|_{\partial\Omega}$ ($\mathbf{n}$ the outward normal) and u is determined up to an additive constant.
Let Λ_σ be the operator that takes $u|_{\partial\Omega} \to \sigma \partial u / \partial \mathbf{n}|_{\partial\Omega}$, the Dirichlet to Neumann map. The inverse conductivity problem (or Calderón problem) is to find σ from Λ_σ.
Practical applications (electrical impedance tomography) include geophysical, medical and industrial process imaging. When only *a* purely imaginary in process tomography called Electrical Capacitance Tomography (ECT)
In some cases, layered rocks, muscle tissue σ is replaced by a symmetric matrix A

$$\nabla \cdot A \nabla u = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i} \left(A_{ij} \frac{\partial u}{\partial x_j} \right) = 0$$

A simple example

Let Ω be the unit disk in $\mathbb{R}^2$ with polar coordinates (r, θ) and consider a concentric anomaly in the conductivity of radius $\rho < 1$

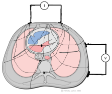
$$\sigma(x) = \begin{cases} a_1 & |x| \leq \rho \\ a_0 & \rho < |x| \leq 1 \end{cases} \tag{1}$$

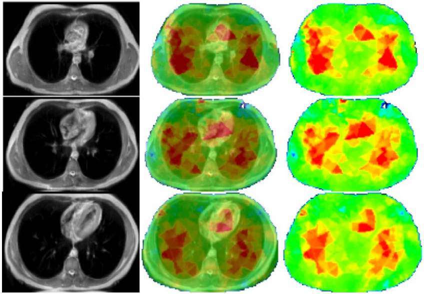
From separation of variables, matching Dirichlet and Neumann boundary conditions at $|x| = \rho$ we find for $n \in \mathbb{Z}$

$$\Lambda_\sigma \mathrm{e}^{\mathrm{i}n\theta} = |n| \frac{1 + \mu\rho^{2|n|}}{1 - \mu\rho^{2|n|}} \mathrm{e}^{\mathrm{i}n\theta} \tag{2}$$

where $\mu = (a_1 - a_0)/(a_1 + a_0)$.

Medical EIT in practice





In medical EIT electrodes are attached to the skin . Current (Neumann data) is applied and voltage measured. This gives a sampled version of Λ_σ. Using a finite element forward model and essentially constrained optimisation, a conductivity image is found that is consistent with the data. Here the EIT image is seen superimposed on an MRI image.

Is the D-to-N sufficient to find the conductivity?

The sufficiency of the data Λ_a to determine the conductivity a for various conditions on a was began by in earnest Alberto Calderón in 1980, although special cases motivated by geophysics had been published since the 1930s, and Tikhonov won a major Soviet medal (Hero of Soviet Labour) for helping use electrical prospecting to locate a massive copper deposit in the 1940s.


Here are some milestones.

- 1980 Calderón showed that for $n > 2$ the linearisation (Fréchet derivative) of $\sigma \to \Lambda_\sigma$ is injective.
- 1985 Kohn and Volgelius uniqueness of solution for $n \geq 2$ and σ piecewise analytic
- 1987 Sylvester and Uhlmann $n > 2, \sigma \in C^\infty(\Omega)$
- 1996 Nachman $n = 2$, $\sigma \in C^2(\Omega)$
- 1997 Brown and Uhlmann $n = 2$ σ Lipschitz.
- 2003 Paivarinta, Panchencko, Uhlmann $n > 2, \sigma \in C^{3/2}(\Omega)$

The results for $n > 2$ mainly follow from Calderón's work in that they construct special "complex geometric optics" solutions, essentially relating the boundary data to the Fourier transform of the conductivity σ. The two dimensional results are more closely related to complex analysis and the $\bar{\partial}$ ("d-bar") operator.

Further recent results have concentrated on problems with limited data, such as current and voltage measured only on part of the boundary.

There are also *conditional stability results*. These say that given you know certain bounds on σ (and maybe its derivatives) then you can control the error in σ by making the error in Λ_σ small, but the *modulus of continuity*, 'δ in terms of ϵ', is horrible. You need an exponential improvement in measurement accuracy to produce a linear improvement in the reconstructed σ.

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Sheffield EIT systems

The first working medical EIT systems came from Sheffield University's David Barber and Brian Brown in the 1980s. They mainly had the following features

- Sixteen electrodes arranged in a plane
- Adjacent pairs of electrodes stimulated
- Linear reconstruction of **difference image** assuming chest is **two dimensional** and **circular**

The systems were fast, safe robust and reliable. They were used for a wide range of applications including heart and lung studies and a number were made for other groups. Sheffield-style systems are available commercially.

Sheffield lung image

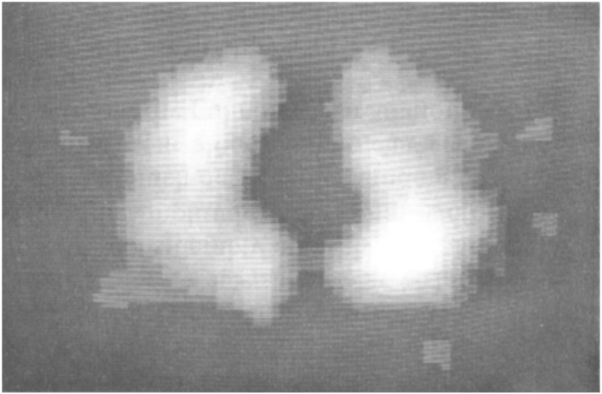


Figure 1. Averaged EIT image of the human lungs showing the increase in lung impedance on deep inspiration with breath-holding, published by Brown *et al* (1985). Ventral is at the bottom and the left side of the body is on the right of the tomogram. (Reproduced with permission of the authors and IOP Publishing, Bristol, UK.)

Adaptive current tomograph (ACT) at RPI

A major advance came from Rensselaer Polytechnic Institute from the group of David Isaacson and John Newell

- All electrodes could be driven with and desired current
- Optimal patterns could be used, in particular trigonometric patterns
- Linear reconstruction of **absolute** conductivity image

The ACT systems were expensive one-offs, carefully tuned to a single frequency. But they produced impressive chest images during respiration. Pulmonary oedema studies were performed by inducing the condition in a goat.

The ACT system

Figure 2. The experimental set-up on a typical subject.

The Move to 3D

- 3D phantom at RPI (Goble, Cheney, Isaccson 1992)
- Sheffield: phantom and then thorax (Metherall Smallwood Barber 1996)
- Non-linear reconstruction using complete electrode model at Kuopio on phantoms (P Vauhkonen 1999)
- 3D reconstruction of time series of chest images during breathing at RPI (Mueller 2001)

Regularization

EIT is ill-posed so need to incorporate some prior information (regularization).

- **Generalized Tikhonov** regularization is easiest to implement and works well on smooth changes.
- **Total Variation Regularization** allows sharp changes but more computationally intensive.
- For two component systems (ie piece wise constant with two values) methods seeking a jump change can be used.

Ideal approach is statistical, eg Markov Chain Monte Carlo MCMC method gives posterior distribution with an assumed prior and error distribution but may not be computationally feasible.

‘Two phase’ methods

While Total Variation is good for multi-component mixtures (eg in Process Tomography), and sharp changes while also allowing for gradients it is computationally expensive. Where only the boundary between two phases is needed consider:

- **Monotonicity** method of Tamburrino and Rubinacci. Used for two component mixtures where properties of components known. Requires measurements at driven electrodes not taken by all EIT systems but OK for ECT. Also works for MIT with three or more frequencies.
- **Shape reconstruction**. When the boundaries between phases are known to be smooth shape based methods such as level sets are useful.
- **Factorization and Linear sampling** methods are useful for detecting jump changes. May not work with small number of measurements, and not clear how to incorporate systematic a priori information.

In some applications volume fraction estimation is more important than imaging and the estimates of Alessandrini and Rosset may be useful.

TV – the idea

Let $F(\sigma) = V$ be the forward problem the a typical regularization method is to solve

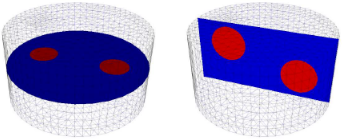
$$\arg\min \|V - F(\sigma)\|^2 + G(\sigma)$$

for a penalty function G. In generalized Tikhonov regularization $G(\sigma) = \alpha^2 \|L(\sigma - \sigma_0)\|^2$ for a differential operator L. The penalty term is smooth so standard (eg Gauss-Newton) optimization will work fine. This regularization incorporates the a priori information that the conductivity is smooth.

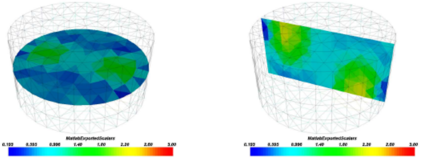
The total variation functional $G(\sigma) = \alpha \|\nabla(\sigma - \sigma_0)\|_1$ still prevents wild fluctuations in σ but allows step changes. The optimization is now of a non-smooth function. One method for solving this is the Primal Dual Interior Point Method. This method tracks a solution between a primal and dual problem avoiding the singularity. It is still more computationally costly than Gauss Newton for a smooth penalty. Now there are more efficient algorithms.

TV Regularization for 3D EIT

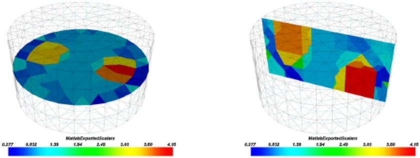
The following work was done in collaboration with A Borsic and N Polydorides.



Two spheres test object

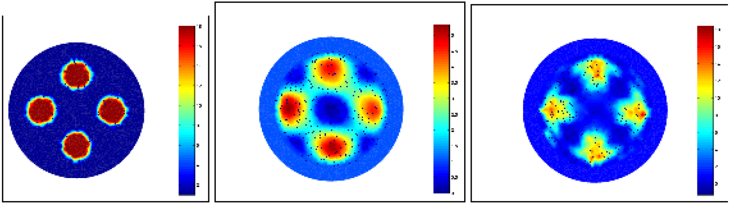


Tikhonov reg. GN reconstruction



TV reconstruction.

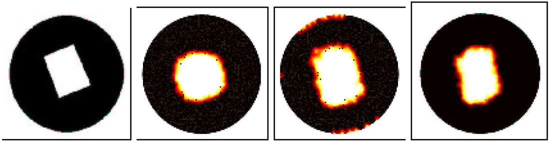
ECT reconstruction ...



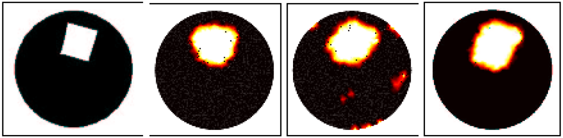
(a) (b) (c)

Results of reconstruction of simulated data (a): True model, (b): Image reconstruction using Tikhonov regularisation and (c): TV regularisation with noise free data

Capacitance Tomography (ECT) reconstruction ...



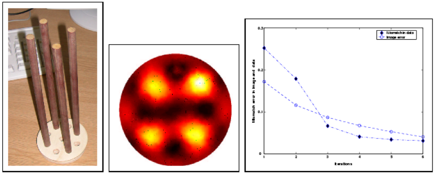
(a) True (b) Tikhonov (c) TV (d) TV+Tikh



(e) True (f) Tikhonov (g) TV (h) TV+Tikh

Reconstruction of wood with square cross section (permittivity 2), using TV and Tikhonov regularisation, We would like to thank M. Byars of Process Tomography Ltd. Wilmslow Manchester UK, for this ECT experimental data.

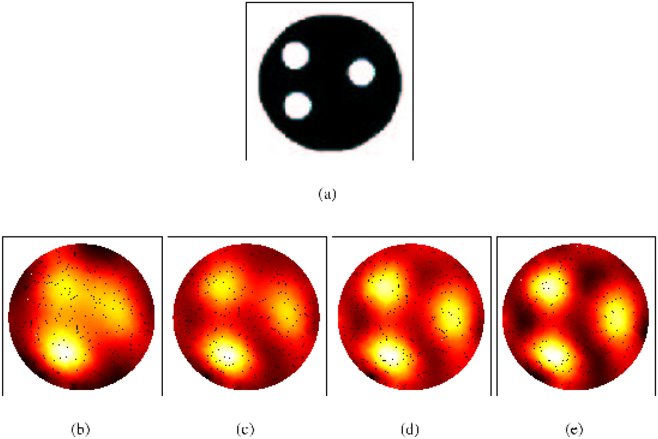
ECT reconstruction ...



(a) (b) (c)

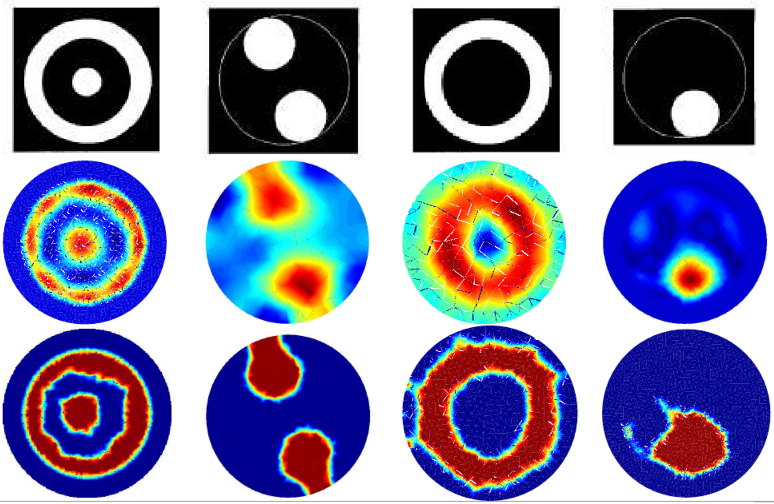
Reconstruction of 4 plastic rods shown in (a) from experimental data shown in (b) and the norm of the mismatch error between measured and simulated normalised capacitance shown in (c)

Iterative ECT reconstruction



Improvement of the image quality using nonlinear steps: (a) Real phantom, (b): Step 1, (c): Step 3, (d): Step 8 and (e): Step 12. Thanks to Bastian Mahr and colleagues from Institute of Process Engineering at university of Hannover in Germany for the experimental ECT data for this test

Level set method for ECT (Dorn, Soleimani,L)



Level set reconstruction for experimental ECT data. The region of interest is the interior of the pipe. In the images 2 and 4 only the inside of the pipe shown, whereas in the images 1 and 2 the exterior part is also shown. The top row shows the real object, the center row the pixel based reconstructions, and the bottom row the shape based reconstructions.

Monotonicity

The monotonicity method of Tamburrino exploits the monotonic dependence of transfer impedance (Neuman-to-Dirichlet map) on resistivity ($\rho = 1/\sigma$)

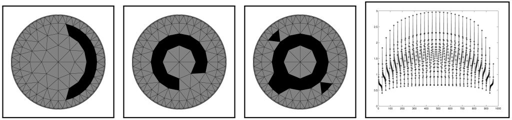
$$\rho_1 \geq \rho_2 \implies \mathbf{R}_{\rho_1} \geq \mathbf{R}_{\rho_2},$$

where $\mathbf{A} \geq 0$ means that the matrix is positive semi-definite. In a situation where the resistivity is known to take one of two values in each pixel or voxel (two 'phases'), the monotonicity criterion gives a test that may show a pixel is definately in one or the the other phase. **Test may be inconclusive depending** on the size of the pixels.

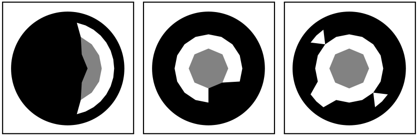
Transfer impedance matrices are pre-computed for each pixel set to the second phase with a background in the first phase. The reconstruction procedure requires only the calculation of the smallest eigenvalue of M $N \times N$ matrices, for N electrodes and M pixels. The algorithm is one-pass and gives an absolute non-linear reconstruction, without the need to solve the forward problem during the reconstruction.

When data has noise, more pixels be inconclusive or even mis classified, but the technique can be combined with other methods, in particular Markov Chain Monte Carlo method (MCMC), to estimate the resistivities of unknown pixels.

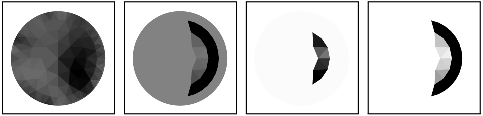
Monotonicity results (Aykroyd, Soleimani, L)



True inclusions: (a) *Barrier*, (b) *C-Shape* and (c) *Ring*, and (d) measurements from Barrier for a 32 electrode system.



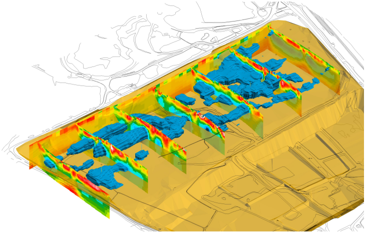
Classification maps for noise-free data from 32 electrode systems, showing the interior set Ω_{Int} (white), exterior set Ω_{Ext} (grey and white), and background (black).



Barrier example: (a) Gauss-Newton reconstruction and then (b) posterior mean resistivity, (c) posterior standard deviation, and (d) posterior "probability of being inclusion".

Geophysics

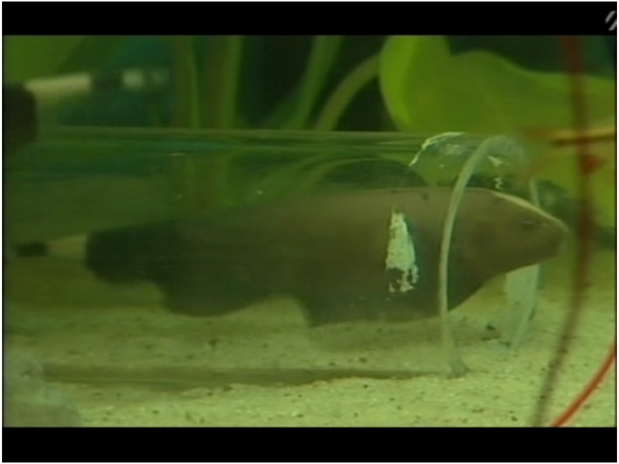
3D EIT reconstruction used in geophysics for example for detecting chemicals leaching from rubbish in land fill sites.



A three dimensional ERT survey of a commercial landfill site to map the volumetric distribution of leachate (opaque blue). Leachate is abstracted and re-circulated to further enhance the production of landfill gas, and subsequently the generation of electricity. This image was provided by the Geophysical Tomography Team, BGS, and is reproduced with the permission of the British Geological Survey ©NERC. All rights Reserved.

A word from the fish..

Weakly electric fish have evolved in the murkey rivers of South America and Africa. They have a ‘dipole source’ between nose and tail, and hundreds of voltage sensors on their body. They use electrosensing to locate prey and to navigate.
It is though the electric organ initially evolved to help locate a mate.
The voltage sensors map in to several separate areas of the brain for long and short range sensing.



A Black Ghost Knifefish here in the Alan Turing Building. He is “hiding” in an insulating tube.

Video from Mark Nelson

Switch to videos by Mark Nelson from Beckman Institute Neuroscience Program University of Illinois, Urbana-Champaign (albifrons use VLC as viewer)

Golden Rules of Inverse Problems

Here are some useful questions to ask in a practical inverse problem.

- Clarify what the problem owner really wants to know. They often ask for something imposible (like a high resolution image), when they need something simpler like volume of air in left and right lungs
- What is already known? (a priori information). Such as smoothness of a solution, max and min values, known structures (eg most people have a spine!)
- What do they actually measure and with what accuracy (what distribution of errors)?

You are then in a position to ask about the sufficiency of the data they measure to determine what they need to know (to the accuracy they need). When talking to engineers, doctors etc it is more useful to talk about *sufficiency of data* – they understand that better than *uniqueness of solution*. A non-uniqueness result often has much more impact than a uniqueness result. Is the reason obvious?

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Inverse Crimes

Often one sees some dodgy practise is Inverse Problems research. These are called *Inverse Crimes*. The first two are associated with work not using real data

- Using the same forward model to generate simulated data as is used in the reconstruction
- Not adding simulated noise to synthetic data
- Showing only reconstructions of a few special cases which worked well and claiming this to be an indication of general performance.
- Tuning reconstruction parameters by hand using a knowledge of the correct answer, but not presenting any 'blind trials' where the parameters are not specifically tweaked. (cf training a neural net and only testing it on the training set)

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Suddenly chest EIT is useful?

- In around 2007 respiratory intensive care specialists became interested in using EIT. Developments in ventilators with more settings to control the wave form of the air pressure meant that real time monitoring of lungs, even with very low resolution became attractive. This activity is associated with lung protective ventilation, avoiding *ventilator associated lung injury*, optimizing *lung recruitment*
- As well as a flush of academic papers, we noticed that intensive care specialists were organizing *their own* EIT meetings! A major was studying regional lung ventillation under Positive End Expiratory Pressure (PEEP).

Draeger

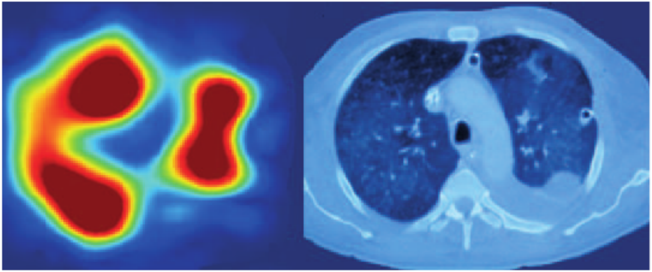

Fig. 5: Functional EIT image (left) and CT scan from a patient with a pleural effusion after rupture of the diaphragm, resulting in a significantly reduced ventilation of the lower left lung. The red color represents regions with the highest volume changes, the non-ventilated regions are displayed in deep blue.

An image used in Draeger publicity

Suddenly chest EIT is useful? 2

- The activity was encouraged by Viasys Healthcare and by Draeger medical loaning their Sheffield style EIT systems.
- But the reconstruction algorithm was one published by the Sheffield group before they adopted a rigorously derived algorithm, and the 2D, circular geometry, adjacent drive approach from the 1980s was still being used.
- Currently study of lung function, especially during mechanical ventilation, is the the most popular potential clinical application of EIT. (eg at EIT2009 Manchester there was around 25 papers and a special session related to lung EIT, more than twice the number of the next most popular, breast cancer)

The challenge: can the last 20 years EIT research help?

Maybe we have been wasting our time for the last 20 years and circa 1988 EIT systems are all that are needed?

Probably not. But now the EIT community is focused on an application with a *genuine clinical pull* we are starting to see modern techniques being applied to these specific applications.

Here are some key challenges:

- Can accurate absolute images be formed (in an intensive care situation)? Can EIT give quantitative, *repeatable* measures of lung function?
- What is the best electrode configuration and stimulation pattern for ventilation monitoring?
- How can we remove or be sure we are not sensitive to artefacts caused by changing chest shape?

Future work..

- Shape and electrode measurement leading to accurate forward models to enable accurate reconstruction
- Shape correction using EIT data
- Simulation study to find best electrode positions and stimulation patterns using realistic anatomical model
- Try combinations of advanced non-linear algorithms: Total Variation, Level Set, monotonicity, factorization method etc. Evaluate their performance with respect to reconstructing clinically useful parameters **with error bars**.
- Design and build data collection systems optimized for studying lung function.
- Testing and evaluation in collaboration with clinicians.