Explicit Model Predictive Control of the Non-inverting Buck Boost DC-DC Converter

Mahmood Mirzaei¹ and Ali A. Afzalian²
Department of Electrical Engineering
Shahid Abbaspour University of Technology, Tehran, Iran
¹ (Tel : +98-912-4245122; E-mail: mirzaei@stud.pwut.ac.ir)
² (Tel : +98-44171971; E-mail: afzalian@pwut.ac.ir)

Abstract: In this paper recent works on hybrid control of DC-DC converters is extended to the Non-inverting Buck Boost converter. Hybrid automaton model of the converter is obtained to check the behavior of the converter and also to be used in simulations. Piecewise affine (PWA) model of the converter is derived as a predictive model and model predictive control is formulized on the obtained model to fulfill defined control objectives. The explicit solution to the problem of the defined optimal control is presented and the performance of the obtained controller is evaluated in three scenarios namely startup, change in the load resistance and the source voltage. The simulations are provided for Buck and Boost operational modes of the converter to check the performance of the controlled system.

Keywords: Explicit Solution, Model Predictive Control, Non-inverting Buck Boost, Piecewise Affine Model.

1 INTRODUCTION

In the past decades the growing demand for delivering electric power in various forms and with high performance and reliability has had a great effect on Power Electronics. Various areas of Power Electronics including power devices, circuit design and control methods have undergone rapid development. In this stream, various control strategies have been proposed to achieve high performance and at the same time low cost power converters and this made a bridge between Automatic Control and Power Electronics communities. One of the mostly used classes of power converters is the class of DC-DC converters. These systems are extensively used in different applications ranging from space and medical applications to DC motor derives and many others because of their advantages such as light weight, small size and high reliability.

Because of the type of nonlinearity in the DC-DC converters mathematical model, the controller design for these converters has become an attractive topic in automatic control community. Various control techniques ranging from linear control based on linearized model [1] to passivity based control [2] and sliding mode [3] have been proposed to deal with the problem. For more information on various control methods the reader is referred to [4] and [5]. Most of the methods use simplified model of the converter which in turn reduce the validity of the controller for real applications and also except some of them, constraints are not generally tackled in the design process.

The difficulty of controlling DC-DC converters stems from their hybrid nature. External signals change discrete variables of converters between two or more states. In each of the states the system has a specific continuous dynamics, thus according to [6] these systems can be categorized as hybrid systems with controlled switching. Considering hybrid nature of the converters, authors of [7] proposed a controller based on hybrid automaton model of the Boost converter. A Nonlinear Model Predictive Control (NMPC) was used based on nonlinear average model [8] of the converter in [9]. Another problem in designing DC-DC converters rises from introducing constraints in the design process. These constraints can be hard such as constraints on duty cycle value or soft such as security constraints imposed on the inductor current.

Model Predictive Control of hybrid systems has proved its power in controlling systems with hybrid nature and subject to various constraints [10]. Online MPC has been successfully tested on Buck and Boost DC-DC converters in [11] and [12]. The main drawback of this method is the problem of solving an optimization problem in each step time and because of high rate of sampling and the demand for low cost converters this approach is not realistic. Solution to the problem of online computational burden is explicit hybrid control [13]. Explicit hybrid control was checked on Buck and Boost converters in [14] and [15] respectively. Control of DC-DC converters can be achieved only based on output voltage measurements but some knowledge of inductor current can substantially improve the performance of the system. All of the proposed methods of hybrid control use inductor current in feedback loop that in turn increase hardware complexity and cost of the converter. Two methods that use computations based on input and output voltage measurements to estimate inductor current were proposed in [16]. The validity and performance of the explicit method was proved by experimental results for Buck and Boost in [16] and [17] respectively.

This paper sketches the works that have been done on Buck and Boost DC-DC converters in [14, 15] to Non-inverting Buck-Boost converter. This converter can provide an output voltage below and above the source voltage and its main advantage is that because of its specific topology utilizing four switches (Fig. 1), the provided output voltage has the same polarity of the...
source. The switches are dependent and make two distinct topologies, thus the converter can be categorized according to [4] as a mono-variable converter. The main control objective here is to control the semiconductor switches (Fig. 1) such that the DC component of the output voltage reaches a specific reference value. This must be done in the presence of changes of source voltage and load resistance.

The paper is organized as follows. Physical setup of the converter is presented in 2.1. Schematic of the circuit with parasitic elements is shown and hybrid automaton of the system is obtained. In this paper we use normalized models and normalization is addressed in 2.1 section. In 2.2 least squares fitting (LSF) is employed to derive piecewise affine (PWA) model of the converter to be used as predictive model. Control objectives and control strategy are explained in 3.1. In 3.3, control objectives are transformed into objective function and an optimal control problem is formulated. State feedback law is explained in 3.4. Finally simulation results are presented in 4 for three scenarios and two operational modes.

2 PHYSICAL SETUP AND HYBRID MODEL

2.1 Physical setup

The physical setup of the synchronous Non-inverting Buck-Boost converter is shown in Fig. 1. Switches are considered bidirectional as a result, the converter will work in continuous conduction mode (CCM).

![Fig. 1 Physical setup of the Non-inverting Buck-Boost](image)

In the setup R, L and C denote load resistance, inductance and capacitance respectively. \( r_L \) and \( r_C \) are parasitic elements of the inductor and capacitor and \( v_s \) is the input voltage. The converter with its set of switches has two distinct dynamical modes, and duty cycle which is a variable bounded between zero and one determines how long each of the dynamics is in charge. At the beginning of the first interval \( (kT_s \leq t < (k + 1)T_s) \), the switches are in \( u = 1 \) position (Fig. 2) which means S1 and S3 are ON and S2 and S4 are OFF (Fig. 1). At the end of the first interval all the switches in Fig. 1 toggle i.e. switches in Fig. 2 change to \( u = 0 \) and the dynamic of the system changes. In the second interval inequality of \( (k + d(k))T_s \leq t < ((k + 1)T_s) \) holds. By defining \( x(t) = [i_L(t), v_C(t)]^T \) as the state vector, where \( i_L(t) \) is the inductor current and \( v_C(t) \) is the capacitor voltage, the dynamics of the system can be defined by the following affine continuous time state space equations:

\[
\dot{x}(t) = \begin{cases} 
F_1 x + f_1 v_s & kT_s \leq t < (k + d(k))T_s \\
F_2 x + f_2 v_s & (k + d(k))T_s \leq t < (k + 1)T_s 
\end{cases}
\]

Where matrices \( F_i \) and \( f_i \) can be found by Kirchhoff’s laws and simple mathematical operations.

\[
F_1 = \begin{bmatrix} -\frac{1}{r_L} & 0 \\ 0 \end{bmatrix}, \quad f_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
F_2 = \begin{bmatrix} -\frac{1}{r_L + r_C} & \frac{1}{R} \\ \frac{1}{r_L} & -\frac{1}{r_C} \end{bmatrix}, \quad f_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

At the beginning of each period the first dynamic of the system is active. At the end of the first interval \( (kT_s \leq t < (k + d(k))T_s) \) a transition occurs and the second dynamic becomes active. The hybrid automaton model for the converter can be easily found. This model of the system has two locations [7] (Fig. 3).

![Fig. 3 Hybrid automaton model of the system](image)

In the hybrid automaton model \( G_1 \) is \( kT_s \leq t < (k + d(k))T_s \) and by putting \( k = k + 1 \), \( G_2 \) is \( kT_s \leq t < (k + d(k))T_s \). \( q_1 \) and \( q_2 \) are discrete states and:

\[
\begin{cases} 
q_1 \in q \Rightarrow f_{q_1}(x(t)) = F_1 x(t) + f_1 v_s \\
q_2 \in q \Rightarrow f_{q_2}(x(t)) = F_2 x(t) + f_2 v_s 
\end{cases}
\]

In this paper normalization is used as a systematic time scale and state variable transformation. It has several advantages for the designer [4]. For example it simplifies the mathematical description of the system by...
eliminating superfluous parameters. Only those parameters which are responsible for important qualitative changes in system’s behavior will remain in the mathematical model. Simulation runs on normalized models facilitates mathematical operations for the processor. It eliminates the stiffness that present in most power electronic device models in turn caused by small capacitance and inductance values. Returning from normalized model to non-normalized model is simple and can be achieved by simple multiplications. It also eases the implication of analytic results of the system model and shows only relevant parameters that have effect on results such as equilibrium point, steady state behavior, and control amplitude restrictions. DC-DC converters are somewhat difficult to simulate with their un-normalized parameter values. Small values of inductance and capacitance multiply the left hand side of the differential equation of the system and produces large right hand side. This can make the model numerically stiff for simulations. For more information about normalization we refer the reader to [4]. To normalize parameters of the model, $v_s$, $R$ and $T_s$ are used as reference values and all other parameters are normalized. As a result the state space equation can be reformulated as follows:

$$
\dot{x}(t) = \begin{cases} 
F_1 x + f_1 & k \leq t < (k + d(k)) \\
F_2 x + f_2 & (k + d(k)) \leq t < (k + 1) 
\end{cases}
$$

### 2.2 PWA model

$v$-Resolution model was first introduced in [11] as a modeling strategy to capture the hybrid nature of the synchronous DC-DC converters. This modeling method provides a discrete time model which can be easily implemented in control design strategies. The method of DC-DC converter modeling which is used here results in PWA model of the system. This method was first introduced in [15] for hybrid modeling of the boost converter. The model obtained to be used in an optimal control problem. A discrete time model of the system is obtained by employing a sampling interval equal to the switching period $T_s$. In this method the least square fitting (LSF) over several regions of the exact system update equation is used to obtain PWA description of the nonlinear dynamic. System update equation was obtained by integrating (1) from $t = k$ to $t = k + 1$:

$$
x(k+1) = e^{F_2(1-d(k))} \int_0^{d(k)} e^{F_2(d(k)-s)} f_2 ds + (2)
$$

Equation (2) can be written as:

$$
x(k+1) = \Phi(d(k))x(k) + \Gamma(d(k))
$$

Where $\Phi(d(k))$ and $\Gamma(d(k))$ are matrices that depend nonlinearly on $d(k)$. In PWA model of the system expression (2) is approximated by determining $A_i, B_i$ and $f_i$ in the following formula:

$$
x(k+1) = A_i x(k) + B_i d(k) + f_i
$$

if $d(k) \in D_i$ $i = 1, \ldots, v$

$0 \leq d(k) \leq 1$

To determine $A_i, B_i$ and $f_i$, the following expression is minimized over grided series of points of $x(k)$ in the state space $[i_{L_{\min}} \ldots i_{L_{\max}}] \times [v_{C_{\min}} \ldots v_{C_{\max}}]$. $D_i$ s are the $v$ intervals $[0,1/v]$ , $\ldots$, $[v-1/v,1]$ or a set of intervals which can result in a better model, and $i_{L_{\min}}$, $i_{L_{\max}}$, $v_{C_{\min}}$, and $v_{C_{\max}}$ are determined by checking state variable deviations of the hybrid automaton model of the system which was obtained in Section A under changes of duty cycle.

### 3 The Control Problem

#### 3.1 Control Objectives and Strategy

The main control objective of the converter is to derive DC component of the output voltage to its reference value as fast and with as little overshoot as possible. The control must be done in the presence of source voltage and load resistance changes. These are control objectives for the transient response of the controlled system. It also must have an acceptable error in steady state. Also the controller should produce constant value for duty cycle in the steady state to prevent chattering (subharmonic oscillations).

Basically two control methods can be used for controlling DC-DC converters: voltage mode control and current mode control. The capacitor voltage of the Non-inverting Buck-Boost converter has two possible solutions for a specific reference value of output voltage which results in two optimal points for the cost function. In one of these optimal points system works with high value of inductor current which is undesirable and also decreases performance of the converter. Also $v_C$ exhibits non-minimum phase behavior. Thus current mode control is used to bypass these problems. The controller is designed in order to derive inductor current to its reference value. Reference value of inductor current can be computed based on known values of capacitor voltage reference, source voltage and load resistance. This strategy also makes the use of feedforward loops possible to alleviate disturbance effects.
3.2 Model Predictive Control

MPC is the only advanced control strategy that has had a significant impact on industry and its simplicity made it so popular. MPC has been widely accepted as a control tool because of its ability to deal with constraint and multivariable systems. MPC uses the model of the system to predict future behavior of the plant to use in a cost function to predict future performance. This strategy computes a sequence of control signals by solving an optimization problem based on predicted performance and some measurements on the plant in each step. Thus given the model of the plant one only needs to formulate an objective function based on control objectives. Each sampling time ideally a constraint infinite time optimal control (CITOC) should be solved as the optimization problem. Because in general there does not exist a simple closed form solution for CITOC problem it can be simplified by defining a finite horizon and approximate CITOC problem with a constraint finite time optimal control (CFTOC) problem.

3.3 CFTOC Problem and the Explicit Solution

The control objective is to regulate dc component of the output voltage to its reference value which is equivalent to regulate inductor current to its calculated reference value. In practice this reference value can be achieved by incorporating a PI controller. Thus \( i_{L, err} = i_L - i_{L, ref} \) is in the objective function. To prevent unwanted chattering a term also is added to objective function that penalizes the change of duty cycle in two consecutive steps \( \Delta d(k) = |d(k) - d(k-1)| \). Thus the error vector can be defined as:

\[
\epsilon(k) = [i_{L, err}(k), \Delta d(k)]^T
\]

Thus the objective function can be defined as follows:

\[
J(D(k), x(k), d(k-1)) = \sum_{l=0}^{L-1} \| Q\epsilon(k+l | k) \|_1
\]

in which the penalty matrix is \( Q = \text{diag}(q_1, q_2) \). This penalized \( \epsilon(k+l, k) \) over finite horizon \( L \).

The main drawback of MPC is its computational burden which limits the method to plants whose optimization problem can be solved in one step. The online solution is particularly not applicable for DC-DC converters which mostly work with sampling times around micro seconds. Also using a processor to solve optimization problem is not cost effective. This problem is solved by carrying the computational burden of the optimization problem offline and using explicit solution. Recently it was shown that the problem of CFTOC of PWA systems can be reformulated as a multi-parametric program by treating the state vector as parameters [18, 19].

3.4 The State Feedback Law

Multi-Parametric Toolbox (MPT) [20] is used to find the explicit solution of the MPC problem. MPT also is used to merge regions which contain the same expressions of the control law. This results in a controller with the same laws over reduced number of partitions. Because normalization is used in our approach, the dependence of control law on \( v_s \) is removed, thus the partitions can be viewed in 3-dimensional state space. Simplified partition of the explicit solution is shown in Fig. 4.

![Fig. 4 Partitions of the simplified explicit controller](image)

Fig. 4 Partitions of the simplified explicit controller

![Fig. 5 Visualized state feedback for d(0)=0.6](image)

Fig. 5 Visualized state feedback law for \( d(k-1)=0.6 \)

The optimal state feedback law can be expressed in PWA format [19] as follows:

\[
d^*(k) = F_j x'(k) + g_j
\]

\[
\text{If } \quad H_j x'(k) \leq K_j \quad j = 1, \ldots, N
\]

\( H_j \) and \( K_j \) define polyhedral partitions of state space. \( F_j \) and \( g_j \) are the values of the piecewise affine control law.

Fig. 5 visualizes state feedback law for a specific
value of \( d(k-1) \). This is done to reduce the dimension of the PWA function by removing one of its parameters. The control law is originally of the form \( d(k) = f(i_L, v_C, d(k-1)) \) and can be drawn in 4-dim space.

Implementing this state feedback in real applications is straightforward. This can be simply done by determining the partition of measured state and finding its corresponding affine control law from a lookup table with a micro-control based hardware.

4 Simulation results

4.1 Model and Controller Parameters

Inductance and capacitance are chosen to be 1 p.u. and 5 p.u. respectively. Parasitic elements are 0.005 and 0.08 p.u. for capacitor equivalent series resistor (ESR) and inductor resistance. Source voltage and load resistance are reference values and therefore are 1 p.u. finally switching is chosen to be 20 KHz.

The nonlinear model is approximated by 3 PWA dynamics \( (\nu=3) \) on grided state space of \([-6,12] \times [-2,4] \). The bounds of state space were obtained by running hybrid automaton model of the converter with a sequence of duty cycle to incorporate state variations for different scenarios of duty cycle change. Explicit MPC controller was obtained which resulted in 129 regions for PWA feedback and 70 regions after simplification (Fig. 4).

4.2 Simulation Scenarios

Three scenarios namely startup, load resistance change and source voltage change which are common in practice are chosen [12]. In startup scenario the system starts with zero values for inductor current and capacitor voltage. Disturbances that can affect the system are change in the load resistance and source voltage. The responses of the controller to these disturbances are also shown. In all the figures the quantities are in per unit and one can use simple multiplication to find real values. Also in all the figures, figure A the shows result of state evolution for different scenarios in which the solid line is capacitor voltage, the dashed line is \( v_{C,\text{ref}} \) and the dotted line is inductor current and figure B shows duty cycle value.

5 Conclusion and future works

PWA model of the Non-inverting Buck-Boost converter was obtained. Current mode control was employed and a set of nonlinear equations was solved to find \( i_{L,\text{ref}} \) based on known values of source voltage, load resistance and \( v_{C,\text{ref}} \). Explicit model predictive controller was designed to regulate \( i_L \) to its reference value \( i_{L,\text{ref}} \) which is equivalent to steering \( v_C \) to \( v_{C,\text{ref}} \). Performance of the controller was evaluated with three different scenarios for Buck and Boost operating modes. In this paper current mode control was used to bypass non-minimum phase behavior of the capacitor voltage. This problem also can be by solved by choosing long prediction horizon \( (N=30) \), and move blocking strategy can be used to reduce complexity of the controller [17]. It is also desirable to eliminate inductor current sensor for some reasons [16], thus sensorless control method which was recently introduced [16] can be a topic for future research. Also implementing the obtained controller using MPT code generator on hardware can be a topic of future works.
Fig. 9 50 percent change in the source voltage; it changes from 1 p.u. to 1.5 p.u., Buck operation mode

Fig. 10 100 percent change in the load resistance; it changes from 1 p.u. to 2 p.u., Boost operation mode

Fig. 11 100 percent change in the load resistance; it changes from 1 p.u. to 2 p.u., Buck operation mode

6 REFERENCES


