Combinatorial Designs and High-Performance Computing

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Abstract

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Combinatorial Designs (like Hadamard matrices and D-optimal designs) arise in Combinatorics and have found numerous applications in Statistics, Coding Theory, Cryptography and Wireless Communications to name just a few. We have developed a Computational Algebra formalism in terms of ideals in multivariate polynomial rings to construct various types of combinatorial designs. Our formalism allows us to establish many old and new theoretical results concerning these objects. In addition, our formalism is directly usable for high-performance computing purposes. HPC turned out to be the ideal tool to manage the combinatorial explosion that occurs in these problems. Currently, we are are using the Shared Hierarchical Academic Research Computing Network (SHARCNET) HPC cluster in Ontario, Canada, to compute several millions of Hadamard matrices of orders up to 100, as well as D-optimal designs of big orders through serial and parallel (MPI) programs. These Hadamard matrices are of a certain special structure and they are subsequently examined to search for inequivalent Hadamard matrices. The quest for inequivalent Hadamard matrices resulted in building (on-going project) a database for inequivalent Hadamard matrices in the Computational Algebra System Magma in collaboration with J. J. Cannon and his group in Sydney, Australia. Revisiting our formalism from a binary tree optic, allow us to apply combinatorial optimization algorithms (simulated annealing, hill-climbing, tabu search) and genetic algorithms. D-optimal designs are ideally suited to apply these ideas, because in many cases we know one or two solutions and these can be used as an initial population to discover nearby solutions. From the results gathered thus far, we firmly believe that HPC will transform the landscape of the current state-of-the-art knowledge in the area of combinatorial designs in the next few years.