DFM
Danish National Metrology Institute

Survey of Models for Acquiring the Optical Properties of Translucent Materials:

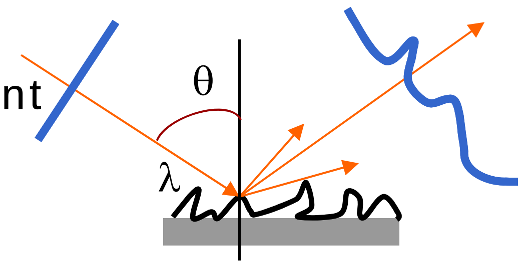
Field models for surface finishing and appearance of objects

Poul-Erik Hansen

The EMPIR initiative is co-funded by the European Union's Horizon 2020 research and innovation programme and the EMPIR Participating States

BxDiff
JRP 18SIB03

How to express wavefront aberration in terms of surface variation

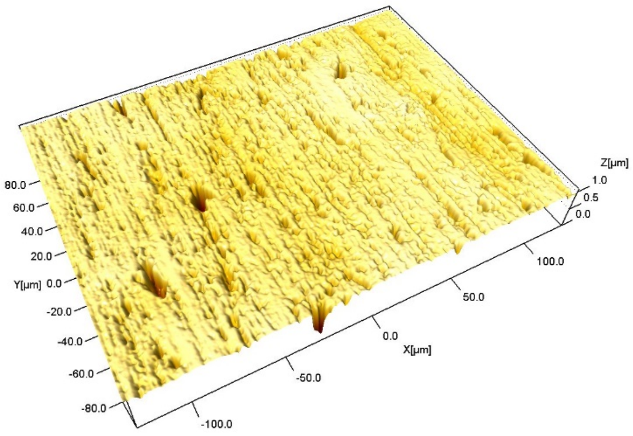


$$U(\hat{x}) = \sqrt{R}\ e^{i\varphi(\hat{x})}$$

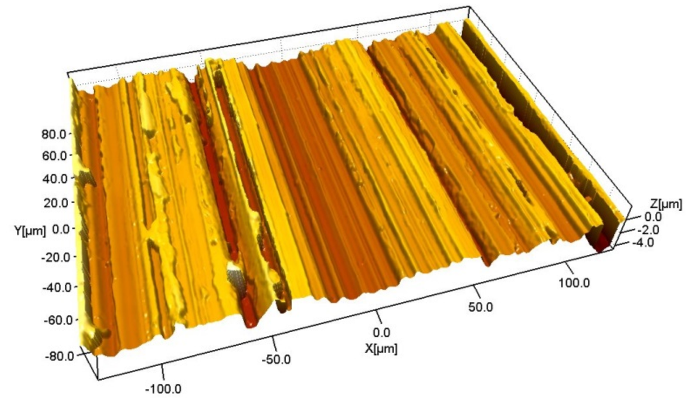
? → **BRDF**

- Surface height distribution and surface spatial distribution
- Wavefront aberration
- How to use the Harvey-Shack method for BRDF
- Surface characterization beyond Harvey-Shack method (RCWA)

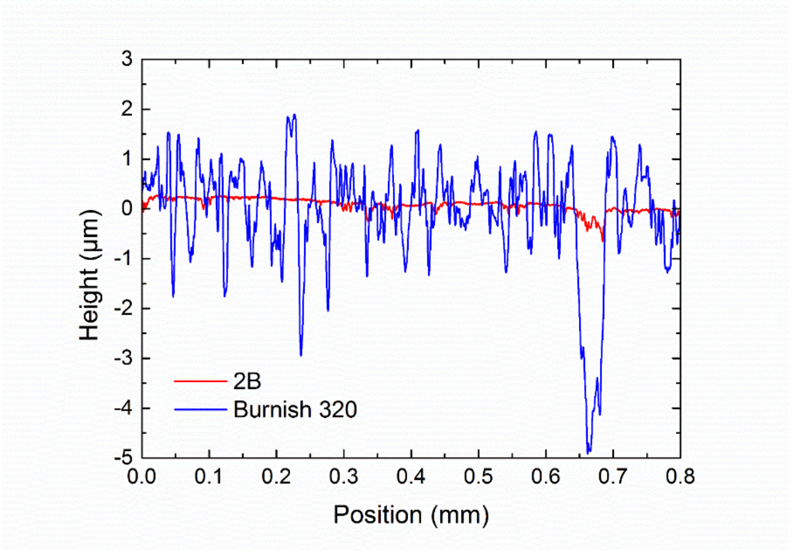
Surface height distribution and surface spatial distribution



Stainless steel 2B

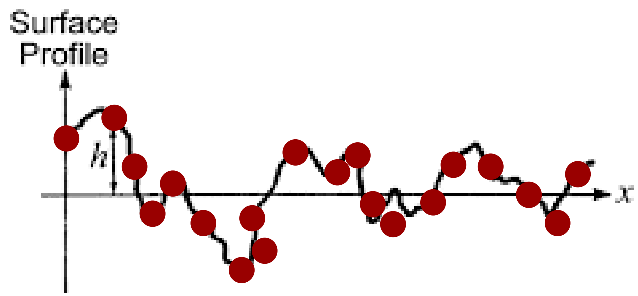


Stainless steel Grinding 320

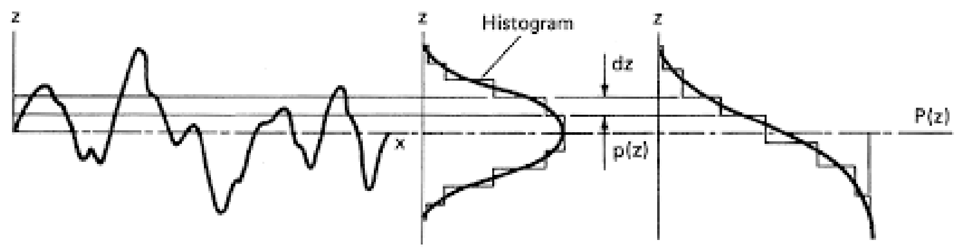


Average profile

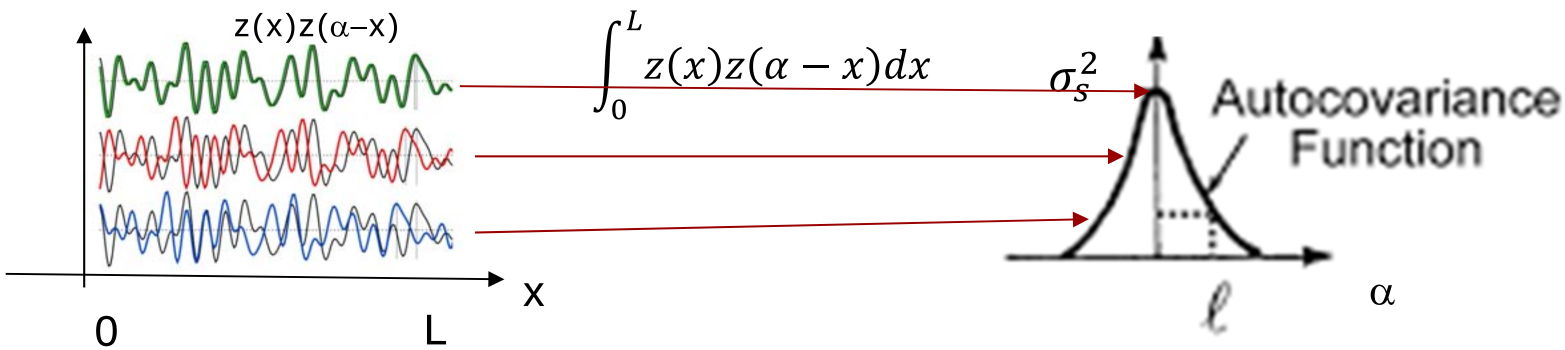
Surface height distribution and surface spatial distribution



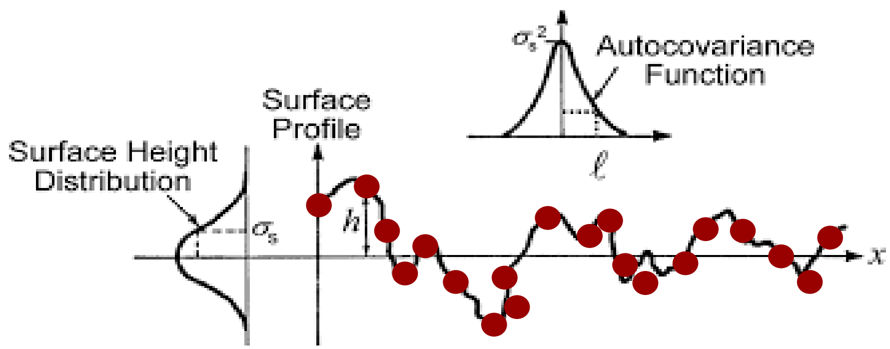
Height of points -> Surface Height Distribution



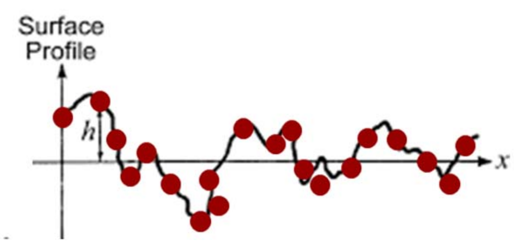
Position of points -> Autocovariance function



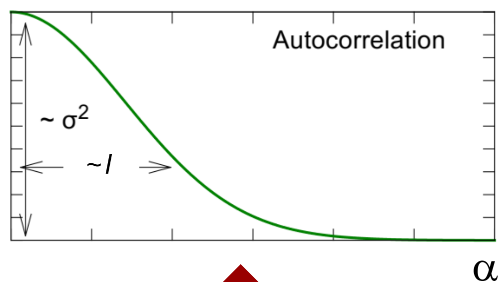
Surface height distribution and surface spatial distribution



l is correlation length
σ is RMS roughness



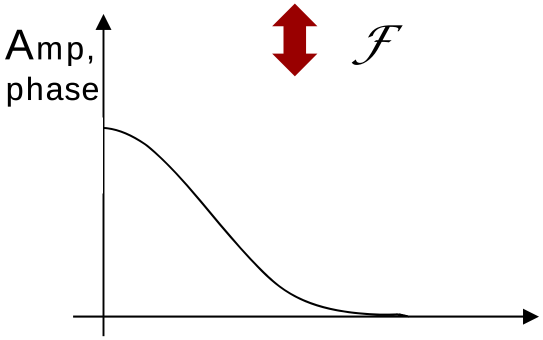
$$\int_0^L z(x)z(\alpha - x)dx$$



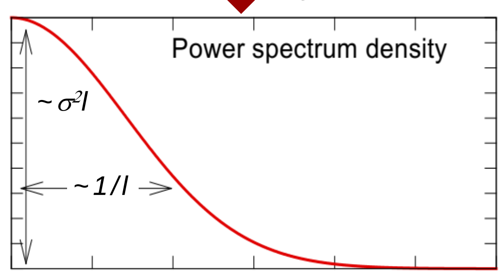
Direct Space

$\mathcal{F}$

$\mathcal{F}$



$| \quad |^2$



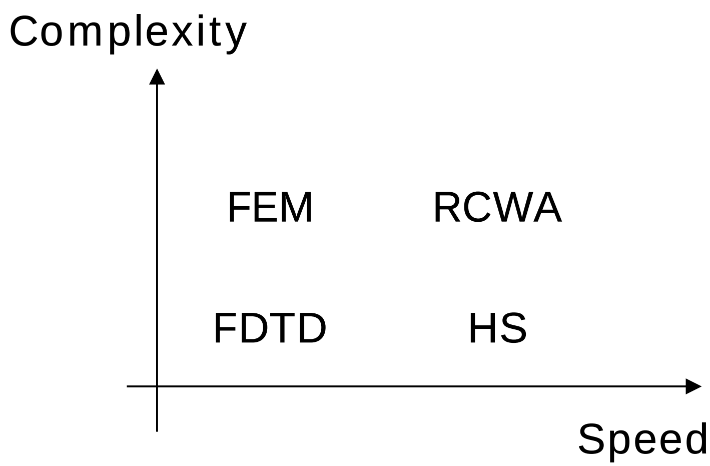
Fourier Space

f_x or k_x

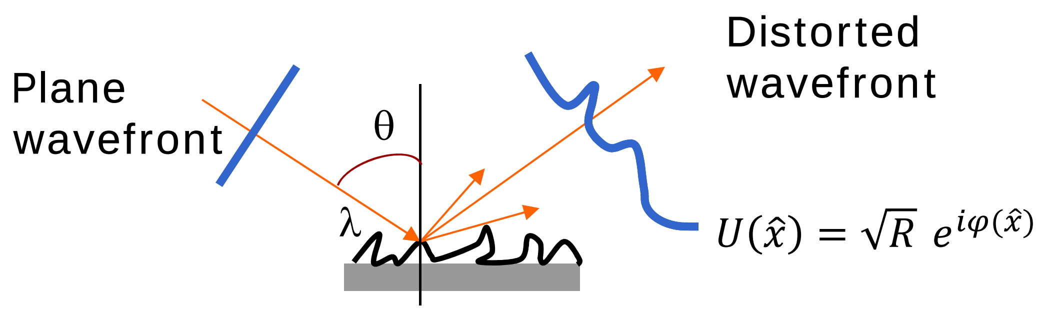
f_x or k_x

Timeline for the development of the different numerical methods for BRDF

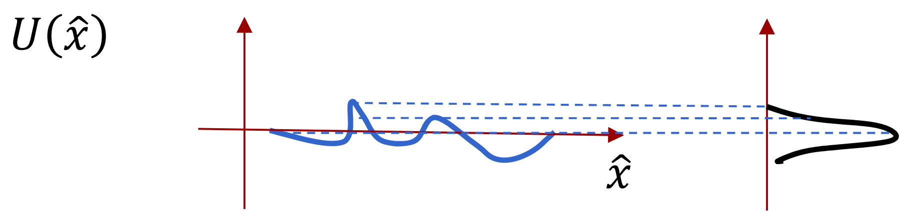
- Finite Element Methods (FEM) 1950->
- Finite Difference Time Domain (FDTD) 1966->
- **Harvey-Shack (HS)** 1976->
- **Rigorous Coupled Wave Analysis (RCWA)** 1995->



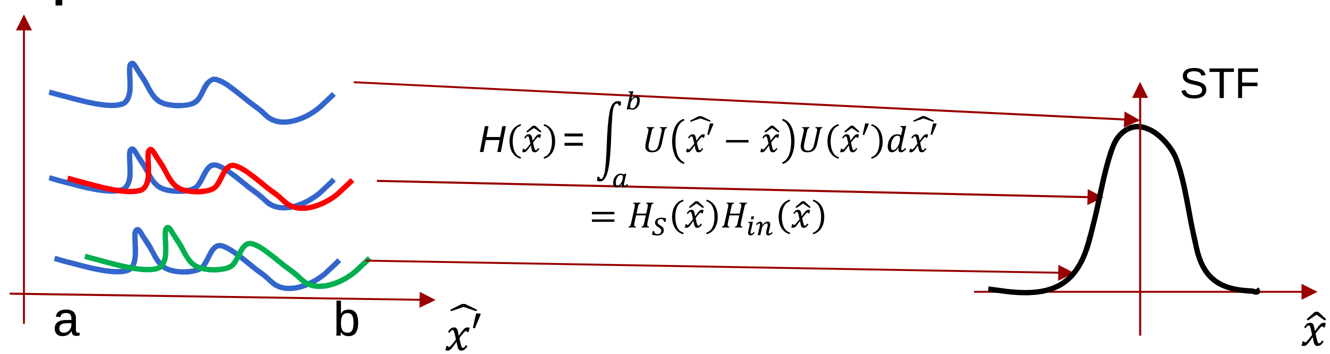
Scalar diffraction models
Light matter interaction



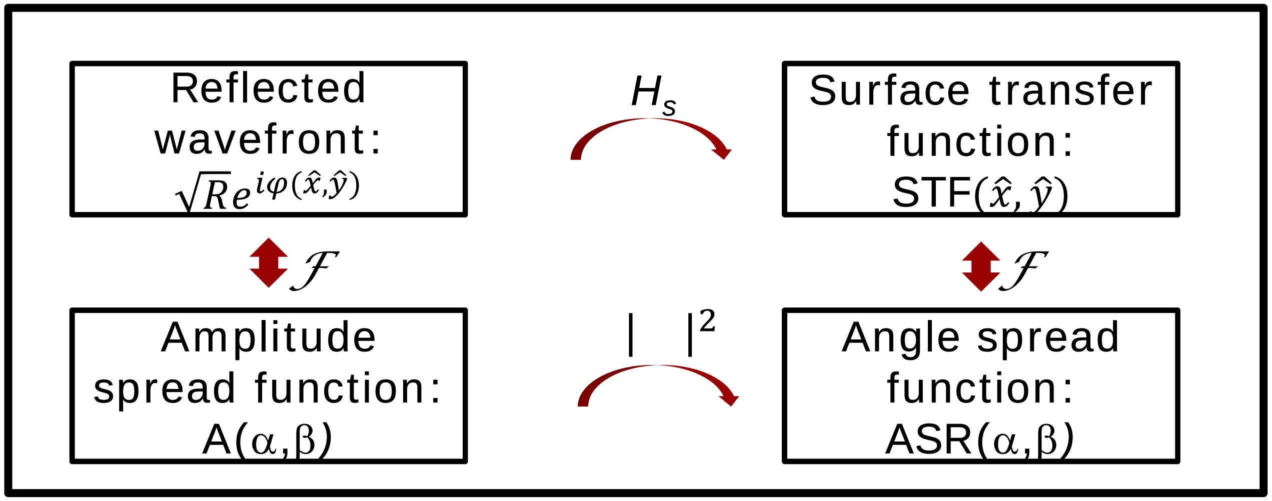
Wavefront points -> Reflectivity ($\sqrt{R}$), phase variation ($e^{i\varphi(\hat{x})}$)



Position of points -> Surface transfer function



Scalar diffraction models
Scattering transfer function



J. Harvey

Autocovariance

Harvey-Shack $$H_S(\hat{x},\hat{y};0,0) = e^{-(4\pi\hat{\sigma}_s)^2}\left(1 - F^{-1}\left(PSD(\alpha,\beta)\right)/\sigma_s^2\right)$$

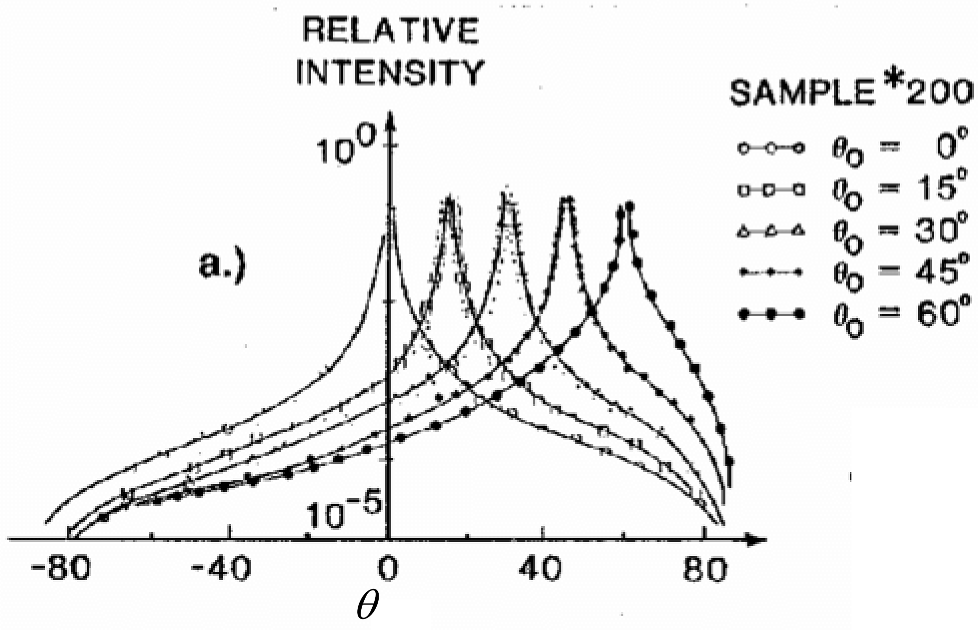
Surface characteristic

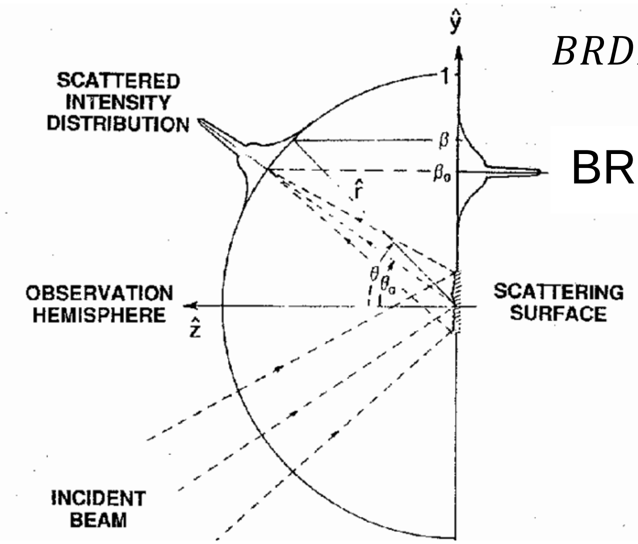
Optical characteristic

$$\boldsymbol{BRDF} = Q \quad F^{-1}(H_s)$$

$$\alpha = \lambda f_x$$
$$\beta = \lambda f_y$$

BRDF any angle of incident



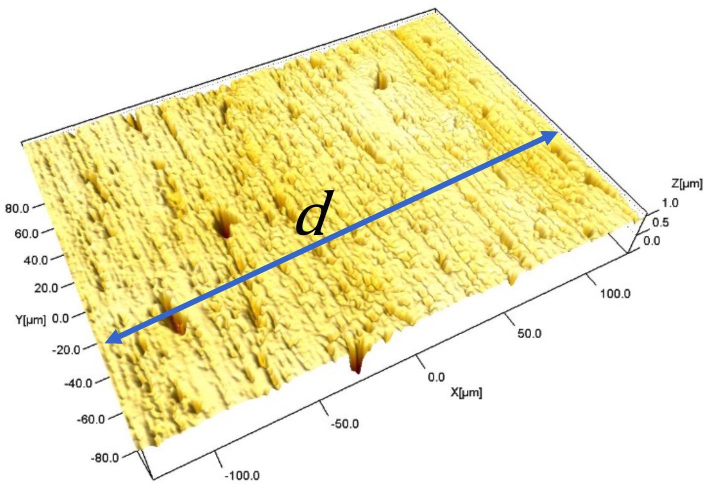


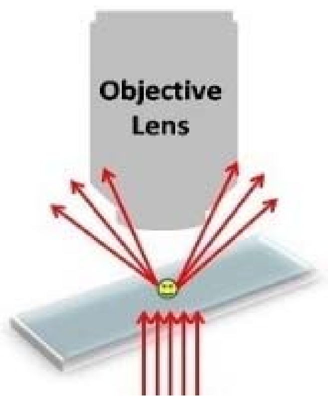
$$BRDF|_{\theta_i,\varphi_i,\theta_s,\varphi_s} = Q\,F\left(H_s\, e^{-i2\pi\hat{y}\beta_o}\right)\Big|_{\alpha.\beta=\alpha_s.\beta_s}$$

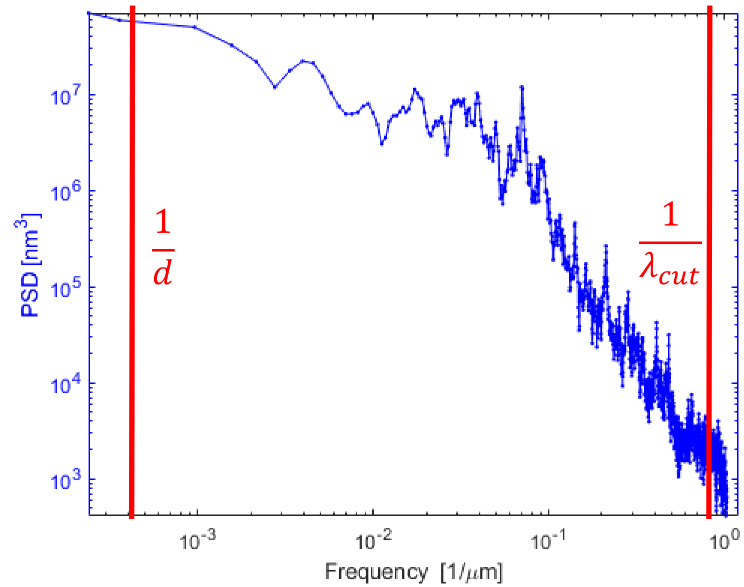
BRDF

Images from: Understanding surface scatter phenomena SPIE 2019

Influence of experiment







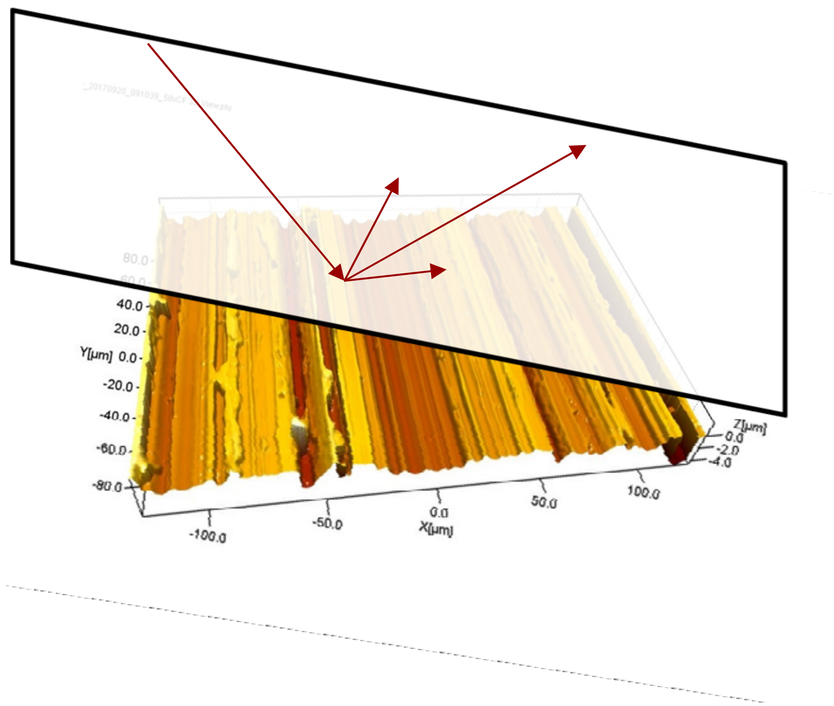
Spatial frequency f (1/µm)

$$\sigma_q^2 = 4\int_{f_{min}}^{f_{max}} PSD(f)\, df \qquad \frac{1}{d} < f < \frac{1}{\lambda_{cut}}$$

Frequency limit depends on experimental setup!

$\frac{1}{\lambda_{cut}}$: Frequency of collected angle

Influence of experiment



Your PSD function should reflect the physical situation!
PSD evaluate along the light direction!

Difficult, should be verified with more rigorous methods

Multiple experiments in one analysis

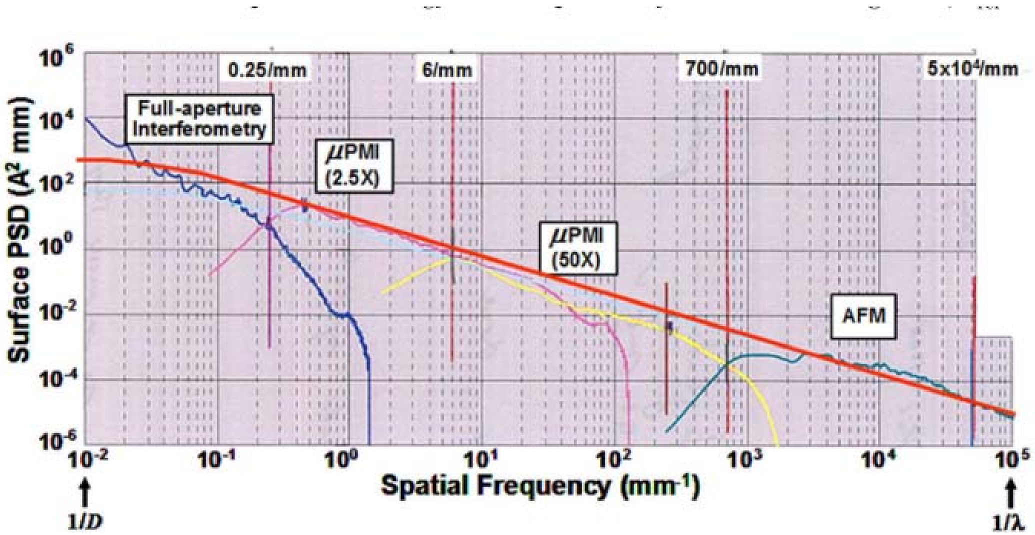


Image from: Understanding surface scatter phenomena SPIE 2019

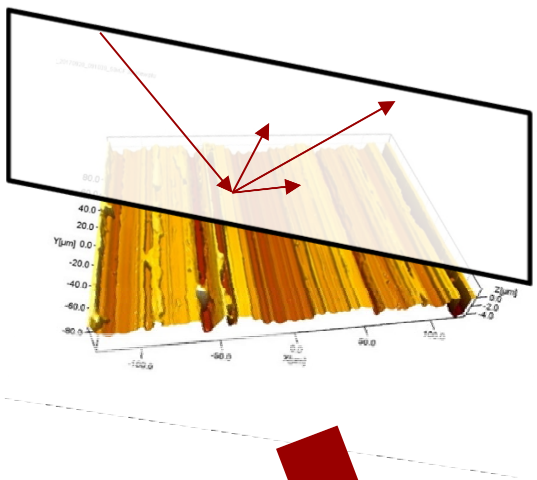
Generalized Harvey Shack

$$BRDF|_{\theta_i,\varphi_i,\theta_s,\varphi_s} = Q\ ARS(\alpha,\beta) = Q\ F^{-1}\left(H_s\ \right)$$

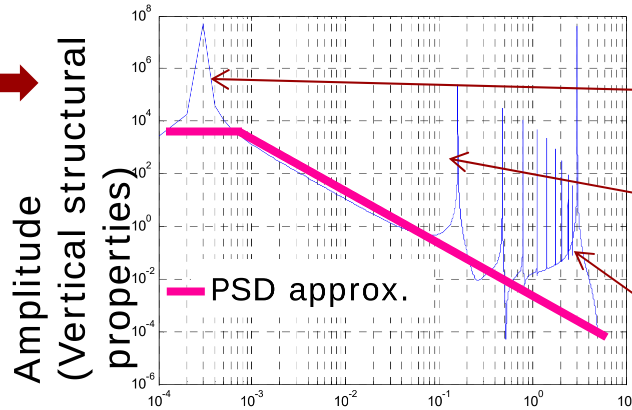
$$H_S(\hat{x},\hat{y};\gamma_i\ ,\gamma_s\ ) = e^{-\left(2\pi(\gamma_i\ +\gamma_s\ )\hat{\sigma}_{rel}\right)^2}\left(1 - F^{-1}\left(PSD(\alpha,\beta)\right)/\sigma_s^2\right)$$

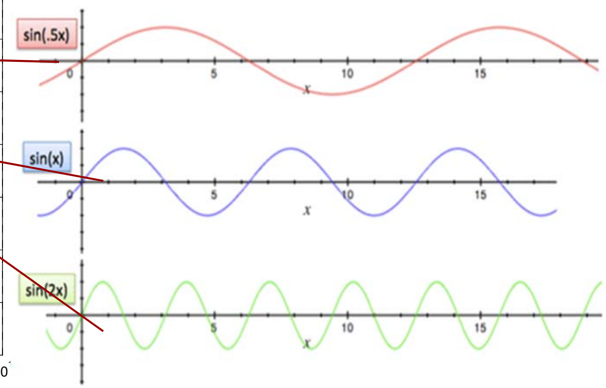
Surface characterization beyond Harvey-Shack method (RCWA)

PSD



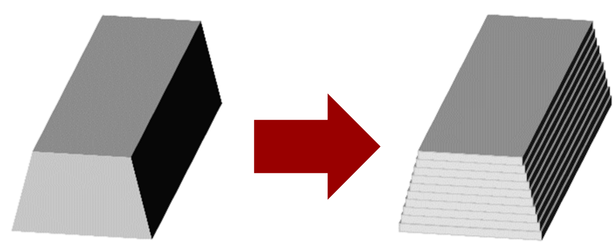
HS





Periodic function (RCWA)

Validation: aRCWA



$$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\varepsilon}$$

$$\nabla \cdot \mathbf{H} = 0$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon \frac{\partial \mathbf{E}}{\partial t}$$

For more information

peh@dfm.dk

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