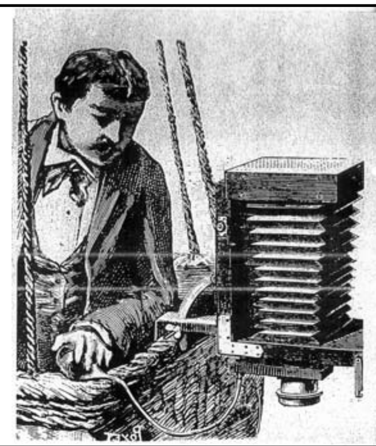
Image analysis, vision and computer graphics.

Pinhole Camera
Homogeneous Coordinates
Outer Orientation



NIÉPCE, Joseph Nicephore

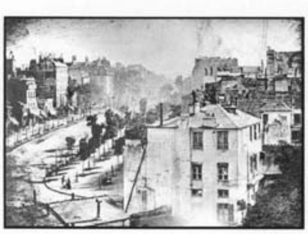
b. 7 March 1765; d. 5 July 1833

Niépce (pronounced Nee-ps) is universally credited with producing the first successful photograph in June/July 1827. He was fascinated with lithography, and worked on this process. Unable to draw, he needed the help of his artist son to make the images. However, when in 1814 his son was drafted into the army to fight at Waterloo, he was left having to look for another way of obtaining images. Eventually he succeeded, calling his product Heliographs (after the Greek "of the sun"). Lady Elizabeth Eastlake, writing in 1857, informs us that he was a man of private means, who had began his researches in 1814. When he eventually succeeded, he came over to England later that year and sought to promote his invention via the Royal Society (then as now regarded as the leading learned body concerned with science). However, the Royal Society had a rule that it would not publicise a discovery that contained an undivulged secret, so Niépce meet with total failure. Returning to France, he teamed up with Louis Daguerre in 1829, a partnership which lasted until his death only four years later, at the age of 69. He left behind him some examples of his heliographs, which are now in the Royal Photographic Society's collection.

This is the first known photograph.** There is little merit in this picture other than that fact. It is difficult to decipher: the building is on the left, a tree a third in from the left, and a barn immediately in front. The exposure lasted eight hours, so the sun had time to move from east to west, appearing to shine on both sides of the building.

For further information on Niepce, see here.

** I have been taken to task by some who point to the picture in the Turin Shroud as being the first photograph. Whether the shroud dates back to the time of Jesus Christ, which most scholars discount, or whether it dates from around 1000AD, it does certainly show an image of a dead person. Whether this was produced intentionally though is more unlikely. The picture shown here is generally acknowledged to be the first image produced intentionally.

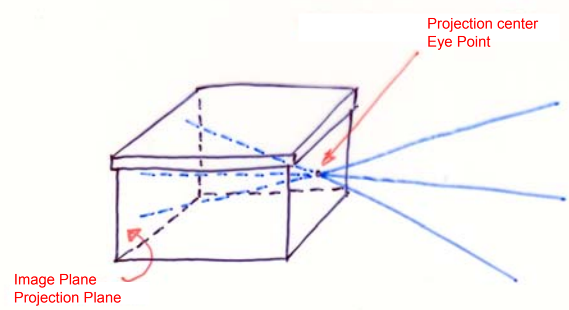


Taken in 1839, this picture of a boulevard gives the impression of empty streets, because with long exposures moving objects would not register.

However, there was an exception when a man stopped to have his shoes shined, (see bottom left of the larger picture) and though he remains anonymous he may have the distinction of being the first person ever to have been photographed.

In 1851 Daguerre died. In a sense this symbolically ended an era, for that very same year a new technique was invented, which was another milestone in photography - the wet collodion process by Frederick Scott Archer.

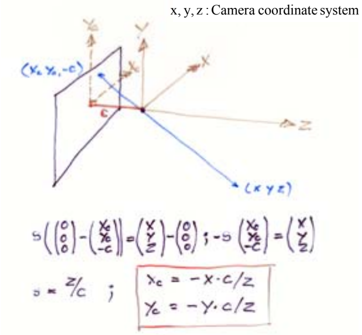
There is considerable material to be found in the Daguerrian Society's web-site. Do have a look.

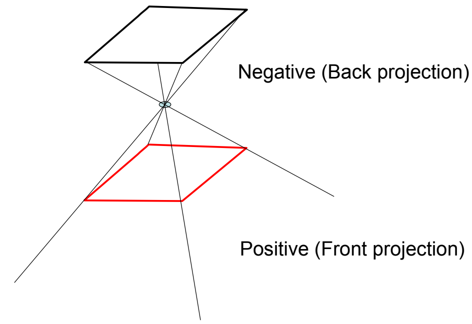


HULKAMERA (pinholecamera)

Pinhole camera – Perspective projection

c : Focal length (camera constant)
x_c, y_c : Image coordinate system
x, y, z : Camera coordinate system





Opgave 10:

Vi betragter et hulkamera med kamerakonstanten 0.01 og et plan, der beskrives ved ligningen x + y + z = 4 i kamerakoordinatsystemet.

Et punkt beliggende i planet afbildes i billedkoordinaterne:

$(x_c, y_c) = (-0.02, -0.01)$

Angiv punktets koordinater i kamerakoordinatsystemet.

1) (x, y, z) = (1, 1, 2)
2) (x, y, z) = (1, 2, 1)
3) (x, y, z) = (2, 1, 1)
4) (x, y, z) = (1, -2, 1)
5) (x, y, z) = (-2, 1, 1)
6) Ved ikke

Opgave 10: Modelløsning

Ved hjælp af formel [illegible].1 i forelæsningsnoterne beregnes en parameterfremstilling for den retlinede stråle:

$$x = -\frac{-0.02}{0.01}\cdot z = 2\cdot z$$
$$y = -\frac{-0.01}{0.01}\cdot z = z$$

Ved at indsætte i planets ligning kan punktets z koordinat findes:

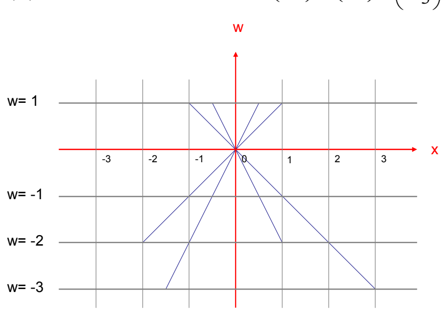
$$2\cdot z + z + z = 4 \Rightarrow z = 1$$

Koordinaterne til punktet findes ved at indsættes z i parameterfremstillingen:

(x, y, z) = (2, 1, 1).

Homogeneous Coordinates

$$\begin{pmatrix}x\\w\end{pmatrix}\rightarrow(x)\qquad \begin{pmatrix}1\\-2\end{pmatrix}\ \begin{pmatrix}3\\-3\end{pmatrix}\ \begin{pmatrix}-\frac{3}{2}\\-3\end{pmatrix}\ \begin{pmatrix}-2\\-2\end{pmatrix}$$



$$\hat{\mathbf{X}}' = \mathbf{A}\cdot\mathbf{X}$$

$$\begin{pmatrix}wX'\\wY'\\wZ'\\w\end{pmatrix}=\begin{pmatrix}a_{11}&a_{12}&a_{13}&a_{14}\\a_{21}&a_{22}&a_{23}&a_{24}\\a_{31}&a_{32}&a_{33}&a_{34}\\a_{41}&a_{42}&a_{43}&a_{44}\end{pmatrix}\cdot\begin{pmatrix}X\\Y\\Z\\1\end{pmatrix}$$

$$\begin{pmatrix} wX' \\ wY' \\ wZ' \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} wX' \\ wY' \\ wZ' \\ w \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\begin{pmatrix} wX' \\ wY' \\ wZ' \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} wX' \\ wY' \\ wZ' \\ w \end{pmatrix} = \begin{pmatrix} X \\ Y \\ 0 \\ 1 \end{pmatrix}$$

Orthographic (orthogonal) projection
ex. map projection

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} X \\ Y \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} wX' \\ wY' \\ wZ' \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & X_0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} wX' \\ wY' \\ wZ' \\ w \end{pmatrix} = \begin{pmatrix} X + X_0 \\ Y \\ Z \\ 1 \end{pmatrix}$$

Translation

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} X_0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} wX' \\ wY' \\ wZ' \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & m \end{pmatrix} \cdot \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} wX' \\ wY' \\ wZ' \\ w \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \\ m \end{pmatrix}$$

Overall scale

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \frac{1}{m} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\begin{pmatrix} wX' \\ wY' \\ wZ' \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & m_y & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} wX' \\ wY' \\ wZ' \\ w \end{pmatrix} = \begin{pmatrix} X \\ m_y \cdot Y \\ Z \\ 1 \end{pmatrix}$$

Individual scale

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} X \\ m_y \cdot Y \\ Z \end{pmatrix}$$

$$\begin{pmatrix} wX' \\ wY' \\ wZ' \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Omega & -\sin\Omega & 0 \\ 0 & \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} wX' \\ wY' \\ wZ' \\ w \end{pmatrix} = \begin{pmatrix} X \\ Y \cdot \cos\Omega - Z \cdot \sin\Omega \\ Y \cdot \sin\Omega + Z \cdot \cos\Omega \\ 1 \end{pmatrix}$$

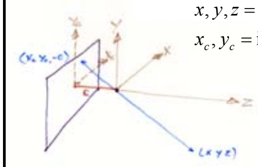
Rotation around X

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\Omega & -\sin\Omega \\ 0 & \sin\Omega & \cos\Omega \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\begin{pmatrix} wX' \\ wY' \\ wZ' \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/c & 0 \end{pmatrix} \cdot \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} wX' \\ wY' \\ wZ' \\ w \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \\ -Z/c \end{pmatrix}$$

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} -X\frac{c}{Z} \\ -Y\frac{c}{Z} \\ -c \end{pmatrix}$$

Perspective projection



x, y, z = camera coordinates

x_c, y_c = image coordinates

$$-s \cdot \begin{pmatrix} x_c \\ y_c \\ -c \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}; \quad x_c = -x \cdot c/z \quad y_c = -y \cdot c/z \quad s = z/c$$

Homogeneous coordinates : $\hat{\mathbf{x}}_c = \mathbf{P} \cdot \hat{\mathbf{x}}$

$$\begin{pmatrix} w \cdot x_c \\ w \cdot y_c \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/c & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Transformation in homogeneous coordinates

$$\hat{\mathbf{X}}' = \mathbf{A} \cdot \hat{\mathbf{X}}$$

Rotation and affine transformation

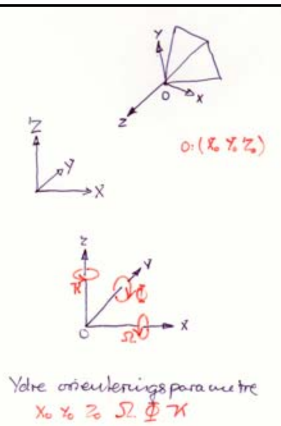
Translation

$$\begin{pmatrix} wX' \\ wY' \\ wZ' \\ w \end{pmatrix} = \begin{pmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix} \cdot \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

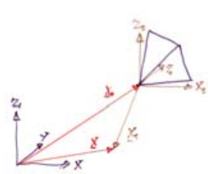
Perspective projection

Scale

Outer orientation



Translation



$$\mathbf{X} = \mathbf{X}_0 + \mathbf{X}_t$$

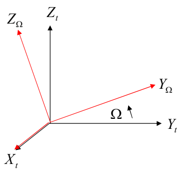
$$\begin{pmatrix} X_t \\ Y_t \\ Z_t \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} - \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix}$$

Homogeneous coordinates

$$\hat{\mathbf{X}}_t = \mathbf{T} \cdot \hat{\mathbf{X}}$$

$$\begin{pmatrix} wX_t \\ wY_t \\ wZ_t \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & -X_0 \\ 0 & 1 & 0 & -Y_0 \\ 0 & 0 & 1 & -Z_0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Rotation



$$\begin{pmatrix} X_{t\Omega} \\ Y_{t\Omega} \\ Z_{t\Omega} \end{pmatrix} = \begin{pmatrix} X_t \\ Y_t \cdot \cos\Omega + Z_t \cdot \sin\Omega \\ -Y_t \cdot \sin\Omega + Z_t \cdot \cos\Omega \end{pmatrix}$$

Homogeneous coordinates $\quad \hat{\mathbf{X}}_{t\Omega} = \mathbf{R}_\Omega \cdot \hat{\mathbf{X}}_t$

$$\begin{pmatrix} wX_{t\Omega} \\ wY_{t\Omega} \\ wZ_{t\Omega} \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Omega & \sin\Omega & 0 \\ 0 & -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X_t \\ Y_t \\ Z_t \\ 1 \end{pmatrix}$$

$$\hat{\mathbf{X}}_{t\Omega} = \mathbf{R}_\Omega \cdot \hat{\mathbf{X}}_t \qquad \begin{pmatrix} wX_\Omega \\ wY_\Omega \\ wZ_\Omega \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Omega & \sin\Omega & 0 \\ 0 & -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X_t \\ Y_t \\ Z_t \\ 1 \end{pmatrix}$$

$$\hat{\mathbf{X}}_{t\Omega\Phi} = \mathbf{R}_\Phi \cdot \hat{\mathbf{X}}_{t\Omega} \qquad \begin{pmatrix} wX_\Omega \\ wY_\Omega \\ wZ_\Omega \\ w \end{pmatrix} = \begin{pmatrix} \cos\Phi & 0 & -\sin\Phi & 0 \\ 0 & 1 & 0 & 0 \\ \sin\Phi & 0 & \cos\Phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X_t \\ Y_t \\ Z_t \\ 1 \end{pmatrix}$$

$$\hat{\mathbf{X}}_{t\Omega\Phi K} = \mathbf{R}_K \cdot \hat{\mathbf{X}}_{t\Omega\Phi} \qquad \begin{pmatrix} wX_\Omega \\ wY_\Omega \\ wZ_\Omega \\ w \end{pmatrix} = \begin{pmatrix} \cos K & \sin K & 0 & 0 \\ -\sin K & \cos K & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X_t \\ Y_t \\ Z_t \\ 1 \end{pmatrix}$$

$$\hat{\mathbf{X}}_{t\Omega\Phi K} = \hat{\mathbf{x}} = \mathbf{R}_K \cdot \mathbf{R}_\Phi \cdot \mathbf{R}_\Omega \cdot \hat{\mathbf{X}}_t \qquad \boxed{\mathbf{R}^T = \mathbf{R}_K \cdot \mathbf{R}_\Phi \cdot \mathbf{R}_\Omega}$$

$$\mathbf{R}^T = \begin{pmatrix} r_{11} & r_{21} & r_{31} \\ r_{12} & r_{22} & r_{32} \\ r_{13} & r_{23} & r_{33} \end{pmatrix}$$

$r_{11} = \cos\Phi\cos K$

$r_{21} = -\cos\Phi\sin K$

$r_{31} = \sin\Phi$

$r_{12} = \cos\Omega\sin K + \sin\Omega\sin\Phi\cos K$

$r_{22} = \cos\Omega\cos K - \sin\Omega\sin\Phi\sin K$

$r_{32} = -\sin\Omega\cos\Phi$

$r_{13} = \sin\Omega\sin K - \cos\Omega\sin\Phi\cos K$

$r_{23} = \sin\Omega\cos K + \cos\Omega\sin\Phi\sin K$

$r_{33} = \cos\Omega\cos\Phi$

Colinearity equations

$$\hat{\mathbf{x}} = \mathbf{P} \cdot \mathbf{R}^T \cdot \mathbf{T} \cdot \hat{\mathbf{X}}$$

$$\begin{pmatrix} wx_c \\ wy_c \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/c & 0 \end{pmatrix} \cdot \begin{pmatrix} r_{11} & r_{21} & r_{31} & 0 \\ r_{12} & r_{22} & r_{32} & 0 \\ r_{13} & r_{23} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & X_0 \\ 0 & 1 & 0 & Y_0 \\ 0 & 0 & 1 & Z_0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$x_c = -c \cdot \frac{r_{11}(X - X_0) + r_{21}(Y - Y_0) + r_{31}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)}$$

$$y_c = -c \cdot \frac{r_{12}(X - X_0) + r_{22}(Y - Y_0) + r_{32}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)}$$

Opgave Ia:

Et positiv billede (front projektion) er optaget med hulkamera. Kamerakonstanten c er 1. Kameraets orientering (rotation og translation) i objektkoordinatsystemet beskrives i homogene koordinater ved hjælp af matricen:

$$\mathbf{D} = \begin{bmatrix} 0 & 1 & 0 & -500 \\ -1 & 0 & 0 & 400 \\ 0 & 0 & 1 & -200 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Beregn billedkoordinater (image coordinates) for et punkt med objektkoordinaterne:

$$\mathbf{X} = \begin{bmatrix} 500 \\ 600 \\ 300 \end{bmatrix}$$

1: (1, -1)
2: (1, 1)
3: (-1, -1)
4: (-2, 1)
5: (2, 2)
6: ved ikke

Opgave Ia: Modelløsning

Ifølge lærebogen side 56 nederst kan den perspektiviske projektion for et positiv billede beskrives i homogene koordinater ved matricen:

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/c & 0 \end{bmatrix}$$

Indsættes kamerakonstanten c = 1, får vi:

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

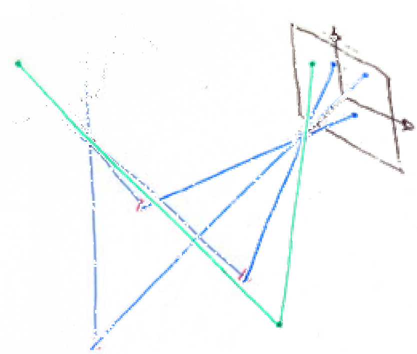
Transformationen fra objektkoordinater til billedkoordinater er givet ved formel 4.18 i lærebogen:

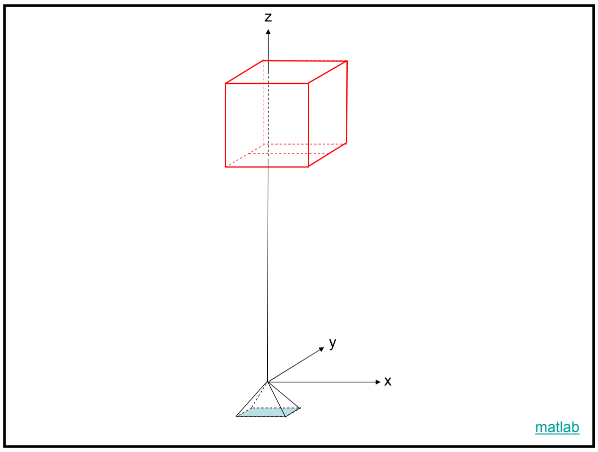
$$\hat{\mathbf{x}}_c = \mathbf{P}\,\mathbf{R}^t\,\mathbf{T}\,\hat{\mathbf{X}}$$

Hvor $\mathbf{R}^t$ beskriver kameraets rotation og T kameraets translation i objektkoordinatsystemet. Altså er $\mathbf{D} = \mathbf{R}^t \cdot \mathbf{T}$. Indsættes P, D, og X får vi:

$$\hat{\mathbf{x}}_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & -500 \\ -1 & 0 & 0 & 400 \\ 0 & 0 & 1 & -200 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 500 \\ 600 \\ 300 \\ 1 \end{bmatrix} = \begin{bmatrix} 100 \\ -100 \\ 100 \end{bmatrix} \Rightarrow \mathbf{x}_c = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Det rigtige svar er 1.




z
y
x
matlab