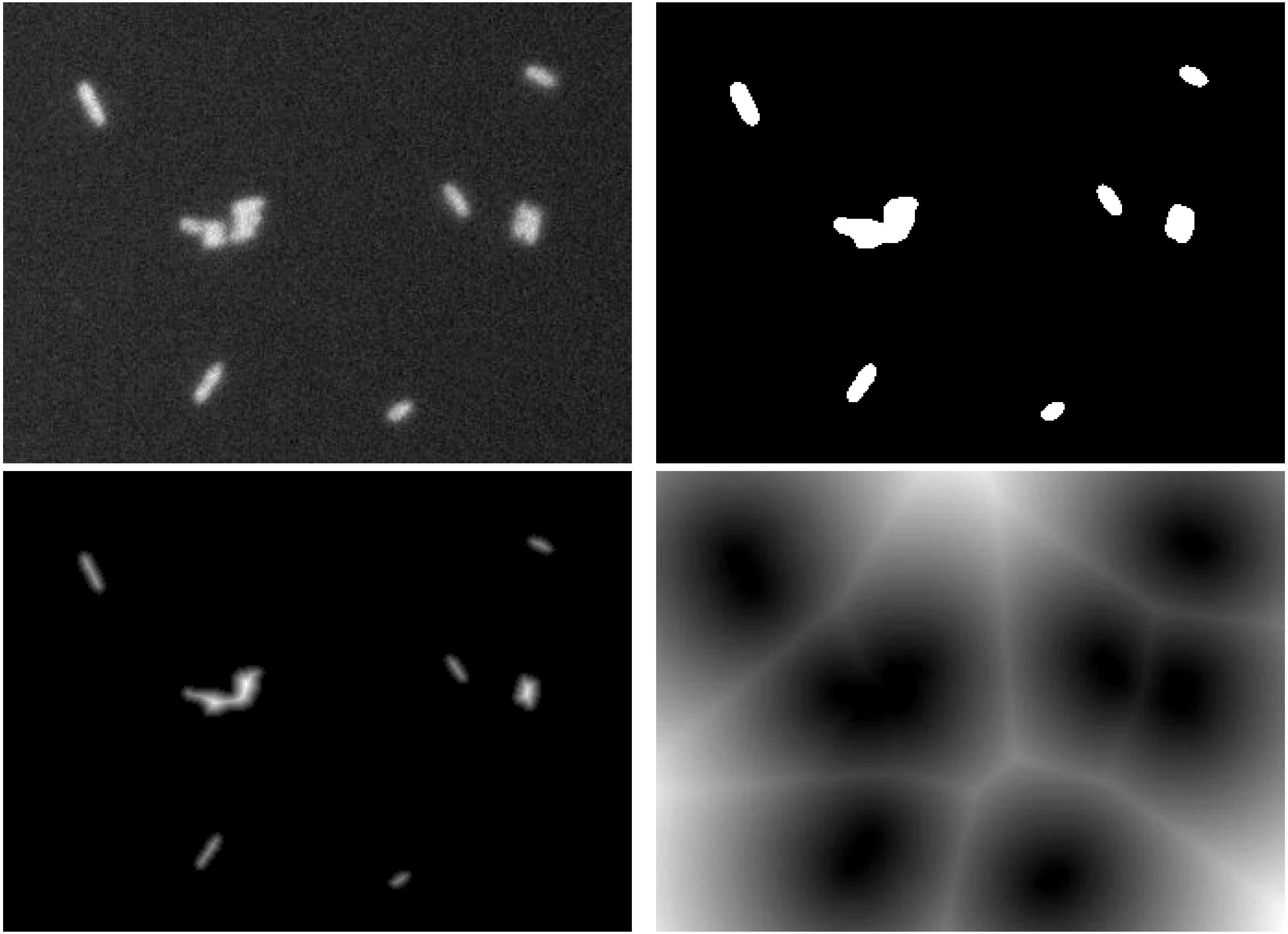
Distance transformation

Binary image → Distance map

Distance transformation

Pixels of one phase are assigned the minimum distance to a pixel (source point) in the other phase

Distance map of bacteria

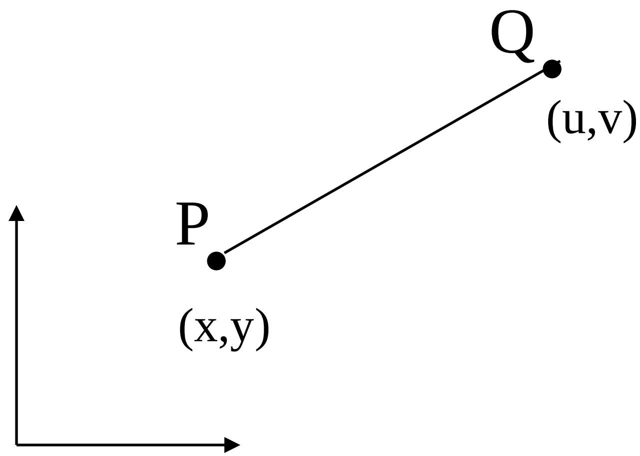


Distance metrics

Definition

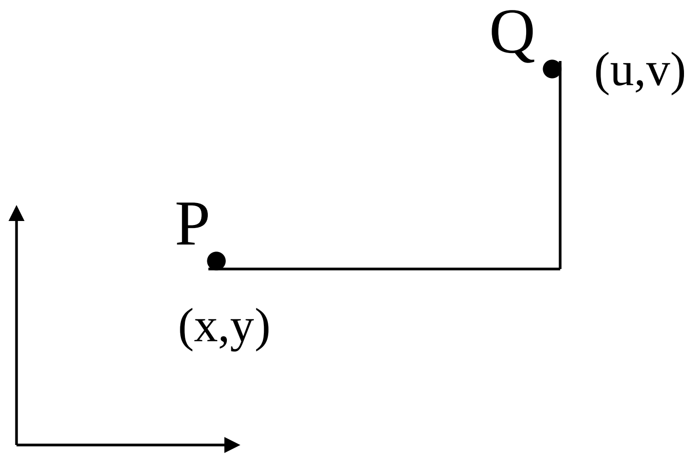
- $d(P,Q) \geq 0$
- $d(P,Q) = d(Q,P)$
- $d(P,R) \leq d(P,Q) + d(Q,R)$

Euclidean distance



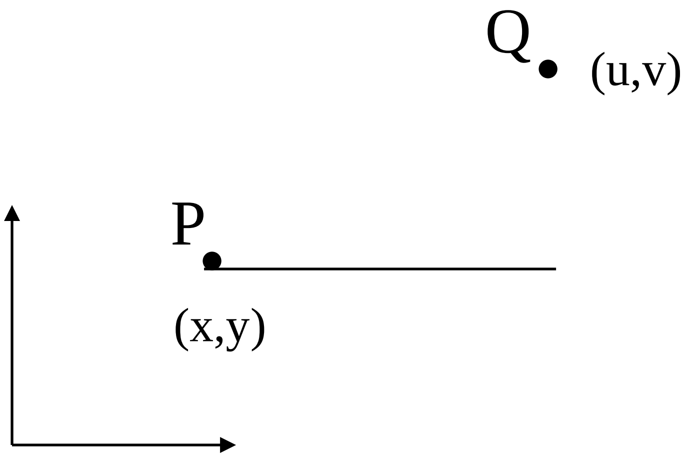
$$d_e(P,Q) = \sqrt{(x-u)^2 + (y-v)^2}$$

City-block distance



$$d_4(P,Q) = |x-u| + |y-v|$$

Chess-board distance



$$d_8(P,Q) = \max(|x-u|, |y-v|)$$

Distance transforms

- Euclidean distance transform (EDT)
- Chamfer distance transform
- Hexagonal distance transform

Euclidean distance transform

- Two component distance map

$$\mathbf{L}(r,c) = (L_r(r,c), L_c(r,c))$$

represented by magnitude

$$\|\mathbf{L}(r,c)\|_2 = \sqrt{L_r^{\,2}(r,c) + L_c^{\,2}(r,c)}$$

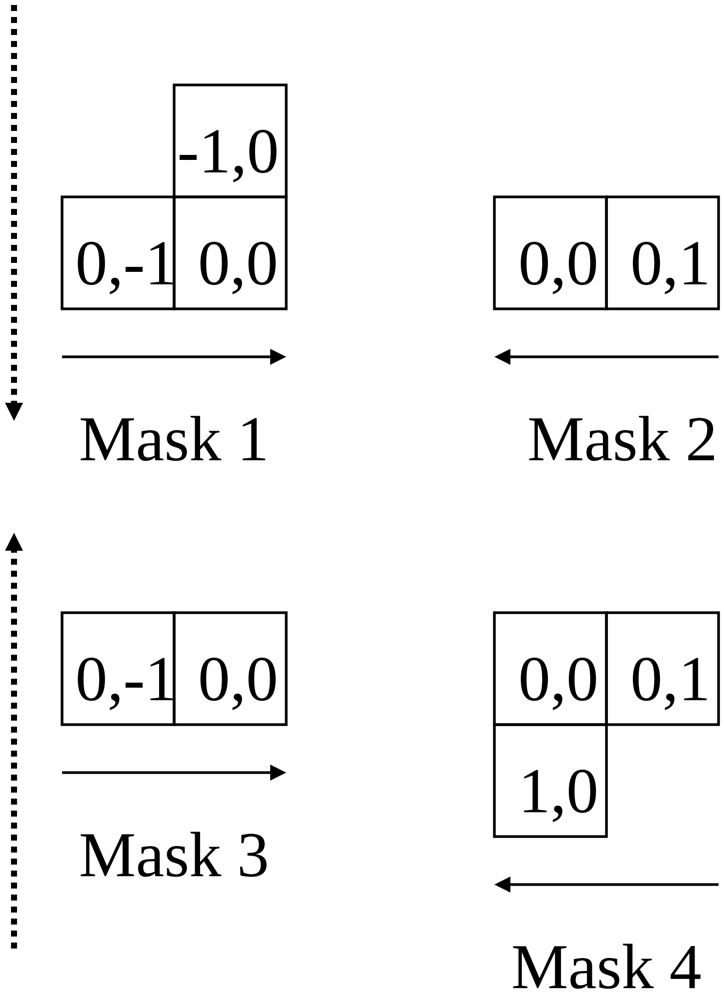
and phase

$$\arg \mathbf{L}(r,c) = \arctan \frac{L_c(r,c)}{L_r(r,c)}$$

- Source point

$$(r_0, c_0) = (r,c) + (L_r(r,c), L_c(r,c))$$

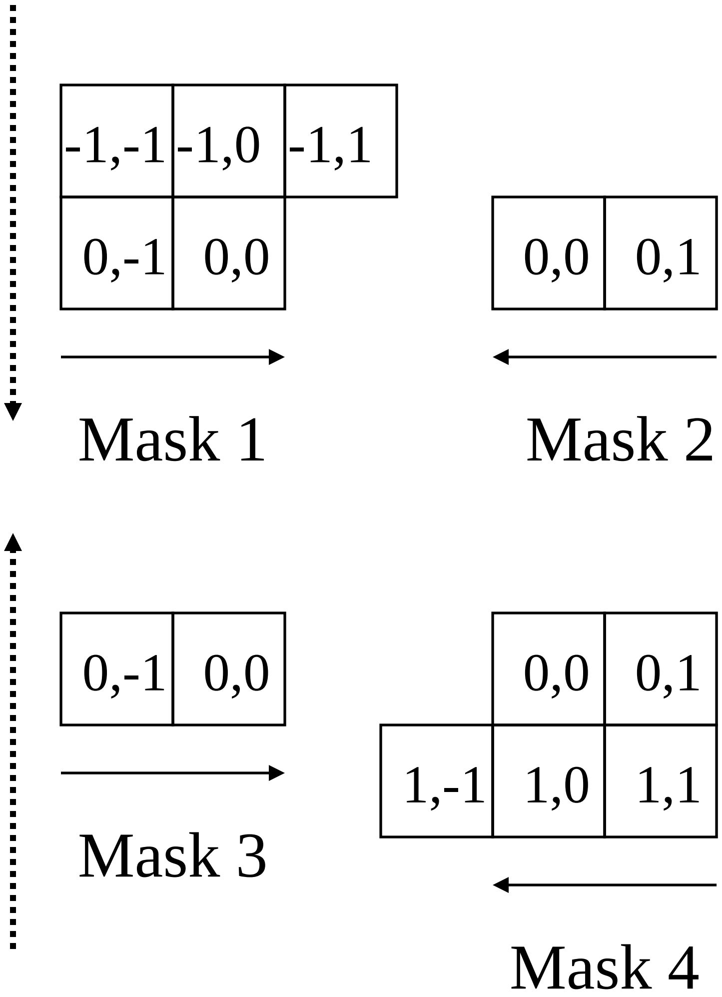
4SSEDT



1. Rowwise down
   1. Mask 1 left to right
   2. Mask 2 right to left
2. Rowwise up
   1. Mask 3 left ro right
   2. Mask 4 right to left

- Recursive update

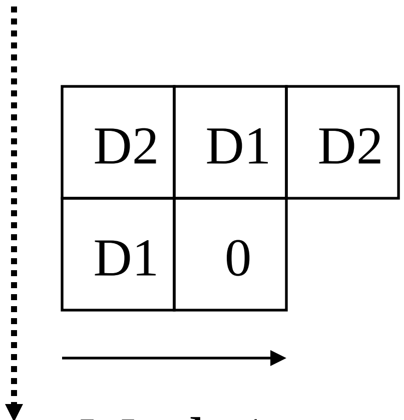
8SSEDT



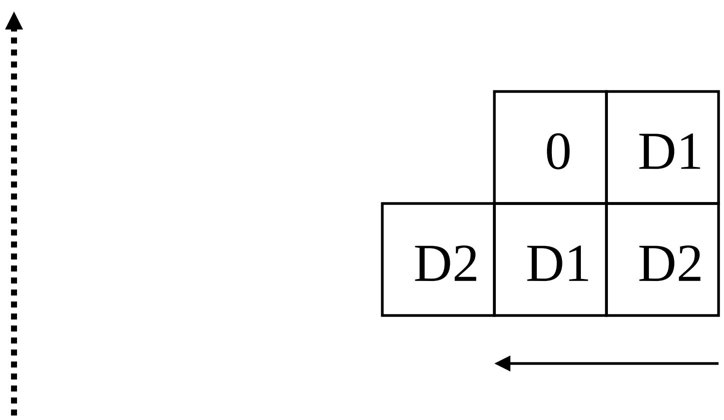
1. Rowwise down
   1. Mask 1 left to right
   2. Mask 2 right to left
2. Rowwise up
   1. Mask 4 right to left
   2. Mask 3 left to right

- Recursive update

Chamfer distance transform



Mask 1



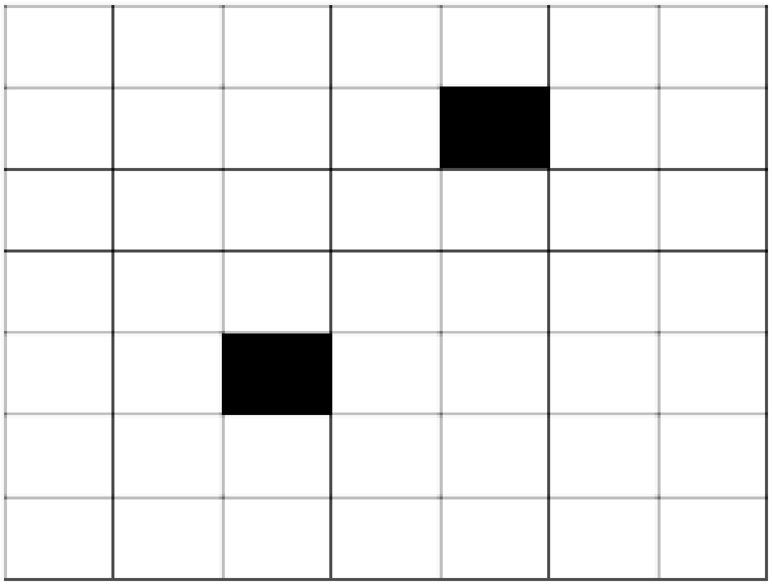
Mask 2

1. Rowwise down
    1. Mask 1 left to right
2. Rowwise up
    1. Mask 2 right to left

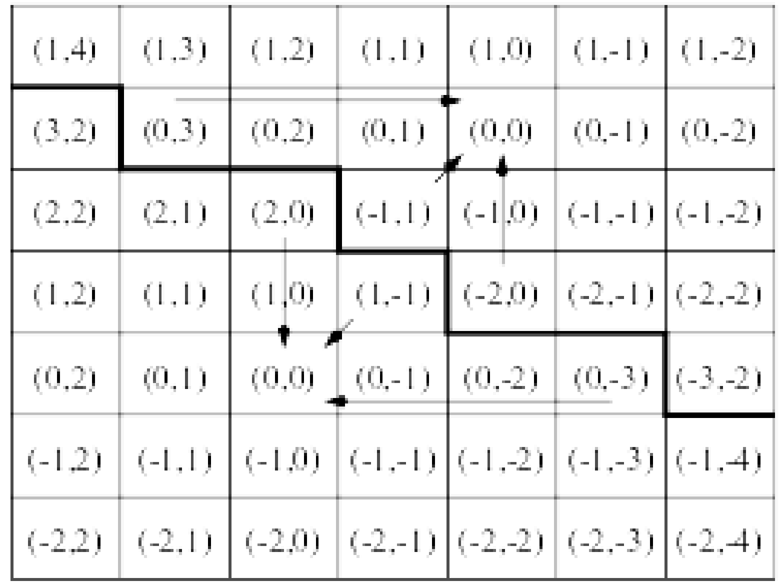
- Recursive update

- Implements City-block DT
- Implements Chess-board DT

EDT example

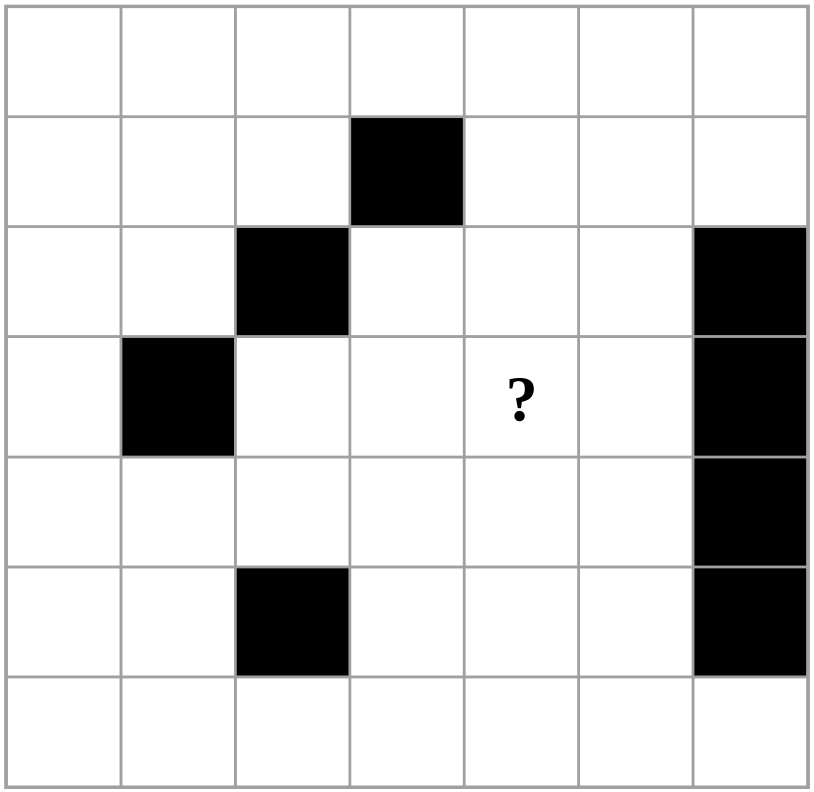


(A)



(B)

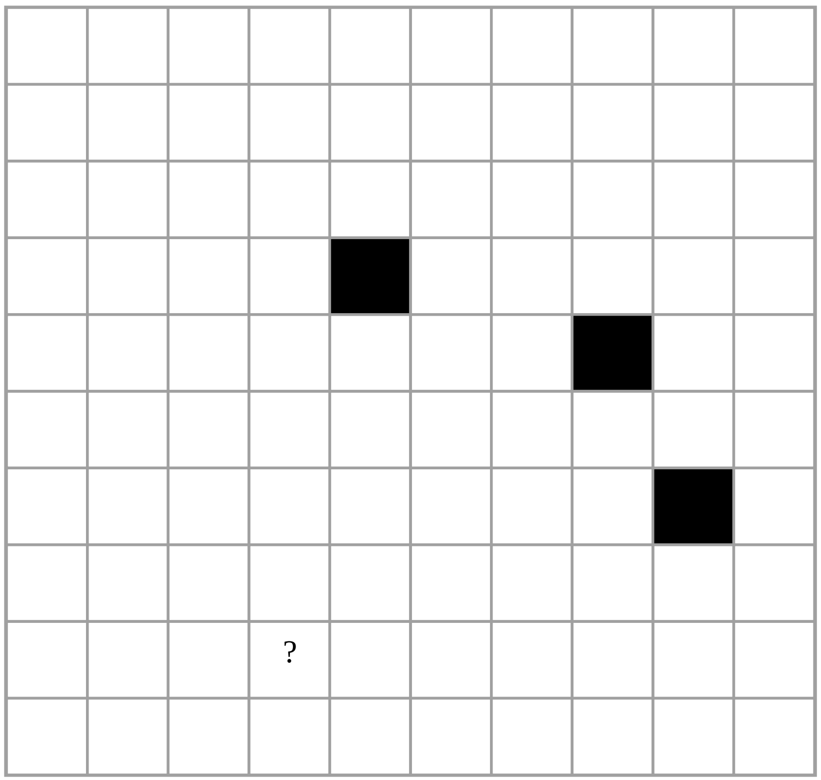
Exam 97.19



The Euclidean distance transform 4SSEDT must be applied to the background (white pixels) in the image above.

What is the value of the pixel marked "?" after two of the four masks have been applied?

Exam 95.11



In the binary image we wish to compute the chamfer(1.1,1.3) distance for the pixel marked with a ?.

What is the value?

Exam 94.15

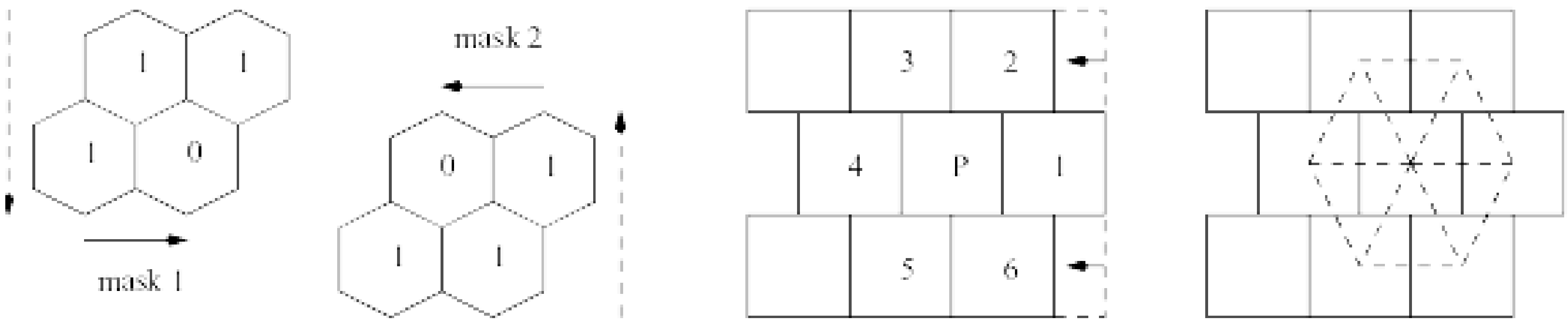
						4
			0			3

This is a partially filled chamfer(a,b) distance map. The shown 0-pixel is the only 0-pixel.

What is (a,b)?

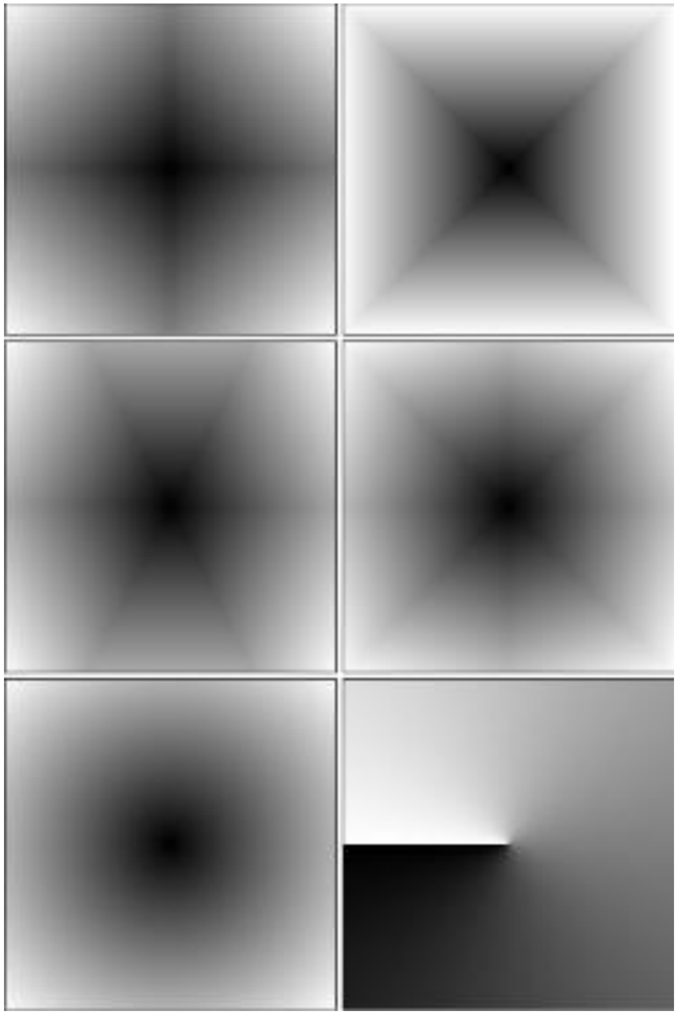
DTU

Hexagonal distance transformation



(A) (B) (C)

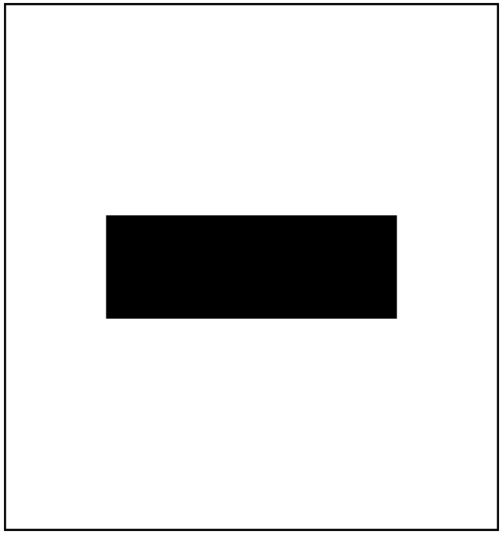
Distance maps

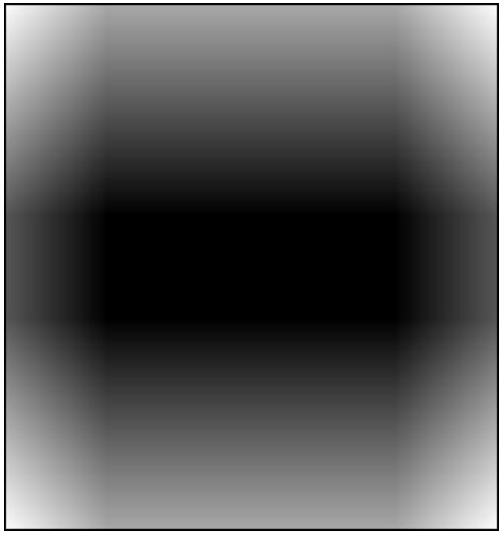


Which is which?

- Euclidean magnitude
- Euclidean phase
- Chamfer(1,1)
- Chamfer(1,2)
- Chamfer(1,1.3507)
- Hexagonal

Exam 96.3

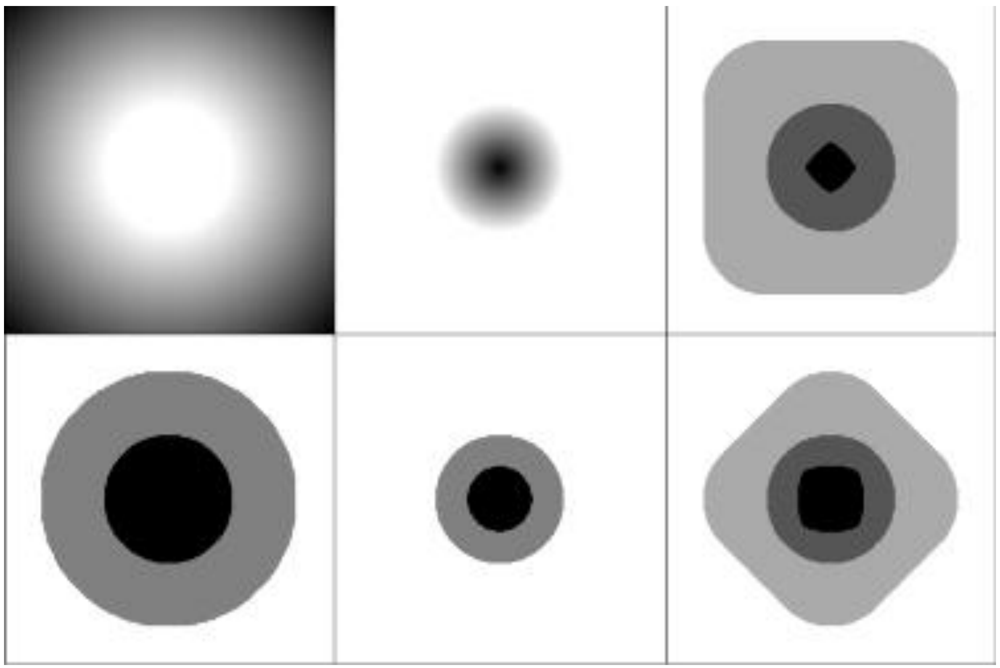




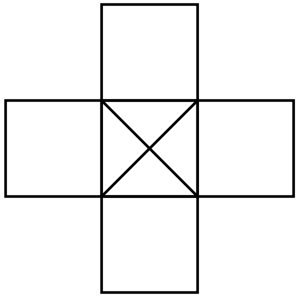
A distance map showing distances to the nearest black pixel is generated for the binary image. The distance map can be displayed as an image and looks like the image below.

Which distance transformation has been used?

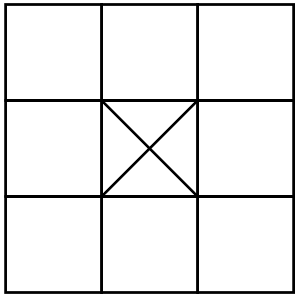
Morphology using DT



5-cross morphology

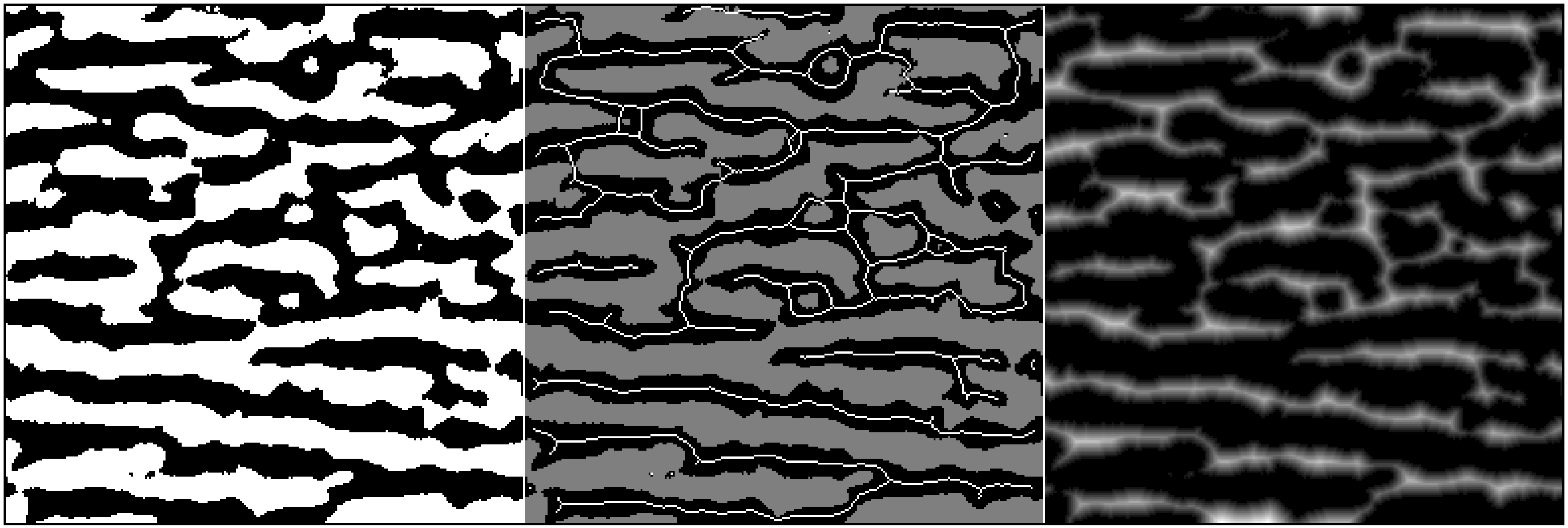


3x3 morphology



Euclidean morphology !

Medial axis transform and EDT

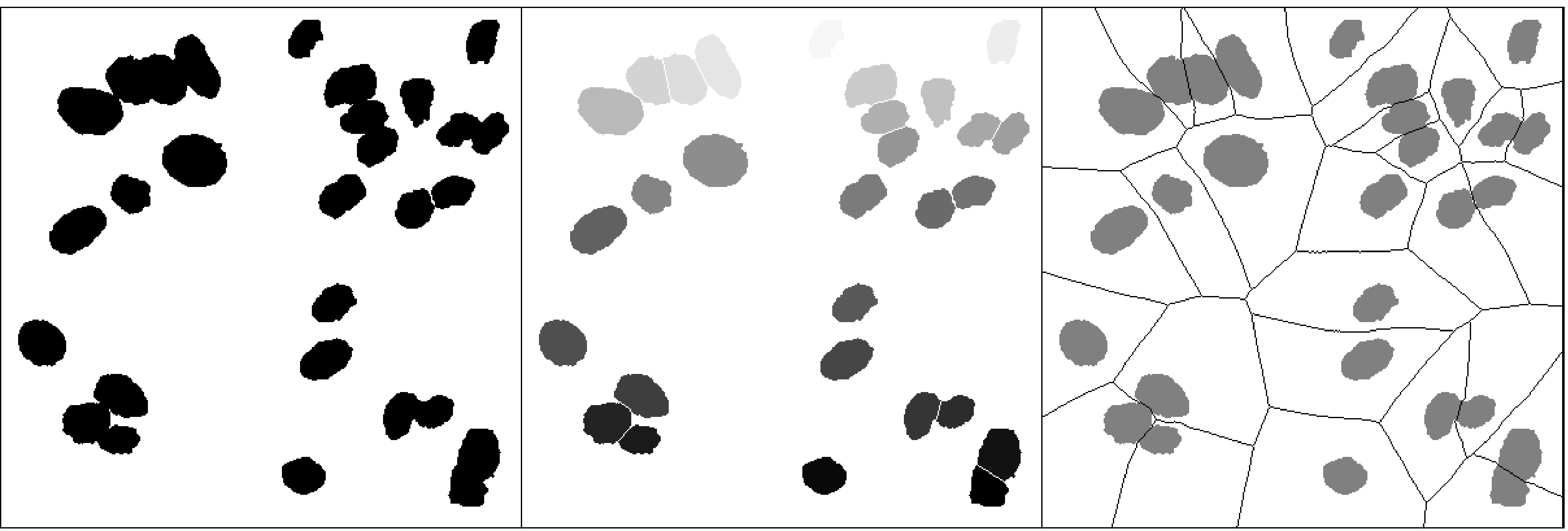


Histogram of EDM-values at EDM points used to express trabecular structures in bones

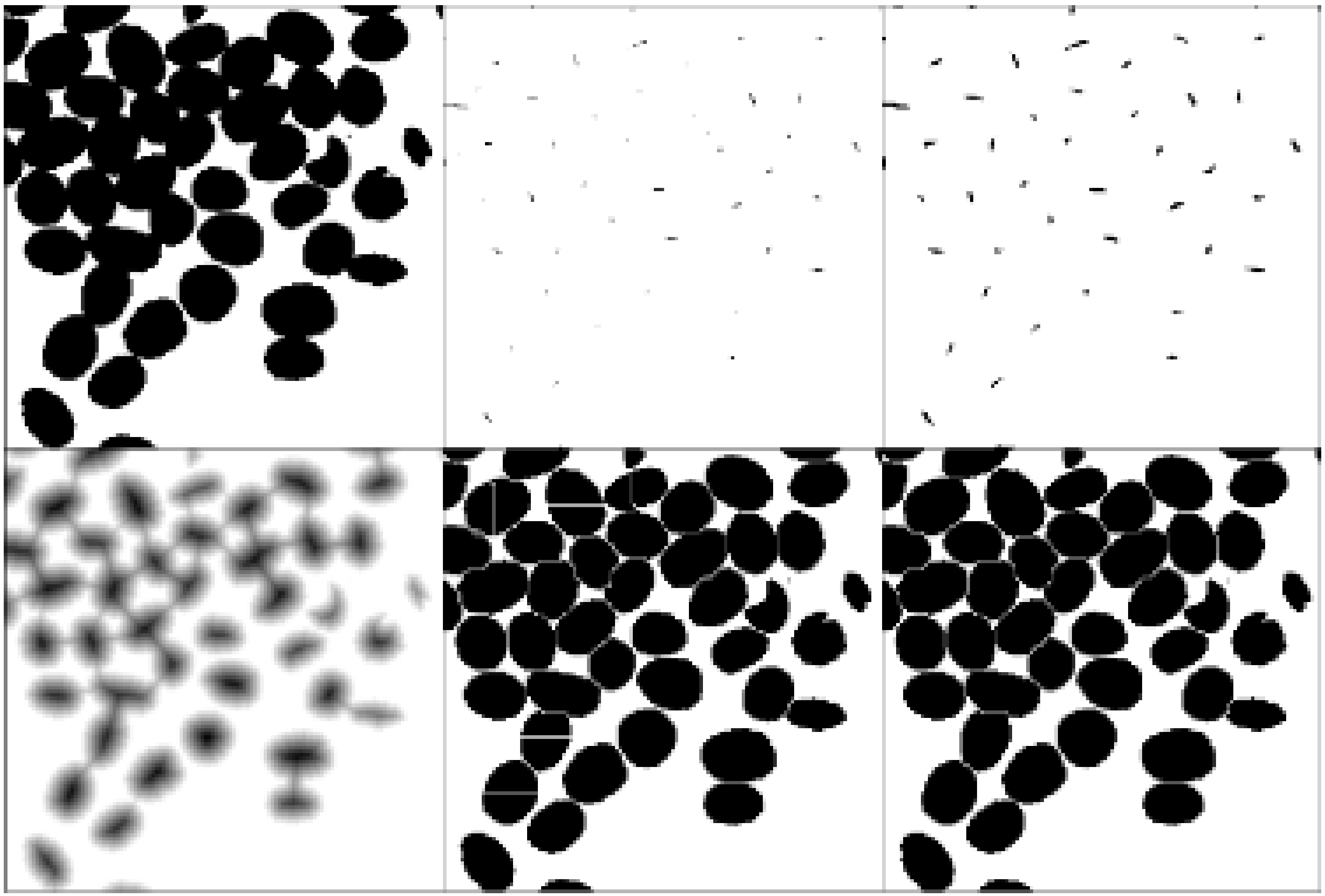
Tesselation by EDT

Assign to every pixel the coordinates of its source point

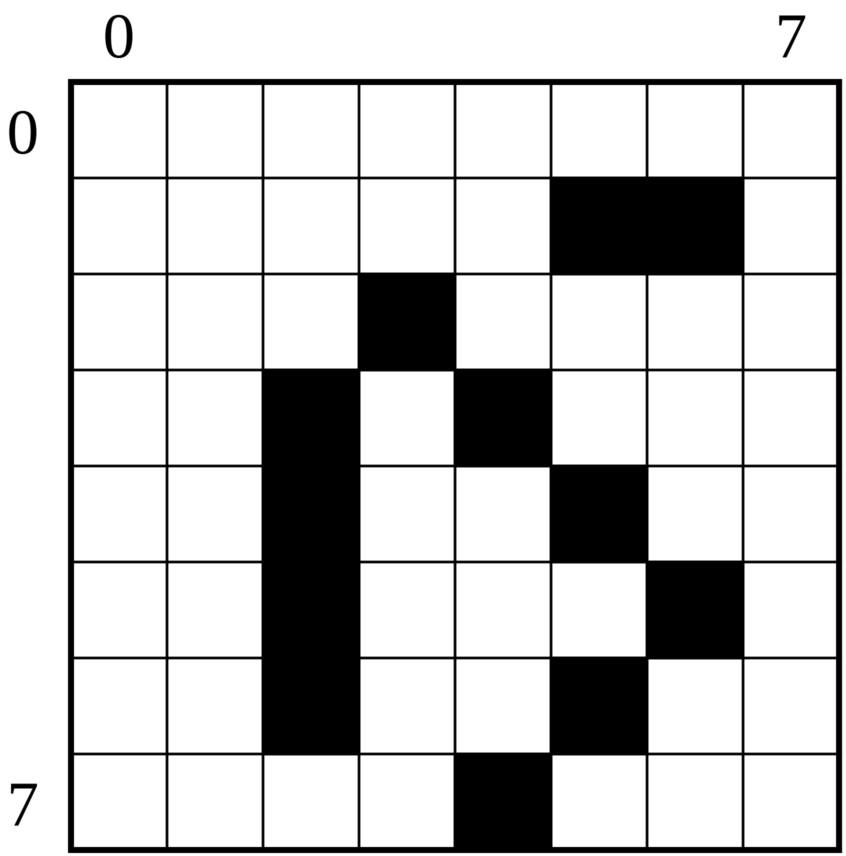
$$(r,c)+(L_r(r,c),L_c(r,c))$$



Separation of cells



Connectivity



How many objects?

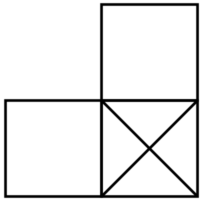
1. 4-connectivity
2. 8-connectivity
3. Object model

Labelling

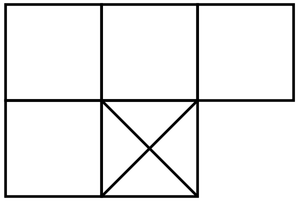
$$L : f \rightarrow l$$

where $f(i, j) \in \{0,1\}$ and $l(i, j) \in \{0,1,\ldots,m\}$

Labelling



4-connectivity



8-connectivity

1. First pass:

   If f(i,j)=0 then l(i,j)=0 else

   If 0 labels assigned then assign new label

   If 1 label assigned in neigborhood then assign same label

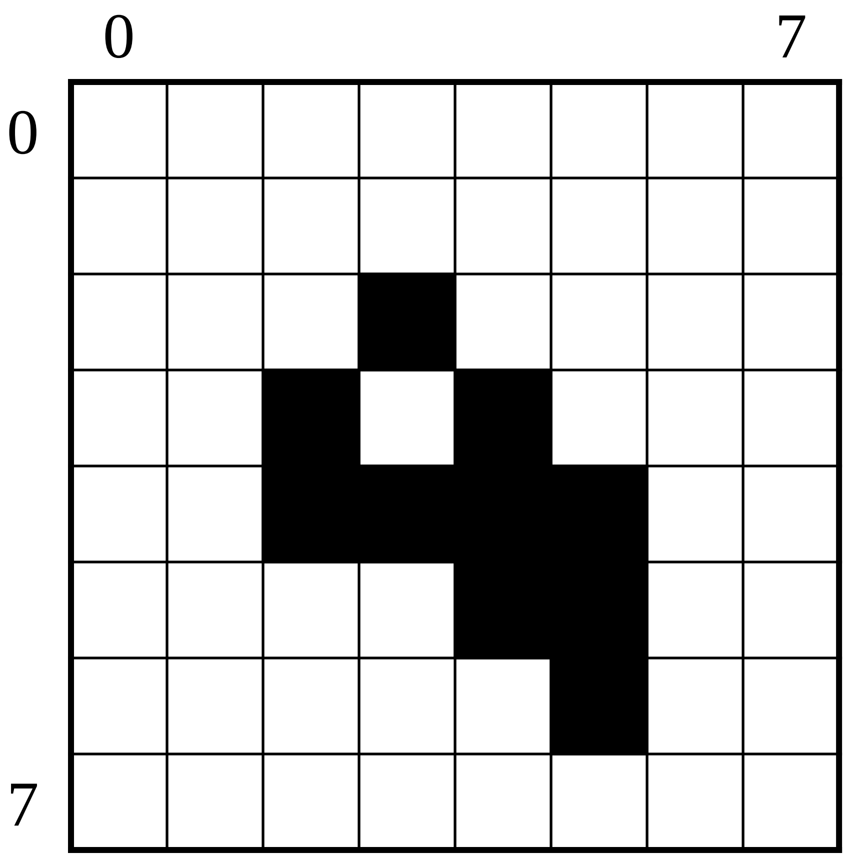
   If >1 label assigned assign one of them and note collision

2. Second pass: apply the equivalence tabel

Characterizing objects

- Invariance
  - Translation
  - Rotation
  - Scale
  - Rubbersheet
  - Noise
- Synthesis
- Missing data
- Contour/region
- Statistical/syntactic

Spatial moments



Definition

$$m_{pq} = \sum_i \sum_j i^p j^q f(i, j)$$

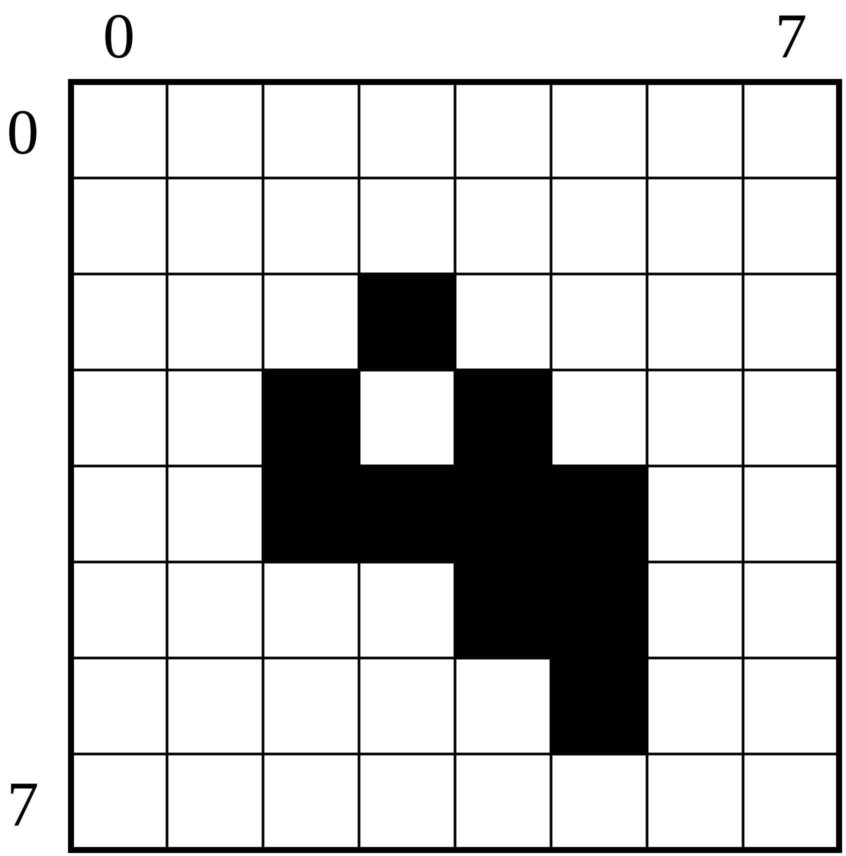
Where p+q is the *order*

0-order moment

$$m_{00} = \sum_i \sum_j f(i, j)$$

is the area e.g. 10

First-order spatial moments



First-order moments

$$m_{10} = \sum_i \sum_j i\, f(i,j)$$

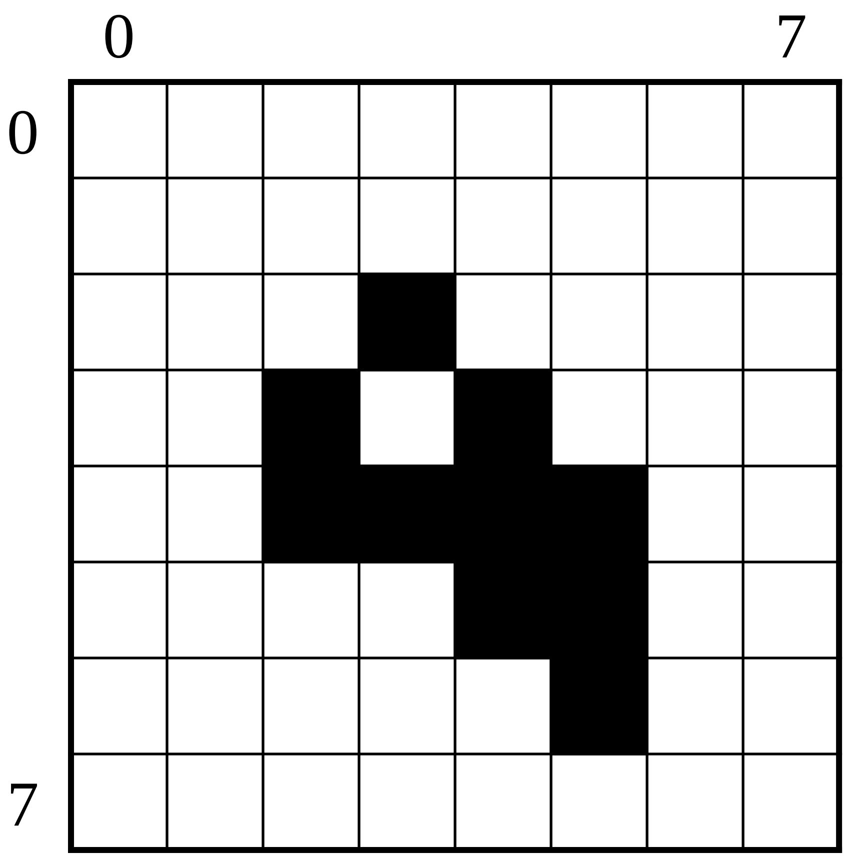
$$m_{01} = \sum_i \sum_j j\, f(i,j)$$

Are used to find the *center of gravity*

$$r_c = \frac{m_{10}}{m_{00}} \qquad c_c = \frac{m_{01}}{m_{00}}$$

Compute the center of gravity for the figure on the left.

First-order spatial moments
Example



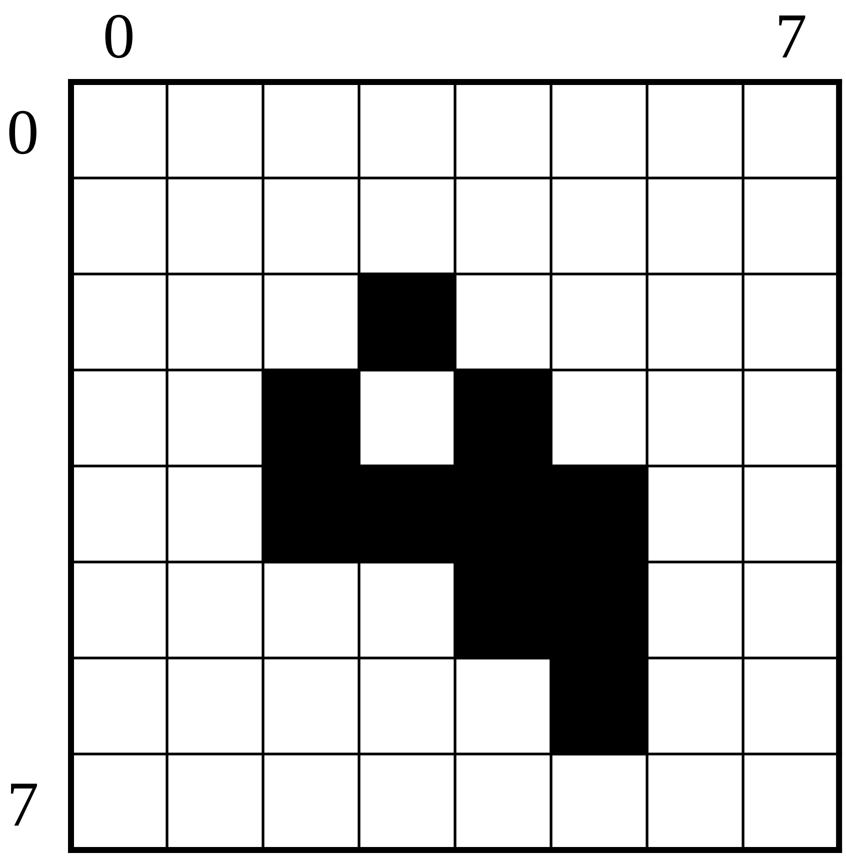
$$m_{00} = \sum_i \sum_j f(i,j) = 10$$

$$m_{10} = \sum_i \sum_j i\,f(i,j) =$$

$$2+3+3+4+4$$

$$+4+4+5+5+6 = 40$$

$$r_c = \frac{m_{10}}{m_{00}} = 4$$



$$m_{00} = \sum_i \sum_j f(i,j) = 10$$

$$m_{01} = \sum_i \sum_j j f(i,j) =$$

$$2+2+3+3+4$$

$$+4+4+5+5+5=37$$

$$c_c = \frac{m_{01}}{m_{00}} = 3.7$$

Central moments

Central moments

$$\mu_{pq} = \sum_i \sum_j (i - r_c)^p (j - c_c)^q f(i, j)$$

Note that

$$\mu_{00} = m_{00} \quad \text{and} \quad \mu_{01} = \mu_{10} = 0$$

Spatial dispersion matrix

$$S = \begin{pmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{pmatrix}$$

Eigenvalues of S

$$\lambda_1 = \frac{1}{2}(\mu_{20} + \mu_{02}) + \frac{1}{2}\sqrt{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}{}^2}$$

$$\lambda_2 = \frac{1}{2}(\mu_{20} + \mu_{02}) - \frac{1}{2}\sqrt{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}{}^2}$$

DTU

Eigenvectors of S

$$\mathbf{v}_1 = \frac{1}{\sqrt{{\mu_{11}}^2 + (\mu_{20} - \lambda_1)^2}} (-\mu_{11}, \mu_{20} - \lambda_1)$$

$$\mathbf{v}_2 = \frac{1}{\sqrt{{\mu_{11}}^2 + (\mu_{20} - \lambda_2)^2}} (-\mu_{11}, \mu_{20} - \lambda_2)$$

Features derived from spatial dispersion matrix

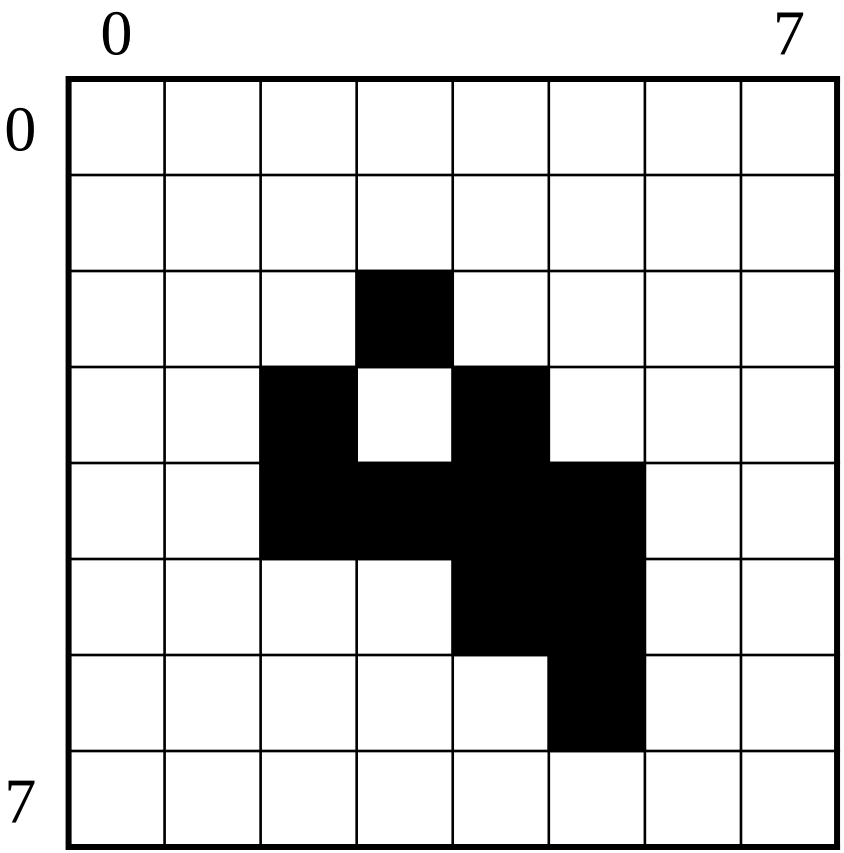
Object orientation

$$\theta = \tan^{-1} \frac{\mu_{11}}{\mu_{20} - \lambda_1}$$

Rotation-and-scale-invariant eccentricity feature

$$R_\lambda = \frac{\lambda_1}{\lambda_2}$$

Example



$$S = \begin{pmatrix} 12 & 7 \\ 7 & 12.1 \end{pmatrix}$$

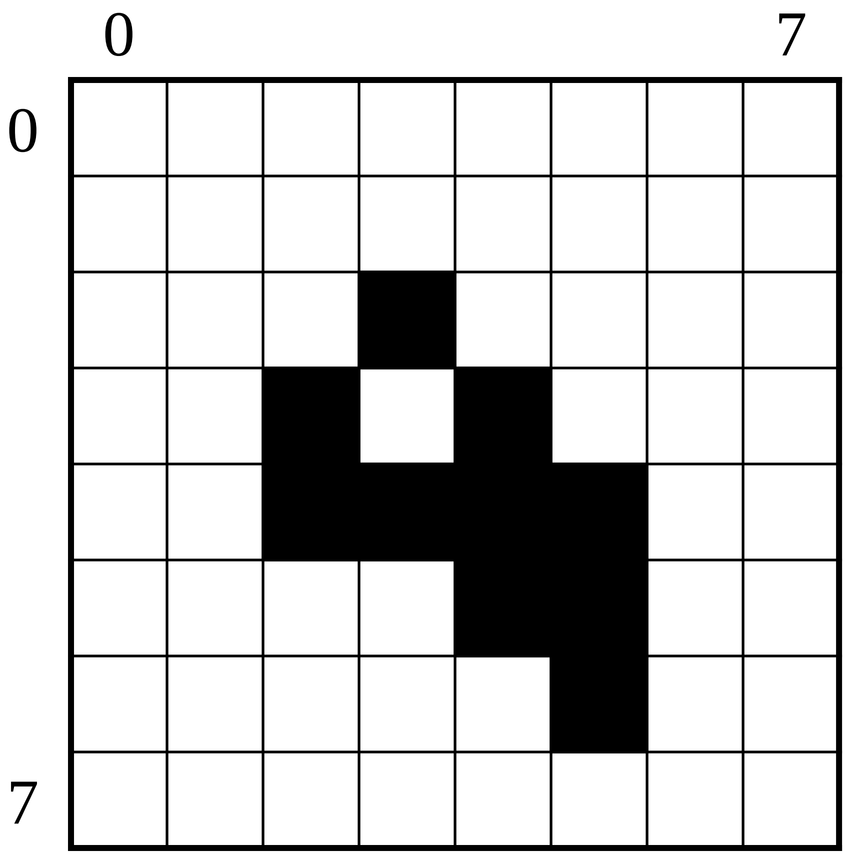
$$\lambda_1 = \frac{1}{2}(12+12.1)$$

$$+\frac{1}{2}\sqrt{(12-12.1)^2+4\cdot 7^2}$$

$$=19.05$$

$$\lambda_2 = \frac{1}{2}(12+12.1)-\frac{1}{2}\sqrt{(12-12.1)^2+4\cdot 7^2} = 5.05$$

Example



$$S = \begin{pmatrix} 12 & 7 \\ 7 & 12.1 \end{pmatrix}$$

$$\lambda_1 = 19.05$$

$$\lambda_2 = 5.05$$

$$\theta = \tan^{-1} \frac{7}{12 - 19.05}$$

$$= -44{,}8$$

$$R_\lambda = \frac{19.05}{5.05} = 3.77$$

Spatial moments used for picking