DIAGRAMMATIC REASONING

with

Class Relationship Logic

Jørgen Fischer Nilsson

DTU Informatics - Technical University of Denmark

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(Logic Diagrams session)

The slides at my homepage

Subject

- We discuss diagrammatic visualization and reasoning for a so-called class relationship logic (CRL)
- - accomplished by extending Euler diagrams with higraphs.
- The diagrams are to afford intuitive appealing inference principles inherent in the visual formalism.
- They aim at facilitating computer assisted reasoning accommodating large amounts of data
- Emphasis on reasoning – contrast e.g. ER-diagrams

Objectives

Pragmatic motivation and desiderata

- Formal ontology engineering and domain modelling call for appropriate abstracted forms of predicate logic
- The screen possesses opportunities for flexible and dynamic visualization enabling management of large amounts of data
- The deductive reasoning capabilities should be reflected in the diagrams in an intuitive manner
- The dynamic visualization capabilities are to be combined with the visual inference abilities for querying and browsing

CRL language forms and levels

- There are 3 forms of language involved
  - Predicate logic as foundation – invisible at the CRL level
  - CRL diagrams (akin to Euler - higraphs)
  - A meta-logic level with variables ranging over classes and relations
- The meta-logic level is in DATALOG definite clauses
- The meta-logic level has a sugared form as stylized NL

The Gist of Class Relationship Logic (CRL)

- Consider relationships between classes $c, d, \ldots$ and binary relations $r, \ldots$.
- Classes are understood as named sets standing in *inter alia* inclusion relationships.
- The prime CRL logical relationship form is the $\forall\exists$-form, explicated in predicate logic as

$$\forall x(c(x) \to \exists y(r(x, y) \wedge d(y)))$$

- This form encompasses as distinguished, important case the extensional class inclusion relationship isa

$$\forall x(c(x) \to d(x))$$

coming about by letting r be identity "=".

Forms of CRL Relationships

- The CRL $\forall\exists$-form

$$\forall x(c(x) \longrightarrow \exists y(r(x,y) \wedge d(y)))$$

may be abstracted as the combinator term

$$\forall\exists(c,r,d)$$

- The 4 different CRL relationships may be abstracted as

$$Q_1 Q_2(c,r,d)$$

where $Q_i \in \{\forall, \exists\}$

- These forms are reminiscent of De Morgan's schemas of categorial propositions (cf. Sánches Valencia (2004))

The pre-dominant $\forall\exists$-relationship form

- is fundamental in ontological domain modelling covering sentences of the principal form:

  common-noun verb common-noun

  understood as All *common-noun verb* some *common-noun*

- – as in

  beta-cell produces insulin

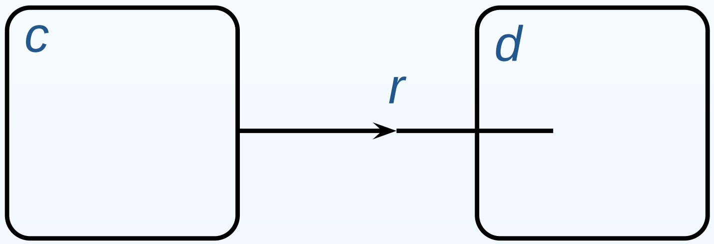
  pancreas has-part beta-cell

  beta-cell isa cell

- The extension sets of classes of no ontological concern
- – but assumed non-empty; thus no notion of empty class
- Distinguished individuals may be lifted to singleton classes

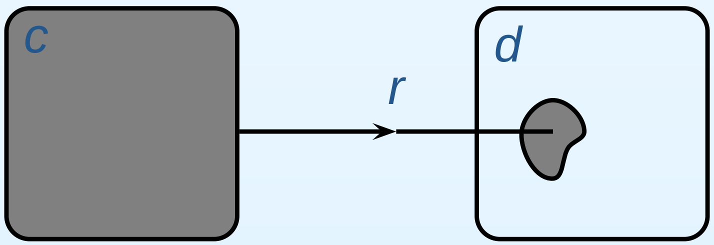
Forms of CRL-diagrams for the $\forall\exists$-relationship

For $\forall\exists(c, r, d)$ we apply the diagram



The arrow tells the direction and does not imply functionality

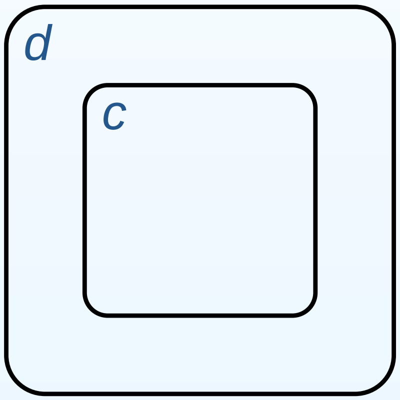
Rationale for this diagram:



$\forall x(c(x) \rightarrow \exists y(r(x, y) \wedge d(y)))$

The case of class inclusion

The case of class inclusion corresponding to r being identity obtains then as Euler diagram form

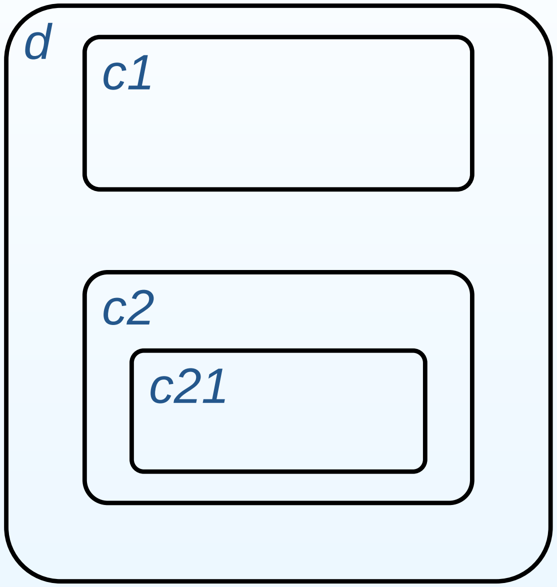


$$isa(c, d)$$

Classes are represented diagrammatically as uniquely named boxes.

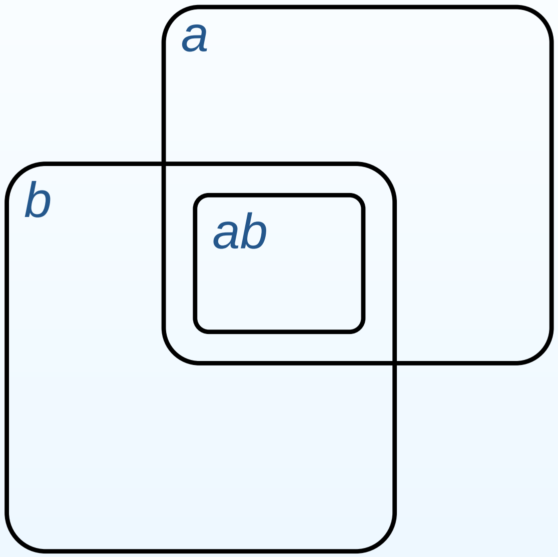
By default convention classes are disjoint unless they have a common subclass (no empty class).

CRL-diagrams for hierarchies and trans-hierarchies



- The underlying inclusion relation, isa, is a partial order
- – but not nescessarilly a lattice (eschewing "Booleanism")
- – might be a meet-semi, if formal extension with $\perp$

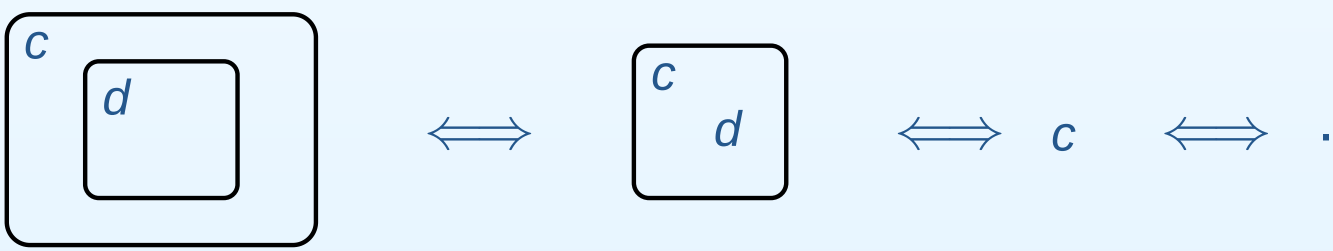
CRL-diagrams for trans-hierarchical structures



- NB At least one common, named sub-class in any overlap
- – since disjointness if no common sub-class
- Disjointness by way of Closed World Assumption for relationships
- No notion of union and intersection.

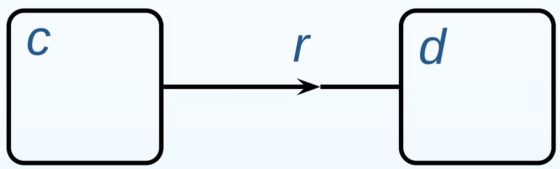
Screen plasticity & dynamics for CRL-diagrams

- boxes are to be made expandable/collabsible for browsing purposes
- a box may be opened/blown-up to reveal inner boxes
- a box may be shrunk to just its name – or nothing
- relation arcs are suppressed according to conventions when boxes become diminished (road map principle)

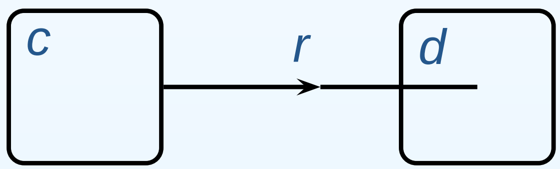


CRL Forms for atomic relationships

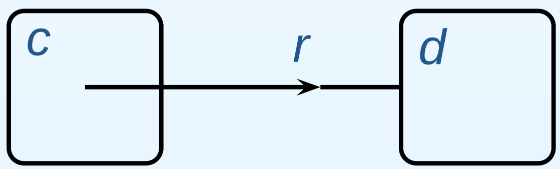
Diagrams for the 4 available atomic relationships $Q_1Q_2(c, r, d)$



$\forall\forall \quad \forall x(c(x) \rightarrow (\forall y(d(y) \rightarrow r(x, y))))$



$\forall\exists \quad \forall x(c(x) \rightarrow (\exists y(d(y) \wedge r(x, y))))$



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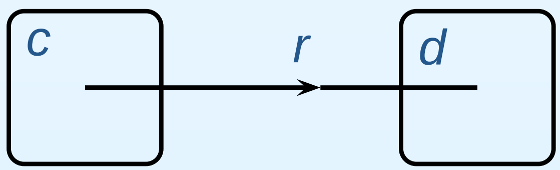
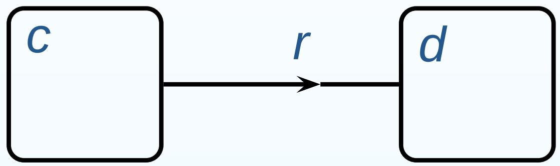
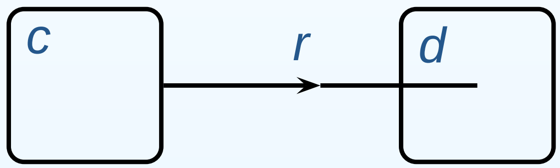


$\exists\exists \quad \exists x(c(x) \wedge (\exists y(d(y) \rightarrow r(x, y))))$

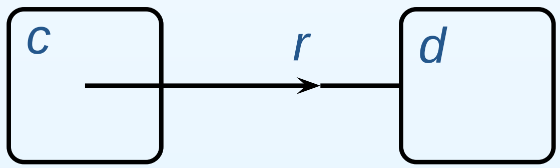
The CRL diagram forms again – with some comments



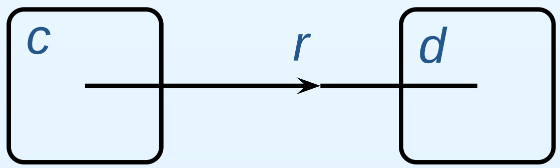
∀∀ "total" cf. Hammer(1995)



∀∃ The prime relationship here

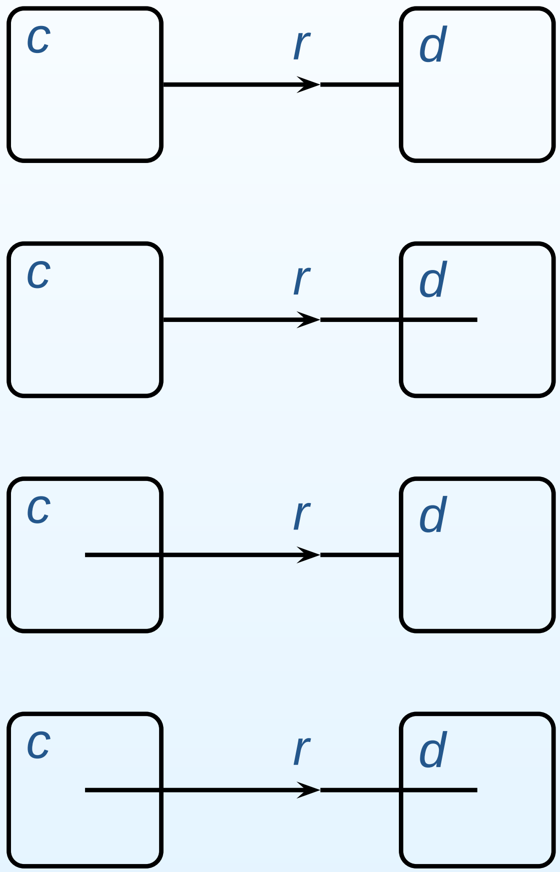


∃∀ cf. Allwein & Barwise(1995)



∃∃ "sparse"

CRL diagram forms again with their NL forms



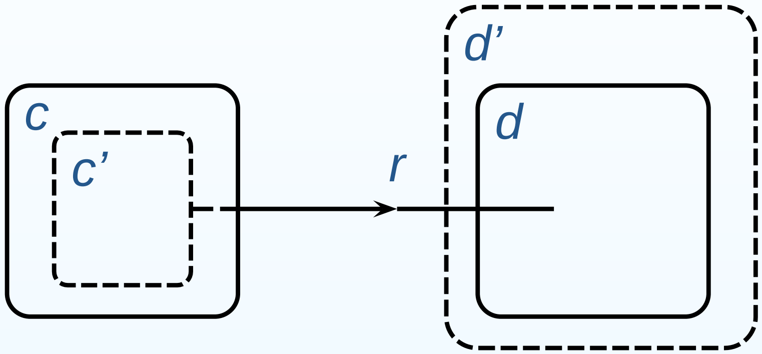
[every] c r every d

[every] c r [some] d

some c r [every] d

some c r [some] d

Key Inference principle for $\forall\exists(c, r, d)$



- –conforming with an inference rule

$$\frac{isa(c',c) \quad \forall\exists(c,r,d) \quad isa(c,d')}{\forall\exists(c',r,d')}$$

- Inclusion as in pure Euler diagrams obtains as special case
- Further diagrammatic support of eg.

$$\frac{\forall\exists(c,r,d)}{\exists\exists(c,r,d)}$$

Diagrammatic reasoning at the meta-logic level

- The diagrammatic reasoning at the level of classes and relationships (above individuals) is formalized in DATALOG
- DATALOG is definite clauses devoid of compound terms

$$p_0(t_{01}, ..., t_{0n_0}) \leftarrow p_1(t_{11}, ..., t_{1n_1}) \wedge ... \wedge p_m(t_{m1}, ..., t_{mn_m})$$

- The predicate argument terms t_{ij} are either constants or variables, with variables being $\forall$-quantified
- DATALOG falls within the Bernays-Schønfinkel subclass of PL – thus effectively propositional
- Possibly also DATALOG$^{\not\vdash}$ (stratified non-provability)
- In the meta-logic the relationship $\forall\exists(c, r, d)$ is re-conceived as an atomic formula with a predicate symbol $\forall\exists$.

Inference rules in Meta-logic

- In the meta-logic the relationships $\forall\exists(c, r, d)$ etc. are conceived as an atomic formula with a predicate symbol $\forall\exists$.
- They form the knowledge base KB$_{meta}$ in DATALOG
- KB$_{meta}$ is extended with inference rules as clauses, e.g.

$$\forall\exists(R, X, Z) \leftarrow \forall\exists(R, X, Y) \wedge isa(Y, Z)$$
$$\forall\exists(R, X, Z) \leftarrow isa(X, Y) \wedge \forall\exists(R, Y, Z)$$

$$\exists\exists(R, X, Y) \leftarrow \forall\exists(R, X, Y)$$

- Correctness criteria, e.g.

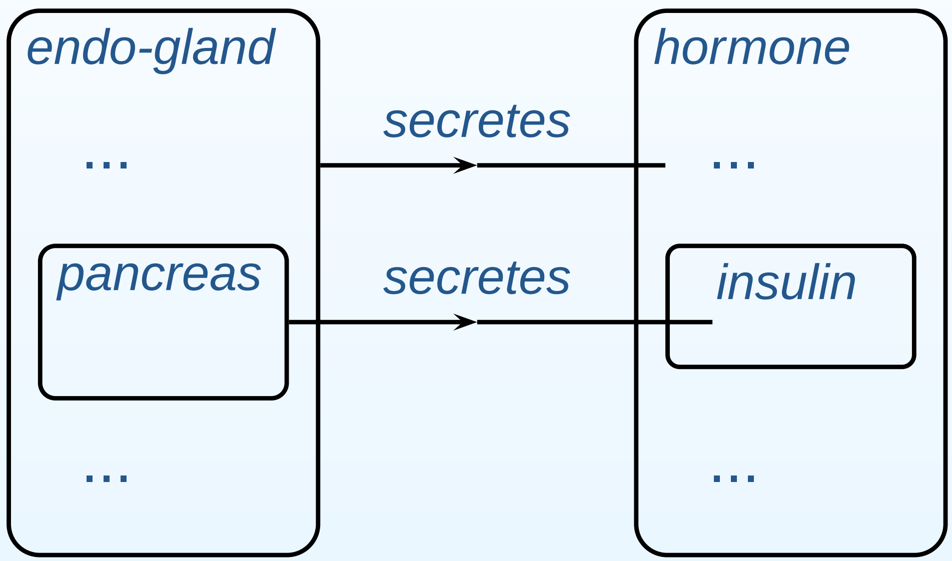
$$KB \models \forall x(c(x) \rightarrow \exists y(r(x, y) \wedge d(y)))$$

$$\mathit{iff}$$

$$KB_{meta} \vdash \forall\exists(c, r, d)$$

Inheritance of ascribed properties

Example

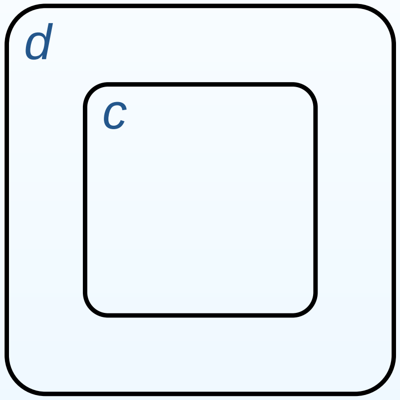


Sample deducibles

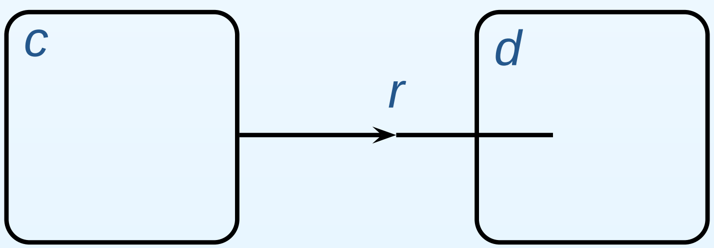
∀∃(pancreas,secretes,hormone)

∃∃(endogland,secretes,insulin)

Comparison CRL-diagrams and Description logic (DL)



for $c \sqsubseteq d$



for $c \sqsubseteq \exists r.d$

where the term $\exists r.d$ corresponds to $\lambda x.\exists y(d(y) \wedge r(x,y))$

Comp. of CRL-diagrams and Description logic (DL) II

Fundamental difference

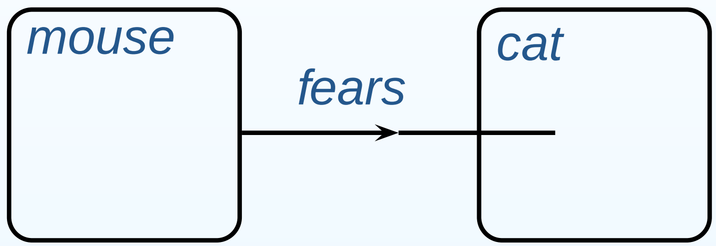
- CRL appeals to closed world assumption (CWA)
- Thus classes are disjoint unless there is a common subclass
- DL appeals to open world assumption (OWA)

To mention also

- $c \sqsubseteq \forall r.d$ differs from the total relship $\forall\forall(c, r, d)$
- $c \sqsubseteq \forall r.d$ may release inconsistency in DL
  – thereby acting as constraint.
- $c \sqsubseteq \forall r.d$ may be achieved in CRL as structural tie.

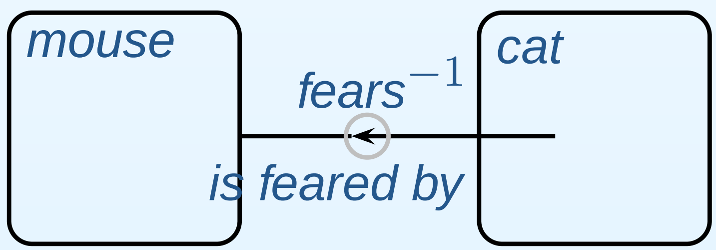
Sample class relationship with the inverse relation

Active form sentence



$\forall\exists$ *every mouse fears some cat*

Corresponding sentence passive form (Katz (1972)
– not equivalent logically, cf. *Skolemization*

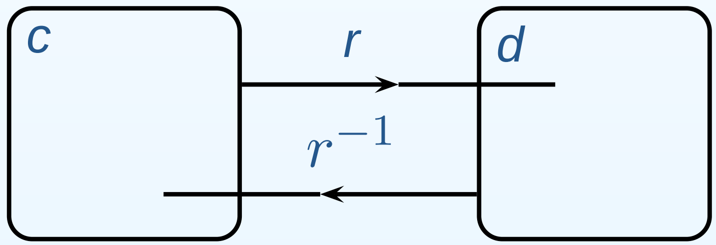


$\exists\forall$ *Some cat is feared by every mouse*

This leads to considering "tight" relationships ... $\Rightarrow$

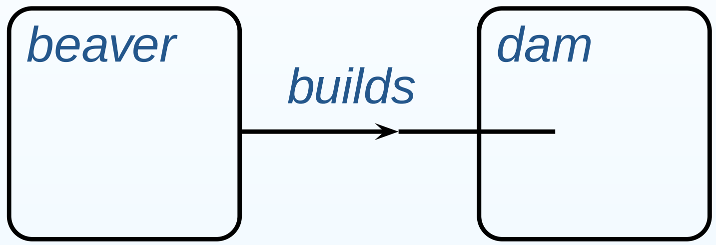
”Tight” class relationship

- Inverse <u>class</u> relationships (reciprocals) usually absent
- Contrast inverse relations r coming with r^{-1}
- The case of co-presence of $\forall\exists(c, r, d)$ and $\forall\exists(d, r^{-1}, c)$ for r:

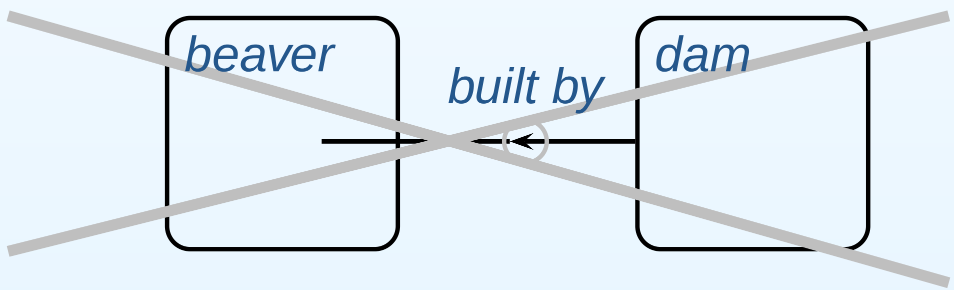


- Plays a special role for partonomic relationships
- – where a pair of inverses, say, *partfor* and *haspart* may or may not form an integral (= tight) part relationship (cf. Smith *et al.*)

Example tight relationship (via active + passive voice)

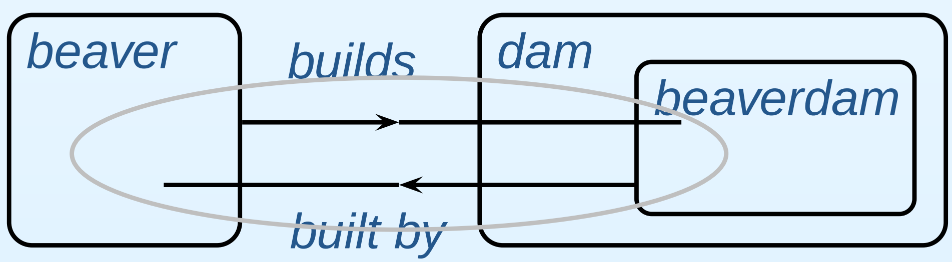


(All) beavers build dams

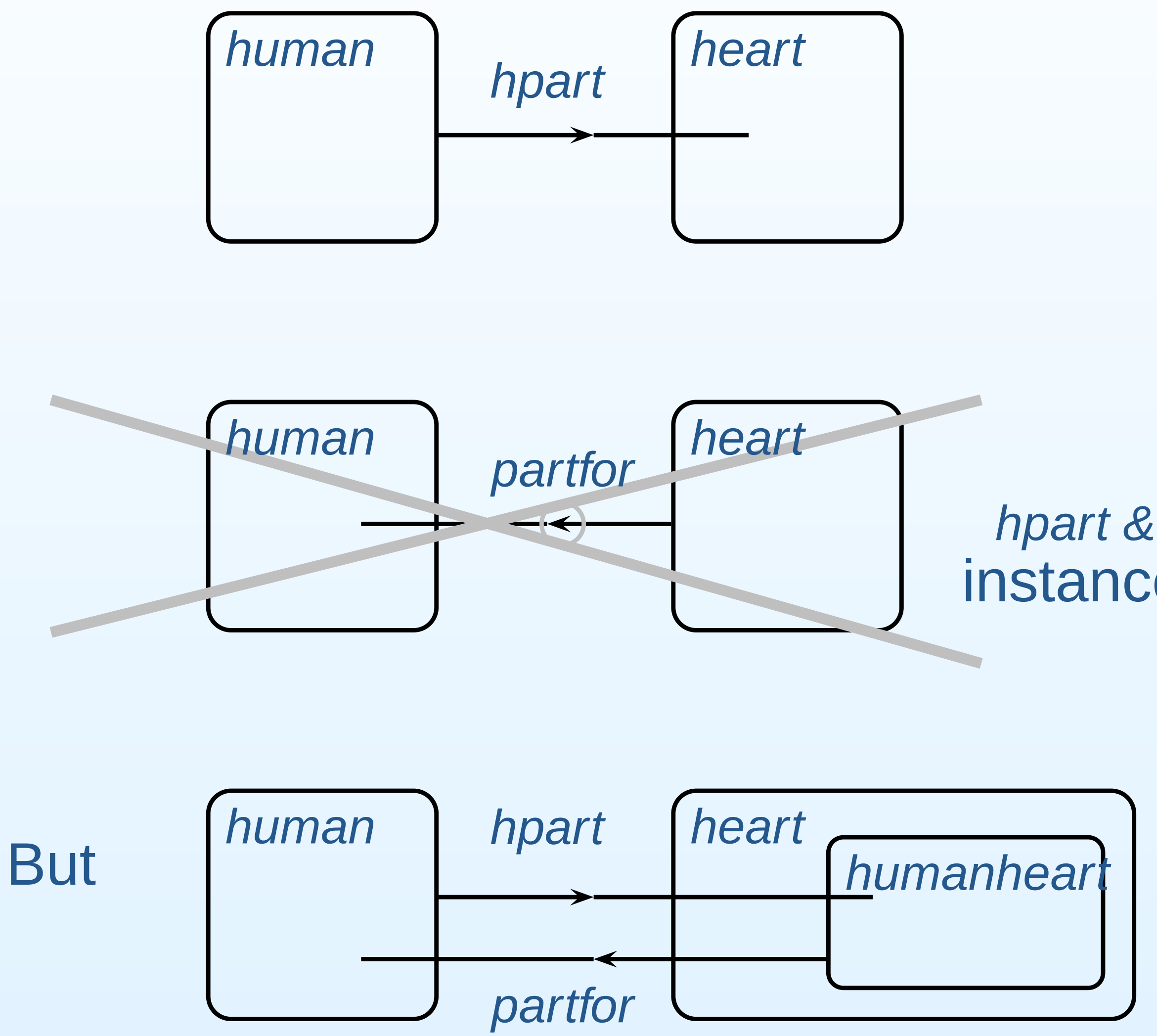


(All) dams are built by beavers

But



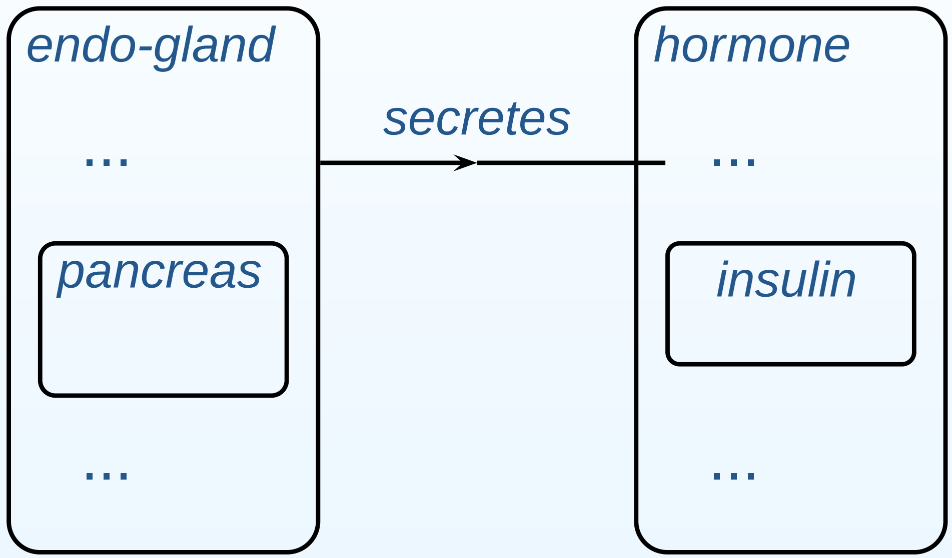
Analogous example with partonomic relationships – cf. Smith *et al.*



hpart & partfor inverses at the instance (mereological) level

Weakening CWA effect of deducing denials

bmof 3 relationship polarities: yes - no - maybe



Here we can not deduce *pancreas secretes insulin*, but we don't want the denial bwof negation as non-provability.

Suggestion: a polarity "maybe" for such cases.

Summary

- We have presented a class relationship logic CRL for practical reasoning purposes, e.g. in formal ontologies.
- We have devised diagram forms which afford computational reasoning capabilities for applications with CRL
- We have suggested DATALOG as computational basis

THANK YOU FOR YOUR ATTENTION