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# Lecture 0: Seminar Overview

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# **Towards a Theory of Domain Descriptions**

**— a gentle introduction —**

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## Summary

- We seek foundations for a possible theory of domain descriptions.
  - Part 2 informally outlines what we mean by a domain.
  - Part 3 informally outlines the entities whose description form a description of a domain.
  - Part 4 then suggests one way of formalising such description parts<sup>1</sup>. There are other ways of formally describing domains<sup>2</sup>, but the one exemplified can be taken as generic for other description approaches.
  - Part 5 outlines a theory of domain mereology.
  - Part 6 suggests some ‘domain discoverers’.

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<sup>1</sup>The exemplified description approach is model-oriented, specifically the **RAISE** cum **RSL** approach.

<sup>2</sup>Other model-oriented approaches are those of **Alloy**, **Event B**, **VDM** and **Z**. Property-oriented description approaches include **CafeOBJ**, **Cas1** and **Maude**

- 
- These lectures reflect our current thinking.
  - Through
    - seminar presentations,
    - their preparation and
    - post-seminar revisions
- it is expected that they will be altered and honed.

## Lecture Overview

1. Introduction	5–41
2. Domains	42–65
3. Entities	66–111
4. Describing Domain Entities	112–197
(a) Parts, Actions, Events	112–172
(b) Behaviours	173–197
5. Discovering Domain Entities	198–273
6. Conclusion	274–280

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# End Lecture 0: Seminar Overview

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# Lecture 1: Introduction

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# 1. Introduction

- In this section we shall cover a number of concepts that lie at the foundation of
  - the theory and practice of
  - domain science and engineering.
- These are general issues such as
  - (i) software engineering as consisting of
    - \* domain engineering,
    - \* requirements engineering, and
    - \* software design,
  - (ii) types and values, and
  - (iii) algebras.



## 1.1. Rôles of Domain Engineering

- By domain engineering we shall understand
  - the engineering<sup>3</sup> of domain descriptions,
  - their study, use and maintenance.
- In this section
  - we shall focus on the use of domain descriptions
    - \* (i) in the construction of requirements and in the design of software, and
    - \* (ii) more generally
      - in the study of man-made domains
      - in a search for possible laws.

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<sup>3</sup>Engineering is the discipline, art, skill and profession of acquiring and applying scientific, mathematical, economic, social, and practical knowledge, in order to design and build structures, machines, devices, systems, materials and processes ... [<http://en.wikipedia.org/wiki/Engineering>]

## 1.1.1. Software Development

- We see domain engineering as a first in a triptych phased software engineering:
  - (I) domain engineering,
  - (II) requirements engineering and
  - (III) software design.
- Parts 3–4 of these lectures cover some engineering aspects of domain engineering.

### 1.1.1.1. Requirements Construction

- As shown elsewhere<sup>4</sup> domain descriptions,  $\mathcal{D}$ , can serve as a firm foundation for requirements engineering.
  - This done is by systematically “deriving” major part of the requirements from the domain description.
  - The ‘derivation’ is done in steps of refinements and extensions.
  - Typical steps reflect such ‘algebraic operations’ as
    - \* projection,                      \* determination,                      \* fitting,
    - \* instantiation,                      \* extension,                      \* etcetera

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<sup>4</sup>From Domains to Requirements. LNCS 5065, Springer

- In “injecting” a domain description,  $\mathcal{D}$ , in a requirements prescription,  $\mathcal{R}$ ,
  - the requirements engineer endeavors to satisfy *goals*,  $\mathcal{G}$ ,
  - where goals are meta-requirements, that is,
  - are a kind of higher-order requirements
  - which can be uttered, that is, postulated,
  - but cannot be formalised in a way from which we can “derive” a software design.
- So, to us, domain engineering becomes an indispensable part of software engineering.

## 1.1.1.2. Software Design

- Finally, from the requirements prescription,  $\mathcal{R}$ ,
  - software,  $\mathcal{S}$ , can be designed
  - through a series of refinements and transformations
  - such that one can prove  $\mathcal{D}, \mathcal{S} \models \mathcal{R}$ ,
  - that is, the software design,  $\mathcal{S}$ , models, i.e.,
  - is a correct implementation of the requirements,  $\mathcal{R}$ ,
  - where the proof makes assumptions about the domain,  $\mathcal{D}$ .

## 1.1.2. Domain Studies “In Isolation”

- But one can pursue developments of domain descriptions whether or not one subsequently wishes to pursue requirements and software design.
  - Just as physicists study “mother nature” in order just to understand,
  - so domain scientists cum engineers can study, for example, man-made domains — just to understand them.

- Such studies of man-made domains seem worthwhile.
  - Health care systems appear to be quite complex, embodying hundreds or even thousands of phenomena and concepts: parts, actions, events and behaviours.
  - So do
    - \* container lines,
    - \* manufacturing,
    - \* financial services,
    - \* liquid and gaseous material distribution (pipelines),
    - \* etcetera.
  - Proper studies of each of these entails many, many years of work.

## 1.2. Additional Preliminary Notions

- We first dwell on the “twinned” notions ‘type’ and ‘value’.
- And then we summarise the notions of
  - (universal, or abstract) algebras,
  - heterogeneous algebras and
  - ‘behavioural’ algebras.
- The latter notion, behavioural algebra, is a “home-cooked” term.
- The algebra section is
  - short on definitions and
  - long on examples.



## 1.2.1. Types and Values

- Values (0, 1, 2, ...) have types (**integer**).
  - We observe values (**false, true**),
  - but we speak of them by their types (**Boolean**);
  - that is:
    - \* types are abstract concepts
    - \* whereas (actual) values are (usually) concrete phenomena.
- By a type we shall here, simplifying, mean a way of characterising a set of entities (of similar “kind”).
- Entity values and types are related:
  - when we observe an entity we observe its value;
  - and when we say that an entity is of a given type, then we (usually) mean that the observed entity is but one of several entities *of that type*.

## Example 1 (Types and Values of Parts) Three naïve examples

When we say, or write, *the* [or *that*] *net*, we mean

1. an entity, a specific value,  $n$ ,
2. of type *net*,  $N$ .

**type**

2.  $N$

**value**

1.  $n:N$

When we say, or write, *the* [or *that*] *account*, we mean

3. an entity, a specific value,  $a$ ,
4. of type *account*,  $A$ .

**type**

4.  $A$

**value**

3.  $a:A$

When we say, or write, *the* [or *that*] *container*, we mean

5. an entity, a specific value,  $c$ ,
6. of type *container*,  $C$ .

**type**

6.  $C$

**value**

5.  $c:C$



**Example 2 (Types and Values of Actions, Events and Behaviours)** We continue the example above:

- A set of actions that all insert hubs in a net have the common signature:

**value**

insert:  $H \rightarrow N \overset{\sim}{\rightarrow} N$

- The type expression  $H \rightarrow N \overset{\sim}{\rightarrow} N$  demotes an infinite set of
  - functions from **Hubs**
  - to partial functions from **Nets**
  - to **Nets**.
- The **value** clause insert:  $H \rightarrow N \overset{\sim}{\rightarrow} N$ 
  - names a function value in that infinite set **insert**
  - and non-deterministically selects an arbitrary value in that infinite set.

- The functions are partial ( $\overset{\sim}{\rightarrow}$ )
  - since an argument **Hub**
  - may already “be” in the **N**
  - in which case the **insert** function is not defined.
- A set of events that all result in a link of a net being broken can be characterised by the same predicate signature:

### value

link\_disappearance:  $N \times N \rightarrow \mathbf{Bool}$

- The set of behaviours that focus only on the insertion and removal of hubs and links in a net have the common signature:

### type

Maintain = Insert\_H | Remove\_H | Insert\_L | remove\_L

### value

maintain\_N:  $N \rightarrow \text{Maintain}^* \rightarrow N$

maintain\_N:  $N \rightarrow \text{Maintain}^\omega \rightarrow \mathbf{Unit}$



## 1.2.2. Algebras

### 1.2.2.1. Abstract Algebras

- By an *abstract algebra* we shall understand
  - a set of parts  $(e_1, e_2, \dots)$  called the *carrier*,  $A$  (a type), of the algebra, and
  - a set of functions,  $f_1, f_2, \dots, f_n$ , [each] in  $\Omega$ , over these.
- Writing  $f_i(e_{j_1}, e_{j_2}, \dots, e_{j_m})$ ,
  - where  $f_i$  is in  $\Omega$  of signature:
 
$$\text{signature } \omega : A^n \rightarrow A$$
  - and each  $e_{j_\ell}$  ( $\ell : \{1..m\}$ ) is in  $A$ .
- The operation  $f_i(e_{j_1}, e_{j_2}, \dots, e_{j_m})$  is then meant to designate
  - either **chaos** (a totally undefined quantity)
  - or some  $e_k$  in  $A$ .

## 1.2.2.2. Heterogeneous Algebras

- A *heterogeneous algebra*

- has its carrier set,  $A$ , consist of a number of usually disjoint sets,
- also referred to as sub-types of  $A$ :  $A_1, A_2, \dots, A_n$ , and
- a set of operations,  $\omega:\Omega$ , such that each operation,  $\omega$ , has a *signature*:

$$\text{signature } \omega : A_i \times A_j \times \dots \times A_k \rightarrow A_r$$

- where  $A_i, A_j, \dots, A_k$  and  $A_r$  are in  $\{A_1, A_2, \dots, A_n\}$ .

**Example 3 (Heterogeneous Algebras: Platoons)** We leave it to the reader to fill in missing narrative and to decipher the following formalisation.

7. There are vehicles.

8. A platoon is a set of one or more vehicles.

**type**

7.  $V$

8.  $P = \{ | p \cdot p:V\text{-set} \wedge p \neq \{\} | \}$

9. A vehicle can join a platoon.

10. A vehicle can leave a platoon.

11. Two platoons can be merged into one platoon.

12. A platoon can be split into two platoons.

9.  $join\_0: V \times P \rightarrow P$

9.  $join\_0(v,p) \equiv p \cup \{v\}$  **pre:**  $v \notin p$

10.  $leave\_0: V \times P \rightarrow P$

10.  $leave\_0(v,p) \equiv p \setminus \{v\}$  **pre:**  $v \in p$

11.  $merge\_0: P \times P \rightarrow P$

11.  $merge\_0(p,p') \equiv p \cup p'$  **pre:**  $p \neq \{\} \neq p' \wedge p \cap p' = \{\}$

12.  $split\_0: P \rightarrow P\text{-set}$

12.  $split\_0(p) \equiv \mathbf{let} \ p',p'':P \cdot p' \cup p'' = p \ \mathbf{in} \ \{p',p''\} \ \mathbf{end} \ \mathbf{pre:} \ \mathbf{card} \ p \geq 2$

- The above formulas define a heterogeneous algebra with
  - types  $V$  and  $P$  and
  - operations (or actions)  $join\_0$ ,  $leave\_0$ ,  $merge\_0$ , and  $split\_0$ .





### 1.2.2.3. Behavioral Algebras

- An abstract algebra is characterised
  - by the one type,  $A$ , of its parts and
  - by its operations all of whose signatures are of the form  $A \times A \times \dots \times A \rightarrow A$ .
- A heterogeneous algebra is an abstract algebra and is further characterised
  - by two or more types,  $A_1, A_2, \dots, A_m$ , and
  - by a set of operations of usually distinctly typed signatures.
- A behavioral algebra is a heterogeneous algebra and is further characterised
  - by a set of events and
  - by a set of behaviours where
    - \* events are like actions and
    - \* behaviours are sets of sequences of actions, events and behaviours.

## Example 4 (A Behavioural Algebra: A System of Platoons and Vehi

Our example may be a bit contrived.

- We have yet to unfold, as we do in this paper, enough material to give more realistic examples.

13. A well-formed platoon/vehicle system consists of a pair:

- (a) *convoys* which is a varying set of [non-empty] platoons and
- (b) *reservoir* which is a varying set of vehicles —
- (c) such that the *convoys* platoons are disjoint, no vehicles in common, and
- (d) such that *reservoir* have no vehicle in common with any platoon in *convoys*.

14. Platoons are characterised by unique platoon identifiers.
15. These identifiers can be observed from platoons.
16. Vehicles from the *reservoir* behaviour may join [leave] a platoon whereby they leave [respectively join] the pool.
17. Two platoons may merge into one, and a platoon may split into two.
18. Finally, vehicles may enter [exit] the system by entering [exiting] *reservoir*.

**type**

$$13. \quad S = \{ | (c,r):C \times R \cdot r \cap \cup c = \{\} | \}$$

$$13(a). \quad C = \{ | c:P\text{-set} \cdot \text{wf}_C(c) | \}$$

**value**

$$13(c). \quad \text{wf}_C: C \rightarrow \mathbf{Bool}$$

$$13(c). \quad \text{wf}_C(c) \equiv \forall p,p':P \cdot \{p,p'\} \subseteq c \Rightarrow p \neq \{\} \neq p' \wedge p \cap p' = \{\}$$

**type**

$$13(b). \quad R = V\text{-set}$$

**value**

$$16. \quad \text{join}_1: S \xrightarrow{\sim} S$$

$$16. \quad \text{leave}_1: S \xrightarrow{\sim} S$$

$$17. \quad \text{merge}_1: S \xrightarrow{\sim} S$$

$$17. \quad \text{split}_1: S \xrightarrow{\sim} S$$

$$18. \quad \text{enter}_1: S \xrightarrow{\sim} S$$

$$18. \quad \text{exit}_1: S \xrightarrow{\sim} S$$

19. **join\_1** selects an arbitrary vehicle in  $r:R$  and an arbitrary platoon  $p$  in  $c:C$ , joins  $v$  to  $p$  in  $c$  and removes  $v$  from  $r$ .
20. **leave\_1** selects a platoon  $p$  in  $c$  and a vehicle  $v$  in  $p$ , removes  $v$  from  $p$  in  $c$  and joins  $v$  to  $r$ .
21. **merge\_1** selects two distinct platoons  $p, p'$  in  $c$ , removes them from  $c$ , takes their union and adds to  $c$ .
22. **split\_1** selects a platoon  $p$  in  $c$ , one which has at least two vehicles,
23. and partitions  $p$  into  $p'$  and  $p''$ , removes  $p$  from  $c$  and joins  $p'$  and  $p''$  to  $c$ .
24. **enter\_1** joins a fresh vehicle  $v$  to  $r$ .
25. **exit\_1** removes a vehicle  $v$  from a non-empty  $r$ .

19.  $\text{join}_1(c,r) \equiv$   
 19. **let**  $v:V \cdot v \in r, p:P \cdot p \in c$  **in**  
 19.  $(c \setminus \{p\} \cup \{\mathbf{join\_0}(v,p)\}, r \setminus \{v\})$  **end**
20.  $\text{leave}_1(c,r) \equiv$   
 20. **let**  $v:V, p:P \cdot p \in c \wedge v \in p$  **in**  
 20.  $(c \setminus \{p\} \cup \{\mathbf{leave\_0}(v,p)\}, r \cup \{v\})$  **end**
21.  $\text{merge}_1(c,r) \equiv$   
 21. **let**  $p,p':P \cdot p \neq p' \wedge \{p,p'\} \subseteq c$  **in**  
 21.  $(c \setminus \{p,p'\} \cup \{\mathbf{merge\_0}(p,p')\}, r)$  **end**
22.  $\text{split}_1(c,r) \equiv$   
 23. **let**  $p:P \cdot p \in c \wedge \text{card } p \geq 2$  **in**  
 23. **let**  $p',p'':P \cdot p \cup p' = p$  **in**  
 23.  $(c \setminus \{p\} \cup \mathbf{split\_0}(p), r)$  **end end**
24.  $\text{enter}_1(c,r) \equiv (c, \mathbf{let } v:V \cdot v \notin r \cup c \mathbf{in } r \cup \{v\} \mathbf{end})$   
 25.  $\text{exit}_1(c,r) \equiv (c, \mathbf{let } v:V \cdot v \in r \mathbf{in } r \setminus \{v\} \mathbf{end}) \mathbf{pre: } r \neq \{\}$

- The above model abstracts an essence of the non-deterministic behaviour of a platooning system.
- We make no assumptions about
  - which vehicles are joined to or leave which platoons,
  - which platoons are merged,
  - which platoon is split nor into which sub-platoons, and
  - which vehicle enters and exits the reservoir state.

26. We model the above *system* as a behaviour which is composed from a pair of concurrent behaviours:

(a) a *convoys* behaviour and

(b) a *reservoir* behaviour

(c) where these behaviours interact via a **channel** *cr\_ch* and

(d) where the entering of “new” and exiting of “old” vehicles occur on a **channel** *io\_ch*

27. Hence the communications between the *reservoir* behaviour and the *convoys* behaviour are of three kinds: *Joining* (moving) a vehicle to a (“magically”<sup>5</sup>) named platoon from the *reservoir* behaviour, *Removing* [moving] a vehicle from a named platoon to (*mkV(v)*) the *reservoir* behaviour

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<sup>5</sup>In this example we skip the somewhat ‘technical’ details as to how the *reservoir* behaviour obtains knowledge of platoon names.



**type**

27.  $M ::= \text{mkJ}(v:V) \mid \text{mkR} \mid \text{mkV}(v:V)$

**channel**

26(c).  $\text{cr\_ch}:M$

26(d).  $\text{io\_ch}:V$

**value**

26.  $\text{system}: S \rightarrow \mathbf{Unit}$

26.  $\text{system}(c,r) \equiv \text{convoys}(c) \parallel \text{reservoir}(r)$

28. The *convoys* behaviour non-deterministically ( $\sqcap$ ) chooses either to
- (a) merge platoons, or to
  - (b) split platoons, or to
  - (c) interact with the *reservoir* behaviour via channel *ct\_ch*
  - (d) and based on that interactions
    - i. to either join a[n arbitrary] vehicle *v* to a platoon, or
    - ii. to remove a named vehicle, *v*, from a platoon
    - iii. while “moving” that vehicle to *reservoir*.

28. convoys:  $C \rightarrow \mathbf{in, out} \text{ cr\_ch } \mathbf{Unit}$

28.  $\text{convoys}(c) \equiv \text{convoys}(\text{merge}(c)) \sqcap \text{convoys}(\text{split}(c)) \sqcap \text{convoys}(\text{interact}(c))$

28(c).  $\text{interact}: C \rightarrow \mathbf{in, out} \text{ cr\_ch } C$

28(c).  $\text{interact}(c) \equiv$

28(c).     **let**  $m = \text{cr\_ch} ?$  **in**

28(d).     **case**  $m$  **of**

28((d))i.      $\text{mkJ}(v) \rightarrow \text{join\_vehicle}(v, c),$

28((d))ii.      $\text{mkR} \rightarrow \mathbf{let} (c', v) = \text{remove\_vehicle}(c) \mathbf{in}$

28((d))iii.      $\text{ct\_ch!mkV}(v) ; c'$

28(c).     **end end end**

## 29. The *merge platoons* behaviour

- (a) non-deterministically chooses two platoons of *convoys*  $(p, p')$ ,
- (b) removes the two platoons from *convoys* and adds the *merge* of these two platoons to *convoys*.
- (c) If *convoys* contain less than two platoons then *merge platoons* is undefined.

29.  $\text{merge\_platoons}: C \rightarrow C$

29.  $\text{merge\_platoons}(c) \equiv$

29(a). **let**  $p, p', p'': P \cdot p \neq p' \wedge \{p, p'\} \subseteq c$  **in**

29(b).  $c \setminus \{p, p'\} \cup \{\text{merge\_0}(p, p')\}$  **end**

29(b). **pre:**  $\text{card } c \geq 2$

### 30. The *split\_platoons* function

- (a) non-deterministically chooses a platoon,  $p$ , of two or more vehicles in *convoys*,
- (b) removes the chosen platoon from *convoys* and inserts the split platoons into *convoys*.
- (c) If there are no platoons in  $c$  with two or more vehicles then *split\_platoons* is undefined.

30.  $\text{split\_platoons}: C \xrightarrow{\sim} C$

30.  $\text{split\_platoons}(c) \equiv$

30(a). **let**  $p:P \cdot p \in c \wedge \mathbf{card} \ p \geq 2$  **in**

30(b).  $c \setminus \{p\} \cup \{\mathbf{split\_0}(p)\}$  **end**

30(c). **pre:**  $\exists p:P \cdot p \in c \wedge \mathbf{card} \ p \geq 2$

31. The *reservoir* behaviour interacts with the *convoys* behaviour and with “an external”, that is, undefined behaviour through channels *ct\_ch* and *io\_ch*.

The *reservoir* behaviour [external] non-deterministically chooses between

- (a) importing a vehicle from “the outside”,
- (b) exporting a vehicle to “the outside”,
- (c) moving a vehicle to the *convoys* behaviour, and
- (d) moving a vehicle from the *convoys* behaviour.

31. reservoir:  $R \rightarrow \mathbf{in, out}$  cr\_ch, io\_ch **Unit**

31. reservoir( $r$ )  $\equiv$

31(a).  $(r \cup \{\text{io\_ch?}\})$ ,

31(b).  $\square \mathbf{let} v:V \cdot v \in t \mathbf{in} \text{io\_ch!mkV}(v) ; \text{reservoir}(r \setminus \{v\}) \mathbf{end}$

31(c).  $\square \mathbf{let} v:V \cdot v \in t \mathbf{in} \text{ct\_ch!mkJ}(v) ; \text{reservoir}(r \setminus \{v\}) \mathbf{end}$

31(d).  $\square \mathbf{let} \text{mkV}(v) = \text{ct\_ch?} \mathbf{in} \text{reservoir}(r \cup \{v\}) \mathbf{end}$

- We may consider Items 31(a)–31(b) as designating events.
- This example designates a behavioural algebra.



## 1.3. On 'Method' and 'Methodology'

- By a *method* we shall understand
  - a set of *principles, techniques* and *tools*
  - where the principles help
    - \* *select* and
    - \* *apply*
  - these *techniques* and *tools*
  - such that an *artifact*, here a domain description, can be constructed.
- By *methodology* we shall understand
  - the *knowledge* and *study* of one or more methods.
- Languages,
  - whether informal, as English,
  - or formal, as RSL,are *tools*.



## 1.4. An Ontology of Descriptions

- *“By ontology we mean*
  - *the philosophical study*
    - \* *of the nature of being, existence, or reality as such,*
    - \* *as well as the basic categories of being and their relations.*
- *Traditionally listed as a part of the major branch of philosophy known as metaphysics,*
  - *ontology deals with questions concerning*
  - *what entities exist or can be said to exist,*
  - *and how such entities can be grouped,*
  - *related within a hierarchy,*
  - *and subdivided according to similarities and differences.”*<sup>6</sup>

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<sup>6</sup><http://en.wikipedia.org/wiki/Ontology>

## 1.4.1. Entities and Properties

- A main stream of philosophers  
[MellorOliver1997,ChierchiaEtAl1998,ChrisFox2000]  
appear to agree that there are two categories of discourse:
  - entities<sup>7</sup> and
  - properties.
- Once we say that, a number of questions arise:
  - ( $Q_1$ ) What counts as an entity ?
  - ( $Q_2$ ) What counts as a property ?
  - ( $Q_3$ ) Are properties entities ?
  - ( $Q_4$ ) Can properties predicate properties ?
- We shall take **no** and **yes** to be answers to  $Q_3$  and  $Q_4$ .
- These lectures shall answer  $Q_1$  and  $Q_2$

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<sup>7</sup>The literature [LeonardGoodman1940,BowmanLClarke81,BowmanLClarke85,  
MellorOliver1997,ChierchiaEtAl1998,ChrisFox2000,MarcusRossberg2008]  
alternatively refer to entities by the term individuals.

## 1.4.2. Categories of Entities

- We shall promulgate the following classes of entities:
  - parts, and – operations.

where we further “sub-divide” operations into

  - actions, – events and – behaviours
- That is, we can predicate entities,  $e$ , as follows:
  - $IS\_PART(e)$ , \*  $IS\_EVENT(e)$  and
  - $IS\_OPERATION(e)$ , that is, \*  $IS\_BEHAVIOUR(e)$ .
  - \*  $IS\_ACTION(e)$ ,
- We shall justify the above categorisation through these lectures.
- So parts, actions, events and behaviours form an ontology of descriptions.

## 1.5. Structure of These Lectures

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# End Lecture 1: Introduction

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