

## Documents An Experimental Domain Description

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### Abstract

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We speculate on the concept of documents. Documents are here considered abstract enduring entities: we stipulate, or assume, no concrete form for documents, whether as printed or electronic material, but we do impose some restrictions that may seem peculiar: copies of a document are distinct from this, have an own, unique identity, and occupy distinct locations. Documents are subject to operations: creation, editing, copying, reading and shredding. Each and every one of these operations leave an indelible mark on the document: the identity of the actor, the time and location of the operation; and the names and explicit argument values of the operations. The ensemble of all such document marks form a trace. If one can speak of a document property then it must be described with no concern for its implementability.

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In this document (!) we shall describe such documents as hinted at above, informally, in the form of increasingly terser narratives, and formally, in the RAISE [2] Specification Language, RSL [1].

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## 1 Documents

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What Are Documents? To a beginning we shall consider the concept of ‘document’ to be an algebra: an indefinite set of enduring entities, “the documents” (!), and a finite set of perdurant entities: the operations resulting in and/or performable on documents. Thus we are left to describe these entities and their operations. We shall start out by a narrative description of operations resulting in or on documents. Through such a description we approach also a description of the enduring document entities by describing their properties and how they may change. 6

### 1.1 Document Operations

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Documents are *created*, *edited*, *read*, *copied* and *shredded*.

- **type** DocOp = Cre | Edi | Rea | Cop | Shr

We shall later describe these operations in detail.

### 1.2 Document Space

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Without committing ourselves too much we can consider the entirety of all documents to reside in a space,  $\omega: \Omega$  of uniquely identified such:

- **type**  $\Omega = \text{DI} \xrightarrow{\text{m}} \text{D}$

We exemplify expressions involving  $\omega$ ,  $u$  and  $d$ .

- **dom**  $\omega$ :

The expression **dom**  $\omega$  stands for the set of unique document identifiers of all documents in the space  $\omega$ .

- $\omega(u)$ :

Let  $u$  stand for a unique document identifier in **dom**  $\omega$  then  $\omega(u)$  stands for a document. 9

- $\omega \cup [u \mapsto d]$ :

Let  $u$  stand for a unique document identifier not in **dom**  $\omega$  and  $d$  for a document, then  $\omega \cup [u \mapsto d]$  stands for a space, say  $\omega'$  which is like  $\omega$  except that a new document, with a fresh, unique document identifier  $u$ , has been joined to  $\omega$ .

- $\omega \dagger [u \mapsto d']$ :

Let  $u$  stand for a unique document identifier in **dom**  $\omega$  and  $d'$  for a document, then  $\omega \dagger [u \mapsto d']$  stands for a space, say  $\omega'$  which is like  $\omega$  except that  $u$  now maps to document  $d'$ .

- $\omega \setminus \{u\}$ :

Let  $u$  stand for a unique document identifier in **dom**  $\omega$  then  $\omega \setminus \{u\}$  stands for a document space that no longer records a document by name  $u$ , that is,

$$\diamond \text{ axiom } \omega \setminus \{u\} \cup [u \mapsto \omega(u)] = \omega.$$

## 1.3 Document Properties

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### 1.3.1 Unique Document Identifiers

Consider a set of documents:  $\{d_i, d_j, \dots, d_k\}$ . Documents,  $d:D$ , are uniquely identified,  $u:DI$ . Thus documents in  $\{d_i, d_j, \dots, d_k\}$  all have *the unique identifier property*, i.e., the unique identifiers:  $\{u_i, u_j, \dots, u_k\}$ , respectively. We can therefore associate a *unique identifier function* which from every document observes its *unique identifier*;

- **value**  $uid.D: D \rightarrow DI$

### 1.3.2 Actors

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Actors,  $\alpha : A$ , perform these operations on documents at times,  $\tau:T$ , and at locations,  $\ell:L$ . Actors also have the *the unique identifier property*, i.e., have unique identifiers  $\alpha u : AI$ .

### 1.3.3 Time

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Time,  $T$ , is here considered a simple, dense set of points with simple functions:

- **value**  $<, \leq, =, \geq, >, \neq, =: T \times T \rightarrow \mathbf{Bool}$

### 1.3.4 Locations

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Locations,  $\ell:L$ , are considered, for simplicity, a set of points:

- **value**  $\neq, =: L \times L \rightarrow \mathbf{Bool}$

### 1.3.5 Authorisation

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We augment the set of the above (non-create) operations (edit, read, copy and shred) with an additional operation: *authorisation*.

With documents we associate, for each (non-create) operation, sets of authorised persons, i.e., identification of actors, who are allowed to perform these operations.

- **type**  $\text{Auth} = \text{DocOp} \xrightarrow{\overline{m}} \text{AI-set}$

$$\text{aut:Auth} : \left[ \begin{array}{l} \text{edi} \mapsto \{\alpha_{l_{i_1}}, \dots, \alpha_{l_{i_p}}\}, \\ \text{rea} \mapsto \{\alpha_{l_{j_1}}, \dots, \alpha_{l_{j_q}}\}, \\ \text{cop} \mapsto \{\alpha_{l_{k_1}}, \dots, \alpha_{l_{k_r}}\}, \\ \text{aut} \mapsto \{\alpha_{l_{\ell_1}}, \dots, \alpha_{l_{\ell_s}}\}, \\ \text{shr} \mapsto \{\alpha_{l_{m_1}}, \dots, \alpha_{l_{m_t}}\} \end{array} \right],$$

where either of  $i_p, j_q, k_r, \ell_s$  or  $m_t$  may be zero, i.e., the operation name maps into an empty set, i.e., no-one is authorised for the given operation.

### 1.3.6 Document Contents

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A primary purpose of documents is to “carry” document contents:  $c:C$ . Documents are further undefined quantities: either  $c:C$  (also further undefined), or “nil” designating an “empty” document:

**type**  $C = C \mid \{\text{"nil"}\}$

With document contents,  $c:C$ , we associate the following functions:

- **value** *is empty*  $C \rightarrow \mathbf{Bool}$
- value** *forward*  $C \rightarrow C$
- value** *undo*  $C \rightarrow C$

such that:

- **axiom**  $forward \circ undo = \lambda c:C. c = undo \circ forward$

## 1.3.7 Document Traces

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With every document we associate a trace of quintuplets: an actor identification, a time, a location, the name of the operation and arguments. The trace is a list of length one after the initial create operation. Each operation and function appends one element to the list. 17

**type**

Trace = Mark=list

Mark = s\_AI:AI s\_T:T s\_L:L s\_DocOp:DocOp a\_Args:Args

**value**

attr\_Trace: D  $\rightarrow$  Trace

**axiom**

$\forall u\iota:DI, op:DocOp \setminus \{Cre\}, ai:AI, t:T, l:L, \omega:\Omega \bullet$

**let**  $\omega' = \text{int\_DocOp}(op)(u\iota)(args)(ai, t, l)(\omega)$  **in**

$\text{attr\_Trace}(\omega'(u\iota)) = \langle (ai, t, l, op, args) \rangle \hat{\ } \text{attr\_Trace}(\omega(u\iota))$  **end**

## 1.4 The Operations — Rough Narratives

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### 1.4.1 Create Document

When a document,  $d$ , is first created it receives (from an oracle) its unique identifier. In addition one can from  $d$  observe the identity,  $\alpha$ , of the actor who created  $d$ , at which time,  $\tau$ ,  $d$  was created, and at which location,  $\ell$ ,  $d$  was created. Since it is an initial operation no authorisation is required. When creating a document the actor may refer to other documents as being a source for the created document – provided, of course, that the creator has at least reading right to these documents. There is not much else to observe from  $d$  as it is initially created. We thus say that an initially created document is [otherwise] empty: its document contents is “nil”.

### 1.4.2 Edit Document

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Editing a document,  $d$ , results in a document,  $d'$ ;  $d$  no longer exists, but  $d'$  has the same unique identifier as  $d$  had. In addition one can from  $d$  observe the identity,  $\alpha$ , of the actor who created  $d$ , at which time,  $\tau$ ,  $d$  was edited, and at which location,  $\ell$ ,  $d$  was edited. Let  $c$  be the document contents of  $d$ . text with which  $d$  was edited. Editing  $d$  results in a new document contents,  $c'$ . Editing is here understood as a process, abstracted as a function,  $\epsilon$ , from  $c$  to  $c'$ . That function,  $\epsilon$  always has an inverse,  $\epsilon^{-1}$ , which when applied to  $d'$ ,  $\epsilon^{-1}(d')$ , 20

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<sup>1</sup>We here assume that  $\alpha$  is in the set of those actors who are edit-authorized

yields  $d$ , such that  $\epsilon(\epsilon^{-1}(d'))=d'$ . That is  $\epsilon^{-1} \cdot \epsilon = \lambda x \bullet x$ . Editing may refer to other documents,  $\{d_a, d_b, \dots, d_c\}$ , in addition to any specified (initial) source documents, as a source of editing. You may think of  $\epsilon$  and references to  $\{d_a, d_b, \dots, d_c\}$ , as well as, of course,  $d$ , being arguments of the edit operation.

### 1.4.3 Read Document

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Reading a document,  $d$ , does not change its unique identifier nor the operation-authorisations, but “reveals” the document contents,  $c$ . But, as other operations do, reading leaves a “foot-print”: who, *alpha*, did the reading<sup>2</sup>, where,  $\ell$ , was the reading done and at which time.  $\tau$ . The operation-authorisations is unchanged. When reading a document, i.e., basically its contents,  $c$ , the reader may come across references to initial or editing source documents. Reading these is subject to the same constraints as listed just above – and are seen as separate, in a sense, “parallel” reading.

### 1.4.4 Copy Document

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Copying a document,  $d$ , results in two documents,  $d'$  and  $d''$ , the ‘copy’,  $d''$ , which is a copy of the ‘master’, but with a new unique identifier; the ‘copied master’,  $d'$ , with the same unique identifier as  $d$  had; and with  $d$  no longer existing. From  $d'$  and  $d''$  one can observe the appropriate  $\alpha$ ,  $\tau$ , and  $\ell$ <sup>3</sup>. From  $d'$ , the copied master, one can also observe the unique identifier of the copy, and From  $d''$ , the copy, one can also observe the unique identifier of the copied master, i.e., the master. Operation-authorisations for  $d'$  and  $d''$  are unchanged (inherited) from  $d$ .

### 1.4.5 Shred Document

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The shredding operation

### 1.4.6 Authorise Document

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The authorisation operation

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<sup>2</sup>We here assume that  $\alpha$  is in the set of those actors who are read-authorised

<sup>3</sup>We here assume that  $\alpha$  is in the set of those actors who are copy-authorised

## 2 Formalisation, I/II 25

### 2.1 States 26

#### 2.1.1 Narrative

1. There are documents,  $d : D$ , unique [document] identifiers,  $ui : UI$ , actor names  $alpha : A$ , times,  $\tau, T$ , locations,  $\ell : L$ , and
2. document contents,  $c : C$ , which are either "nil" or some proper contents (in  $\mathbb{C}$ ).
3. There are spaces of documents,  $\omega : \Omega$ .

#### 2.1.2 Formalisation

1. **type**
  1.  $D, UI, AI, T, L$
  7.  $C = \mathbb{C} \mid \{\text{"nil"}\}$
  3.  $\Omega = UI \xrightarrow{m} D$

### 2.2 Trace Wellformedness 27

#### 2.2.1 Narrative

4. A trace is a sequence of zero or more actor, time and location triplets which is
5. which is wellformed if triplet times are ordered in increasing sequence.

#### 2.2.2 Formalisation

4. **type**
  4.  $AITL = s\_AI:AI \times s\_T:T \times s\_L:L$
  4.  $Trace' = AITL^*$
  4.  $Trace = \{ \mid tr:Trace' \bullet wf\_Trace(wf) \mid \}$
5. **value**
  5.  $wf\_Trace: Trace \rightarrow \mathbf{Bool}$
  5.  $wf\_Trace(tr) \equiv$
  5.  $\forall i:\mathbf{Nat} \bullet \{i, i+1\} \subseteq \mathbf{dom} \ tr \Rightarrow s\_T(tr(tr[i])) < s\_T(tr(tr[i+1]))$

### 2.3 Operations: Syntax and Signatures 28

#### 2.3.1 Narrative

- 6.
7. There are operations:

- |            |                 |
|------------|-----------------|
| a create,  | d copy;         |
| b edit,    | e authorise and |
| c read and | f shred.        |

## 2.3.2 Formalisation

### 7. value

- 7a. create:  $\text{UI-set} \times \text{Auth} \rightarrow \text{AITL} \rightarrow \Omega \rightarrow \Omega \times \text{UI}$
- 7b. edit:  $\text{UI} \rightarrow (\text{C} \rightarrow \text{C}) \rightarrow \text{AITL} \rightarrow \Omega \rightarrow \Omega$
- 7c. read:  $\text{UI} \rightarrow \text{AITL} \rightarrow \Omega \rightarrow \Omega$
- 7d. copy:  $\text{UI} \rightarrow \text{AITL} \rightarrow \text{D} \times \Omega \rightarrow \Omega$
- 7e. authorise:  $\text{UI} \rightarrow \text{Auth} \rightarrow \text{AITL} \rightarrow \Omega \rightarrow \Omega$
- 7f. shred:  $\text{UI} \rightarrow \text{AITL} \rightarrow \Omega \rightarrow \Omega$

## 2.4 Authorisation

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### 2.4.1 Narrative

- 8. And there are authorisations. *auth*, which we model as maps from operation names to sets of actor names.
- 9. These are wellformed if all authorisations contain all operation names.

### 2.4.2 Formalisation

#### 8. type

- 8.  $\text{OpNam} = \text{edi|rea|cop|aut|shr}$
- 8.  $\text{Auth}' = \text{OpNm} \xrightarrow{\text{m}} \text{AI-set}$
- 9.  $\text{Auth} = \{ | \text{auth} : \text{Auth}' \cdot \text{wf\_Auth}(\text{auth}) | \}$

#### 9. value

- 9.  $\text{wf\_Auth} : \text{Auth}' \rightarrow \text{Bool}$
- 9.  $\text{wf\_Auth}(\text{auth}) \equiv \text{dom } \text{auth} = \{\text{edi,rea,cop,aut,shr}\}$

## 2.5 Operation Semantics

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### 2.5.1 Create

We refer to section 1.4.1 on Page 5.

- 10. The create operation takes as arguments
  - a possibly empty set of unique document identifiers,
  - a possibly empty map of authorisations, and
  - a triplet of actor names, times and locations;
 applies
  - to a state, the space of documents;



and yields

- a possibly updated state and
- the unique document identifier

11. The create operation satisfies its pre-condition

- if unique document identifiers of the argument are indeed those of documents in the state (space).

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12. The create operation now yields a new state and a new, unique document identifier

- a with a unique document identifier not of any document in the argument state space but in the resulting state
- b which is like the argument state except that the unique document identifier is bound to a document,  $d$ ,
- c with a one element trace of the argument triplet of actor names, times and locations,
- d with the argument authorisations, and
- e with no contents.

13. It updates the state space to now contain also this document and yields its new, unique document identifier.

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10. **value**

10. create:  $UI\text{-set} \times Auth \rightarrow AITL \rightarrow \Omega \rightarrow \Omega \times UI$

10. create( $u\iota s, auth, aitl$ )( $\omega$ ) **as** ( $\omega', u\iota$ )

11. **pre**: known\_ $u\iota s$ ( $u\iota s$ )( $\omega$ )

12a. **post**:  $\sim$ known\_ $u\iota$ ( $u\iota$ )( $\omega$ )  $\wedge$  known\_ $u\iota$ ( $u\iota$ )( $\omega'$ )

12a.  $\wedge$  **let**  $d = \omega'(u\iota)$  **in**  $\omega' = \omega \cup [u\iota \mapsto d]$

12d.  $\wedge$  inspect\_ $Auth$ ( $d$ ) =  $auth$

12c.  $\wedge$  inspect\_ $Trace$ ( $d$ ) =  $\langle aitl \rangle$

12e.  $\wedge$  inspect\_ $C$ ( $d$ ) = " $nil$ " **end**

## 2.5.2 Edit

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We refer to section 1.4.2 on Page 5.

14. By an editing function we mean a pair,  $e_f, i_f$ , of functions from document contents to document contents such that  $i_f$  is an inverse of  $e_f$  and, vice versa,  $e_f$  is an inverse of  $i_f$ .

15. The edit operation takes as arguments

- a unique document identifier, an editing function and
- a triplet of actor names, times and locations.

16. The edit operation satisfies its pre-condition
- a if the argument unique document identifier is that of a document in the argument state,
  - b if the actor is authorised for the edit operation on that document.
17. The edit operation yields a new state
- a where all documents in the argument and the yielded states other than the one document identified by the unique document identifier are unchanged, and
  - b where the document identified by the unique document identifier in the argument state differs from document identified by the unique document identifier in the yielded state as follows:
    - i. the trace of the yielded document is that of the argument document with the triplet of actor name, time and location appended to its front;
    - ii. the argument and the yielded authorisations are unchanged; and
    - iii. the yielded contents is the forward editing of the argument contents.
  - c Undoing the yielded contents results in the argument contents.

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14. **type**
14.  $EF = (C \rightarrow C) \times (C \rightarrow C)$
14. **axiom**
14.  $\forall (fwd, und): EF \cdot fwd \circ und = \lambda x. x = und \circ fwd$
15. **value**
15.  $edit: UI \times EF \rightarrow AITL \rightarrow \Omega \rightarrow \Omega$
15.  $edit(u, (fwd, und), aitl: (ai, t, l))(\omega) \text{ as } \omega'$
- 16a. **pre:**  $u \in \mathbf{dom} \omega$
- 16b.  $\wedge edi \in \mathbf{dom}(inspect\_Auth(\omega(u)))(ai)$
- 17a. **post:**  $u \in \mathbf{dom} \omega' \wedge \mathbf{dom} \omega \setminus \{u\} = \omega' \setminus \{u\}$
- 17a.  $\wedge \mathbf{let} d = \omega(u), d' = \omega'(u) \mathbf{in} \omega' = \omega \uparrow [u \mapsto d']$
- 17(b)i.  $\wedge inspect\_Trace(d') = \langle aitl \rangle \hat{\ } inspect\_Trace(d)$
- 17(b)ii.  $\wedge inspect\_Auth(d') = inspect\_Auth(d)$
- 17(b)iii.  $\wedge inspect\_C(d') = fwd(inspect\_C(d)) \mathbf{end}$
- 17c. **axiom:**
- 17c.  $und(inspect\_C(\omega'(u))) = inspect\_C(\omega(u))$

### 3 Legal Documents

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### 4 Bibliographical Notes

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