2. Lecture 2: Domain Analysis: Meaning and Syntax

2.1. Formal Concept Analysis

2.1.1. FCA: Theory

- This section is a transcription of
  - Ganter & Wille's [GanterWille:ConceptualAnalysis1999] *Formal Concept Analysis, Mathematical Foundations*, the 1999 edition, Pages 17–18.

Some Notation:

- By $\mathcal{E}$ we shall understand the type of entities;
- by $\mathbb{E}$ we shall understand a value of type $\mathcal{E}$;
- by $\mathcal{Q}$ we shall understand the type of qualities;
- by $\mathbb{Q}$ we shall understand a value of type $\mathcal{Q}$;
- by $\mathcal{E}$**-set** we shall understand the type of sets of entities;
- by $\mathbb{ES}$ we shall understand a value of type $\mathcal{E}$**-set**;
- by $\mathcal{Q}$**-set** we shall understand the type of sets of qualities; and
- by $\mathbb{QS}$ we shall understand a value of type $\mathcal{Q}$**-set**.

Definition: 1 Formal Context:

- A formal context $\mathbb{K} := (\mathbb{ES}, \mathbb{I}, \mathbb{QS})$ consists of two sets;
  - $\mathbb{ES}$ of entities,
  - $\mathbb{QS}$ of qualities, and a
  - relation $\mathbb{I}$ between $\mathbb{E}$ and $\mathbb{Q}$. ■

- To express that $\mathbb{E}$ is in relation $\mathbb{I}$ to a Quality $\mathbb{Q}$ we write
  - $\mathbb{E} \cdot \mathbb{I} \cdot \mathbb{Q}$, which we read as
  - *"entity $\mathbb{E}$ **has** quality $\mathbb{Q}$".*

$\mathbb{ES} \setminus \mathbb{QS}$	$\mathbb{Q}_1$	$\mathbb{Q}_1$	$\mathbb{Q}_2$	$\mathbb{Q}_3$	$\mathbb{Q}_4$	$\mathbb{Q}_5$	$\mathbb{Q}_6$	$\mathbb{Q}_7$
$\mathbb{E}_a$		$\oplus$					$\oplus$	
$\mathbb{E}_b$				$\oplus$				
$\mathbb{E}_c$			$\oplus$	$\oplus$				
$\mathbb{E}_d$			$\oplus$	$\oplus$				
$\mathbb{E}_e$		$\oplus$	$\oplus$	$\oplus$				
$\mathbb{E}_f$	$\oplus$		$\oplus$					
$\mathbb{E}_g$	$\oplus$				$\oplus$	$\oplus$		
$\mathbb{E}_h$							$\oplus$	
$\mathbb{E}_i$								$\oplus$

- Example endurant entities are
  - a specific vehicle,
  - another specific vehicle,
  - etcetera;
  - a specific street segment (link),
  - another street segment,
  - etcetera;
  - a specific road intersection (hub),
  - another specific road intersection,
  - etcetera,
  - a monitor.

  One can also list perdurant entities.

- Example endurant entity qualities are
  - has mobility,
  - has velocity ($\geq$0),
  - has acceleration ($\geq$0),
  - has length ($>$0),
  - has location,
  - has traffic state,
  - etcetera.

  One can also list perdurant entity qualities.

Definition: 2 Qualities Common to a Set of Entities:

- For any subset, $s\mathbb{ES} \subseteq \mathbb{ES}$, of entities we can define $\mathcal{DQ}$ for "derive set of qualities".

$$\mathcal{DQ} : \mathcal{E}\textbf{-set} \to (\mathcal{E}\textbf{-set} \times \mathcal{I} \times \mathcal{Q}\textbf{-set}) \to \mathcal{Q}\textbf{-set}$$
$$\mathcal{DQ}(s\mathbb{ES})(\mathbb{ES}, \mathbb{I}, \mathbb{QS}) \equiv \{\mathbb{Q} \mid \mathbb{Q}{:}\mathcal{Q}, \mathbb{E}{:}\mathcal{E} \cdot \mathbb{E}{\in}s\mathbb{ES} \wedge \mathbb{E} \cdot \mathbb{I} \cdot \mathbb{Q}\}$$
$$\textbf{pre}: s\mathbb{ES} \subseteq \mathbb{ES}$$

"the set of qualities common to entities in $s\mathbb{ES}$". ■

Definition: 3 Entities Common to a Set of Qualities:

- For any subset, $s\mathbb{QS} \subseteq \mathbb{QS}$, of qualities we can define $\mathcal{DE}$ for "derive set of entiites".

$$\mathcal{DE}: \mathcal{Q}\textbf{-set} \to (\mathcal{E}\textbf{-set} \times \mathcal{I} \times \mathcal{Q}\textbf{-set}) \to \mathcal{E}\textbf{-set}$$
$$\mathcal{DE}(s\mathbb{QS})(\mathbb{ES}, \mathbb{I}, \mathbb{QS}) \equiv \{\mathbb{E} \mid \mathbb{E}{:}\mathcal{E}, \mathbb{Q}{:}\mathcal{Q} \cdot \mathbb{Q}{\in}s\mathbb{Q} \wedge \mathbb{E} \cdot \mathbb{I} \cdot \mathbb{Q}\},$$
$$\textbf{pre}: s\mathbb{QS} \subseteq \mathbb{QS}$$

"the set of entities which have all qualities in $s\mathbb{Q}$". ■

Definition: 4 Formal Concept:

- A **formal concept** of a **context** $\mathbb{K}$ is a pair:
  - $(s\mathbb{Q}, s\mathbb{E})$ where
    - $\mathcal{DQ}(s\mathbb{E})(\mathbb{E}, \mathbb{I}, \mathbb{Q}) = s\mathbb{Q}$ and
    - $\mathcal{DE}(s\mathbb{Q})(\mathbb{E}, \mathbb{I}, \mathbb{Q}) = s\mathbb{E}$;
  - $s\mathbb{Q}$ is called the **intent** of $\mathbb{K}$ and $s\mathbb{E}$ is called the **extent** of $\mathbb{K}$. ■

- $s\mathbb{Q} = \{\mathcal{Q}_2, \mathcal{Q}_3\}, s\mathbb{E} = \{\mathcal{E}_c, \mathcal{E}_d\}$

$\mathbb{ES} \setminus \mathbb{QS}$	$\mathbb{Q}_1$	$\mathbb{Q}_1$	$\mathbb{Q}_2$	$\mathbb{Q}_3$	$\mathbb{Q}_4$	$\mathbb{Q}_5$	$\mathbb{Q}_6$	$\mathbb{Q}_7$
$\mathbb{E}_a$								
$\mathbb{E}_b$								
$\mathbb{E}_c$			$\oplus$	$\oplus$				
$\mathbb{E}_d$			$\oplus$	$\oplus$				
$\mathbb{E}_e$								
$\mathbb{E}_f$								
$\mathbb{E}_g$								
$\mathbb{E}_h$								
$\mathbb{E}_i$								$\oplus$

- Now comes the "crunch":
  - *In the* `TripTych` *domain analysis*
  - *we strive to find formal concepts*
  - *and, when we think we have found one,*
  - *we assign a type*
  - *and properties:*
    - *unique identification,*
    - *mereology and*
    - *attributes*

  to it !

- In mathematical terms it turns out that **formal concepts** are **Galois connections**.
- We can, in other words, characterise **domain analysis** to be the "hunting" for **Galois connections**.
- Or, even more "catchy":
  - **domain types**,
  - whether they be **endurant entity types**
  - or they be **perdurant entity signatures**
  - are **Galois connections**.

2.1.2. Formal Concepts: Practice

- Usually a concept, as understood above, has been given a name:
  - that name have developed it more-or-less precise
  - and commonly accepted meaning over the years.
- **Examples of Concepts:**
  - street segment (**link**),
  - street crossing (**hub**),
  - **vehicle**,
  - **pipe**,
  - **valve** and
  - **pump**.

- In our domain analysis we shall therefore take a two-pronged approach.
  - (i) For commonly accepted and identified entity class names we immediately suggest a type name and identify qualities etc.
  - (ii) For "novel" entities,
    - for which no commonly agreed concept name are available,
    - one must carefully analyse a suitable set of entities claimed to "represent that concept", and
    - then suggest a concept cum type name and a suitable set of qualities.

- That is, we reverse matters.
  - Postulate a concept, whether concrete or abstract,
  - endow it with a name an properties,
  - and, if challenged, point to instances, i.e., entities.
- We may be forced to retract a postulated concept.[4]

[4] *"There are no theories; there are no proofs. There may be bold conjectures; and someties there are sad refutations." [A Sir Karl Popper essence.]*..

- Therefore, if an entity has quality $\mathcal{Q}$,
  - that is, $has\_\mathcal{Q}$ holds,
  - then values of that quality are obtained by either
    - **uid_P**,
    - **mereo_P** or
    - **attr_A**,

2.2. Basic Domain Concepts

- Before software can be designed
  - one must have a reasonable understanding of its requirements.
- Before requirements can be prescribed
  - one must have a reasonable understanding of the domain
  - within which the requirements "reside".

[1] Domain

- By a **domain** we shall[5] understand
  - *an area of human activity*
  - *characterised by observable phenomena, that is,*
    - *entities whether*
      - *endurants (manifest parts and materials)*
      - *or perdurants (actions, events or behaviours),*

      whether
      - *discrete or*
      - *continuous,*
    - *and of their [further] qualities.*

[5] This characterisation is additional to that of the introductory summary.

- For practical reasons we name domains.
  - In the below example we rely on your intuition
    - in "filling out the details"
    - when given the domain names.

Example: 4 Areas of Human Activity. Informal examples of domain names are:

- air traffic,
- banks,
- container line,
- hospitals,
- manufacturing,
- pipelines,
- railways,
- waste management,
- etcetera. ■

- A key term above was that of entity. Other terms for the same are:
  - ‘thing’,
  - ‘individual’,
  - ‘term’,
  - ‘quantity’.
  - ‘object’,
  - ‘unit’,
  - ‘particular’,
  - etcetera.

- Henceforth we shall think of domains without any reference
  - to requirements to software
  - let alone software.
- That is, we shall consider
  - the study of domains
  - like the study of physics,
  - something that is of importance in and of itself.

- The above characterisation of **domain** hinged on the concept of
  - observable phenomena, that is
  - entities.
- Thus we "equate"
  - observable phenomena
  - with entities.

[2] Entity

- By a **domain entity** we shall understand
  - *a manifest domain phenomenon*

  or

  - *a domain concept,*
    - *i.e., an abstraction,*
    - *derived from a domain entity.* ●
- The distinction between
  - a **manifest domain phenomenon** and
  - a **concept** thereof, i.e., a **domain concept**,

  is important.

- Really, what we describe are the **domain concepts** derived
  - from **domain phenomena** or
  - from other **domain concepts**.

Example: 5 Entity Instances versus Entity Types. Thus we do not specifically describe

- *that street segment there*,
- *that vehicle passing us*,
- *this barrel of oil here*,
- etcetera,

but focus on

- *street segments (links)*,
- *vehicle*,
- *oil*,
- etcetera. ■

- The above characterisation of domain also hinged on
  - ◈ the concept of [entity] qualities.

  We "loosely" distinguish between
  - ◈ extensional qualities (entity form),
    - ⊚ which we shall call entity syntax
      (i.e., entity form or entity structure),

    and
  - ◈ intensional qualities (entity attribute),
    - ⊚ which we shall call entity properties
      (i.e., entity content or just property).

[3] Phenomena

- By a **domain phenomenon** we shall understand
  - *something that can be observed by the human senses*
  - *or by equipment based on laws of physics and chemistry.*

- We shall make a distinction between
  - spatial and
  - temporal

  phenomena, respectively concepts derived from
  - spatial and
  - temporal

  phenomena.
- The former we shall call **endurants**, the latter **perdurants**.

[4] Endurants

- By an **endurant** we shall understand
  - *a specific kind of phenomenon, that is,*
  - *an entity that can be observed, i.e., perceived or conceived,*
  - *as a complete physical entity or as a concept*
  - *at no matter which given snapshot of time;*
    - *were we to freeze time*
    - *we would still be able to observe the entire endurant*[6]*.*
- Colloquially you may think of endurants
  - as data (structures)
  - should a domain description subsequently lead to software
  - wherein some of the domain endurants are also represented.

[6] edited from Wikipedia

Example: 6 Endurant Entities. Rephrasing Example 5 we get:

- *road nets*,
- *links*,
- *hubs*,
- *pipelines*,
- *valves*,
- *pumps*,
- *oil*,
- *gas*,
- etcetera, ■

[5] Discrete Endurants: Parts

- By a **discrete endurant**, that is, a **part**, we shall understand
  - *an endurant which is*
  - *separate or distinct in form or concept,*
  - *consisting of distinct or separate parts.*

Example: 7 **Parts.** Examples of parts:

- *container*,
- *freight item*,
- *vessel*,
- *truck*,
- *crane*,
- *port*,
- *person*,
- *vehicle*,
- etcetera. ■

[6] Continuous Endurants: Materials

- By a continuous endurant, that is, a material, we shall understand
  - *an endurant whose spatial characteristics are*
  - *prolonged, without interruption,*
  - *in an unbroken spatial series or pattern.* •

Example: 8 **Materials.** Examples of materials:

- *oil*,
- *gas*,
- *water*,
- *sand*,
- *gravel*,
- *garbage*,
- *grain*,
- *milk*,
- etcetera. ■

[7] Endurant Attributes

- By an **endurant attribute** we shall understand
  - *a phenomenon that can be observed of a part or a material,*
  - *not by manifest means,*
  - *but by using equipment based on laws of physics (incl. chemistry),*
  - *or by being related to other parts and materials,*
  - *or by being counted !*

Example: 9 Endurant Attributes. Some examples are:

- *length of a street segment,*
- *setting of a street signal,*
- *colour of current signal,*
- *velocity of a vehicle,*
- *colour of a car,*
- *decibel level of car horn,*
- *frequency of alarm signal,*
- *flow capacity of a pipe,*
- *denomination of a bank note,*
- *viscosity of oil,*
- *temperature of water,*
- *gender of a person,*
- *weight of a person,*
- etcetera. ■

[8] Perdurants

- By a **perdurant** we shall understand
  - *an entity which exists*
    - *only instantaneously, at some point in time,*
    - *or during a time interval*
      - *such that if we freeze time, in that interval,*
      - *then we only see a proper fragment of the perdurant.*

Example: 10 Perdurant Entities. Examples of perdurant entities:

- *inserting a link between two hubs* (an action),
- *disappearance of a link* (an event),
- *movement (traffic) of vehicles* (a behaviour).
- etcetera. ■

[9] Discrete Perdurant

- By a **discrete perdurant** we shall understand
  - *a perdurant which we consider as taking place*
  - *either instantaneously, in no time,*
  - *or where whatever time interval it may take to complete that time interval is considered immaterial,*
  - *or a sequence [of sets] of discrete perdurants where intervals between discrete perdurants are considered immaterial.*

Example: 11 Discrete Perdurants. Example discrete perdurants are:

- *the* `deposit` *of funds into a bank account (an* ***action****),*
- *the overdraft on a deposit/withdrawal account (an* ***event****),*
- *the sequence of actions and events with respect to a bank account (a* ***behaviour****):*
  - `opening` *the account,*
  - `deposits` *and* `withdrawals`*,*
  - *occasional* `statements`*, ending with*
  - *the* `closing` *of the account.*

■

[10] Continuous Perdurant

- By a continuous perdurant we shall understand
  - *a perdurant whose temporal characteristics are likewise*
  - *prolonged, without interruption,*
  - *in an unbroken temporal series or pattern.*

Example: 12 Continuous Perdurants. Example of continuous perdurants are:

- *the flow of oil in a pipeline,*
- *the traffic of vehicles on a road net,*
- *the change of weather at a given geographical spot,*
- etcetera. ■

[11] Atomic Parts

- By an **atomic part** we shall understand
  - *is a part which,*
  - *in a given context,*
  - *is deemed not to consist of meaningful, separately observable proper sub-parts,*
  - *where sub-parts are parts.*

Example: 13 Atomic Parts. We consider

- *street segments,*
- *street intersections and*
- *vehicles*

to be atomic parts [of transport systems]. ■

[12] Composite Parts

- By a **composite part** we shall understand
  - *a part which, in a given context,*
  - *is deemed to indeed consist of meaningful, separately observable proper sub-parts,*
  - *where sub-parts are parts.*

Example: 14 Composite Parts. We consider

- *road transport systems,*
- *hospitals and*
- *pipelines*

to be composite parts. ■

[13] Entity Properties

- By an entity property we shall understand *a quality such as*
  - *whether the entity is an endurant or a perdurant,*
  - *whether the entity is discrete or continuous,*
  - *whether an endurant entity is atomic or composite, or*
  - *whether a perdurant entity is an action, or an event or a behaviour.*

[14] Entity Qualities

- By an **entity quality** we shall understand *a proposition*
  - *such as a property,*
  - *or as an attribute.*

[15] Domain Description

- By a **domain description** we shall understand
  - *a narrative description*
  - *tightly coupled (say line-number-by-line-number)*
  - *to a formal description*
  - *of a domain:*
    - *its entities*
    - *and their qualities.*

Example: 15 A Domain Description. The following is a tiny fragment of a domain description:

Narrative:

72. *A road net is a composition of*
    - *a composition of hubs and*
    - *a composition of links.*
73. *A composition of hubs is a set of hubs.*
74. *A composition of links is a set of links.*
75. *Hubs and links are here considered atomic endurants.*

Formalisation:

type
72. N, HS, LS, Hs, Ls
75. H, L,
value
72. **obs_**HS: N → HS,
72. **obs_**LS: N → LS,
73. **obs_**Hs: HS → H-**set**,
74. **obs_**Ls: LS → L-**set**,

- The example does not illustrate a full complement of road net properties and attributes.
- This example will therefore reappear in many forms and extensions in this seminar. ■

[16] Domain Engineering

- By **domain engineering** we shall understand
  - *the engineering of a domain description,*
  - *that is,*
    - *the rigorous construction of domain descriptions, and*
    - *the further analysis of these, creating theories of domains, etc.*
- We are not engineering a domain, but [only] its description.

Example: 16 Domain Engineering. Examples of facets of domain engineering are:

- (i) *the planning, selection, scheduling and allocation of resources for the development of a domain description,*
- (ii) *the selection of proper tools and techniques for domain description,*
- (iii) *the decisions made in abstraction and description choices,*
- (iv) *the instrumentation of proofs, model checks and test data,*
- (v) *the decisions made to possibly redo a description section,*
- (vi) *the decisions made when regrettably replacing domain engineering staff,*
- etcetera.

- In this seminar we shall not cover such aspects as
  - planning, selection, scheduling and allocation of resources,
  - selection of proper tools and techniques for domain description and
  - instrumentation of proofs, model checks and test data.
- but shall focus on
  - analysis techniques,
  - abstraction and
  - description.

[17] Domain Science

- By **domain science** we shall understand
  - *either*
    - *(i) the general study and knowledge of*
      - *how to create and handle domain descriptions*
      - *(a general theory of domain descriptions)*

  or
    - *(ii) the specific study and knowledge of a particular domain.*
  - *The two studies intertwine.* ●

Example: 17 Domain Science. Examples of possible domain science elements:

- general
  - *laws of domain descriptions* and
  - a possible *calculus of domain description operators*

  are of the first kind (i), and
- specific, proven
  - *properties of a domain*

  are of the second kind (ii). ■

[18] Extensionality

- By **extensionality** we shall mean
  - *something which relates to, or is marked by extension,*
  - *that is, concerned with objective reality.*[7]
- Our use basically follows this characterisation:
  - We think of extensionality as a syntactic notion,
  - one that characterises an exterior appearance or form
- We shall therefore think of
  - part types and material types
  - whether parts are atomic or composite, and
  - how composite parts are composed

as extensional features.

[7] Extensionality. Merriam-Webster.com. 2011, http://www.merriam-webster.com (16 August 2012).

[19] Intentionality

- By **intentionality** we shall mean
  - *done by intention or design,*
  - *intended,*
  - *of or relating to epistemological intention,*
  - *having external reference.*[8]
- Our use basically follows this characterisation:
  - we think of intentionality as a semantic notion,
  - one that characterises an intention.
- We shall therefore think of
  - **part attribute**s and **material attribute**s

  as **intentional feature**s.

[8] Intentionality. Merriam-Webster.com. 2011, http://www.merriam-webster.com (16 August 2012).

2.3. Discussion

- The crucial characterisation (above) is that of domain entity (Slide 99).
  - It is pivotal since all we describe are domain entities and their qualities.
  - If we get the characterisation wrong we get everything wrong!
  - What might get the characterisation, or its interpretation, wrong is the interpretation of domain entities:
    - *those phenomena that can be observed by*
      - *the human eye or*
      - *touched, for example, by human hands,*

    and
    - *manifest domain phenomena or*
    - *domain concepts, i.e., abstractions,*
    - *derived from domain entities.*

- The whole thing hinges of
  - *what can be described,*
  - *what constitutes a description and*
  - *when is a text a bona fide description.*
- Another set of questions are
  - *of what we have chosen to constitute entities*
  - *which should we describe,*
  - *which not ?*

- Philosophers have dealt with these questions.
  - Recent writings are [Badiou1988,BarrySmith1993,ChrisFox2000] and [CasatiVarzi2010,HenryLaycock2011,WilsonScpall2012].
  - Going back in time we find [LeonardGoodman1940,Kripke1980,BowmanLClarke81].
  - Among the classics we mention [Russell1905,Russell1922,RudolfCarnap1928,StanislawLesniewksi1927-19

- We shall only indirectly contribute to this philosophical discussion and do so by presenting the material of this paper.
  - We have studied, over the years, fragments of the above cited publications.
  - And we humbly suggest that
    - following the principles, techniques and tools presented here
    - can lead the **domain engineer** to
    - a large class of **domain descriptions**s,
    - large enough for our "immediate future" needs !
- We shall, in the conclusion, return to the questions of
  - what can be described,
  - what constitutes a description and
  - when is a text a bona fide description ?