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Begin of Lecture 7: Last Session — Calculus II

Function Signature Discoverers and Laws

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

Tutorial Schedule

| • Lectures 1–2 | 9:00-9:40 + 9:50-10:30 | |
|--|-----------------------------|------------------|
| 1 Introduction | | Slides 1–35 |
| 2 Endurant Entities: Parts | | Slides 36–110 |
| • Lectures 3–5 11:00–11:15 | + 11:20-11:45 + 11:50-12:30 | |
| 3 Endurant Entities: Materials, States | | Slides 111–142 |
| 4 Perdurant Entities: Actions and Events | | Slides 143–174 |
| 5 Perdurant Entities: Behaviours | | Slides 175–285 |
| Lunch | 12:30-14:00 | |
| • Lecture 6–7 | 14:00-14:40 + 14:50-15:30 | |
| 6 A Calculus: Analysers, Parts and Materials | | Slides 286–339 |
| $\sqrt{7}$ A Calculus: Function Signatures and Laws | S | lides 340–377 |
| • Lecture 8–9 | 16:00-16:40 + 16:50-17:30 | |
| 8 Domain and Interface Requirements | | Slides 378–424 |
| 9 Conclusion: Comparison to Other Work | | Slides 428–460 |
| Conclusion: What Have We Achieved | Slides 425 | -427 + 461 - 472 |

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11.3.7. ACTION_SIGNATURES

- We really should discover actions, but actually analyse function definitions.
- And we focus, in this tutorial, on just "discovering" the function signatures of these actions.
- By a function signature, to repeat, we understand

 a functions name, say fct, and
 a function type expression (te), say dte→rte where
 dte defines the type of the function's definition set
 and rte defines the type of the function's image, or range set.

- We use the term 'functions' to cover actions, events and behaviours.
- We shall in general find that the signatures of actions, events and behaviours depend on types of more than one domain.

 \otimes Hence the schematic index set $\{\ell_1 \land \langle t_1 \rangle, \ell_2 \land \langle t_2 \rangle, \dots, \ell_n \land \langle t_n \rangle\}$ \otimes is used in all action, event and behaviour discoverers.

ACTION_SIGNATURES |/||

120. The ACTION_SIGNATURES meta-function, besides narrative texts, yields

- (a) a set of auxiliary sort or concrete type definitions and
- (b) a set of action signatures each consisting of an action name and
 - an action name and
 - a pair of definition set and range type expressions where
- (c) the type names that occur in these type expressions are defined by in the domains indexed by the index set.

| 100 | $A (\mathbf{m}) = A (\mathbf{m}) + A (\mathbf{m}) = A (\mathbf{m}) + A (\mathbf{m}) $ |
|--------|--|
| 120 | $ACTION_SIGNATURES: Index \rightarrow Index-set \rightarrow (Text \times RSL)$ |
| 120 | $\texttt{ACTION_SIGNATURES}(\ell^{(t_1)})(\{\ell_1^{(t_1)}, \ell_2^{(t_2)}, \dots, \ell_n^{(t_n)}\}):$ |
| 120 | [narrative, possibly enumerated texts ; |
| 120 | type $t_a, t_b, \dots, t_c,$ |
| 120(b) | value |
| 120(b) | $\operatorname{act}_i:\operatorname{te}_{i_d} \xrightarrow{\sim} \operatorname{te}_{i_r}, \operatorname{act}_j:\operatorname{te}_{j_d} \xrightarrow{\sim} \operatorname{te}_{j_r}, \dots, \operatorname{act}_k:\operatorname{te}_{k_d} \xrightarrow{\sim} \operatorname{te}_{k_r}$ |
| 120(c) | where: |
| 120(c) | type names in $te_{(i j k)_d}$ and in $te_{(i j k)_r}$ are either |
| 120(c) | type names $t_a, t_b, \dots t_c$ or are type names defined by the |
| 120(c) | indices which are prefixes of $\ell_m \widehat{\langle} T_m \widehat{\rangle}$ and where T_m is |
| 120(c) | in some signature $act_{i j \dots k}$] |
| | |

Example: 55 Transport Nets: Action Signatures.

- $ACTION_SIGNATURES(\langle \Delta, N, HS, Hs, H \rangle)(\{\langle \Delta, N, LS, Ls, L \rangle \rangle\}):$ insert_H: N \rightarrow H $\xrightarrow{\sim}$ N remove_H: N \rightarrow HI $\xrightarrow{\sim}$ N
- $\mathbb{ACTION_SIGNATURES}(\langle \Delta, N, LS, Ls, L \rangle)(\{\langle \Delta, N, HS, Hs, H \rangle \rangle\}):$ insert_L: $\mathbb{N} \to \mathbb{L} \xrightarrow{\sim} \mathbb{N}$ remove_L: $\mathbb{N} \to \mathbb{LI} \xrightarrow{\sim} \mathbb{N}$
- \bullet where \cdots refer to the possibility of discovering further action signatures "rooted" in

$$\ll \langle \Delta, N, HS, Hs, H \rangle$$
, respectively
 $\ll \langle \Delta, N, LS, Ls, L \rangle$.

. . .

. . .

11.3.8. EVENT_SIGNATURES

EVENT_SIGNATURES |/||

121. The EVENT_SIGNATURES meta-function, besides narrative texts, yields

(a) a set of auxiliary event sorts or concrete type definitions and

(b) a set of event signatures each consisting of

- an event name and
- \bullet a pair of definition set and range type expressions where
- (c) the type names that occur in these type expressions are defined either in the domains indexed by the indices or by the auxiliary event sorts or types.

| EVENT_SIGNATURES / | |
|----------------------|--|
|----------------------|--|

| 121 | $\mathbb{EVENT}_SIGNATURES: Index \rightarrow Index-set \xrightarrow{\sim} (Text \times RSL)$ |
|-----|---|
| 121 | EVENT_SIGNATURES $(\ell^{(t)})(\{\ell_1^{(t_1)}, \ell_2^{(t_2)}, \dots, \ell_n^{(t_n)}\})$: |

- 121(a) [narrative, possibly enumerated texts omitted ;
- 121(a) **type** $t_a, t_b, \dots, t_c,$

121(b) **value**

- 121(b) $\operatorname{evt_pred}_i: \operatorname{te}_{d_i} \times \operatorname{te}_{r_i} \to \operatorname{\mathbf{Bool}}$
- 121(b) $\operatorname{evt_pred}_j: \operatorname{te}_{d_j} \times \operatorname{te}_{r_j} \to \operatorname{\mathbf{Bool}}$
- 121(b) ...

121(b)
$$\operatorname{evt_pred}_k: \operatorname{te}_{d_k} \times \operatorname{te}_{r_k} \to \operatorname{\mathbf{Bool}}]$$

121(c) where: t is any of $t_a, t_b, ..., t_c$ or type names listed in in indices; type names of the 'd'efinition set and 'r'ange set type expressions te_d and te_r are type names listed in domain indices or are in $t_a, t_b, ..., t_c$, the auxiliary discovered event types.

Example: 56 Transport Nets: Event Signatures.

We refer to Example 34 on page 169. The omitted narrative text would, if included, as it should, be a subset of the Items 23–26 texts on Slide 167.

- EVENT_SIGNATURES($\langle \Delta, N, LS, Ls, L \rangle$)({ $\langle \Delta, N, HS, Hs, H \rangle$ }): value
 - link_disappearance: $N \times N \xrightarrow{\sim} Bool$ link_disappearance(n,n') \equiv
 - $\exists \ \ell: L \cdot l \in obs_Ls(n) \Rightarrow pre_cond(n,\ell) \land post_cond(n,\ell,n')$

... [possibly further, discovered event]

- ... [signatures "rooted" in $\langle \Delta, N, LS, Ls, L \rangle$]
- The undefined **pre_** and **post_cond**itions were "fully discovered" on Slides 169 and 171.

11.3.9. BEHAVIOUR_SIGNATURES

- We choose, in this tutorial, to model behaviours in CSP^{28} .
- \bullet This means that we model (synchronisation and) communication between behaviours by means of messages m of type M, CSP channels (channel ch:M) and CSP
 - ∞ output: ch!e [offer to deliver value of e on channel ch], and∞ input: ch? [offer to accept a value on channel ch].

²⁸Other behaviour modelling languages are Petri Nets, MSCs: Message Sequence Charts, Statechart etc.

• We allow for the declaration of single channels as well as of one, two, ..., n dimensional arrays of channels with indexes ranging over channel index types

 \otimes type Idx, Cldx, Rldx ...:

etcetera.

• We assume some familiarity with CSP [Hoare85+2004] (or even RSL/CSP [TheSEBook1wo] [Chapter 21]).

• A behaviour usually involves two or more distinct sub-domains.

Example: 57 Vehicle Behaviour. Let us illustrate that behaviours usually involve two or more distinct sub-domains.

- A vehicle behaviour, for example, involves
 - \otimes the vehicle sub-domain,
 - « the hub sub-domain (as vehicles pass through hubs),
 - « the link sub-domain (as vehicles pass along links) and,
 - \otimes for the road pricing system, also the monitor sub-domain.

BEHAVIOUR_SIGNATURES |/||

- 122. The BEHAVIOUR_SIGNATURES meta-function, besides narrative texts, yields
- 123. It applies to a set of indices and results in a pair,
 - (a) a narrative text and
 - (b) a formal text:
 - i. a set of one or more message types,
 - ii. a set of zero, one or more channel index types,
 - iii. a set of one or more channel declarations,
 - iv. a set of one or more process signatures with each signature containing a behaviour name, an argument type expression, a result type expression, usually just **Unit**, and
 - v. an input/output clause which refers to channels over which the signatured behaviour may interact with its environment.

BEHAVIOUR_SIGNATURES ||/|| 122. BEHAVIOUR_SIGNATURES: Index \rightarrow Index-set \rightarrow (Text \times RSL) BEHAVIOUR_SIGNATURES $(\ell^{(t_1)}, \ell_2^{(t_2)}, \dots, \ell_n^{(t_n)})$: 122. 123(a). [narrative, possibly enumerated texts ; **type** $m = m_1 | m_2 | ... | m_{\mu}, \mu \ge 1$ 123((b))i. 123((b))ii. $i = i_1 | i_2 | ... | i_n, n > 0$ 123((b))iii. **channel** c:m, $\{vc[x]|x:i_a\}:m, \{mc[x,y]|x:i_b,y:i_c\}:m,...$ 123((b))iv.value 123((b))iv. bhv₁: ate₁ \rightarrow *inout*₁ rte₁, 123((b))iv. ••• , 123((b))iv. bhv_m: ate_m \rightarrow inout_m rte_m. 123((b))iv. where type expressions $atei_i$ and rte_i for all i involve at least 123((b))iv.two types t'_i, t''_j of respective indexes $\ell_i (t_i), \ell_j (t_j),$ 123((b))v. where Unit may appear in either ate_i or rte_i or both. 123((b))v. **where** $inout_i$: in k | out k | in,out k 123((b))v. **where** k: c or vc[x] or {vc[x]|x: i_a ·x \in xs} or 123((b))v. $\{ \operatorname{mc}[\mathbf{x},\mathbf{y}] | \mathbf{x}: \mathbf{i}_b, \mathbf{y}: \mathbf{i}_c \cdot \mathbf{x} \in \mathbf{xs} \land \mathbf{y} \in \mathbf{ys} \}$ **or** ...

Example: 58 Vehicle Transport: Behaviour Signatures. We refer to Examples 35 and 36.

 $\begin{array}{l} \mathbb{BEHAVIOUR_SIGNATURES}(\langle \Delta, F, VS, Vs, V \rangle)(\{\langle \Delta, M \rangle\}):\\ [With each vehicle we associate behaviour with the following arguments: the vehicle identifier, the vehicle parts, and the vehicle position. The vehicle communicates with the monitor process over a vehicle to monitor array of channels, one for each vehicle ...; \end{array}$

type

VPos

channel

 $\{vm[vi]|vi:VI \cdot vi \in vis\}:VPos$

value

veh: vi:VI \rightarrow v:V \rightarrow vp:VPos \rightarrow **out** vm[vi] **Unit**]

$$\begin{split} \mathbb{B}\mathbb{E}\mathbb{H}\mathbb{A}\mathbb{V}\mathbb{I}\mathbb{O}\mathbb{U}\mathbb{R}_\mathbb{S}\mathbb{I}\mathbb{G}\mathbb{N}\mathbb{A}\mathbb{T}\mathbb{U}\mathbb{R}\mathbb{E}\mathbb{S}(\langle\Delta,\mathbf{M}\rangle)(\{\langle\Delta,\mathbf{F},\mathbf{V}\mathbf{S},\mathbf{V}\mathbf{s},\mathbf{V}\rangle\}):\\ & [\text{ With the monitor part we associate a behaviour with the monitor part as only argument. The monitor accepts communications from vehicle behaviours ... ;} \end{split}$$

value

mon: $M \rightarrow in \{vm[vi]|vi:VI \cdot vi \in vis\}$ Unit]

that "discovery", is left for further analysis.

We refer to Slide 192 Items 31–31(d),

11.4. Order of Analysis and "Discovery"

- Analysis and "discovery", that is, the "application" of
 - \otimes the analysis meta-functions % f(x)=f(x) and
 - \otimes the "discovery" meta-functions
- has to follow some order:
 - \otimes starts at the "root", that is with index $\langle \Delta \rangle$,
 - and proceeds with indices appending part domain type names already discovered.

11.5. Analysis and "Discovery" of "Leftovers"

- The analysis and discovery meta-functions focus on types, that is, the types
 - \otimes of abstract parts, i.e., sorts,
 - « of concrete parts, i.e., concrete types,
 - \otimes of unique identifiers,
 - \otimes of mereologies, and of
 - \otimes attributes where the latter has been largely left as sorts.

- In this tutorial we do not suggest any meta-functions for such analyses that may lead to
 - « concrete types from non-part sorts, or to
 - action, event and behaviour definitions
 say in terms of pre/post-conditions,
 etcetera.
 - So, for the time, we suggest, as a remedy for the absence of such "helpers", good "old-fashioned" domain engineer ingenuity.

11.6. Laws of Domain Descriptions

- By a **domain description law** we shall understand
 - « some desirable property
 - \otimes that we expect (the 'human') results of
 - the (the 'human') use of the domain description calculus to satisfy.

Notational Shorthands:

- $\bullet \; (f;g;h)(\Re) = h(g(f(\Re)))$
- $(f_1; f_2; \ldots; f_m)(\Re) \simeq (g_1; g_2; \ldots; g_n)(\Re)$ means that the two "end" states are equivalent modulo appropriate renamings of types, functions, predicates, channels and behaviours.
- $[f; g; \ldots; h; \alpha]$ stands for the Boolean value yielded by α (in state \Re).

11.6.1. 1st Law of Commutativity

• We make a number of assumptions:

 \otimes the following two are well-formed indices of a domain:

where ℓ' and ℓ'' may be different or empty $(\langle \rangle)$ and A and B are distinct;

- \circledast that ${\mathcal F}$ and ${\mathcal G}$ are two, not necessarily distinct discovery functions; and
- \otimes that the domain at ι' and at ι'' have not yet been explored.

• We wish to express,

« as a desirable property of **domain description development** \otimes that exploring domain Δ at ∞ either ι' first and then ι'' ∞ or at ι'' first and then ι' , \otimes the one right after the other (hence the ";"), ∞ ought yield the same partial description fragment: 124. $(\mathcal{G}(\iota''); (\mathcal{F}(\iota')))(\Re) \simeq (\mathcal{F}(\iota'); (\mathcal{G}(\iota'')))(\Re)$ When a domain description development satisfies Law 124., under the above assumptions,

 \otimes then we say that the development,

modulo type, action, event and behaviour name "assignments",satisfies a mild form of commutativity.

11.6.2. 2nd Law of Commutativity

• Let us assume

 \ll that we are exploring the sub-domain at index $\ll \iota: \langle \Delta \rangle^{\widehat{}} \ell^{\widehat{}} \langle \mathsf{A} \rangle.$

• Whether we

 \otimes first "discover" \mathcal{A} ttributes

 \otimes and then \mathcal{M} ereology (including \mathcal{U} nique identifiers)

or

 \otimes first "discover" \mathcal{M} ereology (including \mathcal{U} nique identifiers) \otimes and then \mathcal{A} ttributes

should not matter.

- We make some abbreviations:
 - $\otimes \mathcal{A}$ stand for the ATTRIBUTES,
 - $\otimes \mathcal{U}$ stand for the UNIQUE_IDENTIFIER,
 - $\otimes \mathcal{M}$ stand for the MEREOLOGY,
 - $\ll \iota$ for index $\langle \Delta \rangle \hat{\ell} \langle \mathsf{A} \rangle$, and
 - $\ll \iota \mathbf{s}$ for a suitable set of indices.
- Thus we wish the following law to hold:

125.
$$(\mathcal{A}(\iota); \mathcal{U}(\iota); \mathcal{M}(\iota)(\iota s))(\Re) \simeq$$

 $(\mathcal{U}(\iota); \mathcal{M}(\iota)(\iota s); \mathcal{A}(\iota))(\Re) \simeq$
 $(\mathcal{U}(\iota); \mathcal{A}(\iota); \mathcal{M}(\iota)(\iota s))(\Re).$

« here modulo attribute and unique identifier type name renaming.

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11.6.3. 3rd Law of Commutativity

• Let us again assume

 \otimes that we are exploring the sub-domain at index

 $\ll \iota: \langle \Delta \rangle \hat{\ell} \langle \mathsf{A} \rangle$

 \otimes where $\iota \mathbf{s}$ is a suitable set of indices.

• Whether we are

 \otimes exploring actions, events or behaviours at that domain index \otimes in that order,

 \otimes or some other order

ought be immaterial.

• Hence with

 $\ll \mathcal{A}$ now standing for the ACTION_SIGNATURES,

- $\otimes \mathcal{E}$ standing for the EVENT_SIGNATURES,
- $\otimes \ensuremath{\mathcal{B}}$ standing for the <code>BEHAVIOUR_SIGNATURES</code>,
- discoverers, we wish the following law to hold:

126.
$$(\mathcal{A}(\iota)(\iota s); \mathcal{E}(\iota)(\iota s); \mathcal{B}(\iota)(\iota s))(\Re) \simeq$$

 $(\mathcal{A}(\iota)(\iota s); \mathcal{B}(\iota)(\iota s); \mathcal{E}(\iota)(\iota s))(\Re) \simeq$
 $(\mathcal{E}(\iota)(\iota s); \mathcal{A}(\iota)(\iota s); \mathcal{B}(\iota)(\iota s))(\Re) \simeq$
 $(\mathcal{E}(\iota)(\iota s); \mathcal{B}(\iota)(\iota s); \mathcal{A}(\iota)(\iota s))(\Re) \simeq$
 $(\mathcal{B}(\iota)(\iota s); \mathcal{A}(\iota)(\iota s); \mathcal{E}(\iota)(\iota s))(\Re) \simeq$
 $(\mathcal{B}(\iota)(\iota s); \mathcal{E}(\iota)(\iota s); \mathcal{A}(\iota)(\iota s))(\Re).$

 where modulo action function, event predicate, channel, message type and behaviour (and all associated, auxiliary type) renamings.

11.6.4. 1st Law of Stability

• Re-performing

the same discovery function that is with identical indices, over the same sub-domain, one or more times,

ought not produce any new description texts.

• That is:

$$\begin{array}{ll} 127. \ (\mathcal{D}(\iota)(\iota \mathbf{s}); \mathcal{A}_\mathsf{and}_\mathcal{D}_\mathsf{seq})(\Re) \ \simeq \\ (\mathcal{D}(\iota)(\iota \mathbf{s}); \mathcal{A}_\mathsf{and}_\mathcal{D}_\mathsf{seq}; \mathcal{D}(\iota)(\iota \mathbf{s}))(\Re) \end{array}$$

- where
 - $\circledast \mathcal{D}$ is any discovery function,
 - $\ll \mathcal{A}_and_\mathcal{D}_seq$ is any specific sequence of
 - intermediate analyses and discoveries, and where
 - $\ll \iota$ and $\iota {\bf s}$ are suitable indices, respectively sets of indices.

11.6.5. **2nd Law of Stability**

• Re-performing

the same analysis functions that is with identical indices, over the same sub-domain, one or more times,

ought not produce any new analysis results.

• That is:

128.
$$[\mathcal{A}(\iota)] = [\mathcal{A}(\iota); \ldots; \mathcal{A}(\iota)]$$

- $\ll \mathcal{A}$ is any analysis function,
- \otimes "..." is any sequence of intermediate analyses and discoveries, and where
- $\ll \iota$ is any suitable index.

11.6.6. Law of Non-interference

- \bullet When performing a discovery meta-operation, ${\cal D}$
 - \ll on any index, $\iota,$ and possibly index set, $\iota \mathbf{s},$ and
 - \otimes on a repository state, $\Re,$
 - \otimes then using the $[\mathcal{D}(\iota)(\iota \mathbf{s})]$ notation
 - \otimes expresses a pair of a narrative text and some formulas, [txt,rsl],
 - \otimes whereas using the $(\mathcal{D}(\iota)(\iota s))(\Re)$ notation
 - \otimes expresses a next repository state, \Re' .
- What is the "difference" ?
- Informally and simplifying we can say that the relation between the two expressions is:

129.
$$[\mathcal{D}(\iota)(\iota \mathbf{s})]$$
: [txt,rsl]
 $(\mathcal{D}(\iota)(\iota \mathbf{s}))(\Re) = \Re'$
where $\Re' = \Re \cup \{[txt,rsl]\}$

• We say that when 129. is satisfied

 \otimes for any discovery meta-function \mathcal{D} ,

 \otimes for any indices ι and ι s

 \otimes and for any repository state $\Re,$

then the repository is not interfered with,

∞ that is, "what you see is what you get:"

and therefore that

 \otimes the discovery process satisfies the law on non-interference.

11.7. **Discussion**

- The above is just a hint at **domain development laws** that we might wish orderly developments to satisfy.
- We invite the audience to suggest other laws.
- The laws of the analysis and discovery calculus
 - \otimes forms an ideal set of expectations
 - \circledast that we have of not only one domain describer
 - \circledast but from a domain describer team
 - \circledast of two or more domain describers
 - ∞ whom we expect to work, i.e., loosely collaborate,
 - « based on "near"-identical domain development principles.

- These are quite some expectations.
 - \otimes But the whole point of
 - ∞ a highest-level
 - ${\scriptstyle \scriptsize \odot}$ academic scientific education and
 - engineering training
 - \otimes is that one should expect commensurate development results.

- The laws of the analysis and discovery calculus
 « expressed some properties that we wish the repository to exhibit.

- \bullet We expect further
 - research into, or possible changes to or development of, or and use
 - of the calculus to yield such insight as to lead to
 - \otimes a firmer understanding of
 - \otimes the nature of repositories.

• In the analysis and discovery calculus

 \otimes such as we have presented it

• we have emphasised

∞ the types of parts, sorts and immediate part concrete types, and
∞ the signatures of actions, events and behaviours —
∞ as these predominantly featured type expressions.

• We have therefore, in this tutorial, not investigated, for example,

« pre/post conditions of action function,

« form of event predicates, or

∞ behaviour process expressions.

• We leave that, substantially more demanding issue, for future explorative and experimental research.



End of Lecture 7: Last Session — Calculus II

Function Signature Discoverers and Laws

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012



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