



**HAD A GOOD LUNCH?**

# **Begin of Lecture 6: First Session — Calculus I**

## **Part and Material Discoverers**

**FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012**

# Tutorial Schedule

- **Lectures 1–2** 9:00–9:40 + 9:50–10:30
- 1 **Introduction** Slides 1–35
- 2 **Endurant Entities: Parts** Slides 36–110
- **Lectures 3–5** 11:00–11:15 + 11:20–11:45 + 11:50–12:30
- 3 **Endurant Entities: Materials, States** Slides 111–142
- 4 **Perdurant Entities: Actions and Events** Slides 143–174
- 5 **Perdurant Entities: Behaviours** Slides 175–285
- Lunch** **12:30–14:00**
- **Lectures 6–7** 14:00–14:40 + 14:50–15:30
- ✓ 6 **A Calculus: Analysers, Parts and Materials** **Slides 286–339**
- 7 **A Calculus: Function Signatures and Laws** Slides 340–377
- **Lectures 8–9** 16:00–16:40 + 16:50–17:30
- 8 **Domain and Interface Requirements** Slides 378–424
- 9 **Conclusion: Comparison to Other Work** Slides 428–460
- Conclusion: What Have We Achieved** Slides 425–427 + 461–472

## 11. Towards a Calculus of Domain Discoverers

- The ‘towards’ term is significant.
- We are not presenting
  - ❖ a “ready to serve”
  - ❖ comprehensive,
  - ❖ tested and triedcalculus.
- We hope that the one we show you is interesting.
- It is, we think, the first time such a calculus is presented.

- By a domain description calculus

- ◇ or, as we shall also call it,

- ⊗ either a domain discovery calculus

- ⊗ or a calculus of domain discoverers

we shall understand an **algebra**, that is,

- ◇ a set of meta-operations and

- ◇ a pair of

- ⊗ a fixed domain and

- ⊗ a varying **repository**.

- The meta-operations will be outlined in this section.

- The fixed domain is of the kind of domains alluded to in the previous section of this tutorial.

- The varying repository contains fragments of a description of the fixed domain.

- The meta-operators are referred to as
  - ❖ either **domain analysis meta-functions**
  - ❖ or **domain discovery meta-functions**.
- The former are carried out by the **domain analyser** when inquiring (the domain) as to its properties.
- The latter are carried out by the **domain describer** when deciding upon which descriptions “to go for” !
- The two persons can be the same one **domain engineer**.
- The operators are referred to as meta-functions,
  - ❖ or meta-linguistic functions,
  - ❖ since they are
    - ⊗ applied and ⊗ calculated
  - ❖ by humans, i.e., the **domain describers**.
- They are directives which can be referred to by the **domain describers** while carrying out their analytic and creative work.

## 11.1. Introductory Notions

- In order to present the operators of the calculus
  - ⋄ we must clear a few concepts.

### 11.1.1. Discovery

- By a **domain discovery calculus** we shall understand
  - ⋄ a set of operations (the **domain discoverers**),
    - ⊗ which when applied to a domain
    - ⊗ by a human agent, the **domain describer**,
  - and
    - ⊗ yield **domain description texts**.

- The **domain discoverers** are applied “mentally”.
  - ❖ That is, not in a mechanisable way.
    - ⊗ It is not like when procedure calls
    - ⊗ invoke computations
    - ⊗ of a computer.
  - ❖ But they are applied by the **domain describer**.
  - ❖ That person is to follow the ideas laid down for
  - ❖ these **domain discoverers**
    - ⊗ (as they were in the earlier parts of this talk).
  - ❖ They serve to guide the **domain engineer**
    - ⊗ to discoverer the desired domain entities
    - ⊗ and their properties.
- In this section we shall review an ensemble of (so far) nine **domain discoverers** and (so far) four **domain analysers**.



We list the nine domain discoverers.

- [ Slide 320] PART\_SORTS,  
[ Slide 317] MATERIEAL\_SORTS,  
[ Slide 324] PART\_TYPES,  
[ Slide 327] UNIQUE\_ID,  
[ Slide 328] MEREOLOGY,  
[ Slide 332] ATTRIBUTES,  
[ Slide 341] ACTION\_SIGNATURES,  
[ Slide 346] EVENT\_SIGNATURES and  
[ Slide 349] BEHAVIOUR\_SIGNATURES.

## 11.1.2. Analysis

- In order to “apply” these domain discoverers certain conditions must be satisfied.
- Some of these condition inquiries can be represented by (so far) four domain analysers.
  - ◇ [ Slide 306] `IS_MATERIALS_BASED`,
  - [ Slide 308] `IS_ATOM`,
  - [ Slide 308] `IS_COMPOSITE` and
  - [ Slide 312] `HAS_A_CONCRETE_TYPE`.

### 11.1.3. Domain Indexes

- In order to **discover**, the domain describer must decide on “*where & what in the domain*” to analyse and describe.
- One can, for this purpose, think of the domain as **semi-lattice-structured**.
  - ❖ The **root** of the lattice is then labelled  $\Delta$ .
  - ❖ Let us refer to the domain as  $\Delta$ .
  - ❖ We say that it has index  $\langle \Delta \rangle$ .
  - ❖ Initially we analyse the usually composite  $\Delta$  domain to consist of one or more distinctly typed parts  $\mathbf{p}_1:\mathbf{t}_1, \mathbf{p}_2:\mathbf{t}_2, \dots, \mathbf{p}_m:\mathbf{t}_m$ .
  - ❖ Each of these have indexes  $\langle \Delta, t_i \rangle$ .
  - ❖ So we view  $\Delta$ , in the semi-lattice, to be the **join** of  $m$  **sub-semi-lattices** whose roots we shall label with  $\mathbf{t}_1, \mathbf{t}_2, \dots, \mathbf{t}_m$ .

- ❖ And so forth for any composite part type  $\mathbf{t}_i$ , etcetera.
- ❖ It may be that any two or more such **sub-semi-lattice root types**,  $\mathbf{t}_{i_j}, \mathbf{t}_{i_j'}, \dots, \mathbf{t}_{i_k}$  designate the same, **shared type**  $\mathbf{t}_{i_x}$ , that is  $\mathbf{t}_{i_j} = \mathbf{t}_{i_j'} = \dots = \mathbf{t}_{i_k} = \mathbf{t}_{i_x}$ .
- ❖ If so then the  $k$  sub-semi-lattices are “collapsed” into one sub-semi-lattice.
- ❖ The building of the semi-lattice terminates when one can no longer analyse part types into further sub-semi-lattices, that is, when these part types are atomic.

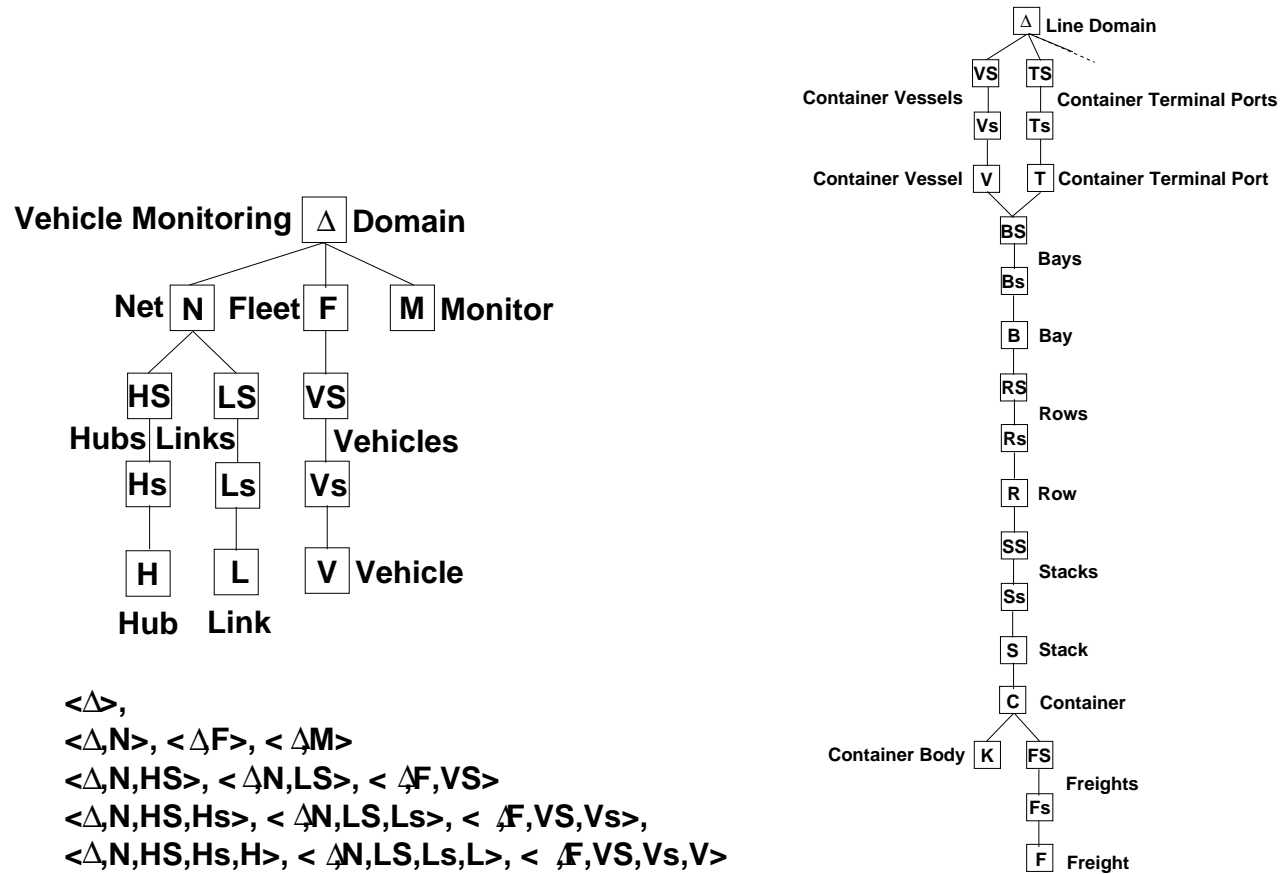


Figure 3: Domain indices

- That is, the roots of the **sub-trees** of the  $\Delta$  tree are labelled with **type names**.
  - ⊠ Every point in the semi-lattice can be identified by a **domain index**.
    - ⊗ The root is defined to have index  $\langle \Delta \rangle$ .
    - ⊗ The immediate **sub-semi-lattices** of  $\Delta$  have domain indexes  $\langle \Delta, \mathbf{t}_1 \rangle, \langle \Delta, \mathbf{t}_2 \rangle, \dots, \langle \Delta, \mathbf{t}_m \rangle$ .
    - ⊗ And so forth.
    - ⊗ If  $\ell^{\wedge} \langle \mathbf{t} \rangle$  is a prefix of another **domain index**, say  $\ell^{\wedge} \langle \mathbf{t}, \mathbf{t}' \rangle$ , then  $\mathbf{t}$  designates a composite type.

- For every domain index,  $\ell^{\langle t \rangle}$ , that index designates the type **t** domain type texts.
- These texts consists of several sub-texts.
- There are the texts directly related to the parts,  $p:P$ :
  - ⋄ the observer functions,  $obs_{\dots}$ , if type **t** is composite,
  - ⋄ the unique identifier functions,  $uid_P$ ,
  - ⋄ the mereology function,  $mereo_P$ , and
  - ⋄ the attribute functions,  $attr_{\dots}$ .
  - ⋄ To the above “add”
    - ⊗ possible auxiliary types and auxiliary functions
    - ⊗ as well as possible axioms.

- Then there are the texts related to
  - ❖ actions,
  - ❖ events, and
  - ❖ behaviours“based” (primarily) on parts  $p:P$ .
- These texts consists of
  - ❖ function signatures (for actions, events, and behaviours),
  - ❖ function definitions for these, and
  - ❖ channel
    - ⊗ declarations and
    - ⊗ channel message type definitionsfor behaviours.

We shall soon see examples of the above.



- But not all can be “discovered” by just examining the domain from the point of view of a sub-semi-lattice type.
  - ❖ Many interesting action, event and behaviour signatures depend on **domain type texts** designated by “roots” of disjoint sub-trees of the semi-lattice.
  - ❖ Each such root has its own **domain index**.
  - ❖ Together a **meet** of the semi-lattice is defined by the set of disjoint domain indices:  $\{\ell_i, \ell_j, \dots, \ell_k\}$ .
- It is thus that we arrive at a proper semi-lattice structure relating the various entities of the domain rooted in  $\Delta$ .

- The **domain discoverers** are therefore provided with arguments:
  - ⋄ either a single **domain index**, `DOMAIN_FUNCTION( $\ell$ )`,
  - ⋄ or a pair, `DOMAIN_FUNCTION( $\ell$ )( $\{\ell_i, \ell_j, \dots, \ell_k\}$ )`,
    - ⊗ the single **domain index**  $\ell$  and
    - ⊗ a set of **domain indices**,  $\{\ell_i, \ell_j, \dots, \ell_k\}$where `DOMAIN_FUNCTION` is any of the
  - ⊗ **domain discoverers** or
  - ⊗ **domain analysers**listed earlier.

## 11.1.4. The Repository

- We have yet to give the full signature of the domain discoverers and domain analysers.
  - ⊠ One argument of these meta-functions
    - ⊗ was parts of the actual domain
    - ⊗ as designated by the domain indices.
  - ⊠ Another argument
    - ⊗ is to be the **Repository** of **description texts**
    - ⊗ being inspected (together with the sub-domain) when
      - \* analysing that sub-domain and
    - ⊗ being updated
      - \* when “generating” the “discovered” description texts.

- ❖ We can assume, without loss of generality, that
  - ⊗ the **Repository** of **description texts**
  - ⊗ is the **description texts** discovered so far.
- ❖ The result of **domain analysis** is either undefined or a truth value. We can assume, without any loss of generality that that result is not recorded.
- ❖ The result of **domain discovery** is either undefined or is a **description text** consisting of two well-defined fragments:
  - ⊗ a **narrative text**, and
  - ⊗ a **formal text**.
- ❖ Those well-defined texts are “added” to the text of the **Repository** of **description texts**.
  - ⊗ For pragmatic reasons,
  - ⊗ when we explain the positive effect of domain discovery,
  - ⊗ then we show just this “addition” to the **Repository**.

102. The proper type of the discover functions is therefore:

$$102. \text{ DISCOVER\_FUNCTION: } \text{Index} \rightarrow \text{Index-set} \rightarrow \mathfrak{R} \xrightarrow{\sim} \mathfrak{R}$$

- In the following we shall omit the  $\mathfrak{R}$ epository argument and result.

103. So, instead of showing the discovery function invocation and result as:

$$103. \text{ DISCOVER\_FUNCTION}(\ell)(\ell\text{set})(\rho) = \rho'$$

- where  $\rho'$  incorporates a pair of texts and RSL formulas,

104. we shall show the discover function signature, the invocation and the result as:

$$104. \text{ DISCOVER\_FUNCTION: } \text{Index} \rightarrow \text{Index-set} \xrightarrow{\sim} (\text{Narr\_Text} \times \text{RSL\_Text})$$

$$104. \text{ DISCOVER\_FUNCTION}(\ell)(\ell\text{set}): (\text{narr\_text}, \text{RSL\_text})$$

## 11.2. Domain Analysers

- Currently we identify four analysis functions.
- As the discovery calculus evolves
  - ❖ (through further practice and research)
  - ❖ we expect further analysis functions to be identified.

### 11.2.1. IS\_MATERIALS\_BASED

- You are reminded of the *Continuous Endurant Modelling* frame on Slide 132.

#### IS\_MATERIALS\_BASED

- An early decision has to be made as to whether a domain is significantly based on materials or not:

105. IS\_MATERIALS\_BASED( $\langle \Delta_{\text{Name}} \rangle$ ).

- If Item 105 holds of a domain  $\Delta_{\text{Name}}$ 
  - ◊ then the **domain describer** can apply
  - ◊ MATERIAL\_SORTS (Item 107 on page 317).

## Example: 45 Pipelines and Transports: Materials or Parts.

- $IS\_MATERIALS\_BASED(\langle \Delta_{\text{Pipeline}} \rangle) = \text{true}$ .
- $IS\_MATERIALS\_BASED(\langle \Delta_{\text{Transport}} \rangle) = \text{false}$ .



## 11.2.2. IS\_ATOM, IS\_COMPOSITE

- During the discovery process
  - ❖ **discrete part types** arise (i.e., the names are yielded)
  - ❖ and these may either denote **atomic** or **composite** parts.
- The domain describer
  - ❖ must now decide as to
  - ❖ whether a named, **discrete type** is **atomic** or is **composite**.

## IS\_ATOM

- The IS\_ATOM analyser serves that purpose:

**value**

IS\_ATOM: Index  $\xrightarrow{\sim}$  **Bool**

IS\_ATOM( $\ell^{\wedge}\langle t \rangle$ )  $\equiv$  **true** | **false** | **chaos**

- The analysis is undefined for ill-formed indices.

**Example: 46 Transport Nets: Atomic Parts (II).** We refer to Example 3 (Slide 16).

IS\_ATOM( $\langle \Delta, N, HS, Hs, H \rangle$ ),      IS\_ATOM( $\langle \Delta, N, LS, Ls, L \rangle$ )  
 $\sim$  IS\_ATOM( $\langle \Delta, N, HS, Hs \rangle$ ),       $\sim$  IS\_ATOM( $\langle \Delta, N, LS, Ls \rangle$ )



## IS\_COMPOSITE

- The IS\_COMPOSITE analyser is
  - ◇ similarly applied by the domain describer
  - ◇ to a **part type t**
  - ◇ to help decide whether **t** is a **composite type**.

### value

IS\_COMPOSITE: Index  $\xrightarrow{\sim}$  **Bool**

IS\_COMPOSITE( $\ell^{\wedge}\langle t \rangle$ )  $\equiv$  **true** | **false** | **chaos**

**Example: 47 Transport Nets: Composite Parts.** We refer to Example 3 (Slide 16)

IS\_COMPOSITE( $\langle \Delta \rangle$ ),  
IS\_COMPOSITE( $\langle \Delta, N \rangle$ )  
IS\_COMPOSITE( $\langle \Delta, N, HS, Hs \rangle$ ),  
IS\_COMPOSITE( $\langle \Delta, N, LS, Ls \rangle$ )  
 $\sim$  IS\_COMPOSITE( $\langle \Delta, N, HS, Hs, H \rangle$ ),  
 $\sim$  IS\_COMPOSITE( $\langle \Delta, N, LS, Ls, L \rangle$ )



### 11.2.3. HAS\_A\_CONCRETE\_TYPE

- Sometimes we find it expedient
  - ◊ to endow a “discovered” sort with a concrete type expression, that is,
  - ◊ “turn” a sort definition into a concrete type definition.

#### HAS\_A\_CONCRETE\_TYPE

106. Thus we introduce the analyser:

```

106  HAS_A_CONCRETE_TYPE: Index  $\xrightarrow{\sim}$  Bool
106  HAS_A_CONCRETE_TYPE( $\ell^{\langle t \rangle}$ ): true | false | chaos

```

**Example: 48 Transport Nets: Concrete Types .** We refer to Example 3 (Slide 16) while exemplifying four cases:

HAS\_A\_CONCRETE\_TYPE( $\langle \Delta, N, HS, Hs \rangle$ )

HAS\_A\_CONCRETE\_TYPE( $\langle \Delta, N, LS, Ls \rangle$ )

$\sim$  HAS\_A\_CONCRETE\_TYPE( $\langle \Delta, N, HS, Hs, H \rangle$ )

$\sim$  HAS\_A\_CONCRETE\_TYPE( $\langle \Delta, N, LS, Ls, L \rangle$ )



- We remind the listener that
  - ◇ it is a decision made by the domain describer
  - ◇ as to whether a part type is
    - ⊗ to be considered a sort or
    - ⊗ be given a concrete type.
- We shall later cover a domain discoverer related to the positive outcome of the above inquiry.

## 11.3. Domain Discoverers

- A domain discoverer is a mental tool.
  - ❖ It takes a written form shown earlier.
  - ❖ It is to be “applied” by a human, the domain describer.
  - ❖ The domain describer applies the domain discoverer to a fragment of the domain, as it is: “out there” !



- ‘Application’ means the following.
  - ❖ The **domain describer** examines the domain as directed by the explanation given for the **domain discoverer** — as here, in these lectures.
  - ❖ As the brain of the **domain describer** views, examines, analyses, a domain index-designated fragment of the domain,
    - ⊗ ideas as to which domain concepts to capture arise
    - ⊗ and these take the form of pairs of **narrative** and **formal texts**.

## 11.3.1. MATERIAL\_SORTS

### MATERIAL\_SORTS – I/II

107. The MATERIAL\_SORTS discovery function applies to a domain, usually designated by  $\langle \Delta \text{Name} \rangle$  where **Name** is a pragmatic hinting at the domain by name.
108. The result of the **domain discoverer** applying this meta-function is some narrative text
109. and the **types** of the discovered **materials**
110. usually affixed a comment
  - (a) which lists the “**somehow related**” part types
  - (b) and their related materials observers.

## MATERIAL\_SORTS II/II

```

107. MATERIAL_SORTS:  $\langle \Delta \rangle \rightarrow (\mathbf{Text} \times \mathbf{RSL})$ 
107. MATERIAL_SORTS( $\langle \Delta_{\text{Name}} \rangle$ ):
108.   [ narrative text ;
109.     type  $M_a, M_b, \dots, M_c$  materials
110.     comment: related part types:  $P_i, P_j, \dots, P_k$ 
110.            $\text{obs\_}M_n : P_m \rightarrow M_n, \dots ]$ 
105.   pre: IS_MATERIALS_BASED( $\langle \Delta_{\text{Name}} \rangle$ )

```

## Example: 49 Pipelines: Material.

- MATERIAL\_SORTS( $\langle \Delta$ Oil Pipeline System $\rangle$ ):  
[ The oil pipeline system is focused on oil ;  
**type**  $O$  **material**  
**comment** related part type:  $U$ ,  $obs\_O: U \rightarrow O$  ]

## 11.3.2. PART\_SORTS

### PART\_SORTS I/II

11.1. The part type discoverer PART\_SORTS

(a) applies to a simply indexed domain,  $\ell^{\langle \mathbf{t} \rangle}$ ,

(b) where  $\mathbf{t}$  denotes a composite type, and yields a pair

i. of narrative text and

ii. formal text which itself consists of a pair:

A. a set of type names

B. each paired with a part (sort) observer.

## PART\_SORTS II/II

**value**

111. PART\_SORTS: Index  $\xrightarrow{\sim}$  (**Text**  $\times$  RSL)

111(a). PART\_SORTS( $\ell^{\langle t \rangle}$ ):

111((b))i. [ narrative, possibly enumerated texts ;

111((b))iiA. **type**  $t_1, t_2, \dots, t_m,$

111((b))iiB. **value**  $\text{obs\_}t_1:t \rightarrow t_1, \text{obs\_}t_2:t \rightarrow t_2, \dots, \text{obs\_}t_m:t \rightarrow t_m$

111(b). **pre:** IS\_COMPOSITE( $\ell^{\langle t \rangle}$ ) ]

**Example: 50 Transport: Part Sorts.** We apply a concrete version of the above sort discoverer to the vehicle monitoring domain  $\Delta$ . See Example 36 (Slide 188).

- **PART\_SORTS( $\langle\Delta\rangle$ ):**

[ the vehicle monitoring domain contains three sub-parts:  
net, fleet and monitor ;

**type** N, F, M,

**value** obs\_N:  $\Delta \rightarrow N$ , obs\_F:  $\Delta \rightarrow F$ , obs\_M:  $\Delta \rightarrow M$  ]

- **PART\_SORTS( $\langle\Delta, N\rangle$ ):**

[ the net domain contains two sub-parts:  
sets of hubs and sets of link ;

**type** HS, LS,

**value** obs\_HS:  $N \rightarrow HS$ , obs\_LS:  $N \rightarrow LS$  ]

- $\text{PART\_SORTS}(\langle \Delta, F \rangle)$ :  
[ the fleet domain consists of one sub-domain:  
set of vehicles;  
**type** VS,  
**value** obs\_VS:  $F \rightarrow VS$  ]





### 11.3.3. PART\_TYPES

#### PART\_TYPES I/II

112. The PART\_TYPES discoverer applies to a composite sort,  $t$ , and yields a pair
- (a) of narrative, possibly enumerated texts [omitted], and
  - (b) some formal text:
    - i. a type definition,  $\mathbf{t}_c = \mathbf{te}$ ,
    - ii. together with the sort definitions of so far undefined type names of  $\mathbf{te}$ .
    - iii. An observer function observes  $\mathbf{t}_c$  from  $\mathbf{t}$ .
    - iv. The PART\_TYPES discoverer is not defined if the designated sort is judged to not warrant a concrete type definition.

## PART\_TYPES II/II

112. PART\_TYPES: Index  $\xrightarrow{\sim}$  (**Text**  $\times$  RSL)
112. PART\_TYPES( $\ell^{\langle t \rangle}$ ):
- 112(a). [ narrative, possibly enumerated texts ;
- 112((b))i. **type**  $t_c = t_e$ ,
- 112((b))ii.  $t_\alpha, t_\beta, \dots, t_\gamma$ ,
- 112((b))iii. **value**  $\text{obs}_{t_c}: t \rightarrow t_c$
- 112((b))iv. **pre:** HAS\_CONCRETE\_TYPE( $\ell^{\langle t \rangle}$ ) ]
- 112((b))ii. **where:** type expression  $t_e$  contains
- 112((b))ii. type names  $t_\alpha, t_\beta, \dots, t_\gamma$

**Example: 51 Transport: Concrete Part Types.** Continuing Examples 36–50 and Example 3 – we omit narrative informal texts.

PART\_TYPES( $\langle \Delta, F, VS \rangle$ ):

type V, Vs=V-set, value obs\_Vs: VS  $\rightarrow$  Vs

PART\_TYPES( $\langle \Delta, N, HS \rangle$ ):

type H, Hs=H-set, value obs\_Hs: HS  $\rightarrow$  Hs

PART\_TYPES( $\langle \Delta, N, LS \rangle$ ):

type L, Ls=L-set, value obs\_Ls: LS  $\rightarrow$  Ls



## 11.3.4. UNIQUE\_ID

### UNIQUE\_ID

113. For every part type  $t$  we postulate a unique identity analyser function  $\text{uid}_t$ .

**value**

113.  $\text{UNIQUE\_ID}$ :  $\text{Index} \rightarrow (\mathbf{Text} \times \text{RSL})$

113.  $\text{UNIQUE\_ID}(\ell^{\langle t \rangle})$ :

113. [ narrative, possibly enumerated text ;

113.     **type**  $t_i$

113.     **value**  $\text{uid}_t: t \rightarrow t_i$  ]

**Example: 52 Transport Nets: Unique Identifiers.** Continuing Example 3:

$\text{UNIQUE\_ID}(\langle \Delta, \text{HS}, \text{Hs}, \text{H} \rangle)$ : **type**  $\text{H}$ ,  $\text{Hl}$ , **value**  $\text{uid}_\text{H} \rightarrow \text{Hl}$

$\text{UNIQUE\_ID}(\langle \Delta, \text{LS}, \text{Ls}, \text{L} \rangle)$ : **type**  $\text{L}$ ,  $\text{Ll}$ , **value**  $\text{uid}_\text{L} \rightarrow \text{Ll}$  ■

### 11.3.5. MEREOLGY

- Given a part,  $p$ , of type  $t$ , the mereology, MEREOLGY, of that part
  - ◇ is the set of all the unique identifiers of the other parts to which part  $p$  is part-ship-related
  - ◇ as “revealed” by the  $\text{mereo\_ti}_i$  functions applied to  $p$ .
- Henceforth we omit the otherwise necessary narrative texts.

## MEREOLOGY I/II

114. Let type names  $t_1, t_2, \dots, t_n$  denote the types of all parts of a domain.
115. Let type names  $ti_1, ti_2, \dots, ti_n$ <sup>26</sup>, be the corresponding type names of the unique identifiers of all parts of that domain.
116. The mereology analyser MEREOLOGY is a generic function which applies to a pair of an index and an index set and yields some structure of unique identifiers.  
We suggest two possibilities,  
but otherwise leave it to the **domain analyser** to formulate the mereology function.
117. Together with the “discovery” of the **mereology function** there usually follows some **axioms**.

## MEREOLGY II/II

### type

114.  $t_1, t_2, \dots, t_n$

115.  $t_{idx} = ti_1 \mid ti_2 \mid \dots \mid ti_n$

116. MEREOLGY: Index  $\xrightarrow{\sim}$  Index-set  $\xrightarrow{\sim}$  (**Text**  $\times$  RSL)

116. MEREOLGY( $\ell \hat{\langle} t \rangle$ )( $\{\ell_i \hat{\langle} t_j \rangle, \dots, \ell_k \hat{\langle} t_l \rangle\}$ ):

116. [ narrative, possibly enumerated texts ;

116.     **either:**  $\{\}$

116.     **or:**     **value** mereo\_t:  $t \rightarrow ti_x$

116.     **or:**     **value** mereo\_t:  $t \rightarrow ti_x\text{-set} \times ti_y\text{-set} \times \dots \times ti_x\text{-set}$

117.     **axiom**  $\mathcal{P}$  Predicate over values of  $t'$  and  $t_{idx}$  ]

where none of the  $ti_x, ti_y, \dots, ti_z$  are equal to  $ti$ .

<sup>26</sup>We here assume that all parts have unique identifications.

## Example: 53 Transport Net Mereology. Examples:

- $\text{MEREOLGY}(\langle \Delta, N, HS, Hs, H \rangle)(\{\langle \Delta, N, LS, Ls, L \rangle\})$ :  
value mereo\_H  $\rightarrow$  LI-set
- $\text{MEREOLGY}(\langle \Delta, N, LS, Ls, L \rangle)(\{\langle \Delta, N, HS, Hs, H \rangle\})$ :  
value mereo\_L  $\rightarrow$  HI-set  
axiom see Example 10 Slide 83.



## 11.3.6. ATTRIBUTES

- A general attribute analyser analyses parts beyond their unique identities and possible mereologies.
  - ❖ Part attributes have names.
  - ❖ We consider these names to also abstractly name the corresponding attribute types.

## ATTRIBUTES I/II

118. Attributes have types.

We assume **attribute type names** to be distinct from **part type names**.

119. **ATTRIBUTES** applies to parts of type **t** and yields a pair of

(a) narrative text and

(b) formal text, here in the form of a pair

i. a set of one or more attribute types, and

ii. a set of corresponding attribute observer functions **attr\_at**, one for each attribute sort **at** of **t**.

## ATTRIBUTES II/II

### type

118.  $at = at_1 \mid at_2 \mid \dots \mid at_n$

### value

119. ATTRIBUTES:  $\text{Index} \rightarrow (\mathbf{Text} \times \text{RSL})$

119. ATTRIBUTES( $\ell^{\langle t \rangle}$ ):

119(a). [ narrative, possibly enumerated texts ;

119((b))i. **type**  $at_1, at_2, \dots, at_m$

119((b))ii. **value**  $\text{attr\_at}_1:t \rightarrow at_1, \text{attr\_at}_2:t \rightarrow at_2, \dots, \text{attr\_at}_m:t \rightarrow at_m$  ]

- where  $m \leq n$

**Example: 54 Transport Nets: Part Attributes.** We exemplify attributes of composite and of atomic parts — omitting narrative texts:

ATTRIBUTES( $\langle \Delta \rangle$ ):

**type** Domain\_Name, ...

**value** attr\_Domain\_Name:  $\Delta \rightarrow$  Domain\_Name, ...

● where

◆ Domain\_Name could include *State Roads* or *Rail Net*.

◆ etcetera.

ATTRIBUTES( $\langle \Delta, N \rangle$ ):

**type**

Sub\_Domain\_Name      ex.: *State Roads*

Sub\_Domain\_Location   ex.: *Denmark*

Sub\_Domain\_Owner     ex.: *The Danish Road Directorate*

...

Length                      ex.: *3.786 Kms.*

**value**

attr\_Sub\_Domain\_Name:  $N \rightarrow \text{Sub\_Domain\_Name}$

attr\_Sub\_Domain\_Location:  $N \rightarrow \text{Sub\_Domain\_Location}$

attr\_Sub\_Domain\_Owner:  $N \rightarrow \text{Sub\_Domain\_Owner}$

...

attr\_Length:  $N \rightarrow \text{Length}$

ATTRIBUTES( $\langle \Delta, N, LS, Ls, L \rangle$ ):

**type** LOC, LEN, ...

**value** attr\_LOC:  $L \rightarrow \text{LOC}$ , attr\_LEN:  $L \rightarrow \text{LEN}$ , ...

ATTRIBUTES( $\langle \Delta, N, LS, Ls, L \rangle$ )( $\{ \langle \Delta, N, HS, Hs, H \rangle \}$ ):

**type**

$L\Sigma$ =HI-set

$L\Omega$ = $L\Sigma$ -set

**value**

attr\_ $L\Sigma$ :  $L \rightarrow L\Sigma$

attr\_ $L\Omega$ :  $L \rightarrow L\Omega$

- where

- ◆ LOC might reveal some Bézier curve<sup>27</sup> representation of the possibly curved three dimensional location of the link in question,
- ◆ LEN might designate length in meters,
- ◆  $L\Sigma$  designates the state of the link,
- ◆  $L\Omega$  designates the space of all allowed states of the link. ■

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<sup>27</sup>[http://en.wikipedia.org/wiki/Bézier\\_curve](http://en.wikipedia.org/wiki/Bézier_curve)

## **End of Lecture 6: First Session — Calculus I**

### **Part and Material Discoverers**

**FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012**





**SHORT BREAK**