WELCOME

Domain Science & Engineering
A Precursor for Requirements Engineering

FM 2012 Tutorial, CNAM, 28 August 2012

Dines Bjørner

DTU Informatics

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Begin of Lecture 1: First Session — Introduction

Domains, `TripTych`, Issues

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

Tutorial Schedule

•	**Lectures 1–2**	9:00–9:40 + 9:50–10:30	
√ 1	**Introduction**		**Slides 1–35**
2	**Endurant Entities: Parts**		Slides 36–114
•	**Lectures 3–5**	11:00–11:15 + 11:20–11:45 + 11:50–12:30	
3	**Endurant Entities: Materials, States**		Slides 115–146
4	**Perdurant Entities: Actions and Events**		Slides 147–178
5	**Perdurant Entities: Behaviours**		Slides 179–284
	Lunch	**12:30–14:00**	
•	**Lectures 6–7**	14:00–14:40 + 14:50–15:30	
6	**A Calculus: Analysers, Parts and Materials**		Slides 285–338
7	**A Calculus: Function Signatures and Laws**		Slides 339–376
•	**Lecturs 8–9**	16:00–16:40 + 16:50–17:30	
8	**Domain and Interface Requirements**		Slides 377–423
9	**Conclusion: Comparison to Other Work**		Slides 427–459
	Conclusion: What Have We Achieved		Slides 424–426 + 460–471

Summary

- This tutorial covers
  - a **new science & engineering of domains** as well as
  - a **new foundation for software development**.

  We treat the latter first.
- Instead of commencing with requirements engineering,
  - whose pursuit may involve repeated,
  - but unstructured forms of domain analysis,
  - we propose a predecessor phase of domain engineering.
- That is, we single out domain analysis as an activity to be pursued prior to requirements engineering.

- In emphasising domain engineering as a predecessor phase
  - we, at the same time, introduce a number of facets
  - that are **not present,** we think,
  - in current software engineering studies and practices.
- **One facet** is **the construction of separate domain descriptions.**
  - Domain descriptions are void of any reference to requirements
  - and encompass the modelling of domain phenomena
  - without regard to their being computable.

- **Another facet** is **the pursuit of domain descriptions as a free-standing activity.**
  - In this tutorial we emphasize domain description development need not lead to software development.
  - This gives a new meaning to business process engineering, and should lead to
    - a deeper understanding of a domain
    - and to possible non-IT related business process re-engineering of areas of that domain.
- In this tutorial we shall investigate
  - a method for analysing domains,
  - for constructing domain descriptions
  - and some emerging scientific bases.

- **Our contribution** to **domain analysis** is that we view domains as having the following **ontology**.
  - There are the entities that we can describe and then there is "the rest" which we leave un-described.
  - We analyse entities into
    - endurant entities (Slides 52–146) and
    - perdurant entities (Slides 148–279),

    that is,
    - parts and materials as endurant entities (Slides 52–136) and
    - discrete actions, discrete events and behaviours as perdurant entities (Slides 150–279), respectively.
- Another way of looking at entities is as
  - discrete entities (Slides 52–114 and 148–244), or as
  - continuous entities (Slides 116–136 and Slides 245–279).

- We also contribute to the **analysis** of **discrete endurant**s in terms of the following notions:
  - **part type**s and **material type**s (Slides 55–73 and Slides 116–136),
  - **part unique identifier**s (Slides 79–81),
  - **part mereology** (Slides 82–97) and
  - **part attribute**s and **material attribute**s (Slides 98–109, Slides 125–129) and
  - **material laws** (Slides 130–135).
- Of the above we point to the introduction, into **computing science** and **software engineering** of the notions of
  - **material**s (Slides 116–136) and
  - **continuous behaviour**s (Slides 245–279)

as novel.

1. Introduction

- I remind You of the **abstract**,
  - Slide 7,
  - as for the **contributions** of this tutorial.
- This is primarily a **methodology** paper.
- By a **method** we shall understand
  - a set of **principles**
  - for **selecting** and **applying**
  - a number of **techniques** and **tools**
  - in order to **analyse** a **problem**
  - and **construct** an **artefact**.
- By **methodology** we shall understand
  - the study and knowledge about methods.

- This tutorial contributes to
  - the study and knowledge
  - of software engineering development methods.
- Its contributions are those of suggesting and exploring
  - **domain engineering** and
  - domain engineering as a basis for **requirements engineering**.
- We are not saying
  - *"thou must develop software this way"*,
- but we do suggest
  - that since it is possible
  - and makes sense to do so
  - it may also be wise to do so.

1.1. Domains: Some Definitions

- By a **domain** we shall here understand
  - an area of human activity
  - characterised by observable phenomena:
    - **entities**
      - whether **endurant**s (manifest **part**s and **material**s)
      - or **perdurant**s (**action**s, **event**s or **behaviour**s),
    - whether
      - **discrete** or
      - **continuous**;
    - and of their **properties**.

Example: 1 Some Domains. Some examples are:

air traffic,
airport,
banking,
consumer market,
container lines,
fish industry,
health care,
logistics,
manufacturing,
pipelines,
securities trading,
transportation
etcetera. ■

1.1.1. Domain Analysis

- By **domain analysis** we shall understand
  - an inquiry into the domain,
  - its **entities**
  - and their **properties**.

Example: 2 A Container Line Analysis.
We omit enumerating entity properties.

- *parts:*
  - container,
  - vessel,
  - terminal port, etc.;
- *actions:*
  - container loading,
  - container unloading,
  - vessel arrival in port, etc.;
- *events:*
  - container falling overboard;
  - container afire;
  - etc.;
- *behaviour:*
  - vessel voyage,
  - across the seas,
  - visiting ports, etc.

Length of a container is a container *property*.
Name of a vessel is a vessel *property*.
Location of a container terminal port is a port *property*.

1.1.2. Domain Descriptions

- By a **domain description** we shall understand
  - a **narrative description**
  - tightly coupled (say line-number-by-line-number)
  - to a **formal description**.
- To develop a **domain description**
  requires a thorough amount of **domain analysis**.

Example: 3 A Transport Domain Description.

- *Narrative:*
  - a transport net, n:N, consists of an aggregation of hubs, hs:HS, which we "concretise" as a set of hubs, **H-set**, and an aggregation of links, ls:LS, that is, a set **L-set**,
- *Formalisation:*

```
type N, HS, LS, Hs = H-set, Ls = L-set, H, L
value
    obs_HS: N→HS,
    obs_LS: N→LS.
    obs_Hs: HS→H-set,
    obs_Ls: LS→L-set.
```

■

1.1.3. Domain Engineering

- By domain engineering we shall understand
  - the engineering of a domain description,
  - that is,
    - the rigorous construction of domain descriptions, and
    - the further analysis of these, creating theories of domains[1], etc.

[1] Examples of such theories, albeit in rather rough forms, are given in Appendices B–C.

- The size[2], structure[3] and complexity[4] of interesting domain descriptions is usually such as to put a special emphasis on engineering:
  - the management and organisation of several, typically 5–6 collaborating domain describers,
  - the ongoing check of description quality, completeness and consistency, etcetera.

[2] usually, say a hundred pages

[3] usually a finely sectioned document of may subsub···subsections

[4] having many cross-references between subsub···subsections

1.1.4. Domain Science

- By domain science we shall understand
  - two things:
    - the general study and knowledge of
      - how to create and handle domain descriptions
      - (a general theory of domain descriptions)

      and
    - the specific study and knowledge of a particular domain.
  - The two studies intertwine.

1.2. The Triptych of Software Development

- We suggest a "dogma":
  - before **software** can be **design**ed one must understand[5] the **requirements**; and
  - before **requirements** can be expressed one must understand[6] the **domain**.
- We can therefore view software development as ideally proceeding in three (i.e., TripTych) phases:
  - an initial phase of **domain engineering**, followed by
  - a phase of **requirements engineering**, ended by
  - a phase of **software design**.

[5] Or maybe just: have a reasonably firm grasp of
[6] See previous footnote!

- In the **domain engineering phase** ($\mathcal{D}$)
  - a domain is analysed, described and "theorised",
  - that is, the beginnings of a **specific domain theory** is established.
- In the **requirements engineering phase** ($\mathcal{R}$)
  - a **requirements prescription** is constructed —
  - significant fragments of which are "derived",
  - systematically, from the **domain description**.
- In the **software design phase** ($\mathcal{S}$)
  - a **software design**
  - is derived, systematically, rigorously or formally,
  - from the **requirements prescription**.
- Finally the $\mathcal{S}$oftware is proven correct with respect to the $\mathcal{R}$equirements under assumption of the $\mathcal{D}$omain: $\mathcal{D}, \mathcal{S} \models \mathcal{R}$.

- By a **machine** we shall understand the **hardware** and **software** of a target, i.e., a required **IT system**.
- In [dines:ugo65:2008,psi2009,Kiev:2010ptI] we indicate how one can "derive" significant parts of requirements from a suitably comprehensive domain description – basically as follows.
  - **Domain projection**: from a domain description one **project**s those areas that are to be somehow manifested in the software.
  - **Domain initialisation**: for that resulting projected requirements prescription one **initialise**s a number of part types as well as action and behaviour definitions, from less **abstract** to more **concrete**, specific types, respectively definitions.

  - **Domain determination**: hand-in-hand with domain initialisation a[n interleaved] stage of making values of types less **non-deterministic**, i.e., more **deterministic**, can take place.
  - **Domain extension**: Requirements often arise in the context of new business processes or technologies either placing old or replacing human processes in the domain. Domain extension is now the 'enrichment' of the domain requirements, so far developed, with the description of these new business processes or technologies.
  - Etcetera.
- The result of this part of "**requirements derivation**" is the **domain requirements**.

- A set of domain-to-requirements operators similarly exists for constructing **interface requirements**
  - from the domain description and,
  - independently, also from knowledge of the **machine**
  - for which the required IT system is to be developed.
- We illustrate the **techniques** of **domain requirements** and **interface requirements** in Sect. 4.
- Finally **machine requirements** are "derived"
  - from just the knowledge of the **machine**,
  - that is,
    - the target hardware and
    - the software system tools for that hardware.

- When you review this section ('A Triptych of Software Development')
  - then you will observe how 'the domain'
  - predicates both the requirements
  - and the software design.
- For a specific domain one may develop
  - many (thus related) requirements
  - and from each such (set of) requirements
  - one may develop many software designs.
- We may characterise this multitude of domain-predicated requirements and designs as a **product line** [dines-maurer].
- You may also characterise domain-specific developments as representing another 'definition' of **domain engineering**.

1.3. Issues of Domain Science & Engineering

- We specifically focus on the following issues of domain science &[7] engineering:
  - (i) which are the "things" to be described[8],
  - (ii) how to analyse these "things" into description structures[9],
  - (iii) how to describe these "things" informally and formally,
  - (iv) how to further structure descriptions[10], and a further study of
  - (v) **mereology**[11].

[7] When we put '&' between two terms that the compound term forms a whole concept.

[8] **endurants** [manifest entities henceforth called **part**s and **material**s] and **perdurants** [**action**s, **event**s, **behaviour**s]

[9] **atomic** and **composite**, **unique identifiers**, **mereology**, **attribute**s

[10] *intrinsics, support technology, rules & regulations, organisation & management, human behaviour* etc.

[11] the study and knowledge of parts and relations of parts to other parts and a "whole".

1.4. Structure of Paper

- It is always a good idea to consult and study the *table of contents* listing of the colloquium one is listening to. Therefore one is brought here:

- First (Sect. 1) we introduce the problem. And that was done above.

- Then, in (Sects. 2–10)
  - we bring a rather careful analysis of
  - the concept of the observable, manifest phenomena
  - that we shall refer to as entities.
- We strongly think that these sections of this tutorial
  - brings, to our taste, a simple and elegant
  - reformulation of what is usually called *"data modelling"*,
  - in this case for domains —
  - but with major aspects applicable as well to
  - requirements development and software design.

- That analysis focuses on
  - endurant entities, also called parts and materials,
    - those that can be observed at no matter what time,
    - i.e., entities of substance or continuant, and
  - perdurant entities: action, event and behaviour entities, those
    - that occur,
    - that happen,
    - that, in a sense, are accidents.

- **We think** that this "decomposition" of the "data analysis" problem into
  - discrete parts and continuous materials,
  - atomic and composite parts,
  - their unique identifiers and mereology, and
  - their attributes
  - **is novel**,
  - and differs from past practices in domain analysis.

- In Sect. 11 we suggest
  - for each of the entity categories
    - parts,
    - materials,
    - actions,
    - events and
    - behaviours,
  - a calculus of meta-functions:
    - analytic functions,
      - that guide the domain description developer
      - in the process of selection,

      and
    - so-called discovery functions,
      - that guide that person
      - in "generating" appropriate domain description texts, informal and formal.

- The domain description calculus is to be thought of
  - as directives to the domain engineer,
  - mental aids that help a team of domain engineers
  - to steer it simply through the otherwise daunting task
  - of constructing a usually large domain description.
- Think of the calculus
  - as directing
  - a human calculation
  - of domain descriptions.
- Finally the domain description calculus section
  - suggests a number of laws that the
  - domain description process ought satisfy.

- Finally (Sect. 12) we bring a brief survey of the kind of requirements engineering
  - that one can now pursue based on a reasonably comprehensive domain description.
  - We show how one can systematically, but not automatically
  - "derive" significant fragments
    - of requirements prescriptions
    - from domain descriptions.

- The formal descriptions will here be expressed in the RAISE [RaiseMethod] Specification Language, RSL.
- We otherwise refer to [TheSEBook1wo].
- Appendix brings a short primer, mostly on the syntactic aspects of RSL.
- But other model-oriented formal specification languages can be used with equal success; for example:
  - Alloy [alloy],
  - Event B [JRAbrial:TheBBooks],
  - VDM [e:db:Bj78bwo,e:db:Bj82b,jf-pgl-97] and
  - Z [m:z:jd+jcppw96].

End of Lecture 1: First Session — Introduction

Domains, TripTych, Issues

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SHORT BREAK

NICE TO SEE YOU BACK

Begin of Lecture 2: Last Session — Discrete Endurant Entities

Parts

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Tutorial Schedule

2. Domain Entities

- The world is divisible into two kinds of people:
  - those who divide the population into two kinds of people
  - and the others.
- In this tutorial we shall divide the phenomena we can observe and whose properties we can ascertain into two kinds:
  - the endurant entities and
  - the perdurant entities.
- Another "division" is of the phenomena and their properties into
  - the discrete entities and
  - the continuous entities.
- You can have it, i.e., the the analysis and the presentation, either way.

- By a **domain** we shall understand a suitably delineated set of observable entities and abstractions of these, that is, of
  - discrete parts and
  - continuous materials and,
  - discrete actions
    (operation applications causing state changes),
  - discrete events
    ("spurious" state changes not [intentionally] caused by actions),
  - discrete discrete behaviours
    (seen as sets of sequences of actions, events and behaviours) and
  - continuous behaviours
    (abstracted as continuous functions in space and/or time).

2.1. From Observations to Abstractions

- When we observe a domain we observe **instances** of **entities**;
- but when we describe those instances
  - (which we shall call **value**s)
  - we describe, not the values,
  - but their **type** and **properties**.
    - Parts and materials have **type**s and **value**s;
    - actions, events and behaviours, all, have **type**s and **value**s, namely as expressed by their **signature**s; and
    - actions, events and behaviours have **properties**, namely as expressed by their **function definition**s.

2.2. Algebras

- **Algebra:** Taking a clue from mathematics, an **algebra** is considered
  - a set of **endurant**s:
    - a set of **part**s and
    - a set of **material**s

  and
  - a set of **perdurant**s: operations on entities.

  These operations yield parts or materials.
- With that in mind we shall try view a domain as an algebra, of some kind, of
  - **part**s and
  - **action**s, **event**s and **behaviour**s.

2.3. Domain Phenomena

- By a **domain phenomenon** we shall understand
  - something that can be observed by the **human senses**
  - or by **equipment** based on laws of physics and chemistry.
- Those phenomena that can be observed by
  - the human eye or
  - touched, for example, by human hands,
  - we call **part**s and **material**s.
- Those phenomena that can be observed of parts and materials
  - can usually be measured
  - and we call them **properties** of these **part**s and those **material**s.

2.4. Entities

- By a **domain entity** we shall understand
  - a manifest **domain phenomenon** or
  - a concept, i.e., an **abstraction**,
  - derived from a **domain entity**.
- The distinction between
  - a **manifest domain phenomenon** and
  - a **concept** thereof, i.e., a **domain concept**,

  is important.
- Really, what we describe are the **domain concept**s derived
  - from **domain phenomena** or
  - from other **domain concept**s.

- Ontologically we distinguish between two kinds of domain entities:
  - endurant entities and
  - perdurant entities.
- We shall characterise these two terms:
  - endurants on Slide 50 and
  - perdurants on Slide 148.
- This distinction is supported by current literature on ontology [BarrySmith1993].
- In this section of this lecture we shall not enter a discourse on
  - *"things"*,
  - *objects*,
  - *entities*,
  - etcetera.

2.4.1. A Description Bias

- One of several "twists"
  - that make the `TripTych` form of domain engineering
  - distinct from that of ontological engineering
  - is that we use a model-oriented formal specification approach[12]
  - where usual ontology formalisation languages are variants of `Lisp`'s [Lisp1] S-expressions.
  - `KIF`: Knowledge Interchange Format, `http://www.ksl.stanford.edu/knowledge-sharing/kif/` is a leading example.

[12] `RAISE` [RaiseMethod]. Our remarks in this section apply equally well had we instead chosen either of the `Alloy` [alloy], `Event B` [JRAbrial:TheBBooks], `VDM` [e:db:Bj78bwo,e:db:Bj82b,jf-pgl-97] or `Z` [m:z:jd+jcppw96] formal specification languages.

- The bias is now this:
  - The model-oriented languages mentioned in this section all share the following:
    - (a) a type concept and facilities for defining types, that is: endurants (parts), and
    - (b) a function concept and facilities for defining functions (notably including predicates), that is: perdurants (actions and events).
    - (c) `RSL` further has constructs for defining processes, which we shall use to model behaviours.

2.4.2. An 'Upper Ontology'

- By an upper ontology we shall understand
  - a relatively small, ground set of ontology expressions
  - which form a basis for a usually very much larger set of ontology expressions.

- The need for introducing the notion of an **upper ontology** arose, in the late 1980s to early 1990s as follows:
  - usually an ontology was (is) expressed in some very basic language, viz., Lisp-like S-expressions[13].
  - This was necessitated by the desire to be able to share ontologies between many computing applications worldwide.
  - Then it was found that several ontologies shared initial bases in terms of which the rest of their ontologies were formulated.
  - These shared bases were then referred to as **upper ontologies** — and a need to "standardise" these arose [ontology:guarino97a,StaabStuder2004].

[13] Ontology languages: KIF http://www.ksl.stanford.edu/knowledge-sharing/kif/-#manual, OWL [Ontology Web Language] [OWL:2009], ISO Common Logic [ISO:CL:2007]

- We therefore consider the following **model-oriented specification language** constructs as forming an **upper ontology**:
  - **type**s, **ground type**s, **type expression**s and **type definition**s;
  - **function**s, **function signature**s and **function definition**s;
  - **process**es, **process signature**s and **process definition**s,

  as constituting an **upper level ontology** for TripTych **domain description**s.
- That is, every domain description is structured with respect to:
  - **part**s and **material**s using **type**s,
  - **action**s using **function**s,
  - **event**s using **predicate**s,
  - **discrete**behaviours using **process**es and
  - **continuous behaviour**s using **partial differential**equations.

3. Endurants

- There is sort of a dichotomy buried in our treating endurants before perdurants. The dichotomy is this:
  - one could claim that the perdurants, i.e., the actions, events and behaviours is *"what it, the domain, is all about"*;
- To describe these, however, we need refer to endurants!

3.1. General

Wikipedia:

- *By an **endurant** (also known as a **continuant** or a **substance**) we shall understand an entity*
  - *that can be observed, i.e., perceived or conceived,*
  - *as a complete concept,*
  - *at no matter which given snapshot of time.*
- *Were we to freeze time*
  - *we would still be able to observe the entire endurant.*

3.2. Discrete and Continuous Endurants

- We distinguish between
  - **discrete endurant**s, which we shall call **part**s, and
  - **continuous endurant**s, which we shall call **material**s.

We motivate and characterise this distinction.

- By a **discrete endurant**, that is, a **part**, we shall understand something which is
  - separate or distinct in form or concept,
  - consisting of distinct or separate parts.
- By a **continuous endurant**, that is, a **material**, we shall understand something which is
  - prolonged without interruption,
  - in an unbroken series or pattern.
- We shall
  - first treat the idea of **discrete endurant**, that is, a **part** (Slides 52–114),
  - then the idea of **continuous endurant**, that is, a **material** (Slides 116–136).

4. Discrete Endurants: Parts

4.1. What is a Part?

- By a **part** we mean an **observable manifest endurant**.

4.1.1. Classes of "Same Kind" Parts

- We repeat:
  - the **domain describer** does not describe instances of parts,
  - but seeks to describe classes of parts of the same kind.
- Instead of the term 'same kind' we shall use either the terms
  - **part sort** or
  - **part type**.
- By a **same kind class of parts**, that is a **part sort** or **part type** we shall mean
  - a class all of whose members, i.e., **parts**,
  - enjoy "exactly" the same **properties**
  - where a **property** is expressed as a **proposition**.

Example: 4 Part Properties.

- Examples of **part properties** are:
  - *has unique identity*,
  - *has mereology*,
  - *has length*,
  - *has location*,
  - *has traffic movement restriction*,
  - *has position*,
  - *has velocity* and
  - *has acceleration*. ■

4.1.2. Concept Analysis as a Basis for Part Typing

- The **domain analyser** examines collections of **parts**.
  - In doing so the **domain analyser** discovers and thus identifies and lists a number of **properties**.
  - Each of the **parts** examined usually satisfies only a subset of these properties.
  - The **domain analyser** now groups **parts** into collections
    - such that each collection have its **parts** satisfy the same set of **properties**,
    - such that no two distinct collections are indexed, as it were, by the same set of **properties**, and
    - such that all **parts** are put in some collection.
  - The **domain analyser** now
    - assigns distinct **type names** (same as **sort names**)
    - to distinct collections.
- That is how we assign **types** to **parts**.
- We shall return later to a proper treatment of **formal concept analysis** [Wille:ConceptualAnalysis1999].

4.2. Atomic and Composite Parts

- Parts may be analysed into disjoint sets of
  - atomic parts and
  - composite parts.
- **Atomic parts** are those which,
  - in a given context,
  - are deemed *not* to consist of meaningful, separately observable proper sub-parts.
- **Composite parts** are those which,
  - in a given context,
  - are deemed to *indeed* consist of meaningful, separately observable proper sub-parts.
- A sub-part is a part.

Example: 5 Atomic and/or Composite Parts. To one person a part may be atomic; to another person the same part may be composite.

- It is the domain describer who decides the outcome of this aspect of domain analysis.
  - In some domain analysis a 'person' may be considered an atomic part.
    - For the domain of ferrying cars with passengers
    - persons are considered parts.
  - In some other domain analysis a 'person' may be considered a composite part.
    - For the domain of medical surgery
    - persons may be considered composite parts. ■

Example: 6 Container Lines.

- We shall presently consider containers (as used in container line shipping) to be atomic parts.
- And we shall consider a container vessel to be a composite part consisting of
  - an indexed set of container bays
  - where each container bay consists of indexed set of container rows
  - where each container row consists of indexed set of container stacks
  - where each container stack consists of a linearly indexed sequence of containers.
- Thus container vessels, container bays, container rows and container stacks are composite parts. ■

4.2.1. Atomic Parts

- When we observe
  - what we have decided, i.e., analysed, to be an endurant,
  - more specifically an atomic part, of a domain,
  - we are observing an instance of an atomic part.
- When we describe those instances
  - we describe, not their values, i.e., the instances,
  - but their
    - type and
    - properties.

- In this section on endurant entities
  we shall unfold what these properties might be.
- But, for now, we focus on the type of the observed atomic part.
- So the situation is that we are observing a number of atomic parts
  - and we have furthermore decided that
  - they are all of *"the same kind"*.

- What does it mean for a number of atomic parts to be of *"the same kind"* ?
  - It means
    - that we have decided,
    - for any pair of parts considered of the same kind,
    - that the kinds of properties,
      - ∗ for such two parts,
    - are *"the same"*,
      - ∗ that is, of the same type, but possibly of different values,
    - and that a number of different, other "facets",
    - are not taken into consideration.

- That is,
  - we abstract a collection of atomic parts
  - to be of the same kind,
  - thereby "dividing the domain of endurants" into possibly two distinct sets
    - those that are of the analysed kind, and
    - those that are not.

- It is now our description choice to associate with a set of atomic parts of *"the same kind"*
  - a part type (by suggesting a name for that type, for example, T) and
  - a set of properties (of its values):
    - unique identifier,
    - mereology and
    - attributes.

- Later we shall introduce discrete perdurants (actions, events and behaviours) whose signatures involves (possibly amongst others) type T.
- Now we can characterise *"of the same kind"* atomic part facets[14]
  - being of the same, named part type,
  - having the same unique identifier type,
  - having the same mereology (but not necessarily the same mereology values), and
  - having the same set of attributes (but not necessarily of the same attribute values),
- The *"same kind"* criteria apply equally well to composite part facets.

[14]as well as "of the same kind" composite part facets.

Example: 7 Transport Nets: Atomic Parts (I).

- The types of atomic transportation net parts are:
  - hubs, say of type H, and
  - links, say of type L.
- The chosen mereology associates with every hub and link a
  - distinct unique identifiers
  - (of types HI and LI respectively), and, *vice versa*,
  - how hubs and links are connected:
    - hubs to any number of links and
    - links to exactly two distinct hubs.

- The chosen attributes of
  - hubs include
    - hub location,
    - hub design[15],
    - hub traffic state[16],
    - hub traffic state space[17], etc.;
  - and of links include
    - link location,
    - link length,
    - link traffic state[18],
    - link traffic state space[19], etc.
- With these mereologies and attributes we see that we can consider hubs and links as different kinds of atomic parts. ■

[15]Design: simple crossing, freeway "cloverleaf" interchange, etc.
[16]A hub traffic state is (for example) a set of pairs of link identifiers where each such pair designates that traffic can move from the first designated link to the second.
[17]A hub state space is (for example) the set of all hub traffic states that a hub may range over.
[18]A link traffic state is (for example) a set of zero to two distinct pairs of the hub identifiers of the link mereology.
[19]A link traffic state space is (for example) the set of all link traffic states that a link may range over.

Observers for Atomic Parts

- Let the domain describer decide
  - that a type, A (or Δ), is atomic,
  - hence that it does not consists of sub-parts.
- Hence there are no observer to be associated with A (or Δ).

4.2.2. Composite Parts

- The domain describer has chosen to consider
  - a part (i.e., a part type)
  - to be a composite part (i.e., a composite part type).
- Now the domain describer has to analyse the types of the sub-parts of the composite part.
  - There may be just one *"kind of"* sub-part of a composite part[20],
  - or there may be more than one *"kind of"*[21].
- For each such sub-part type
  - the domain describer decides on
  - an appropriate, distinct type name and
  - a sub-part observer (i.e., a function signature).

[20] that is, only one sub-part type
[21] that is, more than one sub-part type

Example: 8 Container Vessels: Composite Parts. We bring pairs of informal, narrative description texts and formalisations.

- For a container vessel, say of type V, we have
  - *Narrative:*
    - A container vessel, v:V, consists of container bays, bs:BS.
    - A container bay, b:B, consists of container rows, rs:RS.
    - A container row, r:R, consists of container stacks, ss:SS.
    - A container stack, s:S, consists of a linearly indexed sequence of containers.
  - *Formalisation:*

type V,BS, **value** obs_BS: V→BS,
type B,RS, **value** obs_RS: B→RS,
type R,SS, **value** obs_CS: R→SS,
type SS,S, **value** obs_S: SS→S,
type S = C*. ■

4.2.3. Abstract Types, Sorts, and Concrete Types

- By an abstract type, or a sort, we shall understand a type
  - which has been given a name
  - but is otherwise undefined, that is,
    - is a space of undefined mathematical quantities,
      - where these are given properties
      - which we may express in terms of axioms over sort (including property) values.

- By a concrete type we shall understand a type, T,
  - which has been given both a name
  - and a defining type expression of, for example the form
    - T = A-**set**,
    - T = A-**infset**,
    - T = A×B×···×C,
    - $T = A^*$,
    - $T = A^\omega$,
    - $T = A \underset{m}{\to} B$,
    - T = A→B,
    - $T = A \tilde{\to} B$, or
    - T = A|B|···|C.
  - where A, B, ..., C are type names or type expressions.

Example: 9 Container Bays. We continue Example 8 on page 68.

type Bs = BId $\overrightarrow{m}$ B,
value obs_Bs: BS→Bs,

type Rs = RId $\overrightarrow{m}$ R,
value obs_Rs: B→Rs,

type Ss = SId $\overrightarrow{m}$ S,
value obs_Ss: R→Ss,

type S = C*. ■

Observers for Composite Parts I/II

- Let the domain describer decide
  - that a type, A (or Δ), is composite
  - and that it consists of sub-parts of types B, C, ..., D.
- We can initially consider these types B, C, ..., D, as abstract types, or sorts, as we shall mostly call them.
- That means that there are the following formalisations:
  - **type** A, B, C, ..., D;
  - **value** obs_B: A→B, obs_C: A→C, ..., obs_D: A→D.

Observers for Composite Parts II/II

- We can also consider the types B, C, ..., D, as concrete types,
  - **type** Bc = TypBex, Cc = TypCex, ..., Dc = TypDex;
  - **value** obs_Bc: B→Bc, obs_Cc: C→Cc, ..., obs_Dc: D→Dc,
  - where TypBex, TypCex, ..., TypDex are type expressions as, for example, hinted at above.
- The prefix obs_ distinguishes part observers
  - from mereology observers (uid_, mereo_) and
  - attribute observers (attr_).

4.3. Properties

- Endurants have properties.
  - Properties are
    - what makes up a parts (and materials) and,
    - with property values distinguishes
      one part from another part and
      one material from another material.
  - We name properties.
    - Properties of parts and materials can be given distinct names.
    - We let these names also be the property type name.
    - Hence two parts (materials) of the same part type (material type)
      have the same set of property type names.

- Properties are all that distinguishes parts (and materials).
  - The part types (material types) in themselves do not express properties.
  - They express a class of parts (respectively materials).
  - All parts (materials) of the same type
  - have the same property types.
  - Parts (materials) of the different types have different sets of property types,

- For pragmatic reasons we distinguish between three kinds of properties:
  - unique identifiers,
  - mereology, and
  - attributes.
- If you "remove" a property from a part
  - it "looses" its (former) part type,
  - to, in a sense, attain another part type:
    - perhaps of another, existing one,
    - or a new "created" one.
- *But we do not know* how to model *removal of a property* from an endurant value![22]

[22] And we see no need for describing such type-changes. Crude oil does not "morph" into fuel oil, diesel oil, kerosene and petroleum. Crude oil is consumed and the fractions result from distillation, for example, in an oil refinery.

Example: 10 Atomic Part Property Kinds.

- We distinguish between two kinds of persons:
  - 'living persons' and 'deceased persons';
  - they could be modelled by two different part types:
    - LP: living person, with a set of properties,
    - DP: deceased person, with a, most likely, different set of properties.
- All persons have been born, hence have a birth date (static attributes).
- Only deceased persons have a (well-defined) death date.

- All persons also have height and weight profiles (i.e., with dated values, i.e., dynamic attributes).
- One can always associate a unique identifier with each person.
- Persons are related, family-wise:
  - have parents (living or deceased),
  - (up to four known) grandparents, etc.,
  - may have brothers and sisters (zero or more),
  - may have children (zero or more), etc.
  - These family-relations can be considered the mereology for living persons. ■

4.3.1. Unique Identification

- We can assume that all parts
  - of the same part type
  - can be uniquely distinguished,
  - hence can be given unique identifications.

Unique Identification

- With every part, whether atomic or composite we shall associate a unique part identifier, of just unique identifier.
- Thus we shall associate with part type T
  - the unique part type identifier type TI,
  - and a unique part identifier observer function, uid_TI: T→TI.
- These associations (TI and uid_TI) are, however,
  - usually expressed explicitly,
  - whether they are ("subsequently") needed!

- The unique identifier of a part
  - can not be changed;
  - hence we can say that
    - no matter what a given part's property values may take on,
    - that part cannot be confused with any other part.
- Since we can talk about this concept of unique identification,
  - we can abstractly describe it —
    - and do not have to bother about any representation,
    - that is, whether we can humanly observe unique identifiers.

4.3.2. Mereology

- Mereology [CasatiVarzi1999][23] (from the Greek $\mu\epsilon\rho o\varsigma$ 'part') is
  - *the theory of part-hood relations:*
  - *of the relations of part to whole and*
  - *the relations of part to part within a whole.*

[23] Achille Varzi: Mereology, http://plato.stanford.edu/entries/mereology/

- For pragmatic reasons we choose to model the mereology of a domain in either of two ways
  - either by defining a concrete type
    as a model of the composite type,
  - or by endowing the sub-parts of the composite part with structures of unique part identifiers.

  or by suitable combinations of these.

Example: 11 Container Bays, Etcetera: Mereology. First we show how to model indexed set of container bays, rows and stacks for the previous example.

- *Narrative:*
  - (i) An indexed set, bs:BS, of bays is a bijective map
    from unique bay identifiers, bid:BId, to bays, b:B.
  - (ii) An indexed set, rs:RS, of rows is a bijective map
    from unique row identifiers, rid:RId, to rows, r:R.
  - (iii) An indexed set, ss:SS, of stacks is a bijective map
    from unique stack identifiers, sid:SId, to stacks, s:S.
  - (iv) A stack is a linear indexed sequence of containers, c:C.

- *Formalisation:*
  - (i) **type** BS, B, BId,
    Bs=BId $\overrightarrow{m}$ B,
    value obs_Bs: BS→Bs
    (or obs_Bs: BS→(BId $\overrightarrow{m}$ B));
  - (ii) **type** RS, R, RId,
    Rs=RId $\overrightarrow{m}$ R,
    value obs_Rs: RS→Rs
    (or obs_Rs: RS→(RId $\overrightarrow{m}$ R));
  - (iii) **type** SS, S, SId,
    Ss=SId $\overrightarrow{m}$ S;
  - (iv) **type** C,
    S=C*. ■

Example: 12 Transport Nets: Mereology.

- We show how to model a mereology
  - for a transport net of links and hubs.
- *Narrative:*

(i) Hubs and links are endowed with unique hub, respectively link identifiers.

(ii) Each hub is furthermore endowed with a hub mereology which lists the unique link identifiers of all the links attached to the hub.

(iii) Each link is furthermore endowed with a link mereology which lists the set of the two unique hub identifiers of the hubs attached to the link.

(iv) Link identifiers of hubs and hub identifiers of links must designate hubs, respectively links of the net.

- *Formalisation:*

(i) **type** H, HI, L, LI;
value
(ii) uid_HI:H→HI, uid_LI:L→LI,
mereo_H:H→LI-**set**, mereo_L:L→HI-**set**,
axiom
(iii) ∀ l:L · **card** mereo_L(l) = 2
(iv) ∀ n:N, l:L, h:H · l ∈ obs_Ls(obs_LS(n)) ∧ h ∈ obs__Hs(obs_HS(n))
∀ hi:HI · hi ∈ mereo_L(l) ⇒
∃ h′:H·h′ ∈ obs_Hs(obs_HS(n)) ∧ uid_HI(h)=hi
∧ ∀ li:LI · li ∈ mereo_H(h) ⇒
∃ l′:L·l′ ∈ obs_Ls(obs_LS(n)) ∧ uid_LI(l)=li

■

Concrete Models of Mereology

The concrete mereology example models above illustrated maps and sequences as such models.

- In general we can model mereologies in terms of
  - (i) sets: A-**set**,
  - (ii) Cartesians: $A_1 \times A_2 \times \ldots \times A_m$,
  - (iii) lists: A^*, and
  - (iv) maps: $A \underset{m}{\to} B$,

  where A, A_1, A_2,...,A_m and B are types [we assume that they are type names] and where the A_1, A_2,...,A_m type names need not be distinct.
- Additional concrete types, say D, can be defined by concrete type definitions, D=E, where E is either of the type expressions (i–iv) given above or (v) $E_i|E_j$, or (vi) (E_i). where E_k (for suitable k) are either of (i–vi).
- Finally it may be necessary to express well-formedness predicates for concretely modelled mereologies.

Abstract Models of Mereology

Abstractly modelling mereology of parts, to us, means the following.

- With part types P_1, P_2, ..., P_n
  - is associated the unique part identifier types, Π_1, Π_2, ..., Π_n,
  - that is uid_Π_i: $P_i \to \Pi_i$ for $i \in \{1..n\}$,
- and with each part type, P_i,
  - is then associated a mereology observer,
  - mereo_P_i: $P_i \to \Pi_j$-**set**$\times\Pi_k$-**set**$\times\ldots\times\Pi_\ell$-**set**,
- such that for all p:Pi we have that
  - if mereo_P_i(p) = $(\{..., \pi_{j_a}, ...\},\{..., \pi_{k_b}, ...\},...,\{..., \pi_{\ell_c}, ...\})$
  - for $i, j, k, ...\ell \in \{1..n\}$
  - then part p:P_i is connected (related) to the parts identified by $..., \pi_{j_a}, ... \pi_{k_b}, ..., \pi_{\ell_c}, ...$
- Finally it may be necessary to express axioms for abstractly modelled mereologies.

- How parts are related to other parts
  - is really a modelling choice, made by the domain describer.
  - It is not necessarily something that is obvious from observing the parts.

Example: 13 Pipelines: A Physical Mereology.

- Let pipes of a pipe line be composed with valves, pumps, forks and joins of that pipe line.
- Pipes, valves, pumps, forks and joins (i.e., pipe line units) are given unique pipe, valve, pump, fork and join identifiers.
- A mereology for the pipe line could now endow pipes, valves and pumps with
  - one input unique identifier, that of the predecessor successor unit, and
  - one output unique identifier, that of the successor unit.
- Forks would then be endowed with
  - two input unique identifiers, and
  - one out put unique identifier;
- and joins "the other way around".

Example: 14 Documents: A Conceptual Mereology.

- The mereology of, for example, this document,
  - that is, of the tutorial slides,

  is determined by the author.
- There unfolds, while writing the document,
  - a set of unique identifiers
  - for section, subsection, sub-subsection, paragraph, etc., units. and
  - between texts of a "paper version" of the document and slides of a "slides version" of the document.

- This occurs as the author necessarily
  - inserts cross-references,
    - in unit texts to other units, and
    - from unit texts to other documents (i.e., 'citations');
  - and while inserting "page" shifts for the slides.
- From those inserted references there emerges what we could call the document mereology. ■
- So the determination of a, or the, mereology of composite parts
  - is either given by physical considerations,
  - or are given by (more-or-less) logical (or other) considerations,
  - or by combinations of these.
- The "design" of mereologies improves with experience.

Example: 15 Pipelines: Mereology.

- We divert from our line of examples centered around
  - transport nets and, to some degree,
  - container transport,
- to bring a second, in a series of examples
  - on pipelines
  - (for liquid or gaseous material flow).

1. A pipeline consists of connected units, u:U.
2. Units have unique identifiers.
3. And units have mereologies, ui:UI:
   (a) pump, pu:Pu, pipe, pi:Pi, and valve, va:Va, units have one input connector and one output connector;
   (b) fork, fo:Fo, [join, jo:Jo] units have one [two] input connector[s] and two [one] output connector[s];
   (c) well, we:We, [sink, si:Si] units have zero [one] input connector and one [zero] output connector.
   (d) Connectors of a unit are designated by the unit identifier of the connected unit.
   (e) The auxiliary sel_UIs_in selector funtion selects the unique identifiers of pipeline units providing input to a unit;
   (f) sel_UIs_out selects unique identifiers of output recipients.

type
1. U = Pu | Pi | Va | Fo | Jo | Si | We
2. UI

value
2. uid_U: U → UI
3. mereo_U: U → UI-**set** × UI-**set**
3. wf_mereo_U: U → **Bool**
3. wf_mereo_U(u) ≡
3(a). is_(Pu|Pi|Va)(u) → **card** iusi = 1 = **card** ouis,
3(b). is_Fo(u) → **card** iuis = 1 ∧ **card** ouis = 2,
3(b). is_Jo(u) → **card** iuis = 2 ∧ **card** ouis = 1,
3(c). is_We(u) → **card** iuis = 0 ∧ **card** ouis = 1,
3(d). is_Si(u) → **card** iuis = 1 ∧ **card** ouis = 0

3(e). sel_UIs_in
3(e). sel_UIs_in(u) ≡ **let** (iuis,_)=mereo_U(u) **in** iuis **end**
3(f). sel_out: U → UI-**set**
3(f). sel_UIs_out(u) ≡ **let** (_,ouis)=mereo_U(u) **in** ouis **end**

- We omit treatment of axioms for pipeline units
  - being indeed connected to existing other pipeline units.
  - We refer to Example 23 on page 123 and 24 on page 127. ■

4.3.3. Attributes

- By an attribute of a part, p:P, we shall understand
  - some observable property, some phenomenon,
  - that is not a sub-part of p
  - but which characterises p
  - such that all parts of type P have that attribute and
  - such that “removing” that attribute from p (if such was possible) “renders” the type of p undefined.
- We ascribe types to attributes — not, therefore, to be confused with types of (their) parts.

Example: 16 **Attributes.**

- Example attributes of links of a transport net are:
  - length LEN,
  - location LOC,
  - state LΣ and
  - state space LΩ,
- Example attributes of a person could be:
  - name NAM,
  - birth date BID,
  - gender GDR,
  - weight WGT,
  - height HGT and
  - address ADR.

- Example attributes of a transport net could be:
  - name of the net,
  - legal owner of the net,
  - a map of the net,
  - etc.
- Example attributes of a container vessel could be:
  - name of container vessel,
  - vessel dimensions,
  - vessel tonnage (TEU),
  - vessel owner,
  - current stowage plan,
  - current voyage plan, etc. ■

4.3.3.1 Static and Dynamic Attributes

- By a **static attribute** we mean an attribute (of a part) whose value remains fixed.
- By a **dynamic attribute** we mean an attribute (of a part) whose value may vary.

Example: 17 **Static and Dynamic Attributes.**

- The length and location attributes of links are static.
- The state and state space attributes of links and hubs are dynamic.
- The birth-date attribute of a person is considered static.
- The height and weight attributes of a person are dynamic.
- The map of a transport net may be considered dynamic.
- The current stowage and the current voyage plans of a vessel should be considered dynamic. ■

Attribute Types and Observers, I/II

- Let the domain describer decide that parts of type P
- have attributes of types A_1, A_2, ..., A_t.
- This means that the following two formal clauses arise:
  - P, A_1, A_2, ..., A_t and
  - attr_A_1:P→A_1, attr_A_2:P→A_2, ..., attr_A_t:P→A_t

Attribute Types and Observers, II/II

- We may wish to annotate the list of attribute type names as to whether they are static or dynamic, that is,
  - whether values of some attribute type
  - vary or
  - remain fixed.
- The prefix attr_ distinguishes attribute observers from part observers (obs_) and mereology observers (uid_, mereo_).

4.4. Shared Attributes and Properties

- Shared attributes and shared properties
  - play an important rôle in understanding domains.

4.4.1. Attribute Naming

- We now *impose a restriction* on the naming of part attributes.
  - If attributes
    - of two different parts
    - of different part types
    - are identically named
    - then attributes must be somehow related, over time!
  - The "somehow" relationship must be described.

Example: 18 **Shared Bus Time Tables.**

- Let our domain include that of *bus time tables* for *busses* on a *bus transport net* as described in many examples in this tutorial.
- We can then imagine a *bus transport net* as containing the following parts:
  - a *net*,
  - a *management system*,
  - a set of *busses*.
- For the sake of argument we consider a *bus time table* to be an attribute of the *bus management system*.
- And we also consider *bus time tables* to be attributes of *busses*.

- We think of the *bus time table* of a *bus*
  - to be that subset of the *bus management system bus time table*
  - which corresponds to the *bus*' *line number*.
- By saying that *bus time tables*
  - "corresponds" to well-defined subsets of
  - the *bus management system bus time table*

  we mean the following
  - The value of the *bus bus time table*
  - must at every time
  - be equal to the corresponding *bus line entry* in the *bus management system bus time table*. ■

4.4.2. Attribute Sharing

- We say that two parts,
  - of no matter what part type,
  - *share* an attribute,
  - if the following is the case:
    - the corresponding part types (and hence the parts)
    - have identically named attributes.
    - We say that identically named attributes designate shared attributes.
  - We do not present the corresponding invariants over parts with identically named attributes.

4.5. Shared Properties

- We say that two parts,
  - of no matter what part type,
  - *share* a property,
  - if either of the following is the case:
    - (i) either the corresponding part types (and hence the parts) have shared attributes;
    - (ii) or the unique identifier type of one of the parts potentially is in the mereology type of the other part;
    - (iii) or both.
  - We do not present the corresponding invariants over parts with shared properties.

4.6. Summary of Discrete Endurants

- We have introduced the endurant notions of atomic parts and composite parts:
  - part types,
  - part observers (**obs_**),
    - sort observers, and
    - concrete type observers;
  - part properties:
    - unique identifiers:
      - unique part identifier observers (**uid_**),
      - unique part identifier types,
    - mereology:
      - part mereologies,
      - part mereology observers (**mereo_**);

      and
    - attributes:
      - attribute observers (**attr_**) and
      - attribute types.

- The unique identifier property cannot necessarily be observed:
  - it is an abstract concept and
  - can be objectively "assigned".

  That is: unique identifiers are not required to be manifest.
- The mereology property also cannot usually be observed:
  - it is also an abstract concept,
  - but can be deduced from careful analysis.

  That is: mereology is not required to be manifest.
- The attributes can be observed:
  - usually by simple physical measurements,
  - or by deduction from (conceptual) facts,

  That is: attributes are usually only "indirectly" manifest.

Discrete Endurant Modelling I/II

Faced with a phenomenon the domain analyser has to decide

- whether that phenomenon is an entity or not, that is, whether
  - an endurant or
  - a perdurant or
  - neither.
- If endurant and if discrete, then whether it is
  - an atomic part or
  - a composite part.
- Then the domain analyser must decide on its type,
  - whether an abstract type (a sort)
  - or a concrete type, and, if so, which concrete form.

Discrete Endurant Modelling II/II

- Next the unique identifier and the mereology of the part type (e.g., P) must be dealt with:
  - type name (e.g., PI) for and, hence, unique identifier observer name (uid_PI) of unique identifiers and the
  - part mereology types and mereology observer name (mereo_P).
- Finally the designer must decide on the part type attributes for parts p:P:
  - for each such a suitable attribute type name, for example, A_i for suitable i,
  - a corresponding attribute observer signature, $\mathsf{attr\_A}_i\mathsf{:P}\rightarrow\mathsf{A}_i$,
  - and whether an attribute is considered static or dynamic.

End of Lecture 2: Last Session — Discrete Endurant Entities

Parts

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

LONG BREAK

WELCOME BACK

Begin of Lecture 3: First Session — Continuous Endurants

Materials, States

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

Tutorial Schedule

5. Continuous Endurants: Materials

- Let us start with examples of materials.

Example: 19 Materials. Examples of endurant continuous entities are such as

- coal,
- air,
- natural gas,
- grain,
- sand,
- iron ore,
- minerals,
- crude oil,
- solid waste,
- sewage,
- steam and
- water. ■

The above materials are either

- liquid materials (crude oil, sewage, water),
- gaseous materials (air, gas, steam), or
- granular materials (coal, grain, sand, iron ore, mineral, or solid waste).

- Endurant continuous entities, or materials as we shall call them,
  - are the core endurants of process domains,
  - that is, domains in which those materials *form the basis* for their *"raison d'être"*.

Example: 20 Material Processing.

- Oil or gas materials are ubiquitous to pipeline systems.
- Sewage is ubiquitous to, well, sewage systems.
- Water is ubiquitous to systems composed from reservoirs, tunnels and aqueducts which again are ubiquitous to hydro-electric power plants or irrigation systems. ■

- Ubiquitous means 'everywhere'.
- A continuous entity, that is, a material
  - is a core material,
  - if it is "somehow related"
  - to one or more parts of a domain.

5.1. "Somehow Related" Parts and Materials

- We explain our use of the term "somehow related".

Example: 21 "Somehow Related" Parts and Materials.

- Oil is pumped from wells, runs through pipes, is "lifted" by pumps, diverted by forks, "runs together" by means of joins, and is delivered to sinks – *and is hence a core endurant*.
- Grain is delivered to silos by trucks, piped through a network of pipes, forks and valves to vessels, etc. – *and is hence a core endurant*.
- Gravel, minerals (including) iron ore is mined, conveyed by belts to lorries or trains or cargo vessels and finally deposited. For minerals typically in mineral processing plants – *and is hence a core endurant*.
- Iron ore, for example, is conveyed into smelters, roasted, reduced and fluxed, mixed with other mineral ores to produced a molten, pure metal, which is then "collected" into ingots, etc. – *and is hence a core endurant* ■

5.2. Material Observers

- When analysing domains a key question,
  - in view of the above notion of core continuous endurants (i.e., materials)

  is therefore:
  - does the domain embody a notion of core continuous endurants (i.e., materials);
  - if so, then identify these "early on" in the domain analysis.
- Identifying materials —
  - their types and
  - attributes —

  is slightly different from identifying discrete endurants, i.e., parts.

Example: 22 Pipelines: Core Continuous Endurant.

- The core continuous endurant, i.e., material,
- of (say oil) pipelines is, yes, oil:

type
  O **material**
value
  obs_Materials: PLS → O

- The keyword **material** is a pragmatic. ■
- Materials are "few and far between" as compared to parts,
  - we choose to mark the type definitions which designate materials with the keyword **material**.
  - In contrast, we do not mark the type definitions which designate parts with the keyword **discrete**.

- First we do not associate the notion of atomicity or composition with a material. Materials are continuous.
- Second, amongst the attributes, none have to do with geographic (or cadestral) matters. Materials are moved.
- And materials have no unique identification or mereology. No "part" of a material distinguishes it from other "parts".
- But they do have other attributes when occurring in connection with, that is, related to parts, for example,
  - volume or
  - weight.

Example: 23 Pipelines: Parts and Materials. We refer to Example 15 on page 94.

4. From an oil pipeline system one can, amongst others,
   (a) observe the finite set of all its pipeline bodies,
   (b) units are composite and consists of a unit,
   (c) and the oil, even if presently, at time of observation, empty of oil.
5. Whether the pipeline is an oil or a gas pipeline is an attribute of the pipeline system.
   (a) The volume of material that can be contained in a unit is an attribute of that unit.
   (b) There is an auxiliary function which estimates the volume of a given "amount" of oil.
   (c) The observed oil of a unit must be less than or equal to the volume that can be contained by the unit.

type
4. PLS, B, U, O, Vol
value
4(a). obs_Bs: PLS → B-**set**
4(b). obs_U: B → U
4(c). obs_O: B → O
5. attr_PLS_Type: PLS → {"oil"|"gas"}
5(a). attr_Vol: U → Vol
5(b). vol: O → Vol
axiom
5(c). ∀ pls:PLS,b:B•b ∈ obs_Bs(pls)⇒vol(obs_O(b))≤attr_Vol(obs_U(b))

- Notice how bodies are composite and consists of
  - a discrete, atomic part, the unit, and
  - a material endurant, the oil.
- We refer to Example 24 on page 127. ■

5.3. Material Properties

- These are some of the key concerns in domains focused on materials:
  - transport, flows, leaks and losses, and
  - input to systems and output from systems,
- Other concerns are in the direction of
  - dynamic behaviours of materials focused domains (mining and production), including
  - stability, periodicity, bifurcation and ergodicity.
- In this tutorial we shall, when dealing with systems focused on materials, concentrate on modelling techniques for
  - transport, flows, leaks and losses, and
  - input to systems and output from systems.

- Formal specification languages like
  - `Alloy` [alloy],
  - `Event B` [JRAbrial:TheBBooks],
  - `CASL` [CoFI:2004:CASL-RM]
  - `CafeOBJ` [futatsugi2000a],
  - `RAISE` [RaiseMethod],
  - `VDM` [e:db:Bj78bwo,e:db:Bj82b,jf-pgl-97] and
  - `Z` [m:z:jd+jcppw96]

  do not embody the mathematical calculus notions of
  - continuity, hence do not "exhibit"
  - neither differential equations
  - nor integrals.
- Hence cannot formalise dynamic systems within these formal specification languages.
- We refer to Sect. 9 where we discuss these issues at some length.

Example: 24 Pipelines: Parts and Material Properties. We refer to Examples 15 on page 94 and 23 on page 123.

6. Properties of pipeline units additionally include such which are concerned with flows (F) and leaks (L) of materials:
   (a) current flow of material into a unit input connector,
   (b) maximum flow of material into a unit input connector while maintaining laminar flow,
   (c) current flow of material out of a unit output connector,
   (d) maximum flow of material out of a unit output connector while maintaining laminar flow,
   (e) current leak of material at a unit input connector,
   (f) maximum guaranteed leak of material at a unit input connector,
   (g) current leak of material at a unit input connector,
   (h) maximum guaranteed leak of material at a unit input connector,
   (i) current leak of material from "within" a unit,
   (j) maximum guaranteed leak of material from "within" a unit.

type

6. F, L

value

6(a). attr_cur_iF: U → UI → F
6(b). attr_max_iF: U → UI → F
6(c). attr_cur_oF: U → UI → F
6(d). attr_max_oF: U → UI → F
6(e). attr_cur_iL: U → UI → L
6(f). attr_max_iL: U → UI → L
6(g). attr_cur_oL: U → UI → L
6(h). attr_max_oL: U → UI → L
6(i). attr_cur_L: U → L
6(j). attr_max_L: U → L

- The maximum flow attributes are static attributes and are typically provided by the manufacturer as indicators of flows below which laminar flow can be expected.
- The current flow attributes as dynamic attributes.

7. Properties of pipeline materials may additionally include
   (a) kind of material[24],
   (b) paraffins,
   (c) naphtenes,
   (d) aromatics,
   (e) asphatics,
   (f) viscosity,
   (g) etcetera.

- We leave it to the student to provide the formalisations. ■

[24] For example **Brent Blend** Crude Oil

5.4. Material Laws of Flows and Leaks

- It may be difficult or costly, or both
  - to ascertain flows and leaks in materials-based domains.
  - But one can certainly speak of these concepts.
  - This casts new light on domain modelling.
  - That is in contrast to
    - incorporating such notions of flows and leaks
    - in requirements modelling
  - where one has to show implementability.
- Modelling flows and leaks is important to the modelling of materials-based domains.

Example: 25 Pipelines: Intra Unit Flow and Leak Law.

8. For every unit of a pipeline system, except the well and the sink units, the following law apply.
9. The flows into a unit equal
   (a) the leak at the inputs
   (b) plus the leak within the unit
   (c) plus the flows out of the unit
   (d) plus the leaks at the outputs.

axiom

8. ∀ pls:PLS,b:B\We\Si,u:U ·
8. b ∈ obs_Bs(pls)∧u=obs_U(b)⇒
8. **let** (iuis,ouis) = mereo_U(u) **in**
9. sum_cur_iF(iuis)(u) =
9(a). sum_cur_iL(iuis)(u)
9(b). ⊕ attr_cur_L(u)
9(c). ⊕ sum_cur_oF(ouis)(u)
9(d). ⊕ sum_cur_oL(ouis)(u)
8. **end**

10. The sum_cur_iF (cf. Item 9) sums current input flows over all input connectors.
11. The sum_cur_iL (cf. Item 9(a)) sums current input leaks over all input connectors.
12. The sum_cur_oF (cf. Item 9(c)) sums current output flows over all output connectors.
13. The sum_cur_oL (cf. Item 9(d)) sums current output leaks over all output connectors.

10. sum_cur_iF: UI-**set** → U → F
10. sum_cur_iF(iuis)(u) ≡ ⊕ {attr_cur_iF(ui)(u)|ui:UI·ui ∈ iuis}
11. sum_cur_iL: UI-**set** → U → L
11. sum_cur_iL(iuis)(u) ≡ ⊕ {attr_cur_iL(ui)(u)|ui:UI·ui ∈ iuis}
12. sum_cur_oF: UI-**set** → U → F
12. sum_cur_oF(ouis)(u) ≡ ⊕ {attr_cur_iF(ui)(u)|ui:UI·ui ∈ ouis}
13. sum_cur_oL: UI-**set** → U → L
13. sum_cur_oL(ouis)(u) ≡ ⊕ {attr_cur_iL(ui)(u)|ui:UI·ui ∈ ouis}
⊕: (F|L) × (F|L) → F

- where ⊕ is both an infix and a distributed-fix function which adds flows and or leaks. ■

Example: 26 Pipelines: Inter Unit Flow and Leak Law.

14. For every pair of connected units of a pipeline system the following law apply:
 (a) the flow out of a unit directed at another unit minus the leak at that output connector
 (b) equals the flow into that other unit at the connector from the given unit plus the leak at that connector.

14. ∀ pls:PLS,b,b′:B,u,u′:U·
14. {b,b′}⊆obs_Bs(pls)∧b≠b′∧u′=obs_U(b′)
14. ∧ **let** (iuis,ouis)=mereo_U(u),(iuis′,ouis′)=mereo_U(u′),
14. ui=uid_U(u),ui′=uid_U(u′) **in**
14. ui ∈ iuis ∧ ui′ ∈ ouis′ ⇒
14(a). attr_cur_oF(us′)(ui′) − attr_leak_oF(us′)(ui′)
14(b). = attr_cur_iF(us)(ui) + attr_leak_iF(us)(ui)
14. **end**
14. **comment:** b′ precedes b

- From the above two laws one can prove the **theorem:**
  - what is pumped from the wells equals
  - what is leaked from the systems plus what is output to the sinks.
- We need formalising the flow and leak summation functions. ■

Continuous Endurant Modelling

As one of the first steps

- in domain analysis
- determine if the domain is materials-focused.

If so, then determine

- the material types,

  type M1, M2, ... Mn **material**

- the parts, that is, the part types, with which the materials are "somehow related",

  value obs_Mi: Pi → Mi, obs_Mj: Pj → Mj, ..., obs_Mk: Pk → Mk

- the relevant flow or transport and/or leak or loss attributes, if any,
- and the possible laws related to these attributes.

6. States

6.1. General

- The above Wikipedia characterisation of the concept of perdurant
  - mentioned time,
  - but implied a concept that we shall call state.
- In this version of this tutorial
  - we shall not cover the modelling of time phenomena —
  - but we shall model that some actions occur before others.

- By a state we shall understand a collection of parts
  - such that each of these parts have dynamic attributes.
- We can characterise the state
  - by giving it a type,
  - for example, Σ, where the state type definition
  - $\Sigma = \mathsf{S}_1 \times \mathsf{S}_2 \times \cdots \times \mathsf{S}_s$
  - assembles the types of the parts making up the state —
  - where we assume that types $\mathsf{S}_1, \mathsf{S}_2, \ldots, \mathsf{S}_s$
    - are types of parts
    - such that no S_i is a sub-part (of a subpart, ...) of some S_j,
    - and such that each part has dynamic attributes.

Example: 27 Net and Vessel States.

- We may consider a transport net, n:N, to represent a state (subject to the actions of maintaining a net: adding or removing a hub, adding or removing a link, etc.).
- We may also consider a hub, h:H, to represent a state (subject to the changing of a hub traffic signal: from red to green, etc., for specific directions through the hub).
- We may consider a container vessel to represent a state (subject to adding or removing containers from, respectively onto the top of stacks). ■

Thus the context determines how wide a scope the domain designer chooses for the state concept.

6.2. State Invariants

- States are subject to invariants.

Example: 28 State Invariants: Transport Nets. Nets, hubs and links were first introduced in Example 3 on page 16 – and were and will be prominent in this tutorial, to wit, Examples 7–16 and 29– **??** on page ??.

- Net hubs and links may be inserted into and removed from nets.
- Thus is also introduced changes to the net mereology.
- Yet, the axioms, as illustrated in Example 12, must remain invariant.
- Likewise changes to dynamic attributes may well be subject to the holding of certain well-formedness constraints.
- We will illustrate this claim.

With each hub we associate a hub [link] state and a hub [link] state space.

15. A hub [link] state models the permissible routes from hub input links to (same) hub output links [respectively through a link].
16. A hub [link] state space models the possible set of hub [link] states that a hub [link] is intended to "occupy".

type
15. $H\Sigma = (LI \times LI)$**-set**, $L\Sigma = HI$**-set**
16. $H\Omega = H\Sigma$**-set**, $L\Omega = L\Sigma$**-set**
value
15. attr_HΣ: H → HΣ, attr_LΣ: L → LΣ
16. attr_HΩ: H → HΩ, attr_LΩ: L → LΩ

17. For any given hub, h, with links, $l_1, l_2, ..., l_n$ incident upon (i.e., also emanating from) that hub, each hub state in the hub state space
18. must only contain such pairs of (not necessarily distinct) link identifiers that are identifiers of $l_1, l_2, ..., l_n$.

value
17. wf_HΩ: H → **Bool**
17. wf_HΩ(h) ≡ ∀ hσ:HΣ · hσ ∈ attr_HΩ(h) ⇒ wf_HΣ(h)

17. wf_HΣ: H → **Bool**
17. wf_HΣ(h) ≡
18. ∀ (li,li′):(LI×LI)·(li,li′)∈ attr_HΣ(h) ⇒ {li,li′} ⊆ mereo_H(h)

- This well-formedness criterion is part of the state invariant over nets.
  - We never write down the full state invariant for nets.
  - It is tacitly assume to be the collection of all the axioms and well-formedness predicates over net parts. ■

7. A Final Note on Endurant Properties

- The properties of parts and materials are fully captured by
  - (i) the unique part identifiers,
  - (ii) the part mereology and
  - (iii) the full set ofpart attributes and material attributes
- We therefore postulate a property function
  - when when applied to a part or a material
  - yield this triplet, (i–iii), of properties
  - in a suitable structure.

type
  Props = {|PI|**nil**|} × {|(PI-**set**×...×PI-**set**)|**nil**|} × Attrs
value
  props: Part|Material → Props

- where
  - Part stands for a part type,
  - Material stands for a material type,
  - PI stand for unique part identifiers and
  - PI-set×...×PI-set for part mereologies.
- The {|...|} denotes a proper specification language sub-type and **nil** denotes the empty type.

End of Lecture 3: First Session — Continuous Endurants

Materials, States

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

MINI BREAK

HAPPY TO SEE YOU AGAIN

Begin of Lecture 4: Middle Session — Perdurant Entities

Actions and Events

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

Tutorial Schedule

8. Discrete Perdurants

8.1. General

- From Wikipedia:
  - *Perdurant: Also known as occurrent, accident or happening.*
  - *Perdurants are those entities for which only a fragment exists if we look at them at any given snapshot in time.*
  - *When we freeze time we can only see a fragment of the perdurant.*
  - *Perdurants are often what we know as processes, for example 'running'.*
  - *If we freeze time then we only see a fragment of the running, without any previous knowledge one might not even be able to determine the actual process as being a process of running.*
  - *Other examples include an activation, a kiss, or a procedure.*

- We shall consider **actions** and **events**
  - to occur instantaneously,
  - that is, in time, but taking no time
- Therefore we shall consider **actions** and **events** to be **perdurants**.

8.2. Discrete Actions

- By a **function** we understand
  - a thing
  - which when **applied** to a **value**, called its **argument**,
  - **yields** a **value**, called its **result**.
- An **action** is
  - a **function**
  - **invoked** on a **state value**
  - and is one that potentially changes that value.

Example: 29 Transport Net and Container Vessel Actions.

- *Inserting* and *removing* hubs and links in a net are considered actions.
- *Setting* the traffic signals for a hub (which has such signals) is considered an action.
- *Loading* and *unloading* containers from or unto the top of a container stack are considered actions. ■

8.2.1. An Aside on Actions

Think'st thou existence doth depend on time?
It doth; but actions are our epochs.
George Gordon Noel Byron,
Lord Byron (1788-1824) Manfred. Act II. Sc. 1.

- *"An action is*
  - *something an agent does*
  - *that was 'intentional under some description'"* [Davidson1980].
- That is, actions are performed by agents.
  - We shall not yet go into any deeper treatment of **agency** or **agents**. We shall do so later.
    - **Agents** will here, for simplicity, be considered **behaviours**,
    - and are treated later in this lecture.

- As to the relation between **intention** and **action**
  - we note that Davidson wrote: 'intentional under some description'
  - and take that as our cue:
    - the agent follows a script,
    - that is, a behaviour description,
    - and invokes actions accordingly,
    - that is, follow, or honours that script.
- The philosophical notion of 'action' is over-viewed in [sep-action].
- We
  - observe actions in the domain
  - but describe "their underlying" functions.
- Thus we abstract from the **time**s at which actions occur.

8.2.2. Action Signatures

- By an **action signature** we understand a quadruple:
  - a **function name**,
  - a **function definition set type expression**,
  - a **total** or **partial function** designator ($\rightarrow$, respectively $\xrightarrow{\sim}$), and
  - a **function image set type expression**:
    fct_name: A $\rightarrow$ Σ ($\rightarrow$|$\xrightarrow{\sim}$) Σ [$\times$ R],

  where $(X \mid Y)$ means either X or Y, and $[Z]$ means optional Z.

Example: 30 Action Signatures: Nets and Vessels.

insert_Hub: N$\rightarrow$H$\xrightarrow{\sim}$N;
remove_Hub: N$\rightarrow$HI$\xrightarrow{\sim}$N;
set_Hub_Signal: N$\rightarrow$HI$\xrightarrow{\sim}$H$\Sigma$$\xrightarrow{\sim}$N
load_Container: V$\rightarrow$C$\rightarrow$StackId$\xrightarrow{\sim}$V; and
unload_Container: V$\rightarrow$StackId$\xrightarrow{\sim}$(V$\times$C). ■

8.2.3. Action Definitions

- There are a number of ways in which to characterise an action.
- One way is to characterise its underlying function by a pair of predicates:
  - **precondition**: a predicate over function arguments — which includes the state, and
  - **postcondition**: a predicate over function arguments, a proper argument state and the desired result state.
  - If the precondition holds, i.e., is **true**, then the arguments, including the argument state, forms a proper 'input' to the action.
  - If the postcondition holds, assuming that the precondition held, then the resulting state [and possibly a yielded, additional "result" (R)] is as they would be had the function been applied.

Example: 31 Transport Nets: Insert Hub Action. We give one example.

19. The **insert** action applies to a net and a hub and conditionally yields an updated net.
    (a) The condition is that there must not be a hub in the "argument" net with the same unique hub identifier as that of the hub to be inserted and
    (b) the hub to be inserted does not initially designate links with which it is to be connected.
    (c) The updated net contains all the hubs of the initial net "plus" the new hub.
    (d) and the same links.

value

19. insert_H: N → H $\stackrel{\sim}{\to}$ N
19. insert_H(n)(h) **as** n′, **pre**: pre_insert_H(n)(h), **post**: post_insert_H(n)(h)

19(a). pre_insert_H(n)(h) ≡
19(a). ∼∃ h′:H • h′ ∈ obs_Hs(n) ∧ uid_HI(h)=uid_HI(h′)
19(b). ∧ mereo_H(h) = {}

19(c). post_insert_H(n)(h)(n′) ≡
19(c). obs_Hs(n) ∪ {h} = obs_Hs(n′)
19(d). ∧ obs_Ls(n) = obs_Ls(n′)

- We refer to the notes accompanying these lectures.
- There you will find definitions of insert_link, remove_hub and remove_link action functions. ■

- What is not expressed, but tacitly assume in the above pre- and post-conditions is
  - that the state, here n, satisfy invariant criteria before (i.e. n) and after (i.e., n') actions,
  - whether these be implied by axioms
  - or by well-formedness predicates.

  over parts.
- This remark applies to any definition of actions, events and behaviours.

Example: 32 Action: Remove Container from Vessel. We give the second of two examples.

20. The remove_Container_from_Vessel action applies to a vessel and a stack address and conditionally yields an updated vessel and a container.
    (a) We express the 'remove from vessel' function primarily by means of an auxiliary function remove_C_from_BS, remove_C_from_BS(obs_BS(v))(stid), and some further post-condition on the before and after vessel states (cf. Item 20(d)).
    (b) The remove_C_from_BS function yields a pair: an updated set of bays and a container.
    (c) When obs_erving the BayS from the updated vessel, v′, and pairing that with what is assumed to be a vessel, then one shall obtain the result of remove_C_from_BS(obs_BS(v))(stid).
    (d) Updating, by means of remove_C_from_BS(obs_BS(v))(stid), the bays of a vessel must leave all other properties of the vessel unchanged.

21. The pre-condition for remove_C_from_BS(bs)(stid) is
    (a) that stid is a valid_address in bs, and
    (b) that the stack in bs designated by stid is non_empty.
22. The post-condition for remove_C_from_BS(bs)(stid) wrt. the updated bays, bs′, is
    (a) that the yielded container, i.e., c, is obtained, get_C(bs)(stid), from the top of the non-empty, designated stack,
    (b) that the mereology of bs′ is unchanged, unchanged_mereology(bs,bs′). wrt. bs. ,
    (c) that the stack designated by stid in the "input" state, bs, is popped, popped_designated_stack(bs,bs′)(stid), and
    (d) that all other stacks are unchanged in bs′ wrt. bs, unchanged_non_designated_stacks(bs,bs′)(stid).

value

20. remove_C_from_V: V → StackId $\stackrel{\sim}{\to}$ (V×C)
20. remove_C_from_V(v)(stid) **as** (v′,c)
20(c). (obs_BS(v′),c) = remove_C_from_BS(obs_BS(v))(stid)
20(d). ∧ props(v)=props(v″)

20(b). remove_C_from_BS: BS → StackId → (BS×C)
20(a). remove_C_from_BS(bs)(stid) **as** (bs′,c)
21(a). **pre**: valid_address(bs)(stid)
21(b). ∧ non_empty_designated_stack(bs)(stid)
22(a). **post**: c = get_C(bs)(stid)
22(b). ∧ unchanged_mereology(bs,bs′)
22(c). ∧ popped_designated_stack(bs,bs′)(stid)
22(d). ∧ unchanged_non_designated_stacks(bs,bs′)(stid)

- This example hints at *a theory of container vessel bays, rows and stacks*.
- More on that is found in Appendix C. ■

- There are other ways of defining functions.
- But the form of these are not material to the aims of this tutorial.

Modelling Actions, I/III

- The domain describer has decided that an entity is a perdurant and is, or represents an action: was *"done by an agent and intentionally under some description"* [Davidson1980].
  - The domain describer has further decided that the observed action is of a class of actions — of the "same kind" — that need be described.
  - By actions of the 'same kind' is meant that these can be described by the same function signature and function definition.

Modelling Actions, II/III

- First the domain describer must decide on the underlying function signature.
  - The argument type and the result type of the signature are those of either previously identified
    - parts and/or materials,
    - unique part identifiers, and/or
    - attributes.

Modelling Actions, III/III

- Sooner or later the domain describer must decide on the function definition.
  - The form must be decided upon.
  - For pre/post-condition forms it appears to be convenient to have developed, "on the side", a theory of mereology for the part types involved in the function signature.

8.3. Discrete Events

- By an event we understand
  - a state change
  - resulting indirectly from an unexpected application of a function,
  - that is, that function was performed "surreptitiously".
- Events can be characterised by a pair of (before and after) states, a predicate over these and, optionally, a time or time interval.
- Events are thus like actions:
  - change states,
  - but are usually
    - either caused by "previous" actions,
    - or caused by "an outside action".

Example: 33 Events.

- *Container vessel:* A container falls overboard sometimes between times t and t'.
- *Financial service industry:* A bank goes bankrupt sometimes between times t and t'.
- *Health care:* A patient dies sometimes between times t and t'.
- *Pipeline system:* A pipe breaks sometimes between times t and t'.
- *Transportation:* A link "disappears" sometimes between times t and t'.

8.3.1. An Aside on Events

- We may observe an event, and
  - then we do so at a specific time or
  - during a specific time interval.
- But we wish to describe,
  - not a specific event
  - but a class of events of "the same kind".
- In this tutorial
  - we therefore do not ascribe
  - time points or time intervals
  - with the occurrences of events.

8.3.2. Event Signatures

- An event signature
  - is a predicate signature
  - having an event name,
  - a pair of state types $(\Sigma \times \Sigma)$,
  - a total function space operator $(\rightarrow)$
  - and a **Bool**ean type constant:
  - evt: $(\Sigma\times\Sigma) \rightarrow$ **Bool**.
- Sometimes there may be a good reason
  - for indicating the type, ET, of an event cause value,
  - if such a value can be identified:
  - evt: ET $\times (\Sigma \times \Sigma) \rightarrow$ **Bool**.

8.3.3. Event Definitions

- An event definition takes the form of a predicate definition:
  - A predicate name and argument list, usually just a state pair,
  - an existential quantification
    - over some part (of the state) or
    - over some dynamic attribute of some part (of the state)
    - or combinations of the above
  - a pre-condition expression over the input argument(s),
  - an implication symbol $(\Rightarrow)$, and
  - a post-condition expression over the argument(s).
- evt$(\sigma,\sigma') = \exists$ (ev:ET) • pre_evt(ev)$(\sigma) \Rightarrow$ post_evt(ev)(σ,σ').
- There may be variations to the above form.

Example: 34 Narrative of Link Event. The disappearance of a link in a net, for example due to a mud slide, or a bridge falling down, or a fire in a road tunnel, can, for example be described as follows:

23. Link disappearance is expressed as a predicate on the "before" and "after" states of the net. The predicate identifies the "missing" ℓink (!).
24. Before the disappearance of link ℓ in net n
    (a) the hubs h' and h'' connected to link ℓ
    (b) were connected to links identified by $\{l'_1, l'_2, \ldots, l'_p\}$ respectively $\{l''_1, l''_2, \ldots, l''_q\}$
    (c) where, for example, l'_i, l''_j are the same and equal to uid_$\Pi(\ell)$.

25. After link ℓ disappearance there are instead
    (a) two separate links, ℓ_i and ℓ_j, "truncations" of ℓ
    (b) and two new hubs h''' and h''''
    (c) such that ℓ_i connects h' and h''' and
    (d) ℓ_j connects h'' and h'''';
    (e) Existing hubs h' and h'' now have mereology
        i. $\{l'_1, l'_2, \ldots, l'_p\} \setminus \{$uid_$\Pi(\ell)\} \cup \{$uid_$\Pi(\ell_i)\}$ respectively
        ii. $\{l''_1, l''_2, \ldots, l''_q\} \setminus \{$uid_$\Pi(\ell)\} \cup \{$uid_$\Pi(\ell_j)\}$
26. All other hubs and links of n are unaffected. ■

Example: 35 Formalisation of Link Event. Continuing Example 34 above:

23. link_disappearance: N × N → **Bool**
23. link_disappearance(n,n′) ≡
23. $\exists\ \ell$:L · pre_link_dis(n,ℓ) ⇒ post_link_dis(n,ℓ,n′)

24. pre_link_dis: N × L → **Bool**
24. pre_link_dis(n,ℓ) ≡ $\ell \in$ obs_Ls(n)

27. We shall “explain” *link disappearance* as the combined, instantaneous effect of
    (a) first a remove link “event” where the removed link connected hubs hi_j and hi_k;
    (b) then the insertion of two new, “fresh” hubs, h_α and h_β;
    (c) “followed” by the insertion of two new, “fresh” links $l_{j\alpha}$ and $l_{k\beta}$ such that
        i. $l_{j\alpha}$ connects hi_j and h_α and
        ii. $l_{k\beta}$ connects hi_k and $h_{k\beta}$

value
27. post_link_dis(n,ℓ,n′) ≡
27(a). **let** n″ = remove_L(n)(uid_L(ℓ)) **in**
27(b). **let** h_α,h_β:H · {h_α,h_β}∩obs_Hs(n)={} **in**
27(b). **let** n‴ = insert_H(n″)(h_α) **in**
27(b). **let** n⁗ = insert_H(n‴)(h_β) **in**
27(c). **let** $l_{j\alpha}$,$l_{k\beta}$:L · {$l_{j\alpha}$,$l_{k\beta}$}∩obs_Ls(n)={} **in**
27((c))i. **let** n′′′′′ = insert_L(n⁗)($l_{j\alpha}$) **in**
27((c))ii. n′ = insert_L(n′′′′′)($l_{k\beta}$) **end end end end end end**

- We refer to the notes accompanying these lectures.
- There you will find definitions of insert_link, remove_hub and remove_link action functions. ■

Modelling Events I/II

- The domain describer has decided that an entity is a perdurant and is, or represents an event: occurred surreptitiously, that is, was not an action that was *“done by an agent and intentionally under some description”* [Davidson1980].
  - The domain describer has further decided that the observed event is of a class of events — of the “same kind” — that need be described.
  - By events of the ‘same kind’ is meant that these can be described by the same predicate function signature and predicate function definition.

Modelling Events, II/II

- First the domain describer must decide on the underlying predicate function signature.
  - The argument type and the result type of the signature are those of either previously identified
    - parts,
    - unique part identifiers, or
    - attributes.
- Sooner or later the domain describer must decide on the predicate function definition.
  - For predicate function definitions it appears to be convenient to have developed, "on the side", a theory of mereology for the part types involved in the function signature.

End of Lecture 4: Middle Session — Perdurant Entities

Actions and Events

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

MINI BREAK

LAST HAUL BEFORE LUNCH

Begin of Lecture 5: Last Session — Perdurant Entities

Behaviours, Discussion Entities

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

Tutorial Schedule

8.4. Discrete Behaviours

- We shall distinguish between
  - discrete behaviours (this section) and
  - continuous behaviours (Sect.).
- Roughly discrete behaviours
  - proceed in discrete (time) steps —
  - where, in this tutorial, we omit considerations of time.
  - Each step corresponds to an action or an event or a time interval between these.
  - Actions and events may take some (usually inconsiderable time),
  - but the domain analyser has decided that it is not of interest to understand what goes on in the domain during that time (interval).
  - Hence the behaviour is considered discrete.

- Continuous behaviours
  - are continuous in the sense of the calculus of mathematical;
  - to qualify as a continuous behaviour time must be an essential aspect of the behaviour.
  - We shall treat continuous behaviours in Sect. 9.
- Discrete behaviours can be modelled in many ways, for example using
  - CSP [Hoare85+2004].
  - MSC [MSCall],
  - Petri Nets [m:petri:wr09] and
  - Statechart [Harel87].
- We refer to Chaps. 12–14 of [TheSEBook2wo].
- In this tutorial we shall use RSL/CSP.

8.4.1. What is Meant by 'Behaviour' ?

- We give two characterisations of the concept of 'behaviour'.
  - a "loose" one and
  - a "slanted one.
- A loose characterisation runs as follows:
  - by a **behaviour** we understand
    - a set of sequences of
    - **action**s, **event**s and **behaviour**s.

- A "slanted" characterisation runs as follows:
  - by a **behaviour** we shall understand
    - either a **sequential behaviour** consisting of a possibly infinite sequence of zero or more actions and events;
    - or one or more **communicating behaviour**s whose **output action**s of one behaviour may **synchronise** and **communicate** with **input action**s of another behaviour; and
    - or two or more **behaviour**s acting either as **internal non-deterministic behaviour**s (⌈⌉) or as **external non-deterministic behaviour**s ([]).

- This latter characterisation of behaviours
  - is "slanted" in favour of a `CSP`, i.e., a **communicating sequential behaviour**, view of behaviours.
  - We could similarly choose to "slant" a behaviour characterisation in favour of
    - `Petri Nets`, or
    - `MSC`s, or
    - `Statechart`s, or other.

8.4.2. Behaviour Narratives

- **Behaviour narratives** may take many forms.
  - A behaviour may best be seen as composed from several interacting behaviours.
    - Instead of narrating each of these,
    - as will be done in Example **??**,
    - one may proceed by first narrating the interactions of these behaviours.
  - Or a behaviour may best be seen otherwise,
    - for which, therefore, another style of narration may be called for,
    - one that "traverses the landscape" differently.
  - Narration is an art.
  - Studying narrations – and practice – is a good way to learn effective narration.

8.4.3. An Aside on Agents, Behaviours and Processes

- "In philosophy and sociology, agency is the capacity of an agent (a person or other entity) to act in a world."
- "In philosophy, the agency is considered as belonging to that agent even if that agent represents a fictitious character, or some other non-existent entity."
- That is, we consider agents to be those persons or other entities that
  - are in the domain and
  - observes the domain
  - evaluates what is being observed
  - and invokes actions.
- We describe agents by describing behaviours.

- A behaviour description denotes a process, that is, a set of
  - actions,
  - events and
  - processes.
- We shall not enter into any further speculations on
  - agency,
  - agents and
  - how agents observe, including
    - what they know and believe (epistemic logic),
    - what is necessary and possible (deontic logic) and
    - what is true at some tie and what is always true (temporal logic).
  - A proper domain science and engineering must, however, eventually examine these (modal logic) issues.

8.4.4. On Behaviour Description Components

- When narrating plus, at the same time, formalising,
  - i.e., textually alternating between
  - narrative texts and
  - formal texts,
- one usually starts with what seems to be the most important behaviour concepts of the given domain:
  - which are the important part types characterising the domain;
  - which of these parts will become a basis for behaviour processes;
  - how are these behaviour processes to interact,
  - that is, which channels and what messages may possibly be communicated.

Example: 36 A Road Traffic System. We continue our long line of examples around transport nets. The present example interprets these as road nets.

8.4.4.1 Continuous Traffic

- For the road traffic system
  - perhaps the most significant example of a behaviour
  - is that of its traffic

28. the continuous time varying discrete positions of vehicles, vp:VP[25],
29. where time is taken as a dense set of points.

type

29. $c\mathbb{T}$

28. $cRTF = c\mathbb{T} \rightarrow (V \overrightarrow{m} VP)$

[25] For VP see Item 47(a) on page 197.

8.4.4.2 Discrete Traffic

- We shall model, not continuous time varying traffic, but

30. discrete time varying discrete positions of vehicles,
31. where time can be considered a set of linearly ordered points.

31. d$\mathbb{T}$
30. dRTF = d$\mathbb{T}$ $\overrightarrow{m}$ (V $\overrightarrow{m}$ VP)

32. The road traffic that we shall model is, however, of vehicles referred to by their unique identifiers.

type
32. RTF = d$\mathbb{T}$ $\overrightarrow{m}$ (VI $\overrightarrow{m}$ VP)

8.4.4.3 Time: An Aside

- We shall take a rather simplistic view of time [wayne.d.blizard.90,mctaggart-t0,prior68,J.van.Benthem.Logic.Time91].

33. We consider d$\mathbb{T}$, or just $\mathbb{T}$, to stand for a totally ordered set of time points.
34. And we consider $\mathbb{TI}$ to stand for time intervals based on $\mathbb{T}$.
35. We postulate an infinitesimal small time interval δ.
36. $\mathbb{T}$, in our presentation, has lower and upper bounds.
37. We can compare times and we can compare time intervals.
38. And there are a number of "arithmetics-like" operations on times and time intervals.

type
33. $\mathbb{T}$
34. $\mathbb{TI}$

value
35. δ:$\mathbb{TI}$
36. MIN, MAX: $\mathbb{T} \rightarrow \mathbb{T}$
36. $<,\leq,=,\geq,>$: $(\mathbb{T}\times\mathbb{T})|(\mathbb{TI}\times\mathbb{TI}) \rightarrow$ **Bool**
37. $-$: $\mathbb{T}\times\mathbb{T} \rightarrow \mathbb{TI}$
38. $+$: $\mathbb{T}\times\mathbb{TI},\mathbb{TI}\times\mathbb{T} \rightarrow \mathbb{T}$
38. $-,+$: $\mathbb{TI}\times\mathbb{TI} \rightarrow \mathbb{TI}$
38. $*$: $\mathbb{TI}\times$**Real** $\rightarrow \mathbb{TI}$
38. $/$: $\mathbb{TI}\times\mathbb{TI} \rightarrow$ **Real**

39. We postulate a global clock behaviour which offers the current time.
40. We declare a channel clk_ch.

value
39. clock: $\mathbb{T} \rightarrow$ **out** clk_ch **Unit**
39. clock(t) $\equiv$ ... clk_ch!t ... clock(t $\sqcap$ t+δ)

channel
40. clk_ch:$\mathbb{T}$

8.4.4.4 Road Traffic System Behaviours

41. Thus we shall consider our road traffic system, rts, as
    (a) the concurrent behaviour of a number of vehicles and, to "observe", or, as we shall call it, to monitor their movements,
    (b) the monitor behaviour.

value

41. trs() =
41(a). ‖ {veh(uid_V(v))(v)|v:V•v ∈ vs}
41(b). ‖ mon(m)([])

- where the "extra" monitor argument ([])
  - records the discrete road traffic, RTF,
  - initially set to the empty map (of, "so far no road traffic"!).

8.4.4.5 Globally Observable Parts

- There is given

42. a net, n:N,
43. a set of vehicles, vs:V-**set**, and
44. a monitor, m:M.

- The n:N, vs:V-**set** and m:M are observable from the road traffic system domain.

value

42. n:N = obs_N(Δ)
42. ls:L-**set** = obs_Ls(obs_LS(n)), hs:H-**set** = obs_Hs(obs_HS(n)),
42. lis:LI-**set** = {uid_L(l)|l:L•l ∈ ls}, his:HI-**set** = {uid_H(h)|h:H•h ∈ hs}
43. vs:V-**set** = obs_Vs(obs_VS(obs_F(Δ))), vis:V-**set** = {uid_V(v)|v:V•v ∈ v
44. m:obs_M(Δ)

8.4.4.6 Channels

- In order for the monitor behaviour to assess the vehicle positions
  - these vehicles communicate their positions
  - to the monitor
  - via a vehicle to monitor channel.
- In order for the monitor to time-stamp these positions
  - it must be able to "read" a clock.

45. Thus we declare a set of channels indexed by the unique identifiers of vehicles and communicating vehicle positions; and
46. a single clock to monitor channel.

channel

45. {vm_ch[vi]|vi:VI•vi ∈ vis}:VP
46. clkm_ch:dT

8.4.4.7 An Aside: Attributes of Vehicles

47. Dynamic attributes of vehicles include
    (a) position
        i. at a hub (about to enter the hub — referred to by the link it is coming from, the hub it is at and the link it is going to, all referred to by their unique identifiers or
        ii. some fraction "down" a link (moving in the direction from a from hub to a to hub — referred to by their unique identifiers)
        iii. where we model fraction as a real between 0 and 1 included.
    (b) velocity, acceleration, etcetera.

type

47(a). VP = atH | onL
47((a))i. atH :: fli:LI × hi:HI × tli:LI
47((a))ii. onL :: fhi:HI × li:LI × frac:FRAC × thi:HI
47((a))iii. FRAC = **Real**, **axiom** ∀ frac:FRAC • 0 ≤ frac ≤ 1
47(b). Vel, Acc, ...

8.4.4.8 Behaviour Signatures

48. The road traffic system behaviour, rts, takes no arguments; and "behaves", that is, continues forever.
49. The vehicle behaviours are indexed by the unique identifier, uid_V(v):VI, the vehicle part, v:V and the vehicle position; offers communication to the monitor behaviour; and behaves "forever".
50. The monitor behaviour takes monitor part, m:M, as argument and also the discrete road traffic, drtf:dRTF; the behaviour otherwise runs forever.

value

48. rts: **Unit** → **Unit**

49. veh: vi:VI → v:V → VP → **out** vm_ch[vi] **Unit**

50. mon: m:M → RTF → **in** {vm_ch[vi]|vi:VI•vi ∈ vis},clkm_ch **Unit**

8.4.4.9 The Vehicle Behaviour

51. A vehicle process
    - is indexed by the unique vehicle identifier vi:VI,
    - the vehicle "as such", v:V and
    - the vehicle position, vp:VP.

    The vehicle process communicates
    - with the monitor process on channel vm[vi]
    - (sends, but receives no messages), and
    - otherwise evolves "infinitely" (hence **Unit**).

52. We describe here an abstraction of the vehicle behaviour at a Hub (hi).
    (a) Either the vehicle remains at that hub informing the monitor,
    (b) or, internally non-deterministically,
        i. moves onto a link, tli, whose "next" hub, identified by thi, is obtained from the mereology of the link identified by tli;
        ii. informs the monitor, on channel vm[vi], that it is now on the link identified by tli,
        iii. whereupon the vehicle resumes the vehicle behaviour positioned at the very beginning (0) of that link,
    (c) or, again internally non-deterministically,
    (d) the vehicle "disappears — off the radar" !

52. veh(vi)(v)(vp:atH(fli,hi,tli)) ≡

52(a). vm_ch[vi]!vp ; veh(vi)(v)(vp)

52(b). ⌈⌉

52((b))i. **let** {hi′,thi}=mereo_L(get_L(tli)(n)) **in assert:** hi′=hi

52((b))ii. vm_ch[vi]!onL(tli,hi,0,thi) ;

52((b))iii. veh(vi)(v)(onL(tli,hi,0,thi)) **end**

52(c). ⌈⌉

52(d). **stop**

53. We describe here an abstraction of the vehicle behaviour on a Link (ii).
Either
   (a) the vehicle remains at that link position informing the monitor,
   (b) or, internally non-deterministically,
   (c) if the vehicle's position on the link has not yet reached the hub,
      i. then the vehicle moves an arbitrary increment δ along the link informing the monitor of this, or
      ii. else, while obtaining a "next link" from the mereology of the hub (where that next link could very well be the same as the link the vehicle is about to leave),
         A. the vehicle informs the monitor that it is now at the hub identified by thi,
         B. whereupon the vehicle resumes the vehicle behaviour positioned at that hub.
54. or, internally non-deterministically,
55. the vehicle "disappears — off the radar" !


```
51.        veh(vi)(v)(vp:onL(fhi,li,f,thi)) ≡
53(a).          vm_ch[vi]!vp ; veh(vi)(v)(vp)
53(b).       ⌈⌉
53(c).          if f + δ<1
53((c))i.          then vm_ch[vi]!onL(fhi,li,f+δ,thi) ;
53((c))i.               veh(vi)(v)(onL(fhi,li,f+δ,thi))
53((c))ii.         else let li′:LI•li′ ∈ mereo_H(get_H(thi)(n)) in
53((c))iiA.             vm_ch[vi]!atH(li,thi,li′);
53((c))iiB.             veh(vi)(v)(atH(li,thi,li′)) end end
54.          ⌈⌉
55.          stop
```


8.4.4.10 The Monitor Behaviour

56. The monitor behaviour evolves around the attributes of an own "state", m:M, a table of traces of vehicle positions, while accepting messages about vehicle positions and otherwise progressing "in[de]finitely".
57. Either the monitor "does own work"
58. or, internally non-deterministically accepts messages from vehicles.
   (a) A vehicle position message, vp, may arrive from the vehicle identified by vi.
   (b) That message is appended to that vehicle's movement trace,
   (c) whereupon the monitor resumes its behaviour —
   (d) where the communicating vehicles range over all identified vehicles.


```
56.     mon(m)(rtf) ≡
57.          mon(own_mon_work(m))(rtf)
58.       ⌈⌉
58(a).       ⌈⌉ { let ((vi,vp),t) = (vm_ch[vi]?,clkm_ch?), in
58(b).            let rtf′ = rtf † [t ↦ rtf(max dom rtf) † [vi ↦ vp]] in
58(c).            mon(m)(rtf′) end
58(d).            end | vi:VI · vi ∈ vis }

57.     own_mon_work: M → TBL → M
```

- We do not describe the clock behaviour by other than stating that it continually offers the current time on channel clkm_ch. ■

8.4.5. A Model of Parts and Behaviours

- How often have you not "confused"
  - the perdurant notion of a train process: progressing from railway station to railway station,
  - with the endurant notion of the train, say as it appears listed in a train time table, or as it is being serviced in workshops, etc.
- There is a reason for that — as we shall now see:
  parts may be considered syntactic quantities
  denoting semantic quantities.
  - We therefore describe a general model of parts of domains
  - and we show that for each instance of such a model
  - we can 'compile' that instance into a CSP'program'.

A Model of Parts

59. The *whole* contains a set of *parts*.
60. *Parts* are either *atomic* or *composite*.
61. From *composite parts* one can observe a set of *parts*.
62. All *parts* have *unique identifiers*

type
59. W, P, A, C
60. P = A | C
value
61. obs_Ps: $(W|C) \to$ P-**set**

type
62. PI
value
62. uid_Π: $P \to \Pi$

63. From a *whole* and from any *part* of that *whole* we can e**xtr**act all contained *parts*.
64. Similarly one can e**xtr**act the *unique identifiers* of all those contained *parts*.
65. Each part may have a *mereology* which may be "empty".
66. A *mereology*'s *unique part identifiers* must refer to some other parts other than the part itself.

value
63. xtr_Ps: $(W|P) \to$ P-**set**
63. xtr_Ps(w) ≡
63. $\{$xtr_Ps(p)$|$p:P·p $\in$ obs_Ps(p)$\}$
63. **pre**: is_W(p)
63. xtr_Ps(p) ≡
63. $\{$xtr_Ps(p)$|$p:C·p$\in$ obs_Ps(p)$\}\cup\{$p$\}$
63. **pre**: is_P(p)
64. xtr_Πs: $(W|P) \to \Pi$-**set**
64. xtr_Πs(wop) ≡
64. $\{$uid_P(p)$|$p $\in$ xtr_Ps(wop)$\}$
65. mereo_P: $P \to \Pi$-**set**
axiom
66. $\forall$ w:W
66. **let** ps = xtr_Ps(w) **in**
66. $\forall$ p:P · p $\in$ ps ·
66. $\forall\ \pi:\Pi \cdot \pi \in$ mereo_P(p) $\Rightarrow$
66. $\pi \in$ xtr_Πs(p) **end**

67. An attribute map of a *part* associates with *attribute names*, i.e., *type names*, their *values*, whatever they are.
68. From a *part* one can extract its attribute map.
69. Two *parts share attributes* if their respective attribute maps share *attribute names*.
70. Two *parts share properties* if the y
    (a) either *share attributes*
    (b) or the *unique identifier* of one is in the *mereology* of the other.

type
67. AttrNm, AttrVAL,
67. AttrMap = AttrNm $\overrightarrow{m}$ AttrVAL
value
68. attr_AttrMap: $P \to$ AttrMap
69. share_Attributes: $P \times P \to$ **Bool**
69. share_Attributes(p,p′) ≡
69. **dom** attr_AttrMap(p) $\cap$
69. **dom** attr_AttrMap(p′) $\neq \{\}$
70. share_Properties: $P \times P \to$ **Bool**
70. share_Properties(p,p′) ≡
70(a). share_Attributes(p,p′)
70(b). $\vee$ uid_P(p) $\in$ mereo_P(p′)
70(b). $\vee$ uid_P(p′) $\in$ mereo_P(p)

Conversion of Parts into CSP Programs

71. We can define the set of two element sets of *unique identifiers* where
    - one of these is a *unique part identifier* and
    - the other is in the mereology of some other *part*.
    - We shall call such two element "pairs" of *unique identifiers* connectors.
    - That is, a connector is a two element set, i.e., "pairs", of *unique identifiers*
      - for which the identified parts share properties.
72. Let there be given a 'whole', w:W.
73. To every such "pair" of *unique identifiers* we associate a *channel*
    - or rather a position in a matrix of *channels* indexed over the "pair sets" of *unique identifiers*.
    - and communicating messages m:M.

type
71. K = Π-**set** **axiom** ∀ k:K•**card** k=2
value
71. xtr_Ks: (W|P) → K-**set**
71. xtr_Ks(wop) ≡
71. **let** ps = xtr_Ps(w) **in**
71. {{uid_P(p),π}|p:P,π:Π•p∈ ps
71. ∧ ∃ p′:P•p′≠p∧π=uid_P(p′)
71. ∧ uid_P(p)∈uid_P(p′)} **end**
72. w:W
73. **channel** {ch[k]|k:xtr_Ks(w)}:M

74. Now the 'whole' *behaviour* whole is the parallel composition of *part processes*, one for each of the immediate parts of the *whole*.
75. A *part process* is
    (a) either an *atomic part process*, atom, if the *part* is an *atomic part*,
    (b) or it is a *composite part process*, comp, if the *part* is a *composite part*.

74. whole: W → **Unit**
74. whole(w) ≡
74. ‖ {part(uid_P(p))(p) |
74. p:P•p ∈ xtr_Ps(w)}

75. part: π:Π → P → **Unit**
75. part(π)(p) ≡
75(b). is_A(p) → atom(π)(p),
75(b). _ → comp(π)(p)

76. A *composite process*, part, consists of
    (a) a *composite core process*, comp_core, and
    (b) the parallel composition of *part processes* one for each *contained part* of part.
77. An *atomic process* consists of just an *atomic core process*, *atom_core*.

value
76. comp: π:Π → p:P →
76. **in**,**out** {ch[{π,π′}|{π′∈ mereo_P(p)}]}
76. **Unit**
76. comp(π)(p) ≡
76(a). comp_core(π)(p) ‖
76(b). ‖ {part(uid_P(p′))(p′) |
76(b). p′:P•p′ ∈ obs_Ps(p)}
77. atom: π:Π → p:P →
77. **in**,**out** {ch[{π,π′}|{π′∈ mereo_P(p)}]}
77. **Unit**
77. atom(π)(p) ≡ atom_core(π)(p)

78. The core behaviours both
    (a) update the part properties and
    (b) recurses with the updated properties,
    (c) without changing the part identification.

We leave the update action undefined.

value
78. core: π:Π → p:P →
78. **in**,**out** {ch[{π,π′}|{π′∈ mereo_P(p)}]}
78. **Unit**
78. core(π)(p) ≡
78(a). **let** p′ = update(π)(p)
78(b). **in** core(π)(p′) **end**
78(b). **assert:** uid_P(p)=π=uid_P(p′)

- The model of parts can be said to be a syntactic model.
  - No meaning was "attached" to parts.
- The conversion of parts into CSP programs can be said to be a semantic model of parts,
  - one which to every part associates a behaviour
  - which evolves "around" a state
  - which is that of the properties of the part.

8.4.6. Sharing Properties ≡ Mutual Mereologies

- In the model of the tight relationship between parts and behaviours
  - we "equated" two-element set of unique identifiers of parts that share properties
  - with the concept of connectors, and these again with channels.
- We need secure that this relationship,
  - between the two-element connector sets of unique identifiers of parts that share properties
  - and the channels

  with the following theorem:

79. For every *whole*, i.e., domain,
80. if two distinct *parts* share properties
81. then their respective mereologies refer to one another,
82. and vice-versa
    - if two distinct *parts*
    - have their respective mereologies refer to one another,
    - then they share properties.

theorem:

79. $\forall$ w:W,p,p′:P•p≠p′∧{p,p′}⊆xtr_Ps(w) ⇒
80. share_Properties(p,p′)
82. ≡
81. uid_P(p)∈mereo_P(p′)∧uid_P(p′)∈mereo_P(p)

8.4.7. Behaviour Signatures

- By a behaviour signature we shall understand the combination of three clauses:
  - a message type clause,
    - **type** M,
  - possibly a channel index type clause,
    - **type** Idx,
  - a channel declaration clause
    - **channel** ch:M or
      channel {ch[i]|i:Idx•i ∈is}:M

    where is is a set of Idx values (defined somehow, e.g., **value** is:Idx-**set** = ... where ... is an expression of Idx values), and, finally,
  - a behaviour function signature:
    - **value** beh: Π → P → **out** ch **Unit** or
      value beh: Π → P → **out** ch **Unit** or
      value beh: Π → P → **in, out** ch **Unit** or
      value beh: Π → P → **in, out** {ch[i]|i:Idx• ∈is′} **Unit** or
      value beh: Π → P → **in** {ch[i]|i:Idx• ∈is′} **out** {ch[j]|j:Idx• ∈is′} **Unit**,
      etc.

- The Conversion of Parts into CSP Programs "story" gives the general idea:
  - To associate, in principle, with every part an own behaviour.
  - (Example **??** (Slides **??**–**??**) did not do that:
    - in principle it did, but then it omitted describing
    - behaviours of "un-interesting" parts!)
  - Tentatively each behaviour signature, that is, each part behaviour, is
    - specified having a unique identifier type, respectively
    - given a unique identifier argument.

    Whether this tentative provision
    - for unique identifiers is necessary
    - will soon be revealed by further domain analysis.

- Before defining the behaviour process signatures
  - the domain analyser examines each of the chosen behaviours
  - with respect to its interaction with other chosen behaviours
  - in order to decide on
    - interaction message types and
    - "dimensionality" of channels,
    - whether singular or an array.
- Then the
  - message types can be *defined*,
  - the channels *declared*, and
  - the behaviour function signature can be *defined*,

  i.e., the full behaviour signature can be *defined*.

8.4.8. Behaviour Definitions

- We observe from the 'Conversion of Parts into CSP Programs' section, Slide 210,
  - that the "generation" of the core processes was syntax directed,
  - yet "delivered" a "flat" structure of parallel processes,
  - that is, no processes "running", *embedded*, within other processes.
- We make this remark since parts did not follow that prescription:
  - parts can, indeed, be *embedded* within one another.

- So our first "conclusion"[26], with respect to the structure of domain behaviours, is
  - that we shall model all behaviours of the "whole" domain
  - as a flat structure of concurrent behaviours —
    - one for each part contained in the whole —
  - which, when they need refer to properties of
  - behaviours of parts within which the part
    - on which "their" behaviour

    is embedded
  - then they interact with the behaviours of those parts,
  - that is, communicate messages.

[26] We put double quotes around the term 'conclusion' (above) since that conclusion was and is a choice, that is, not governed by necessity.

- The ‘Conversion of Parts into CSP Programs’ section, Slide 210,
  - then suggested that there be
    - one **atom core** behaviour for each **atomic part**, and
    - one **composite core** behaviour for each **composite part**

    of the domain.
- The **domain analyser** may find that some of these **core behaviour**s
  - are not necessary,
  - that is, that they — for the chosen scope of the domain model —
  - do not play a meaningful rôle.

Example: 37 “Redundant” Core Behaviours. We refer to the series of examples around the transport net domain.

- Transport nets, n:N, consist of
  - sets, hs:HS, of hubs and
  - sets, ls:LS, of links.
- Yet we may decide, for one domain scope,
  - to model only
    - hub,
    - link and
    - vehicle

    behaviours,
- and not ‘set of hubs’ and ‘set of links’ behaviours. ■

- Then the **domain analyser** can focus on exploring each individual **process behaviour**.
- Again the **Conversion of Parts into CSP Programs** “story” gives the general ideas that motivate the following:
- For each of the **part**s, p,
  a **behaviour expression** can be “generated”:
  - beh_p(uid_P(p))(p).

  The idea is
  - that (uid_P(p)) uniquely identifies the **part behaviour** and
  - that the **part properties** of (p) serve as the **local state** for beh_p.

- Now we present an **analysis** of **part behaviour**s around three ‘alternatives’:
  - (i) a **part behaviour** which basically represents a **proactive behaviour**;
  - (ii) one which basically represents a **reactive behaviour**; and
  - (iii) one which, so-to-speak alternates between **proactive** and **reactive behaviours.**
- What we are doing now is to examine
  - the form of the **core behaviour**s,
  - cf. Item 78 (Slide 213).

- (i) A proactive behaviour is characterised by three facets.
  - (i.1) taking the initiative to interact with other part behaviours by offering output,
  - (i.2) internally non-deterministically ($\sqcap$) ranging interactions over several alternatives, and
  - (i.3) externally non-deterministically ($\square$) selecting which other behaviour to interact with, i.e., to offer output to.
- (i.1) A proactive behaviour takes the initiative to interact by expressing output clauses:

83. $\mathcal{O}_P$: ch ! val or ch[i] ! val or ch[i,j] ! val etc.

- (i.2) The proactive behaviour interaction request
  - may range over either of a finite number of alternatives,
  - one for each alternative, a_i, "kind" of interaction.
  - We may express such a non-deterministic (alternative) choice *either* as follows:

84. $\mathcal{NI}_P$: **type** Choice = $a_1 \sqcap a_2 \sqcap \ldots \sqcap a_n$
value let c:Choice **in**
case c **of** $a_1 \rightarrow \mathcal{E}_1$, $a_2 \rightarrow \mathcal{E}_2$, ..., $a_n \rightarrow \mathcal{E}_n$ **end end**

  - *or*, which is basically the same,

85. $\mathcal{NI}_P$: **value** ... $\mathcal{E}_1 \sqcap \ldots \sqcap \mathcal{E}_n$ ...

  - where each $\mathcal{E}_i$ usually contains an input clause, for example, ch ?.

- (i.3) The proactive external non-deterministic choice is directed at either of a number of other part behaviours.
  - This proactive selection is expressed

86. $\mathcal{NX}_P$: $\mathcal{C}_i \;\square\; \mathcal{C}_j \;\square \ldots \square\; \mathcal{C}_k$

    - where each of the $\mathcal{C}$lauses
    - express respective output clauses
    - (usually) directed at different part behaviours,
    - say ch[i] ! val. ch[j] ! val, etc., ch[k] ! val.
  - Another way of expressing external non-deterministic choice selection is

87. $\mathcal{NX}_P$: $\square$ { ...; ch[i] ! fct(i) ; ... | i:Idx·i ∈ is }

- $\mathcal{O}$utput clauses [(i.1)], Item 84 $\mathcal{O}_P$,
  - may [(i.2)] occur in the $\mathcal{E}_i$ clauses of $\mathcal{NI}_P$, Items 85 and 86 and
  - must [(i.3)] occur in each of the $\mathcal{C}_i$ clauses of $\mathcal{NX}_P$, Item 87.

- (ii) A reactive behaviour is characterised by three
  - (ii.1) offering to interact with other part behaviours by offering to accept input,
  - (ii.2) internally non-deterministically ($\sqcap$) ranging interactions over several alternatives, and
  - (ii.3) externally non-deterministically ($\square$) selecting which other behaviour to interact with, i.e., to accept input from.
- (ii.1) A reactive behaviour expresses input clauses:

88. $\mathcal{I}_R$: ch ? or ch[i] ? or ch[i,j] ? etc.

- (ii.2) The **reactive behaviour**
  - may range over either of a finite number of alternatives,
  - one for each alternative, a_i, "kind" of interaction.
  - We may express such a non-deterministic (alternative) choice *either* as follows:

    89. $\mathcal{NI}_R$: **value let** c:Choice **in**
    case c **of** $a_1 \rightarrow \mathcal{E}_1$, ..., $a_n \rightarrow \mathcal{E}_n$ **end end**

    where each of the expressions, $\mathcal{E}_i$, may, and usually contains a input clause ($\mathcal{I}$, Item 88 on the preceding page).
  - Thus the $\mathcal{NI}_R$ clause is almost identical to the $\mathcal{NI}_P$ clause, Item 85 on page 227.
  - Hence another way of expressing external non-deterministic choice is

    90. $\mathcal{NX}_R$: $\lceil\rceil$ { ...; ch[i] ! fct(i) ; ... | i:Idx·i ∈ is }.

- (ii.3) The **reactive behaviour** selection is directed at either of a number of other **part behaviours**.
  - This **external non-deterministic choice** is expressed

    91. $\mathcal{NX}_R$: $\mathcal{C}_i$ $[]$ $\mathcal{C}_j$ $[]$ ... $[]$ $\mathcal{C}_k$
    - where each of the $\mathcal{C}$lauses
    - express respective input clauses
    - (usually) directed at different **part behaviours**,
    - say ch[i] ?. ch[j] ?, etc., ch[k] ?.
  - Another way of expressing **external non-deterministic choice** selection is

    92. $\mathcal{NX}_R$: $[]$ { ...; ch[i] ? ; ... | i:Idx·i ∈ is }
  - Thus the $\mathcal{NX}_R$ clauses are almost identical to the $\mathcal{NX}_P$ clauses, Items 86–87.

- $\mathcal{I}$nput clauses [(ii.1)], Item 88 $\mathcal{I}_R$,
  - may [(ii.2)] occur in the $\mathcal{E}_i$ clauses of $\mathcal{NI}_R$, Items 89–90 and
  - must [(ii.3)] occur in each of the $\mathcal{C}_i$ clauses of $\mathcal{NX}_R$, Items 91–92.

- (iii) An alternating **proactive behaviour** and **reactive behaviour**
  - is characterised by expressing both
    - **reactive behaviour** and
    - **proactive behaviours**

    combined by either
    - **non-deterministic internal choice** ($\lceil\rceil$) or
    - **non-deterministic external choice** ($[]$) combinators.

    For example:

    93. $(\mathcal{NI}_{P_i}[\lceil\rceil \text{or} []]\mathcal{NX}_{P_j})[\lceil\rceil \text{or} []](\mathcal{NI}_{R_k}[\lceil\rceil \text{or} []]\mathcal{NX}_{R_\ell})$.
- The meta-clause $[\lceil\rceil \text{or} []]$ stands for either $\lceil\rceil$ or $[]$.
- Here there usually is a disciplined use of input/output clauses.

Example: 38 A Pipeline System Behaviour.

- We refer to Examples
  - 15 (Slide 94) and
  - 22–24 (Slides
  - 121–129)
  - and especially Examples 25–26 (Slides 131–135).

- We consider (cf. Example 23) the pipeline system units to represent also the following behaviours:
  - pls:PLS, Item 4(a) on page 123, to also represent the system process, pipeline_system, and for each kind of unit, cf. Example 15, there are the unit processes:
    - unit,
    - well (Item 3(c) on page 95),
    - pipe (Item 3(a)),
    - pump (Item 3(a)),
    - valve (Item 3(a)),
    - fork (Item 3(b)),
    - join (Item 3(b)) and
    - sink (Item 3(d) on page 95).


```
channel
   { pls_u_ch[ ui ]:ui:UI•i ∈ UIs(pls) } MUPLS
   { u_u_ch[ ui,uj ]:ui,uj:UI•{ui,uj}⊆UIs(pls) } MUU
type
   MUPLS, MUU
value
   pipeline_system: PLS → in,out { pls_u_ch[ ui ]:ui:UI•i ∈ UIs(pls) }  Unit
   pipeline_system(pls) ≡ ‖ { unit(u)|u:U•u ∈ obs_Us(pls) }
   unit: U → Unit
   unit(u) ≡
3(c).    is_We(u) → well(uid_U(u))(u),
3(a).    is_Pu(u) → pump(uid_U(u))(u),
3(a).    is_Pi(u) → pipe(uid_U(u))(u),
3(a).    is_Va(u) → valve(uid_U(u))(u),
3(b).    is_Fo(u) → fork(uid_U(u))(u),
3(b).    is_Jo(u) → join(uid_U(u))(u),
3(d).    is_Si(u) → sink(uid_U(u))(u)
```


- We illustrate essentials of just one of these behaviours.

```
3(b).  fork: ui:UI → u:U → out,in pls_u_ch[ ui ],
                 in  { u_u_ch[ iui,ui ] | iui:UI • iui ∈ sel_UIs_in(u) }
                 out { u_u_ch[ ui,oui ] | iui:UI • oui ∈ sel_UIs_out(u) }  Unit
3(b).  fork(ui)(u) ≡
3(b).     let u′ = core_fork_behaviour(ui)(u) in
3(b).     fork(ui)(u′) end
```

- The core_fork_behaviour(ui)(u) distributes
  - what oil (or gas) in receives,
    - on the one input sel_UIs_in(u) = {iui},
    - along channel u_u_ch[iui]
  - to its two outlets
    - sel_UIs_out(u) = $\{oui_1, oui_2\}$,
    - along channels u_u_ch[oui_1], u_u_ch[oui_2].

- The core_fork_behaviour(ui)(u) also communicates with the pipeline_system behaviour.
  - What we have in mind here is to model a traditional supervisory control and data acquisition, SCADA system.

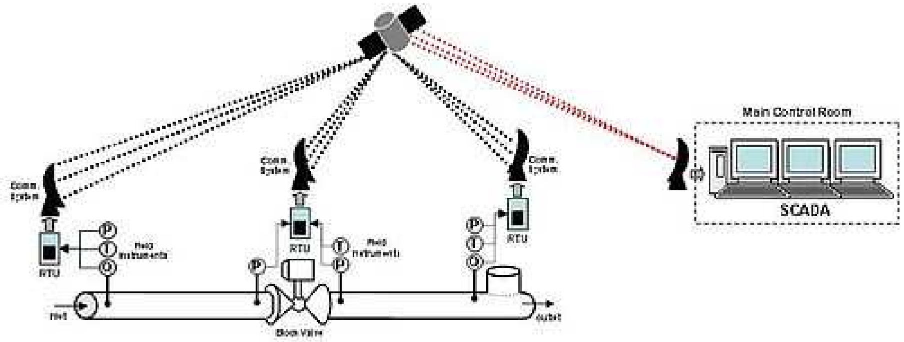


Figure 1: A supervisory control and data acquisition system

- SCADA is then part of the pipeline_system behaviour.

94.

94. pipeline_system: PLS → **in**,**out** { pls_u_ch[ui]:ui:UI·i ∈ UIs(pls) } **Unit**
94. pipeline_system(pls) ≡ scada(props(pls)) ∥ ∥{ unit(u)|u:U·u ∈ obs_Us(pl

- props was defined on Slide 144.

95. scada non-deterministically (internal choice, ⊓), alternates between continually
  (a) doing own work,
  (b) acquiring data from pipeline units, and
  (c) controlling selected such units.

type
95. Props
value
95. scada: Props → **in**,**out** { pls_ui_ch[ui] | ui:UI·ui ∈ ∈ uis } **Unit**
95. scada(props) ≡
95(a). scada(scada_own_work(props))
95(b). ⊓ scada(scada_data_acqui_work(props))
95(c). ⊓ scada(scada_control_work(props))

- We leave it to the listeners imagination to describe scada_own_work.

96. The scada_data_acqui_work
  (a) non-deterministically, external choice, ⫿, offers to accept data,
  (b) and scada_input_updates the scada state —
  (c) from any of the pipeline units.

value
96. scada_data_acqui_work: Props → **in**,**out** { pls_ui_ch[ui] | ui:UI·ui ∈ ∈ u
96. scada_data_acqui_work(props) ≡
96(a). ⫿ { **let** (ui,data) = pls_ui_ch[ui] ? **in**
96(b). scada_input_update(ui,data)(props) **end**
96(c). | ui:UI · ui ∈ uis }

96(b). scada_input_update: UI × Data → Props → Props
type
96(a). Data

97. The scada_control_work

(a) analyses the scada state (props) thereby selecting a pipeline unit, ui, and the controls, ctrl, that it should be subjected to;
(b) informs the units of this control, and
(c) scada_output_updates the scada state.

```
97.    scada_control_work: Props → in,out { pls_ui_ch[ui] | ui:UI•ui ∈ ∈ uis }
97.    scada_control_work(props) ≡
97(a).     let (ui,ctrl) = analyse_scada(ui,props) in
97(b).     pls_ui_ch[ui] ! ctrl ;
97(c).     scada_output_update(ui,ctrl)(props) end

97(c).  scada_output_update UI × Ctrl → Props → Props
type
97(a).  Ctrl
```


Modelling Behaviours, I/II

- The domain describer has decided that an entity is a perdurant and is, or represents a behaviour.
  - The domain describer has further decided that the observed behaviour is of a class of behaviours — of the “same kind” — that need be described.
  - By behaviours of the ‘same kind’ is meant that these can be described by the same channel declarations, function signature and function definition.

Modelling Behaviours, II/II

- First the domain describer must decide on the underlying function signature.
  - It must be decided which synchronisation and communication
    - inputs and
    - outputs

    this behaviour requires, i.e., the **in,out** clause of the signature,
  - that also includes the “discovery” of necessary channel declarations.
- Finally the function definition must be decided upon.

9. Continuous Perdurants

- By a continuous perdurant we shall understand a continuous behaviour.
- This section serves two purposes:
  - to point out that believable system descriptions must entail both
    - a discrete phenomena domain description and
    - a continuous phenomena mathematical model.
  - and this poses some semantics problems:
    - the formal semantics of the discrete phenomena description language and
    - the meta-mathematics of, for example, differential equations,

    at least as of today, August 10, 2012, are not commensurable!
  - That is, we have a problem — as will be outlined later in this lecture.

9.1. Some Examples

Example: 39 Continuous Behaviour: The Weather. We give a familiar example of continuous behaviour.

- The *weather* — understood as the time-wise evolution of a number of attributes of the *weather* material:
  - *temperature*,
  - *wind direction*,
  - *wind force*,
  - *atmospheric pressure*,
  - *humidity*,
  - *sky formation* (`clear, cloudy, ...`),
  - *precipitation*,
  - etcetera.
- That is, *weather* is seen as the state of the *atmosphere* as it evolves over time.■

Example: 40 Continuous Behaviour: Road Traffic. We give another familiar example of continuous behaviour.

- The *automobile traffic* is the time-wise evolution of cars along a net has the following additional attributes:
  - *car identity* (CI),
  - *position* (P, on the net),
  - *direction* (D),
  - *velocity* (V),
  - *acceleration* (A),
  - etcetera (...).
- The equation below captures this:

$$\mathsf{TF} = \mathsf{T} \rightarrow (\mathsf{CI} \xrightarrow{m} (\mathsf{P}\times\mathsf{D}\times\mathsf{V}\times\mathsf{A}\times...))$$

- We refer to Example **??**
  - specifically the veh, hub and mon behaviours.
  - These "mimic" a discretised version of the above:

$$\mathsf{TF} = \mathsf{T} \xrightarrow{m} (\mathsf{CI} \xrightarrow{m} (\mathsf{P}\times\mathsf{D}\times\mathsf{V}\times\mathsf{A}\times...))$$

■

Example: 41 Pipeline Flows. A last example of continuous behaviour.

- We refer to Examples 13, 15, 22–26, 41–45 and 49.
- These examples focused on
  - the atomicparts and the composite parts of pipelines,
  - and dealt with the liquid or gas materials as they related to pipeline units.
- In the present example we shall focus on
  - the overall material flow "across" a pipeline.
  - in particular the continuity as
  - as contrasted with the pipeline unit discrete
  - aspects of flow.

- Which, then, are these pipeline system continuity concerns ?
  - In general we are interested in
    1. *whether the flow is laminar or turbulent:*
       (a) *within a unit, or*
       (b) *within an entire, possibly intricately networked pipeline*;
    2. *what the shear stresses are*;
    3. *whether there are undesirable pressures*;
    4. *whether there are leaks above normal values*;

    etcetera.
- To answer questions like those posed in
  - Items 1(a) and 2, we need not build up the models sketched in Examples 13, 15, 25, 26, 41–45 and 49.
  - But for questions like those posed in Items 1(b), 3 and 4 we need such models.

- To answer any of the above questions, and many others, we need establish, in the case of pipelines, *fluid dynamics models* [Batchelor1967,Thorley1991,Wendt1992,Coulbeck2010].
- These models involve such mathematical as are based, for example, on
  - *Newtonian Fluid Behaviours*,
  - *Bernoulli Equations*,
  - *Navier–Stokes Equations*,
  - etcetera.
- Each of these mathematical models
  - capture the dynamics of one specific pipeline unit,
  - not assemblies of two or more. ■

9.2. Two Kinds of Continuous System Models

- There are at least two different kinds of mathematical models for continuous systems.
  - There are the models which are based on physics models mentioned above, for example
    - the dynamics of flows in networks,
  - and there are the models which builds on control theory to express automatic control solutions to the monitoring & control of pipelines, for example:
    - the opening, closing and setting of pumps, and
    - the opening, closing and setting of valves

    depending on monitored values of dynamic well, pipe, pump, valve, fork, join and sink attributes.

- Example 41 on page 248 assumes
  - the fluid mechanics domain models
  - to complement the discrete domain model of Example 38 on page 234,

  whereas
- Example 44 on page 271
  - builds on Examples 41 and 38
  - but assumes that automatic monitoring & control requirements prescriptions
  - have been derived, in the usual way from the former fluid mechanics domain models.

9.3. Motivation for Consolidated Models

- By a consolidated model
  - we shall understand a formal description
  - that brings together both
    - discrete
      - for example `TripTych` style domain description

    and
    - continuous
      - for example classical mathematical description
  - models of a system.

- We shall **motivate** the need for consolidated models, that is **for building both**
  - the novel **domain descriptions,**
    - such as this tutorial suggests,
    - with its many aspects of discreteness,

    and the
  - the **classical mathematical models**,
    - as this section suggests,
    - including, for example, as in the case of Example 41, fluid dynamics mathematics.

- This motivation really provides the justification for bringing the two disciplines together:
  - discrete system domain modelling with
  - continuous system physics modelling

  in this tutorial.

- The classical mathematical models of, for example, pipelines,
  - model physical phenomena within parts or within materials;
  - and also combinations of *neighbouring*,
    - parts with parts and
    - parts with materials.
  - But classical mathematical modelling
    - cannot model continuous phenomena
    - for other than definite concrete, specific combinations of parts and/or materials.

- The kind of domain modelling,
  - that is brought forward in this tutorial can,
  - within one domain description
  - model a whole class,
  - indeed an indefinite,
  - class of systems.

9.4. Generation of Consolidated Models

- The idea is therefore this
  - create a domain description
    for a whole, the indefinite class of "alike" systems, to wit
    - for an indefinite class of pipelines,
    - for an indefinite class of container lines,
    - for an indefinite class of health care systems,
  - and then "adorn" such a description
    - first with classical mathematical models
      of simple parts of such systems; and
    - then "replicate" these mathematical models across the
      indefinite class of discrete models
    - by "pairing"
      - each definite classical concrete mathematical model
      - with an, albeit abstract general discrete model.

9.4.1. The Pairing Process

- The "pairing process" depends on a notion of boundary condition.
  - The boundary conditions for mereology-related parts are, yes,
    - expressed by their mereology,
    - that is, by how the parts fit together.
  - The boundary conditions for continuous models are understood as
    - the set of conditions specified for the solution
    - to a set of differential equations at the boundary
      between the parts being individually modelled.

- In pairing we take the "cue", i.e., directives, from
  - the discrete domain model
    for the generic part and its related material
  - since it is the more general, and
  - "match" its mereology with
  - the continuous mathematics model
    of a part and its related material

9.4.2. Matching

- Matching now means the following.
  - Let $\mathcal{D}_{\mathsf{P,M}}$
    - designate a $\mathcal{D}$omain $\mathcal{D}$escription
    - for a part and/or a material, of type P, respectively M,
    - zero or one part type and zero or one material type(s).
  - Let $\mathcal{M}_{\mathsf{P,M}}$
    - designate a $\mathcal{M}$athematical $\mathcal{M}$odel
    - for a part and/or a material of type P, respectively M,
    - zero or one part type and zero or one material type(s).

Example: 42 **A Transport Behaviour Consolidation.**

- An example $\mathcal{D}_{\mathsf{P,M}}$ could be
  - the one, for vehicles, shown in Example ?? (Slides ??–205)
  - as specifically expressed in the two frames:
    - 'The Vehicle Behaviour at Hubs' on Slide 201 and
    - 'The Vehicle Behaviour along Links' on Slide 203.
- On Slide 201 of Example ?? notice vehicle vi movement at hub in formula line
  - 52(a) — apparently not showing any movement and
  - 52((b))iii — showing movement from hub onto link.
- On Slide 203 notice vehicle vi movements along link in formula lines
  - 53(a) — no movement (stopped or parked),
  - 53((c))i — incremental movement along link, and
  - 53((c))iiB — movement from link into hub.

- The corresponding example $\mathcal{M}_{\mathsf{P,M}}$ might then be
  - modelling these movements and no movements
  - requiring access to such attributes as
    - link length,
    - vehicle velocity,
    - vehicle position,
    - vehicle acceleration,

  etcetera.
- This model would need to abstract the non-deterministic behaviour of the driver:
  - accelerating,
  - decelerating or
  - steady velocity.
- Example ??'s model of vehicles' link position in terms of a fragment (δ) can be expected to appear in $\mathcal{M}_{\mathsf{P,M}}$ as an x, viewing the link as an x-axis. ■

Example: 43 **A Pipeline Behaviour Consolidation.** We continue the line of exemplifying formalisations of pipelines, cf. Examples 15 (Slide 94) and 22–24 (Slides 121–129) and especially Examples 25–26 (Slides 131–135).

- Let the $\mathcal{D}_{\mathsf{P,M}}$ model be focused on the flows and leaks of pipeline units, cf. Examples 25 and 26.
- The $\mathcal{M}_{\mathsf{P,M}}$ model would then $\mathcal{M}$athematically model the fluid dynamics of the pipeline material per pipeline unit: flow and part actions and reactions for any of the corresponding $\mathcal{D}$omain models:
  - *wells*, $\mathcal{D}^{\mathsf{well}}_{\mathsf{U,O}} \rightarrow \mathcal{M}^{\mathsf{well}}_{\mathsf{U,O}}$,
  - *pipes*, $\mathcal{D}^{\mathsf{pipe}}_{\mathsf{U,O}} \rightarrow \mathcal{M}^{\mathsf{pipe}}_{\mathsf{U,O}}$,
  - *pumps*, $\mathcal{D}^{\mathsf{pump}}_{\mathsf{U,O}} \rightarrow \mathcal{M}^{\mathsf{pump}}_{\mathsf{U,O}}$,
  - *valves*, $\mathcal{D}^{\mathsf{valve}}_{\mathsf{U,O}} \rightarrow \mathcal{M}^{\mathsf{valve}}_{\mathsf{U,O}}$,
  - *forks*, $\mathcal{D}^{\mathsf{fork}}_{\mathsf{U,O}} \rightarrow \mathcal{M}^{\mathsf{fork}}_{\mathsf{U,O}}$,
  - *joins*, $\mathcal{D}^{\mathsf{join}}_{\mathsf{U,O}} \rightarrow \mathcal{M}^{\mathsf{join}}_{\mathsf{U,O}}$, and
  - *sinks* $\mathcal{D}^{\mathsf{sink}}_{\mathsf{U,O}} \rightarrow \mathcal{M}^{\mathsf{sink}}_{\mathsf{U,O}}$. ■

- Some more model annotations,
  - reflecting the match between $\mathcal{D}_{\mathsf{P,M}}$ and $\mathcal{M}_{\mathsf{P,M}}$,

  seem relevant.
  - Thus we further subscript $\mathcal{D}_{\mathsf{P,M}}$ optionally with
    - a unique identifier variable, π, and
    - the properties $p_i, p_j, ..., p_k$ where
      - p_i is a property name of part type P or of material type M,
      - and where these property names typically are the distinct attribute names of P and/or M,

    to arrive at $\mathcal{D}^{\pi}_{\mathsf{P,M}_{p_i,p_j,...,p_k}}$.
  - Here π is a variable name for p:P, i.e., π is uid_P(p).
  - Do not confuse property names, p_i etc., with part names, p.

- And we likewise adorn $\mathcal{M}_{\mathsf{P,M}}$ optionally with
  - superscripts $p_i, p_j, ..., p_k$ and
  - subscripts $x_i, x_j, ..., x_k$ where
    - $p_i, p_j, ..., p_k$ are as for $\mathcal{D}^{\pi}_{\mathsf{P,M}_{p_i,p_j,...,p_k}}$ and
    - $x_i, x_j, ..., x_k$ are the names of the variables occurring in $\mathcal{M}_{\mathsf{P,M}}$
      - possibly in its partial differential equations,
      - possibly in its difference equations,
      - possibly in its other mathematical expressions of the $\mathcal{M}_{\mathsf{P,M}}$ model.

  to arrive at $\mathcal{M}^{\pi}{}^{p_i,p_j,...,p_k}_{\mathsf{P,M}_{x_i,x_j,...,x_k}}$

- The "adornments" are the result of an analysis which
  - identifies the variables of $\mathcal{M}_{\mathsf{P,M}}$
  - with the properties of $\mathcal{D}_{\mathsf{P,M}}$.
- Common to all conventional mathematical models
  - is that they all operate with a very simple type concept:
    - **Real**s, **Int**egers,
    - arrays (vectors, matrices, and tensors),
    - sets of the above and sets.
- Common to all domain model descriptions
  - is that they all operate with a rather sophisticated type concept:
    - abstract types and concrete types,
    - union $(\mathsf{T}_i|\mathsf{T}_j...)$ of these,
    - sets, Cartesians, lists, maps, and partial functions and total functions over these, etcetera.

9.4.3. Model Instantiation

- The above models, $\mathcal{D}_{\mathsf{P,M}}$ and $\mathcal{M}_{\mathsf{P,M}}$, differ as follows.
  - The $\mathcal{D}_{\mathsf{P,M}}$ models (are claimed to) hold for indefinite sets of domains "of the same kind":
    - The axioms and invariants, cf.
      - Example 12 on page 86,
      - Examples 25–26 (Slides 131–134) and
      - Example 28 on page 140,

      are universally quantified over all transport nets.
- The $\mathcal{M}_{\mathsf{P,M}}$ models express no such logic.

- The above difference can, however, be ameliorated.
  - For a given, that is, an instantiated domain,
    - we can "compile" the $\mathcal{D}_{\mathsf{P,M}}$ models
    - into a set of models,
    - one per part of that domain;
  - similarly, with the binding of model $\mathcal{M}_{\mathsf{P,M}}$ variables to instantiated model $\mathcal{D}_{\mathsf{P,M}}$ attributes,
    - we can "compile" the $\mathcal{M}_{\mathsf{P,M}}$ models
    - into as set of — instantiated $\mathcal{M}_{\mathsf{P,M}}$ models,
    - one per part of that domain.

9.4.3.1 Model Instantiation – in Principle

- Since this **partial evaluation compilation** can be (almost) automated,
  - there is really no reason to actually perform it;
  - all necessary theorems should be derivable from the annotated models.
    - $\mathcal{D}^{\pi}_{\mathsf{P,M}_{p_i,p_j,\ldots,p_k}}$ and
    - $\mathcal{M}^{\pi}_{\mathsf{P,M}^{p_i,p_j,\ldots,p_k}_{x_i,x_j,\ldots,x_k}}$.
- That is, as far as a **domain understanding** concerns
  - we might, with
    - **continuous mathematical modelling** and
    - mostly **discrete domain modelling**
  - very well have achieved all we can possibly, today, achieve.

9.4.3.2 Model Instantiation – in Practice

- We continue Example 38 (Slides 234–242).
  - The definition of **pipeline_system** function (Slide 239) indicates the basis for an instantiation.

Example: 44 An Instantiated Pipeline System.

- Figure 2 indicates an instantiation.

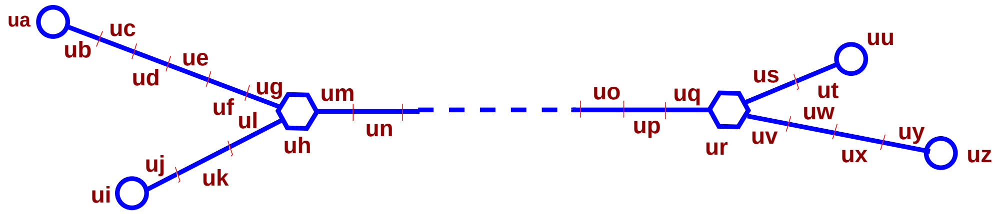


Figure 2: A specific pipeline

- That pipeline system gives rise to the following instantiation.

```
scada(pro)‖
unit(ua)‖unit(ub)‖unit(uc)‖unit(ud)‖unit(ue)‖unit(uf)‖unit(ug)‖
unit(uh)‖
unit(ui)‖unit(uj)‖unit(uk)‖unit(ul)‖
unit(um)‖unit(un)‖...‖unit(uo)‖unit(up)‖unit(uq)‖
unit(ur)‖
unit(us)‖unit(ut)‖unit(uu)‖
unit(uv)‖unit(uw)‖unit(ux)‖unit(uy)‖unit(uz)
```

- It is in the **scada** behaviour, that each of the $\mathcal{M}^{\mathsf{uid\_U(u)}}_{\mathrm{U,O}}$ models are 'instantiated'.
- The above instantiated model
  - is not a domain model of a generic pipeline system
  - but is a requirements model for the monitoring & control of a specific pipeline system. ■

9.5. An Aside on Time

- An important aspect of **domain modelling** is the description of **time phenomena**:
  - **absolute time** (or just **time**) and
  - **time intervals**.
- We shall, regrettably, not cover this facet in this tutorial, but refer to
  - a number of specifications expressed in combined uses of
    - the `RAISE` [RaiseMethod] combined with
    - the `DC: Duration Calculus` [zcc+mrh2002].
  - We could also express these specifications using `TLA+` [Lamport-TLA+02]: Lamport's `Temporal Logic of Actions`.
- We otherwise refer to [TheSEBook2wo] (Chap. 15.).

9.6. A Research Agenda

- This section opens two main lines of research problems;
  - methodology problems cum computing science problems and
  - computer science cum mathematics problems.

9.6.1. Computing Science cum Programming Methodology Problems

- Some of the methodology problems are
  - techniques for developing continuous mathematics models — which we leave to the relevant fields of
    - physics and
    - control theory

    to "deliver";
  - contained in this are more detailed techniques for matching $\mathcal{D}_{\mathsf{D,M}}$ and $\mathcal{M}_{\mathsf{D,M}}$ models,
    - that is, for identifying and pairing the p_is and x_is in
      - $\mathcal{D}^{\pi}_{\mathsf{P,M}_{p_i,p_j,\ldots,p_k}}$ and
      - $\mathcal{M}^{\pi}_{\mathsf{P,M}^{p_i,p_j,\ldots,p_k}_{x_i,x_j,\ldots,x_k}}$

      and
    - for instantiating these.

- A problem of current programming methodology in
  - that it has for most of its "existence"
  - relied on discrete mathematics
  - and not sufficiently educated and trained
  - its candidates in continuous mathematics.

9.6.2. Mathematical Modelling Problems

- Some of the open mathematics problems are
  - the lack of well-understood interfaces between
    - discrete mathematics models and
    - continuous mathematics models;
  - and the lack of proof systems across the two modes of expression.

- By well-understood interfaces between the two modes of expression,
  - the discrete mathematics models and
  - the continuous mathematics models;

  we mean that the semantics models of
  - the discrete mathematics formal specification languages and
  - the continuous mathematics specification notations,

  at this time, August 10, 2012, are not commensurate, that is, do not "carry over":
  - a variable, a of some, even abstract type, say A,
  - cannot easily be related to what it has to be related to, namely
  - a variable, x of some concrete, mathematical type, say **Real** or **Int**eger, or arrays of these, etc.

- Lack of proof systems across the two modes of expression.
  - the discrete mathematics models and
  - the continuous mathematics models;

  we mean,
  - firstly, that the former problem of lack of clear a↔x relations is taken to prevent such proof systems,
  - secondly, that mathematics essentially does not embody a "formal language".
- But nobody is really looking into, that is, researching possible "solutions" to these problems.

10. Discussion of Entities

- We have examined the concepts of entities, endurant and perdurant.
- We have not examined those "things" (of a domain) which "fall outside" this categorisation.
  - That would lead to a rather lengthy discourse.
  - In the interest of "really understanding" what can be described such a computer science study should be made.
  - Philosophers have clarified the issues in centuries of studies.
    - Their interest is in
      - identifying the issues and
      - clarifying the questions.
    - Computer scientists are interested in answers.

- We see entities as either
  - endurants or
  - perdurants

  or as either
  - discrete or
  - continuous.
- We analyse discrete endurants into atomic and composite parts with
  - observers,
  - unique identifiers,
  - mereology and
  - attributes.
- And we analyse perdurants into actions, events and behaviours.

- This **domain ontology** is entirely a pragmatic one:
  - it appears to work;
  - it has been used in the description of numerous cases;
  - it leads to descriptions which in a straightforward manner lend
    - themselves to the "derivation"
    - of significant fragments of requirements;
  - and appears not to stand in the way of obtaining remaining requirements.

- Most convincingly to us is that the concepts of our approach
  - **endurants** and **perdurants**,
  - **atomic** and **composite parts**,
  - **mereology** and **attributes**,
  - **actions**, **events** and **behaviours**

  fit it with major categories of philosophically analyses.

End of Lecture 5: Last Session — Perdurant Entities

Behaviours, Discussion Entities

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

HAVE A GOOD LUNCH – SEE YOU BACK AT 2 PM

HAD A GOOD LUNCH ?

Begin of Lecture 6: First Session — Calculus I

Part and Material Discoverers

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

Tutorial Schedule

11. Towards a Calculus of Domain Discoverers

- The ‘towards’ term is significant.
- We are not presenting
  - a “ready to serve”
  - comprehensive,
  - tested and tried

  calculus.
- We hope that the one we show you is interesting.
- It is, we think, the first time such a calculus is presented.

- By a domain description calculus
  - or, as we shall also call it,
    - either a domain discovery calculus
    - or a calculus of domain discoverers

  we shall understand an algebra, that is,
  - a set of meta-operations and
  - a pair of
    - a fixed domain and
    - a varying repository.
- The meta-operations will be outlined in this section.
- The fixed domain is of the kind of domains alluded to in the previous section of this tutorial.
- The varying repository contains fragments of a description of the fixed domain.

- The meta-operators are referred to as
  - either domain analysis meta-functions
  - or domain discovery meta-functions.
- The former are carried out by the domain analyser when inquiring (the domain) as to its properties.
- The latter are carried out by the domain describer when deciding upon which descriptions "to go for" !
- The two persons can be the same one domain engineer.
- The operators are referred to as meta-functions,
  - or meta-linguistic functions,
  - since they are
    - applied and
    - calculated
  - by humans, i.e., the domain describers.
- They are directives which can be referred to by the domain describers while carrying out their analytic and creative work.

11.1. Introductory Notions

- In order to present the operators of the calculus
  - we must clear a few concepts.

11.1.1. Discovery

- By a domain discovery calculus we shall understand
  - a set of operations (the domain discoverers),
    - which when applied to a domain
    - by a human agent, the domain describer,

    and
    - yield domain description texts.

- The domain discoverers are applied "mentally".
  - That is, not in a mechanisable way.
    - It is not like when procedure calls
    - invoke computations
    - of a computer.
  - But they are applied by the domain describer.
  - That person is to follow the ideas laid down for
  - these domain discoverers
    - (as they were in the earlier parts of this talk).
  - They serve to guide the domain engineer
    - to discoverer the desired domain entities
    - and their properties.
- In this section we shall review an ensemble of (so far) nine domain discoverers and (so far) four domain analysers.

We list the nine domain discoverers.

- [Slide 319] PART_SORTS,
  [Slide 316] MATEREIAL_SORTS,
  [Slide 323] PART_TYPES,
  [Slide 326] UNIQUE_ID,
  [Slide 327] MEREOLOGY,
  [Slide 331] ATTRIBUTES,
  [Slide 340] ACTION_SIGNATURES,
  [Slide 345] EVENT_SIGNATURES and
  [Slide 348] BEHAVIOUR_SIGNATURES.

11.1.2. Analysis

- In order to “apply” these domain discoverers certain conditions must be satisfied.
- Some of these condition inquiries can be represented by (so far) four domain analysers.
  - [Slide 305] IS_MATERIALS_BASED,
    [Slide 307] IS_ATOM,
    [Slide 307] IS_COMPOSITE and
    [Slide 311] HAS_A_CONCRETE_TYPE.

11.1.3. Domain Indexes

- In order to discover, the domain describer must decide on *“where & what in the domain”* to analyse and describe.
- One can, for this purpose, think of the domain as semi-lattice-structured.
  - The root of the lattice is then labelled Δ.
  - Let us refer to the domain as Δ.
  - We say that it has index $\langle\Delta\rangle$.
  - Initially we analyse the usually composite Δ domain to consist of one or more distinctly typed parts p_1:t_1, p_2:t_2, ..., p_m:t_m.
  - Each of these have indexes $\langle\Delta, t_i\rangle$.
  - So we view Δ, in the semi-lattice, to be the join of m sub-semi-lattices whose roots we shall label with t_1, t_2, ..., t_m.

  - And so forth for any composite part type t_i, etcetera.
  - It may be that any two or more such sub-semi-lattice root types, t_{i_j}, t_{i_j}, ..., t_{i_k} designate the same, shared type t_{i_x}, that is $t_{i_j} = t_{i_j} = \ldots = t_{i_k} = t_{i_x}$.
  - If so then the k sub-semi-lattices are “collapsed” into one sub-semi-lattice.
  - The building of the semi-lattice terminates when one can no longer analyse part types into further sub-semi-lattices, that is, when these part types are atomic.

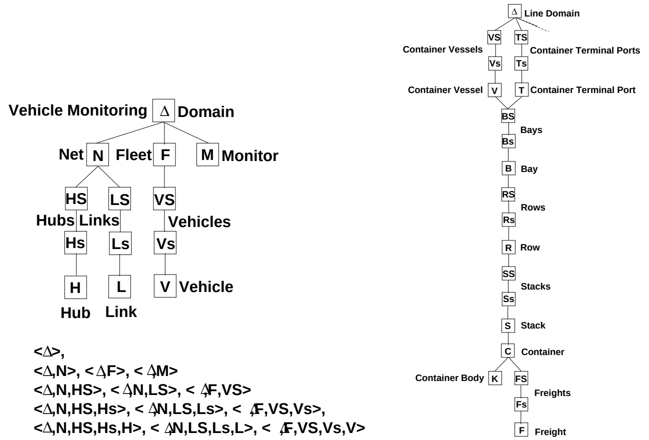


Figure 3: Domain indices

- That is, the roots of the sub-trees of the Δ tree are labelled with type names.
  - Every point in the semi-lattice can be identified by a domain index.
    - The root is defined to have index $\langle\Delta\rangle$.
    - The immediate sub-semi-lattices of Δ have domain indexes $\langle\Delta,t_1\rangle$, $\langle\Delta,t_2\rangle$, $\ldots$, $\langle\Delta,t_m\rangle$.
    - And so forth.
    - If $\ell^\frown\langle t\rangle$ is a prefix of another domain index, say $\ell^\frown\langle t,t'\rangle$, then t designates a composite type.

- For every domain index, $\ell^\frown\langle t\rangle$, that index designates the type t domain type texts.
- These texts consists of several sub-texts.
- There are the texts directly related to the parts, p:P:
  - the observer functions, obs_$\cdots$, if type t is composite,
  - the unique identifier functions, uid_P,
  - the mereology function, mereo_P, and
  - the attribute functions, attr_$\cdots$.
  - To the above "add"
    - possible auxiliary types and auxiliary functions
    - as well as possible axioms.

- Then there are the texts related to
  - actions,
  - events, and
  - behaviours

  "based" (primarily) on parts p:P.
- These texts consists of
  - function signatures (for actions, events, and behaviours),
  - function definitions for these, and
  - channel
    - declarations and
    - channel message type definitions

    for behaviours.

We shall soon see examples of the above.

- But not all can be "discovered" by just examining the domain from the point of view of a sub-semi-lattice type.
  - Many interesting action, event and behaviour signatures depend on domain type texts designated by "roots" of disjoint sub-trees of the semi-lattice.
  - Each such root has its own domain index.
  - Together a meet of the semi-lattice is defined by the set of disjoint domain indices: $\{\ell_i, \ell_j, \cdots, \ell_k\}$.
- It is thus that we arrive at a proper semi-lattice structure relating the various entities of the domain rooted in Δ.

- The domain discoverers are therefore provided with arguments:
  - either a single domain index, $\mathbb{DOMAIN\_FUNCTION}(\ell)$,
  - or a pair, $\mathbb{DOMAIN\_FUNCTION}(\ell)(\{\ell_i, \ell_j, \cdots, \ell_k\})$,
    - the single domain index ℓ and
    - a set of domain indices, $\{\ell_i, \ell_j, \cdots, \ell_k\}$

  where $\mathbb{DOMAIN\_FUNCTION}$ is any of the
    - domain discoverers or
    - domain analysers

  listed earlier.

11.1.4. The ℜepository

- We have yet to give the full signature of the domain discoverers and domain analysers.
  - One argument of these meta-functions
    - was parts of the actual domain
    - as designated by the domain indices.
  - Another argument
    - is to be the ℜepository of description texts
    - being inspected (together with the sub-domain) when
      - analysing that sub-domain and
    - being updated
      - when "generating" the "discovered" description texts.

- We can assume, without loss of generality, that
  - the ℜepository of description texts
  - is the description texts discovered so far.
- The result of domain analysis is either undefined or a truth value. We can assume, without any loss of generality that that result is not recorded.
- The result of domain discovery is either undefined or is a description text consisting of two well-defined fragments:
  - a narrative text, and
  - a formal text.
- Those well-defined texts are "added" to the text of the ℜepository of description texts.
  - For pragmatic reasons,
  - when we explain the positive effect of domain discovery,
  - then we show just this "addition" to the ℜepository.

98. The proper type of the discover functions is therefore:

98. $\mathbb{DISCOVER\_FUNCTION}$: Index→Index-set→ℜ$\stackrel{\sim}{\to}$ℜ

- In the following we shall omit the ℜepository argument and result.

99. So, instead of showing the discovery function invocation and result as:

99. $\mathbb{DISCOVER\_FUNCTION}(\ell)(\ell\text{set})(\rho) = \rho'$

- where ρ' incorporates a pair of texts and RSL formulas,

100. we shall show the discover function signature, the invocation and the result as:

100. $\mathbb{DISCOVER\_FUNCTION}$: Index→Index-set$\stackrel{\sim}{\to}$(Narr_Text×RSL_Text)

100. $\mathbb{DISCOVER\_FUNCTION}(\ell)(\ell\text{set})$: (narr_text,RSL_text)

11.2. Domain Analysers

- Currently we identify four analysis functions.
- As the discovery calculus evolves
  - (through further practice and research)
  - we expect further analysis functions to be identified.

11.2.1. IS_MATERIALS_BASED

- You are reminded of the *Continuous Endurant Modelling* frame on Slide 136.

IS_MATERIALS_BASED

- An early decision has to be made as to whether a domain is significantly based on materials or not:

101. $\mathbb{IS\_MATERIALS\_BASED}(\langle\Delta_{Name}\rangle)$.

- If Item 101 holds of a domain Δ_{Name}
  - then the domain describer can apply
  - $\mathbb{MATERIAL\_SORTS}$ (Item 103 on page 316).

Example: 45 Pipelines and Transports: Materials or Parts.

- $\mathbb{IS\_MATERIALS\_BASED}(\langle\Delta_{\text{Pipeline}}\rangle)$ = **true**.
- $\mathbb{IS\_MATERIALS\_BASED}(\langle\Delta_{\text{Transport}}\rangle)$= **false**.

11.2.2. IS_ATOM, IS_COMPOSITE

- During the discovery process
  - discrete part types arise (i.e., the names are yielded)
  - and these may either denote atomic or composite parts.
- The domain describer
  - must now decide as to
  - whether a named, discrete type is atomic or is composite.

IS_ATOM

- The IS_ATOM analyser serves that purpose:

value
IS_ATOM: Index $\xrightarrow{\sim}$ **Bool**
IS_ATOM($\ell^\frown\langle t\rangle$) $\equiv$ **true** | **false** | **chaos**

- The analysis is undefined for ill-formed indices.

Example: 46 Transport Nets: Atomic Parts (II). We refer to Example 3 (Slide 16).

IS_ATOM(⟨Δ,N,HS,Hs,H⟩), IS_ATOM(⟨Δ,N,LS,Ls,L⟩)
∼IS_ATOM(⟨Δ,N,HS,Hs⟩), ∼IS_ATOM(⟨Δ,N,LS,Ls⟩) ■

IS_COMPOSITE

- The IS_COMPOSITE analyser is
  - similarly applied by the domain describer
  - to a part type t
  - to help decide whether t is a composite type.

value
IS_COMPOSITE: Index $\xrightarrow{\sim}$ **Bool**
IS_COMPOSITE($\ell^\frown\langle t\rangle$) $\equiv$ **true** | **false** | **chaos**

Example: 47 Transport Nets: Composite Parts. We refer to Example 3 (Slide 16)

IS_COMPOSITE(⟨Δ⟩),
IS_COMPOSITE(⟨Δ,N⟩)
IS_COMPOSITE(⟨Δ,N,HS,Hs⟩),
IS_COMPOSITE(⟨Δ,N,LS,Ls⟩)
∼IS_COMPOSITE(⟨Δ,N,HS,Hs,H⟩),
∼IS_COMPOSITE(⟨Δ,N,LS,Ls,L⟩) ■

11.2.3. HAS_A_CONCRETE_TYPE

- Sometimes we find it expedient
  - to endow a "discovered" sort with a concrete type expression, that is,
  - "turn" a sort definition into a concrete type definition.

HAS_A_CONCRETE_TYPE

102. Thus we introduce the analyser:

102 HAS_A_CONCRETE_TYPE: Index $\stackrel{\sim}{\rightarrow}$ **Bool**
102 HAS_A_CONCRETE_TYPE($\ell^\frown\langle t\rangle$): **true** | **false** | **chaos**

Example: 48 Transport Nets: Concrete Types . We refer to Example 3 (Slide 16) while exemplifying four cases:

HAS_A_CONCRETE_TYPE($\langle\Delta$,N,HS,Hs$\rangle$)
HAS_A_CONCRETE_TYPE($\langle\Delta$,N,LS,Ls$\rangle$)
$\sim$ HAS_A_CONCRETE_TYPE($\langle\Delta$,N,HS,Hs,H$\rangle$)
$\sim$ HAS_A_CONCRETE_TYPE($\langle\Delta$,N,LS,Ls,L$\rangle$) ■

- We remind the listener that
  - it is a decision made by the domain describer
  - as to whether a part type is
    - to be considered a sort or
    - be given a concrete type.
- We shall later cover a domain discoverer related to the positive outcome of the above inquiry.

11.3. Domain Discoverers

- A domain discoverer is a mental tool.
  - It takes a written form shown earlier.
  - It is to be "applied" by a human, the domain describer.
  - The domain describer applies the domain discoverer to a fragment of the domain, as it is: "out there" !

- 'Application' means the following.
  - The domain describer examines the domain as directed by the explanation given for the domain discoverer — as here, in these lectures.
  - As the brain of the domain describer views, examines, analyses, a domain index-designated fragment of the domain,
    - ideas as to which domain concepts to capture arise
    - and these take the form of pairs of narrative and formal texts.

11.3.1. MATERIAL_SORTS

MATERIAL_SORTS – I/II

103. The MATERIAL_SORTS discovery function applies to a domain, usually designated by $\langle\Delta_{\mathsf{Name}}\rangle$ where Name is a pragmatic hinting at the domain by name.
104. The result of the domain discoverer applying this meta-function is some narrative text
105. and the **type**s of the discovered materials
106. usually affixed a comment
    (a) which lists the "somehow related" part types
    (b) and their related materials observers.

MATERIAL_SORTS II/II

```
103. MATERIAL_SORTS: ⟨Δ⟩ → (Text × RSL)
103. MATERIAL_SORTS(⟨Δ_Name⟩):
104.    [ narrative text ;
105.      type M_a, M_b, ..., M_c  materials
106.      comment: related part types: P_i, P_j, ..., P_k
106.                   obs_M_n : P_m → M_n, ... ]
101.    pre: IS_MATERIALS_BASED(⟨Δ_Name⟩)
```


Example: 49 Pipelines: Material.

- MATERIAL_SORTS($\langle\Delta_{\mathsf{Oil\ Pipeline\ System}}\rangle$):
  [The oil pipeline system is focused on oil ;
  type O **material**
  comment related part type: U, obs_O: U → O]

11.3.2. PART_SORTS

PART_SORTS I/II

107. The part type discoverer PART_SORTS
   (a) applies to a simply indexed domain, ℓ⁀⟨t⟩,
   (b) where t denotes a composite type, and yields a pair
      i. of narrative text and
      ii. formal text which itself consists of a pair:
         A. a set of type names
         B. each paired with a part (sort) observer.

PART_SORTS II/II

value
107. PART_SORTS: Index $\stackrel{\sim}{\rightarrow}$ (**Text**×RSL)
107(a). PART_SORTS(ℓ⁀⟨t⟩):
107((b))i. [narrative, possibly enumerated texts ;
107((b))iiA. **type** t_1,t_2,...,t_m,
107((b))iiB. **value** obs_t_1:t→t_1,obs_t_2:t→t_2,...,obs_t_m:t→t_m
107(b). **pre**: IS_COMPOSITE(ℓ⁀⟨t⟩)]

Example: 50 Transport: Part Sorts. We apply a concrete version of the above sort discoverer to the road traffic system domain Δ. See Example 36.

- PART_SORTS(⟨Δ⟩):
  [the vehicle monitoring domain contains three sub-parts:
  net, fleet and monitor ;
  type N, F, M,
  value obs_N: Δ → N, obs_F: Δ → F, obs_M: Δ → M]
- PART_SORTS(⟨Δ,N⟩):
  [the net domain contains two sub-parts:
  sets of hubs and sets of link ;
  type HS, LS,
  value obs_HS: N → HS, obs_LS: N → LS]

- PART_SORTS(⟨Δ,F⟩):
  [the fleet domain consists of one sub-domain:
  set of vehicles;
  type VS,
  value obs_VS: F → VS] ■

11.3.3. PART_TYPES

PART_TYPES I/II

108. The PART_TYPES discoverer applies to a composite sort, t, and yields a pair
    (a) of narrative, possibly enumerated texts [omitted], and
    (b) some formal text:
        i. a type definition, t_c = te,
        ii. together with the sort definitions of so far undefined type names of te.
        iii. An observer function observes t_c from t.
        iv. The PART_TYPES discoverer is not defined if the designated sort is judged to not warrant a concrete type definition.

PART_TYPES II/II

108. PART_TYPES: Index $\xrightarrow{\sim}$ (**Text**×RSL)
108. PART_TYPES(ℓ^⟨t⟩):
108(a). [narrative, possibly enumerated texts ;
108((b))i. **type** t_c = te,
108((b))ii. t_α, t_β, ..., t_γ,
108((b))iii. **value** obs_t_c: t → t_c
108((b))iv. **pre**: HAS_CONCRETE_TYPE(ℓ^⟨t⟩)]
108((b))ii. **where:** type expression te contains
108((b))ii. type names t_α, t_β, ..., t_γ

Example: 51 Transport: Concrete Part Types. Continuing Examples ??–50 and Example 3 – we omit narrative informal texts.

PART_TYPES(⟨Δ,F,VS⟩):
  type V, Vs=V-**set**, **value** obs_Vs: VS→Vs
PART_TYPES(⟨Δ,N,HS⟩):
  type H, Hs=H-**set**, **value** obs_Hs: HS→Hs
PART_TYPES(⟨Δ,N,LS⟩):
  type L, Ls=L-**set**, **value** obs_Ls: LS→Ls ■

11.3.4. UNIQUE_ID

UNIQUE_ID

109. For every part type t we postulate a unique identity analyser function uid_t.

value
109. UNIQUE_ID: Index → (**Text**×RSL)
109. UNIQUE_ID(ℓ^⟨t⟩):
109. [narrative, possibly enumerated text ;
109. **type** ti
109. **value** uid_t: t → ti]

Example: 52 Transport Nets: Unique Identifiers. Continuing Example 3:

UNIQUE_ID(⟨Δ,HS,Hs,H⟩): **type** H, HI, **value** uid_H→HI
UNIQUE_ID(⟨Δ,LS,Ls,L⟩): **type** L, LI, **value** uid_L→LI ■

11.3.5. MEREOLOGY

- Given a part, p, of type t, the mereology, MEREOLOGY, of that part
  - is the set of all the unique identifiers of the other parts to which part p is part-ship-related
  - as "revealed" by the mereo_ti$_i$ functions applied to p.
- Henceforth we omit the otherwise necessary narrative texts.

MEREOLOGY I/II

110. Let type names t_1, t_2, ..., t_n denote the types of all parts of a domain.
111. Let type names ti_1, ti_2, ..., ti_n[27], be the corresponding type names of the unique identifiers of all parts of that domain.
112. The mereology analyser MEREOLOGY is a generic function which applies to a pair of an index and an index set and yields some structure of unique identifiers. We suggest two possibilities, but otherwise leave it to the domain analyser to formulate the mereology function.
113. Together with the "discovery" of the mereology function there usually follows some axioms.

MEREOLOGY II/II

type
110. t_1, t_2, ..., t_n
111. $t_{idx} = ti_1 \mid ti_2 \mid ... \mid ti_n$

112. MEREOLOGY: Index $\xrightarrow{\sim}$ Index-**set** $\xrightarrow{\sim}$ (**Text**×RSL)
112. MEREOLOGY(ℓ⌢⟨t⟩)({ℓ_i⌢⟨t_j⟩,...,ℓ_k⌢⟨t_l⟩}):
112. [narrative, possibly enumerated texts ;
112. **either:** {}
112. **or:** **value** mereo_t: t → ti_x
112. **or:** **value** mereo_t: t → ti_x-**set** × ti_y-**set** × ... × ti_z-**set**
113. **axiom** $\mathcal{P}$redicate over values of t′ and t_{idx}]

where none of the ti_x, ti_y, ..., ti_z are equal to ti.

[27] We here assume that all parts have unique identifications.

Example: 53 Transport Net Mereology. Examples:

- MEREOLOGY(⟨Δ,N,HS,Hs,H⟩)({⟨Δ,N,LS,Ls,L⟩}):
  value mereo_H→LI-**set**
- MEREOLOGY(⟨Δ,N,LS,Ls,L⟩)({⟨Δ,N,HS,Hs,H⟩}):
  value mereo_L→HI-**set**
  axiom see Example 11 Slide 87.

11.3.6. ATTRIBUTES

- A general attribute analyser analyses parts beyond their unique identities and possible mereologies.
  - Part attributes have names.
  - We consider these names to also abstractly name the corresponding attribute types.

ATTRIBUTES I/II

114. Attributes have types.
We assume **attribute type names** to be distict from **part type names**.
115. ATTRIBUTES applies to parts of type **t** and yields a pair of
  (a) narrative text and
  (b) formal text, here in the form of a pair
    i. a set of one or more attribute types, and
    ii. a set of corresponding attribute observer functions **attr_at**, one for each attribute sort **at** of **t**.

ATTRIBUTES II/II

type
114. $at = at_1 \mid at_2 \mid ... \mid at_n$
value
115. ATTRIBUTES: Index → (**Text**×RSL)
115. ATTRIBUTES(ℓ^⟨t⟩):
115(a). [narrative, possibly enumerated texts ;
115((b))i. **type** at_1, at_2, ..., at_m
115((b))ii. **value** $attr\_at_1$:t→at_1,$attr\_at_2$:t→at_2,...,$attr\_at_m$:t→at_m]

- where m≤n

Example: 54 Transport Nets: Part Attributes. We exemplify attributes of composite and of atomic parts — omitting narrative texts:

ATTRIBUTES(⟨Δ⟩):
  type Domain_Name, ...
  value attr_Domain_Name: Δ → Domain_Name, ...

- where
  - Domain_Name could include *State Roads* or *Rail Net*.
  - etcetera.

ATTRIBUTES(⟨Δ,N⟩):
type
Sub_Domain_Name ex.: *State Roads*
Sub_Domain_Location ex.: *Denmark*
Sub_Domain_Owner ex.: *The Danish Road Directorate*
...
Length ex.: *3.786 Kms.*
value
attr_Sub_Domain_Name: N → Sub_Domain_Name
attr_Sub_Domain_Location: N → Sub_Domain_Location
attr_Sub_Domain_Owner: N → Sub_Domain_Owner
...
attr_Length: N → Length

ATTRIBUTES(⟨Δ,N,LS,Ls,L⟩):
type LOC, LEN, ...
value attr_LOC: L → LOC, attr_LEN: L → LEN, ...

ATTRIBUTES(⟨Δ,N,LS,Ls,L⟩)({,⟨Δ,N,HS,Hs,H⟩}):
type
LΣ=HI-**set**
LΩ=LΣ-**set**
value
attr_LΣ:L→LΣ
attr_LΩ:L→LΩ

- where
  - LOC might reveal some Bézier curve[28] representation of the possibly curved three dimensional location of the link in question,
  - LEN might designate length in meters,
  - LΣ designates the state of the link,
  - LΩ designates the space of all allowed states of the link. ■

[28] http://en.wikipedia.org/wiki/Bézier_curve

End of Lecture 6: First Session — Calculus I

Part and Material Discoverers

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

SHORT BREAK

HELLO THERE!

Begin of Lecture 7: Last Session — Calculus II

Function Signature Discoverers and Laws

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

Tutorial Schedule

11.3.7. ACTION_SIGNATURES

- We really should discover actions, but actually analyse function definitions.
- And we focus, in this tutorial, on just "discovering" the function signatures of these actions.
- By a function signature, to repeat, we understand
  - a functions name, say fct, and
  - a function type expression (te), say dte $\tilde{\to}$ rte where
    - dte defines the type of the function's definition set
    - and rte defines the type of the function's image, or range set.

- We use the term 'functions' to cover actions, events and behaviours.
- We shall in general find that the signatures of actions, events and behaviours depend on types of more than one domain.
  - Hence the schematic index set $\{\ell_1\frown\langle t_1\rangle,\ell_2\frown\langle t_2\rangle,...,\ell_n\frown\langle t_n\rangle\}$
  - is used in all action, event and behaviour discoverers.

ACTION_SIGNATURES I/II

116. The ACTION_SIGNATURES meta-function, besides narrative texts, yields
   (a) a set of auxiliary sort or concrete type definitions and
   (b) a set of action signatures each consisting of an action name and a pair of definition set and range type expressions where
   (c) the type names that occur in these type expressions are defined by in the domains indexed by the index set.

ACTION_SIGNATURES II/II

116 ACTION_SIGNATURES: Index → Index-**set** $\tilde{\to}$ (**Text**×RSL)
116 ACTION_SIGNATURES($\ell\frown\langle t\rangle$)($\{\ell_1\frown\langle t_1\rangle,\ell_2\frown\langle t_2\rangle,...,\ell_n\frown\langle t_n\rangle\}$):
116 [narrative, possibly enumerated texts ;
116 **type** $t_a, t_b, ... t_c$,
116(b) **value**
116(b) $act_i:te_{i_d}\tilde{\to}te_{i_r}, act_j:te_{j_d}\tilde{\to}te_{j_r}, ..., act_k:te_{k_d}\tilde{\to}te_{k_r}$
116(c) **where:**
116(c) type names in $te_{(i|j|...|k)_d}$ and in $te_{(i|j|...|k)_r}$ are either
116(c) type names t_a, t_b, ... t_c or are type names defined by the
116(c) indices which are prefixes of $\ell_m\frown\langle T_m\rangle$ and where T_m is
116(c) in some signature $act_{i|j|...|k}$]

Example: 55 Transport Nets: Action Signatures.

- ACTION_SIGNATURES(⟨Δ,N,HS,Hs,H⟩)({⟨Δ,N,LS,Ls,L⟩}):
  insert_H: N → H $\stackrel{\sim}{\to}$ N
  remove_H: N → HI $\stackrel{\sim}{\to}$ N
  ...
- ACTION_SIGNATURES(⟨Δ,N,LS,Ls,L⟩)({⟨Δ,N,HS,Hs,H⟩}):
  insert_L: N → L $\stackrel{\sim}{\to}$ N
  remove_L: N → LI $\stackrel{\sim}{\to}$ N
  ...
- where ··· refer to the possibility of discovering further action signatures "rooted" in
  - ⟨Δ,N,HS,Hs,H⟩, respectively
  - ⟨Δ,N,LS,Ls,L⟩. ■

11.3.8. EVENT_SIGNATURES

EVENT_SIGNATURES I/II

117. The EVENT_SIGNATURES meta-function, besides narrative texts, yields
   (a) a set of auxiliary event sorts or concrete type definitions and
   (b) a set of event signatures each consisting of
   - an event name and
   - a pair of definition set and range type expressions

   where
   (c) the type names that occur in these type expressions are defined either in the domains indexed by the indices or by the auxiliary event sorts or types.

EVENT_SIGNATURES II/II

117 EVENT_SIGNATURES: Index → Index-**set** $\stackrel{\sim}{\to}$ (**Text**×RSL)
117 EVENT_SIGNATURES(ℓ^⟨t⟩)({ℓ_1^⟨t_1⟩,ℓ_2^⟨t_2⟩,...,ℓ_n^⟨t_n⟩}):
117(a) [narrative, possibly enumerated texts omitted ;
117(a) **type** t_a,t_b,... t_c,
117(b) **value**
117(b) evt_pred$_i$: te_{d_i} × te_{r_i} → **Bool**
117(b) evt_pred$_j$: te_{d_j} × te_{r_j} → **Bool**
117(b) ...
117(b) evt_pred$_k$: te_{d_k} × te_{r_k} → **Bool**]

117(c) **where:** t is any of t_a,t_b,...,t_c or type names listed in in indices; type names of the 'd'efinition set and 'r'ange set type expressions te_d and te_r are type names listed in domain indices or are in t_a,t_b,...,t_c, the auxiliary discovered event types. ■

Example: 56 Transport Nets: Event Signatures.

We refer to Example 35 on page 173. The omitted narrative text would, if included, as it should, be a subset of the Items 23–26 texts on Slide 171.

- EVENT_SIGNATURES(⟨Δ,N,LS,Ls,L⟩)({⟨Δ,N,HS,Hs,H⟩}):
  value
  link_disappearance: N × N $\stackrel{\sim}{\to}$ **Bool**
  link_disappearance(n,n′) ≡
  ∃ ℓ:L · l ∈ obs_Ls(n) ⇒ pre_cond(n,ℓ) ∧ post_cond(n,ℓ,n′)

  ... [possibly further, discovered event]
  ... [signatures "rooted" in ⟨Δ,N,LS,Ls,L⟩] ■

- The undefined pre_ and post_conditions were "fully discovered" on Slides 173 and 175.

11.3.9. BEHAVIOUR_SIGNATURES

- We choose, in this tutorial, to model behaviours in CSP[29].
- This means that we model (synchronisation and) communication between behaviours by means of messages m of type M, CSP channels (**channel** ch:M) and CSP
  - output: ch!e [offer to deliver value of e on channel ch], and
  - input: ch? [offer to accept a value on channel ch].

[29]Other behaviour modelling languages are Petri Nets, MSCs: Message Sequence Charts, Statechart etc.

- We allow for the declaration of single channels as well as of one, two, ..., n dimensional arrays of channels with indexes ranging over channel index types
  - **type** Idx, CIdx, RIdx . . . :
  - **channel** ch:M, { ch_v[vi]:M′|vi:Idx }, { ch_m[ci,ri]:M′′|ci:CIdx,ri:RIdx }, . . .

  etcetera.
- We assume some familiarity with CSP [Hoare85+2004] (or even RSL/CSP [TheSEBook1wo] [Chapter 21]).

- A behaviour usually involves two or more distinct sub-domains.

Example: 57 Vehicle Behaviour. Let us illustrate that behaviours usually involve two or more distinct sub-domains.

- A vehicle behaviour, for example, involves
  - the vehicle sub-domain,
  - the hub sub-domain (as vehicles pass through hubs),
  - the link sub-domain (as vehicles pass along links) and,
  - for the road pricing system, also the monitor sub-domain. ■

BEHAVIOUR_SIGNATURES I/II

118. The BEHAVIOUR_SIGNATURES meta-function, besides narrative texts, yields
119. It applies to a set of indices and results in a pair,
    (a) a narrative text and
    (b) a formal text:
        i. a set of one or more message types,
        ii. a set of zero, one or more channel index types,
        iii. a set of one or more channel declarations,
        iv. a set of one or more process signatures with each signature containing a behaviour name, an argument type expression, a result type expression, usually just **Unit**, and
        v. an input/output clause which refers to channels over which the signatured behaviour may interact with its environment.

BEHAVIOUR_SIGNATURES II/II

118. BEHAVIOUR_SIGNATURES: Index→ Index-**set** $\xrightarrow{\sim}$ (**Text**×RSL)
118. BEHAVIOUR_SIGNATURES(ℓ⌢⟨t⟩)({ℓ_1⌢⟨t_1⟩,ℓ_2⌢⟨t_2⟩,...,ℓ_n⌢⟨t_n⟩}):
119(a). [narrative, possibly enumerated texts ;
119((b))i. **type** m = m_1 | m $_2$ | ... | m_μ, $\mu \geq 1$
119((b))ii. i = i_1 | i_2 | ... | i_n, $n \geq 0$
119((b))iii. **channel** c:m, {vc[x]|x:i_a}:m, {mc[x,y]|x:i_b,y:i_c}:m,...
119((b))iv. **value**
119((b))iv. bhv_1: ate_1 → *$inout_1$* rte_1,
119((b))iv. ... ,
119((b))iv. bhv_m: ate_m → *$inout_m$* rte_m.]
119((b))iv. **where** type expressions $atei_i$ and rte_i for all i involve at least
119((b))iv. two types t'_i, t''_j of respective indexes ℓ_i⌢⟨t_i⟩, ℓ_j⌢⟨t_j⟩,
119((b))v. **where** **Unit** may appear in either ate_i or rte_j or both.
119((b))v. **where** *$inout_i$*: **in** k | **out** k | **in**,**out** k
119((b))v. **where** k: c **or** vc[x] **or** {vc[x]|x:i_a·x ∈ xs} **or**
119((b))v. {mc[x,y]|x:i_b,y:i_c · x ∈ xs ∧ y ∈ ys} **or** ...

Example: 58 Vehicle Transport: Behaviour Signatures. We refer to Example 36.

BEHAVIOUR_SIGNATURES(⟨Δ,F,VS,Vs,V⟩)({⟨Δ,M⟩}):
[With each vehicle we associate behaviour with the following arguments: the vehicle identifier, the vehicle parts, and the vehicle position. The vehicle communicates with the monitor process over a vehicle to monitor array of channels, one for each vehicle ... ;
type
VP
channel
{vm[vi]|vi:VI · vi ∈ vis}:VP
value
veh: vi:VI → v:V → vp:VP → **out** vm[vi] **Unit**]

BEHAVIOUR_SIGNATURES(⟨Δ,M⟩)({⟨Δ,F,VS,Vs,V⟩}):
[With the monitor part we associate a behaviour with the monitor part as only argument. The monitor accepts communications from vehicle behaviours ... ;
value
mon: M → **in** {vm[vi]|vi:VI · vi ∈ vis},clkm_ch **Unit**]

- The "discovery" of vehicle positions into positions
  - on a link, some fraction down that link, or
  - at a hub,

  that "discovery", is left for further analysis.

We refer to Slide 197 (Items 47(a)–47((a))iii). ■

11.4. Order of Analysis and "Discovery"

- Analysis and "discovery", that is, the "application" of
  - the analysis meta-functions and
  - the "discovery" meta-functions
- has to follow some order:
  - starts at the "root", that is with index ⟨Δ⟩,
  - and proceeds with indices appending part domain type names already discovered.

11.5. Analysis and “Discovery” of “Leftovers”

- The analysis and discovery meta-functions focus on types, that is, the types
  - of abstract parts, i.e., sorts,
  - of concrete parts, i.e., concrete types,
  - of unique identifiers,
  - of mereologies, and of
  - attributes – where the latter has been largely left as sorts.

- In this tutorial we do not suggest any meta-functions for such analyses that may lead to
  - concrete types from non-part sorts, or to
  - action, event and behaviour definitions
    - say in terms of pre/post-conditions,
    - etcetera.
  - So, for the time, we suggest, as a remedy for the absence of such “helpers”, good “old-fashioned” domain engineer ingenuity.

11.6. Laws of Domain Descriptions

- By a domain description law we shall understand
  - some desirable property
  - that we expect (the ‘human’) results of
  - the (the ‘human’) use of the domain description calculus
  - to satisfy.
- We may think of these laws as axioms
  - which an ideal domain description ought satisfy,
  - something that domain describers should strive for.

Notational Shorthands:

- $(f;g;h)(\Re) = h(g(f(\Re)))$
- $(f_1;f_2;\ldots;f_m)(\Re) \simeq (g_1;g_2;\ldots;g_n)(\Re)$
  means that the two “end” states are equivalent modulo appropriate renamings of types, functions, predicates, channels and behaviours.
- $[f;g;\ldots;h;\alpha]$
  stands for the Boolean value yielded by α (in state $\Re$).

11.6.1. 1st Law of Commutativity

- We make a number of assumptions:
  - the following two are well-formed indices of a domain:
    - ι': $\langle\Delta\rangle^\frown\ell'^\frown\langle\mathsf{A}\rangle$,
    - ι'': $\langle\Delta\rangle^\frown\ell''^\frown\langle\mathsf{B}\rangle$,

    where ℓ' and ℓ'' may be different or empty ($\langle\rangle$) and A and B are distinct;
  - that $\mathcal{F}$ and $\mathcal{G}$ are two, not necessarily distinct discovery functions; and
  - that the domain at ι' and at ι'' have not yet been explored.

- We wish to express,
  - as a desirable property of **domain description development**
  - that exploring domain Δ at
    - either ι' first and then ι''
    - or at ι'' first and then ι',
  - the one right after the other (hence the ";"),
  - ought yield the same partial description fragment:

120. $(\mathcal{G}(\iota'')\ ;\ (\mathcal{F}(\iota')))(\Re) \simeq (\mathcal{F}(\iota')\ ;\ (\mathcal{G}(\iota'')))(\Re)$

When a **domain description development** satisfies Law 120., under the above assumptions,

- then we say that the development,
- modulo type, action, event and behaviour name "assignments",
- satisfies a mild form of commutativity.

11.6.2. 2nd Law of Commutativity

- Let us assume
  - that we are exploring the sub-domain at index
  - ι: $\langle\Delta\rangle^\frown\ell^\frown\langle\mathsf{A}\rangle$.
- Whether we
  - first "discover" $\mathcal{A}$ttributes
  - and then $\mathcal{M}$ereology (including $\mathcal{U}$nique identifiers)

  or
  - first "discover" $\mathcal{M}$ereology (including $\mathcal{U}$nique identifiers)
  - and then $\mathcal{A}$ttributes

  should not matter.

- We make some abbreviations:
  - $\mathcal{A}$ stand for the $\mathbb{ATTRIBUTES}$,
  - $\mathcal{U}$ stand for the $\mathbb{UNIQUE\_IDENTIFIER}$,
  - $\mathcal{M}$ stand for the $\mathbb{MEREOLOGY}$,
  - ι for index $\langle\Delta\rangle^\frown\ell^\frown\langle\mathsf{A}\rangle$, and
  - $\iota\mathsf{s}$ for a suitable set of indices.
- Thus we wish the following law to hold:

121. $(\mathcal{A}(\iota);\mathcal{U}(\iota);\mathcal{M}(\iota)(\iota s))(\Re) \simeq$
$(\mathcal{U}(\iota);\mathcal{M}(\iota)(\iota s);\mathcal{A}(\iota))(\Re) \simeq$
$(\mathcal{U}(\iota);\mathcal{A}(\iota);\mathcal{M}(\iota)(\iota s))(\Re)$.

  - here modulo attribute and unique identifier type name renaming.

11.6.3. 3rd Law of Commutativity

- Let us again assume
  - that we are exploring the sub-domain at index
  - ι: $\langle\Delta\rangle\hat{}\ell\hat{}\langle\mathsf{A}\rangle$
  - where $\iota\mathsf{s}$ is a suitable set of indices.
- Whether we are
  - exploring actions, events or behaviours at that domain index
  - in that order,
  - or some other order

  ought be immaterial.

- Hence with
  - $\mathcal{A}$ now standing for the ACTION_SIGNATURES,
  - $\mathcal{E}$ standing for the EVENT_SIGNATURES,
  - $\mathcal{B}$ standing for the BEHAVIOUR_SIGNATURES,
- discoverers, we wish the following law to hold:

122. $(\mathcal{A}(\iota)(\iota s);\mathcal{E}(\iota)(\iota s);\mathcal{B}(\iota)(\iota s))(\Re) \simeq$
$(\mathcal{A}(\iota)(\iota s);\mathcal{B}(\iota)(\iota s);\mathcal{E}(\iota)(\iota s))(\Re) \simeq$
$(\mathcal{E}(\iota)(\iota s);\mathcal{A}(\iota)(\iota s);\mathcal{B}(\iota)(\iota s))(\Re) \simeq$
$(\mathcal{E}(\iota)(\iota s);\mathcal{B}(\iota)(\iota s);\mathcal{A}(\iota)(\iota s))(\Re) \simeq$
$(\mathcal{B}(\iota)(\iota s);\mathcal{A}(\iota)(\iota s);\mathcal{E}(\iota)(\iota s))(\Re) \simeq$
$(\mathcal{B}(\iota)(\iota s);\mathcal{E}(\iota)(\iota s);\mathcal{A}(\iota)(\iota s))(\Re)$.

  - here modulo action function, event predicate, channel, message type and behaviour (and all associated, auxiliary type) renamings.

11.6.4. 1st Law of Stability

- Re-performing
  - the same discovery function
  - over the same sub-domain,
  - that is with identical indices,
  - one or more times,

  ought not produce any new description texts.
- That is:

123. $(\mathcal{D}(\iota)(\iota\mathsf{s});\mathcal{A}\_\mathsf{and}\_\mathcal{D}\_\mathsf{seq})(\Re) \simeq$
$(\mathcal{D}(\iota)(\iota\mathsf{s});\mathcal{A}\_\mathsf{and}\_\mathcal{D}\_\mathsf{seq};\mathcal{D}(\iota)(\iota\mathsf{s}))(\Re)$

- where
  - $\mathcal{D}$ is any discovery function,
  - $\mathcal{A}\_\mathsf{and}\_\mathcal{D}\_\mathsf{seq}$ is any specific sequence of intermediate **a**nalyses **and** **d**iscoveries, and where
  - ι and $\iota\mathsf{s}$ are suitable indices, respectively sets of indices.

11.6.5. 2nd Law of Stability

- Re-performing
  - the same analysis functions
  - over the same sub-domain,
  - that is with identical indices,
  - one or more times,

  ought not produce any new analysis results.
- That is:

124. $[\mathcal{A}(\iota)] \ = \ [\mathcal{A}(\iota);\ldots;\mathcal{A}(\iota)]$

- where
  - $\mathcal{A}$ is any analysis function,
  - "…" is any sequence of intermediate analyses and discoveries, and where
  - ι is any suitable index.

11.6.6. Law of Non-interference

- When performing a discovery meta-operation, $\mathcal{D}$
  - on any index, ι, and possibly index set, ιs, and
  - on a repository state, $\Re$,
  - then using the $[\mathcal{D}(\iota)(\iota s)]$ notation
  - expresses a pair of a narrative text and some formulas, [txt,rsl],
  - whereas using the $(\mathcal{D}(\iota)(\iota s))(\Re)$ notation
  - expresses a next repository state, $\Re'$.
- What is the "difference" ?
- Informally and simplifying we can say that the relation between the two expressions is:

125. $[\mathcal{D}(\iota)(\iota s)]$: [txt,rsl]
$(\mathcal{D}(\iota)(\iota s))(\Re) = \Re'$
where $\Re' = \Re \cup \{[\text{txt,rsl}]\}$

- We say that when 125. is satisfied
  - for any discovery meta-function $\mathcal{D}$,
  - for any indices ι and ιs
  - and for any repository state $\Re$,

  then the repository is not interfered with,
  - that is, *"what you see is what you get:"*

  and therefore that
  - the discovery process satisfies the law on non-interference.

11.7. Discussion

- The above is just a hint at domain development laws that we might wish orderly developments to satisfy.
- We invite the audience to suggest other laws.
- The laws of the analysis and discovery calculus
  - forms an ideal set of expectations
  - that we have of not only one domain describer
  - but from a domain describer team
  - of two or more domain describers
  - whom we expect to work, i.e., loosely collaborate,
  - based on "near"-identical domain development principles.

- These are quite some expectations.
  - But the whole point of
    - a highest-level
    - academic scientific education and
    - engineering training
  - is that one should expect commensurate development results.

- Now, since the ingenuity and creativity in the analysis and discovery process does differ between domain developers
  - we expect that a daily process of *"buddy checking"*,
  - where individual team members present their findings
  - and where these are discussed by the team
  - will result in adherence to the laws of the calculus.
- The laws of the analysis and discovery calculus
  - expressed some properties that we wish the repository to exhibit.

- We have deliberately abstained from "over-defining"
  - the structure of repositories and
  - the "hidden" operations (i.e., 'update', etc.)

  repositories.
- We expect further
  - research into,
  - development of,
  - possible changes to
  - and use

  of the calculus to yield such insight as to lead to
  - a firmer understanding of
  - the nature of repositories.

- In the analysis and discovery calculus
  - such as we have presented it
- we have emphasised
  - the types of parts, sorts and immediate part concrete types, and
  - the signatures of actions, events and behaviours —
  - as these predominantly featured type expressions.

- We have therefore, in this tutorial, not investigated, for example,
  - pre/post conditions of action function,
  - form of event predicates, or
  - behaviour process expressions.
- We leave that, substantially more demanding issue, for future explorative and experimental research.

End of Lecture 7: Last Session — Calculus II

Function Signature Discoverers and Laws

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

LONG BREAK

DRAWING TO A CLOSE

Begin of Lecture 8: First Session — Requirements Engineering

Domain and Interface Requirements

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

Tutorial Schedule

12. Requirements Engineering

- We shall present a terse overview of
  - how one can "derive" essential fragments of requirements prescriptions
  - from a domain description.
- First we give,
  - in the next section,
  - a summary of the net domain, N,
  - as developed in earlier sections.

12.1. The Transport Domain — a Resumé

12.1.1. Nets, Hubs and Links

126. From a transport net one can observe sets of hubs and links.

type
126. N, HS, Hs = H-**set**, H, LS, Ls = L-**set**, L
127. HI, LI
15. $L\Sigma$ = HI-**set**, $H\Sigma$ = (LI×LI)-**set**
16. $L\Omega$ = $L\Sigma$-**set**, $H\Omega$ = $H\Sigma$-**set**
value
126. obs_HS: N → HS, obs_LS: N → LS
126. obs_Hs: N → H-**set**, obs_Ls: N → L-**set**
15. attr_$L\Sigma$: L → $L\Sigma$, attr_$H\Sigma$: H → $H\Sigma$
16. attr_$L\Omega$: L → $L\Omega$, attr_$H\Omega$: H → $H\Omega$

12.1.2. Mereology

127. From hubs and links one can observe their unique hub, respectively link identifiers and their respective mereologies.
128. The mereology of a link identifies exactly two distinct hubs.
129. The mereologies of hubs and links must identify actual links and hubs of the net.

value

127. uid_H: H → HI, uid_L: L → LI
127. mereo_H: H → LI-**set**, mereo_L: L → HI-**set**

axiom

128. ∀ l:L•**card** mereo_L(l)=2
129. ∀ n:N,l:L•l ∈ obs_Ls(n) ⇒
129. ∧ ∀ hi:HI•hi ∈ mereo_L(l)
129. ⇒ ∃ h:h•h ∈ obs_Hs(n)∧uid_H(h)=hi
129. ∧ ∀ h:H•h ∈ obs_Hs(n) ⇒
129. ∀ li:LI•li ∈ mereo_H(h)
129. ⇒ ∃ l:L•l ∈ obs_Ls(n)∧uid_L(l)=li

12.2. A Requirements "Derivation"

12.2.1. Definition of Requirements

IEEE Definition of 'Requirements'

- By a requirements we understand (cf. IEEE Standard 610.12 [ieee-610.12]):
  - *"A condition or capability needed by a user to solve a problem or achieve an objective".*

12.2.2. The Machine = Hardware + Software

- By 'the machine' we shall understand the
  - software to be developed and
  - hardware (equipment + base software) to be configured

  for the domain application.

12.2.3. Requirements Prescription

- The core part of the requirements engineering of a computing application is the requirements prescription.
  - A requirements prescription tells us which parts of the domain are to be supported by 'the machine'.
  - A requirements is to satisfy some goals.
  - Usually the goals cannot be prescribed in such a manner that they can serve directly as a basis for software design.
  - Instead we derive the requirements from the domain descriptions and then argue (incl. prove) that the goals satisfy the requirements.
  - In this colloquium we shall not show the latter but shall show the former.

12.2.4. Some Requirements Principles

The "Golden Rule" of Requirements Engineering

- Prescribe only such requirements
  - that can be objectively shown to hold
  - for the designed software.

An "Ideal Rule" of Requirements Engineering

- When prescribing (including formalising) requirements,
  - also formulate tests (theorems, properties for model checking)
  - whose actualisation should show adherence to the requirements.

- We shall not show adherence to the above rules.

12.2.5. A Decomposition of Requirements Prescription

- We consider three forms of requirements prescription:
  - the domain requirements,
  - the interface requirements and
  - the machine requirements.
- Recall that the machine is the hardware and software (to be required).
  - Domain requirements are those whose technical terms are from the domain only.
  - Machine requirements are those whose technical terms are from the machine only.
  - Interface requirements are those whose technical terms are from both.

12.2.6. An Aside on Our Example

- We shall continue our "ongoing" example.
- Our requirements is for a tollway system.
- By a requirements goal we mean
  - *an objective*
  - *the system under consideration*
  - *should achieve* [LamsweerdeIEEE2001].
- The goals of having a tollway system are:
  - to decrease transport times between selected hubs of a general net; and
  - to decrease traffic accidents and fatalities while moving on the tollway net as compared to comparable movements on the general net.

- The tollway net, however, must be paid for by its users.
  - Therefore tollway net entries and exits occur at tollway plazas
  - with these plazas containing entry and exit toll collectors
  - where tickets can be issued, respectively collected and travel paid for.
- We shall very briefly touch upon these toll collectors, in the Extension part (as from Slide 404) below.
- So all the other parts of the next section serve to build up to the Extension section.

12.3. Domain Requirements

- Domain requirements cover all those aspects of the domain —
  - parts and materials,
  - actions,
  - events and
  - behaviours —
- which are to be supported by 'the machine'.

- Thus domain requirements are developed by systematically "revising" cum "editing" the domain description:
  - which parts are to be **projected:** left in or out;
  - which general descriptions are to be **instantiated** into more specific ones;
  - which non-deterministic properties are to be made more **determinate**; and
  - which parts are to be **extended** with such computable domain description parts which are not feasible without IT.

- Thus
  - projection,
  - instantiation,
  - determination and
  - extension

  are the basic engineering tasks of domain requirements engineering.

- An example may best illustrate what is at stake.
- The example is that of a tollway system —
  - in contrast to the general nets covered by description Items 126–129
  - (Slides 379–380).
  - See Fig. 4 on the next page.

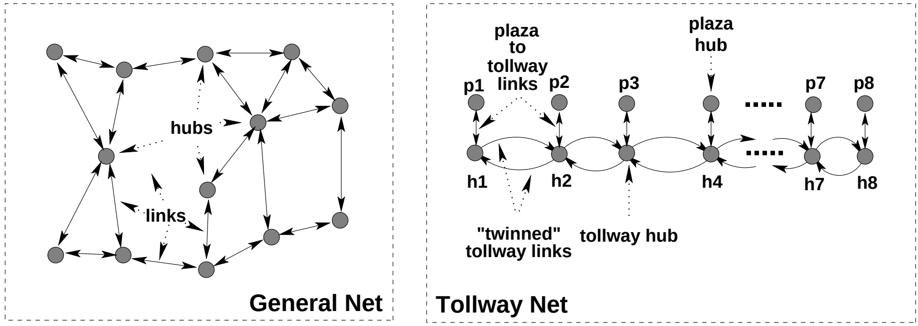


Figure 4: General and Tollway Nets

12.3.1. Projection

We keep what is needed to prescribe the tollway system and leave out the rest.

130. We keep the description, narrative and formalisation,
    (a) nets, hubs, links,
    (b) hub and link identifiers,
    (c) hub and link states,
131. as well as related observer functions.

type
130(a). N, H, L
130(b). HI, LI
130(c). HΣ, LΣ
value
131. obs_Hs,obs_Ls,obs_HI,obs_LI,
131. obs_HIs,obs_LIs,obs_HΣ,obs_L Σ

- We omit bringing the composite part concepts
- of HS, LS, Hs and Ls
- into the requirements.

12.3.2. Instantiation

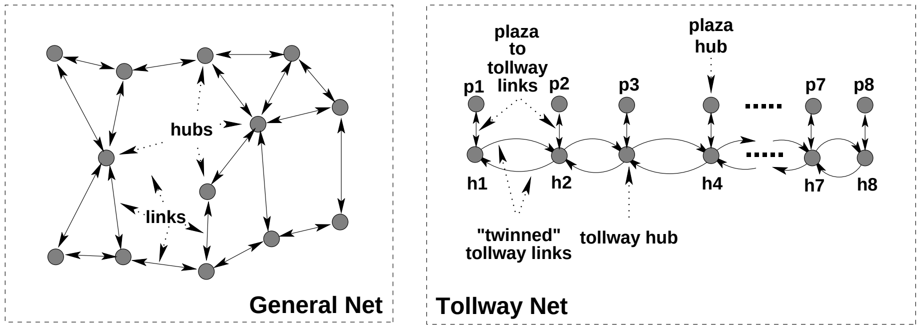


Figure 5: General and Tollway Nets

- From the general net model of earlier formalisations
  we instantiate, that is, make more concrete,
  the tollway net model now described.

132. The net is now concretely modelled as a pair of sequences.
133. One sequence models the plaza hubs, their plaza-to-tollway link and the connected tollway hub.
134. The other sequence models the pairs of “twinned” tollway links.
135. From plaza hubs one can observe their hubs and the identifiers of these hubs.
136. The former sequence is of m such plaza “complexes” where $m \geq 2$; the latter sequence is of $m-1$ “twinned” links.
137. From a tollway net one can abstract a proper net.
138. One can show that the posited abstraction function yields well-formed nets, i.e., nets which satisfy previously stated axioms.

```
type
132. TWN = PC* × TL*
133. PC = PH × L × H
134. TL = L × L
value
133. obs_H: PH → H, obs_HI: PH → HI
axiom
136. ∀ (pcl,tll):TWN •
136.    2≤len pcl∧len pcl=len tll+1
value
137. abs_N: TWN → N
137. abs_N(pcl,tll) as n
137.    pre: wf_TWN(pcl,tll)
137.    post:
137.     obs_Hs(n) =
137.       {h,h′|(h,_,h′):PC
137.       •(h,_,h′)∈ elems pcl}
137.    ∧ obs_Ls(n) =
137.       {l|(_,l,_):PC
137.        •(_,l,_)∈ elems pcl} ∪
137.       {l,l′|(l,l′):TL•(l,l′)∈ elems tll}

theorem:
138. ∀ twn:TWN • wf_TWN(twn)
138.       ⇒ wf_N(abs_N(twn))
```

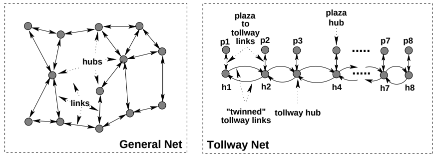


Figure 6: General and tollway Nets

12.3.2.1 Model Well-formedness wrt. Instantiation

- Instantiation restricts general nets to tollway nets.
- Well-formedness deals with proper mereology: that observed identifier references are proper.
- The well-formedness of instantiation of the tollway system model can be defined as follows:

139. The i'plaza complex, (p_i, l_i, h_i), is instantiation-well-formed if
    (a) link l_i identifies hubs p_i and h_i, and
    (b) hub p_i and hub h_i both identifies link l_i; and if
140. the i'th pair of twinned links, tl_i, tl'_i,
    (a) has these links identify the tollway hubs of the i'th and i+1'st plaza complexes ((p_i, l_i, h_i) respectively $(p_{i+1}, l_{i+1}, h_{i_1})$).


```
value
  Instantiation_wf_TWN: TWN → Bool
  Instantiation_wf_TWN(pcl,tll) ≡
139.    ∀ i:Nat · i ∈ inds pcl⇒
139.      let (pi,li,hi)=pcl(i) in
139(a).     obs_LIs(li)={obs_HI(pi),obs_HI(hi)}
139(b).    ∧ obs_LI(li)∈ obs_LIs(pi)∩ obs_LIs(hi)
140.     ∧ let (li′,li″) = tll(i) in
140.        i < len pcl ⇒
140.          let (pi′,li‴,hi′) = pcl(i+1) in
140(a).        obs_HIs(li) = obs_HIs(li′)
140(a).        = {obs_HI(hi),obs_HI(hi′)}
    end end end
```


12.3.3. Determination

- Determination, in this example, fixes states of hubs and links.
- The state sets contain only one set.
  - Twinned tollway links allow traffic only in opposite directions.
  - Plaza to tollway hubs allow traffic in both directions.
  - tollway hubs allow traffic to flow freely from
    - plaza to tollway links
    - and from incoming tollway links
    - to outgoing tollway links
    - and tollway to plaza links.
- The determination-well-formedness of the tollway system model can be defined as follows[30]:

[30] i ranges over the length of the sequences of twinned tollway links, that is, one less than the length of the sequences of plaza complexes. This "discrepancy" is reflected in out having to basically repeat formalisation of both Items 142(a) and 142(b).

12.3.3.1 Model Well-formedness wrt. Determination

- We need define well-formedness wrt. determination.
- Please study Fig. 7.

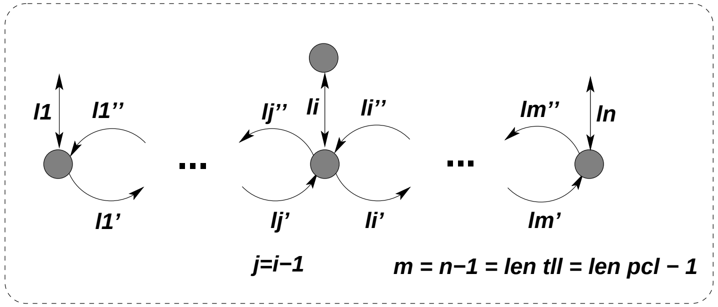


Figure 7: Hubs and Links

141. All hub and link state spaces contain just one hub, respectively link state.
142. The i'th plaza complex, pcl(i):(p_i, l_i, h_i) is determination-well-formed if
    - (a) l_i is open for traffic in both directions and
    - (b) p_i allows traffic from h_i to "revert"; and if
143. the i'th pair of twinned links (li', li'') (in the context of the i+1st plaza complex, pcl(i+1):$(p_{i+1}, l_{i+1}, h_{i+1})$) are determination-well-formed if
    - (a) link l'_i is open only from h_i to h_{i+1} and
    - (b) link l''_i is open only from h_{i+1} to h_i; and if
144. the jth tollway hub, h_j (for $1 \leq j \leq \mathbf{len}\,$pcl) is determination-well-formed if, depending on whether j is the first, or the last, or any "in-between" plaza complex positions,
    - (a) [the first:] hub $i = 1$ allows traffic in from l_1 and l''_1, and onto l_1 and l'_1.
    - (b) [the last:] hub $j = i + 1 = \mathbf{len}\,$pcl allows traffic in from $l_{\mathbf{len}\,\mathsf{tll}}$ and $l''_{\mathbf{len}\,\mathsf{tll}-1}$, and onto $l_{\mathbf{len}\,\mathsf{tll}}$ and $l'_{\mathbf{len}\,\mathsf{tll}-1}$.
    - (c) [in-between:] hub $j = i$ allows traffic in from l_i, l''_i and l'_i and onto l_i, l'_{i-1} and l''_i.


```
value
142. Determination_wf_TWN: TWN → Bool
142. Determination_wf_TWN(pcl,tll) ≡
142.    ∀ i:Nat• i ∈ inds tll ⇒
142.      let (pi,li,hi) = pcl(i),
142.          (npi,nli,nhi) = pcl(i+1), in
142.          (li′,li″) = tll(i) in
141.        obs_HΩ(pi)={obs_HΣ(pi)}∧obs_HΩ(hi)={obs_HΣ(hi)}
141.     ∧ obs_LΩ(li)={obs_LΣ(li)}∧obs_LΩ(li′)={obs_LΣ(li′)}
141.     ∧ obs_LΩ(li″)={obs_LΣ(li″)}
142(a).     ∧ obs_LΣ(li)
142(a).          = {(obs_HI(pi),obs_HI(hi)),(obs_HI(hi),obs_HI(pi))}
142(a).     ∧ obs_LΣ(nli)
142(a).          = {(obs_HI(npi),obs_HI(nhi)),(obs_HI(nhi),obs_HI(npi))}
142(b).     ∧ {(obs_LI(li),obs_LI(li))}⊆obs_HΣ(pi)
142(b).     ∧ {(obs_LI(nli),obs_LI(nli))}⊆obs_HΣ(npi)
143(a).     ∧ obs_LΣ(li′)={(obs_HI(hi),obs_HI(nhi))}
143(b).     ∧ obs_LΣ(li″)={(obs_HI(nhi),obs_HI(hi))}
144.     ∧ case i+1 of
144(a).        2 → obs_HΣ(h_1)=
144(a).              {(obs_LΣ(l_1),obs_LΣ(l_1)), (obs_LΣ(l_1),obs_LΣ(l_1″)),
144(a).              (obs_LΣ(l″_1),obs_LΣ(l_1)), (obs_LΣ(l″_1),obs_LΣ(l′_1))},
144(b).        len pcl → obs_HΣ(h_i+1)=
144(b).             {(obs_LΣ(l_len pcl),obs_LΣ(l_len pcl)),
144(b).              (obs_LΣ(l_len pcl),obs_LΣ(l′_len tll)),
144(b).              (obs_LΣ(l″_len tll),obs_LΣ(l_len pcl)),
144(b).              (obs_LΣ(l″_len tll),obs_LΣ(l′_len tll))},
144(c).        _ → obs_HΣ(h_i)=
144(c).             {(obs_LΣ(l_i),obs_LΣ(l_i)), (obs_LΣ(l_i),obs_LΣ(l′_i)),
144(c).              (obs_LΣ(l_i),obs_LΣ(l″_i−1)), (obs_LΣ(l″_i),obs_LΣ(l′_i)),
144(c).              (obs_LΣ(l″_i),obs_LΣ(l′_i−1)), (obs_LΣ(l″_i),obs_LΣ(l′_i))}
142.      end end
```


12.3.4. Extension

- By **domain extension** we understand the
    - *introduction of domain entities, actions, events and behaviours that were not feasible in the original domain,*
    - *but for which, with computing and communication,*
    - *there is the possibility of feasible implementations,*
    - *and such that what is introduced become part of the emerging domain requirements prescription.*

12.3.4.1 Narrative

- The **domain extension** is that of the controlled access of vehicles to and departure from the tollway net:
  - the entry to (and departure from) tollgates from (respectively to) an `"an external"` net — which we do not describe;
  - the new entities of tollgates with all their machinery;
  - the user/machine functions:
    - upon entry:
      - ∗ driver pressing entry button,
      - ∗ tollgate delivering ticket;
    - upon exit:
      - ∗ driver presenting ticket,
      - ∗ tollgate requesting payment,
      - ∗ driver providing payment, etc.

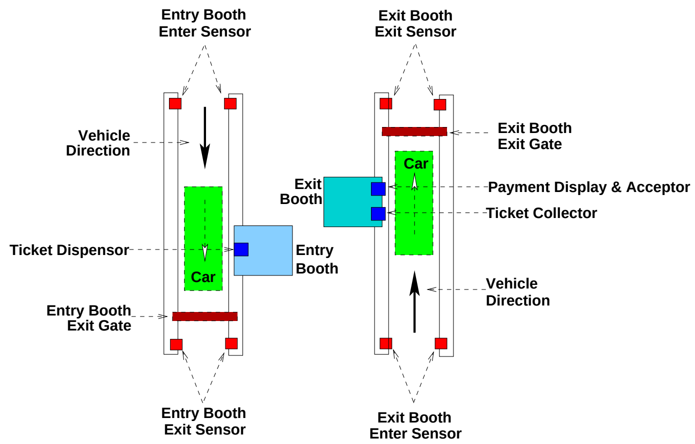


Figure 8: Entry and Exit Tollbooths

- One added (extended) domain requirements:
  - as vehicles are allowed to cruise the entire net
  - payment is a function of the totality of links traversed, possibly multiple times.
- This requires, in our case,
  - that tickets be made such as to be sensed somewhat remotely,
  - and that hubs be equipped with sensors which can record
  - and transmit information about vehicle hub crossings.
    - (When exiting, the tollgate machine can then access the exiting vehicles' sequence of hub crossings — based on which a payment fee calculation can be done.)
    - All this to be described in detail — including all the things that can go wrong (in the domain) and how drivers and tollgates are expected to react.

- We omit details of narration and formalisation.
  - In this case the extension description would entail a number of formalisations:
    - An initial one which relies significantly on the use of `RSL/CSP` [CARH:Electronic,TheSEBook1wo].
      It basically models tollbooth and vehicle behaviours.
    - A "derived" one which models temporal properties.
      It is expressed, for example, in the `Duration Calculus, DC` [zcc+mrh2002].
    - And finally a timed-automata [AluDil:94,olderogdirks2008] model which "implements" the `DC` model.

12.4. Interface Requirements Prescription

- A systematic reading of the domain requirements shall
  - result in an identification of all shared
    - parts and materials,
    - actions,
    - events and
    - behaviours.
- An entity is said to be a shared entity if it is mentioned in both
  - the domain description and
  - the requirements prescription.
- That is, if the entity
  - is present in the domain and
  - is to be present in the machine.

- Each such shared phenomenon shall then be individually dealt with:
  - **part** and **materials sharing** shall lead to interface requirements for **data initialisation and refreshment;**
  - **action sharing** shall lead to interface requirements for **interactive dialogues between the machine and its environment;**
  - **event sharing** shall lead to interface requirements for **how events are communicated between the environment of the machine and the machine.**
  - **behaviour sharing** shall lead to interface requirements for **action and event dialogues between the machine and its environment.**

12.4.1. Shared Parts

- The main shared parts of the main example of this section are
  - the net, hence the hubs and the links.
- As domain parts they repeatedly undergo changes with respect to the values of a great number of attributes and otherwise possess attributes — most of which have not been mentioned so far:
  - length, cadestral information, namings,
  - wear and tear (where-ever applicable),
  - last/next scheduled maintenance (where-ever applicable),
  - state and state space, and
  - many others.

- We “split” our interface requirements development into two separate steps:
  - the development of $d_{r.net}$
    - (the common domain requirements for the shared hubs and links),
  - and the co-development of $d_{r.db:i/f}$
    - (the common domain requirements for the interface between $d_{r.net}$ and DB_{rel} —
- under the assumption of an available relational database system $DB_{\rm rel}$

- When planning the common domain requirements for the net, i.e., the hubs and links,
  - we enlarge our scope of requirements concerns beyond the two so far treated ($d_{r.toll}$, $d_{r.maint.}$)
  - in order to make sure that the shared relational database of nets, their hubs and links, may be useful beyond those requirements.

- We then come up with something like
  - hubs and links are to be represented as tuples of relations;
  - each net will be represented by a pair of relations
    - a hubs relation and a links relation;
    - each hub and each link may or will be represented by several tuples;
  - etcetera.
- In this database modelling effort it must be secured that "standard" actions on nets, hubs and links can be supported by the chosen relational database system DB_{rel}.

12.4.1.1 Data Initialisation

- As part of $d_{r.net}$ one must prescribe **data initialisation**, that is provision for
  - an interactive user interface dialogue with a set of proper display screens,
    - one for establishing net, hub or link attributes (names) and their types and,
    - for example, two for the input of hub and link attribute values.
  - Interaction prompts may be prescribed:
    - next input,
    - on-line vetting and
    - display of evolving net, etc.
  - These and many other aspects may therefore need prescriptions.
- Essentially these prescriptions concretise the insert link action.

12.4.1.2 Data Refreshment

- As part of $d_{r.net}$ one must also prescribe **data refreshment**:
  - an interactive user interface dialogue with a set of proper display screens
    - one for updating net, hub or link attributes (names) and their types and,
    - for example, two for the update of hub and link attribute values.
  - Interaction prompts may be prescribed:
    - next update,
    - on-line vetting and
    - display of revised net, etc.
  - These and many other aspects may therefore need prescriptions.
- These prescriptions concretise remove and insert link actions.

12.4.2. Shared Actions

- The main **shared actions** are related to
  - the entry of a vehicle into the tollway system and
  - the exit of a vehicle from the tollway system.

12.4.2.1 Interactive Action Execution

- As part of $d_{r.toll}$ we must therefore prescribe
  - the varieties of successful and less successful sequences
  - of interactions between vehicles (or their drivers) and the toll gate machines.
- The prescription of the above necessitates determination of a number of external events, see below.
- (Again, this is an area of embedded, real-time safety-critical system prescription.)

12.4.3. Shared Events

- The main **shared external events** are related to
  - the entry of a vehicle into the tollway system,
  - the crossing of a vehicle through a tollway hub and
  - the exit of a vehicle from the tollway system.
- As part of $d_{r.toll}$ we must therefore prescribe
  - the varieties of these events,
  - the failure of all appropriate sensors and
  - the failure of related controllers:
    - gate opener and closer (with sensors and actuators),
    - ticket “emitter” and “reader” (with sensors and actuators),
    - etcetera.
- The prescription of the above necessitates extensive fault analysis.

12.4.4. Shared Behaviours

- The main **shared behaviours** are therefore related to
  - the journey of a vehicle through the tollway system and
  - the functioning of a toll gate machine during “its lifetime”.
- Others can be thought of, but are omitted here.
- In consequence of considering, for example, the journey of a vehicle behaviour, we may “add” some further, extended requirements:
  - requirements for a vehicle statistics “package”;
  - requirements for tracing supposedly “lost” vehicles;
  - requirements limiting tollway system access in case of traffic congestion; etcetera.

12.5. Machine Requirements

- The machine requirements
  - make hardly any concrete reference to the domain description;
  - so we omit its treatment altogether.

12.6. Discussion of Requirements "Derivation"

- We have indicated
  - how the **domain engineer**
  - and the **requirements engineer**
  - can work together
  - to "derive" significant fragments
  - of a **requirements prescription**.

- This puts **requirements engineering** in a new light.
  - Without a previously existing **domain description**s
  - the **requirements engineer** has to do double work:
    - both **domain engineering**
    - and **requirements engineering**
  - but without the **principles** of **domain description**,
    - as laid down in this tutorial
  - that job would not be so straightforward as we now suggest.

End of Lecture 8: First Session — Requirements Engineering

Domain and Interface Requirements

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

SHORT BREAK

FINAL LAST HAUL !

Begin of Lecture 9: Last Session — Conclusion

Comparisons and What Have We Achieved

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

Tutorial Schedule

13. Conclusion

- This document,
  - meant as the basis for my tutorial
  - at FM 2012 (CNAM, Paris, August 28),
  - "grew" from a paper being written for possible journal publication.
    - Sections 2–3 possibly represent two publishable journal papers.
    - Section 4 has been "added" to the 'tutorial' notes.

- The style of the two tutorial "parts",
  - Sects. 2–3 and
  - Sect. 4
  - are, necessarily, different:
    - Sects. 2–3 are in the form of research notes,
    - whereas Sect. 4 is in the form of "lecture notes" on methodology.
  - Be that as it may. Just so that you are properly notified !

13.1. Comparison to Other Work

- In this section we shall only compare
  - our contribution to domain engineering as presented in the section on domain entities
  - to that found in the broader literature with respect to the software engineering term 'domain'.
- We shall not compare
  - our contribution to requirements engineering
  - as surveyed in the section on requirements engineering.
  - to that, also, found in the broader requirements engineering literature.
- Finally we shall also not compare
  - our work on a description calculus
  - as we find no comparable literature!

13.1.1. Ontological Engineering:

- Ontological engineering is described mostly on the Internet, see however [Benjamins+Fensel98].
- Ontology engineers build ontologies.
- And ontologies are, in the tradition of ontological engineering, *"formal representations of a set of concepts within a domain and the relationships between those concepts"* — expressed usually in some logic.
- Published ontologies usually consists of thousands of logical expressions.
- These are represented in some, for example, low-level mechanisable form so that they can be interchanged between ontology groups building upon one-anothers work and processed by various tools.

- There does not seem to be a concern for "deriving" such ontologies into requirements for software.
- Usually ontology presentations
  - either start with the presentation
  - or makes reference to its reliance

  of an upper ontology.
- Instead the ontology databases
  - appear to be used for the computerised
  - discovery and analysis
  - of relations between ontologies.

- The TripTych form of domain science & engineering differs from conventional **ontological engineering** in the following, essential ways:
  - The TripTych domain descriptions rely essentially on a "built-in" **upper ontology**:
    - types, abstract as well as model-oriented (i.e., concrete) and
    - actions, events and behaviours.
  - Domain science & engineering is not, to a first degree, concerned with modalities, and hence do not focus on the modelling of
    - knowledge and belief,
    - necessity and possibility, i.e., alethic modalities,
    - epistemic modality (certainty),
    - promise and obligation (deontic modalities),
    - etcetera.

13.1.2. Knowledge and Knowledge Engineering:

- The concept of **knowledge** has occupied philosophers since Plato.
  - No common agreement on what 'knowledge' is has been reached.
  - From Wikipedia we may learn that
    - *knowledge is a familiarity with someone or something;*
      - *it can include facts, information, descriptions, or skills acquired through experience or education;*
      - *it can refer to the theoretical or practical understanding of a subject;*
    - *knowledge is produced by socio-cognitive aggregates*
      - *(mainly humans)*
      - *and is structured according to our understanding of how human reasoning and logic works.*

- The aim of **knowledge engineering** was formulated, in 1983, by an originator of the concept, Edward A. Feigenbaum [Feigenbaum83]:
  - **knowledge engineering** is an engineering discipline
  - that involves integrating knowledge into computer systems
  - in order to solve complex problems
  - normally requiring a high level of human expertise.

- **Knowledge engineering** focuses on
  - continually building up (acquire) large, shared data bases (i.e., **knowledge bases**),
  - their continued maintenance,
  - testing the validity of the stored 'knowledge',
  - continued experiments with respect to **knowledge representation**,
  - etcetera.

- Knowledge engineering can, perhaps, best be understood in contrast to algorithmic engineering:
  - ⟐ In the latter we seek more-or-less conventional, usually imperative programming language expressions of algorithms
    - ⊚ whose algorithmic structure embodies the knowledge
    - ⊚ required to solve the problem being solved by the algorithm.
  - ⟐ The former seeks to solve problems based on an interpreter inferring possible solutions from logical data. This logical data has three parts:
    - ⊚ a collection that "mimics" the semantics of, say, the imperative programming language,
    - ⊚ a collection that formulates the problem, and
    - ⊚ a collection that constitutes the knowledge particular to the problem.
- We refer to [BjornerNilsson1992].

- The concerns of TripTych domain science & engineering is based on that of algorithmic engineering.
  - ⟐ Domain science & engineering is not aimed at
    - ⊚ letting the computer solve problems based on
    - ⊚ the knowledge it may have stored.
  - ⟐ Instead it builds models based on knowledge of the domain.
- Further references to seminal exposés of knowledge engineering are [Studer1998,Kendal2007].

13.1.3. Domain Analysis:

- There are different "schools of domain analysis".
  - ⟐ Domain analysis, or product line analysis (see below), as it was first conceived in the early 1980s by James Neighbors
    - ⊚ is the analysis of related software systems in a domain
    - ⊚ to find their common and variable parts.
    - ⊚ It is a model of wider business context for the system.
  - ⟐ This form of domain analysis turns matters "upside-down":
    - ⊚ it is the set of software "systems" (or packages)
    - ⊚ that is subject to some form of inquiry,
    - ⊚ albeit having some domain in mind,
    - ⊚ in order to find common features of the software
    - ⊚ that can be said to represent a named domain.

- In this section we shall mainly be comparing the TripTych approach to domain analysis to that of Reubén Prieto-Dĩaz's approach [Prieto-Diaz:1987,Prieto-Diaz:1990,Prieto-Diaz:1991].
- Firstly, the two meanings of domain analysis basically coincide.
- Secondly, in, for example, [Prieto-Diaz:1987], Prieto-Dĩaz's domain analysis is focused on the very important stages that precede the kind of domain modelling that we have described:
  - ⟐ major concerns are
    - ⊚ selection of what appears to be similar, but specific entities,
    - ⊚ identification of common features,
    - ⊚ abstraction of entities and
    - ⊚ classification.
  - ⟐ Selection and identification is assumed in our approach, but we suggest to follow the ideas of Prieto-Dĩaz.
  - ⟐ Abstraction (from values to types and signatures) and classification into parts, materials, actions, events and behaviours is what we have focused on.

- All-in-all we find Prieto-Dĩaz's work very relevant to our work:
  - relating to it by providing guidance to pre-modelling steps,
  - thereby emphasising issues that are necessarily informal,
  - yet difficult to get started on by most software engineers.
- Where we might differ is on the following:
  - although Prieto-Dĩaz does mention a need for domain specific languages,
  - he does not show examples of domain descriptions in such DSLs.
  - We, of course, basically use mathematics as the DSL.
- In the TripTych approach to domain analysis
  - we provide a full ontology — cf. Sects. 2.–10. and
  - suggest a domain description calculus.
- In our approach
  - we do not consider requirements, let alone software components,
  - as do Prieto-Dĩaz,

  but we find that that is not an important issue.

13.1.4. Software Product Line Engineering:

- Software product line engineering, earlier known as domain engineering,
  - is the entire process of reusing domain knowledge in the production of new software systems.
- Key concerns of software product line engineering are
  - reuse,
  - the building of repositories of reusable software components, and
  - domain specific languages with which to, more-or-less automatically build software based on reusable software components.

- These are not the primary concerns of TripTych domain science & engineering.
  - But they do become concerns as we move from domain descriptions to requirements prescriptions.
  - But it strongly seems that software product line engineering is not really focused on the concerns of domain description — such as is TripTych domain engineering.
  - It seems that software product line engineering is primarily based, as is, for example, FODA: Feature-oriented Domain Analysis, on analysing features of software systems.
  - Our [dines-maurer] puts the ideas of software product lines and model-oriented software development in the context of the TripTych approach.
- Notable sources on software product line engineering are [dom:Bayer:1999,dom:Weiss:1999,dom:Ardis:2000,dom:Thiel:2000,dom:Ha

13.1.5. Problem Frames:

- The concept of problem frames is covered in [mja2001a].
- Jackson's prescription for software development focuses on the "triple development" of descriptions of
  - the problem world,
  - the requirements and
  - the machine (i.e., the hardware and software) to be built.
- Here domain analysis means, the same as for us, the problem world analysis.

- In the problem frame approach the software developer plays three, that is, all the TripTych rôles:
  - domain engineer,
  - requirements engineer and
  - software engineer

  "all at the same time",
- well, iterating between these rôles repeatedly.
- So, perhaps belabouring the point,
  - domain engineering is done only to the extent needed by the prescription of requirements and the design of software.
- These, really are minor points.

- But in "restricting" oneself to consider
  - only those aspects of the domain which are mandated by the requirements prescription
  - and software design

  one is considering a potentially smaller fragment [Jackson2010Facs] of the domain than is suggested by the TripTych approach.
- At the same time one is, however, sure to
  - consider aspects of the domain
  - that might have been overlooked when pursuing domain description development
  - the TripTych, "more general", approach.

13.1.6. Domain Specific Software Architectures (DSSA):

- It seems that the concept of DSSA
  - was formulated by a group of ARPA[31] project "seekers"
  - who also performed a year long study (from around early-mid 1990s);
  - key members of the DSSA project were Will Tracz, Bob Balzer, Rick Hayes-Roth and Richard Platek [dom:Trasz:1994].
- The [dom:Trasz:1994] definition of domain engineering is *"the process of creating a DSSA:*
  - *domain analysis and domain modelling*
  - *followed by creating a software architecture*
  - *and populating it with software components."*

[31] ARPA: The US DoD Advanced Research Projects Agency

- This definition is basically followed also by [Mettala+Graham:1992,Shaw+Garlan:1996,Medvidovic+Colbert:2004].
- Defined and pursued this way, DSSA appears,
  - notably in these latter references, to start with the
  - with the analysis of software components, "per domain",
  - to identify commonalities within application software,
  - and to then base the idea of software architecture
  - on these findings.

- Thus DSSA turns matter "upside-down" with respect to TripTych requirements development
  - by starting with software components,
  - assuming that these satisfy some requirements,
  - and then suggesting domain specific software
  - built using these components.
- This is not what we are doing:
  - We suggest that requirements
    - can be "derived" systematically from,
    - and related back, formally to domain descriptionss
    - without, in principle, considering software components,
    - whether already existing, or being subsequently developed.

- Of course, given a domain descriptions
  - it is obvious that one can develop, from it, any number of requirements prescriptions
  - and that these may strongly hint at shared, (to be) implemented software components;
- but it may also, as well, be the case
  - two or more requirements prescriptions
  - "derived" from the same domain description
  - may share no software components whatsoever !
- So that puts a "damper" of my "enthusiasm" for DSSA.

- It seems to this author that had the DSSA promoters
  - based their studies and practice on also using formal specifications,
  - at all levels of their study and practice,
  - then some very interesting insights might have arisen.

13.1.7. Domain Driven Design (DDD)

- Domain-driven design (DDD)[32]
  - *"is an approach to developing software for complex needs*
  - *by deeply connecting the implementation to an evolving model of the core business concepts;*
  - *the premise of domain-driven design is the following:*
    - *placing the project's primary focus on the core domain and domain logic;*
    - *basing complex designs on a model;*
    - *initiating a creative collaboration between technical and domain experts to iteratively cut ever closer to the conceptual heart of the problem."*[33]

[32] Eric Evans: http://www.domaindrivendesign.org/
[33] http://en.wikipedia.org/wiki/Domain-driven_design

- We have studied some of the DDD literature,
  - mostly only accessible on The Internet, but see also [Haywood2009],
  - and find that it really does not contribute to new insight into domains such as wee see them:
  - it is just "plain, good old software engineering cooked up with a new jargon.

13.1.8. Feature-oriented Domain Analysis (FODA):

- Feature oriented domain analysis (FODA)
  - is a domain analysis method
  - which introduced feature modelling to domain engineering
  - FODA was developed in 1990 following several U.S. Government research projects.
  - Its concepts have been regarded as critically advancing software engineering and software reuse.
- The US Government supported report [KyoKang+et.al.:1990] states: *"FODA is a necessary first step"* for software reuse.

- To the extent that
  - TripTych domain engineering
  - with its subsequent requirements engineering

  indeed encourages reuse at all levels:
  - domain descriptions and
  - requirements prescription,

  we can only agree.
- Another source on FODA is [Czarnecki2000].
- Since FODA "leans" quite heavily on 'Software Product Line Engineering' our remarks in that section, above, apply equally well here.

13.1.9. Unified Modelling Language (UML)

- Three books representative of UML are [Booch98,Rumbaugh98,Jacobson99].
- The term domain analysis appears numerous times in these books,
  - yet there is no clear, definitive understanding
  - of whether it, the domain, stands for entities in the domain such as we understand it,
  - or whether it is wrought up, as in several of the 'approaches' treated in this section, to wit, Items [3,4,6,7,8], with
    - either software design (as it most often is),
    - or requirements prescription.

- Certainly, in UML,
  - in [Booch98,Rumbaugh98,Jacobson99] as well as
  - in most published papers claiming "adherence" to UML,
  - that domain analysis usually
    - is manifested in some UML text
    - which "models" some requirements facet.
  - Nothing is necessarily wrong with that;
  - but it is therefore not really the TripTych form of domain analysis
    - with its concepts of abstract representations of endurant and perdurants, and
    - with its distinctions between domain and requirements, and
    - with its possibility of "deriving"
      - requirements prescriptions from
      - domain descriptions.

- There is, however, some important notions of UML
  - and that is the notions of
    - class diagrams,
    - objects, etc.
  - How these notions relate to the discovery
    - of part types, unique part identifiers, mereology and attributes, as well as
    - action, event and behaviour signatures and channels,
  - as discovered at a particular domain index,
  - is not yet clear to me.
  - That there must be some relation seems obvious.
- We leave that as an interesting, but not too difficult, research topic.

13.1.10. Requirements Engineering:

- There are in-numerous books and published papers on requirements engineering.
  - A seminal one is [AvanLamsweerde2009].
  - I, myself, find [SorenLauesen2002] full of very useful, non-trivial insight.
  - [Dorfman+Thayer:1997:IEEEComp.Soc.Press] is seminal in that it brings a number or early contributions and views on requirements engineering.

- Conventional text books, notably [Pfleeger2001,Pressman2001,Sommerville2006] all have their "mandatory", yet conventional coverage of requirements engineering.
  - None of them "derive" requirements from domain descriptions,
    - yes, OK, from domains,
    - but since their description is not mandated
    - it is unclear what "the domain" is.
  - Most of them repeatedly refer to domain analysis
    - but since a written record of that domain analysis is not mandated
    - it is unclear what "domain analysis" really amounts to.

- Axel van Laamsweerde's book [AvanLamsweerde2009] is remarkable.
  - Although also it does not mandate descriptions of domains
  - it is quite precise as to the relationships between domains and requirements.
  - Besides, it has a fine treatment of the distinction between goals and requirements,
  - also formally.
- Most of the advices given in [SorenLauesen2002]
  - can beneficially be followed also in
  - TripTych requirements development.
- Neither [AvanLamsweerde2009] nor [SorenLauesen2002] preempts TripTych requirements development.

13.1.11. Summary of Comparisons

- It should now be clear from the above that
  - basically only Jackson's *problem frames* really take
    - the same view of domains and,
    - in essence, basically maintain similar relations between
      - requirements prescription and
      - domain description.
  - So potential sources of, we should claim, mutual inspiration
    - ought be found in one-another's work —
    - with, for example, [ggjz2000,Jackson2010Facs],
    - and the present document,
    - being a good starting point.

13.2. What Have We Achieved and Future Work

- Sect. 13.1 has already touched upon, or implied,
  - a number of 'achievement' points and
  - issues for future work.
- Here is a summary of 'achievement' and future work items.

- We claim that there are three major contributions being reported upon:
  - (i) the separation of domain engineering from requirements engineering,
  - (ii) the separate treatment of domain science & engineering:
    - as "free-standing" with respect, ultimately, to computer science,
    - and endowed with quite a number of domain analysis principles and domain description principles; and
  - (iii) the identification of a number of techniques
    - for "deriving" significant fragments of requirements prescriptions from domain descriptions —
    - where we consider this whole relation between domain engineering and requirements engineering to be novel.

- Yes, we really do consider the possibility of a systematic
  - 'derivation' of significant fragments of requirements prescriptions from domain descriptions
  - to cast a different light on requirements engineering.
- What we have not shown in this tutorial is
  - the concept of domain facets;
  - this concept is dealt with in [dines:facs:2008] —
  - but more work has to be done to give a firm theoretical understanding of domain facets of
    - domain intrinsics,
    - domain support technology,
    - domain scripts,
    - domain rules and regulations,
    - domain management and organisation, and
    - human domainbehaviour.

13.3. General Remarks

- Perhaps belaboring the point:
  - one can pursue creating and studying domain descriptions
  - without subsequently aiming at requirements development,
  - let alone software design.
- That is, domain descriptions
  - can be seen as
    - "free-standing",
    - of their "own right",
    - useful in simply just understanding
    - domains in which humans act.

- Just like it is deemed useful
  - that we study "Mother Nature",
  - the physical world around us,
  - given before humans "arrived";
- so we think that
  - there should be concerted efforts to study and create domain models,
  - for use in
    - studying "our man-made domains of discourses";
    - possibly proving laws about these domains;
    - teaching, from early on, in middle-school, the domains in which the middle-school students are to be surrounded by;
    - etcetera

- How far must one formalise such domain descriptions ?
  - Well, enough, so that possible laws can be mathematically proved.
  - Recall that domain descriptions usually will or must be developed by domain researchers — not necessarily domain engineers —
    - in research centres, say universities,
    - where one also studies physics.

- And, when we base **requirements development** on **domain descriptions**,
  - as we indeed advocate,
  - then the **requirements engineers**
  - must understand the formal **domain descriptions**,
  - that is, be able to perform formal
    - **domain projection**,
    - **domain determination**,
    - **domain instantiation**,
    - **domain extension**,

    etcetera.

- This is similar to the situation in classical engineering
  - which rely on the sciences of physics,
  - and where, for example,
    - *Bernoulli's equations*,
    - *Maxwell's equations*,
    - *Navier-Stokes equations*,
    - etcetera
  - were developed by physicists and mathematicians,
  - but are used, daily, by engineers:
    - read and understood,
    - massaged into further differential equations, etcetera,
    - in order to calculate (predict, determine values), etc.

- Nobody would hire non-skilled labour
  - for the engineering development of airplane designs
    - unless that "labourer" was skilled in *Navier-Stokes equations*,

  or
  - for the design of mobile telephony transmission towers
    - unless that person was skilled in *Maxwell's equations*.

- So we must expect a future, we predict,
  - where a subset of the software engineering candidates from universities
    - are highly skilled in the development of
      - formal **domain descriptions**
      - formal **requirements prescriptions**
  - in at least one domain, such as
    - *transportation*, for example,
      - air traffic,
      - road traffic and
      - railway systems,
      - shipping;

      or
    - *manufacturing*,
    - *services* (health care, public administration, etc.),
    - *financial industries*, or the like.

13.4. Acknowledgements

- I thank the tutorial organisers of the `FM 2012` event for accepting my Dec. 31. 2011 tutorial proposal.
- I thank that part of participants
  - who first met up for this tutorial this morning (Tuesday 28 August, 2012)
  - to have remained in this room for most, if not all of the time.
- I thank colleagues and PhD students around Europe
  - for having listened to previous,
  - somewhat less polished versions of this tutorial.
  - I in particular thank Drs. **Magne Haveraaen** and **Marc Bezem** of the University of Bergen for providing an important step in the development of the present material.
- And I thank my wife
  - for her patience during the spring and summer of 2012
  - where I ought to have been tending to the garden, etc. !

End of Lecture 9: Last Session — Conclusion

Comparisons and What Have We Achieved

FM 2012 Tutorial, Dines Bjørner, Paris, 28 August 2012

THANKS AGAIN — HAVE A NICE CONFERENCE

14. On A Theory of Transport Nets

- This section is under development.
  - The idea of this section is
    - not so much to present a transport domain description,
    - but rather to present fragments, "bits and pieces", of a theory of such a domain.
- The purpose of having a theory
  - is to "draw" upon the 'bits and pieces'
  - when expressing
    - properties of endurants and
    - definitions of
      - actions,
      - events and
      - behaviours.
- Again: this section is very much in embryo.

14.1. Some Pictures

- Nets can either be
  - rail nets,
  - road nets,
  - shipping lanes, or
  - air traffic nets.
- The following pictures illustrate some of these nets.


A rail net; a traffic light

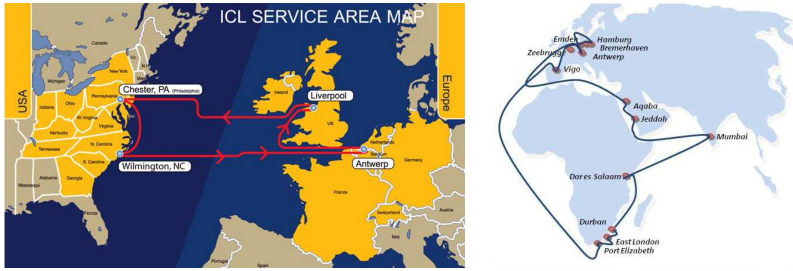
A freeway hub

Another freeway hub

- The left side of the road roundabout below is rather special.
  - Its traffic lights are also located in the inner circle of the roundabout.
  - One drives in,
    - at green light,
    - and may be guided by striping,
    - depending on where one is driving,
    - either directly to an outgoing link,
    - or is queued up against a red light
    - awaiting permission to continue.

A roundabout

- The map below left is for a container line serving one route between Liverpool (UK), Chester (PA, USA), Wilmington (NC, USA) and Antwerp (Belgium), an so forth, circularly.
- The map below right is an "around Africa" Mitsui O.S.K. Line.



Two shipping line nets

14.2. Parts

14.2.1. Nets, Hubs and Links

145. From a transport net one can observe sets of hubs and links.

type
145. N, H, L
value
145. obs_Hs: N → H-**set**, obs_Ls: N → L-**set**

14.2.2. Mereology

146. From hubs and links one can observe their unique hub, respectively link identifiers and their respective mereologies.
147. The mereology of a link identifies exactly two distinct hubs.
148. The mereologies of hubs and links must identify actual links and hubs of the net.

type
146. HI, LI
value
146. uid_H: H → HI, uid_L: L → LI
146. mereo_H: H → LI-**set**, mereo_L: L → HI-**set**
axiom
147. ∀ l:L•**card** mereo_L(l)=2
148. ∀ n:N,l:L•l ∈ obs_Ls(n) ⇒
148. ∧ ∀ hi:HI•hi ∈ mereo_L(l)
148. ⇒ ∃ h:h•h ∈ obs_Hs(n)∧uid_H(h)=hi
148. ∧ ∀ h:H•h ∈ obs_Hs(n) ⇒
148. ∀ li:LI•li ∈ mereo_H(h)
148. ⇒ ∃ l:L•l ∈ obs_Ls(n)∧uid_L(l)=li

14.2.3. An Auxiliary Function

149. For every net we can define functions which
   (a) extracts all its link identifiers,
   (b) and all its hub identifiers.

value
149(a). xtr_HIs: N → HI-**set**
149(a). xtr_HIs(n) ≡ {uid_H(h)|h:H·h ∈ obs_Hs(n)}
149(b). xtr_LIs: N → LI-**set**
149(b). xtr_LIs(n) ≡ {uid_L(l)|l:L·l ∈ obs_Ls(n)}

14.2.4. Retrieving Hubs and Links

150. We can also define functions which
   (a) given a net and a hub identifier obtains the designated hub, respectively
   (b) given a net and a link identifier obtains the designated link.

value
150(a). get_H: N → HI $\xrightarrow{\sim}$ H
150(a). get_H(n)(hi) **as** h
150(a).   **pre** hi ∈ xtr_HIs(n)
150(a).   **post** h ∈ obs_Hs(n)∧hi=uid_H(h)
150(b). get_L: N → LI $\xrightarrow{\sim}$ L
150(b).   **pre** li ∈ xtr_LIs(n)
150(b).   **post** l ∈ obs_Ls(n)∧li=uid_L(l)

14.2.5. Invariants over Link and Hub States and State Spaces

151. Links include two attributes:
   (a) Link states. These are sets of pairs of the identifiers of the hubs to which the links are connected.
   (b) Link state spaces. These are the sets of link states that a link may attain.
152. The link states must mention only those hub identifiers of the two hubs to which the link is connected.
153. The link state spaces must likewise mention only such link states as are defined in Items 151(a) and 152.

type
151(a). LΣ = (HI×HI)-**set** **axiom** ∀ lσ:LΣ·**card** lσ≤2
151(b). LΩ = LΣ-**set**
value
151(a). attr_LΣ: L → LΣ
151(b). attr_LΩ: L → LΩ
axiom
152. ∀ l:L, lσ':LΣ · lσ' ∈ attr_LΩ(l)
152.   ⇒ lσ' ⊆ {(hi,hi′)|hi,hi′:HI·{hi,hi′}⊆mereo_L(l)}
152.   ∧ attr_LΣ(l) ∈ attr_LΩ(l)

154. Hubs include two attributes:
    (a) Hub states. These are sets of pairs of identifiers of the links to which the hubs are connected.
    (b) Hub state spaces. These are the sets of hub states that a hub may attain.
155. The hub states must mention only those link identifiers of the links to which the hub is connected.
156. The hub state spaces must likewise mention only such hub states as are defined in Items 154(a) and 155.

type
154(a). $H\Sigma = (LI \times LI)$**-set**
154(b). $H\Omega = H\Sigma$**-set**
value
154(a). attr_HΣ: $H \rightarrow H\Sigma$
154(b). attr_HΩ: $H \rightarrow H\Omega$
axiom
155. $\forall$ h:H, $h\sigma'$:HΣ $\cdot$ $h\sigma' \in$ attr_HΩ(h)
155. $\Rightarrow h\sigma' \subseteq \{(li,li')|li,li':LI \cdot \{li,li'\} \subseteq$ mereo_H(h)$\}$
155. $\wedge$ attr_HΣ(h) $\in$ attr_HΩ(h)

14.2.6. Maps

- A map is an abstraction of a net.
    - The map just shows the hub and link identifiers of the net, and hence its mereology.

type
  Map′ = HI $\overrightarrow{m}$ (LI $\overrightarrow{m}$ HI)
  Map = {|m:Map′·wf_Map(m)|}
value
  wf_Map: Map′ → **Bool**
  wf_Map(m) ≡ **dom** m = ∪ { **rng** lhm | lhm:(LI $\overrightarrow{m}$ HI) · lhm ∈ **rng** m }

- Let m be a map.
- The *definition set* of the map is domṁ.
- Let hi be in the definition set of map m.
- Then m(hi) is the *image* of hi in m.
- Let li be in the image of m(hi), that is, liISINdom (m(hi)), then hi'=(m(hi))(li) is the *target* of li in m(hi).

- Given a net which satisfies the axiom concerning mereology
- one can extract from that net a corresponding map.

```
value
  xtr_Map: N → Map
  xtr_Map(n) ≡
    [ hi ↦ [ li ↦ uid_H(retr_H(n)(hi)(li))
            | li:LI · li ∈ mere_H(get_H(n)(hi)) ]
      | h:H,hi:HI · h ∈ obs_Hs(n) ∧ hi = uid_H(h) ]
```


- The retrieve hub function
  - retrieve the "second" hub, i.e., "at the other end", of
  - a link wrt. a "first" hub.

```
retr_H: N → HI → LI → H
retr_H(n)(hi)(li) ≡
  let h = get_H(n)(hi) in
  let l = get_L(n)(li) in
  let {hi″} = mereo_L(l)\{hi} in
  get_H(n)(hi″) end end end
  pre: hi ∈ mereo_L(get_L(n)(li))

xtr_LIs: Map → LI-set
xtr_LIs(m) = ∪ {dom(m(hi))|hi:HI · hi ∈ dom m}
```


14.2.7. Routes

157. A route is an alternating sequence of hub and link identifiers.

```
157. R′ = (HI|LI)^ω, R = {|r:R′·wf_R(r)|}
value
157. wf_R: R′ → Bool
157. wf_R(r) ≡
157.   ∀ i:Nat · {i,i+1}⊆inds r ⇒
157.     is_HI(r(i))∧is_LI(r(i+1)) ∨ is_LI(r(i))∧is_HI(r(i+1))
```


158. A route of a map, m, is a route as follows:
  (a) An empty sequence is a route.
  (b) A sequence of just a single hub identifier or of hubs of the map is a route.
  (c) A sequence of just a single link identifier of links of the map is a route.
  (d) If r⌢⟨hi⟩ and ⟨li⟩⌢r′ are routes of the map and li is in the definition set of m(hi) then r⌢⟨hi,li⟩⌢r′ is a route of the map.
  (e) If r⌢⟨li⟩ and ⟨hi⟩⌢r′ are routes of the map and hi is the target of (m(hi′))(li) then r⌢⟨li,hi⟩⌢r′ is a route of the map.
  (f) Only such routes are routes of a net if they result from a finite [possibly infinite] set of uses of Items 158(a)-158(e).

type
type
158. MR′ = R, MR = {r:MR′•∃ m:Map • r ∈ routes(m)|}
value
158. routes: N → MR-**infset**
158. routes(n) ≡ routes(xtr_Map(n))
158. routes: Map → MR-**infset**
158. routes(m) ≡
158(a). **let** rs = {⟨⟩}
158(b). ∪ ∪ {⟨hi⟩|hi:HI • hi ∈ **dom** m}
158(c). ∪ ∪ {⟨li⟩|li:LI,hi:HI • li ∈ xtr_LIs(m)}
158(d). ∪ ∪ {r⌢⟨hi,li⟩⌢r′|r,r′:MR,hi:HI,li:LI•{r,r′}⊆rs∧li ∈ **dom** m(hi)}
158(e). ∪ ∪ {r⌢⟨li,hi⟩⌢**tl** r′|r,r′:MR,li:LI,hi:HI•{r,r′}⊆rs∧is_target(m)(hi)(li)}
158(f). **in** rs **end**

158(e). is_target: Map → HI × LI
158(e). is_target(m)(hi)(li) ≡
158(e). ∃ h″:HI•h″∈ **dom** m∧li ∈ **dom** m(hi″)∧hi=(m(hi″))(li)

14.2.8. Special Routes

14.2.8.1 Acyclic Routes

159. A route of a map is acyclic if no hub identifier appears twice or more.

value
159. is_Acyclic: MR → Map $\stackrel{\sim}{\to}$ **Bool**
159. is_Acyclic(mr)(m) ≡ ∼∃ hi:HI,i,j:**Nat**•{i,j}⊆**inds** mr ∧ i≠j ⇒ mr(i)=hi=mr(j)
159. **pre** mr ∈ routes(m)

14.2.8.2 Direct Routes

160. A route, r, of a map (from hub hi or linkli to hub hi′ or linkli′) is a direct route if r is acyclic.

160. direct_route: MR → Map $\stackrel{\sim}{\to}$ **Bool**
160. direct_route(mr) ≡ is_Acyclic(mr)
160. **pre** mr ∈ routes(m)

14.2.8.3 Routes Between Hubs

161. Let there be given two distinct hub identifiers of a route map. Find the set of acyclic routes between them, including zero if no routes.

value
161. find_MR: Map → (HI×HI) $\stackrel{\sim}{\to}$ MR-**set**
161. find_MR(m)(hi,hi′) ≡
161. **let** rs = routes(m) **in**
161. {mr | mr,mr′:MR • mr ∈ rs
161. ∧ mr ∈ mr = ⟨hi⟩⌢mr′⟨hi′⟩ ∧ is_Acyclic(mr)(m) }
161. **end**
161. **pre**: {hi,hi′}⊆**dom** m

14.2.9. Special Maps

14.2.9.1 Isolated Hubs

162. A net, n, consists of two or more isolated hubs
   (a) if there exists two hub identifiers, hi_1,hi_2, of the map of the net
   (b) such that there is no route from hi_1 to hi_2.

value
162. are_isolated_hubs: Map → **Bool**
162. are_isolated_hubs(m) ≡
162(a). ∃ hi_1,hi_2:HI • {hi_1,hi_2}⊆**dom** m ⇒
162(b). ∼∃ mr,mr_i:MR • mr ∈ routes(m) ⇒ mr = ⟨hi_1⟩⌢mr_i⌢⟨hi_2⟩

14.2.9.2 Isolated Maps

163. If there are isolated hubs in a net then the net can be seen as two or more isolated nets.

value
163. are_isolated_nets: Map → **Bool**
163. are_isolated_nets(m) ≡ are_isolated_hubs(m)

14.2.9.3 Sub_Maps

164. Given a map one can identify the set of all sub_maps which which contains a given hub identifier.
165. Given a map one can identify the sub_map which contains a given hub identifier.

value

164. sub_maps: Map → Map-**set**
164. sub_maps(m) **as** ms
164. { xtr_Map(m)(hi) | hi:HI · hi ∈ **dom** m }

165. sub_Map: Map → HI $\stackrel{\sim}{\to}$ Map
165. sub_Map(m)(hi) ≡
165. **let** his = { hi′ | hi′:HI ∧ hi′ ∈ **dom** m ∧ find_MRs(m)(hi,hi′)≠{} } **in**
165. [hi″ ↦ m(hi″) | hi″ ∈ his] **end**

theorem: are_isolated_nets(m) ⇒ sub_maps(m) ≠ { m }

14.3. Actions

14.3.1. Insert Hub

166. The insert action
    (a) applies to a net and a hub and conditionally yields an updated net.
    (b) The condition is that there must not be a hub in the initial net with the same unique hub identifier as that of the hub to be inserted and
    (c) the hub to be inserted does not initially designate links with which it is to be connected.
    (d) The updated net contains all the hubs of the initial net "plus" the new hub.
    (e) and the same links.

value

166. insert_H: N → H $\stackrel{\sim}{\to}$ N
166(a). insert_H(n)(h) **as** n′
166(a). **pre**: pre_insert_H(n)(h)
166(a). **post**: post_insert_H(n)(h)(n′)

166(b). pre_insert_H(n)(h) ≡
166(b). ∼∃ h′:H · h′ ∈ obs_Hs(n) ∧ uid_H(h)=uid_H(h′)
166(c). ∧ mereo_H(h) = {}

166(d). post_insert_H(n)(h)(n′) ≡
166(d). obs_Hs(n) ∪ {h} = obs_Hs(n′)
166(e). ∧ obs_Ls(n) = obs_Ls(n′)

14.3.2. Insert Link

167. The insert link action
    (a) is given a "fresh" link, that is, one not in the net (before the action)
    (b) but where the two distinct hub identifiers of the mereology of the inserted link are of hubs in the net.
    (c) The link is inserted.
    (d) These two hubs
    (e) have their mereologies updated to reflect the new link
    (f) and nothing else; all other links and hubs of the net are unchanged.

value

167. insert_L: N → L $\stackrel{\sim}{\to}$ N
167. insert_L(n)(l) **as** n′
167. ∃ l:L · pre_insert_L(n)(l) ⇒ pre_insert_L(n)(l) ∧ post_insert_L(n,n′)(l)

167. pre_insert_L: N → L → **Bool**
167. pre_insert_L(n)(l) ≡
167(a). uid_L(l) ∉ xtr_LIs(n)
167(b). ∧ mereo_L(l)⊆xtr_HIs(n)

167. post_insert_L: N × N → L → **Bool**
167. post_insert_L(n,n′)(l) ≡
167(c). obs_Ls(n) ∪ {l} = obs_Ls(n′)
167(d). ∧ **let** {hi1,hi2} = mereo_L(l) **in**
167(d). **let** (h1,h2) = (get_H(n)(hi1),get_H(n)(hi2)),
167(d). (h1′,h2′) = (get_H(n′)(hi1),get_H(n′)(hi2)) **in**
167(e). mereo_H(h)∪{uid_L(l)}=mereo_H(h′)
167(f). ∧ obs_Hs(n)\{h1,h2} = obs_Hs(n′)\{h1′,h2′}
167(f). ∧ [all other properties of h1 and h2 unchanged]
167(f). [that is, same as h1′ and h2′]
167. **end end**

- The insert link post-condition has too many lines.
- I will instead compose the post-condition
  - from the conjunction of a number of invocations
  - of predicates with "telling" names.
- For these action function definitions
  - such "small" predicates
  - amount to building a nicer theory.

14.3.3. Remove Hub

168. remove hub
   (a) where a hub, known by its hub identifier, is given,
   (b) where the [to be] removed hub is indeed in the net (before the action),
   (c) where the removed hub's mereology is empty (that is, the [to be] removed hub) is not connected to any links in the net (before the action)).
   (d) All other links and hubs of the net are unchanged.

value
168. remove_H: N → HI $\xrightarrow{\sim}$ N
168(a). remove_H(n)(hi) **as** n′
168(b). ∃ h:H • uid_H(h)=hi ∧ h ∈ obs_Hs(n) ⇒
168(c). pre_remove_H(n)(hi) ∧ post_remove_H(n,n′)(hi)

- We leave the definitions of the pre/post conditions of this and the next action function to the listener.

14.3.4. Remove Link

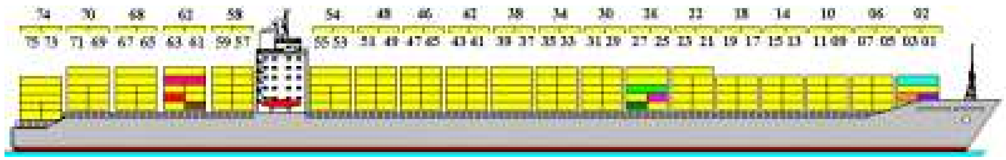
169. remove link
   (a) where a link, known by its link identifier, is given,
   (b) where that link is indeed in the net (before the action),
   (c) where hubs to which the link is connected after the action has the only change to their mereologies changed be that they do not list the [to be] removed link.
   (d) All other links and hubs of the net are unchanged.

value
169. remove_L: N → LI $\xrightarrow{\sim}$ N
169(a). remove_L(n)(li) **as** n′
169(b). ∃ l:L • uid_L(l)=li ∧ l ∈ obs_Ls(n) ⇒
169(c). pre_remove_L(n)(li) ∧ post_remove_L(n,n′)(li)

15. On A Theory of Container Stowage

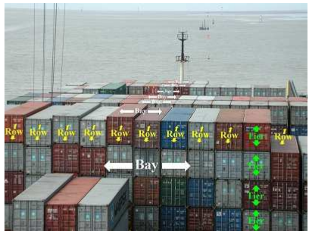
- This section is under development.
  - The idea of this section is
    - not so much to present a container domain description,
    - but rather to present fragments, "bits and pieces", of a theory of such a domain.
- The purpose of having a theory
  - is to "draw" upon the 'bits and pieces'
  - when expressing
    - properties of endurants and
    - definitions of
      - actions,
      - events and
      - behaviours.
- Again: this section is very much in embryo.

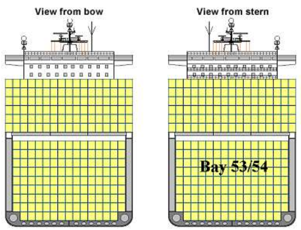
15.1. Some Pictures



A container vessel with 'bay' numbering

- Container vessels ply the seven seas and in-numerous other waters.
- They carry containers from port to port.
- The history of containers goes back to the late 1930s.
- The first container vessels made their first transports in 1956.
- Malcolm P. McLean is credited to have invented the container.
- To prove the concept of container transport he founded the container line `Sea-Land Inc.` which was sold to `Maersk Lines` at the end of the 1990s.





Bay numbers. Ship stowage cross section

- Down along the vessel, horisontally,
  - from front to aft,
  - containers are grouped, in numbered bays.

Row and tier numbers

- Bays are composed from rows, horisontally, across the vessel.
- Rows are composed from stacks, horisontally, along the vessel.
- And stacks are composed, vertically, from [tiers of] containers

15.2. Parts

15.2.1. A Basis

170. From a container vessel (cv:CV) and from a container terminal port (ctp:CTP) one can observe their bays (bays:BAYS).

type
170. CV, CTP, BAYS
value
170. obs_BAYS: (CV|CTP) $\rightarrow$ BAYS

171. The bays, bs:BS, (of a container vessel or a container terminal port) are mereologically structured as an (BId) indexed set of individual bays (b:B).

type
171. BId, B
171. BS = BId $\overrightarrow{m}$ B
value
171. obs_BS: BAYS $\rightarrow$ BS (i.e., BId $\overrightarrow{m}$ B)

172. From a bay, b:B, one can observe its rows, rs:ROWS.

173. The rows, rs:RS, (of a bay) are mereologically structured as an (RId) indexed set of individual rows (r:R).

type
172. ROWS, RId, R
173. RS = RId $\overrightarrow{m}$ R
value
172. obs_ROWS: B $\rightarrow$ ROWS
173. obs_RS: ROWS $\rightarrow$ RS (i.e., RId $\overrightarrow{m}$ R)

174. From a row, r:R, one can observe its stacks, STACKS.

175. The stacks, ss:SS (of a row) are mereologically structured as an (SId) indexed set of individual stacks (s:S).

type
174. STACKS, SId, S
175. SS = SId $\overrightarrow{m}$ S
value
174. obs_STACKS: R $\rightarrow$ STACKS
175. obs_SS: STACKS $\rightarrow$ SS (i.e., SId $\overrightarrow{m}$ S)

176. A stack (s:S) is mereologically structured as a linear sequence of containers (c:C).

type
176. C
176. S = C*

- The containers of the same stack index across stacks are called the tier at that index, cf. photo on Page 508..

177. A container is here considered a composite part
    (a) of the container box, k:K
    (b) and freight, f:F.
178. Freight is considered composite
    (a) and consists of zero, one or more colli (package, indivisible unit of freight),
    (b) each having a unique colli identifier (over all colli of the entire world!).
    (c) Container boxes likewise have unique container identifiers.

type
177. C, K, F, P
value
177(a). obs_K: C → K
177(b). obs_F: C → F
178(a). obs_Ps: F → P-**set**
type
178(b). PI
178(c). CI
value
178(b). uid_P: P → PI
178(c). uid_C: C → CI

15.2.2. Mereological Constraints

179. For any bay of a vessel the index sets of its rows are identical.
180. For a bay of a vessel the index sets of its stacks are identical.

axiom
179. ∀ cv:CV ·
179. ∀ b:B·b ∈ **rng** obs_BS(obs_BAYS(cv))⇒
179. **let** rws=obs_ROWS(b) **in**
179. ∀ r,r′:R·{r,r′}⊆**rng** obs_RS(b)⇒**dom** r=**dom** r′
180. ∧ **dom** obs_SS(r) = **dom** obs_SS(r′) **end**

15.2.3. Stack Indexes

181. A container stack (and a container) is designated by an index triple: a bay index, a row index and a stack index.
182. A container index triple is valid, for a vessel, if its indices are valid indices.

type
181. StackId = BId×RId×SId
value
182. valid_address: BS → StackId → **Bool**
182. valid_address(bs)(bid,rid,sid) ≡
182. bid ∈ **dom** bs
182. ∧ rid ∈ **dom** (obs_RS(bs))(bid)
182. ∧ sid ∈ **dom** (obs_SS((obs_RS(bs))(bid)))(rid)

- The above can be defined in terms of the below.

type
BayId = BId
RowId = BId×RId
value
182. valid_BayId: V → BayId → **Bool**
182. valid_BayId(v)(bid) ≡ bid ∈ **dom** obs_BS(obs_BAYS(v))

182. get_B: V → BayId $\xrightarrow{\sim}$ B
182. get_B(v)(bid) ≡ (get_B(bs))(bid) **pre**: valid_BId(v)(bid)

182. get_B: BS → BayId $\xrightarrow{\sim}$ B
182. get_B(bs)(bid) ≡ (obs_BS(obs_BAYS(v)))(bid) **pre**: bid ∈ **dom** bs

182. valid_RowId: V → RowId → **Bool**
182. valid_RowId(v)(bid,rid) ≡ rid ∈ **dom** obs_RS(get_B(v)(bid))
182. **pre**: valid_BayId(v)(bid)

182. get_R: V → RowId $\xrightarrow{\sim}$ R
182. get_R(v)(bid,rid) ≡ get_R(obs_BS(v))(bid,rid) **pre**: valid_RowId(v)(bid

182. get_R: BS → RowId $\xrightarrow{\sim}$ R
182. get_R(bs)(bid,rid) ≡ (obs_RS(get_RS(bs(bid))))(rid)
182. **pre**: valid_RowId(v)(bid,rid)

182. get_S: V → StackId $\xrightarrow{\sim}$ S
182. get_S(v)(bid,rid,sid) ≡ (obs_SS(get_R(get_B(v)(bid,rid))))(sid)
182. **pre**: valid_address(v)(bid,rid,sid)

182. get_C: V → StackId $\xrightarrow{\sim}$ C
182. get_C(v)(stid) ≡ get_C(obs_BS(v))(stid) **pre**: get_S(v)(bid,rid,sid) ≠ ⟨⟩

182. get_C: BS → StackId $\xrightarrow{\sim}$ C
182. get_C(bs)(bid,rid,sid) ≡ **hd**(obs_SS(get_R((bs(bid))(rid))))(sid)
182. **pre**: get_S(bs)(bid,rid,sid) ≠ ⟨⟩

182. valid_addresses: V → StackId-**set**
182. valid_addresses(v) ≡ {adr|adr:StackId•valid_address(adr)(v)}

183. The predicate non_empty_designated_stack checks whether the designated stack is non-empty.

183. non_empty_designated_stack: V → StackId → **Bool**
183. non_empty_designated_stack(v)(bid,rid,sid) ≡ get_S(v)(bid,rid,sid) ≠ ⟨⟩

184. Two vessels have the same mereology if they have the same set of valid-addresses.

value
184. unchanged_mereology: BS × BS → **Bool**
184. unchanged_mereology(bs,bs′) ≡ valid_addresses(bs) = valid_addresses(b

185. The designated stack, s′, of a vessel, v′ is popped with respect the "same designated" stack, s, of a vessel, v
   (a) if the ordered sequence of the containers of s′ are identical to the ordered sequence of containers of all but the first container of s.

185. popped_designated_stack: BS × BS → StackId → **Bool**
185. popped_designated_stack(bs,bs′)(stid) ≡
185(a). **tl** get_S(v)(stid) = get_S(bs′)(stid)

186. For a given stack index, valid for two bays (bs, bs′) of two vessels or two container terminal ports, and say stid, these two bays enjoy the unchanged_non_designated_stacks(bs,bs′)(stid) property

(a) if the stacks (of the two bays) not identified by stid are identical.

186. unchanged_non_designated_stacks: BS $\times$ BS $\rightarrow$ StackId $\rightarrow$ **Bool**
186. unchanged_non_designated_stacks(bs,bs′)(stid) $\equiv$
186(a). $\forall$ adr:StackId$\cdot$adr $\in$ valid_addresses(v)$\setminus$\{stid\}$\Rightarrow$
186(a). get_S(bs)(adr) = get_S(bs′)(adr)
186. **pre**: unchanged_mereology(bs,bs′)

15.2.4. Stowage Schemas

187. By a stowage schema of a vessel we understand a "table"

(a) which for every bay identifier of that vessel records a bay schema

(b) which for every row identifier of an identified bay records a row schema

(c) which for every stack identifier of an identified row records a stack schema

(d) which for every identified stack records its tier schema.

(e) A stack schema records for every tier index (which is a natural number) the type of container (contents) that may be stowed at that position.

(f) The tier indexes of a stack schema form a set of natural numbers from one to the maximum number in the index set.

value
187. obs_StoSchema: V $\rightarrow$ StoSchema
type
187(a). StoSchema = BId $\overrightarrow{m}$ BaySchema
187(b). BaySchema = RId $\overrightarrow{m}$ RowSchema
187(c). RowSchema = SId $\overrightarrow{m}$ StaSchema
187(d). StaSchema = **Nat** $\overrightarrow{m}$ C_Type
187(e). C_Type
axiom
187(f). $\forall$ stsc:StaSchema $\cdot$ **dom** stsc = \{1..**max dom** stsc\}

188. One can define a function which from an actual vessel "derives" its "current stowage schema".

188. cur_sto_schema: V $\rightarrow$ StoSchema
188. cur_sto_schema(v) $\equiv$
188. **let** bs = obs_BS(obs_BAYS(v)) **in**
188. [bid $\mapsto$ **let** rws = obs_RS(obs_ROWS(bs(bid))) **in**
188. [rid $\mapsto$ **let** ss = obs_SS(obs_STACKS(rws)(rid)) **in**
188. [sid $\mapsto$ $\langle$ analyse_container(ss(i))|i:**Nat**$\cdot$i $\in$ **inds** ss $\rangle$
188. | sid:SId$\cdot$sid $\in$ ss] **end**
188. | rid:RId$\cdot$rid $\in$ **dom** rws] **end**
188. | bid:BId$\cdot$bid $\in$ **dom** ds] **end**

188. analyse_container: C $\rightarrow$ C_Type

189. Given a stowage schema and a current stowage schema one can check the latter for conformance wrt. the former.

189. conformance: StoSchema × StoSchema → **Bool**
189. conformance(stosch,cur_stosch) ≡
189. **dom** cur_stosch = **dom** stosch
189. ∧ ∀ bid:BId · bid ∈ **dom** stosch ⇒
189. **dom** cur_stosch(bid) = **dom** stosch(bid)
189. ∧ ∀ rid:RId · rid ∈ **dom**(stosch(bid))(rid) ⇒
189. **dom**(cur_stosch(bid))(rid) = **dom**(stosch(bid))(rid)
189. ∧ ∀ sid:SId · sid ∈ **dom**(cur_stosch(bid))(rid)
189. ∀ i:**Nat** · i ∈ **inds**((cur_stosch(bid))(rid))(sid) ⇒
189. conform((((cur_stosch(bid))(rid))(sid))(i),
189. (((stosch(bid))(rid))(sid))(i))

189. conform: C_Type × C_Type → **Bool**

190. From a vessel one can observe its mandated stowage schema.

191. The current stowage schema of a vessel must always conform to its mandated stowage schema.

value
190. obs_StoSchema: V → StoSchema

191. stowage_conformance: V → **Bool**
191. stowage_conformance(v) ≡
191. **let** mandated = obs_StoSchema(v),
191. current = cur_sto_schema(v) **in**
191. conformance(mandated,current) **end**

15.3. Actions
15.3.1. Remove Container from Vessel

20. The remove_Container_from_Vessel action applies to a vessel and a stack address and conditionally yields an updated vessel and a container.

20(a). We express the 'remove from vessel' function primarily by means of an auxiliary function remove_C_from_BS, remove_C_from_BS(obs_BS(v))(stid), and some further post-condition on the before and after vessel states (cf. Item 20(d)).

20(b). The remove_C_from_BS function yields a pair: an updated set of bays and a container.

20(c). When obs_erving the BayS from the updated vessel, v′, and pairing that with what is assumed to be a vessel, then one shall obtain the result of remove_C_from_BS(obs_BS(v))(stid).

20(d). Updating, by means of remove_C_from_BS(obs_BS(v))(stid), the bays of a vessel must leave all other properties of the vessel unchanged.

21. The pre-condition for remove_C_from_BS(bs)(stid) is

21(a). that stid is a valid_address in bs, and

21(b). that the stack in bs designated by stid is non_empty.

22. The post-condition for remove_C_from_BS(bs)(stid) wrt. the updated bays, bs′, is

22(a). that the yielded container, i.e., c, is obtained, get_C(bs)(stid), from the top of the non-empty, designated stack,

22(b). that the mereology of bs′ is unchanged, unchanged_mereology(bs,bs′). wrt. bs. ,

22(c). that the stack designated by stid in the "input" state, bs, is popped, popped_designated_stack(bs,bs′)(stid), and

22(d). that all other stacks are unchanged in bs′ wrt. bs, unchanged_non_designated_stacks(bs,bs′)(stid).

value

20. remove_C_from_V: V → StackId $\xrightarrow{\sim}$ (V×C)
20. remove_C_from_V(v)(stid) **as** (v′,c)
20(c). (obs_BS(v′),c) = remove_C_from_BS(obs_BS(v))(stid)
20(d). ∧ props(v)=props(v″)

20(b). remove_C_from_BS: BS → StackId → (BS×C)
20(a). remove_C_from_BS(bs)(stid) **as** (bs′,c)
21(a). **pre**: valid_address(bs)(stid)
21(b). ∧ non_empty_designated_stack(bs)(stid)
22(a). **post**: c = get_C(bs)(stid)
22(b). ∧ unchanged_mereology(bs,bs′)
22(c). ∧ popped_designated_stack(bs,bs′)(stid)
22(d). ∧ unchanged_non_designated_stacks(bs,bs′)(stid)

15.3.2. Remove Container from CTP

- We define a remove action similar to that of the previous section.

192. Instead of vessel bays we are now dealing with the bays of container terminal ports.

We omit the narrative — which is very much like that of narrative Items 20(c) and 20(d).

value

192. remove_C_from_CTP: CTP → StackId $\xrightarrow{\sim}$ (CTP×C)
192. remove_C_from_CTP(ctp)(stid) **as** (ctp′,c)
20(c). (obs_BS(ctp′),c) = remove_C_from_BS(obs_BS(ctp))(stid)
20(d). ∧ props(ctp)=props(ctp″)

15.3.3. Stack Container on Vessel

193. Stacking a container at a vessel bay stack location
    (a)
    (b)
    (c)

value

193. stack_C_on_vessel: BS → StackId $\xrightarrow{\sim}$ C $\xrightarrow{\sim}$ BS
193(a). stack_C_on_vessel(bs)(stid)(c) **as** bs′
193(a). **comment:** bs is bays of a v:V, i.e., bs = obs_BS(v)
193(b). **pre**:
193(c). **post**:

15.3.4. Stack Container in CTP

194.
195.
196.
197.

value

194. stack_C_in_CTP: CTP → StackId → C $\xrightarrow{\sim}$ CTP
195. stack_C_in_CTP(ctp)(stid)(c) **as** ctp′
196. **pre**:
197. **post**:

15.3.5. Transfer Container from Vessel to CTP

198.

199.

200.

201.

value

198. transfer_C_from_V_to_CTP: V→StackId $\stackrel{\sim}{\rightarrow}$ CTP→StackId $\stackrel{\sim}{\rightarrow}$ (V×CTP)
199. transfer_C_from_V_to_CTP(v)(v_stid)(ctp)(ctp_stid) ≡
200. **let** (c,v′) = remove_C_from_V(v)(v_stid) **in**
200. (v′,stack_C_in_CTP(ctp)(ctp_stid)(c)) **end**

15.3.6. Transfer Container from CTP to Vessel

202.

203.

204.

value

202. transfer_C_from_CTP_to_V: CTP→StackId $\stackrel{\sim}{\rightarrow}$ V→StackId $\stackrel{\sim}{\rightarrow}$ (CTP×V)
203. transfer_C_from_CTP_to_V(ctp)(ctp_stid)(v)(v_stid) ≡
204. **let** (c,ctp′) = remove_C_from_CTP(ctp)(ctp_stid) **in**
204. (ctp′,stack_C_in_CTP(ctp)(ctp_stid)(c)) **end**

Any Questions ?

16. RSL: The Raise Specification Language

16.1. Type Expressions

- Type expressions are expressions whose value are type, that is,
- possibly infinite sets of values (of "that" type).

16.1.1. Atomic Types

- Atomic types have (atomic) values.
- That is, values which we consider to have no proper constituent (sub-)values,
- i.e., cannot, to us, be meaningfully "taken apart".

type
[1] **Bool** **true**, **false**
[2] **Int** ... , −2, −2, 0, 1, 2, ...
[3] **Nat** 0, 1, 2, ...
[4] **Real** ..., −5.43, −1.0, 0.0, 1.23· · · , 2,7182· · · , 3,1415· · · , 4.56, ...
[5] **Char** ”a”, ”b”, ..., ”0”, ...
[6] **Text** ”abracadabra”

16.1.2. Composite Types

- Composite types have composite values.
  - That is, values which we consider to have proper constituent (sub-)values,
  - i.e., can be meaningfully “taken apart”.
- There are two ways of expressing composite types:
  - either explicitly, using concrete type expressions,
  - or implicitly, using sorts (i.e., abstract types) and observer functions.

16.1.2.1 Concrete Composite Types

[7] A-**set**
[8] A-**infset**
[9] A × B × ... × C
[10] A^*
[11] A^ω
[12] $A \xrightarrow{m} B$
[13] A → B
[14] $A \stackrel{\sim}{\to} B$
[15] (A)
[16] A | B | ... | C
[17] mk_id(sel_a:A,...,sel_b:B)
[18] sel_a:A ... sel_b:B

16.1.2.2 Sorts and Observer Functions

type
A, B, C, ..., D
value
obs_B: A → B, obs_C: A → C, ..., obs_D: A → D

- The above expresses
  - that values of type A
  - are composed from at least three values —
  - and these are of type B, C, . . . , and D.
- A concrete type definition corresponding to the above
  - presupposing material of the next section

type
B, C, ..., D
A = B × C × ... × D

16.2. Type Definitions
16.2.1. Concrete Types

- Types can be concrete
- in which case the structure of the type is specified by type expressions:

type
  A = Type_expr

- Schematic type definitions:

[1] Type_name = Type_expr /∗ without |s or subtypes ∗/
[2] Type_name = Type_expr_1 | Type_expr_2 | ... | Type_expr_n
[3] Type_name ==
    mk_id_1(s_a1:Type_name_a1,...,s_ai:Type_name_ai) |
    ... |
    mk_id_n(s_z1:Type_name_z1,...,s_zk:Type_name_zk)
[4] Type_name :: sel_a:Type_name_a ... sel_z:Type_name_z
[5] Type_name = {| v:Type_name′ • $\mathcal{P}$(v) |}

- where a form of [2–3] is provided by combining the types:

Type_name = A | B | ... | Z
A == mk_id_1(s_a1:A_1,...,s_ai:A_i)
B == mk_id_2(s_b1:B_1,...,s_bj:B_j)
...
Z == mk_id_n(s_z1:Z_1,...,s_zk:Z_k)

axiom
  ∀ a1:A_1, a2:A_2, ..., ai:Ai •
    s_a1(mk_id_1(a1,a2,...,ai))=a1 ∧ s_a2(mk_id_1(a1,a2,...,ai))=a2 ∧
    ... ∧ s_ai(mk_id_1(a1,a2,...,ai))=ai ∧
  ∀ a:A • **let** mk_id_1(a1′,a2′,...,ai′) = a **in**
    a1′ = s_a1(a) ∧ a2′ = s_a2(a) ∧ ... ∧ ai′ = s_ai(a) **end**

16.2.2. Subtypes

- In RSL, each type represents a set of values. Such a set can be delimited by means of predicates.
- The set of values b which have type B and which satisfy the predicate $\mathcal{P}$, constitute the subtype A:

type
  A = {| b:B • $\mathcal{P}$(b) |}

16.2.3. Sorts — Abstract Types

- Types can be (abstract) sorts
- in which case their structure is not specified:

type
  A, B, ..., C

16.3. The RSL Predicate Calculus

16.3.1. Propositional Expressions

- Let identifiers (or propositional expressions) a, b, ..., c designate Boolean values (**true** or **false** [or **chaos**]).
- Then:

false, **true**
a, b, ..., c ∼a, a∧b, a∨b, a⇒b, a=b, a≠b

- are propositional expressions having Boolean values.
- ∼, ∧, ∨, ⇒, = and ≠ are Boolean connectives (i.e., operators).
- They can be read as: *not*, *and*, *or*, *if then* (or *implies*), *equal* and *not equal*.

16.3.2. Simple Predicate Expressions

- Let identifiers (or propositional expressions) a, b, ..., c designate Boolean values,
- let x, y, ..., z (or term expressions) designate non-Boolean values
- and let i, j, . . ., k designate number values,
- then:

false, **true**
a, b, ..., c
∼a, a∧b, a∨b, a⇒b, a=b, a≠b
x=y, x≠y,
i<j, i≤j, i≥j, i≠j, i≥j, i>j

- are simple predicate expressions.

16.3.3. Quantified Expressions

- Let X, Y, . . ., C be type names or type expressions,
- and let $\mathcal{P}(x)$, $\mathcal{Q}(y)$ and $\mathcal{R}(z)$ designate predicate expressions in which x, y and z are free.
- Then:

∀ x:X · $\mathcal{P}(x)$
∃ y:Y · $\mathcal{Q}(y)$
∃ ! z:Z · $\mathcal{R}(z)$

- are quantified expressions — also being predicate expressions.

16.4. Concrete RSL Types: Values and Operations

16.4.1. Arithmetic

type
 Nat, **Int**, **Real**
value
 +,−,∗: **Nat**×**Nat**→**Nat** | **Int**×**Int**→**Int** | **Real**×**Real**→**Real**
 /: **Nat**×**Nat**$\stackrel{\sim}{\to}$**Nat** | **Int**×**Int**$\stackrel{\sim}{\to}$**Int** | **Real**×**Real**$\stackrel{\sim}{\to}$**Real**
 <,≤,=,≠,≥,> (**Nat**|**Int**|**Real**) → (**Nat**|**Int**|**Real**)

16.4.2. Set Expressions

16.4.2.1 Set Enumerations

Let the below a's denote values of type A, then the below designate simple set enumerations:

{{}, {a}, {e_1,e_2,...,e_n}, ...} ∈ A-**set**
{{}, {a}, {e_1,e_2,...,e_n}, ..., {e_1,e_2,...}} ∈ A-**infset**

16.4.2.2 Set Comprehension

- The expression, last line below, to the right of the ≡, expresses set comprehension.
- The expression "builds" the set of values satisfying the given predicate.
- It is abstract in the sense that it does not do so by following a concrete algorithm.

type
  A, B
  P = A → **Bool**
  Q = A $\xrightarrow{\sim}$ B
value
  comprehend: A-**infset** × P × Q → B-**infset**
  comprehend(s,P,Q) ≡ { Q(a) | a:A • a ∈ s ∧ P(a)}

16.4.3. Cartesian Expressions

16.4.3.1 Cartesian Enumerations

- Let e range over values of Cartesian types involving A, B, ..., C,
- then the below expressions are simple Cartesian enumerations:

type
  A, B, ..., C
  A × B × ... × C
value
  (e1,e2,...,en)

16.4.4. List Expressions

16.4.4.1 List Enumerations

- Let a range over values of type A,
- then the below expressions are simple list enumerations:

{⟨⟩, ⟨e⟩, ..., ⟨e1,e2,...,en⟩, ...} ∈ A^*
{⟨⟩, ⟨e⟩, ..., ⟨e1,e2,...,en⟩, ..., ⟨e1,e2,...,en,... ⟩, ...} ∈ A^ω

⟨ a_i .. a_j ⟩

- The last line above assumes a_i and a_j to be integer-valued expressions.
- It then expresses the set of integers from the value of e_i to and including the value of e_j.
- If the latter is smaller than the former, then the list is empty.

16.4.4.2 List Comprehension

- The last line below expresses list comprehension.

type
A, B, P = A → **Bool**, Q = A $\stackrel{\sim}{\to}$ B
value
comprehend: $A^\omega \times P \times Q \stackrel{\sim}{\to} B^\omega$
comprehend(l,P,Q) ≡
⟨ Q(l(i)) | i **in** ⟨1..**len** l⟩ · P(l(i))⟩

16.4.5. Map Expressions
16.4.5.1 Map Enumerations

- Let (possibly indexed) u and v range over values of type $T1$ and $T2$, respectively,
- then the below expressions are simple map enumerations:

type
T1, T2
M = T1 $\overrightarrow{m}$ T2
value
u,u1,u2,...,un:T1, v,v1,v2,...,vn:T2
[ ], [u↦v], ..., [u1↦v1,u2↦v2,...,un↦vn] ∀ ∈ M

16.4.5.2 Map Comprehension

- The last line below expresses map comprehension:

type
U, V, X, Y
M = U $\overrightarrow{m}$ V
F = U $\stackrel{\sim}{\to}$ X
G = V $\stackrel{\sim}{\to}$ Y
P = U → **Bool**
value
comprehend: M×F×G×P → (X $\overrightarrow{m}$ Y)
comprehend(m,F,G,P) ≡
[F(u) ↦ G(m(u)) | u:U · u ∈ **dom** m ∧ P(u)]

16.4.6. Set Operations
16.4.6.1 Set Operator Signatures

value
205 ∈: A × A-**infset** → **Bool**
206 ∉: A × A-**infset** → **Bool**
207 ∪: A-**infset** × A-**infset** → A-**infset**
208 ∪: (A-**infset**)-**infset** → A-**infset**
209 ∩: A-**infset** × A-**infset** → A-**infset**
210 ∩: (A-**infset**)-**infset** → A-**infset**
211 \: A-**infset** × A-**infset** → A-**infset**
212 ⊂: A-**infset** × A-**infset** → **Bool**
213 ⊆: A-**infset** × A-**infset** → **Bool**
214 =: A-**infset** × A-**infset** → **Bool**
215 ≠: A-**infset** × A-**infset** → **Bool**
216 **card**: A-**infset** $\stackrel{\sim}{\to}$ **Nat**

16.4.6.2 Set Examples

examples

a ∈ {a,b,c}
a ∉ {}, a ∉ {b,c}
{a,b,c} ∪ {a,b,d,e} = {a,b,c,d,e}
∪{{a},{a,b},{a,d}} = {a,b,d}
{a,b,c} ∩ {c,d,e} = {c}
∩{{a},{a,b},{a,d}} = {a}
{a,b,c} \ {c,d} = {a,b}
{a,b} ⊂ {a,b,c}
{a,b,c} ⊆ {a,b,c}
{a,b,c} = {a,b,c}
{a,b,c} ≠ {a,b}
card {} = 0, **card** {a,b,c} = 3

16.4.6.3 Informal Explication

205. ∈: The membership operator expresses that an element is a member of a set.
206. ∉: The nonmembership operator expresses that an element is not a member of a set.
207. ∪: The infix union operator. When applied to two sets, the operator gives the set whose members are in either or both of the two operand sets.
208. ∪: The distributed prefix union operator. When applied to a set of sets, the operator gives the set whose members are in some of the operand sets.
209. ∩: The infix intersection operator. When applied to two sets, the operator gives the set whose members are in both of the two operand sets.
210. ∩: The prefix distributed intersection operator. When applied to a set of sets, the operator gives the set whose members are in some of the operand sets.

211. \: The set complement (or set subtraction) operator. When applied to two sets, the operator gives the set whose members are those of the left operand set which are not in the right operand set.
212. ⊆: The proper subset operator expresses that all members of the left operand set are also in the right operand set.
213. ⊂: The proper subset operator expresses that all members of the left operand set are also in the right operand set, and that the two sets are not identical.
214. =: The equal operator expresses that the two operand sets are identical.
215. ≠: The nonequal operator expresses that the two operand sets are *not* identical.
216. **card**: The cardinality operator gives the number of elements in a finite set.

16.4.6.4 Set Operator Definitions

value

s′ ∪ s″ ≡ { a | a:A • a ∈ s′ ∨ a ∈ s″ }
s′ ∩ s″ ≡ { a | a:A • a ∈ s′ ∧ a ∈ s″ }
s′ \ s″ ≡ { a | a:A • a ∈ s′ ∧ a ∉ s″ }
s′ ⊆ s″ ≡ ∀ a:A • a ∈ s′ ⇒ a ∈ s″
s′ ⊂ s″ ≡ s′ ⊆ s″ ∧ ∃ a:A • a ∈ s″ ∧ a ∉ s′
s′ = s″ ≡ ∀ a:A • a ∈ s′ ≡ a ∈ s″ ≡ s⊆s′ ∧ s′⊆s
s′ ≠ s″ ≡ s′ ∩ s″ ≠ {}
card s ≡
  if s = {} **then** 0 **else**
  let a:A • a ∈ s **in** 1 + **card** (s \ {a}) **end end**
  pre s /∗ is a finite set ∗/
card s ≡ **chaos** /∗ tests for infinity of s ∗/

16.4.7. Cartesian Operations

type
A, B, C
g0: G0 = A × B × C
g1: G1 = (A × B × C)
g2: G2 = (A × B) × C
g3: G3 = A × (B × C)

value
va:A, vb:B, vc:C, vd:D
(va,vb,vc):G0,
(va,vb,vc):G1
((va,vb),vc):G2
(va3,(vb3,vc3)):G3

decomposition expressions
let (a1,b1,c1) = g0,
(a1′,b1′,c1′) = g1 **in** .. **end**
let ((a2,b2),c2) = g2 **in** .. **end**
let (a3,(b3,c3)) = g3 **in** .. **end**

16.4.8. List Operations

16.4.8.1 List Operator Signatures

value
hd: $A^\omega \stackrel{\sim}{\to} A$
tl: $A^\omega \stackrel{\sim}{\to} A^\omega$
len: $A^\omega \stackrel{\sim}{\to}$ **Nat**
inds: $A^\omega \to$ **Nat-infset**
elems: $A^\omega \to$ A**-infset**
.(.): $A^\omega \times$ **Nat** $\stackrel{\sim}{\to} A$
⌢=≠: $A^* A^\omega \times A^\omega \times A^\omega \to A^\omega$ **BBool**

16.4.8.2 List Operation Examples

examples
hd⟨a1,a2,...,am⟩=a1
tl⟨a1,a2,...,am⟩=⟨a2,...,am⟩
len⟨a1,a2,...,am⟩=m
inds⟨a1,a2,...,am⟩={1,2,...,m}
elems⟨a1,a2,...,am⟩={a1,a2,...,am}
⟨a1,a2,...,am⟩(i)=ai
⟨a,b,c⟩⌢⟨a,b,d⟩ = ⟨a,b,c,a,b,d⟩
⟨a,b,c⟩=⟨a,b,c⟩
⟨a,b,c⟩ ≠ ⟨a,b,d⟩

16.4.8.3 Informal Explication

- **hd**: Head gives the first element in a nonempty list.
- **tl**: Tail gives the remaining list of a nonempty list when Head is removed.
- **len**: Length gives the number of elements in a finite list.
- **inds**: Indices give the set of indices from **1** to the length of a nonempty list. For empty lists, this set is the empty set as well.
- **elems**: Elements gives the possibly infinite set of all distinct elements in a list.
- $\ell(i)$: Indexing with a natural number, i larger than 0, into a list ℓ having a number of elements larger than or equal to i, gives the ith element of the list.

- $\frown$: Concatenates two operand lists into one. The elements of the left operand list are followed by the elements of the right. The order with respect to each list is maintained.
- $=$: The equal operator expresses that the two operand lists are identical.
- $\neq$: The nonequal operator expresses that the two operand lists are *not* identical.

The operations can also be defined as follows:

16.4.8.4 List Operator Definitions

value
  is_finite_list: $A^\omega \to$ **Bool**

  len q $\equiv$
    case is_finite_list(q) **of**
      true $\to$ **if** q $= \langle\rangle$ **then** 0 **else** 1 + **len tl** q **end**,
      false $\to$ **chaos end**

  inds q $\equiv$
    case is_finite_list(q) **of**
      true $\to$ { i | i:**Nat** $\cdot$ $1 \leq i \leq$ **len** q },
      false $\to$ { i | i:**Nat** $\cdot$ i$\neq$0 } **end**

  elems q $\equiv$ { q(i) | i:**Nat** $\cdot$ i $\in$ **inds** q }

  q(i) $\equiv$
    if i=1
      then
        if q$\neq\langle\rangle$
          then let a:A,q′:Q $\cdot$ q=$\langle$a$\rangle\frown$q′ **in** a **end**
          else chaos end
      else q(i−1) **end**

  fq $\frown$ iq $\equiv$
      $\langle$ **if** $1 \leq i \leq$ **len** fq **then** fq(i) **else** iq(i − **len** fq) **end**
      | i:**Nat** $\cdot$ **if len** iq$\neq$**chaos then** i $\leq$ **len** fq+**len end** $\rangle$
    pre is_finite_list(fq)

  iq′ = iq″ $\equiv$
    inds iq′ = **inds** iq″ $\wedge$ $\forall$ i:**Nat** $\cdot$ i $\in$ **inds** iq′ $\Rightarrow$ iq′(i) = iq″(i)

  iq′ $\neq$ iq″ $\equiv$ $\sim$(iq′ = iq″)

16.4.9. Map Operations

16.4.9.1 Map Operator Signatures and Map Operation Examples

value
  m(a): M $\to$ A $\overset{\sim}{\to}$ B, m(a) = b

  dom: M $\to$ A-**infset** [domain of map]
    dom [a1$\mapsto$b1,a2$\mapsto$b2,...,an$\mapsto$bn] = {a1,a2,...,an}

  rng: M $\to$ B-**infset** [range of map]
    rng [a1$\mapsto$b1,a2$\mapsto$b2,...,an$\mapsto$bn] = {b1,b2,...,bn}

  $\dagger$: M $\times$ M $\to$ M [override extension]
    [a$\mapsto$b,a′$\mapsto$b′,a″$\mapsto$b″] $\dagger$ [a′$\mapsto$b″,a″$\mapsto$b′] = [a$\mapsto$b,a′$\mapsto$b″,a″$\mapsto$b′]

$\cup$: M $\times$ M $\rightarrow$ M [merge $\cup$]

$$[a \mapsto b, a' \mapsto b', a'' \mapsto b''] \cup [a''' \mapsto b'''] = [a \mapsto b, a' \mapsto b', a'' \mapsto b'', a''' \mapsto b''']$$

$\backslash$: M $\times$ A-**infset** $\rightarrow$ M [restriction by]

$$[a \mapsto b, a' \mapsto b', a'' \mapsto b''] \backslash \{a\} = [a' \mapsto b', a'' \mapsto b'']$$

/: M $\times$ A-**infset** $\rightarrow$ M [restriction to]

$$[a \mapsto b, a' \mapsto b', a'' \mapsto b''] / \{a', a''\} = [a' \mapsto b', a'' \mapsto b'']$$

$=, \neq$: M $\times$ M $\rightarrow$ **Bool**

$\circ$: (A $\overrightarrow{m}$ B) $\times$ (B $\overrightarrow{m}$ C) $\rightarrow$ (A $\overrightarrow{m}$ C) [composition]

$$[a \mapsto b, a' \mapsto b'] \circ [b \mapsto c, b' \mapsto c', b'' \mapsto c''] = [a \mapsto c, a' \mapsto c']$$

16.4.9.2 Map Operation Explication

- $m(a)$: Application gives the element that a maps to in the map m.
- **dom**: Domain/Definition Set gives the set of values which *maps to* in a map.
- **rng**: Range/Image Set gives the set of values which *are mapped to* in a map.
- †: Override/Extend. When applied to two operand maps, it gives the map which is like an override of the left operand map by all or some "pairings" of the right operand map.
- $\cup$: Merge. When applied to two operand maps, it gives a merge of these maps.
- $\backslash$: Restriction. When applied to two operand maps, it gives the map which is a restriction of the left operand map to the elements that are not in the right operand set.

- /: Restriction. When applied to two operand maps, it gives the map which is a restriction of the left operand map to the elements of the right operand set.
- =: The equal operator expresses that the two operand maps are identical.
- $\neq$: The nonequal operator expresses that the two operand maps are *not* identical.
- $\circ$: Composition. When applied to two operand maps, it gives the map from definition set elements of the left operand map, m_1, to the range elements of the right operand map, m_2, such that if a is in the definition set of m_1 and maps into b, and if b is in the definition set of m_2 and maps into c, then a, in the composition, maps into c.

16.4.9.3 Map Operation Redefinitions

```
value
  rng m ≡ { m(a) | a:A • a ∈ dom m }

  m1 † m2 ≡
    [ a↦b | a:A,b:B •
      a ∈ dom m1 \ dom m2 ∧ b=m1(a) ∨ a ∈ dom m2 ∧ b=m2(a) ]

  m1 ∪ m2 ≡ [ a↦b | a:A,b:B •
            a ∈ dom m1 ∧ b=m1(a) ∨ a ∈ dom m2 ∧ b=m2(a) ]

  m \ s ≡ [ a↦m(a) | a:A • a ∈ dom m \ s ]
  m / s ≡ [ a↦m(a) | a:A • a ∈ dom m ∩ s ]

  m1 = m2 ≡
    dom m1 = dom m2 ∧ ∀ a:A • a ∈ dom m1 ⇒ m1(a) = m2(a)
  m1 ≠ m2 ≡ ∼(m1 = m2)

  m°n ≡
    [ a↦c | a:A,c:C • a ∈ dom m ∧ c = n(m(a)) ]
    pre rng m ⊆ dom n
```

16.5. λ-Calculus + Functions

16.5.1. The λ-Calculus Syntax

type /∗ A BNF Syntax: ∗/
⟨L⟩ ::= ⟨V⟩ | ⟨F⟩ | ⟨A⟩ | (⟨A⟩)
⟨V⟩ ::= /∗ variables, i.e. identifiers ∗/
⟨F⟩ ::= λ⟨V⟩ • ⟨L⟩
⟨A⟩ ::= (⟨L⟩⟨L⟩)
value /∗ Examples ∗/
⟨L⟩: e, f, a, ...
⟨V⟩: x, ...
⟨F⟩: λ x • e, ...
⟨A⟩: f a, (f a), f(a), (f)(a), ...

16.5.2. Free and Bound Variables

Let x, y be variable names and e, f be λ-expressions.

- ⟨V⟩: Variable x is free in x.
- ⟨F⟩: x is free in $\lambda y \cdot e$ if $x \neq y$ and x is free in e.
- ⟨A⟩: x is free in $f(e)$ if it is free in either f or e (i.e., also in both).

16.5.3. Substitution

- **subst**([N/x]x) ≡ N;
- **subst**([N/x]a) ≡ a,
  for all variables a≠ x;
- **subst**([N/x](P Q)) ≡ (**subst**([N/x]P) **subst**([N/x]Q));
- **subst**([N/x]($\lambda x \cdot P$)) ≡ λ y•P;
- **subst**([N/x](λ y•P)) ≡ $\lambda y \cdot$ **subst**([N/x]P),
  if x≠y and y is not free in N or x is not free in P;
- **subst**([N/x](λy•P)) ≡ λz•**subst**([N/z]**subst**([z/y]P)),
  if y≠x and y is free in N and x is free in P
  (where z is not free in (N P)).

16.5.4. α-Renaming and β-Reduction

- α-renaming: λx•M
  If x, y are distinct variables then replacing x by y in λx•M results in λy•**subst**([y/x]M). We can rename the formal parameter of a λ-function expression provided that no free variables of its body M thereby become bound.
- β-reduction: (λx•M)(N)
  All free occurrences of x in M are replaced by the expression N provided that no free variables of N thereby become bound in the result. (λx•M)(N) ≡ **subst**([N/x]M)

16.5.5. Function Signatures

For sorts we may want to postulate some functions:

type
  A, B, C
value
  obs_B: A → B,
  obs_C: A → C,
  gen_A: B×C → A

16.5.6. Function Definitions

Functions can be defined explicitly:

value
  f: Arguments → Result
  f(args) ≡ DValueExpr

  g: Arguments $\stackrel{\sim}{\rightarrow}$ Result
  g(args) ≡ ValueAndStateChangeClause
  pre P(args)

Or functions can be defined implicitly:

value
  f: Arguments → Result
  f(args) **as** result
  post P1(args,result)

  g: Arguments $\stackrel{\sim}{\rightarrow}$ Result
  g(args) **as** result
  pre P2(args)
  post P3(args,result)

16.6. Other Applicative Expressions

16.6.1. Simple let Expressions

Simple (i.e., nonrecursive) **let** expressions:

  let a = $\mathcal{E}_d$ **in** $\mathcal{E}_b$(a) **end**

is an “expanded” form of:

  $(\lambda a.\mathcal{E}_b(a))(\mathcal{E}_d)$

16.6.2. Recursive let Expressions

Recursive **let** expressions are written as:

let f = λa:A · E(f) **in** B(f,a) **end**

is "the same" as:

let f = **Y**F **in** B(f,a) **end**

where:

F ≡ λg·λa·(E(g)) and YF = F(YF)

16.6.3. Predicative let Expressions

Predicative **let** expressions:

let a:A · $\mathcal{P}$(a) **in** $\mathcal{B}$(a) **end**

express the selection of a value a of type A which satisfies a predicate $\mathcal{P}$(a) for evaluation in the body $\mathcal{B}$(a).

16.6.4. Pattern and "Wild Card" let Expressions

Patterns and *wild cards* can be used:

let {a} ∪ s = set **in** ... **end**
let {a,_} ∪ s = set **in** ... **end**

let (a,b,...,c) = cart **in** ... **end**
let (a,_,...,c) = cart **in** ... **end**

let ⟨a⟩⌢ℓ = list **in** ... **end**
let ⟨a,_,b⟩⌢ℓ = list **in** ... **end**

let [a↦b] ∪ m = map **in** ... **end**
let [a↦b,_] ∪ m = map **in** ... **end**

16.6.5. Conditionals

if b_expr **then** c_expr **else** a_expr
end

if b_expr **then** c_expr **end** ≡ /∗ same as: ∗/
  if b_expr **then** c_expr **else skip end**

if b_expr_1 **then** c_expr_1
elsif b_expr_2 **then** c_expr_2
elsif b_expr_3 **then** c_expr_3
...
elsif b_expr_n **then** c_expr_n **end**

case expr **of**
  choice_pattern_1 → expr_1,
  choice_pattern_2 → expr_2,
  ...
  choice_pattern_n_or_wild_card → expr_n
end

16.6.6. Operator/Operand Expressions

⟨Expr⟩ ::=
    ⟨Prefix_Op⟩ ⟨Expr⟩
  | ⟨Expr⟩ ⟨Infix_Op⟩ ⟨Expr⟩
  | ⟨Expr⟩ ⟨Suffix_Op⟩
  | ...
⟨Prefix_Op⟩ ::=
  − | ∼ | ∪ | ∩ | **card** | **len** | **inds** | **elems** | **hd** | **tl** | **dom** | **rng**
⟨Infix_Op⟩ ::=
  = | ≠ | ≡ | + | − | ∗ | ↑ | / | < | ≤ | ≥ | > | ∧ | ∨ | ⇒
  | ∈ | ∉ | ∪ | ∩ | \ | ⊂ | ⊆ | ⊇ | ⊃ | ^ | † | ◦
⟨Suffix_Op⟩ ::= !

16.7. Imperative Constructs

16.7.1. Statements and State Changes

Unit
value
stmt: **Unit** → **Unit**
stmt()

- Statements accept no arguments.
- Statement execution changes the state (of declared variables).
- **Unit** → **Unit** designates a function from states to states.
- Statements, stmt, denote state-to-state changing functions.
- Writing () as "only" arguments to a function "means" that () is an argument of type **Unit**.

16.7.2. Variables and Assignment

0. **variable** v:Type := expression
1. v := expr

16.7.3. Statement Sequences and skip

2. **skip**
3. stm_1;stm_2;...;stm_n

16.7.4. Imperative Conditionals

4. **if** expr **then** stm_c **else** stm_a **end**
5. **case** e **of**: p_1→S_1(p_1),...,p_n→S_n(p_n) **end**

16.7.5. Iterative Conditionals

6. **while** expr **do** stm **end**
7. **do** stmt **until** expr **end**

16.7.6. Iterative Sequencing

8. **for** e **in** list_expr · P(b) **do** S(b) **end**

16.8. Process Constructs

16.8.1. Process Channels

Let A and B stand for two types of (channel) messages and i:KIdx for channel array indexes, then:

channel c:A
channel { k[i]:B · i:KIdx }

16.8.2. Process Composition

- Let P and Q stand for names of process functions,
- i.e., of functions which express willingness to engage in input and/or output events,
- thereby communicating over declared channels.
- Let P() and Q stand for process expressions, then:

P ‖ Q	Parallel composition
P ⌷ Q	Nondeterministic external choice (either/or)
P ⊓ Q	Nondeterministic internal choice (either/or)
P ╫ Q	Interlock parallel composition

16.8.3. Input/Output Events

Let c, k[i] and e designate channels of type A and B, then:

c ?, k[i] ?	Input
c ! e, k[i] ! e	Output

- expresses the willingness of a process to engage in an event that
  - "reads" an input, respectively
  - "writes" an output.

16.8.4. Process Definitions

The below signatures are just examples. They emphasise that process functions must somehow express, in their signature, via which channels they wish to engage in input and output events.

value
P: **Unit** → **in** c **out** k[i]
Unit
Q: i:KIdx → **out** c **in** k[i] **Unit**

P() ≡ ... c ? ... k[i] ! e ...
Q(i) ≡ ... k[i] ? ... c ! e ...

The process function definitions (i.e., their bodies) express possible events.

16.9. Simple RSL Specifications

type
...
variable
...
channel
...
value
...
axiom
...