Domain Analysis & Description
The Implicit and Explicit Semantics Problem

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The domain analysis & description calculi introduced in [1] is shown to alleviate the issue of implicit semantics [2, 3]. The claim is made that domain descriptions, whether informal, or as also here, formal, amount to an explicit semantics for what is otherwise implicit if not described! I claim that [1] provides an answer to the claim in both [2, 3] that “The contexts of the systems in these cases are treated as second-class citizens . . . ”, respectively “In general, modeling languages are not equipped with resources, concepts or entities handling explicitly domain engineering features and characteristics (domain knowledge) in which the modeled systems evolve”.

1 Introduction

1.1 On the Issues of Implicit and Explicit Semantics

In [2] the issues of implicit and explicit semantics are analysed. It appears, from [2], that when an issue of software requirements or of the context, or, as we shall call it, the domain, is not prescribed or described to the extent that is relied upon in the software design, then it is referred to as an issue of implicit semantics. Once prescribed, respectively described, that issue becomes one of explicit semantics. In this paper we offer a calculus for analysing & describing domains (i.e., contexts), a calculus that allows you to systematically and formally describe domains.

1.2 A Triptych of Software Engineering

The dogma is:

• before software can be designed we must understand its requirements;
• and before we can prescribe the requirements we must understand the domain, that is, describe the domain.

A strict, but not a necessary, interpretation of this dogma thus suggests that software development “ideally” proceeds in three phases:

• First a phase of domain engineering in which an analysis of the application domain leads to a description of that domain.
• Then a phase of requirements engineering in which an analysis of the domain description leads to a prescription of requirements to software for that domain.

• And, finally, a phase of software design in which an analysis of the requirements prescription leads to software for that domain.

Proof of program, i.e., software code, correctness can be expressed as:

\[ D, S |\Rightarrow R \]

which we read as: proofs that software is correct with respect to requirements implies references to the domain.

1.3 **Contexts** \([2] \equiv \text{Domains} \[1]\)

Often the domain is referred to as the **context**. We treat contexts, i.e., domain descriptions as first class citizens \([2]\) Abstract, Page 1, lines 9–10]. By emphasizing the formalisation of domain descriptions we thus focus on the **explicit** semantics. Our approach, \([1]\), summarised in Sect.2 of this paper, thus represents a formal approach to the description of contexts (i.e., domains) \([2]\) Abstract, Page 1, line 12]. By a **domain**, i.e., a context, **description**, we shall here understand an **explicit semantics** of what is usually not specified and, when not so, referred to as **implicit semantics**\[3\].

1.4 **Semantics**

I use the term ‘semantics’ rather than the term ‘knowledge’. The reason is this: The entities are what we can meaningfully speak about. That is, the names of the endurants and perdurants, of their being atomic or composite, discrete or continuous, parts, components or materials, their unique identifications, mereologies and attributes, and the types, values and use of operations over these, form the language spoken by practitioners in the domain. It is this language its base syntactic quantities and semantic domains we structure and ascribe a semantics.

1.5 **Method & Methodology**

By a **method** I understand a set of principles for selecting and applying techniques and tools for constructing a manifest or an abstract artifact. By **methodology** I understand the study and knowledge of methods. **My work is almost exclusively in the area of methods and methodology.**

1.6 **Computer & Computing Sciences**

By **computer science** I understand the study and knowledge about the things that can exist inside computing devices.

By **computing science** I understand the study and knowledge about how to construct the things that can exist inside computing devices. Computing science is also often referred to as **programming methodology. My work is almost exclusively in the area of computing science.**

1.7 **Software and Systems Engineering**

By **software engineering** I understand the triplet of **domain engineering**, **requirements engineering**, and **software design**. **My work has almost exclusively been in the area of methodologies for large scale software – beginning with compilers (CHILL and Ada, \[4,5,6,7,8]\).**

\[2\] “The contexts . . . are treated as second-class citizens: in general, the modelling is implicit and usually distributed between the requirements model and the system model.” \([2]\) Abstract, Page 1, lines 9–12].
2 The Analysis & Description Prompts

We present a calculus of analysis and description prompts\(^3\). The presentation here is a very short, 12 pages, version of [1] Sects. 2–4, 31 pages]. These prompts are tools that the domain analyser & describer uses. The domain analyser & describer is in the domain, sees it, can touch it, and then applies the prompts, in some orderly fashion, to what is being observed. So, on one hand, there is the necessarily informal domain, and, on the other hand, there are the seemingly formal prompts and the “suggestions for something to be said”, i.e., written down: narrated and formalised. See Fig. 1. The

![An Ontology for Manifest Domains](image)

**Figure 1: An Ontology for Manifest Domains**

figure suggests a number of analysis and description prompts. The domain analyser & describer is “positioned” at the top, the “root”. If what is observed can be conceived and described then it is an entity. If it can be described as a “complete thing” at no matter which given snapshot of time then it is an endurant. If it is an entity but for which only a fragment exists if we look at or touch them at any given snapshot in time, then it is a perdurant. If it is an entity but for which only a fragment exists if we look at or touch them at any given snapshot in time, then it is a perdurant.

The concepts of endurants and perdurants may seem novel to some readers. So we elaborate a bit. First we must recall that we are trying to describe aspects a real worlds. That is, to model, in narrative and in formal terms, what has puzzled philosophers since antiquity. One can therefore not expect to define the terms ‘endurants’ and ‘perdurants’ as one define terms in computer science and

\(^3\)Prompt, as a verb: to move or induce to action; to occasion or incite; inspire; to assist (a person speaking) by “suggesting something to be said”. 
mathematics. Here, then are some “definitions”, i.e. some delineations, some “encirclings” of crucial concepts.

**Definition 1 Entity:** By an entity we shall understand a phenomenon, i.e., something that can be observed, i.e., be seen or touched by humans, or that can be conceived as an abstraction of an entity; alternatively, a phenomenon is an entity, if it exists, it is “being”, it is that which makes a “thing” what it is: essence, essential nature [9, Vol. I, pg. 665].

**Definition 2 Endurant:** By an endurant we shall understand an entity that can be observed or conceived and described as a “complete thing” at no matter which given snapshot of time; alternatively an entity is endurant if it is capable of enduring, that is persist, “hold out” [9, Vol. I, pg. 656]. Were we to “freeze” time we would still be able to observe the entire endurant.

**Definition 3 Perdurant:** By a perdurant we shall understand an entity for which only a fragment exists if we look at or touch them at any given snapshot in time, that is, were we to freeze time we would only see or touch a fragment of the perdurant, alternatively an entity is perdurant if it endures continuously, over time, persists, lasting [9, Vol. II, pg. 1552].

2.1 **Endurants: Parts, Components and Materials**

Endurants are either discrete or continuous. With discrete endurants we can choose to associate, or to not associate mereologies. If we do we shall refer to them as parts, else we shall call them components. With continuous endurants we do not associate mereologies. The continuous endurants we shall also refer to as (gaseous or liquid) materials. Parts are either atomic or composite and all parts have unique identifiers, mereology and attributes. If the observed part, \( p : P \), is composite then we can observe the part sorts and values, \( P_1, P_2, ..., P_m \) respectively \( p_1, p_2, ..., p_m \) of \( p \). “Applying” observe part sorts to \( p \) yields an informal (i.e., a narrative) and a formal description:

<table>
<thead>
<tr>
<th>Schema: Composite Parts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Narrative:</strong></td>
</tr>
<tr>
<td>⋄ ⋄ ⋄</td>
</tr>
<tr>
<td><strong>Formal:</strong></td>
</tr>
<tr>
<td>⋄ ⋄ type</td>
</tr>
<tr>
<td>⋄ ( P_1, P_2, ..., P_m ),</td>
</tr>
<tr>
<td>⋄ value</td>
</tr>
<tr>
<td>⋄ ( obs_P_i: P \rightarrow P_i ),</td>
</tr>
<tr>
<td>repeated for all ( m ) part sorts ( P_i )</td>
</tr>
</tbody>
</table>

**Aircraft Example 1: The Pragmatics**

The pragmatics of this ongoing example is this: We are dealing with ordinary passenger aircraft. We are focusing on that tiny area of concern that focus on passengers being informed of the progress of the flight, once in the air: where is the aircraft: its current position somewhere above the earth; its current speed and direction and possible acceleration (or deceleration); We do not bother about what time it is – etc. We abstract from the concrete presentation of this information.

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\(^4\) — ‘mereology’ will be explained next
Aircraft \textbf{Example 2:} Parts

1 An \textit{aircraft} is composed from several parts of which we focus on
   \begin{itemize}
     \item a \textit{position} part,
     \item b \textit{travel dynamics} part, and
     \item c \textit{display} part.
   \end{itemize}

\begin{tabular}{|c|c|}
\hline
\textbf{type} & AC, PP, TD, DP \\
\hline
\textbf{value} & \begin{itemize}
   \item \texttt{obs\_PP}: AC $\rightarrow$ PP
   \item \texttt{obs\_TD}: AC $\rightarrow$ TD
   \item \texttt{obs\_DP}: AC $\rightarrow$ DP
\end{itemize} \\
\hline
\end{tabular}

We have just summarised the analysis and description aspects of endurants in \textit{extension} (their “form”). We now summarise the analysis and description aspects of endurants in \textit{intension} (their “contents”). There are three kinds of intensional \textit{qualities} associated with parts, two with components, and one with materials. Parts and components, by definition, have \textit{unique identifiers}; parts have \textit{mereologies}, and all endurants have \textit{attributes}.

\section*{2.2 \textbf{Internal Qualities}}

\subsection*{2.2.1 Unique Identifiers}

Unique identifiers are further undefined tokens that uniquely identify parts and components. The description language observer \texttt{uid\_P}, when applied to parts \textit{p}: \textit{P} yields the unique identifier, $\pi$:$\Pi$, of \textit{p}. So the \texttt{observe\_part\_sorts}(\textit{p}) invocation also yields the description text:

\begin{itemize}
  \item ... [added to the narrative and]
  \item \textbf{type} \\
      $\Pi_1, \Pi_2, \ldots, \Pi_m$;
  \item \textbf{value} \\
      $\texttt{uid\_} \Pi_i : P_i \rightarrow \Pi_i$,
      repeated for all \textit{m} part sorts \textit{P}_i's and added to the formalisation.
\end{itemize}

\textbf{Aircraft \textbf{Example 3:} Unique Identifiers}

2 position, travel dynamic and display parts have unique identifiers.

\begin{tabular}{|c|c|}
\hline
\textbf{type} & PPI, TDI, DPI \\
\hline
\textbf{value} & \\
\hline
\end{tabular}

\footnote{Pragmatics is here used in the sense outlined in \cite{10} Chapter 7, Pages 145--148}
2.2.2 Mereology

*Mereology is the study and knowledge of parts and part relations.* The mereology of a part is an expression over the unique identifiers of the (other) parts with which it is related, hence \( \text{mero}_p : P \rightarrow \mathcal{E}(\Pi_j, ..., \Pi_k) \) where \( \mathcal{E}(\Pi_j, ..., \Pi_k) \) is a type expression. So the \texttt{observe\_part\_sorts}(p) invocation also yields the description text:

\[
\begin{align*}
\text{Schema: Mereology} & \\
\text{• ... [added to the narrative and]} \\
\text{• value} & \\
& \bowtie \text{mero}_p : P \rightarrow \mathcal{E}(\Pi_j, ..., \Pi_k) \ [\text{added to the formalisation}]
\end{align*}
\]

Aircraft Example 4: Mereology

We shall omit treatment of aircraft mereologies.

3 The position part is related to the display part.
4 The travel dynamics part is related to the display part.
5 The display part is related to both the position and the travel dynamics parts.

\[
\begin{align*}
\text{value} & \\
& \bowtie \text{mero}_p : PP \rightarrow DPI \\
& \bowtie \text{mero}_t : TP \rightarrow DPI \\
& \bowtie \text{mero}_d : DP \rightarrow DPI \times TDI
\end{align*}
\]

2.2.3 Attributes

Attributes are the remaining qualities of endurants. The analysis prompt \texttt{obs\_attributes} applied to an endurant yields a set of type names, \( A_1, A_2, ..., A_t \), of attributes. They imply the additional description text:

\[
\begin{align*}
\text{Schema: Attributes} & \\
\text{• Narrative:} & \\
& \bowtie ... \\
\text{• Formal:} & \\
& \bowtie \text{type} \\
& \bowtie \text{type} \ A_1, A_2, ..., A_t \\
& \bowtie \text{value} \\
& \bowtie \text{attr}_A : E \rightarrow A_i
\end{align*}
\]
repeated for all \( t \) attribute sorts \( A_i \)’s!

---

**Aircraft Example 5:** Position Attributes

6 Position parts have longitude, latitude and altitude attributes.

<table>
<thead>
<tr>
<th>type</th>
<th>LO, LA, AL</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td></td>
</tr>
<tr>
<td>attr_( LO ): PP ( \rightarrow ) LO</td>
<td></td>
</tr>
<tr>
<td>attr_( LA ): PP ( \rightarrow ) LA</td>
<td></td>
</tr>
<tr>
<td>attr_( AL ): PP ( \rightarrow ) AL</td>
<td></td>
</tr>
</tbody>
</table>

These quantities: longitude, latitude and altitude are “actual” quantities, they mean what they express, they are not recordings or displays of these quantities; to express those we introduce separate types.

---

**Aircraft Example 6:** Travel Dynamics Attributes

7 Travel dynamics parts have velocity\(^6\) and acceleration\(^7\).

<table>
<thead>
<tr>
<th>type</th>
<th>VEL, ACC</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td></td>
</tr>
<tr>
<td>attr_( VEL ): TD ( \rightarrow ) VEL</td>
<td></td>
</tr>
<tr>
<td>attr_( ACC ): TD ( \rightarrow ) ACC</td>
<td></td>
</tr>
</tbody>
</table>

These quantities: velocity and acceleration, are “actual” quantities, they mean what they express, they are not recordings or displays of these quantities; to express those we introduce separate types.

---

**Aircraft Example 7:** Quantity Recordings

8 On one hand there are the actual location and dynamics quantities (i.e., values),

9 on the other hand there are their recordings,

10 and there are conversion functions from actual to recorded values.

<table>
<thead>
<tr>
<th>type</th>
<th>LO, LA, AL, VEL, ACC</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td></td>
</tr>
<tr>
<td>a2r_( LO ): LO ( \rightarrow ) rLO, a2r_( LA ): LA ( \rightarrow ) rLA, a2r_( AL ): AL ( \rightarrow ) rAL</td>
<td></td>
</tr>
<tr>
<td>a2r_( VEL ): VEL ( \rightarrow ) rVEL, a2r_( ACC ): ACC ( \rightarrow ) rACC</td>
<td></td>
</tr>
</tbody>
</table>

There are, of course, no functions that convert recordings to actual values!

---

\(^6\)Velocity is a vector of speed and orientation (i.e., direction).

\(^7\)Acceleration is a vector of change of speed per time unit and orientation.
Aircraft Example 8: Display Attributes

Display parts have display-modified longitude, latitude and altitude, and velocity and acceleration attributes – with functions that convert between these, recorded and displayed, attributes.

<table>
<thead>
<tr>
<th>type</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>dLO, dLA, dAL</td>
<td>attr_dLO: DP → dLO</td>
</tr>
<tr>
<td>dVEL, dACC</td>
<td>attr_dLA: DP → dLA</td>
</tr>
<tr>
<td></td>
<td>attr_dAL: DP → dAL</td>
</tr>
<tr>
<td></td>
<td>attr_dVEL: DP → dVEL</td>
</tr>
</tbody>
</table>

2.2.4 Attribute Categories

Michael A. Jackson categorizes and defines attributes as either static or dynamic, with dynamic attributes being either inert, reactive or active. The latter are then either autonomous, biddable or programmable. This categorization has a strong bearing on how these (f.ex., part) attributes are dealt with when now interpreting parts as behaviours.

Aircraft Example 9: Attribute Categories

Longitude, latitude, altitude, velocity and acceleration are all reactive attributes – they change in response to the bidding of aircraft attributes that we have not covered.

<table>
<thead>
<tr>
<th>attribute categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>reactive: LO, LA, AL, VEL, ACC</td>
</tr>
<tr>
<td>programmable: dLO, dLA, dAL, dVEL, dACC</td>
</tr>
</tbody>
</table>

2.3 Description Axioms and Proof Obligations

In we show that the description prompts may result in axioms or proof obligations. We refer to for details. Here we shall, but show one example of an axiom.

Aircraft Example 10: An Axiom

The displayed attributes must at any time be displays of the corresponding recorded position and travel dynamics attributes.

\[
\begin{align*}
\Box \forall \text{ac:AC} \cdot \\
\text{let } (pp, \text{TD}, \text{DP}) &= (\text{obs\_PP(ac)}, \text{obs\_TD(ac)}, \text{obs\_DP(ac)}) \text{ in} \\
\text{let } (lo, la, at) &= (\text{attr\_LO(pp), attr\_LA(pp), attr\_AT(pp)}),
\end{align*}
\]

*– for example: thrust, weight, lift, drag, rudder position, and aileron position – plus dozens of other – attributes
2.4 From Manifest Parts (Endurants) to Domain Behaviours (Perdurants)

then presents a compiler which to manifest parts associate behaviours. These are then specified as CSP processes. We choose CSP for the following reasons: it is a well-established formalism for expressing the behaviour of cooperating sequential processes; it has withstood the test of time: first articles appeared in 1978 and research and industrial use is still at a high; it has a well-founded theory and a failures–divergence-refinement proof system with proof rules; we have shown, in [20] To Every Manifest Domain a CSP Expression – A Role for Mereology in Computer Science], how to relate a world of space-based endurants with a world of time-based perdurants; in [21] A Philosophy of Domain Science & Engineering –An Interpretation of Kai Sørlander’s Philosophy]. The latter two reasons, to us, are rather convincing: The concept of mereology is well-established, both in philosophy and in logic. To “connect” the concepts of mereology with ontological concepts of describable domains, by some neo-Kantian transcendental deduction is quite surprising – and pleasing.

2.4.1 The Idea — by means of an example

The term aircraft can have the following “meanings”: the aircraft, as an endurant, parked at the airport gate, i.e., as a composite part; the aircraft, as a perdurant, as it flies through the skies, i.e., as a behaviour; and the aircraft, as an attribute, of an airline timetable.
The six actual position and travel dynamics attribute values longitude, latitude, altitude, velocity and acceleration are recorded, by appropriate instruments. In the above figure this is indicated by input channels attr\_LO\_ch, attr\_LA\_ch, attr\_AL\_ch, attr\_VEL\_ch and attr\_ACC\_ch.

### 2.4.2 Channels and Communication

Behaviours sometimes synchronise and usually communicate. We use the CSP [12] notation (adopted by RSL) to model behaviour communication. Communication is abstracted as the sending, ch ! m, and receipt, ch ? m, of messages, m:M, over channels, ch.

\[
\text{type M} \\
\text{channel ch:M}
\]

**Aircraft Example 12: Channels**

For this example we focus only on communications from the position and travel dynamics behaviours to the display behaviour.

15 The messages sent from the position behaviour to the display behaviour are triplets of recorded longitude, latitude and altitude values.

16 The messages sent from the travel dynamics behaviour to the display behaviour are droplets of recorded velocity and acceleration values.

17 There is a channel, po\_di\_ch, that allows communication of messages from the position behaviour to the display behaviour.

18 There is a channel, td\_di\_ch, that allows communication of messages from the travel dynamics behaviour to the display behaviour.

19 For each of the reactive attributes there is a corresponding channel.

\[
\text{type PM} = rLO \times rLA \times rAL \\
\text{channel TDM} = rVEL \times rACC
\]
2.4.3 Behaviour Signatures

We shall only cover behaviour signatures when expressed in RSL/CSP [22]. The behaviour functions are now called processes. That a behaviour function is a never-ending function, i.e., a process, is “revealed” in the function signature by the “trailing” Unit:

\[
\text{behaviour: } \ldots \rightarrow \ldots \text{ Unit}
\]

That a process takes no argument is “revealed” by a “leading” Unit:

\[
\text{behaviour: } \text{Unit} \rightarrow \ldots
\]

That a process accepts channel, viz.: \text{ch}, inputs, including accesses an external attribute \text{A}, is “revealed” in the function signature as follows:

\[
\text{behaviour: } \ldots \rightarrow \text{in ch } \ldots, \text{ resp. in attr}_A \text{ch}
\]

That a process offers channel, viz.: \text{ch}, outputs is “revealed” in the function signature as follows:

\[
\text{behaviour: } \ldots \rightarrow \text{out ch } \ldots
\]

That a process accepts other arguments is “revealed” in the function signature as follows:

\[
\text{behaviour: } \text{ARG} \rightarrow \ldots
\]

where \text{ARG} can be any type expression:

\[
T, T \rightarrow T, T \rightarrow T \rightarrow T, \text{ etcetera}
\]

where \text{T} is any type expression.

2.4.4 Translation of Part Qualities

Part qualities, that is: unique identifiers, mereologies and attributes, are translated into behaviour arguments – of one kind or another, i.e., (…). Typically we can choose to index behaviour names, \(b\) by the unique identifier, \(id\), of the part based on which they were translated, i.e., \(b_{id}\). Mereology values are usually static, and can, as thus, be treated like we treat static attributes (see next), or can be set by their behaviour, and are then treated like we treat programmable attributes (see next), i.e., (…). Static attributes become behaviour definition (body) constant values. Inert, reactive and autonomous attributes become references to channels, say \(ch\_dyn\), such that when an inert, reactive and autonomous attribute value is required it is expressed as \(ch\_dyn\ ?\). Programmable and biddable attributes become arguments which are passed on to the tail-recursive invocations of the behaviour, and possibly updated as specified [with] in the body of the definition of the behaviour, i.e., (…).
2.4.5 Part Behaviour Signatures

We can, without loss of generality, associate with each part a behaviour; parts which share attributes (and are therefore referred to in some parts’ mereology), can communicate (their “sharing”) via channels. A behaviour signature is therefore:

\[
\text{beh}_{\pi}: \text{me:MT} \times \text{sa:SA} \rightarrow \text{ca:CA} \rightarrow \text{ichns}(\text{ea:EA})\in\text{out ichns}(\text{me}) \text{ Unit}
\]

where (i) \(\pi: II\) is the unique identifier of part \(p\), i.e., \(\pi=\text{uid}_P(p)\), (ii) \(\text{me:ME}\) is the mereology of part \(p\), \(\text{me}=\text{obs mereo}_P(p)\), (iii) \(\text{sa:SA}\) lists the static attribute values of the part, (iv) \(\text{ca:CA}\) lists the biddable and programmable attribute values of the part, (v) \(\text{ichns}(\text{ea:EA})\) refer to the external attribute input channels, and where (vi) \(\text{iochs}(\text{me})\) are the input/output channels serving the attributes shared between the part \(p\) and the parts designated in its mereology \(me\).

We focus, for a little while, on the expression of \(\text{sa:SA}\), \(\text{ea:EA}\) and \(\text{ca:CA}\), that is, on the concrete types of \(\text{SA}\), \(\text{EA}\) and \(\text{CA}\). \(\text{sa:SA}\) lists the static value types, \((svT_1, \ldots, svT_s)\), where \(s\) is the number of static attributes of parts \(p:P\). \(\text{ea:EA}\) lists the external attribute value channels of parts \(p:P\) in the behaviour signature and as input channels, \(\text{ichns}\), see 9 lines above. \(\text{ca:CA}\) lists the controllable value expression types of parts \(p:P\). A controllable attribute value expression is an expression involving one or more attribute value expressions of the type of the biddable or programmable attribute.

--- Aircraft Example 13: Part Behaviour Signatures, I/II ---

We omit the signature of the aircraft behaviour.

20 The signature of the position behaviour lists its unique identifier, mereology, no static and no controllable attributes, but its three reactive attributes (as input channels) and its (output) channel to the display behaviour.

21 The signature of the travel dynamics behaviour lists its unique identifier, mereology, no static and no controllable attributes, but its three reactive attributes (as input channels) and its (output) channel to the display behaviour.

22 The signature of the display behaviour lists its unique identifier, its mereology, no static attribute, but the programmable display attributes, assembled in a pair of a triplet and doublet, and its two input channels from the position, respectively the travel dynamics behaviours.

--- Aircraft Example 14: Part Behaviour Signatures, I/II ---

<table>
<thead>
<tr>
<th>type</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DA</td>
<td>((dLA \times dLO \times dAL) \times (dVEL \times dACC))</td>
</tr>
<tr>
<td>position: PI \times DPI \rightarrow</td>
<td>\text{in attr}_L O _ch, \text{attr}_L A _ch, \text{attr}_AL _ch, \text{out po}_di _ch Unit</td>
</tr>
<tr>
<td>travel_dynamics: TDI \times DPI \rightarrow</td>
<td>\text{in attr}_VEL _ch, \text{attr}_ACC _ch, \text{attr}_DIR _ch, \text{out td}_di _ch Unit</td>
</tr>
<tr>
<td>display: DI \times (PPI \times TDI) \rightarrow DA \rightarrow \text{in po}_di _ch, \text{td}_di _ch Unit</td>
<td></td>
</tr>
</tbody>
</table>

2.4.6 Behaviour Compilations

Composite Behaviours Let \(P\) be a composite sort defined in terms of sub Sorts \(P_1, P_2, \ldots, P_n\). The process definition compiled from \(p:P\), is composed from a process description, \(Mc_P \text{uid}_P(p)\).
relying on and handling the unique identifier, mereology and attributes of part \( p \) operating in parallel with processes \( p_1, p_2, \ldots, p_n \) where \( p_1 \) is compiled from \( p_1 : P_1 \), \( p_2 \) is compiled from \( p_2 : P_2 \), \ldots, and \( p_n \) is compiled from \( p_n : P_n \). The domain description “compilation” schematic below “formalises” the above.

\[
\text{Transcendental Schema: Abstract is\_composite}(p)
\]

\[
\text{value}
\]
\[
\begin{align*}
\text{compile\_process}: & \ P \rightarrow \text{RSL\_Text} \\
\text{compile\_process}(p) & \equiv \\
& M_{\text{uid\_P}(p)}(\text{obs\_mereo\_P}(p), \mathcal{S}(p))(\mathcal{C}(p)) \\
& \parallel \text{compile\_process}(\text{obs\_part\_P}_1(p)) \\
& \parallel \text{compile\_process}(\text{obs\_part\_P}_2(p)) \\
& \ldots \\
& \parallel \text{compile\_process}(\text{obs\_part\_P}_n(p))
\end{align*}
\]

The text macros: \( \mathcal{S} \) and \( \mathcal{C} \) were informally explained above. Part sorts \( P_1, P_2, \ldots, P_n \) are obtained from the observe part sorts prompt.

\[
\text{Aircraft Example 15: Aircraft Behaviour, I/II}
\]

23 Compiling a composite aircraft part results in the parallel composition
\begin{enumerate}
\item the compilation of the atomic position part,
\item the compilation of the atomic travel dynamics part, and
\item the compilation of the atomic display part.
\end{enumerate}

We omit compiling the aircraft core behaviour.

24 Compilation of atomic parts entail no further compilations.

\[
\text{Aircraft Example 15: Aircraft Behaviour, II/II}
\]

\[
\begin{align*}
\text{value}
\end{align*}
\]
\[
\begin{align*}
\text{compile}(ac) & \equiv \\
\text{compile}(\text{obs\_PP}(p)) \\
\parallel \text{compile}(\text{obs\_TD}(p)) \\
\parallel \text{compile}(\text{obs\_DI}(p))
\end{align*}
\]

\[
\text{Atomic Behaviours}
\]

\[
\text{Transcendental Schema: is\_atomic}(p)
\]

\[
\text{value}
\]
\[
\begin{align*}
\text{compile\_process}: & \ P \rightarrow \text{RSL\_Text} \\
\text{compile\_process}(p) & \equiv \\
& M_{\text{uid\_P}(p)}(\text{obs\_mereo\_P}(p), \mathcal{S}(p))(\mathcal{C}(p))
\end{align*}
\]
We initialise the display behaviour with a further undefined value.

```
23a compile(obs_PP(p)) ≡ position(uid_PP(p),mereo_PP(p))
23b compile(obs_TD(p)) ≡ travel_dynamics(uid_TD(p),mereo_TD(p))
23c compile(obs_DI(p)) ≡ display(.uid_DI(p),mereo_DI(p))(init_DA)
```

In the above we have already subsumed the atomic behaviour definitions, see next, and directly inserted the $\mathcal{F}$ definitions.

### 2.4.7 Atomic Behaviour Definitions

#### Transcendental Schema IV: Atomic Core Processes

```
\text{value}
\begin{align*}
\mathcal{M}_{P_{\pi,\Pi}} : & \text{me:MT} \times \text{sa:SA} \rightarrow \text{ca:CA} \\
& \text{in ichns(ea:EA) in out iochs(me)} \quad \text{Unit} \\
\mathcal{M}_{P_{\pi,\Pi}}(\text{me,sa})(\text{ca}) & \equiv \\
& \text{let } (\text{me}',\text{ca}') = \mathcal{F}_{\pi,\Pi}(\text{me,sa})(\text{ca}) \text{ in} \\
& \mathcal{M}_{P_{\pi,\Pi}}(\text{me}',\text{sa})(\text{ca}') \quad \text{end}
\end{align*}
\mathcal{F}_{\pi,\Pi} : & \text{me:MT} \times \text{sa:SA} \rightarrow \text{CA} \\
& \text{in ichns(ea:EA) in out iochs(me)} \rightarrow \text{MT} \times \text{CA}
```

#### Aircraft Example 17: Position Behaviour Definition

26 The *position* behaviour offers to receive the *longitude*, *latitude* and the *altitude* attribute values
27 and to offer them to the *display* behaviour,
28 whereupon it resumes being the *position* behaviour.

```
26 position(p\pi,d\pi) ≡ \\
26 \text{let } (lo,la,al) = (\text{attr}_\text{LOch?},\text{attr}_\text{LAch?},\text{attr}_\text{ALch?}) \text{ in} \\
27 \text{po_dil.ch ! (a2rLO(lo),a2rLA(la),a2rAL(al)) ;} \\
27 \text{position(p\pi,d\pi) \text{ end}}
```
Aircraft Example 18: Travel Dynamics Behaviour Definition

29 The travel_dynamics behaviour offers to receive the recorded velocity and the acceleration attribute values
30 and to offer these to the display behaviour,
31 whereupon it resumes being the travel_dynamics behaviour.

\text{value}
\begin{verbatim}
29 travel_dynamics(tdπ,dπ) ≡
29  let (vel,acc)=(attr_VEL_ch?,attr_ACC_ch?) in
30  td_di_ch ! (a2rVEL(vel),a2rACC(acc)) ;
31  travel_dynamics(tdπ,dπ) end
\end{verbatim}

Aircraft Example 19: Display Behaviour Definition

32 The display behaviour offers to receive the reactive attribute doublet s from the position and the travel_dynamics behaviours while
33 resuming to be that behaviour albeit now with these as their updated display.
34 The conversion functions are extensions of the ones introduced earlier.

\text{value}
\begin{verbatim}
32 display(dπ,(dπ,tdπ))(d_pos,d_tdy) ≡
32  let (pos_d′,tdy_d′) = (pos_di_ch?,td_di_ch?) in
33  display(dπ,(dπ,tdπ))(conv(pos_d′),conv(tdy_d′)) end
\end{verbatim}
\text{type}
\begin{verbatim}
34 dMPD = dLO × dLA × dAL
34 dMTD = dVEL × dACC
\end{verbatim}
\text{value}
\begin{verbatim}
34 conv: MPD → dMPD
34 conv(rlo,rla,ral) ≡ (r2dLO(rlo),r2dLA(rla),r2dAL(ral))
34 conv: MTD → dMTD
34 conv(rvel,racc) ≡ (r2dVEL(rvel),r2dACC(racc))
\end{verbatim}

2.5 A Proof Obligation

We refer, again, to [1] for more on proof obligations.

Aircraft Example 20: A Proof Obligation

The perdurant descriptions of Items 15–34 is a model of the axiom expressed in Item 14.
Calculations in Classical Domains: Some Simple Observations

This section covers three loosely related topics: Sect. 3.1 muses over properties of some attribute values. Then, Sect. 3.2 we recall some facts about types, scales and values of measurable units in physics. The previous leads us, in Sect. 3.3 to consider further detailing the concept of attributes such as we have covered it in Sect. 2.2.3 Pages and in [1]. The reason for covering these topics is that most attribute values are represented in “final” programs as numbers of one kind or another and that type checking in most software is with respect to these numbers.

3.1 Some Observations on Some Attribute Values

Let us, seemingly randomly, examine some simple, e.g., arithmetic, operations in classical domains. By time is often meant absolute time. So a time could be March 29, 2018: 06:41 am. One can not add two times. One can speak of a time being earlier, or before another time. October 23, 2017: 10:01 am is earlier, ≤, than March 29, 2018: 06:41 am. One can speak of the time interval between October 23, 2016: 8:01 am and October 24, 2017: 10:05 am being 1 year, 1 day, 2 hours and 4 minutes, that is: October 24, 2017: 10:05 am ⊕ October 23, 2016: 8:01 am = 1 year, 1 day, 2 hours and 4 minutes. One can add a time interval to a time and obtain a time. One can multiply a time interval with a real number.

We can formalize the above:

\[
\begin{align*}
T & = \text{Month} \times \text{Day} \times \text{Year} \times \text{Hour} \times \text{Minute} \times \text{Sec} \\
\text{TII} & = \text{Days} \times \text{Hours} \times \text{Minutes} \times \text{Seconds} \times ...
\end{align*}
\]

\[
\begin{align*}
\text{Month} & = \{1,2,3,4,5,6,7,8,9,10,11,12\} \\
\text{Day} & = \{1,2,3,4,...,28,29,30,31\} \\
\text{Hour,Hours} & = \{0,1,2,3,...,21,22,23\} \\
\text{Minute,Minutes} & = \{0,1,2,3,...,56,57,58,59\} \\
\text{Second,Seconds} & = \{0,1,2,3,...,56,57,58,59\}
\end{align*}
\]

One can not add temperatures – makes no sense in physics! But one can take the mean value of two (or more) temperatures. One can subtract temperatures obtaining positive or negative temperature intervals. One can take the mean of any number of temperature, but would probably be well advised to have these represent regular sampling, or at least time-stamped. One can also define rate of change of temperature.

\[
\begin{align*}
\text{Temp, MeanTemp, Degrees, TempIntv} & = \text{Degrees} \\
\text{mean: Temp}-\text{set} \times \text{Nat} & \to \text{MeanTemp} \\
\text{−: Temp} \times \text{Temp} & \to \text{TempIntv}
\end{align*}
\]

\[
\begin{align*}
\text{TST} & = \text{(Temp} \times T)-\text{set} \\
\text{avg: TST} & \to \text{MeanTemp}
\end{align*}
\]

\text{Days} = \text{Nat}

\[
\begin{align*}
\text{value} & <,\leq,=,\geq,>: T \times T \to \text{Boole} \\
\text{−: } T \times T & \to \text{TII pre } t-t': t' \leq t \\
\text{<,\leq,=,\geq,>: } TII \times TII & \to \text{Bool} \\
\text{−,+: } TII \times TII & \to TII \\
\text{*: } TII \times \text{Real} & \to TII \\
/\!: TII \times TII & \to \text{Real}
\end{align*}
\]

The time interval could, e.g., be converted into seconds, then the integer number standing for seconds can be multiplied by \( r \) and the result be converted “back” into years, days, hours, minutes and seconds — whatever it takes!
type
\[ \text{TimeUnit} = \{ \text{"year","month","day"},\text{"hour"},... \} \]
\[ \text{RoTC} = \text{TempIntv} \times \text{TimeUnit} \]

Etcetera. We leave it to the reader to speculate on which operations one can perform on a persons’ attributes: height, weight, birth date, name, etc. And similarly for other domains. It is time to “lift” these observations. After the examples above we should inquire as to which kind of units we may operate upon. For the sake of our later exposition it is enough that we look in some detail at the “universe” of physics.

3.2 Physics Attributes

3.2.1 SI: The International System of Quantities

In physics we operate on values of attributes of manifest, i.e., physical phenomena. The type of some of these attributes are recorded in well known tables, cf. Tables 1–3.

Table 1 shows the base units of physics.

<table>
<thead>
<tr>
<th>Base quantity</th>
<th>Name</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>meter</td>
<td>m</td>
</tr>
<tr>
<td>mass</td>
<td>kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>time</td>
<td>second</td>
<td>s</td>
</tr>
<tr>
<td>electric current</td>
<td>ampere</td>
<td>A</td>
</tr>
<tr>
<td>thermodynamic temperature</td>
<td>kelvin</td>
<td>K</td>
</tr>
<tr>
<td>amount of substance</td>
<td>mole</td>
<td>mol</td>
</tr>
<tr>
<td>luminous intensity</td>
<td>candela</td>
<td>cd</td>
</tr>
</tbody>
</table>

Table 1: Base Units

Table 2 on the following page shows the units of physics derived from the base units.
Table 3 on page 19 shows further units of physics derived from the base units.
Table 4 on page 19 shows standard prefixes for SI units of measure.
Table 5 on page 20 shows fractions of SI units of measure.
These “pictures” are meant as an eye opener, a “teaser”.

And these formulas likewise!
Table 2: Derived Units

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Derived Quantity</th>
<th>Derived Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>radian</td>
<td>rad</td>
<td>angle</td>
<td>m/m</td>
</tr>
<tr>
<td>steradian</td>
<td>sr</td>
<td>solid angle</td>
<td>m²×m⁻²</td>
</tr>
<tr>
<td>Hertz</td>
<td>Hz</td>
<td>frequency</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>newton</td>
<td>N</td>
<td>force, weight</td>
<td>kg×m×s⁻²</td>
</tr>
<tr>
<td>pascal</td>
<td>Pa</td>
<td>pressure, stress</td>
<td>N/m²</td>
</tr>
<tr>
<td>joule</td>
<td>J</td>
<td>energy, work, heat</td>
<td>N×m</td>
</tr>
<tr>
<td>watt</td>
<td>W</td>
<td>power, radiant flux</td>
<td>J/s</td>
</tr>
<tr>
<td>coulomb</td>
<td>C</td>
<td>electric charge</td>
<td>s×A</td>
</tr>
<tr>
<td>volt</td>
<td>V</td>
<td>voltage, electromotive force</td>
<td>W/A (kg×m²×s⁻³×A⁻¹)</td>
</tr>
<tr>
<td>farad</td>
<td>F</td>
<td>capacitance</td>
<td>C/V (kg⁻¹×m⁻²×s⁴×A²)</td>
</tr>
<tr>
<td>ohm</td>
<td>Ω</td>
<td>electrical resistance</td>
<td>V/A (kg×m²×s³×A²)</td>
</tr>
<tr>
<td>siemens</td>
<td>S</td>
<td>electrical conductance</td>
<td>A/V (kg1×m²×s³×A²)</td>
</tr>
<tr>
<td>weber</td>
<td>Wb</td>
<td>magnetic flux</td>
<td>V×s (kg×m²×s⁻²×A⁻¹)</td>
</tr>
<tr>
<td>tesla</td>
<td>T</td>
<td>magnetic flux density</td>
<td>Wb/m² (kg×s²×A⁻¹)</td>
</tr>
<tr>
<td>henry</td>
<td>H</td>
<td>inductance</td>
<td>Wb/A (kg×m²×s⁻²×A²)</td>
</tr>
<tr>
<td>degree Celsius</td>
<td>°C</td>
<td>temperature relative to 273.15 K</td>
<td>K</td>
</tr>
<tr>
<td>lumen</td>
<td>lm</td>
<td>luminous flux</td>
<td>cd×sr (cd)</td>
</tr>
<tr>
<td>lux</td>
<td>lx</td>
<td>illuminance</td>
<td>lm/m² (m²×cd)</td>
</tr>
</tbody>
</table>

The point in bringing this material is that when modelling, i.e., describing domains we must be extremely careful in not falling into the trap of modelling physics, etc., types as we do in programming!

3.2.2 What Are We to Learn from this Exposition?

We see from the previous section, Sect.3.2, that physics units can be highly “structured”. What Are We to Learn from this Exposition? I think it is this: It is customary, in programs of languages from Algol 60 via Pascal to Java, to assign float or double types, as in Java, to [constants or] variables that for example represent values of physics. So rather completely different types of physics units are all cast into a same, simple-minded, “number” type. No chance, really, for any meaningful type checking.

---

10For example, Newton: kg×m×s⁻², Volt = kg×m²×s⁻³×A⁻¹, etc.
11representing single-, resp. double-precision 32-bit IEEE 754 floating point values
<table>
<thead>
<tr>
<th>Name</th>
<th>Explanation</th>
<th>Derived Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>area</td>
<td>square meter</td>
<td>m²</td>
</tr>
<tr>
<td>volume</td>
<td>cubic meter</td>
<td>m³</td>
</tr>
<tr>
<td>speed, velocity</td>
<td>meter per second</td>
<td>m/s</td>
</tr>
<tr>
<td>acceleration</td>
<td>meter per second squared</td>
<td>m/s²</td>
</tr>
<tr>
<td>wave number</td>
<td>reciprocal meter</td>
<td>m⁻¹</td>
</tr>
<tr>
<td>mass density</td>
<td>kilogram per cubic meter</td>
<td>kg/m³</td>
</tr>
<tr>
<td>specific volume</td>
<td>cubic meter per kilogram</td>
<td>m³/kg</td>
</tr>
<tr>
<td>current density</td>
<td>amperes per square meter</td>
<td>A/m²</td>
</tr>
<tr>
<td>magnetic field strength</td>
<td>amperes per meter</td>
<td>A/m</td>
</tr>
<tr>
<td>amount-of-substance concentration</td>
<td>mole per cubic meter</td>
<td>mol/m³</td>
</tr>
<tr>
<td>luminance</td>
<td>candela per square meter</td>
<td>cd/m²</td>
</tr>
<tr>
<td>mass fraction</td>
<td>kilogram per kilogram</td>
<td>kg/kg = 1</td>
</tr>
</tbody>
</table>

Table 3: Further Units

<table>
<thead>
<tr>
<th>Prefix name</th>
<th>deca</th>
<th>hecto</th>
<th>kilo</th>
<th>mega</th>
<th>giga</th>
<th>tera</th>
<th>peta</th>
<th>exa</th>
<th>zetta</th>
<th>yotta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prefix symbol</td>
<td>da</td>
<td>h</td>
<td>k</td>
<td>M</td>
<td>G</td>
<td>T</td>
<td>P</td>
<td>E</td>
<td>Z</td>
<td>Y</td>
</tr>
<tr>
<td>Factor</td>
<td>10⁰</td>
<td>10¹</td>
<td>10²</td>
<td>10³</td>
<td>10⁶</td>
<td>10⁹</td>
<td>10¹²</td>
<td>10¹⁵</td>
<td>10¹⁸</td>
<td>10²¹</td>
</tr>
</tbody>
</table>

Table 4: Standard Prefixes for SI Units of Measure

3.3 Attribute Types, Scales and Values: Some Thoughts

This section further elaborates on the treatment of attributes given in Sect. 2.2.3, Pages 6–8. The elaboration is only sketched. It need be studied, in detail.

The elaboration is this: The attr_A observer function, for a part p of sort P, such as defined in Sect. 2.2.3 (Page 6) yields values of type A. In the revised understanding of attributes the attr_A observer is now to yield both the type, AT, and the value, AV, of attribute A:

\[
\text{type} \quad \text{AT, AV} \\
\text{value} \quad \text{attr}_A : P \to AT \times AV
\]

You may think of A being defined by AT \times AV.

The revision is further that a domain analysis & description of the operations over attributes values, \( \theta \):

\[
\theta : A_i \times A_j \times \ldots \times A_k \to V
\]

be carefully checked – such as hinted at in Sect. 3.1 on page 16.

Whether such operator-checks be researched and documented “once-and-for-all” for given “standard” domains, by domain scientists, or per domain model, by domain engineers, in connection with specific software development projects is left for you to decide! These operator-checks, together with an otherwise appropriate domain analysis & description, if not pursued, results in implicit semantics, and if pursued, results in explicit semantics. It is as simple as that!
Table 5: Fractions

<table>
<thead>
<tr>
<th>Prefix name</th>
<th>deci</th>
<th>centi</th>
<th>milli</th>
<th>micro</th>
<th>nano</th>
<th>pico</th>
<th>femto</th>
<th>attol</th>
<th>zeptol</th>
<th>yoctol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prefix symbol</td>
<td>d</td>
<td>c</td>
<td>m</td>
<td>μ</td>
<td>n</td>
<td>p</td>
<td>f</td>
<td>a</td>
<td>z</td>
<td>y</td>
</tr>
<tr>
<td>Factor</td>
<td>$10^0$</td>
<td>$10^{-1}$</td>
<td>$10^{-2}$</td>
<td>$10^{-3}$</td>
<td>$10^{-6}$</td>
<td>$10^{-9}$</td>
<td>$10^{-12}$</td>
<td>$10^{-15}$</td>
<td>$10^{-18}$</td>
<td>$10^{-21}$</td>
</tr>
</tbody>
</table>

4 Conclusion

4.1 What Have We Achieved?

We have suggested that the issue of implicit semantics [2] be resolved by providing a carefully analysed and described domain model [1] prior to requirements capture and software design, a both informally annotated and formally specified model that goes beyond [1] in its treatment of attributes in that these are now endowed with types [and possibly scales (or fractions)] and that each specific domain model analyses and formalises the constraints that operations upon attribute values are carefully analysed, statically.

4.2 Domain Descriptions as Basis for Requirements Prescriptions

This paper covers but one aspect of software development.

- [23] covers additional facets of domain analysis & description.
- [24] offers a systematic approach to requirements engineering based on domain descriptions. It is this approach that justifies our claim that domain modelling “alleviate the issue of implicit semantics.”
- [25] presents an operational/denotational semantics of the manifest domain analysis & description calculus of [1].
- [20] shows that to every manifest mereology there corresponds a CSP expression.
- [26] muses over issues of software simulators, demos, monitors and controllers.

4.3 What Next?

Well, there is a lot of fascinating research to be done now. Studying analysis & description techniques for attribute types, values and constraints. And for engineering their support.

4.4 Thanks

to J. Paul Gibson and Dominique Méry for inviting me, to J. Paul Gibson for organising my flights, hotel and registration, and to Dominique Méry for his patience in waiting for my written contribution.

5 Bibliographical Notes

5.1 References to Draft Domain Descriptions

I apologise for the numerous references to own reports and publications.

**References**


Domain Analysis & Description


