Simulated annealing in FWA-network planning

Master thesis report by

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Abstract

The planning of FWA-networks is a time consuming process. The aim with this paper is to find a model that can locate base stations in designspace and connect end-user to the base stations and solve the model within one hour. This paper describes the exact mathematical model for the base station location problem in FWA-networks and why it is not possible to solve the problem using this exact method within the given timeframe. Instead the base station location problem is solved using the Metaheuristic Simulated Annealing while minimizing the number of not connected end-users.

Preface

During the time I was writing this master thesis I was pleased with the assistance form not only my supervisors. I was also supported by the people at both O/ZA group at L.M. Ericsson Denmark A/S and Operational Research (OR) -group at Informatics, Mathematics and Modeling (IMM) at the Technical University of Denmark (DTU). I am very grateful for this support and I would like to thank all the people who have contributed to my paper and have been very helpful when things have appeared impossible. At IMM I would like to thank my co-students Michael Løve, Kim Riis Sørensen, Mette Krogh-Jespersen and the Ph.D.-students Thomas K. Stidsen and Jesper A. Hansen for their assistance in all aspects of the project. At L.M. Ericsson I would like to thank Ulrik Heins and Nils Rinaldi for explaining the mysteries of radio transmission and Mazeyar Firouzi for helping in mastering C-programming.

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Abbreviations

FWA	Fixed Wireless Access
PMP	Point-to-Multipoint
IP	Internet Protocol
ATM	Asynchronous Transfer Mode
PSTN	Public Switching Telephone Network
BAS	Broadband Access System
bps	bits pr. Second
GSM	'Global System of Mobile communications' or 'Groupe Spéciale Mobile'
UMTS	Universal Mobile Telecommunication System
LOS	Line of Sight
C/I	Carrier to interference signal ratio
ISP	Internet Service Provider

1 Introduction

A typical connection from the end-users to a plain old telephone system or an Internet service provider (ISP) is via fixed lines. Another option is to use Fixed Wireless Access (FWA) which provides a fast establishment and/or expansion of the connection between operator and end-user. This report considers the planning of networks starting at a level where location and demand of each end-user and location of potential base station sites within the service area are known. Today, the radio network planning is done manually. Depending of the size of the desired network the process takes between 3 to 5 days for one person. This gives rise to promote use of computers utilizing operational research to speed-up this process.

The aim for this project is to create mathematical models for tasks of the planning process, identifying the necessary input data and defining planning parameters that identify the quality of the network. Another important part is deciding the criteria that make it possible to detect success when planning.

The FWA system applied in this master thesis is the Ericsson MINI-LINK Broadband Access System (MINI-LINK BAS). The MINI-LINK BAS system provides connection between IP/PSTN/ATM backbone network and the end-user service terminals. The backbone network is connected to the base stations and one base station can be connected to a number of end-users (Point to Multipoint access (PMP)). One base station can host up to 6 sectors and each sector has a capacity of 37 Mbps Gross bit rate full duplex. One sector covers the end-users in an area within an angle of 90° with a maximum transmission range at approximately 5 km. This means that a base station with 4 sectors have a total potential coverage area that can be approximated by a circle with center at the base station and a radius of maximum 5 km.

1.1 Planning tasks

One of the major tasks is identifying the best location of base stations. The primary cost of the network is the cost of establishing base stations. Hence, the object is to minimize the number of base stations while maintaining 'sufficient coverage'. 'Sufficient coverage' is a matter of definition similar to the success parameter. The operator decides whether the network requested has to cover all potential end-users, or that the success parameter of e.g. 80% capacity utilization for base stations is acceptable. There exists other ways of defining the best network for the actual operator but these two are the most common. In the typical real-life planning process the first question in the inquiry is the cost of covering all end-users with sufficient capacity. Second question is how many end-users are covered with a lower number of base stations, and is this number of end-users above operator's minimum service limit. This means that it is desirable to design the planning model in a way where both planning with unlimited and fixed numbers of base stations are possible.

Current computerbased tools for assisting in the planning process are only capable of computing the coverage once the base stations have been placed, and not placing the base stations themselves.

This gives the primary task of this project; find a model that can place base stations and connect end-users to base stations. Firstly it is necessary to define limitations and assumptions in the model in order to make the problem manageable.

When dealing with radio signals it is important to take radio propagation loss between transmitter and receiver into account. Unfortunately it is very complicated to compute this value due to its dependency of the topography of the surface in the coverage area e.g. vegetation, buildings etc. Hence, in this model it is decided to reduce the planning area from a 3-dimensional into a 2-dimensional plan. Connections between base stations and end-users are pre-computed in the 3-dimensional plan in order to identify where line of sight (LOS) between transmitting and receiving antennas exists. Finally, in the 2-dimensional model the propagation loss is assumed linear dependent of transmission distance and transmission power, measured on a logarithmic scale.

End-users are located in the planning area and identified by the coordinates and their demand measured in bps. Potential sites for base stations are also given in the plan, identified by their coordinates.

Connecting end-users to base stations is done by connecting the end-users to the nearest base station. This is done while monitoring whether the sum of end-user demand is less than or equal to, the maximal capacity of the base station and that the distance between the base station and the end-user is less than 5 km. To do this it is necessary to simplify the model of the base station in a way where the coverage area of each base station is assumed to be one circular area instead of four sectors of 90 degrees. The maximal capacity of the base station is simply computed as the product of the number of sectors and the maximal capacity of each sector. The distance to the most distant end-user from each base station defines the radius of the coverage circle of the base station. It is desirable that the overlap between any pair of coverage circles is minimal due to interference.

The actual planning of the network can be divided into two separate steps as follows:

1.1.1 Step 1

Create a network with total coverage and sufficient capacity for all end-users in the area while maximizing the minimal load on each base station and minimizing the overlap between coverage circles.

The output of this step is a plan that gives the number of base stations, the location of each base station and all base station-end-user connections. This information gives the operator an opportunity to decide if the number of base stations is acceptable or that another optimization with a fixed number of base stations has to be made. An optimization with a fixed number of base stations is as follows:

1.1.2 Step 2

Find maximal coverage with a fixed number of base stations while maximizing the minimal load on each base station, and minimizing the overlap between circles.

When step 2 has been performed it should be checked whether the number of end-users connected is above the minimum service limit.

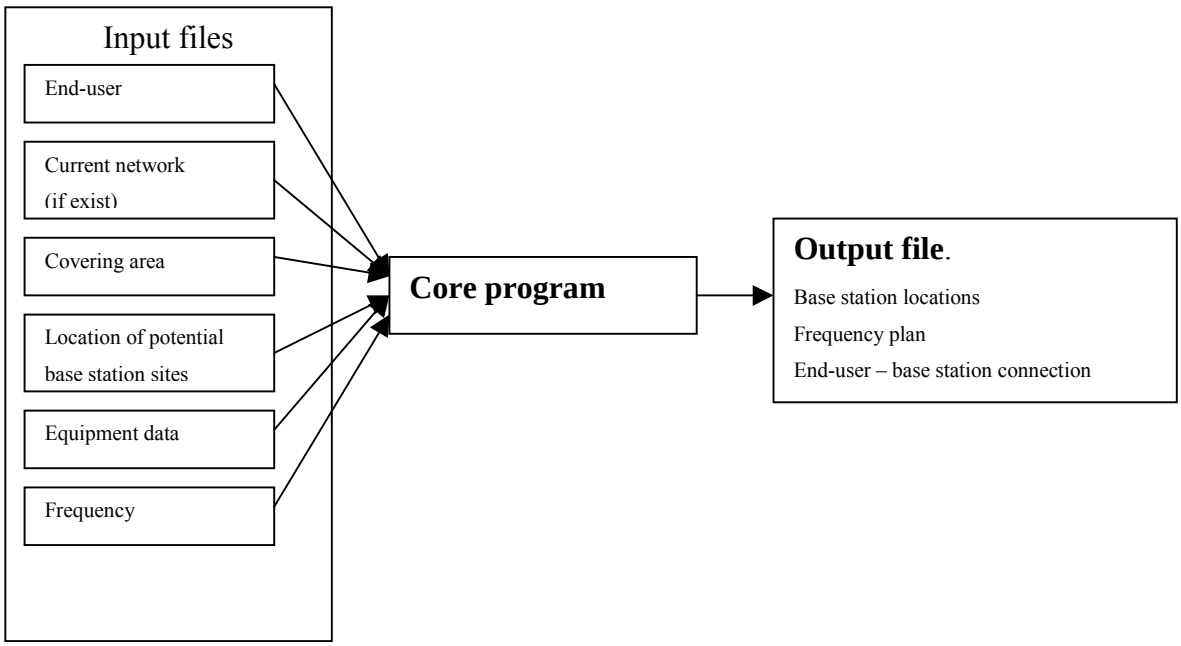
The planning is performed, aiming at covering the expected demand after a period of e.g. 8 years. Identifying milestones for the rollout plan is done by ranking the base stations by the load on each base station. Base stations are then established successively starting with the one having the most load. This means that milestones for year 3 and 5 are given by the pace of the rollout progress and not by a separate optimization aiming at year 3 and 5.

1.2 Thesis statement

Summing up, the formulation of the aim of this project is:
Develop and implement a mathematical model that can locate base stations, connect end-users to base stations and find a way to solve the model spending less time than solving the location problem manually.

1.3 Project visions

The overall perspective with this project is to do the first step in the development of a planning tool that can assist in the planning of any wireless communication network such as GSM, UMTS, FWA and others. The idea is to create a core program that, with the correct input files is able to assist in the planning process. *(See below)*



2 Other researchers work

So far it has not been possible to find articles describing the base station location problem especially for FWA networks. The search has been performed at The Technical Knowledge Center & Library of Denmark (DTV). The searches were performed using keywords like 'Fixed Wireless Access', 'Broadband', 'Coverage', 'Location' and 'Radio network'. Instead a number of articles about general base station location and about base station location for cellular networks such as GSM 900 and UMTS emerged. The question is if the experience in these articles is applicable in FWA networks.

The aspects considered in the articles spans from modeling the movements of the mobiles in design space to describe different solution methods of base station location models. Additionally the design space can be represented as either continuous or discretized in 2- or 3-dimensional space.

All results and illustrations in this section is from the respective articles.

Shih-Tsung Yang and Anthony Epremides have in the article [1] worked with the problems related to select location and transmission power of base stations while maximizing the minimum throughput among the mobiles. The major issue of the article is to model the movements of the mobile terminals and argue that this simulation is correct. The design space is discretized into a grid of legal points. The movement of the mobile terminals is simulated by a random walk between points. Over time the movements of the mobile terminals converge towards a steady state. The traffic to each terminal is assumed to be a Bernoulli packet arrival process with a given rate. The maximal coverage area with respect to load is the area where the transmission capacities of the base stations can match the demand of all terminals in the coverage area. In areas where there is low traffic the limiting factor is the attenuation of signal power from the base station. The modeling of the signal loss is like the questions about solving the model only briefly touched.

The article [2] by Calégari P et al. focus on problems related to base station location in UMTS networks. The article uses a graph theoretical representation of the problem structure. The design space is tiled using a grid. A node represents each tile in the grid and an edge connects the node with each of the base stations that can cover the node. See *Figure 1*. This gives a bipartite graph with tile nodes on one side and base station nodes on the other. The solution method suggested in the articles is the graph theoretical problem 'Minimum dominating set'. This means find the minimum set of base stations that covers all end-users. This problem has been proved to be NP-hard. The primary aim of the article is to describe a solution method based on a genetic algorithm. The results from a test run on a 70 km x 70 km planning area did not produce satisfactory solutions but the algorithms are still under development. One area of special interest is to reduce the size of bipartite graph.

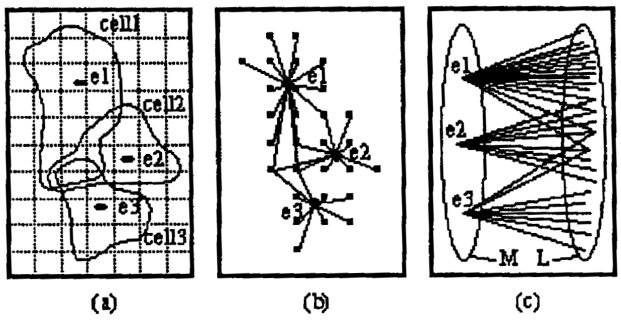


Fig. 2 The three cells associated to the BSs *e1*, *e2*, and *e3*, are discretized on a grid (a). Each BS is then connected to the locations it covers (b) in order to build a bipartite graph (c).

Figure 1

The graph theoretical approach in article [3] by Charmaret B. et al. is different. Here the propagation model added to each potential base station site is based on one type of antenna. A graph is created where a node represents each potential base station site. If two sites have a coverage area in common that is over a given threshold, an edge is added between the respective nodes see *Figure 2*. In this graph the solution is given by the graph theoretical problem 'Maximum independent set'. This means find a maximum set of base stations that are not connected. The problem is solved using 7 different stepwise heuristic methods and the results are analyzed to find the method

that gives the best coverage. The article pays most attention to select the best overlap threshold value. According to the article the success parameter when evaluating a given overlap threshold value is either the number of selected base stations or the relative coverage.

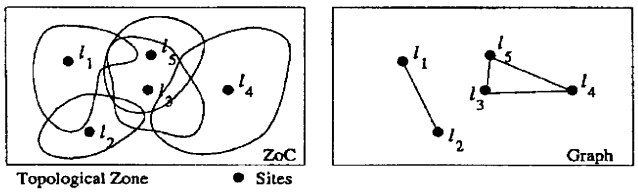


Fig. 1 Initial graph construction

Figure 2

For indoor applications using micro cells the location problems is in many ways similar to the base station location problem. In [4] Ivan Howitt and Seung-Yong Ham proposes a solution method for the indoor location problem. This is a heuristic way using a Wiener process to model the objective function. The Wiener process is a model where optimality is defined using a stochastic model and then the stochastic model is optimized.

The overall idea of the method is the following: The process is based on evaluating a number of solutions found by guessing. Due to the fact that the Wiener process is a stochastical process it is assumed that the process in some way ensures that the guesses are evenly distributed in design space. When the solutions are evaluated local search is performed from the best of the global search solutions. Furthermore, the article is dealing with indoor applications utilizing two base stations. In the FWA-network the number of base stations is typically in the size 10-20.

In article [5] by Margaret H. Wright, no model is proposed but it applies the base station location problem as an example of complex problems where exact methods are rarely usable. To solve these problems a direct search method is suggested. This solution method is suitable if the objective value is a scalar as a function of a number

of parameters. The method is called Nelder–Mead 'simplex'[1] method. This method is a non-exact method able to solve problems in continuos solution space though it is not possible to determine how close the given solution is to the optimal solution.

Both [4] and [5] do only pay attention to solve large location problems. The actual modeling of the problem is either limited or non-existing. The primary reason why the authors focus on base station location problem is to show how these solving methods handle hard optimization problems. No numeric results are shown in any of the articles.

Hanif D. Sherali et al. describes in [6] a model that locates *n* base stations in an indoor environment. The design space is discretized into a grid where each tile represents an end-user. The location of base stations is found by a search in continuous solution space. The final locations of the base stations are where a convex combination of two values is minimal. The first value is the minimum average of all path losses between base station and end-user. Second value is minimum maximal path loss between base stations and end-users. The article proposes three methods to solve the problem. All methods are heuristic search algorithms and hence require an initial solution. The initial solution is found by minimizing the sum of weighted squared Euclidean distances between base stations and end-users. The methods to improve the solution are Hooke and Jeeves' method, Quasi Newton method and Conjugated Gradient search. In order to reduce the computational work the problem is solved as a number of problems. Initially the problem is solved where the grid density is sparse, later the grid is increased. Thereby the number of points to be evaluated in the beginning is reduced, later the solution is refining by increasing the grid density. The results of test runs using the threes methods placing 2 base stations show a final solution within 8% of an optimal solution. Best is Conjugated gradient search where all test runs are within 4% from an optimal solution.

[1] The Nelder-Mead "simplex" have no relation to the simplex methods of linear optimization

In the article [7] Dimitris Stamatelos and Anthony Ephremides describe their work with placing base station in an indoor environment. The design space is discretizied into a grid. Mobiles can only be placed at points in the grid, whereas the location problem of base stations is solved in both continuous and discrete space and both omnidirectional and adaptive antennas at base stations are investigated. The location problem is modeled in a way where the design space is divided into 4 types of area. Cell area, Covered area, Interfered area and Uncovered area. Cell area is the collection of grid points where the signal strength from at least one base station is greater than a given receiver threshold. Covered area is a collection of points in the Cell area where the carrier to interference relation is greater than a given threshold. Interference area is Cell area minus Covered area and Uncovered area is the rest of the design space. The objective of the model is to minimize a convex combination of Uncovered and Interference area by finding the location of each base station and the transmission power. The methods proposed for solving the model in continuous space are Steepest descent and Downhill simplex. The Steepest descent method makes use of the gradient of the objective function to find the next step of the algorithm. The downhill simplex is similar to the Nelder-Mead simplex method mentioned in [5]. In the discrete space where most work in this article has been done, the model is solved using the Hopfield and Tank neural network. In test runs placing 3 base station with omnidirectional antennas the steepest descent method gave worst results. Best results were achieved with the Hopfield and Tank neural network. Of the 200 simulations with neural network only 3% did not converge. All results were within 4% from optimal solution. In the test run with adaptive antennas at the base stations the Nelder-Mead simplex method produced a solution 50 % from optimal solution. The steepest descent algorithm failed to converge but the neural network produced solutions between 17 and 28 % from optimum.

Summing up the result of the article search. At best the articles can be used as a collection of ideas when designing models for FWA. In general the articles only focus on a small part of the entire process of creating the models for layout and propagation and solve the layout problem whereas the rest of the process is only sparsely described.

The steady state of the moving mobiles in article [1] illustrates the theoretical similarities between mobile systems and FWA systems.

In article [2] the similarity between FWA and cellular networks where tiles are replaced by end-users is obvious using this representation. As in the FWA case the objective in the UMTS case is to find a set of base stations that cover a maximum number of tiles/end-users using a minimum number of base stations.

In article [3] the method described appear to be directly applicable in FWA despite the article describe UMTS planning.

The articles [4] and [5] describe methods for optimization of all kinds of large integer problems which means methods that also must be usable to solve FWA problems. Article [6] and [7] deal with problems related to indoor application. These problems usually only need a lower number of base stations compared to FWA. This gives the opportunity to perform exhaustive search in order to find optimal solution for comparison with the heuristic solution.

3 Conceptual model

3.1 Design space

Design space for a FWA network is the geographical area that is sought covered. In the model this area is represented as a 2-D coordinate system.

3.2 End-users

End-users are the final consumers in the network. Their demands are voice, video and data services. The traffic load for these services is dynamic and it is represented by the average traffic load and the peak traffic, both in bits/sec. The load is the expected load of the end-user at a given year. The location of the end-user is either a company or a private domicile. In the model, each end-user address is converted into a coordinate set in the 2-D model design space. End-users are identified by the index *i* in the model. The demand of each end-user is given by his/her average load and is the one used in this planning.

3.3 Base stations sites

Potential base station sites are locations where it is possible to place a base station. These sites are indexed by *j* and *k* and represented by a geographical coordinate set. In the model these values are converted to a set of coordinates in the 2-D model design space. The capacity of each base station is measured in bits/sec. Like in the case of end-users the capacity is converted into a single number.

3.4 Radio propagation

The attenuation of radio signals is the limiting factor when considering the possible distance between base station and end-user. The attenuation is dependent of factors like obstructing buildings, rain attenuation, etc. For FWA the requirement for connection between base station and end-user is line of sight, LOS. Simplified, ‘LOS’ means it is possible for the base station antenna and the end-user antenna to ‘see’ each

other. In the model the radio attenuation measured in dB is assumed linear dependent of the Euclidean distance between base station and end-user. A potential connection between a base station and an end-user requires that the Euclidean distance is below maximum transmission distance due to signal attenuation and where LOS exists. The signal strength at the end-user is assumed to be the transmitted power minus an attenuation constant multiplied by the Euclidean distance between end-user and base station.

3.5 Interference

When the network consists of more than one base station using the same frequency or one of the two adjacent frequencies, there is interference. Interference is measured as the relationship between the Carrier signal from the assigned base station and the sum of Interfering signals from other base stations, C/I. The interference is only interesting at the points where end-users are located. In this model the C/I is computed as the ratio of the signal strength at each end-user from its assigned base station and by the sum of signal strengths at the end-user, from all the rest of the base stations. The signal strength at an end-user from a given base station is computed according to the model mentioned in the preceding section 'Radio propagation' likewise is the interfering signal strength computed.

3.6 Optimization parameters

In step one in the network design process, the aim for the radio planner is to create a base station layout that ensures that all end-users are connected to a base station. This can be done numerous ways. Operators can have different quality targets toward the layout of the network such as "a maximal number of end-users must be covered" or "the load on each base station must be maximal". Most of these targets can be expressed using the three parameters: active base stations, end-users and load on base stations.

The most expensive components in an FWA-network are the base stations including the cost of the sites. This makes it attractive to minimize the number of base stations in order to reduce the costs of the network.

The load on each base station expresses the future options to develop the network, besides the potential revenue on the individual base station. The decision of placing a base station at a given location, or not, is made on the basis of the expected load at the location. Due to the initial cost of establishing base stations it is not attractive to establish a base station if the expected load at the location is less than a given threshold. In order to prepare the network for future expansion in the number of end-users using the established network, is it desirable to have a close to even load on all base stations.

Finally the total number of end-users connected to base stations is an important parameter. The number of connected end-users is obviously closely related to the load on the base stations.

This gives three options for objective functions where the selected parameter is optimized. Instead of utilizing only one of the objective functions it is possible to use all three in a three-criterion solution method. Another option is to add a 'cost' on each parameter and then optimize the total 'cost' of the network. A special case of this function is where the sum of 'cost' constants is *1*. This special case is called a convex combination.

Regardless of the objective function chosen the function is optimized with respect to a lot of constraints. These constraints are imposed by the system limitations and expressed mathematically.

3.6.1 Objective functions

- Maximizes the minimal load variable.
- Minimizes the number of base stations (in step one only)
- Maximizes the number of end-users connected (in step two only)

- Optimize a convex combination of two or three of the parameters, number of active base stations, number of connected end-users and maximal minimal load on a base station.
- Minimize the total 'cost' of the network.

3.6.2 Constraints

- The sum of capacity demands connected to one base station must be larger than or equal to the minimum load variable or a minimum load constant.
- The sum of capacity demands connected to one base station must be less than or equal the maximum base station capacity.
- Each end-user must be connected to only one base station.
- End-users can only be connected to active base stations.
- End-users can only be connected to base stations if it is possible to get line of sight.
- The C/I at each end-user must be larger than or equal to a given threshold value.
- The signal strength at each end-user from the assigned base station must be larger than or equal to a given threshold value.

3.6.3 Model variables

When modeling these objective functions and constraints a number of factors are identifying base stations, capacity, legal connections, etc. The definitions of these factors are as follows [9]:

Decision factors:
The optimization algorithm can adjust these variables within the given range, in order to achieve the best solution.

Conditional factors:
These values vary when decision variable values are altered, identifying the conditions of the system.

Structural factors:

These values are assumed constant within the time interval considered. Hence, the model cannot change these values.

Environmental factors:

Factors that are controlled outside the system considered but can affect the conditions of the system. These factors do not appear in this paper.

The variables in the model are as follows:

Factor	**Symbol**	**Role**	**Range**
End-user id	i	--	$\{1,2,..,(I-1),I\}$
End-user coordinate	x_i,y_i	Structural factor	$[0;X_{max}, 0;Y_{max}]$
End-user demand	d_i	Structural factor	$[0;CAP_{lim}]$
Base station site id	j,k	--	$\{1,2,..,(J-1),J\}$
Base station coordinate	x_j,y_j	Structural factor	$[0;X_{max}, 0;Y_{max}]$
Transmission power at base station	P_j	Decision factor	$[0;P_{max}]$
Base station at site	b_j	Decision factor	$\{0,1\}$
Base station capacity	CAP_{lim}	Structural factor	R_+
Legal connections	k_{ij}	Structural factor	$\{0,1\}$
End-user – base station connec.	c_{ij}	Decision factor	$\{0,1\}$
Minimum number of end-user base station connection	C_{min}	Decision factor	$\{0,1..(I-1),I\}$
Signal at end-user from base station	s_{ij}	Conditional factor	$[0;p_j]$
Minimum C/I	C/I_{lim}	Structural factor	R
Minimum signal strength	SIG_{lim}	Structural factor	R
Load on base station	L	Conditional factor	$[0;CAP]$
Min. load on each base station	L_{lim}	Decision factor	$[0;CAP]$
Distance between end-user and base station	$Dist_{ij}$	Structural factor	R_+
Signal attenuation constant	$Att.$	Structural factor	R
Distance between two Base stations j,k	$O_{j,k}$	Structural factor	R_+
Minimum distance between two base stations	O_{min}	Structural factor	R_+
Objective value	Z	Conditional factor	R
Various costs	Q	--	R
'Big model constant'	M	--	$\rightarrow\infty$
Distance between base station j and most distant end-user connected to j	r_j	Conditional factor	$[0;Max\_dist]$
Density weight factor	w_i	Structural factor	R_+

4 Mathematical model

4.1 Optimization models

When designing mathematical models the aim is to create the equations that identify the constraints of the system and the objective function that expresses the value of the system. When these equations have been formulated the model can give the answer whether a given parameter (decision factor) setting is legal or not and identify the value of the system as a single number. The model can now be used to find the parameter setting that gives the best system value by optimizing the objective function.

As mentioned, three parameters identify the value of the network design. These parameters are 'minimal load on base stations in the solution' named L, 'number of connected end-users' found as the sum of $c_{i,j}$-variables and 'number of active base stations' found as the sum of b_j-variables.

Despite it is only possible to optimize one parameter at a time none of the parameters can be isolated and optimized without considering the two other parameters. When one parameter acts as the objective function at least one of the other parameters has to be inserted into the model with an upper and lower bound. When setting the parameter bounds in the model, care must be taken not to make the model infeasible. This is regardless of the number of parameters inserted in the model is one or two. E.g. consider a situation where the minimal load on base stations is sought maximized. The number of base stations is bounded to a value where the product of base stations capacity and number of base stations is less than the total sum of end-user demands. In this situation the model is infeasible if all end-users must be covered according to the constraint on the number of end-users. However, removing the constraint on the number of base stations will not guarantee that the model is feasible. Depending on the distribution of the end-users the model could still be infeasible due to the constraints regarding interference. When using a multi-criterion solution method the valid combinations and ranges of these bounds are identified as a part of the process.

4.1.1 Maximizing minimal load

In step one of the optimization process the aim is to cover all end-users and to have a maximal minimal load L on each base station. This outlines a model with an object function that maximizes the minimal load and a constraint that set the sum of end-users equal to the total number of end-users. The sum of the number of base stations can be left unconstrained.

One obvious element in getting a maximal minimal load L on the base stations is to distribute the total load over a minimal number of base stations b_j. The minimal number of base stations that can cover all end-users is found by dividing the total sum of end-user capacity demands with the base station capacity. This number is then rounded up to the nearest integer. One upper limit of L is found as the total sum of end-user capacity demands divided by the minimum number of base stations needed. Intuitively, this objective function should provide a solution with a minimum number of base stations but it is not necessarily the case. The reason is that end-users are not evenly distributed in design space and base stations can only be located at discrete locations.

4.1.2 Maximizing number of connected end-users

In step two of the design process the idea is to choose a fixed number B of base stations and then find the maximal number of end-users that can be connected to B base stations among the available. The obvious objective function for this optimization is a model that maximizes the number of connected end-users. This model needs a bound on the number of active base stations.

A constraint that sets a minimum for the load on each base station can be added to the model. However, here it is important to notice that if the minimal load is set too high the model is infeasible. Omitting the load constraint, the model is always feasible. Though, the load on some of the base stations could end with zero.

4.1.3 Minimize the number of base stations

A model that minimizes the total number of base stations while keeping the number of connected end-users above the lower bound, will at the same time implicitly maximize the average load on the base stations. A minimal number of active base stations require as many end-users as possible at each active base station. Whereas getting a result with maximal minimal load is unlikely the case. Again this is due to end-users not being evenly distributed in design space and it is only possible to locate base stations at discrete locations.

This objective function is not directly necessary in none of the two planning steps but is usable if the operator requires a coverage of e.g. 90% of the end-users and wants to know how many base stations are needed.

Summing up on these three relations, they indicate that a good final solution is a tradeoff between these three parameters. Methods considered for finding a good or best solution in this paper are the cost function and the multi-criterion solution method.

4.1.4 Static multi-criterion solution method

A multi-criterion method is a series of optimization. In each optimization one parameter is selected to be in the objective function. Some or all of the remaining parameters are inserted in the model, each one limited by two equations that sets the upper and lower bound for the parameter. This means finding the best solution for parameter *A* while maintaining given values for parameter *B, C, etc. Figure 3* illustrates the result of an imaginary 2-parameter example. The objective function optimizes parameter *B* and a constraint bounds parameter *A* to be greater than or equal to a given value in each optimization process. The figure shows the optimized value of parameter *B* at each fixed value of parameter *A*.

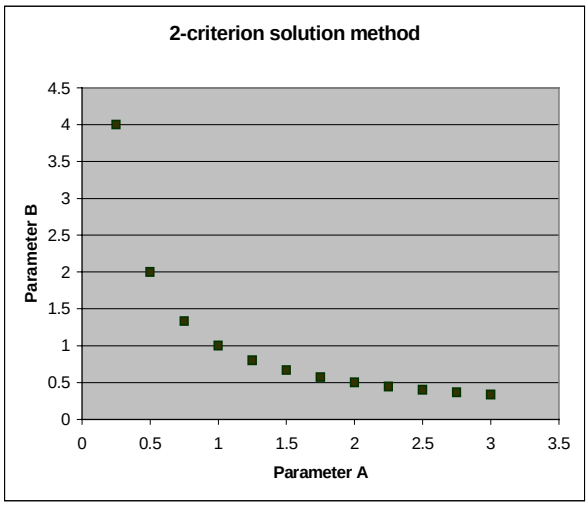


Figure 3

As can be seen from *Figure 3* no solution exists for values of *A* greater than *3*. Equally, for values greater than *4* for parameter *B* the problem is infeasible.

In the actual network *Figure 3* could illustrate the result of a series of optimizations where one of the parameters is uncontrolled. Consider a situation where the aim is to maximize the minimal load while maintaining a fixed number of end-users connected. In this case let parameter *B* illustrate maximal minimal load and parameter *A* be the number of end-users connected. At each optimization the number of end-users is fixed at a value within the desired range and after each optimization the result is plotted as a coordinate set of (end-users connected, maximal minimal load). Due to the fact that end-users are not uniformly distributed in design space, it is very likely that the maximal minimal load is low if the number of end-users connected is high and vice versa. Note that in this situation it is not necessary to control the number of base stations though it is possible.

For the actual problems with 3 parameters the process is technically the same but naturally becomes more time consuming due to the increased number of combinations of parameter settings.

The total model for finding the maximal minimal load in step one in the optimization process is shown in *Model 1*. Notice that *L* in this model is a variable.

$$Z = Max(L)$$

wrt.

$$\sum_i c_{ij} \cdot d_i + (1 - b_j) \cdot M \geq L \quad \forall j \quad i \in I, j \in J \quad Eq.5$$

$$\sum_i c_{ij} \cdot d_i \leq CAP_{lim} \cdot b_j \quad \forall j \quad i \in I, j \in J \quad Eq.6$$

$$\sum_j c_{ij} = 1 \quad \forall i \quad i \in I, j \in J \quad Eq.7$$

$$c_{ij} \leq k_{ij} \cdot b_j \quad \forall i, j \quad i \in I, j \in J \quad Eq.8$$

$$\left(\sum_j s_{ij} \cdot (1 - c_{ij}) \cdot k_{ij} \right) \cdot C / I_{lim} \leq \sum_j (s_{ij} \cdot c_{ij}) \quad \forall i \quad i \in I, j \in J \quad Eq.9$$

$$\sum_j c_{ij} \cdot s_{ij} \geq SIG_{lim} \quad \forall i \quad i \in I, j \in J \quad Eq.10$$

$$b_j \cdot k_{ij} \cdot \left(p_j - dist_{ij} \cdot Att \right) = s_{ij} \quad \forall i, j \quad i \in I, j \in J \quad Eq.11$$

$$b, c \in \{0,1\};\ p \in [0; P_{max}]; s \in \Re; L \in [0; CAP_{lim}]$$

Model 1

Due to multiplication of two or more variables in *Equation 9, 10* and *11* this model is non-linear and hereby not necessarily optimally solvable.

Step two in the optimization process is to find the maximal number of end-users connected to a given number of base stations. A model similar to *Model 1* with a few adjustments is used for this purpose. The objective function in this case is maximizing the number of end-users connected. *Equation 7* is changed from equality to an inequality. Finally, a constraint is added that ensures that the sum of b_j-variables is equal to the desired number of base stations. Additionally, a constraint that controls the minimum load L on base stations is added. Unfortunately it is not possible prior to the optimization to give a value combination for number of base stations and minimum load L that necessarily makes the model feasible.

Rethinking the modeling of step two could as well be a model similar to *Model 1* where *Equation 7* is changed from equality to an inequality and another constraint is added. This constraint set a lower bound to the sum of connected end-users C_{lim} but leaves the number of base stations unconstrained. As mentioned earlier, this model

will implicitly reduce the number of base stations, though not necessarily leading to a global minimum. The operator has to set a range for the desired coverage instead of setting a fixed number of base stations. Using the static multi-criterion solution method, one optimization is made for each value of the C_{lim}-value within the given range set by the operator. For each setting of the C_{lim} the optimization will provide an optimized value for L and the number of base stations. When optimizations with a sufficient number of C_{lim}-values within the given range have been performed, the operator's tradeoff can be performed on the basis of the network options.

4.1.5 Minimal cost function method

The basic idea of the cost function method is to use costs to guide the model towards a solution with the desired qualities. When using the minimal cost function the most difficult task is to put a cost on each parameter. E.g. the cost of a base station can be set relatively easily by using the cost of the base station in money. It gets more complicated when it comes to the cost of the parameter end-user and load.

One option to put a cost on the parameter end-user is to use the loss of income from each not connected end-user. Then the optimization model will weight the cost of adding an extra base station and get the extra income when extra end-users get connected.

Adding costs to load can be done in two ways. One option is to add a cost on load deviation from a given desired load on each base station. The problem here is that the difference between the desired load and the actual load can be either positive or negative. Using 'absolute value' or 'difference squared' functions to avoid this problem makes the objective function non-linear. The effect of this cost is that it finds the minimal sum of deviations from the given load level. Another option is to add a cost to the non-utilized capacity at each base station. This method is relatively easy to implement by multiply the cost value and the difference between capacity and load on each active base station. The cost value in connection with non-utilized demand can be set as the lack of income of each non-utilized kbps.

The major disadvantage with such a formulation of the objective function is that it can be difficult to find the correct weight of the cost values in order to get the desired effect on the network design. Thus, summing disadvantage costs gives a total disadvantage and no information about disadvantage deviation. This hides undesired variations in e.g. load on different base stations. Hence, this objective function maximizes the average load with no respect to deviation, whereas maximizing the minimal load objective function both maximizes the minimal load and minimizes the deviation between load on base station.

The mathematical formulation of the minimum cost model is as follows:

$$min\left(\sum_j \left(CAP_{lim} \cdot b_j - \sum_i c_{ij} \cdot d_i \right) \cdot Q_L + \sum_j b_j \cdot Q_{BS} + \sum_{ij} c_{ij} \cdot Q_{EU} \right)$$

wrt.

$$\sum_i c_{ij} \cdot d_i \leq CAP_{lim} \cdot b_j \quad \forall j \quad i \in I, j \in J \quad Eq.6$$

$$\sum_j c_{ij} = 1 \quad \forall i \quad i \in I, j \in J \quad Eq.7$$

$$c_{ij} \leq k_{ij} \cdot b_j \quad \forall i, j \quad i \in I, j \in J \quad Eq.8$$

$$\left(\sum_j s_{ij} \cdot (1 - c_{ij}) \cdot k_{ij} \right) \cdot C / I_{lim} \leq \sum_j (s_{ij} \cdot c_{ij}) \quad \forall i \quad i \in I, j \in J \quad Eq.9$$

$$\sum_j c_{ij} \cdot s_{ij} \geq SIG_{lim} \quad \forall i \quad i \in I, j \in J \quad Eq.10$$

$$b_j \cdot k_{ij} \cdot \left(p_j - dist_{ij} \cdot Att \right) = s_{ij} \quad \forall i, j \quad i \in I, j \in J \quad Eq.11$$

$$b, c \in \{0,1\}; \; l \in [0; CAP]$$

Model 2

As can be seen, most of this model is equal to the maximizing minimal load model apart from a minimum load limit, *Equation 5*, and the object function, *Equation 4*. Similarly, the model is non-linear due to *Equation 9, 10* and *11*. In this formulation the model will find a solution that covers all end-users at the minimal cost with respect to the given costs *Q*.

4.1.6 Objective functions

The exact mathematical formulations of the object functions mentioned in the previous section are as follows:

$$Z = Max(L)$$

Equation 1

This function maximizes the minimum load variable L. Hence, at least one constraint has to be added to the model that bounds either minimum number of end-users connected or minimum number of active base stations.

$$Z = \underset{j}{Min} \; b_j \quad j \in J$$

Equation 2

This function minimizes the number of active base stations. When using this function with a constraint for minimum number of end-users connected, the average load on base stations is maximized. The difference from using *Equation 1* is that in *Equation 1* the minimum load is maximized whereas with this equation and the minimum number of end-users-constraint, the minimum load is uncontrolled.

Modifying this objective function to maximize the sum of base station variables and use it in connection with a minimum load constraint it finds the maximal set of profitable base stations.

$$Z = \underset{i \quad j}{Max} \; c_{ij} \quad \forall i, j \quad i \in I, j \in J$$

Equation 3

This function maximizes the number of end-users connected. Like *Equation 1* and *Equation 2* this objective function needs at least one constraint to control either the load on each base station or the number of base stations.

$$Z = Min\left(\sum_j \left(\left(\left(CAP_{lim} - \sum_i c_{ij}.d_i \right) \cdot Q_L \cdot b_j \right) + \sum_j b_j \cdot Q_{BS} + \sum_{i,j} c_{ij} \cdot Q_{ij} \right) \right) \quad i \in I, j \in J \quad Eq.4.A$$

$$Z = Min\left(\sum_j \left(CAP_{lim} \cdot b_j - \sum_i c_{ij}.d_i \right) \cdot Q_L + \sum_j b_j \cdot Q_{BS} + \sum_{i,j} c_{ij} \cdot Q_{ij} \right) \quad i \in I, j \in J \quad Eq.4.B$$

Equation 4

These functions minimize the total cost of the network system.

In *Equation 4A*, the first component, the demand connected to a base station *j* is subtracted from the capacity CAP_{lim}. This non-utilized capacity is multiplied a cost constant Q_L and the binary variable b_j that indicates whether a base station capacity is available or not. Finally summed over base stations *j*. Unfortunately, this construction results in a multiplication of the variables c_{ij} and b_j which makes the object function non-linear. It is crucial to have the binary b_j variable present otherwise will non-active base stations contribute in a negative way with the total capacity.

Another way of adding a cost to non-utilized capacity is displayed in *Equation 4B.* Here the total demand connected to the base station *j* is subtracted from the capacity constant multiplied by the base station variable b_j. This formulation is possible due to a constraint that makes sure that demand only can be connected to active base stations. This gives, if the b_j is '0' then the total demand is also '0'.

The rest of *Equation 4A* and *Equation 4B* are identical.

In the second component establishing a base station, $b_j = 1$, is multiplied by a cost and summed over all base stations.

In the last component, the binary variable $c_{i,j,}$ indicating if end-user *i* is connected to base station *j*, is multiplied with the negative cost of not connecting end-user *i*, summed over all combinations of *i* and base station *j*.

The advantage with this formulation is the flexibility of the object function. Any variable in the model can be priced and be added as a component in the object function. The layout of the function in *Equation 4* is only one suggestion on how to price less desired elements in the network.

4.1.7 Constraints

The system constraints imposed by the system like maximal transmission range, total bit rate capacity, etc. must be respected. These constraints formulated mathematically are the following.

$$\sum_i c_{ij} \cdot d_i + \left(1 - b_j\right) \cdot M \geq L \quad \forall j \quad i \in I, j \in J$$

Equation 5

The connections between end-user *i* and base station site *j* multiplied with the demand at end-user *i* summed over end-users *i*, must be greater than or equal to the minimum load variable *L*. If base station *j* is inactive, $b_j = 0$, the second component add a large constant, *M*, to the zero-demand and hereby makes the equation valid. The model generates one equation for each base station site *j*. This constraint ensures that the sum of loads from all end-users connected to base station *j* is above or equal to the minimum limit *L* if a base station exist at site *j*.

$$\sum_i c_{ij} \cdot d_i \leq CAP_{lim} \cdot b_j \quad \forall j \quad i \in I, j \in J$$

Equation 6

The connections between end-user *i* and base station site *j,* multiplied with the demand at end-user *i* summed over end-users *i*, must be less than or equal to CAP_{lim} multiplied by the base station site *j*. This equation generates one inequality for each base station site *j*. This ensures that the sum of loads from all end-users connected to base station *j* is below or equal to the capacity at base station *j*.

$$\sum_j c_{ij} = 1 \quad \forall i \quad i \in I, j \in J$$

Equation 7

The connections between end-user *i* and base station site *j* summed over base station sites *j* must be equal to one. The model generates one equation for each end-user. When c_{ij} is binary, this ensures that each end-user is assigned to exactly one base station site only and due to the equal sign each end-user does get assigned to a base station. In a model that strives toward a solution without all end-users, the equation must be changed from an equality to an inequality.

$$c_{ij} \le k_{ij} \cdot b_j \quad \forall i, j \quad i \in I, j \in J$$

Equation 8

The connection between end-user *i* and base station site *j* must be less than or equal to the product of the legal connection identifier k_{ij} between end-user *i* and base station site *j* and the active base station identifier b_j. The model generates one equation for each combination of *i* and *j*. The equation ensures that end-users only gets connected to active base stations using legal connections.

$$C/I_{lim} \le \frac{s_{ij} \cdot c_{ij}}{\left(\sum_j s_{ij} \cdot (1 - c_{ij}) \cdot k_{ij} \right)} \quad \forall i \quad i \in I, j \in J \quad Eq.9.A$$

$$\left(\sum_j s_{ij} \cdot (1 - c_{ij}) \cdot k_{ij} \right) \cdot C/I_{lim} \le s_{ij} \cdot c_{ij} \quad \forall i \quad i \in I, j \in J \quad Eq.9.B$$

Equation 9

In *Equation 9A*, the signal power at end-user *i* from base station site *j* assigned to the end-user divided by the sum of signal powers at end-user *i* from base station sites *j not* assigned to the end-user *i* must be greater than or equal to C/I_{lim}. However, equations containing divisions with variables are always non-linear. The way to avoid this is to multiply both sides of the inequality sign with the denominator. This has been done in *Equation 9B*. The model generates one equation for each end-user. This ensures that

carrier to interference ratio is above or equal to the threshold value. Due to the multiplication of the two variables c_{ij} and s_{ij}, use of this equation will make the model non-linear.

$$\sum_j c_{ij} \cdot s_{ij} \geq SIG_{lim} \quad \forall i \quad i \in I, j \in J$$

Equation 10

The signal strength at end-user *i* from its assigned base station *j* must be greater than or equal to the minimum signal strength limit. The model generates one inequality for each end-user *i.* This ensures that the signal strength at each end-user from the assigned base station is sufficient. Due to multiplication of the two variables c_{ij} and s_{ij} use of this equation will make the model non-linear.

$$b_j \cdot k_{ij} \cdot \left(p_j - dist_{ij} \cdot Att.\right) \geq s_{ij} \quad \forall i, j \quad i \in I, j \in J$$

Equation 11

The output power at base station *j* minus the attenuation product must be greater than or equal to the signal strength from base station *j* at the end-user *i*. The attenuation product is assumed as a constant multiplied by the distance between base station *j* and end-user *i*. The signal strength is only computed at each end-user from active base stations, $b_j = 1$ and at legal connections, $k_{ij} = 1$. This equation sets the output power value for each base station measured in dBm. Due to the multiplication of the two variables b_j and p_j use of this equation will make the model non-linear.

$$\sum_i \sum_j c_{ij} \geq C_{min} \quad \forall i, j \quad i \in I, j \in J$$

Equation 12

The sum of end-user-base station connections summed over end-users *i* and base stations *j* must be greater than or equal to the value C_{min}. This equation generates only one constraint containing all potential end-user-base station connections. This equation does only have a function if the equality in *Equation 7* is changed to an

inequality. This equation ensures that at least C_{min} end-users get connected to a base station.

5 Data analysis

An analysis of the design space with end-user and base stations sites prior to problem solving can reveal important information that can be very useful when optimizing.

5.1 Input data

The input data for the model is provided by the operator and are typically a spreadsheet with columns containing end-user id, an end-user value proportional to the end-user demand and an (x, y)-coordinate set of the end-user location. Equally, the potential base station sites have an id code and an (x, y)-coordinate set.

The provider of the radio equipment supplies the input data regarding the equipment: The minimum and maximum power output, the need for signal strength at the end-user, the signal attenuation factor and the minimum C/I ration accepted.

5.2 Refining the data

From the given set of data a number of pre-run computations are necessary in order to make the data useable in the model.

5.2.1 Distance table

The distances between end-users and base stations are simply computed as Euclidean distances.

$$dist_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}\ , \forall i, j$$

Equation 13

5.2.2 Legal connection table

A connection in the network is legal if the signal strength at the end-user can be over the signal limit. In this model signal attenuation measured in dB is assumed linearly dependent of the distance with only one attenuation factor for the entire design space.

Hence, it is possible to identify the maximal transmission distance and hereby find the legal connections with respect to signal strength.

$$k_{ij} = \begin{cases} 1 & if\ (dist_{ij} \le dist_{max} \wedge LOS\ exist) \\ 0 & else \end{cases} \forall i, j$$

Equation 14

5.2.3 Graph analysis

One analysis of the design space is performed by creating a graph with base station nodes V_{BS} and end-user nodes V_{EU} and legal base station-end-user edges $E_{BS\text{-}EU}$, $G((V_{BS},V_{EU}),E_{BS\text{-}EU})$. See *Figure 4*. The graph is connected if a path connects any pair of nodes in the graph.

If the aim is to connect all end-users to base stations then it is relatively easy to identify base stations which must be in the solution. All base stations that connect to end-users with valency '*1*' need to be in the solution. . 'Valency *1*' means that the end-user in question can only connect to one base station.

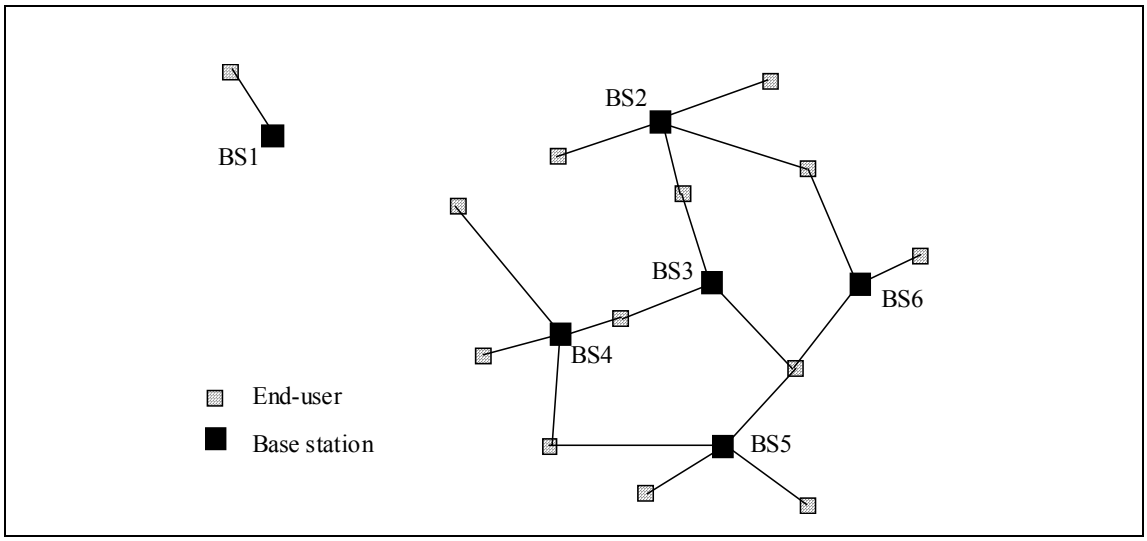


Figure 4

If the graph is not connected the problem related to each connected component should be solved separately for two reasons:

The first reason is that it makes solving the problem unnecessarily complicated if two separate problems are attempted to be solved as one problem. This is closely related to the use of binary variables to indicate if a base station is active or not. Consider an example with 10 potential base stations that can be divided into two sub-problems with 5 base stations in each. Assume 6 of the 10 base stations can cover all end-users, 3 base stations in each sub-problem. Using the binomial coefficients the number of combinations to be evaluated in each case can be computed.

$$As\ one\ problem: \frac{10!}{6!\cdot(10-6)!} = 210$$

$$As\ two\ problems: 2\cdot\frac{5!}{3!\cdot(5-3)!} = 20$$

Example 1

As can be seen in *Example 1* the number of combinations is reduced by a factor of *10* when the problem is split-up into smaller problems. Additionally, when problem size is doubled the factor identifying the difference in size in the numbers of combinations is by far more than doubled.

The second reason is related to using a model with maximizing minimal load object function. If the problem in *Figure 4* is solved as one problem, the end-user connected to BS1 can only connect to BS1. Let demand on each end-user be equal to '*1*'. Then the maximal load on BS1 can not exceed '*1*' due to the fact it is not possible to connect more than one end-user to BS1. Hereby the global minimal load on any base station is '*1*'. Hence, if the model 'Maximizing minimal load' is used; there is no mechanism in the model that maximizes the load on the rest of the base stations.

Finally, when optimizing it is important to know when it is not possible to improve the best solution found, so far. This is especially important when dealing with problems with discrete variables. The straightforward way of solving problems with discrete variables is to 'relax' the integrality requirement and solve this continuos problem. If this method provides a solution where the relaxed variables are integer

values the problem is solved to optimality. Otherwise is it necessary to use other methods to find an optimal integer solution.

If all end-users must be covered, this data analysis can provide an upper bound to the variable *L* in the Maximizing minimal load problem. In this problem an upper bound for *L* is the minimal maximal sum of demands that legally can be connected to any base stations that have to be in the solution. As mentioned earlier the base stations that must be in the solution are those connected to end-users with valency '*1*'. When the optimization has reached a solution where all base stations in the solution have a load greater than or equal to the upper bound for *L* the process can be stopped. At the connected component to the right in *Figure 4* it is easy to see that the base stations BS2, BS4, BS5 and BS6 have to be in a solution. This is due to at least one end-user potentially connected to each of these base stations has valency *'1'*. The minimal maximal potential load on base stations that have to be in the solution is on BS6. The maximal load here is *'3'*. On BS2, BS4 and BS5 the maximal load is *'4'*. Hence, the upper bound for *L* is *'3'*. Hence, the solver can be stopped when a solution that covers all end-users and has minimal load of *'3'* has been found. Furthermore, this makes it potentially profitable to perform a new optimization using a model where the objective function is minimizing the number of active base stations. The minimum load on each base station *L* found in the previous optimization is now added as constraint in the model. Due to the *L*-value being the result of the previous optimization this model is feasible. This second optimization gives the three-criterion solution where all end-users are covered by a minimum number of base stations with the maximal load on each base station.

6 Solving the problem

The timeframe for solving the problem should according to the thesis statement be less than the time spent doing this part of the planning by hand. There fore the model must provide a good solution within one hour. The time is measured from when the input data are read from a file where end-users are listed with x- and y-coordinates and demand, and the potential base stations are listed in a file with x- and y-coordinates. The planning is finished when a file with the selected base stations and a file with base station end-user connections have been stored.

Due to the non-linearities mentioned previously, neither *Model 1* nor *Model 2* can necessarily be solved optimally. However, fixing all binary variables makes the models linear. Therefore, one option is to solve the problem by generating all possible combinations of binary variable settings, solve the linear model in each setting and selecting the best solution. The number of linear problems to be solved or possible binary combinations is relatively difficult to find. Fortunately, it is not necessary to find in order to be convinced that this method is not feasible. Consider a situation where the number of base stations in the final solution is known to be 12 out of 50 potential base stations and end-users must connect to the nearest active base station or not connect at all. In this situation the number of possible solutions is the binomialcoefficient of 12 out of 50, which is about 1.21E11. Notice that due to the requirement of connecting end-users to base stations in this example, the base station end-user variables is set when the active base stations are given. To be able to evaluate this number of solutions within one hour, the number of solutions evaluated each second must be approximately 34 million. Recall that this situation is when information about the final number of base stations is known and the constraints controlling the connections between end-users and base stations are simple. Even in this simple case it appears to be impossible to evaluate the number of solutions needed within the given timeframe.

The goal is now to find what is the best solution that can be found within the given timeframe.

6.1 Relaxed exact model

Finding a good solution can be done in several ways. A first attempt is to use as much information from the models as possible. The idea in this attempt is to remove the equations that make the model non-linear and then solve this linear problem. Then solve the model with the non-linear equations where the variables set in the linear problem are now fixed as constants. If any of the non-linear constraints are violated, new constraints that prevent a solution that causes the violations are added to the linear problem. The linear problem is then re-optimized with the added constraints. This process is repeated until a solution not violating any constraints is identified. This outlines an iterative multi step approach in the attempt to solve the model.

The first step is to find a set of base station location variables b_j using only the linear part of the model. Then set the base stations-end-users connection variables c_{ij} in the same part of the model where the base stations found are fixed. Third step is to find the output level of each base station P_j using the non-linear part of the model where both base stations and end-user-base station connections found previously are fixed. Fourth step is to evaluate the result and give a final objective value for the total network. These four steps are repeated, maximizing or minimizing the objective value, until a stop criterion is reached. When the iterative process is terminated the final result shows one proposal for the final network.

6.1.1 Finding a set of base stations

Finding a set of base stations is done by solving a model similar to *Model 1* without the non-linear constraints. *Equation 9*, *10* and *11* are removed, the base station variables, b_j, are binary and the end-user-base station connection variables, c_{ij}, are continuos in the interval *[0;1]*. When omitting the non-linear constraints, it is necessary to add extra constraints that can simulate the non-linear constraints and hereby guide the model towards a solution that is feasible in step three. As objective function in step one, maximizing the minimum load on each base station, *Equation 1*, minimizing the number of base stations, *Equation 2*, or a multi dimensional solution

method using both, can be used. Depending on the choice of objective function, the correct modification of some of the constraints has to be done before optimizing. In *Equation 5*, setting the minimum load on each base station, *L* has to be set as a variable or a fixed value. If it is not required to cover all end-users *Equation 7* must be changed from an equality to an inequality. If necessary, *Equation 12* that controls the minimum number of end-users connected must be added to the model.

6.1.2 Finding the end-user base station connections

For connecting end-users to the selected base stations the same model with the same objective function as in the first step can be used. The only modification is fixing the set of base stations found in the previous step and letting variables c_{ij} be binary. The primary reason for splitting up the setting of b_j- and c_{ij}-variables into two steps is the number of c_{ij}-variables.

6.1.3 Setting the output power level

The model for setting the output power level is constructed from the three equations *Equation 9, Equation 10* and *Equation 11* where base station variables b_j and end-user base station connection variables c_{ij} found in the previous two steps are fixed. The power output variable P_j is continuos in the interval $[0;P_{max}]$. The objective function for this model could be one minimizing the maximal output power. This requires an extra constraint.

$$p_j \le p_{lim} \forall j \left| b_j = 1 \right.$$

Equation 15

The objective function is defined as follows

$$Z = Min\left(p_{lim}\right)$$

Equation 16

If the model is feasible a solution to the problem is identified. If the model is infeasible, undesired effects due to the stepwise solving of the model can be identified. These effects could be C/I-ratios at end-users that do not meet the limit of 22 dB. Identifying these end-user base station connections and inserting them into

both the model for step one and step two where these variables are forced to be '0' will remove the problems that makes step three infeasible. Unfortunately this procedure is wrong because the connections that cause trouble are those connecting those most distant end-users from the base station. Therefore, when one of the illegal connections has been removed the power p_j at the actual base station can be reduced, hereby making a number of the other illegal connections legal. If all illegal connections are forced to be *0* there is a risk that some of the connections that belong to the optimal solution are prevented from being used when the optimization process is performed again. One way around this problem is to make a branching for each variable c_{ij} that causes the model to be infeasible. The way of doing this is after the first iteration of step one to three; identify the variables, c_{ij}, that makes the model in step three infeasible. For each combination of setting of the 'trouble making' binary variables one optimization is performed using the first three steps. Call this set of variable combinations the primary branching tree. If new binary variables cause trouble when optimizing in branching nodes, a new secondary branching tree is formed emerging from the actual branching node in the primary tree only.

However, initial test runs of the model show execution times at approximately 5 minutes for performing the optimization in step one only. Step two takes about 2-3 minutes. Step three is simple, therefore performing this step is a matter of less than a second. Due to the exponential growth of the size of the branching tree with the number of 'trouble making' binary variables, it is easy to see that even a few variables in the primary branching tree gives a computation time of more than one hour.

Figure 5 is a diagram of the multi step model. In this version the stopping criterion is that the complete model is feasible.

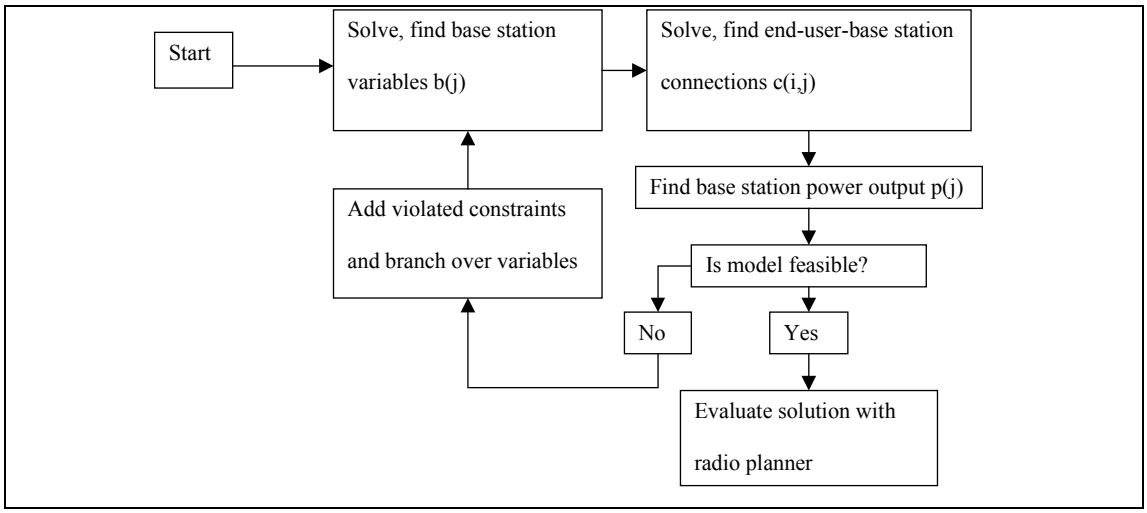


Figure 5

6.2 Artificial constraints

To avoid violating the non-linear constraints when optimizing using only the linear equations it is necessary to add artificial constraints to the model. The idea is to reduce the solution space by adding constraints that prevents the solver from finding solutions that violates the non-linear constraints. The result of an optimization with the linear constraints and the artificial constraints should then be a feasible solution with respect to the non-linear constraints.

6.2.1 Minimum inter base station distance

To avoid violating the non-linear *Equation 9*, that ensures sufficient carrier to interference ratio, sufficient distance between active base stations is a critical parameter. The generic formulation of this constraint is the following:

$$b_j \cdot b_k \cdot o_{j,k} + \left(1 - b_j \cdot b_k\right) \cdot M \geq O_{min} \quad \forall j,k \mid j > k \quad j,k \in J$$

Equation 17

The distance between base station j and base station k multiplied by the binary variables b_j and b_k identifying if base station j or k is active, must be greater than or equal to the minimum distance between two base stations. The number of equations is $(J^2-J)/2$. Due to the multiplication of the two variables b_j and b_k use of this equation will make the model non-linear.

Instead it is possible to add these constraints by pre-calculating a table *bs_dist(j,k)* with distances between any pair of base stations. If the value *bs_dist(j,k)* is less than a given threshold value α, the base stations b_j and b_k cannot both be in the same solution. Formulated mathematically for use in the model, the equation is:

$$b_j + b_k \leq 1, \forall \{j,k\} | bs\_dist(j,k) \leq \alpha$$

Equation 18

The threshold value α gives an indication on how much interference is accepted in the final solution. Setting the α-value is critical for getting a feasible solution when using *Equation 18*. Unfortunately, it is not possible to give the perfect formula to compute α though α is obviously dependent on the transmission power necessary to reach sufficient end-users in order to achieve the desired load on the base stations. I.e. if two base stations are located in a densely populated area of the design space they need only little output power to reach sufficient end-users to 'fill-up' the base station. In this case the α-value must be relatively small. On the other hand, if the base stations are located in a sparsely populated area of the design space then α must be large because the base stations need a large output power to reach sufficient end-users. The point is to make α as large as possible in order to get a solution where base stations are as far away from each other as possible but still close enough to cover all end-users in design space. Due to the fact that end-users are not uniformly distributed in design space there exist an α for each pair of base stations. Therefore the relations can be formulated this way:

$$(r_j + r_k) \cdot Weight\ constant = \alpha_{jk}$$

Equation 19

Here r_j and r_k are the power output converted to transmission range measured in kilometers and this range for each base station is defined to be large enough to enable the base station to reach sufficient number of end-users in order to 'fill-up' the base station.

6.2.2 Demand controlled transmissions power

The maximum transmission distance for the MINI-LINK BAS radio hardware is approximately 5 km in countries with the same rain intensity as Denmark. Depending on the end-user density of the design space, the output power is set individually for each active base station by *Equation 10* and *Equation 11* that ensures sufficient signal strength at any end-user from its assigned base station. It is advantageous to keep the output power at a minimum to get a maximal C/I relationship while maximizing the number of covered end-users. It is easier to cover the design space with smaller circles than with larger circles while keeping the circle overlap at a minimum. One way of simulating these constraints is to perform a pre-computing of a minimum output power that is necessary to reach sufficient end-users to fulfill *Equation 5* where the minimum load on each base station is set. Due to the fact that it is not possible to avoid that transmission areas do overlap, the sum of demands within reach of each base station has to exceed the capacity of the base station hardware. The sum of demands reachable, when setting the output power, together with the minimum load on each active base station, right hand side of *Equation 5*, gives an indication of how much interference is acceptable in the final solution. In model language the idea is to reduce the number of k_{ij}-values equal to '*1*' and hereby to reduce the total number of legal connections between end-user and base stations identified as c_{ij}. See *Equation 20,* a slightly modified version of *Equation 14* below.

$$k_{ij} = \begin{cases} 1 & if\,(dist_{ij} \le r_j \wedge LOS\,exist) \\ 0 & else \end{cases} \forall i, j$$

Equation 20

Identifying the maximal range r_j of base station b_j can be done using a simple program. For each base station the program sums the demand from end-users connected to the base station. When connecting end-users to the base station the program starts with the end-user nearest to the base station, then the second nearest and so on. When the load on the base station has reached the desired value the

maximal range r_j can be identified as the distance between the base station and the last connected end-user.

6.2.3 Transmission area overlap

Another option is to let *Equation 18* be controlled by a function of the area of the transmission circles overlap *bs_ol(j,k)* instead of *bs_dist(j,k)*. *See Figure 6*. The area is computed using the Cosines-relation and the Segment of circle. See *Equation 21* for mathematical definition of *bs_ol(j,k)*.

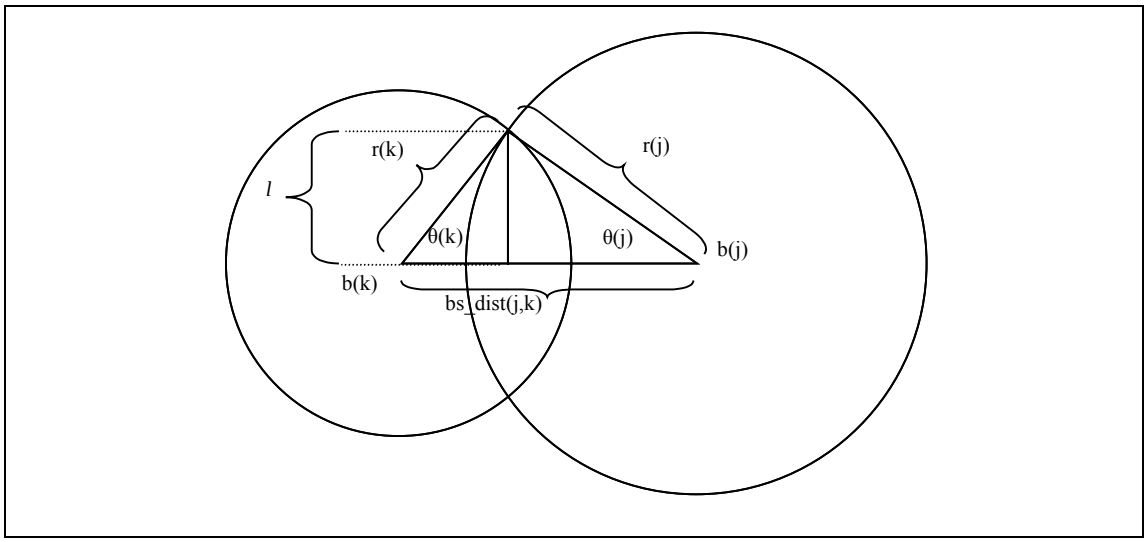


Figure 6

$$Cos(\theta_k) = \frac{r_j^2 - r_k^2 - bs\_dist(j,k)^2}{-2 \cdot r_k \cdot bs\_dist(j,k)}$$

$$\Downarrow$$

$$l = \sqrt{\left(\left(r_k \left(\frac{r_j^2 - r_k^2 - bs\_dist(j,k)^2}{-2 \cdot r_k \cdot bs\_dist(j,k)} \right) \right)^2 + r_k^2 \right)}$$

$$\Downarrow$$

$$Sin(\theta_j) = \frac{l}{r_j}$$

$$\Downarrow$$

$$bs\_ol(j,k) =$$

$$\frac{1}{2} \cdot r_j \left(2 \cdot arcCos \left(\frac{r_r^2 - r_k^2 - bs\_dist(j,k)^2}{-2 \cdot r_j \cdot bs\_dist(j,k)} \right) - Sin \left(2 \cdot arcCos \left(\frac{r_r^2 - r_k^2 - bs\_dist(j,k)^2}{-2 \cdot r_j \cdot bs\_dist(j,k)} \right) \right) \right) +$$

$$\frac{1}{2} \cdot r_k \left(2 \cdot arcSin \left(\frac{l}{r_j} \right) - Sin \left(2 \cdot arcSin \left(\frac{l}{r_j} \right) \right) \right)$$

Equation 21

The critical task is to find the correct value for α. It seems obvious to use a relative overlap in the condition. This could be the relation between the overlap area and the smallest coverage area. α is then reduced to be in the interval *[0;1]*. The problems in finding the correct value for α is similar to those explained with *Equation 19*. Expressed mathematically the constraint is as follows:

$$b_j + b_k \leq 1, \forall \{j,k\} \left| \left(\frac{bs\_ol(j,k)}{\min_{i \in \{j,k\}} (\pi \cdot r_i^2)} \right) \right. \leq \alpha$$

Equation 22

6.3 Test run of relaxed exact model

All data used in these tests from a 20 times 20-kilometer square covering greater Copenhagen. The data simulating end-users are randomly chosen among companies within the design space, with between 25 and 500 employees. For every 5 employees

the company demands 64 kbps. The data simulating potential base stations are randomly chosen among all companies within the design space.

The model is designed in the modeling tool GAMS and solved using CPLEX 6.5. The test run is performed at serv3.imm.dtu.dk, a server with four 440 MHz RISC-processors and 1 GB of RAM. The run time is measured using a simple stopwatch. Other people at the department could have been running other applications and hereby affecting the time measurement. However, the test run is performed on July 16th where summer holiday induced very low activity at the department. Therefore the chance of having at least one processor for this test run only were very good. Furthermore, as the results indicate it is not only the run time that is causing trouble.

6.3.1 First test run

The first test run is made using 'Test0'-data set. The GAMS model used for this test run is in *appendix 1*. As can be seen the GAMS model is made up of the equations *5*, *6*, *7*, *8* and two equations controlling respectively the minimum inter base distance and the sum of b_j-variables. The minimum inter base distance utilizes a pre-computed binary table of legal active pairs of base stations. The table is computed using *Equation 18* where, in this case, the α-value is 5. The equation controlling the sum of base stations is *Equation 2* formulated as a constraint instead of as an objective function. Furthermore the k_{ij}-values in *Equation 8* are computed using *Equation 20*, which are the demand controlled transmission range. The aim of this test is to maximize the number of connected end-users. Hence, the objective function used in the GAMS model is *Equation 3*. The minimum load on active base stations is maintained at 80% of the total capacity at the base station, 96.000 kbps, and a maximal number of base stations allowed in the solution of 12.

The runtime for solving this model is approximately 5 minutes. *Figure 7* is a plot of the result. As can be seen the number of active base stations is 7 and the number of end-users connected is approximately 750 of 1000 potential. The number is 'approximately' due to the fact that the variables c_{ij} are continuous in the interval *[0;1]* and therefore a few of these are not integral. One of the reasons for the low

number of connected end-users is the setting of the minimal load on active base stations at 96.000 kbps. However the most useful information can be extracted from this test run is seen at the locations *(10,12)* and *(6,16)*. Here base station-end-user connections cross each other and introduce unnecessary interference. If it had been possible to solve the non-linear model these crossing would not have existed. The non-linear model would have generated a solution where end-users are connected to the nearest base station.

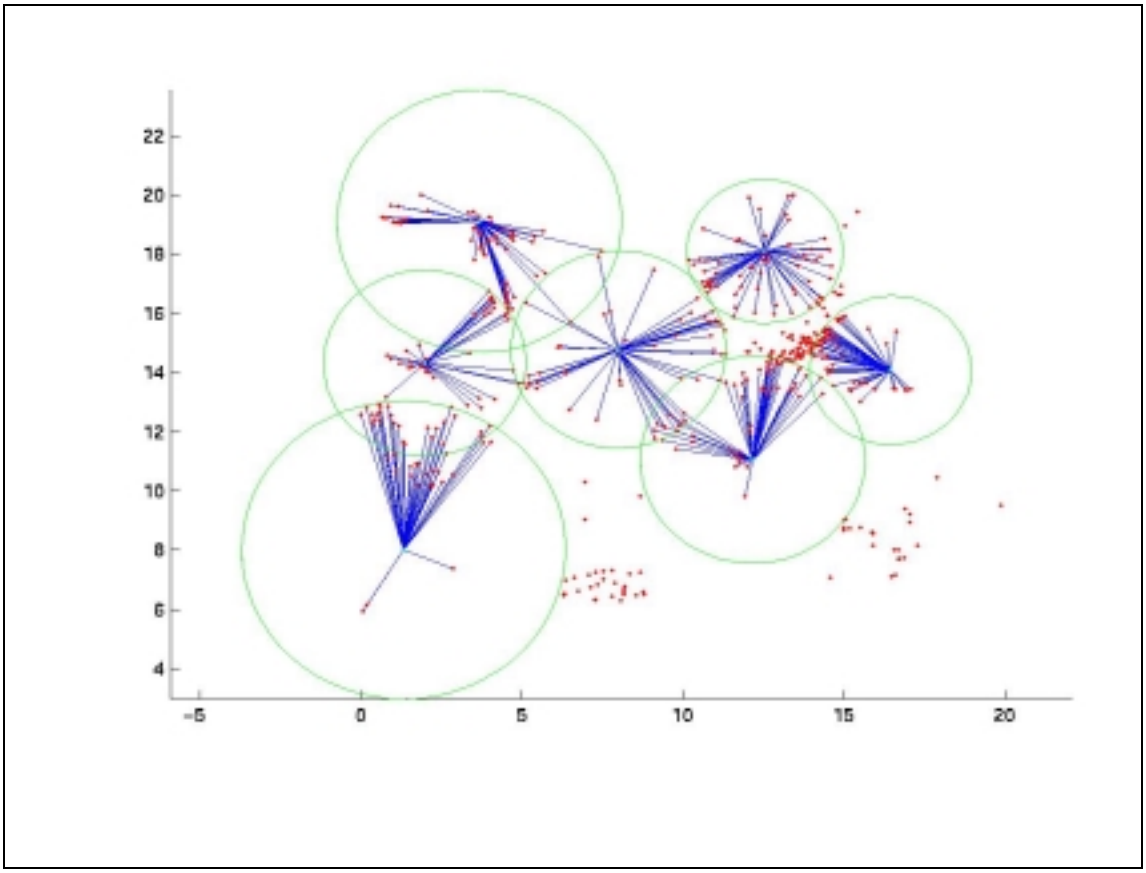


Figure 7

6.3.2 Second test run

In this test run the same model and data as in first test run is used. However, the constraint controlling minimal load on active base stations is set to a value of 9.600. An extra constraint controlling the maximal number of base stations is added the model with a maximal value of 12.

Like in the first test run the runtime is approximately 5 minutes. The result can be seen in *Figure 8*.

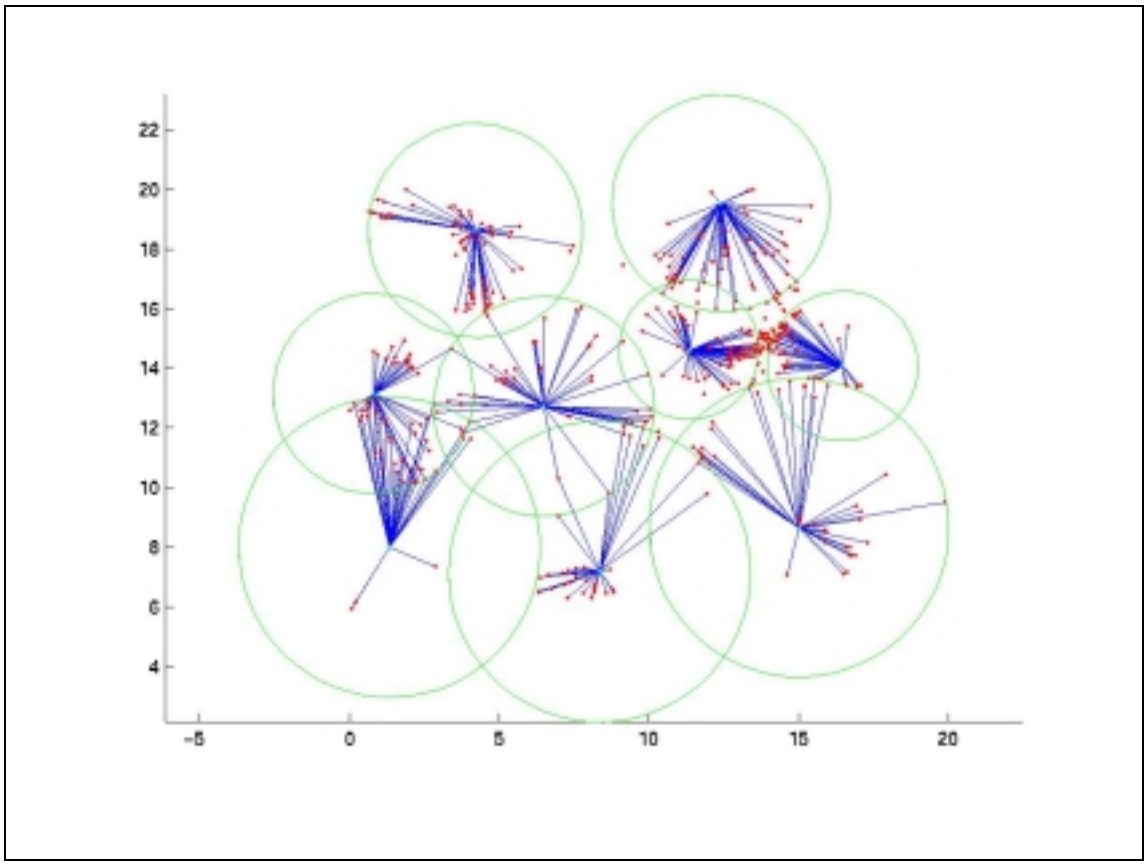


Figure 8

Due to the low value at the minimal load constraint the number of base stations is increased to 9 and the number of end-users connected is increased to around 890. Note that the constraint controlling the number of base stations actually were implicitly redundant like in the first test run.

As can be seen the number of base station end-user connections that cross have increased, especially in the area around the location *(2,11)*. Furthermore, the overlap between two cover circles in the same area appears to be too large. However, it seems that this overlap could be reduced if each end-user in the area is connected to the nearest base station but there is no guarantee that the reduction of the overlap is sufficient to avoid interference.

6.3.3 Third test run

In this test run the aim is to maximize the load on active base stations. As explained in the previous chapter about objective functions, it is necessary to put a constraint on either the minimum number of active base stations or the minimum number of end-users connected. The GAMS model for this test run is in *appendix 2*. As can be seen it is decided to use a constraint with a minimum number of active base stations of 9. Note that now *L* on the right hand side of *Equation 5* is a variable. This variable is the minimal load on any active base station and hence, the variable that is sought maximized in the objective function.

The runtime for this test is 5 minutes. The result of the optimization is a min load 83584 Kbps, 861 end-users out of 1000 and coverage of 80.1% of the demand. A plot of the design space can be seen in *Figure 9*. Like in the previous two test runs the problem is that base station end-user connections cross and that overlap between two base stations are too big. The crossings are at the locations *(0,12)* and *(12,17)*. The overlap violation is at the location *(0,12)*.

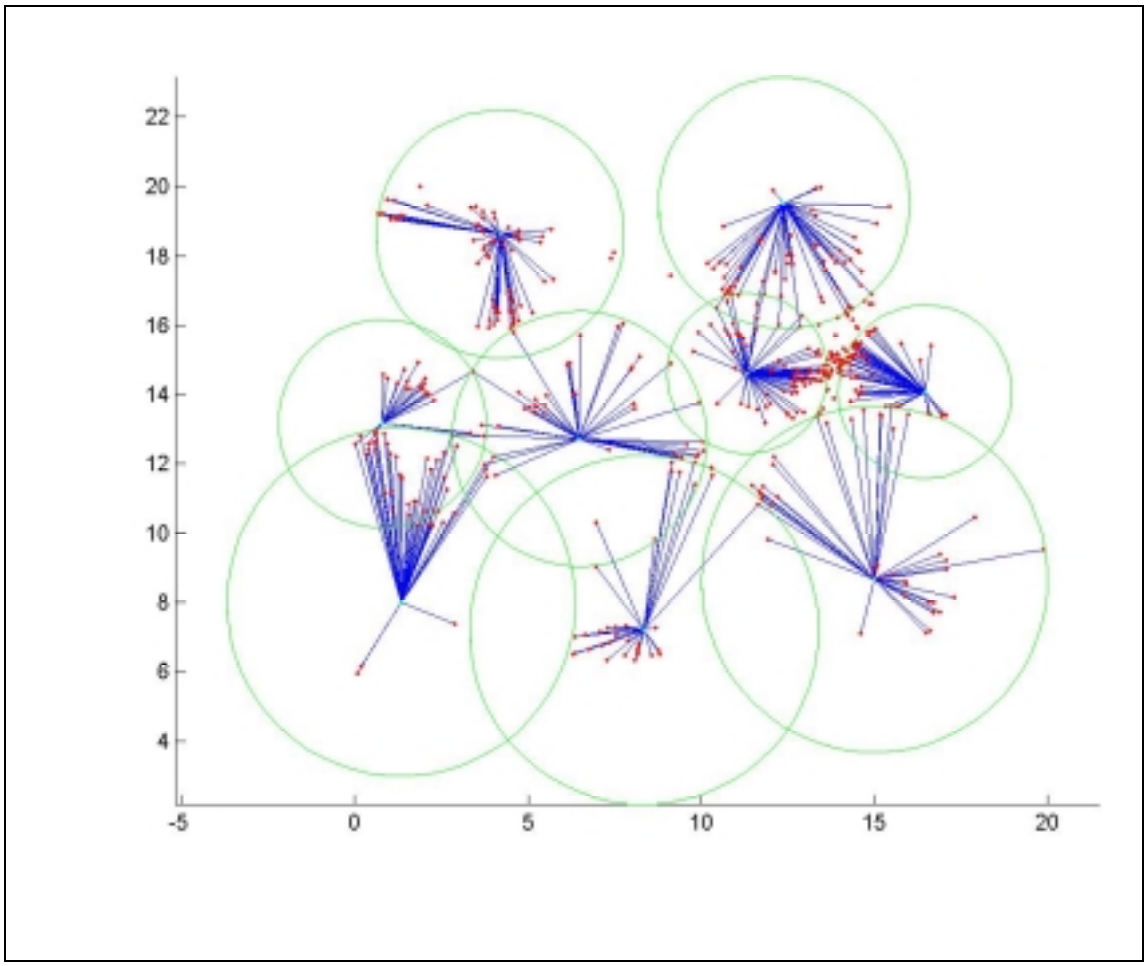


Figure 9

6.3.4 Fourth test run

In this test run the aim is to minimize the number of active base stations while maintaining a fixed number of end-users connected. The GAMS model used for this test run can be seen in *appendix 3*. The difference from the previous test runs is that here is *Equation 5* removed, which means that there is no minimum limit of the load on active base stations. As objective function *Equation 3* that minimizes the number of active base stations is used. Furthermore, a constraint has been added the model that controls that the minimum number of end-users connected is greater than 800.

The runtime for this test is 5 minutes. The result of the optimization is a solution with 7 base stations where the minimum load on an active base station is 66.449. 65.5% of the demand is covered and the number of connected end-users is equal to the

constraint minimum limit of 800. A plot of the solution can be seen in *Figure 10*. In this solution there is no connections that cross. However, a number of end-users are connected to base stations that are different from the nearest base station to the end-user. Furthermore, the base station at location *(15,15)* is significantly smaller than its neighbors, which means that interference at end-users connected to this base station is very likely to occur.

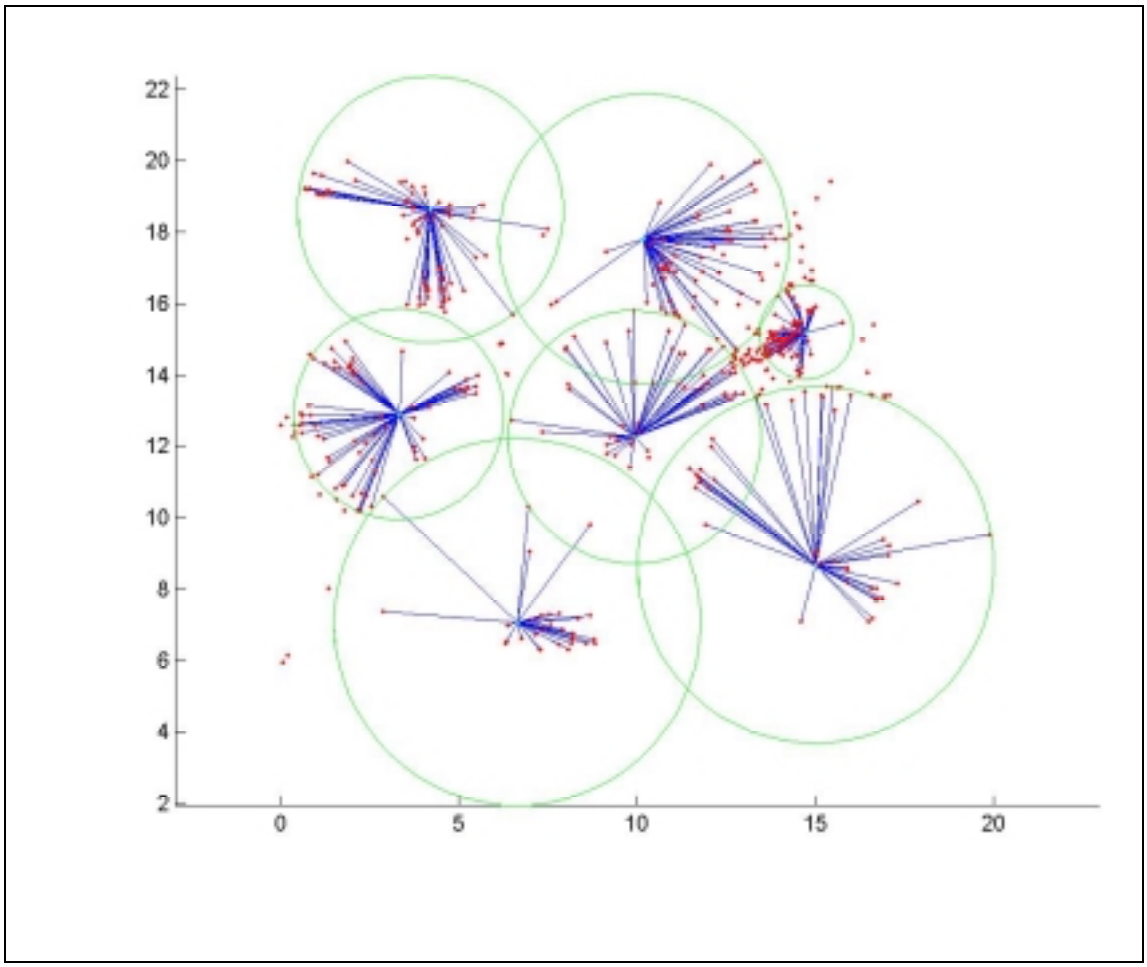


Figure 10

6.4 Relaxed exact model test run discussion

Generalizing the results of the test runs the major problem using this model is that end-users do not get connected to the nearest base station. Moreover, the difference in size in coverage area between overlapping base stations together with the overlap itself are likely to introduce too much of interference. These problems are naturally

caused by the absence of the non-linear constraints. Following the three-step iterative model explained in the beginning of this section would probably provide a good useable solution. However, the number of c_{ij}-variables that are causing problems is relatively large. In this situation more than 3 is considered 'large' due to the fact that finding a solution for each combination of settings of these, say, 4 c_{ij}-variables requires computation of $2^4 = 16$ solutions. In all four test runs the computing time for one solution were approximately 5 minutes. Therefore, the computing time for 16 solutions would exceed the time limit, of one hour, for this optimization. This minor overrun of the time limit is possible to solve by splitting the problem onto two computers. However, the number of 'trouble making' c_{ij}-variables is more likely to be counted in tens or hundreds rather than ones. Resulting in a growth in computing time, which is not possible to overcome with realistic investment in computer hardware.

The obvious question now is if it is possible to improve the model in order to provide better solutions faster. Considering the reason for the non-linearities, the essence is that connecting end-users to base stations is dependent on the set of active base stations. Moreover, simply connecting the end-users to the nearest base station could set the variables identifying the connection between end-users and base stations. However, it has not been possible to find either one linear equation or a set of linear equations to perform this task.

Test runs of a relaxed model minimizing a cost function were also performed. However, the run time for these tests were several days and the results were similar to those illustrated in test run one to four.

At the end of the project period a minor error in the model was discovered. In real life the end-user antenna is directed toward the base station that the end-user is assigned to. Hence, the signal C/I-computation at each end-user must be made with respect to this orientation and the associated antenna gain diagram. However, this error does not make any difference in the conclusion on using the exact model. Taking the

orientation into account would add equations utilizing sinus and cosine functions to the model and hereby making the model even harder to solve.

6.5 Relaxed exact model test run conclusion

In consequence of the results of the test runs and the following discussion, it must be concluded that it is not possible to solve the base station location problem, within the given time frame using the exact models described in the section ‘4 Mathematical model‘.

7 Metaheuristic method

Due to the lack of success in using an exact method the natural step is to use a heuristic method. A heuristic method is a structured way of finding good solutions. In the actual case the largest advantage with the heuristic is that it has no problems handling discrete variables and/or non-linear constraints.

One group of heuristics is called Metaheuristics. These heuristics outline solution search methods in sufficiently general terms in order to make it possible to adapt the heuristic to be able to find solutions to almost any optimization problem. The most common Metaheuristics are Genetic algorithms, Taboo search, Guided local search and Simulated annealing.

Apart from Genetic algorithms, these Metaheuristic methods work the following way. Select a setting of variables that identifies the solution and evaluate this solution. Find a new solution in the neighborhood, evaluate and compare this solution with the previous solution and the best solution. This continues for a 'large' number of solutions. The neighborhood is defined as the group of solutions that can be reached from the actual solution using one move only. The factor that makes the largest difference between Metaheuristics is the way moves are made. All the mentioned Metaheuristics are descent-search algorithms. If the algorithm only moves to better solutions it is most likely to end up in a local minimum which is not the global minimum. The way to avoid this problem differs the last three heuristics, Taboo search, Guided local search and Simulated annealing.

The idea of Taboo search is to store the latest g solutions visited, in the taboo list. In each iteration the algorithm evaluates all solutions in the neighborhood of the current one, and then moves to the best of these solutions not in the taboo list.

For using Guided local search a set of undesired features in the solution is needed. Each time the algorithm finds a solution with one of the undesired features the solution is penalized, hereby pushing the algorithm away from this solution.

Simulated annealing finds a neighborhood solution using a random function. If the neighborhood solution is better than the actual solution, the algorithm moves to that solution. If the neighborhood solution is worse, the algorithm moves to that solution with a given probability. The value for controlling the probability is called a temperature. The temperature is reduced over time, hereby reducing the probability of moving to a worse solution.

Genetic algorithms are inspired by biological evolution and hereby the name. The algorithm works with a population of solutions instead of single solutions. These solutions mix and 'breed children' solutions that hopefully inherit their 'parents' good solution features and develop even better solution features by mutations. Good 'children' solutions are added the population and an equal number of bad solutions leave the population. This process continues over many generations of solutions and hereby develops good solutions.

In any of the Metaheuristics, evaluation of a 'large' number of solutions is required. Hence, in order to be able to go through 'sufficiently many' solutions within 'reasonable' time the Metaheuristics require a very fast method for setting, evaluating and comparing each new solution with the actual solution. In the current case 'reasonable' time is one hour like for the exact model. 'Sufficiently many' is critical in the attempt to get a good solution. However, it is only possible to make the perfect setting of the parameter 'number of solutions evaluated' if the best solution found can be compared with the optimal solution. As mentioned earlier the optimal solution is extremely hard to find for the actual problem.

In this first attempt to use a heuristic it is decided to use Simulated annealing. This is due to observed ability to produce good solutions in other problems combined with the easy implementation of this heuristic. Besides it is assumed that the evaluation is relatively hard and therefore it is attractive to use a heuristic that require few evaluations for each move.

Before setting up the heuristic the following definitions have to be made.

- Problem representation
- Start solution
- Neighborhood
- Solution evaluation
- Cooling scheme
- Stop criterion

7.1 Problem representation

The design of the problem representation is made with focus on ease of computing. The straightforward method is to use the model concept from the exact model. In the exact model the number of variables is in the worst case roughly the number of base stations multiplied by the number of end-users. A large number of these variable combinations are illegal. For instance, each end-user can only be connected to one base station and base stations cannot cover end-users that are farther away from the base station than the maximal transmission distance. When generating a random start the risk of generating an illegal solution is large. Hence, there is a risk of wasting time on evaluating illegal solutions. Alternatively, building up a solution while performing simultaneous check of the legality can be computationally demanding. Therefore the result may be that the number of legal solutions evaluated is the same as if allowing generation of illegal solutions. Hence, it seems potentially profitable to reconsider the model of the network aiming for a solution representation that is easy to evaluate. This model does not necessarily need to be absolutely equivalent to the exact model though it must provide a solution that is legal with respect to this exact model.

7.1.1 Graph models

Inspired by article [2] the legal connections between base stations and end-users can be described as a network *Figure 11*. This network is a bipartite graph with base stations on one side and end-users on the other. Edges between base stations and end-users are legal connections i.e. an edge connects an end-user with a base station if it is

possible to get sufficient signal strength and LOS at the end-user from the base station despite the signal attenuation. This simple network model only describes potential connections. However, the final FWA-network can be considered as a flow network where each end-user have a demand and where base stations have a capacity.

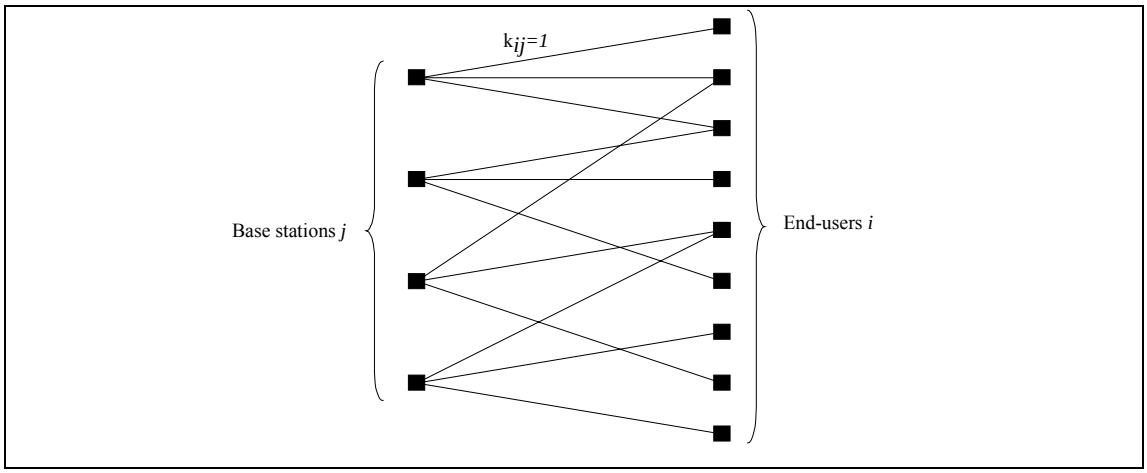


Figure 11

Taking the demand and capacity parameters into account, the flow network can be modeled as in *Figure 12*. When finding the maximal flow from the super source through the network to the super sink, the load on each base station and on each link between base station and end-user is given. The disadvantages with this model are e.g. the end-users can be serviced by more than one base station at the same time and this model does not strive toward a maximization of the load on the active base stations thereby minimizing the number of active base stations.

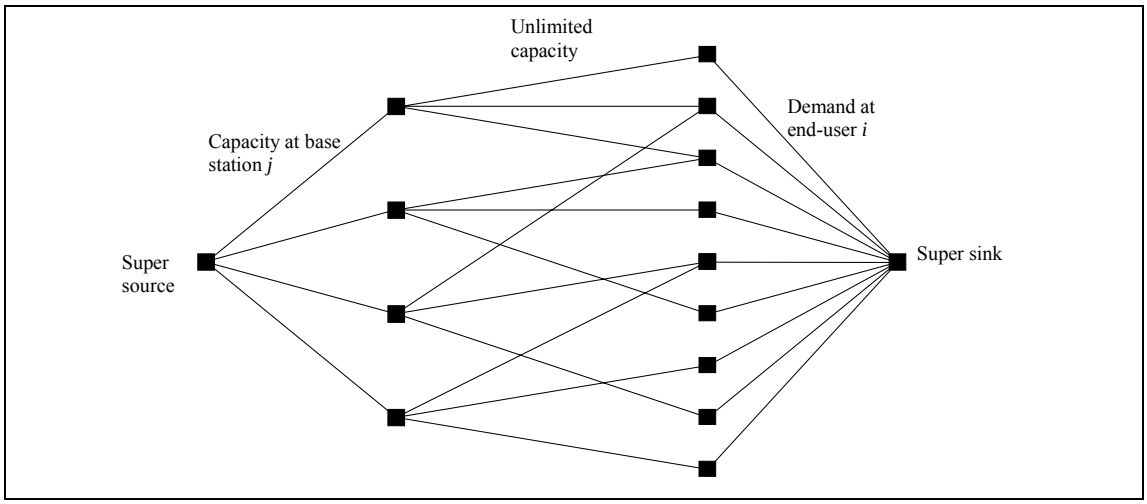


Figure 12

Both *Figure 11* and *Figure 12* give inspiration to formulate the algorithm that connects end-users to base stations.

Consider the non-linear constraint in *Equation 9* controlling that the C/I-value at each end-user is above a threshold value. It is easy to see that the greater the carrier signal and the smaller the interfering signals the better is the C/I relationship. Increasing output power at one base station will result in increased carrier signal at the end-users that are connected to that base station and increased interfering signal at end-users not connected to that base station. Connecting end-users to base stations is practically the same as setting the power output level at the base station. To be able to reach an end-user further away from the base station the output power has to be increased. The output power at a base station can only be increased up to the hardware limit of the output power. As mentioned previously the output power measured in dB can be converted to transmission distance using a linear function.

In the first attempt to create a way of generating solutions, end-users are connected to the nearest base station if this base station have sufficient capacity available. Otherwise it is attempted to connect the end-user to the second nearest base station if possible else the third and so on until the end-user is either connected or the distance to the nearest base station with available capacity is exceeding the maximal transmission limit. If the distance to the nearest base station with vacant capacity is

exceeding the maximal transmission distance the end-user is not connected at all. When connecting end-users all end-users are sorted by the distance to their nearest base station. The algorithm starts with the end-user with the shortest distance to a base station. If an end-user cannot be connected to its nearest base station then the end-user is placed in the list of non-connected end-users with the distance to its second nearest base station. The list is then sorted again and the algorithm continues with the end-user with the shortest distance to a base station.

This way of building up a solution ensures that the solution is legal with respect to the transmission limit and the capacity limit. However, the solution is not necessarily legal with respect to the interference limit but using the shortest possible distance each time an end-user is connected, gives one of the best solutions with the given set of base stations.

Finally, due to the unambiguous way of connecting end-users to base stations the entire solution is given when a set of active base stations is selected.

7.2 Start solution

The initial solution is the starting point for the search for good solutions. In this problem the initial solution can be given as a set of active base stations.

Depending on the target for the optimization the minimum number of active base stations can be pre-computed. If all end-users must be connected, the minimum number of active base station is the total demand divided by the capacity of the base station rounded up to nearest integer. However, it is not certain that all end-user can be connected using any set of base stations of this size or any other size due to interference constraints.

The initial solution for this problem is found by a random selection of base stations. End-users are connected to the selected base stations as indicated in the previous section.

7.3 Neighborhood

The neighborhood is defined as the solutions that can be reached by one move from the actual solution.

In this problem the neighborhood is defined as follows:

- One active base station is turned off and one inactive base station is turned on.

7.4 Solution evaluation

For a given solution it is necessary to be able to evaluate if the solution is better or worse than the previous solution.

In this first attempt where illegal solutions are accepted the evaluation result in a penalty. The penalties represent a cost on both the degree of constraint violations and the network condition. See *Equation 23*. Each penalty is explained later in this section. The penalty function is constructed as a linear sum of total cover circle overlap, load deviation from a given minimum load on each base station if the load is less than the minimum load, tapering between the coverage circle of two base stations and the number of not connected end-users. In order to guide the solver toward a good solution each penalty can be multiplied by a cost constant.

$$Penalty = Load \cdot Q_{load} + Tap \cdot Q_{Tap} + \sum_{j=1}^{J-1} \sum_{k=j+1}^{J} bs\_ol_{jk} \cdot Q_{overlap} + Not\_conn\_eu \cdot Q_{Not\_conn\_eu}$$

Equation 23

7.4.1 Circle overlap

The total circle overlap is computed as the sum of area of design space that is covered by both base stations in any pair of active base stations. The formula for computing this area is given by *Equation 21*.

7.4.2 Load deviation

The load deviation is the sum of difference between utilized capacity and a given minimum load on base stations where the utilized capacity is less than the minimum load '*L*'. Load deviation is computed using *Equation 24*.

$$Load\_deviation = \sum_j \left(L - \sum_i c_{i,j} \cdot d_i \right) \cdot b_j \quad j \in J \mid \left(\sum_i c_{i,j} \cdot d_i < L \right) \quad i \in I$$

Equation 24

7.4.3 Tapering

For any base station the distance to the most distant end-user identifies the radius of the coverage circle. Tapering penalty is the sum of the relationships of the largest radius over the smallest radius of any pairs of base stations where the following apply: The sum of coverage radii r_j and r_k exceeds the distance between the base stations $dist_{jk}$ and the relationship of the largest radius over the smallest radius is greater than a given constant. See the example in *Figure 6*. Here, if r_j/r_k is greater than a given constant '*K*' the relationship r_j/r_k is added the penalty. For convenience *Figure 6* is repeated below.

$$Tapering = \sum_{j,k} \frac{max(r_j, r_k)}{min(r_j, r_k)} \quad j,k \in J \mid \left(r_j + r_k > bs\_dist_{j,k} \wedge \frac{max(r_j, r_k)}{min(r_j, r_k)} > K \right); K \in \Re_+$$

Equation 25

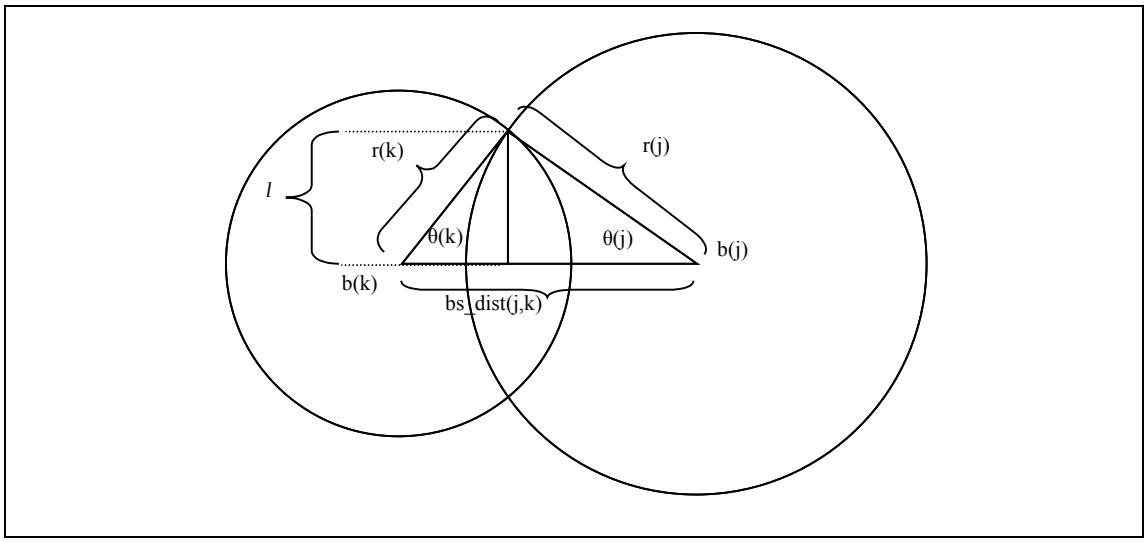


Figure 6

7.4.4 Not connected end-users

The number of not connected end-users are found using the following equation:

$$Not\,connected\,end-users = \sum_i \left(1 - \prod_j c_{i,j}\right) \quad i \in I; j \in J$$

Equation 26

7.5 Cooling scheme

The cooling scheme indicates how the temperature is going to be reduced over time and hereby reducing the probability of going to a solution worse than the actual solution.

The easiest cooling scheme is to reduce the temperature by it with a constant after a specified number of moves. The result of this is a logarithmic temperature curve. The optimization is then done by performing a fixed number of iterations at each temperature level until the stop criterion is reached. However, when the temperature is high the algorithm accepts more or less all neighborhood solutions and performs no actual descent search. Hence, in some cases it can be desirable to reduce the temperature at a faster rate when the temperature is high. Using both 'number of accepted solutions' and 'a fixed number of evaluated solutions' as criterion for

reducing the temperature can do this. The accepted number of solutions is naturally lower than the fixed number of solutions.

At the end of the optimization it is essential that the temperature is not too low before the stop criterion is reached. If the temperature is too low the algorithm will not accept any 'bad' solutions at all and hereby making further search futile. One option is to 'reheat' and hereby make it possible for the algorithm to accept 'bad' solutions again.

In the actual case it is decided to use a simple cooling scheme. After evaluating a fixed number of neighborhood solutions the temperature is lowered by multiplying the temperature by a constant.

7.6 Stop criterion

The stop criterion decides the termination of the heuristic. The aim is to end the heuristic when it is unlikely to find a new solution better than what is already found. One option is to terminate the heuristic when it has been searching for 'some time' without finding any better solution. This can be done using a sliding window, monitoring the number of accepted solutions over a number of iterations. If the number of accepted solutions within the given number of iterations is less than a threshold value the heuristic terminates. The disadvantage with this method is that it is not possible to know prior to start of the heuristic how many iterations are needed before termination and hereby determine how much time the optimization will take.

Another stop criterion is to terminate the heuristic after a fixed number of iterations. This method requires a number of test runs in order to determine how many iterations must be performed until the heuristic does not find any new better solutions within a given 'window'. Naturally, the point when the heuristic does not find any better solutions is in some way dependent on the temperature. If the temperature is low and the heuristic has been searching the local area for some time there is a good chance

that it has found the local minimum and cannot escape from there and therefore cannot get to another local minimum.

Due to the fact that in the current case time is a critical parameter, it is decided to use a stop criterion that terminates the heuristic after a fixed number of iterations. Hereby is it possible prior to optimization to compute how long it takes until the heuristic terminates.

8 First version of heuristic

As mentioned earlier Simulated annealing is a descent-search algorithm. Initialize with a start solution and a start temperature. Find the objective value by evaluating the start solution. Then find a neighborhood solution and evaluate this solution and move to this solution with a probability. If the new objective value is better than the actual value, the probability is '*1*'. Otherwise the probability is found as a function of the difference between the actual objective value and the new objective value and the temperature. After a number of solutions the temperature is lowered hereby lowering the probability of going to a worse solution. When the heuristic reaches the stop criterion it terminates. In pseudo code Simulated annealing looks like this:

- Set t = start temperature
- Find a start solution (x)
- Compute the objective value $F(x)$
- Set global best solution $F(x^*) = F(x)$ and $(x^*) = (x)$
- until global stop criterion is reached do
  - until local stop criterion is reached do
    - find a neighborhood solution (x') and compute objective value $F(x')$
    - if $F(x') <= F(x)$
      - set $(x) = (x')$
    - else $F(x') > F(x)$
      - set p = random number $\in [0;1]$
      - if $exp((F(x)-F(x'))/t) < p$
        - set $(x) = (x')$
      - if $F(x') < F(x^*)$
        - set $(x^*) = (x')$
  - reduce temperature t

8.1 Testing the Metaheuristic

The Metaheuristic is implemented in C. The visualization of the result is made using functions developed in the program Matlab©. The data set used for the test run is the same as those used in the exact case.

8.2 Parameter setting

The following parameters must be set prior to perform the test run:

- Start temperature
- End temperature
- Cooling rate
- Number of local iterations
- Number of global iterations

All five parameters are in some way related to each other. In the current case the optimization must be performed within the given timeframe of one hour. When the time/iteration is known it is simple to determine how many iterations can be performed within this given timeframe. Let '*TI*' indicate the total number of iterations that can be performed. After *TI* iterations the temperature must be reduced from the start temperature to the end temperature.

Start and end temperatures are relatively easy to find using the following procedures. In order to make a good search of the solution space the start temperature must be set at a level where the heuristic accepts many of the generated solutions. This regardless that for each pair of new and actual solutions the new solution can be worse than the actual one. In literature about simulated annealing [8] accept rates at 50-70% of all generated solutions is considered to be sufficient for a good search. For finding the start temperature a first guess of the temperature is made, then 10 test runs with 100 solutions generated in each is made and the average number of accepted solutions in those 10 times 100 solutions are computed. Then the temperature is adjusted and 10 more test runs are made. This is continued until the number of accepts is satisfactory.

When setting the end temperature the task is to ensure that time is not wasted by performing search for solutions when the end temperature is too low to accept any worse solutions at all. However, the end temperature must not be set so high that the heuristic does not perform a thorough search of the neighborhood before terminating. The way of finding the end temperature is to perform a test run where the end temperature is set very low. The setting of the end temperature is then found by a visual inspection of a plot of the actual objective value at each iteration. The end temperature is the temperature at the time where the actual objective value has not changed for 'some time'.

Note that the start and end temperatures are dependent on the numerical size of the difference between the actual and the new objective value. Hence, if the evaluation function is changed it is important to check that the start and end temperatures still are right.

Local iteration is the number of iterations that are performed at each temperature level. Global iteration is the number of times the temperature must be reduced in order to go from start temperature to end temperature. See *Equation 27*. Hence, global iteration multiplied by local iteration must be equal to *TI*.

$$Start\_temperature \cdot cooling\_rate^{global\_iteration} = End\_temperature$$

Equation 27

The combination of cooling rate, global iteration and local iteration can be anything from *TI* local iterations at start temperature and only one temperature drop to end temperature, to one local iteration at each temperature level and reducing the temperature *TI* times. This is possible as long as the product of global and local iterations is equal to *TI*.

The relation between start temperature, end temperature, cooling rate and global iteration can be expressed by *Equation 28*.

$$global\_iteration = \frac{Ln\left(\frac{End\_temperature}{Start\_temperature}\right)}{Ln(cool\_rate)}$$

Equation 28

Setting the size of local iteration must be done with some respect to the size of the neighborhood. Prior to termination the heuristic must be allowed a good probability that the algorithm finds the minimal solution in the neighborhood. The number of solutions in the neighborhood in the actual case is the number of active base stations multiplied by the number of inactive base stations. Hence, identifying the number of iterations that are sufficient for a good probability must be done by performing a number of test runs where different settings of global and local iterations, combined with the appropriate cooling rates are tested.

8.2.1 Cooling rate

The typical value for cooling rate is between '*0.94*' and '*0.98*' and initially the value '*0.96*' is selected.

8.2.2 Global iteration

When start temperature, end temperature and cool rate are decided, global iteration is simple to compute using *Equation 28*:

8.2.3 Local iteration

The local iteration is set in accordance with the results of the test runs, mentioned earlier.

8.3 Test run of first version

The result of the first test run can be seen in *Figure 13*. A visual inspection of the result unveils that this solution is not sufficiently good. This is due to the violation of the two constraints, tapering and overlap.

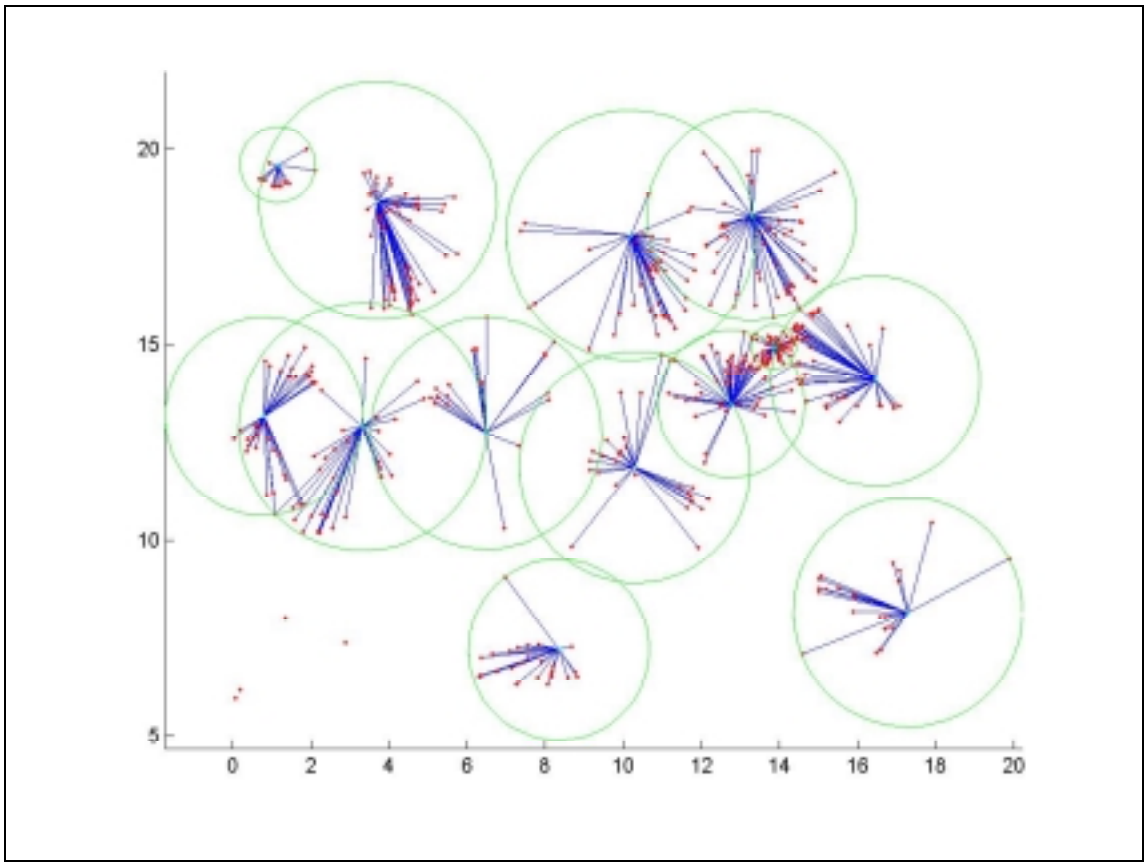


Figure 13

Two obvious violations of the tapering constraint appear in the solution. At location *(14,15)* is a base station located with transmission radius significantly smaller than its neighbors. Similarly at location *(2,20)* is the difference in transmission radii.

The overlap constraint is violated at least 5 times. At the locations *(2,20)*, *(2,13)*, *(5,13)*, *(12,18)* and *(14,15)* the overlap between neighboring transmission areas is too big. At other locations there appears to be violations too but they are not obvious cases.

Additionally, another undesired design error appears. At location *(2,11)* a number of the lines indicating end-user-base station connections cross each other. The result of these crossing is interference and furthermore it is easy to see that if the two end-users swap base station the crossing is removed and, in special cases, the need for

transmission power could be reduced. Finally, these crossings indicate that the algorithm that connects end-users to base stations is not good enough.

Fitting the weights related to each constraint violation could probably solve the trouble with the overlap and tapering. However a number of test runs has been performed with high penalty on overlap and low penalty on tapering, with low penalty on overlap and high on tapering and one test run with high penalty on both tapering and overlap. The results of these test runs showed that it is possible to reduce either tapering, overlap or both. However, the adequate setting of the costs related to the constraints violations are hard to find. Moreover, due to the problems with the connections that cross, the construction of this heuristic is reconsidered. This gave rise to a number of changes and improvements to the heuristic.

9 Second version of heuristic

In this version the problem representation is equal to the representation in the previous. Moreover, in this heuristic only legal solutions are generated. When a set of base stations is found the end-user base station connections are unambiguously given.

The overlap expressed in *Equation 18* is demanding to compute. Due to the fact that, in this version, the check must be performed each time a new end-user is attempted connected, the overlap must be computed one time for each combination of base stations and end-users. In order to keep the computing time low we use a more simple function of the overlap that is not so computationally demanding. We use *simple* overlap as a constraint instead. The constraint is explained later in this section.

In this version a solution is constructed in the following way:

- find a set of base stations
- find nearest active base station for each end-user
- Connect end-users to base stations starting with the shortest end-user base station connection then the second shortest and so on.

Before each end-user is connected a number of checks have to be performed to ensure that the solution still is legal when the actual end-user have been connected. The checks performed are the following:

- Max. transmission distance.
- Max. base station capacity.
- Maximal simple overlap between the actual base station and all neighboring base stations.
- Maximal tapering difference between the actual base station and all neighboring base stations.

If the end-user exceeds the limits in either one of the checks the end-user is not connected at all. The heuristic then proceeds to the next end-user.

9.1 New constraints

9.1.1 Maximal simple overlap

For a given solution where some or all end-users have been connected the distance to the most distant end-user at each base station identify the current radius of the coverage circle. The distance between any two base stations must not be less than a given percentage of the sum of the covering radii of the base stations. See the example in *Figure 6*. Here the sum of r_k and r_j multiplied by a percentage, K, must be less than the distance between the base stations $bs\_dist_{j,k}$.

$$(r_j + r_k) \cdot K < bs\_dist_{j,k} \quad \forall j,k \in J; K \in [0;1]$$

Equation 29

9.1.2 Tapering

Tapering in this version is similar to the tapering in the previous version. However in this version the constraint is formulated slightly different due to the fact that here it is used as a bound instead of a penalty. See *Equation 30*. In this version the tapering constraint is formulated in a way where the latest end-user-base station connection is legal if the following apply. For any pair of the actual connecting base station and all other base stations, the smallest radius is greater than the largest radius multiplied by a constant.

$$min(r_j, r_k) > K \cdot max(r_j, r_k) \quad \forall j,k \in J \mid max(r_j, r_k) > \frac{bs\_dist_{j,k}}{2}; K \in [0;1]$$

Equation 30

In this case the tapering constraint does not apply for any pair of base stations when the algorithm starts. If end-users were evenly distributed it would cause no problem to connect end-users the way used in this version. This is due to the fact that coverage circles then would ‘grow’ at almost the same rate. At the point when two circles starts to overlap they are almost the same size. In the actual problem end-users can be distributed in any fashion. Consider a situation where the border between a densely populated area and a sparsely populated area is narrow *see Figure 14*. Because the

end-user that is about to be connected is the one with the shortest distance to a base station the cover circles at base station *j* and *k* will have approximately the same size when base station *k* have no more vacant capacity. As the process of connecting end-users continues the size of the coverage radius at base station *j* grows. Naturally no end-user located on the right side of the center normal can be connected to base station *j* due to the fact that end-users must only connect to the nearest base station. Therefore no end-user located at the right side of the center normal can induce a cover circle on base station *j* exceeding the center normal. End-users located on the left side of the center normal but more distant from the base station than the center normal, can be connected to base station *j*. Hereby this or these end-users induce a cover circle that also cover parts of the area to the right of the center normal. If the check of tapering is made only when circles overlap then the solution is already illegal. Hence, it is decided to make the tapering check when at least one of the cover radii exceeds the center normal.

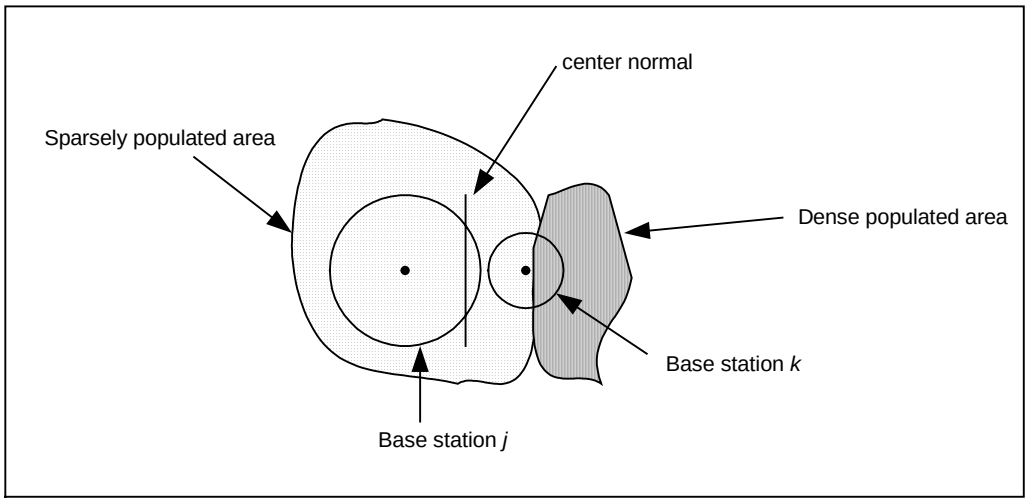


Figure 14

9.2 Neighborhood

In this heuristic the neighborhood is defined as in the first version of the heuristic.

- One active base station is turned off and one inactive base station is turned on.

Furthermore, it is decided to extend the neighborhood by adding the following two operations:

- One active base station is turned off
- One inactive base station is turned on.

These two operations change the final number of base stations. In some cases it is desirable to be able to control the final number of base stations. Hence, these two operations can be set inactive. If they are active a random process controls which one of all three operations that is used in a given iteration.

9.3 Evaluation

In this version of the heuristic like in the previous version the evaluation is initially based on the number of not connected end-users. The function for finding the number of not connected end-users is *Equation 26*. The aim of this optimization is to find a solution where a minimal number of end-users are not connected.

In this version of the heuristic three additional options for optimization is added. These options are minimal sum of not connected demand and minimal sum of either not connected demand or end-users weighted by a density measuring. In *Equation 31* is the general equation for the objective function. If neither demand, weight nor both are desired in the solution the respective variables, d_i and w_i, are set equal to *1*.

$$weighted\_demand = \sum_i \left(1 - \sum_j c_{ij} \right) \cdot d_i \cdot w_i \quad \forall i \in I, j \in J$$

Equation 31

The density weight is made the following way. Design space is tiled into a 20 times 20 grid that makes the boundaries for the density measuring. The demand or number of end-users in each tile is summed. These 400 values are normalized by dividing the each value by the largest value and added a constant of '*0.5*'. The result is a value in the interval [*0.5;1.5*] for each of the 400 tiles. When computing the objective value

each not connected end-user or demand is multiplied by the weight of the tile where the end-user is located.

The advantages using these weights in the objective function is that the heuristic strives toward a solution where covering densely populated areas have a higher priority than covering sparsely populated areas. This is in line with desires expressed by operators.

9.4 Test run of second version

The aim with the test run with this second version of the heuristic is the following:

- Does the improvements actually make the heuristic able to provide a good solution?
- What is the best setting of the parameters cooling rate, local iteration and global iteration?
- What is best: many short optimizations or one long optimization?
- Can the heuristic provide a good solution when setting the number of base stations itself?
- How long time does the heuristic need in order to find the best solution?

The data used in the test runs are the same as used in the exact algorithm or chosen the same way. The following settings have been used in the test runs.

- Tapering constant: 0.70
- Overlap constant: 0.70

The C-program source code of the heuristic can be seen in *appendix 8*.

Initial tests were performed on four different computers in order to measure the performance in run time. The results of the evaluations/second test are the following:

Processor	Operation System	Evaluations/second
Pentium I 100, MHz	Win 95 OSR2	2
Pentium II, 266 MHz	Win NT4	24
Athlon 933 MHz	Win 2000	39
Pentium III, 800 MHz	Win NT4	112

Table 1

Using the Pentium III, 800 Mhz computer it is possible to perform an evaluation of approximately 360.000 solutions within the given timeframe of one hour for the given problem size.

9.4.1 Does the improvements actually make the heuristic able to provide a good solution?

This test run is made using the test 'test0', the same as in the exact case. In order to be able to compare the results with the results from the exact attempt these optimizations are performed with 7 and 9 base stations. The optimizations are performed by minimizing the number of not connected end-users, like in the exact cases 'First test run' and 'Second test run'.

A plot of the result of the test run in the case with 7 base stations can be seen in *Figure 15*. Details about the settings of the heuristic can be seen in *appendix 4*. The number of not connected end-users in this solution is 272 and the minimal load on a base station is 64.512 kbps. The values in the exact case are respectively 250 and of course 96.000.

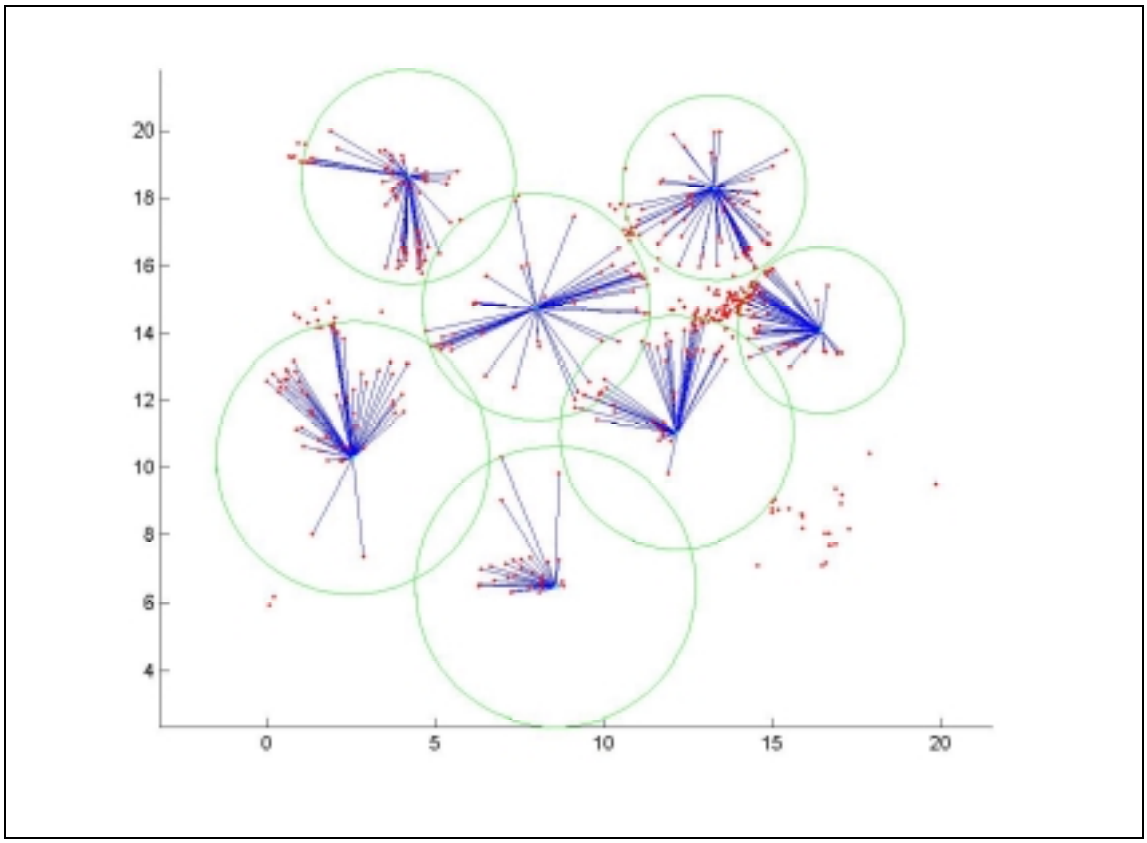


Figure 15

A plot of the result of the test run in the case with 9 base stations can be seen in *Figure 16*. Details about the settings of the heuristic can be seen in *appendix 5*. Here the number of not connected end-users is 198 and the minimal load on a base station is 40.832 kbps. The values in the exact case are 110 and 50.496, respectively.

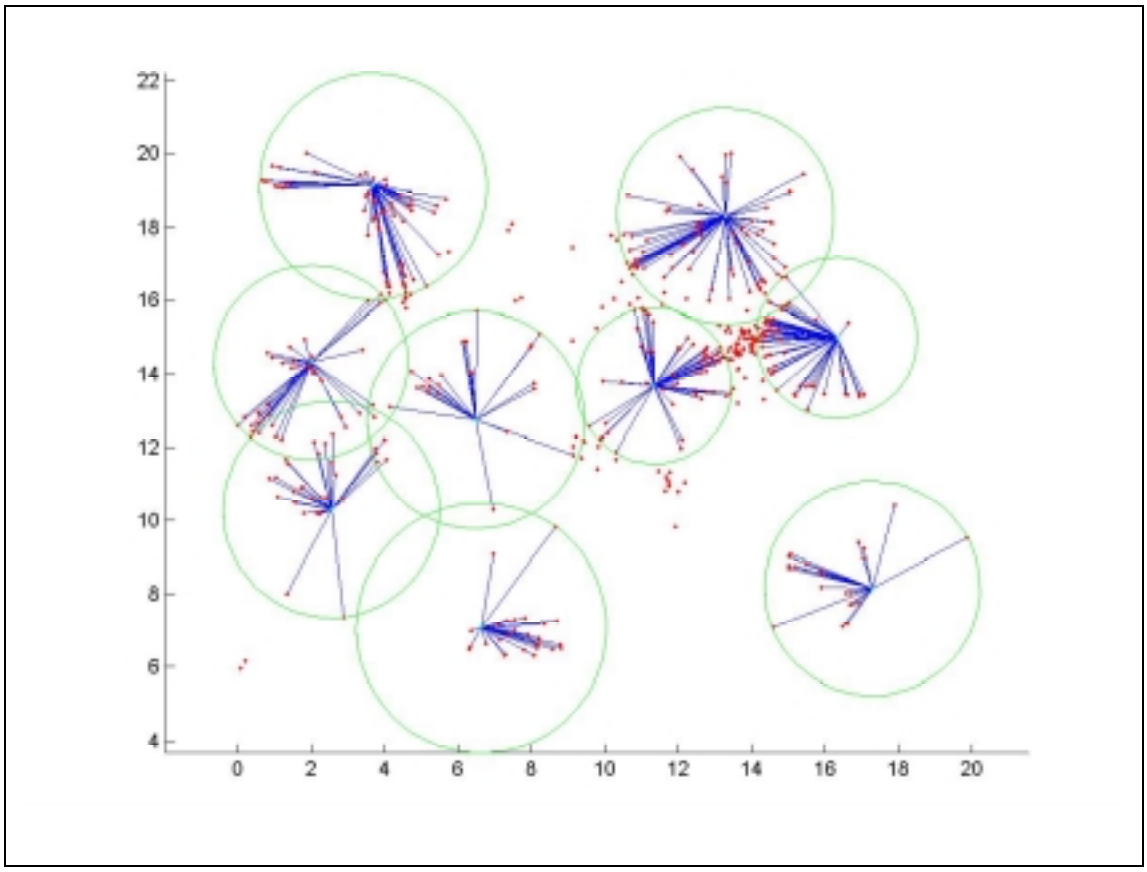


Figure 16

The heuristic solutions are in both cases found within 18 seconds. Comparing the heuristic solution with 7 base stations in *Figure 15* with the relaxed exact solution in *Figure 7*. It can be seen that the heuristic solution is better. Naturally there is no violation of either tapering or overlap in the heuristic solution. Furthermore, there are no base station end-user connections that cross each other. On the other hand, the number of not connected end-users is higher in the heuristic solution than in the relaxed exact one. Equally, the minimal load on a base station has a better value in the relaxed exact case. However, no mechanism in the heuristic is pushing the algorithm toward a solution where the minimal load is maximal. Note that the good result with respect to the number of not connected end-users and minimal load in the relaxed exact case is in a non-legal solution. Hence, it is not possible to determine if these values are achievable in a legal solution.

Finally, a solution evaluation that is closer to real life has been developed. This evaluation computes the C/I ratio at each end-user with respect to the end-user antenna diagram. The number of end-users that has a C/I ratio greater than 22 dB is 680 in the case with 7 base stations. In the case with 9 base stations the number of end-users with a C/I ratio that is greater than 22 dB is 693. These two results are considered realistic by Senior specialist, Phd. Christian Kloch at L.M. Ericsson, Denmark. The function used for computing the C/I ration and the antenna diagram can be seen in *appendix 6*.

9.4.2 What is the best setting of the parameters cooling rate, local iteration and global iteration?

For finding the best setting of the parameters cooling rate and local and global iteration, the following test is performed:
Four test runs each with different seed value are performed. In each run one optimization is performed using each cooling rate from the set {0.91, 0.92…0.98, 0.99}. The total number of iterations in each of the optimizations is selected to be 2000. Hence, the optimization can be performed within 20 seconds. The number of global iterations is computed by the program itself using *Equation 28*. The number of local iterations is also computed by the program as the total number of equations divided by the number of global iterations. Both iteration numbers are naturally rounded to nearest integer value. Hence, the effective total number of iterations in each test run is different from 2000 and is in the range [1748;1988]. The data set for the test run is ‘Test0’. The result of the test run can be seen in *Figure 17*. Here the average of the objective values at each cooling rate is illustrated relative to the total average of all objective values from all cooling rates.

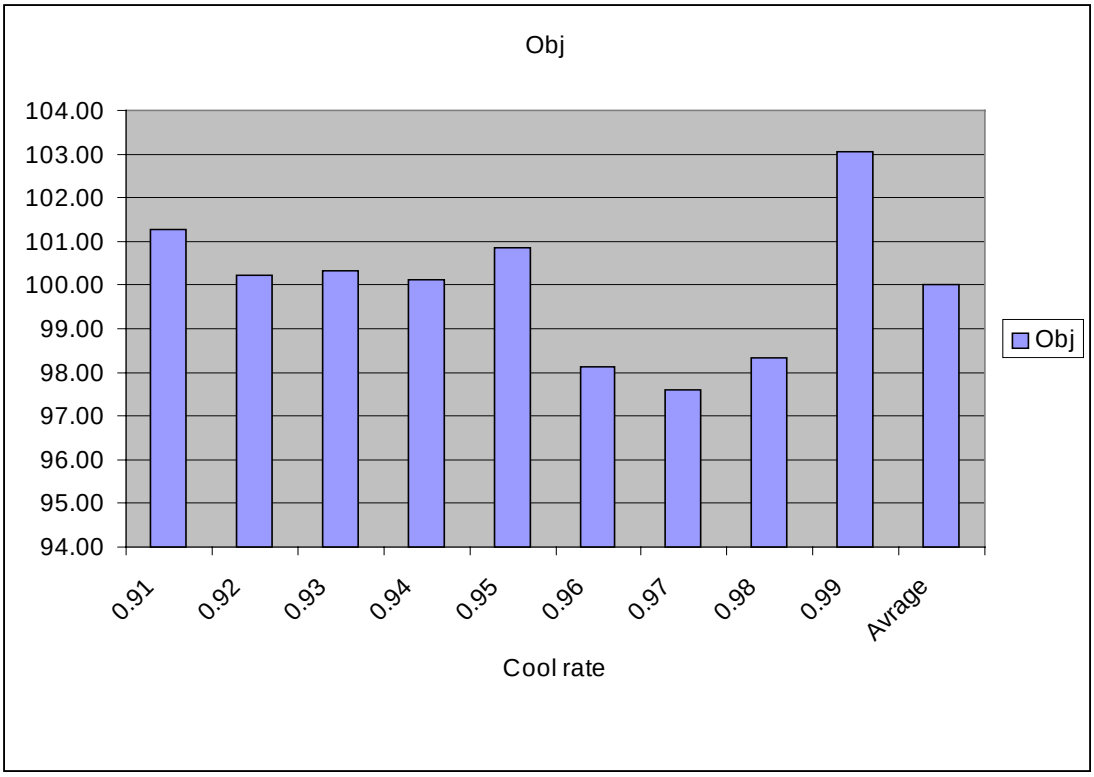


Figure 17

As can be seen from the result the difference in objective values with respect to cooling rate is very small. Therefore is it simply decided to use a cooling rate of *0.96* in the rest of the test runs.

9.4.3 What is best: many short optimizations or one long optimization?

The aim with this test is to find out if it makes a difference to make a number of short optimizations and use the solution with the best objective value rather than making only one long optimization. When making a number of optimizations the heuristic starts each time at a different start solution and hereby the chance of finding a very good solution should be better than if the heuristic only starts once and therefore find only one solution. This is regardless that when the heuristic only perform one long optimization, the number of total iterations is the same as the total number of iterations in the series of short optimizations.

This test run is made up from three series of test runs. The setting in each test run is the following:

Test #	Optimizations	Global iter.	Local iter.
1	10	100	30
2	5	100	60
3	1	100	300

Table 2

In all runs the cooling rate is 0.96. All test runs are made for each of the numbers of base stations in the set {8, 9… 17, 18}. All tests are performed at 5 test sets 'Test1', 'Test2', 'Test3', 'Test4' and 'Test5'. A plot of the result of the test run at the test set 'Test1' can be seen in *Figure 18*. Here the best objective value in each series of test runs is plotted.

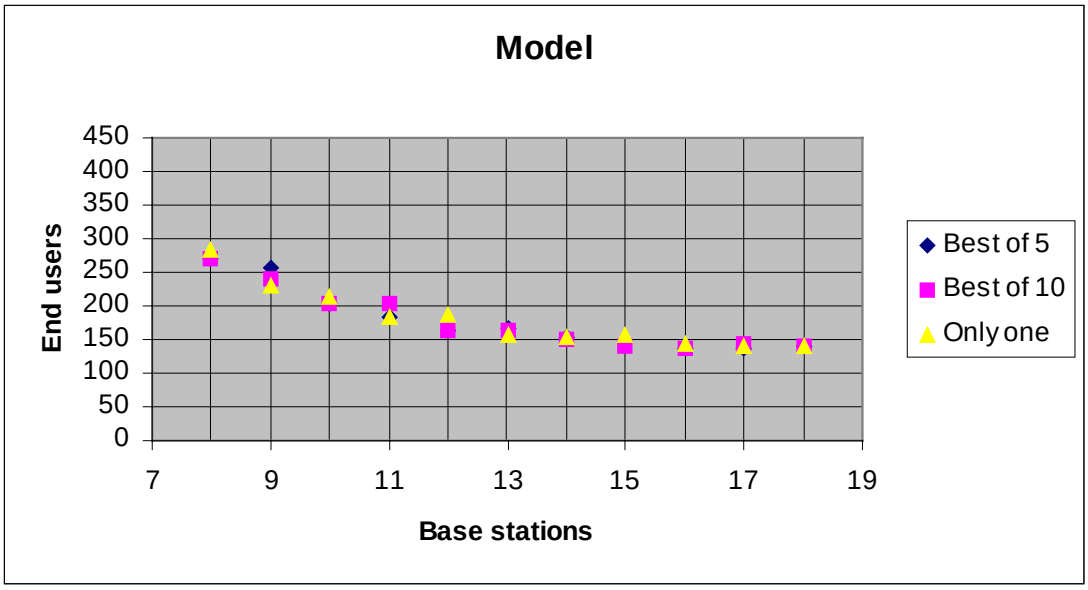


Figure 18

As can be seen from the plot of the result, there is almost no difference in the objective value regardless performing only one optimization or performing a number of optimizations and then picking the solution with the best objective value. The results of the test runs using the test sets 'Test2' to 'Test5' show similar effect and can be seen in *appendix 7*.

Another information that can be extracted from these test runs is that the algorithm also can provide good solutions using other test set than 'test0'. The results of the test

runs in the test sets 'test1' to 'test5' is a number of not connected end-users in the set *{140, 141…280, 281}*.

Note that a series of optimizations as this provides a 2-criterion solution. For each setting of the number of base stations the heuristic provides a number of not connected end-users.

9.4.4 Can the heuristic provide a good solution when setting the number of base stations it self?

The aim with this test is to see if it is possible for the heuristic to find a good solution when setting the number of base stations it self, in order to minimize the number of not connected end-users. In the previous tests the number of base stations has been pre-set. However, the pre-set can only be made on the basis of guess or a desire for a specific number of base stations. If the objective is to find the solution with the overall minimal number of not-connected end-users regardless the number of base stations, the number of base stations can be found by doing a series of optimizations. In each optimization using a number of base stations from the set $\{1,2\ldots(|J|-1), |J|\}$. Recall J is the set of potential base stations. However, some of the optimizations can be omitted if the product of the number of base stations and base station capacity is less than the minimal sum of demand that has to be connected.

Fortunately, in the previous test, 11 tests each with a number of base stations from the set {8,9…17,18} has been made in each of the test sets 'test1' to 'test5'. From *Figure 18* it can be seen that the best solution among the performed optimizations is the one with 16 base stations. Note this is not necessarily the best solution despite it appears that way, for two reasons. First the optimization is made using a heuristic and second there is a chance a better solution can be found using more than 18 base stations.

The test run where the heuristic is suppose to find the number of base stations itself is performed using the test set 'Test1'. The probability controlling which of the following functions to use are initially set to:

- Only add base station 38/506
- Only remove base station 12/506
- Both add and remove base stations 456/506

The values are computed on the basis of the following values:

506 is the number of neighborhood solutions using all three neighborhoods where the number of active base stations is 12.
12 is the number of neighborhood solutions when only removing base stations.
38 is the number of neighborhood solutions when only adding base stations.
456 is the number of neighborhood solution when both adding and removing base stations.

An initial test run showed that the number of base stations increased to 50 active base stations during the test run. The reason for this increase is probably that when the heuristic tries to remove one base station it is very likely that the objective value of the new solution is worse than the actual objective value. Hence, the heuristic rarely removes a base station.

Instead the probabilities is tried to be set as follows:

- Only add base station 2.5%
- Only remove base station 5.0%
- Both add and remove base stations 92.5%

The number of global iterations is 100 and the number of local iterations is 60. Hence, the total number of iterations is 6000. The numbers of base stations is found as part of the test run while minimizing the number of not connected end-users. The result of the test run can be seen in *Table 3*, together with the result where the number of base stations is pre-set:

# of base stations	Not connected end-users found # of base stations	Not connected end-users pre-set # of base stations
13	159	167
15	146	146
16	137	137
17	147	137
21	143	-

Table 3

Comparing the two columns with not connected end-users in *Table 3* it can be seen that the heuristics finds solutions with almost the same objective value.

It is worth noticing that the results in these test runs do not necessarily provide solutions that are profitable for the practical base station location problem.

9.4.5 How long time does the heuristic need in order to find the best solution?

The aim whit this test run is to see how much does the solution improve with more iterations. The test is performed at the test set ‘test1’ with 14 base stations and using a total number of iterations from 500 to 6000 in steps of 500. A plot of the result can be seen in *Figure 19*. As can be seen there is, as expected, a tendency toward better solutions when performing more iterations. Though, with 29916 iterations the objective value is worse than with 4968, 5400 and 5940 iterations! However, these test runs were performed using a cooling rate of *0.96*. If there is time to perform several long optimizations more investigation in parameter setting can be advantageous. Especially, the issues regarding cooling rate, utilization of accept rate when temperature is high and utilization of reheat when temperature is low could be beneficial to use.

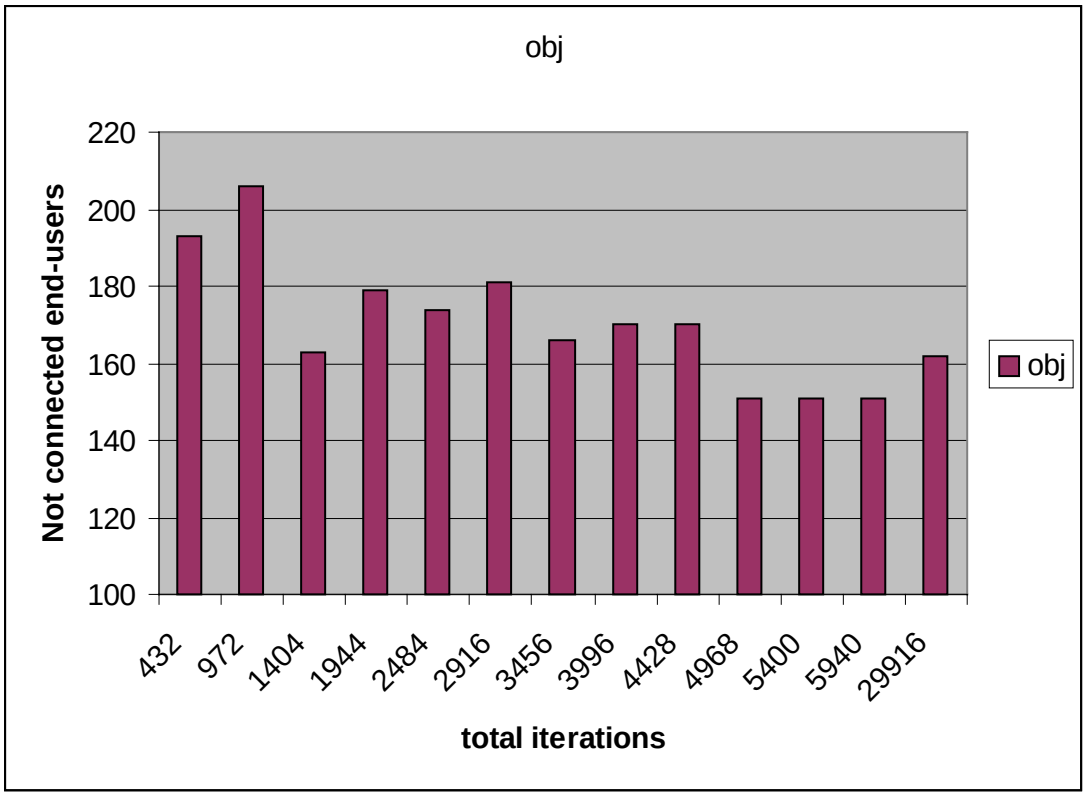


Figure 19

9.5 Heuristic method test run discussion

According to the results in the tests described previously in this section it seems that the second version of the Simulated annealing is able to perform optimizations using the number of not connected end-users as optimizations parameter when the number of base stations is fixed. Setting the number of base stations, as part of the optimization process is also possible according to the results. However, more work needs to be done if this heuristic shall provide solutions that are applicable in real life.

9.6 Heuristic method test run conclusion

As described in the discussion section the second version of Simulated annealing is able to perform optimization using the number of not connected end-users as optimization parameter with success within the given time frame.

10 General discussion

According to the section ‘relaxed exact model test run’ it is not possible to solve the given problem using an exact method within the given time frame. Furthermore, there does not appear to be any options for solving this problem without use of some sort of heuristic method. Fortunately, according to the section ‘test run of second version’, the method developed and implemented is able to place base stations and connect end-users to these base stations. Moreover, the heuristic method is able to perform the location of base stations and connecting end-users in less than 60 seconds. Hereby making it possible to perform a number of optimizations in order to compare different solutions.

Issues interesting for future testing could include the feature that provides the option to weight the end-users according to the density of the area where they are located.

In the second version of the heuristic focus has only been on using the number of not connected end-users as objective value. However, consider the results of the test runs where setting the number of base stations is a part of the optimization. Here, other parameters must be included in the objective function in order to make the heuristic able to weight the marginal improvement of the end-user coverage when adding or removing base stations.

The exact method has the option of optimizing the maximal minimal load on active base stations. This feature is not optional in the heuristic in the current version. However, it appears to be worth consider a function that makes it possible to optimize the network using the minimal load on a base station as objective function. Another option is to use a linear sum of connected end-users and the minimal load on a base station as an objective function. Implementation of these functions is assumed to improve the commercial usability.

11 General conclusion

According to the section ‘General discussion’ and the conclusions in the section ‘Test run of second version’, a method able to place base stations and connect end-users to base stations, spending less than one hour, has been developed and implemented successfully.

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Appendix 1

Full version can be found on CD \Exact1\cut_dk.gms

```
option iterlim=999999999, reslim=3600000, optcr=0.0;
SET   enduser           End_user
      /1*1000/;
      ALIAS(enduser,i);

SET   Basestat    Base stations
      /1*50/;
      ALIAS(Basestat,j,bs);

SET   korr        koordinater
      /X,Y/;
      ALIAS(korr,za);

TABLE lcbs(j,za)

         X        Y
  1    0.798745   13.162966
  2   14.740898   15.195247
  3   12.550226   18.088731
-- CUT --
 48   14.390101   15.416248
 49   16.453164   14.076791
 50   13.722607   15.214297;

TABLE  lceu(i,za)


        X         Y
  1    0.798745   13.162966
  2    0.798745   13.162966
  3   13.933195   15.186057
-- CUT --
 998    1.994665   14.316662
 999    1.994665   14.316662
 1000    5.741703   17.338247;

TABLE  k(i,j)
                 1       2       3       4       5       6       7       8
9       10      11      12      13      14      15      16      17      18
19      20      21      22      23      24      25      26      27      28
29      30      31      32      33      34      35      36      37      38
39      40      41      42      43      44      45      46      47      48
49      50
         1       1       0       0       0       0       0       0       0
0        1       0       0       0       0       0       0       0       0
0        0       0       0       1       0       0       0       1       0
0        0       0       0       0       0       0       1       0       0
0        0       0       0       1       0       0       0       0       0
0        0
         2       1       0       0       0       0       0       0       0
0        1       0       0       0       0       0       0       0       0
0        0       0       0       1       0       0       0       1       0
0        0       0       0       0       0       0       1       0       0
0        0       0       0       1       0       0       0       0       0
0        0
```

```
         3       0       1       0       0       1       0       0       0
0        0       0       1       0       0       0       0       0       0
0        0       0       0       0       0       0       0       0       0
1        0       0       0       0       0       0       0       0       0
0        0       1       0       0       0       0       0       0       1
0        1
         4       0       0       0       0       0       1       0       0
0        0       0       0       0       0       0       0       0       0
0        0       0       0       0       0       0       0       0       0
0        0       0       1       0       0       0       0       0       0
0        0       0       0       0       0       0       0       0       0
0        0
-- CUT --
       997       1       0       0       0       0       0       0       0
0        1       0       0       0       0       0       0       0       0
0        0       0       0       1       0       0       0       1       1
0        0       0       0       0       0       0       1       0       0
0        0       0       0       1       0       0       0       0       0
0        0
       998       1       0       0       0       0       0       0       0
0        1       0       0       0       0       0       0       0       0
0        0       0       0       1       0       0       0       1       1
0        0       0       0       0       0       0       1       0       0
0        0       0       0       1       0       0       0       0       0
0        0
       999       1       0       0       0       0       0       0       0
0        1       0       0       0       0       0       0       0       0
0        0       0       0       1       0       0       0       1       1
0        0       0       0       0       0       0       1       0       0
0        0       0       0       1       0       0       0       0       0
0        0
      1000       0       0       0       0       0       0       0       0
0        0       0       0       0       1       0       1       0       0
0        0       0       0       0       0       0       0       0       1
0        0       0       0       0       0       1       0       0       0
0        0       0       0       0       1       0       1       0       0
0        0;

PARAMETER
      dist(i,j);
      dist(i,j) = (sqrt((lceu(i,'X')-lcbs(j,'X')) * (lceu(i,'X')-lcbs(j,'X')) +
((lceu(i,'Y')-lcbs(j,'Y')) * (lceu(i,'Y')-lcbs(j,'Y'))));


*PARAMETER
*     k(i,j);
*     legal connections
*     k(i,j)$(Dist(i,j) le 5) = 1;


PARAMETER
      tat(j,bs);
      tat(j,bs)$((ORD(j) gt ORD(bs)) and (sqrt((lcbs(j,'X')-lcbs(bs,'X')) *
(lcbs(j,'X')-lcbs(bs,'X')) + (lcbs(j,'Y') - lcbs(bs,'Y')) * (lcbs(j,'Y') -
lcbs(bs,'Y'))) le 5)) = 1;


PARAMETER
      d(i)              Demand on end_user
```

```
       /1      2304
        2      2624
        3      2624
--  CUT  --
      997       384
      998       384
      999      1472
     1000       704/;


SCALARS
      Cap   Capacity at each base station /120000/
      ML    Min. load               /96000/
      M       Big M-constant              /10000000/;


VARIABLES
*     Base station at site
      b(j)

*     Connection between end_user and base station
      c(i,j)

*     Load on base station
      L(j)


*     Objective value
      Z

*     Load limit
      Low;

      c.up(i,j) = 1;
      c.lo(i,j) = 0;

BINARY VARIABLES b;

POSITIVE VARIABLES L;

EQUATIONS
      Eq5(j)            Check minimum load

      Eq6(j)            Check maximum load

      Eq7(i)            only one base station for each end-user

      Eq8(i,j)    only legal connections

      Eq_bs       Max number of base stations

      Eq_cut(j,bs)      min distance between bs

      Eq3         Objectfunction must be maximized;

      Eq5(j) ..   SUM(i,c(i,j) * d(i) ) + M * (1 - b(j) )   =G= ML;

      Eq6(j) ..   SUM(i,c(i,j) * d(i) )               =L= Cap;

      Eq7(i) ..   SUM(j,c(i,j) )          =L= 1;
```

```
      Eq8(i,j) ..       c(i,j)                  =L= k(i,j) * b(j);

      Eq_bs ..          SUM(j,b(j)) =L= 12;

      Eq_cut(j,bs)$((ORD(j) gt ORD(bs)) and (tat(j,bs) eq 1)..   b(j) + b(bs)
=L= tat(j,bs);

      Eq3 ..            Z =E= SUM((i,j),c(i,j));

MODEL test /all/;

OPTION LP=CPLEX;

SOLVE test USING MIP MAXIMIZING Z;

PARAMETER   load(j);
            load(j) = (sum( (i), c.l(i,j) * d(i)));

PARAMETER   cover;
            cover = sum((i,j),c.l(i,j));

PARAMETER   belast;
            belast = sum((i,j)$(c.l(i,j) eq 1), d(i))/ SUM(i,d(i));

Display Z.l, c.l, b.l, load, cover, belast;

file cut_dk /cut_dk.dat/;
put cut_dk;
loop ((i,j)$(c.l(i,j) gt 0), put @1, i.tl, @10 j.tl, @20 c.l(i,j), @30 dist(i,j)
/);
```

Appendix 2

Full version can be found on the CD \Exact3\forsog_max_min_full.gms

```
option iterlim=999999999, reslim=3600000, optcr=0.0;
SET   enduser           End_user
      /1*1000/;
      ALIAS(enduser,i);

SET   Basestat    Base stations
      /1*50/;
      ALIAS(Basestat,j,bs);

SET   korr        koordinater
      /X,Y/;
      ALIAS(korr,za);

TABLE lcbs(j,za)

         X        Y
  1    0.798745   13.162966
  2   14.740898   15.195247
  3   12.550226   18.088731
-- CUT --
 47   11.340857   13.668089
 48   14.390101   15.416248
 49   16.453164   14.076791
 50   13.722607   15.214297;

TABLE  lceu(i,za)


        X         Y
  1    0.798745   13.162966
  2    0.798745   13.162966
  3   13.933195   15.186057
-- CUT --
 997    1.994665   14.316662
 998    1.994665   14.316662
 999    1.994665   14.316662
 1000    5.741703   17.338247;

TABLE  k(i,j)
                1       2       3       4       5       6       7       8
9       10      11      12      13      14      15      16      17      18
19      20      21      22      23      24      25      26      27      28
29      30      31      32      33      34      35      36      37      38
39      40      41      42      43      44      45      46      47      48
49      50
        1       1       0       0       0       0       0       0       0
0       1       0       0       0       0       0       0       0       0
0       0       0       0       1       0       0       0       1       0
0       0       0       0       0       0       0       1       0       0
0       0       0       0       1       0       0       0       0       0
0       0
        2       1       0       0       0       0       0       0       0
0       1       0       0       0       0       0       0       0       0
0       0       0       0       1       0       0       0       1       0
0       0       0       0       0       0       0       1       0       0
```

```
0       0       0       0       1       0       0       0       0       0
0       0
        3       0       1       0       0       1       0       0       0
0       0       0       1       0       0       0       0       0       0
0       0       0       0       0       0       0       0       0       0
1       0       0       0       0       0       0       0       0       0
0       0       1       0       0       0       0       0       0       1
0       1
-- CUT --
      997       1       0       0       0       0       0       0       0
0       1       0       0       0       0       0       0       0       0
0       0       0       0       1       0       0       0       1       1
0       0       0       0       0       0       0       1       0       0
0       0       0       0       1       0       0       0       0       0
0       0
      998       1       0       0       0       0       0       0       0
0       1       0       0       0       0       0       0       0       0
0       0       0       0       1       0       0       0       1       1
0       0       0       0       0       0       0       1       0       0
0       0       0       0       1       0       0       0       0       0
0       0
      999       1       0       0       0       0       0       0       0
0       1       0       0       0       0       0       0       0       0
0       0       0       0       1       0       0       0       1       1
0       0       0       0       0       0       0       1       0       0
0       0       0       0       1       0       0       0       0       0
0       0
     1000       0       0       0       0       0       0       0       0
0       0       0       0       0       1       0       1       0       0
0       0       0       0       0       0       0       0       0       1
0       0       0       0       0       0       1       0       0       0
0       0       0       0       0       1       0       1       0       0
0       0;

PARAMETER
      dist(i,j);
      dist(i,j) = (sqrt((lceu(i,'X')-lcbs(j,'X')) * (lceu(i,'X')-lcbs(j,'X')) +
((lceu(i,'Y')-lcbs(j,'Y')) * (lceu(i,'Y')-lcbs(j,'Y'))));


*PARAMETER
*     k(i,j);
*     legal connections
*     k(i,j)$(Dist(i,j) le 5) = 1;


PARAMETER
      tat(j,bs);
      tat(j,bs)$((ORD(j) gt ORD(bs)) and (sqrt((lcbs(j,'X')-lcbs(bs,'X')) *
(lcbs(j,'X')-lcbs(bs,'X')) + (lcbs(j,'Y') - lcbs(bs,'Y')) * (lcbs(j,'Y') -
lcbs(bs,'Y'))) le 5)) = 1;


PARAMETER
      d(i)              Demand on end_user


       /1      2304
        2      2624
        3      2624
-- CUT --
```

```
      997       384
      998       384
      999      1472
     1000       704/;


SCALARS
      Cap   Capacity at each base station /120000/
      ML    Min. load               /96000/
      M       Big M-constant              /10000000/;


VARIABLES
*     Base station at site
      b(j)

*     Connection between end_user and base station
      c(i,j)

*     Load on base station
      L


*     Objective value
      Z

*     Load limit
      Low;

      c.up(i,j) = 1;
      c.lo(i,j) = 0;

BINARY VARIABLES b;

POSITIVE VARIABLES L;

EQUATIONS
      Eq5(j)            Check minimum load

      Eq6(j)            Check maximum load

      Eq7(i)            only one base station for each end-user

      Eq8(i,j)    only legal connections

      Eq_bs       Max number of base stations

      Eq_cut(j,bs)      min distance between bs

      O           Objectfunction must be maximized;

      Eq5(j) ..   SUM(i,c(i,j) * d(i) ) + M * (1 - b(j) )   =G= L;

      Eq6(j) ..   SUM(i,c(i,j) * d(i) )               =L= Cap;

      Eq7(i) ..   SUM(j,c(i,j) )          =L= 1;

      Eq8(i,j) ..       c(i,j)                  =L= k(i,j) * b(j);

      Eq_bs ..          SUM(j,b(j)) =G= 9;
```

```
      Eq_cut(j,bs)$((ORD(j) gt ORD(bs)) and (tat(j,bs) eq 1))..   b(j) + b(bs)
=L= tat(j,bs);

      O ..        Z =E= L;

MODEL test /all/;

OPTION LP=CPLEX;

SOLVE test USING MIP MAXIMIZING Z;

PARAMETER   load(j);
            load(j) = (sum( (i), c.l(i,j) * d(i)));

PARAMETER   cover;
            cover = sum((i,j),c.l(i,j));

PARAMETER   belast;
            belast = sum((i,j)$(c.l(i,j) eq 1), d(i))/ SUM(i,d(i));

Display Z.l, c.l, b.l, load, cover, belast;

file cut_dk /cut_dk.dat/;
put cut_dk;
loop ((i,j)$(c.l(i,j) gt 0), put @1, i.tl, @10 j.tl, @20 c.l(i,j), @30 dist(i,j)
/);
```

Appendix 3

full version can be found on CD \exact4\forsog_min_bs_full.gms

```
option iterlim=999999999, reslim=3600000, optcr=0.0;
SET   enduser           End_user
      /1*1000/;
      ALIAS(enduser,i);

SET   Basestat    Base stations
      /1*50/;
      ALIAS(Basestat,j,bs);

SET   korr        koordinater
      /X,Y/;
      ALIAS(korr,za);

TABLE lcbs(j,za)

         X        Y
  1    0.798745   13.162966
  2   14.740898   15.195247
  3   12.550226   18.088731
-- CUT --
 48   14.390101   15.416248
 49   16.453164   14.076791
 50   13.722607   15.214297;

TABLE  lceu(i,za)


        X         Y
  1    0.798745   13.162966
  2    0.798745   13.162966
  3   13.933195   15.186057
-- CUT --
 997    1.994665   14.316662
 998    1.994665   14.316662
 999    1.994665   14.316662
 1000    5.741703   17.338247;

TABLE  k(i,j)
                 1       2       3       4       5       6       7       8
9       10      11      12      13      14      15      16      17      18
19      20      21      22      23      24      25      26      27      28
29      30      31      32      33      34      35      36      37      38
39      40      41      42      43      44      45      46      47      48
49      50
        1       1       0       0       0       0       0       0       0
0       1       0       0       0       0       0       0       0       0
0       0       0       0       1       0       0       0       1       0
0       0       0       0       0       0       0       1       0       0
0       0       0       0       1       0       0       0       0       0
0       0
        2       1       0       0       0       0       0       0       0
0       1       0       0       0       0       0       0       0       0
0       0       0       0       1       0       0       0       1       0
0       0       0       0       0       0       0       1       0       0
```

```
0       0       0       0       1       0       0       0       0       0
0       0
        3       0       1       0       0       1       0       0       0
0       0       0       1       0       0       0       0       0       0
0       0       0       0       0       0       0       0       0       0
1       0       0       0       0       0       0       0       0       0
0       0       1       0       0       0       0       0       0       1
0       1
-- CUT --
      998       1       0       0       0       0       0       0       0
0       1       0       0       0       0       0       0       0       0
0       0       0       0       1       0       0       0       1       1
0       0       0       0       0       0       0       1       0       0
0       0       0       0       1       0       0       0       0       0
0       0
      999       1       0       0       0       0       0       0       0
0       1       0       0       0       0       0       0       0       0
0       0       0       0       1       0       0       0       1       1
0       0       0       0       0       0       0       1       0       0
0       0       0       0       1       0       0       0       0       0
0       0
     1000       0       0       0       0       0       0       0       0
0       0       0       0       0       1       0       1       0       0
0       0       0       0       0       0       0       0       0       1
0       0       0       0       0       0       1       0       0       0
0       0       0       0       0       1       0       1       0       0
0       0;

PARAMETER
      dist(i,j);
      dist(i,j) = (sqrt((lceu(i,'X')-lcbs(j,'X')) * (lceu(i,'X')-lcbs(j,'X')) +
((lceu(i,'Y')-lcbs(j,'Y')) * (lceu(i,'Y')-lcbs(j,'Y'))));


*PARAMETER
*     k(i,j);
*     legal connections
*     k(i,j)$(Dist(i,j) le 5) = 1;


PARAMETER
      tat(j,bs);
      tat(j,bs)$((ORD(j) gt ORD(bs)) and (sqrt((lcbs(j,'X')-lcbs(bs,'X')) *
(lcbs(j,'X')-lcbs(bs,'X')) + (lcbs(j,'Y') - lcbs(bs,'Y')) * (lcbs(j,'Y') -
lcbs(bs,'Y'))) le 5)) = 1;


PARAMETER
      d(i)              Demand on end_user


       /1      2304
        2      2624
        3      2624
-- CUT --
      998       384
      999      1472
     1000       704/;


SCALARS
```

```
      Cap   Capacity at each base station /120000/
      ML    Min. load               /96000/
      M       Big M-constant              /10000000/;


VARIABLES
*     Base station at site
      b(j)

*     Connection between end_user and base station
      c(i,j)

*     Load on base station
      L


*     Objective value
      Z

*     Load limit
      Low;

      c.up(i,j) = 1;
      c.lo(i,j) = 0;

BINARY VARIABLES b;

POSITIVE VARIABLES L;

EQUATIONS
*     Eq5(j)            Check minimum load

      Eq6(j)            Check maximum load

      Eq7(i)            only one base station for each end-user

      Eq8(i,j)    only legal connections

      Eq12        Min number of end-users

      Eq_cut(j,bs)      min distance between bs

      Eq2         Objectfunction must be minimized;

*     Eq5(j) ..   SUM(i,c(i,j) * d(i) ) + M * (1 - b(j) )   =G= ML;

      Eq6(j) ..   SUM(i,c(i,j) * d(i) )               =L= Cap;

      Eq7(i) ..   SUM(j,c(i,j) )          =L= 1;

      Eq8(i,j) ..       c(i,j)                  =L= k(i,j) * b(j);

      Eq12 ..           SUM((i,j),c(i,j)) =G= 800;

      Eq_cut(j,bs)$((ORD(j) gt ORD(bs)) and (tat(j,bs) eq 1)).. b(j) + b(bs)
=L= tat(j,bs);

      Eq2 ..            Z =E= SUM(j,b(j));

MODEL test /all/;
```

```
OPTION LP=CPLEX;

SOLVE test USING MIP MINIMIZING Z;

PARAMETER   load(j);
            load(j) = (sum( (i), c.l(i,j) * d(i)));

PARAMETER   cover;
            cover = sum((i,j),c.l(i,j));

PARAMETER   belast;
            belast = sum((i,j)$(c.l(i,j) eq 1), d(i))/ SUM(i,d(i));

Display Z.l, c.l, b.l, load, cover, belast;

file cut_dk /cut_dk.dat/;
put cut_dk;
loop ((i,j)$(c.l(i,j) gt 0), put @1, i.tl, @10 j.tl, @20 c.l(i,j), @30 dist(i,j)
/);
```

Appendix 4

****
****

Out_loop	0
Global iteration	108
Lokal iteration	18
T_start	80.00
Cool rate	0.96
Tapering constant	0.70
Overlap constant	0.70
Validate by density	0
Validate by demand	1
Val. obj by demand	0
Number of base stations	7
Seed	0
Number of not connected end_user	272
Number of End-user hit rate	0.73
End-user demand hit rate	0.75
Best objective value	272.00
Number of end_users with C/I < 22	48
Share of end_users with C/I < 22	0.07
Total number of not connected end_user	320

Fixed base station -1

Base station	Load	x_coordinate	y_coordinate	Radius
10	120000	12.174	11.052	3.475
13	119744	4.198	18.625	3.165
22	120000	2.546	10.299	4.055
23	119808	13.286	18.309	2.736
36	100544	7.999	14.774	3.378
39	64512	8.572	6.475	4.164
48	119936	16.453	14.077	2.473

Appendix 5

****
****

Out_loop	0
Global iteration	108
Lokal iteration	18
T_start	80.00
Cool rate	0.96
Tapering constant	0.70
Overlap constant	0.70
Validate by density	0
Validate by demand	1
Val. obj by demand	0
Number of base stations	9
Seed	0
Number of not connected end_user	198
Number of End-user hit rate	0.80
End-user demand hit rate	0.81
Best objective value	198.00
Number of end_users with C/I < 22	109
Share of end_users with C/I < 22	0.14
Total number of not connected end_user	307

Fixed base station -1

Base station	Load	x_coordinate	y_coordinate	Radius
5	40832	17.289	8.160	2.932
12	119744	16.318	14.981	2.197
15	119808	3.689	19.106	3.096
18	58944	6.636	7.080	3.412
22	61056	2.546	10.299	2.948
23	119808	13.286	18.309	2.958
29	55872	6.485	12.730	2.961
42	118144	1.995	14.317	2.642
46	119936	11.341	13.668	2.113

Appendix 6

Computing C/I

The function for computing C/I values at each end-user is the following:

C/I measured for end-user *i* assigned to base station *k*

$$C = 10^{Z}$$

$$P = \sum_{j \neq k} 10^{Z}$$

$$z = \frac{(27 - (92.5 + 20 * log10(dist_{ij}) + 20 \cdot log10(F)) + gain(angle\_diff) + 33.8)}{10}$$

$$C / I = C / log10(P)$$

Where

F = transmission frequency measured in GHz

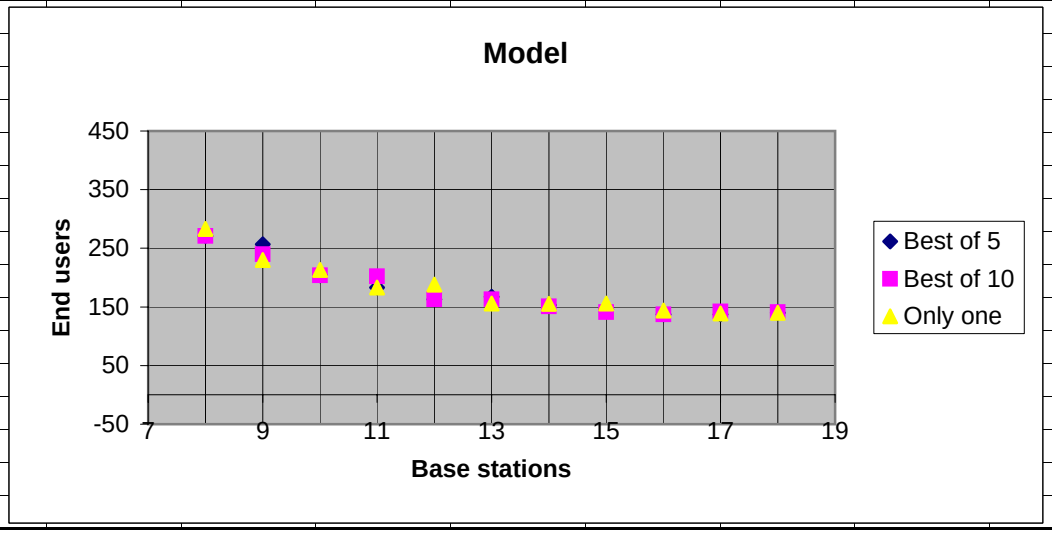
Angle_diff = the angular difference between the direction toward the assigned base station and the actual interfering base station.

The gain diagram is based on the following values at the given angles. The values at angles between the given ones are computed using simple linear interpolation between the adjacent values.

Angle	0	2	8	30	90	100	180
Gain	0	0	-17	-22	-30	-35	-40

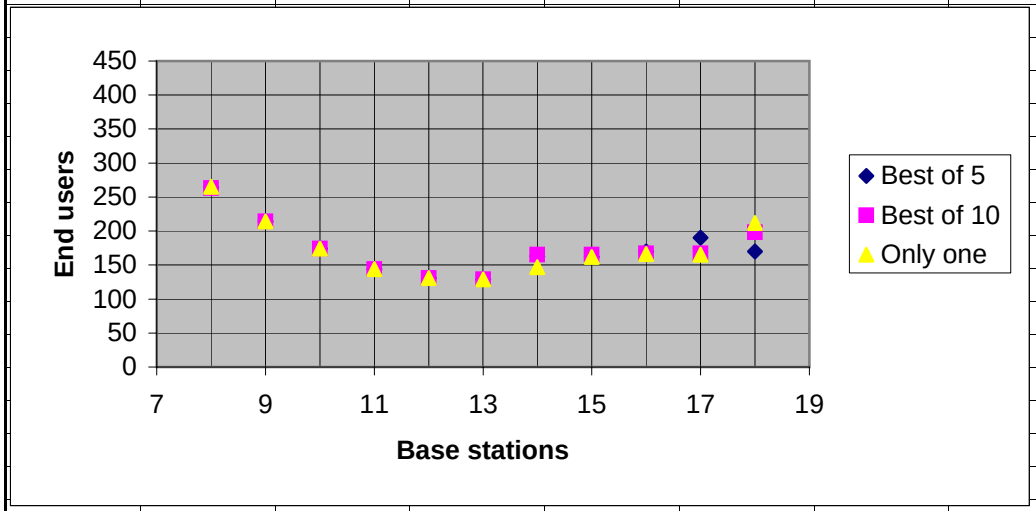
Appendix 7, Test 1

			Best of 5				Best of 10				Only one	
		Global	100			Global	100			Global	100	
	Test1	Local	60			Local	30			Local	300	
	Bs	Not conn	end hit	dem hit		Not conn	end hit	dem hit		Not conn	end hit	dem hit
	8	271	73%	70%		271	73%	70%		283	72%	69%
	9	257	74%	72%		240	76%	74%		230	77%	75%
	10	204	80%	80%		204	80%	80%		213	79%	75%
	11	183	82%	80%		202	80%	79%		183	82%	80%
	12	163	84%	83%		163	84%	83%		188	81%	80%
	13	167	83%	81%		163	84%	81%		156	84%	83%
	14	153	85%	84%		151	85%	85%		155	84%	85%
	15	146	85%	83%		141	86%	85%		156	84%	83%
	16	137	86%	83%		137	86%	83%		144	86%	85%
	17	137	86%	84%		142	86%	85%		139	86%	84%
	18	140	86%	84%		141	86%	85%		140	86%	84%



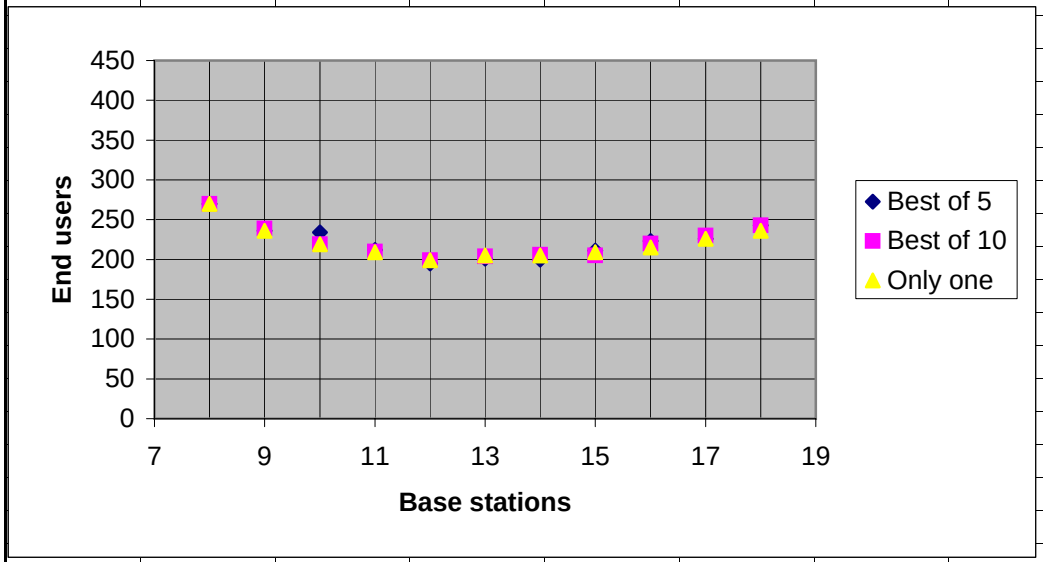
Appendix 7, Test 2

			Best of 5				Best of 10				Only one	
		Global	100			Global	100			Global	100	
	Test2	Local	60			Local	30			Local	300	
	Bs	Not conn	end hit	dem hit		Not conn	end hit	dem hit		Not conn	end hit	dem hit
	8	264	74%	70%		263	74%	69%		265	73%	69%
	9	214	79%	74%		214	79%	76%		214	79%	76%
	10	174	83%	80%		174	83%	80%		174	83%	80%
	11	144	86%	84%		144	86%	84%		144	86%	84%
	12	131	87%	87%		131	87%	85%		131	87%	85%
	13	129	87%	86%		129	87%	86%		129	87%	86%
	14	163	84%	80%		165	83%	80%		147	85%	84%
	15	162	84%	81%		165	83%	80%		162	84%	81%
	16	170	83%	80%		167	83%	81%		166	83%	80%
	17	190	81%	79%		167	83%	81%		165	83%	82%
	18	170	83%	81%		198	80%	78%		212	79%	78%



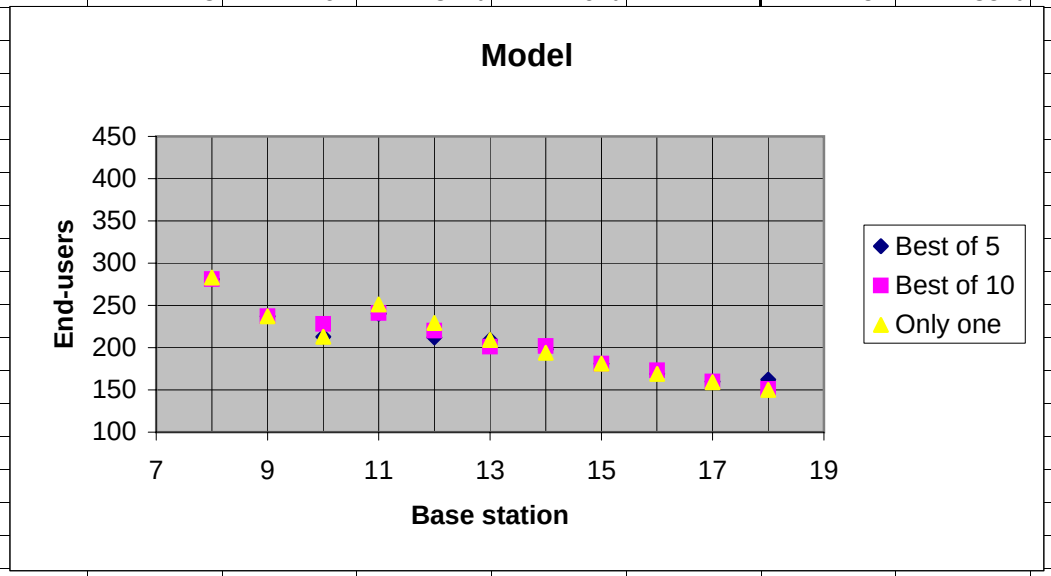
Appendix 7, Test 3

			Best of 5				Best of 10				Only one	
		Global	100			Global	100			Global	100	
	Test3	Local	60			Local	30			Local	300	
	Bs	Not conn	end hit	dem hit		Not conn	end hit	dem hit		Not conn	end hit	dem hit
	8	270	73%	69%		270	73%	70%		270	73	71
	9	236	76%	74%		239	76%	72%		236	76	73
	10	234	77%	75%		219	78%	75%		219	78	73
	11	212	79%	73%		210	79%	77%		209	79	75
	12	195	81%	78%		199	80%	78%		199	80	78
	13	201	80%	76%		204	80%	76%		205	80	77
	14	200	80%	76%		206	79%	77%		205	80	76
	15	211	79%	75%		205	80%	79%		209	79	75
	16	223	78%	75%		220	78%	76%		215	79	76
	17	226	77%	72%		230	77%	73%		226	77	73
	18	243	76%	71%		243	76%	71%		236	76	71



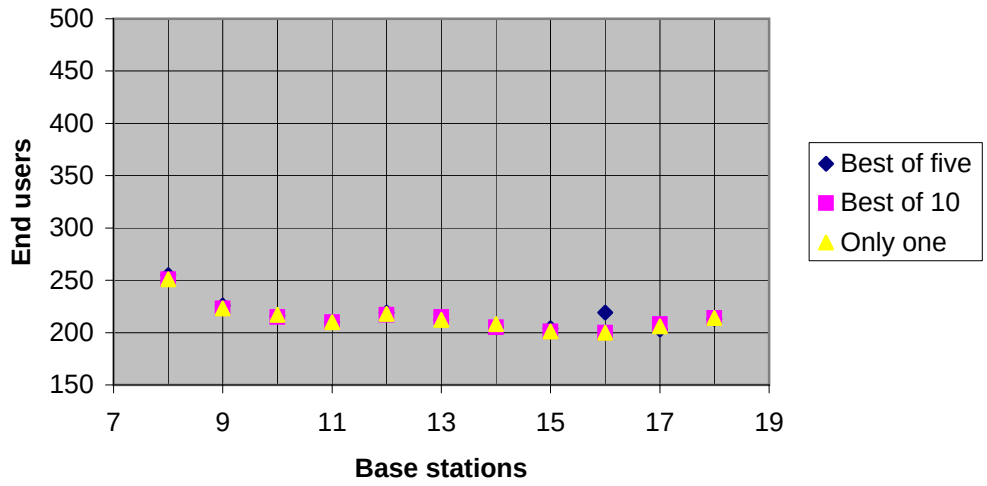
Appendix 7, Test 4

			Best of 5				Best of 10				Only one	
		Global	100			Global	100			Global	100	
	Test4	Local	60			Local	30			Local	300	
	Bs	Not conn	end hit	dem hit		Not conn	end hit	dem hit		Not conn	end hit	dem hit
	8	281	72%	71%		281	72%	69%		283	72%	69%
	9	237	76%	75%		237	76%	72%		237	76%	75%
	10	213	79%	75%		228	77%	77%		213	79%	76%
	11	240	76%	74%		241	76%	75%		251	75%	74%
	12	212	79%	80%		220	78%	77%		229	77%	76%
	13	209	79%	77%		201	80%	77%		209	79%	77%
	14	198	80%	78%		202	80%	78%		194	81%	78%
	15	181	82%	80%		181	82%	79%		181	82%	79%
	16	169	83%	80%		173	83%	78%		169	83%	81%
	17	160	84%	80%		160	84%	81%		159	84%	81%
	18	162	84%	79%		151	85%	83%		150	85%	81%



Appendix 7, Test 5

			Best of five				Best of 10				Only one		
		Global	100			Global	100			Global	100		
	Test5	Local	60			Local	30			Local	300		
	Bs	Not conn	end hit	dem hit		Not conn	end hit	dem hit		Not conn	end hit	dem hit	
	8	255	74%	71%		251	75%	70%		251	75%	70%	
	9	226	77%	73%		223	78%	73%		223	78%	73%	
	10	216	78%	75%		215	79%	75%		217	78%	76%	
	11	210	79%	74%		210	79%	77%		210	79%	76%	
	12	219	78%	76%		217	78%	76%		218	78%	74%	
	13	212	79%	76%		215	79%	73%		212	79%	76%	
	14	205	80%	78%		205	80%	76%		208	79%	7%	
	15	204	80%	78%		201	80%	76%		201	80%	76%	
	16	219	78%	74%		200	80%	76%		200	80%	76%	
	17	203	80%	76%		208	79%	76%		206	79%	77%	
	18	213	79%	75%		214	79%	75%		214	79%	74%	



```
// g++ -O2 -Wall -ansi test.cpp -o indlaes -lm
#include <iostream.h>
#include <iomanip.h>
#include <fstream.h>
#include <assert.h>
#include <stdio.h>
#include <math.h>
#include <stdlib.h>
#include <assert.h>
#include <limits.h>

#define EU 1000
#define BS 50
#define MAX_DIST 5
#define MAX_LOAD 120000
#define PI 3.141592654
#define TAPERING_CONST 0.70
#define OVERLAP_CONST 0.70
#define T_START 80.0
#define T_END 1.0
//#define GLOBAL_ITER 100
//#define LOCAL_ITER 20
//#define COOL_RATE 0.96
#define ACCEPTRATE 700
//#define NUMBER_OF_BS 12
#define GRID 20               // number of entries on each dimension of the density-ma
trix
#define LIM 22
#define GTX 27
#define F 26

bool VAL_BY_DENSITY = false;   // taking density measures into account
bool VAL_BY_DEMAND  = true;    // geo. density of demand or end-users respectivly true
 or false
bool OBJ_BY_DEMAND  = false;   // measuring object function in not-connected demand or
 end-users resp. true or false
bool RESTART_SOL    = false;

int min_bs, max_bs, out_cnt, seed = 0;
int NUMBER_OF_BS, LOCAL_ITER, GLOBAL_ITER, TOTAL_ITER;
int int_solution[]={-1};
int ini = ((sizeof int_solution)/(sizeof int_solution[0]));

double q_p, COOL_RATE;
double data[7][2] = {{180, -40}, {100,-35},{90,-30},{30,-22}, {8,-17},{2,0},{0,0}};
double gain[181];


const char* enduser     = "eu_test.txt";              // contains inputdata of end-us
er
//const char* demand      = "demand_test.txt";          // contains inputdata of dema
nd at end-user
const char* basestation = "bs_test.txt";              // contains inputdata of base s
tations
const char* output      = "outfile.dat";              // contains obj, global_obj, te
mperature
const char* solution    = "solution.dat";             // file containing set of base
 station in global solution
const char* connect     = "connection.dat";           // output contains bs-eu connec
tions
const char* result      = "result1.txt";              // prints the results of the co
mputation
const char* coi_fil     = "coi_table.dat";               // C over I table

typedef struct sortBasestation_t{
  struct sortBasestation_t *next;
```

```
  struct sortBasestation_t *prev;
  int id;
  int endId;
  double x;
  double y;
  double dist;
}kurt;

struct bs_set_t{
  double x;
  double y;
  double dist;
};

struct eu_set_t{
  double x;
  double y;
  double demand;
  double weight;  // weight parameter depending on the density of the area
  int    direct;
};

////////////////////////////////// interpolate ////////////////////////////

void interpolate(){
   int i,j;
   double diff;
//   cout << " int hej " << endl;
   for (i=0;i<6;i++){
      diff = (fabs(data[i][1]-data[i+1][1]))/(data[i][0]-data[i+1][0]);
      for (j=data[i][0];j>data[i+1][0];j--){
         gain[j] = (data[i][0]-j)*diff + data[i][1];
//      cout << j << "  " << gain[j] << "  " << diff << endl;
//      system("pause");
      }
   }
}

////////////////////////////////////  Counter  ////////////////////////////////////
//

int counter(const char* file){
  int j;
  ifstream inFile1(file);
  j = 0;
  while(!inFile1.eof()){
    if(inFile1.get() =='\n')
      j++;
  }
  return j;
}

////////////////////////////////////  ran  ////////////////////////////////////
///

double ran(){
  return ((double) rand())/ ((double) 32767);
}

////////////////////////////////  indlaesEU  ////////////////////////////////////
//

struct eu_set_t* indlaesEU(int j){

  int i, trash;
```

```
  ifstream inFile1(enduser);
  inFile1.close();
  struct eu_set_t *test = new struct eu_set_t [j];
  inFile1.open(enduser);
  for (i=0;i<j;i++){
    inFile1 >> trash >> test[i].x >> test[i].y >> test[i].demand;
    test[i].weight = 1;
  }
  inFile1.close();
return test;
}

////////////////////////////////  indlaesBS  ////////////////////////////////////
//

struct bs_set_t* indlaesBS(int icount){

  int i, trash;
  ifstream inFile1(basestation);
  inFile1.close();

  struct bs_set_t *test = new struct bs_set_t [icount];
  inFile1.open(basestation);
  for (i=0;i<icount;i++){
    inFile1 >> trash >> test[i].x >> test[i].y;
    test[i].dist = MAX_DIST;
  }
  inFile1.close();
  return test;
}


////////////////////////////// alter_dist  ////////////////////////////////////
/

void alter_dist(struct bs_set_t *bs_set, int bs){
  int i=0;
  double dist = 0;
  while(i > -1){
    cout << "Change transmision radii enter base station number (0-" << (bs-1) << "),
end using -1 : " ;
//    cin >> i;
   i = -1;
    if(i > -1){
      cout << "Enter new max. transmission distance at base station "
      <<  i << " in km (0-" << MAX_DIST
      << ") : ";
      cin >> dist;
      bs_set[i].dist = dist;
    }
  }
  cout << "Enter desired min. number of base stations: " ;
  cin >> min_bs;
  cout << "Enter desired max. number of base stations: " ;
  cin >> max_bs;
  cout << "Enter outerloop number: " ;
  cin >> out_cnt;
  cout << "Enter TOTAL iteration count : " ;
  cin >> TOTAL_ITER;
  cout << "Enter cooling rate : ";
  cin >> COOL_RATE;
  GLOBAL_ITER = (int)(log(T_END/T_START)/log(COOL_RATE)) + 1;
  LOCAL_ITER = (int)((double)TOTAL_ITER/(double)GLOBAL_ITER);
//  cout << "Local iter " << LOCAL_ITER << endl;
  cout << "Enter seed value: ";
  cin >> seed;
```

```
  cout << "Enter probability : ";
  cin >> q_p;
}
////////////////////////////////  defDist  ////////////////////////////////////
//

double** defDist(struct eu_set_t *eu_set, struct bs_set_t *bs_set, int eu, int bs){
  int i, j;
  double **dist = new double * [eu];
  for (i=0;i<eu;i++)
    dist[i] = new double [bs];
  for (i=0;i<eu;i++)
    for (j=0;j<bs;j++)
      dist[i][j] = sqrt(pow((eu_set[i].x - bs_set[j].x),2) + pow((eu_set[i].y - bs_set
[j].y),2));
  return dist;
}

////////////////////////////////  defDistBB  ///////////////////////////////////
//

double** defDistBB(struct bs_set_t *bs_set, int bs){
  int i,j;
  double **distBB = new double * [bs];
  for (i=0;i<bs;i++)
    distBB[i] = new double [bs];
  for (i=0;i<bs;i++)
    for (j=i;j<bs;j++)
      distBB[i][j] = distBB[j][i] = sqrt(pow((bs_set[i].x - bs_set[j].x),2) + pow((bs_
set[i].y - bs_set[j].y),2));
  return distBB;
}

////////////////////////////////  defDensity  ///////////////////////////////////
//

void defDensity(int eu, struct eu_set_t *eu_set){
  int i,j,p, i_max_x, i_max_y, i_min_x, i_min_y, x_size, y_size;
  double max_x = 0, max_y = 0, min_x = eu_set[0].x, min_y = eu_set[0].y, x_step, y_ste
p, max_val = 0;

  for (i=0;i<eu;i++){
    if (max_x < eu_set[i].x)
      max_x = eu_set[i].x;
    if (min_x > eu_set[i].x)
      min_x =  eu_set[i].x;
    if (max_y < eu_set[i].y)
      max_y = eu_set[i].y;
    if (min_y > eu_set[i].y)
      min_y = eu_set[i].y;
  }
  i_min_x = ((int)min_x);
  i_min_y = ((int)min_y);
  i_max_x = ((int)max_x)+1;
  i_max_y = ((int)max_y)+1;

  x_size = (i_max_x-i_min_x);        // number of entries in x-direction
  y_size = (i_max_y-i_min_y);        // number of entries in y-direction
  x_step = ((double)(x_size))/GRID;  // size of one interval on x-axis
  y_step = ((double)(y_size))/GRID;  // size of one interval on y-axis

  double **density = new double * [GRID];
  for (i=0;i<GRID;i++)
    density[i] = new double [GRID];
  for(i=0;i<GRID;i++)
    for(j=0;j<GRID;j++)
```

```
        density[i][j] = 0;

    if(VAL_BY_DEMAND){
      for(i=0;i<eu;i++){
        j=((int)((eu_set[i].x-i_min_x)/x_step));
        p=((int)((eu_set[i].y-i_min_y)/y_step));
        density[j][p]+=eu_set[i].demand;
      }
    }
    else{
      for(i=0;i<eu;i++){
        j=((int)((eu_set[i].x-i_min_x)/x_step));
        p=((int)((eu_set[i].y-i_min_y)/y_step));
        density[j][p]+=1;
      }
    }

    for(i=0;i<GRID;i++)
      for(j=0;j<GRID;j++)
        if(max_val < density[i][j])
     max_val = density[i][j];

    for(i=0;i<eu;i++){
      j=((int)((eu_set[i].x-i_min_x)/x_step));
      p=((int)((eu_set[i].y-i_min_y)/y_step));
      eu_set[i].weight = (density[j][p]/max_val) + 0.5;
      //    cout << eu_set[i].weight << " " << endl;
    }
  }

  ////////////////////////////  sortBasestation  ////////////////////////////////
  //

  sortBasestation_t** sortBasestation(int bs, int eu, double **dist, struct bs_set_t *bs
  _set){
    sortBasestation_t **near_eu = new sortBasestation_t * [eu];
    sortBasestation_t *temp_near_eu;
    sortBasestation_t *temp1_near_eu;
    sortBasestation_t *basestationPatch;
    int i,j;
    for (i=0;i<eu;i++)
      near_eu[i] = NULL;

    for (i=0;i<eu;i++)
      for (j=0;j<bs;j++){
        basestationPatch = new sortBasestation_t;
        basestationPatch -> id = j;
        basestationPatch -> endId = i;
        basestationPatch -> x = bs_set[j].x;
        basestationPatch -> y = bs_set[j].y;
        basestationPatch -> dist = dist[i][j];
        basestationPatch -> next = NULL;
        basestationPatch -> prev = NULL;
        temp_near_eu = near_eu[i];
        if (temp_near_eu == NULL){
     near_eu[i] = basestationPatch;
        }
        else{
     while((temp_near_eu -> dist <= basestationPatch -> dist) && (temp_near_eu -> next !
  = NULL))
       temp_near_eu = temp_near_eu->next;
     if ((temp_near_eu -> next == NULL)&&(temp_near_eu -> dist <= basestationPatch -> di
  st)){
       temp_near_eu -> next = basestationPatch;
       basestationPatch -> prev = temp_near_eu;
     }
```

```
    else if((temp_near_eu -> prev == NULL)&&(temp_near_eu -> dist > basestationPatch ->
 dist)){
      temp_near_eu -> prev = basestationPatch;
      near_eu[i] = basestationPatch;
      basestationPatch -> next = temp_near_eu;
    }
    else{
      temp1_near_eu = temp_near_eu -> prev;
      temp1_near_eu -> next = basestationPatch;
      temp_near_eu -> prev = basestationPatch;
      basestationPatch -> prev = temp1_near_eu;
      basestationPatch -> next = temp_near_eu;
    }
       }
     }
  return near_eu;
}

////////////////////////////////////// new_startsol  ////////////////////////////////
///

void new_startsol(bool *usedBS, int bs){
  int i,j=0;

//  cout << "elementer " << ini << endl;
  if (int_solution[0] != -1){
    for(i=0;i<ini;i++)
      usedBS[int_solution[i]] = true;
    while(j<(NUMBER_OF_BS-ini)){
      i = ((int)(ran()*bs));
      while(usedBS[i])
        i = ((int)(ran()*bs));
      usedBS[i] = true;
      j++;
    }
  }
  else{
//    cout << "hello "<< endl;
    while(j < NUMBER_OF_BS){
      i = ((int)(ran()*bs));
      while(usedBS[i])
        i = ((int)(ran()*bs));
      usedBS[i] = true;
      j++;
    }
  }
// cout << " bye " << endl;
}

////////////////////////////////  sort_end_base  ////////////////////////////////
//
// sorts an array with 'eu' entries to point to the shortest enduser base station
// connection for each enduser sorted by distance smallest first.

void sort_end_base(sortBasestation_t **P_short_end_base, int eu){
  int i,j, k;
  double min;
  sortBasestation_t *temp = NULL;
  for(i=0;i<eu;i++){
    min = P_short_end_base[i]->dist;
    k =  -1;
    //    cout << setw(10) << min << endl;
    for(j=i;j<eu;j++){
      if(P_short_end_base[j]->dist < min){
   min = P_short_end_base[j]->dist;
   temp = P_short_end_base[j];
```

```
   k=j;
      }
    }
    if(k>-1){
      P_short_end_base[k] = P_short_end_base[i];
      P_short_end_base[i] = temp;
    }
  }
}

////////////////////////////////////  mini  ////////////////////////////////////////
///

double mini(double final_conn1, double final_conn2){
  if(final_conn1 <= final_conn2)
    return final_conn1;
  else
    return final_conn2;
}

////////////////////////////////////////  maxi ////////////////////////////////////
///

double maxi(double final_conn1, double final_conn2){
  if(final_conn1 >= final_conn2)
    return final_conn1;
  else
    return final_conn2;
}


////////////////////////////////////////  find_nearest  ////////////////////////
///

void find_nearest(sortBasestation_t **P_short_end_base, sortBasestation_t **near_eu, b
ool *usedBS, int eu){
  int i;
  sortBasestation_t *temp_near_eu;
  for(i=0;i<eu;i++){
    temp_near_eu = near_eu[i];
    while(!usedBS[temp_near_eu->id])
      temp_near_eu = temp_near_eu->next;
    P_short_end_base[i] = temp_near_eu;
  }
}

////////////////////////////////////////// legal_conn  ////////////////////////////
//

bool legal_conn(sortBasestation_t *P_short_end_base, bool *usedBS, double **distBB, do
uble *load_bs, double *final_conn, struct eu_set_t *eu_set, int bs, struct bs_set_t *b
s_set){
  int j=0, base, end;
  double load, final;
  bool flag = true;
  base = P_short_end_base->id;
  end  = P_short_end_base->endId;
  final = P_short_end_base->dist;
  if (final > bs_set[base].dist)
    return false;
  load = eu_set[end].demand;
  if(load_bs[base] + load > MAX_LOAD)
    return false;
  while(flag && (j<bs)){
    if((base!=j)&&(usedBS[j])&&(distBB[base][j]<=(bs_set[base].dist + bs_set[j].dist))
){
```

```
      if((distBB[base][j]>=(final + final_conn[j]) * OVERLAP_CONST)){
   if((final>(distBB[base][j]/2))&&(mini(final_conn[j],final) < maxi(final_conn[j],fin
al) * TAPERING_CONST))
     flag = false;
   else
     flag = true;
      }
      else
   flag = false;
    }
    j++;
  }
  return flag;
}

////////////////////////////////////  conn_end  ////////////////////////////////
///

double conn_end(sortBasestation_t **P_short_end_base, struct eu_set_t *eu_set, double
**distBB, double *final_conn, double *load_bs, bool *usedBS, bool *usedEU, int eu, int
 bs, sortBasestation_t **near_eu, struct bs_set_t *bs_set ){
  int i;
  double obj=0;
  bool flag = true;

  find_nearest(P_short_end_base, near_eu, usedBS, eu);
  sort_end_base(P_short_end_base, eu);

  for(i=0;i<bs;i++){
    final_conn[i] = 0;
    load_bs[i]    = 0;
  }
  for(i=0;i<eu;i++)
    usedEU[i] = false;
  while(flag){
    i=0;
    flag = false;
    while(i<eu){
      if(legal_conn(P_short_end_base[i], usedBS, distBB, load_bs, final_conn, eu_set,
bs, bs_set) && !usedEU[i]){
   usedEU[i] = true;
   load_bs[P_short_end_base[i]->id] += eu_set[P_short_end_base[i]->endId].demand;
   final_conn[P_short_end_base[i]->id] = P_short_end_base[i]->dist;
   flag = true;
      }
      i++;
    }
  }
  if(!OBJ_BY_DEMAND){
    for(i=0;i<eu;i++){
      if(!usedEU[i])
   obj+=eu_set[i].weight;
    }
  }
  else{
    for(i=0;i<eu;i++){
      if(!usedEU[i])
   obj+=(eu_set[i].demand * eu_set[i].weight);
    }
  }
  return ((double)obj);
}

////////////////////////////////  indlaesSOL  ////////////////////////////////////
///
```

```
int* indlaesSOL(int *j){

  int i, id;

  ifstream inFile1(solution);
  inFile1.close();
  inFile1.open(solution);
  *j = 0;
  while(!inFile1.eof()){
     if(inFile1.get() =='\n')
       *j+=1;
  }
   int *sol = new int [*j];
   inFile1.close();
   inFile1.open(solution);
   for (i=0;i<*j;i++){
      inFile1 >> id;
      sol[i] = (id-1);
   }
   inFile1.close();
   return sol;
}

////////////////////////////////  indlaesCONN  ////////////////////////////////
////

int** indlaesCONN(int *j){

  int i, id_bs, id_eu;
   double trash;
  ifstream inFile1(connect);
  inFile1.close();
  inFile1.open(connect);
  *j = 0;
  while(!inFile1.eof()){
     if(inFile1.get() =='\n')
       *j+=1;
  }
   int **conn = new int *[*j];
   for (i=0;i<*j;i++)
      conn[i] = new int[2];
   inFile1.close();
   inFile1.open(connect);
   for (i=0;i<*j;i++){
      inFile1 >> id_bs >> id_eu >> trash;
      conn[i][0] = (id_bs-1);
      conn[i][1] = (id_eu-1);
   }
   inFile1.close();
   return conn;
}

////////////////////////////////  angle ////////////////////////////////

int angle(eu_set_t *eu_set, bs_set_t *bs_set, int bs_id, int eu_id, double **dist){
int vinkel;
double ang;
   if (dist[eu_id][bs_id] > 0){
      if ((bs_set[bs_id].x-eu_set[eu_id].x) != 0){
         ang = (atan((bs_set[bs_id].y-eu_set[eu_id].y)/(bs_set[bs_id].x-eu_set[eu_id].
x)))/PI*180;
         if ((bs_set[bs_id].x-eu_set[eu_id].x)<0)
            vinkel = (180.5 + ang);
         else if (ang > 0)
            vinkel = ang + 0.5;
         else
```

```
            vinkel = (360.5 + ang);
         }
         else if ((bs_set[bs_id].y-eu_set[eu_id].y)>0)
            vinkel = 270;
         else if ((bs_set[bs_id].y-eu_set[eu_id].y)<0)
            vinkel = 90;
      }
      else vinkel = -1;
   return (int)vinkel;
   }

   ////////////////////////////////// coi ////////////////////////////////

   double **coi(double **dist, int **conn, eu_set_t *eu_set, bs_set_t *bs_set, int *sol,
   int no_sol, int no_conn){
      int i,j, bs_id, eu_id, inter_bs, direct, in_direct, diff;
      double interferer, carrier;
      double **coi_table = new double * [no_conn];
      for (i=0;i<no_conn;i++){
         coi_table[i] = new double [2];
         bs_id = conn[i][0];
         eu_id = conn[i][1];
         interferer = 0;
         direct = eu_set[eu_id].direct;
         if(direct != -1){
            for(j=0;j<no_sol;j++){
               if(sol[j]!= bs_id){
                  inter_bs = sol[j];
                  in_direct = angle(eu_set, bs_set, inter_bs, eu_id, dist);
                  if(fabs(in_direct-direct) > 180)
                     diff = 360 - fabs(in_direct-direct);
                  else
                     diff = fabs(in_direct-direct);
                  interferer = interferer + pow(10,((GTX - (92.5 + 20*log10(dist[eu_id][i
   nter_bs]) + 20*log10(F)) + gain[diff] + 33.8)/10));
               }
               else{
                  carrier = GTX - (92.5 +20*log10(dist[eu_id][bs_id]) + 20*log10(F)) + ga
   in[0] + 33.8;
               }
            }
            coi_table[i][0] = eu_id;
            coi_table[i][1] = carrier - 10*log10(interferer); //10*log10(pow(10,(carrier/
   10))/interferer);
         }
         else{
            coi_table[i][0] = eu_id;
            coi_table[i][1] = 50;
         }
      }
      return coi_table;

   }
   ////////////////////////////////// defDirect ////////////////////////////

   void defDirect(eu_set_t *eu_set, bs_set_t *bs_set, int **conn, double **dist, int no_c
   onn){
      int i, bs_id, eu_id;
      for (i=0;i<no_conn;i++){
         bs_id = conn[i][0];
         eu_id = conn[i][1];
         eu_set[eu_id].direct = angle(eu_set, bs_set, bs_id, eu_id, dist);
      }
   }

   ////////////////////////////////////////// sortout ////////////////////////////
```

```
void sortout(int no_conn, double **coi_table){
   int i,j,p;
   double temp[2],min;
   for (i=0;i<no_conn-1;i++){
      min = coi_table[i][1];
      p = -1;
      for(j=i+1;j<no_conn;j++){
         if(coi_table[j][1]<min){
            min = coi_table[j][1];
            p=j;
         }
      }
      if (p != -1){
         temp[0]= coi_table[i][0];
         temp[1]= coi_table[i][1];
         coi_table[i][0] = coi_table[p][0];
         coi_table[i][1] = coi_table[p][1];
         coi_table[p][0] = temp[0];
         coi_table[p][1] = temp[1];
      }

   }

}

////////////////////////////////  udskriv  ////////////////////////////
void udskrivcoi(int no_conn, double **coi_table){
   int i;
   FILE *fp1;
   fp1 = fopen(coi_fil, "w");
   for(i=0;i<no_conn;i++){
      fprintf(fp1, "%8.0f %8f \n", coi_table[i][0], coi_table[i][1]);
   }
   fclose(fp1);


}

//////////////////////////////////////////  udskriv  ////////////////////////
///

void udskriv(double **dist, bool *usedBS, bool *usedEU, double *final_conn, sortBasest
ation_t **near_eu, int bs, int eu, struct eu_set_t *eu_set, sortBasestation_t **P_shor
t_end_base, double *load_bs, double **distBB, struct bs_set_t *bs_set, double global_o
bj, int out_loop){
FILE *fp1;
fp1 = fopen(connect, "w");

int i,p=0, no_sol, no_conn, count = 0;
bool flag = true;
double sum_load = 0;
double all_load = 0;

   find_nearest(P_short_end_base, near_eu, usedBS, eu);
   sort_end_base(P_short_end_base, eu);

   for(i=0;i<bs;i++){
      final_conn[i] = 0;
      load_bs[i]    = 0;
   }
   for(i=0;i<eu;i++)
      usedEU[i] = false;

   while(flag){
      i=0;
```

```
      flag = false;
      while(i<eu){
         if(legal_conn(P_short_end_base[i], usedBS, distBB, load_bs, final_conn, eu_se
t, bs, bs_set) && !usedEU[i]){
            fprintf(fp1, "%8d %8d %8f\n", P_short_end_base[i]->id+1, P_short_end_base[
i]->endId+1, P_short_end_base[i]->dist);
            usedEU[i] = true;
            load_bs[P_short_end_base[i]->id] += eu_set[P_short_end_base[i]->endId].dem
and;
            final_conn[ P_short_end_base[i]->id] = P_short_end_base[i]->dist;
            flag = true;
         }
         i++;
      }
   }
   fclose(fp1);
   int *sol    = indlaesSOL(&no_sol);                 // start på evaluering
   int **conn  = indlaesCONN(&no_conn);
   defDirect(eu_set, bs_set, conn, dist, no_conn);
   double **coi_table   = coi(dist, conn, eu_set, bs_set, sol, no_sol, no_conn);
   sortout(no_conn, coi_table);
   for (i=0;i<no_conn;i++){
      if (coi_table[i][1]<LIM)
         count ++;
   }
   cout << "Antal end_user med C/I < " << LIM << "  " << count << endl;
   cout << "Andel af end_user med C/I <" << LIM << "  " << (double)count/(double)no_co
nn << endl;
   udskrivcoi(no_conn, coi_table);                        // slut på evaluering

   for (i=0;i<eu;i++){
      all_load += eu_set[i].demand;
      if(usedEU[i]){
         p++;
         sum_load += eu_set[i].demand;
      }
   }
   FILE *fp4;
   fp4 = fopen(result, "a");
   fprintf(fp4, "**********************************************************************
******************\n\n\n");
   fprintf(fp4, "%-38s %-8d \n","Out_loop",  out_loop);
   fprintf(fp4, "%-38s %-8d \n","Global iteration",  GLOBAL_ITER);
   fprintf(fp4, "%-38s %-8d \n","Lokal iteration",  LOCAL_ITER);
   fprintf(fp4, "%-38s %-8.2f \n","T_start",  T_START);
   fprintf(fp4, "%-38s %-8.2f \n","Cool rate",  COOL_RATE);
   fprintf(fp4, "%-38s %-8.2f \n","Tapering constant",  TAPERING_CONST);
   fprintf(fp4, "%-38s %-8.2f \n","Overlap constant",  OVERLAP_CONST);
   fprintf(fp4, "%-38s %-8d \n","Validate by density",  VAL_BY_DENSITY);
   fprintf(fp4, "%-38s %-8d \n","Validate by demand",  VAL_BY_DEMAND);
   fprintf(fp4, "%-38s %-8d \n","Val. obj by demand",  OBJ_BY_DEMAND);
   fprintf(fp4, "%-38s %-8d \n","Number of base stations",  NUMBER_OF_BS);
   fprintf(fp4, "%-38s %-8d \n","Seed",  seed);
   fprintf(fp4, "%-38s %-8d \n","Number of not connected end_user",  (eu-p));
   fprintf(fp4, "%-38s %-8.2f \n","Number of End-user hit rate",  ((double)p/(double)e
u));
   fprintf(fp4, "%-38s %-8.2f \n","End-user demand hit rate",  (sum_load/all_load));
   fprintf(fp4, "%-38s %-8.2f \n","Best objective value", global_obj);
   fprintf(fp4, "%-30s %d %-4s %-8.2d \n","Number of end_users with C/I <", LIM, "  ",
 count);
   fprintf(fp4, "%-29s %d %-5s %-8.2f \n","Share of end_users with C/I <", LIM, "  ",
(double)count/(double)no_conn);
   fprintf(fp4, "%-38s %-8d \n\n","Total number of not connected end_user",  (eu-p)+ c
ount);
   for(i=0;i<ini;i++)
      fprintf(fp4, "%-15s %-15d \n","Fixed base station", int_solution[i]);
```

```
   fprintf(fp4, "\n %-15s %-15s %-15s","Base station", "Load", "x_coordinate");
   fprintf(fp4, "%-15s %-15s \n","y_coordinate", "Radius");

   for(i=0;i<bs;i++)
      if(usedBS[i]){
         fprintf(fp4, "%-15d %-15.0f %-15.3f", i, load_bs[i], bs_set[i].x);
         fprintf(fp4, "%-15.3f %-15.3f \n",bs_set[i].y, final_conn[i]);
      }
   fprintf(fp4, "\n\n");
   fclose(fp4);
   delete [] sol;
   for (i=0;i<no_conn;i++){
      delete coi_table[i];
      delete conn[i];
   }
   delete [] coi_table;
   delete [] conn;

}
////////////////////////////////////// new2global  ////////////////////////////////
///

void new2global(bool *usedBS, bool *global_sol, int bs, double *obj, double *global_ob
j){
  int i;
  for (i=0;i<bs;i++){
    global_sol[i] = usedBS[i];
//    if(usedBS[i])
//       cout << i << "  ";
  }
//  cout << endl;
  *global_obj = *obj;
}

//////////////////////////////////////  new2prev  ////////////////////////////////
///

void new2prev(double *obj, double *prev_obj){
  *prev_obj = *obj;
}

//////////////////////////////////  find_new_sol  ////////////////////////////////
///

void find_new_sol(bool *usedBS, int bs, int *add, int *remove, bool *bin_solution){
   double p;
   p = ran();
   if(p<=(1-q_p)){
      *add = ((int)(ran()*bs));
      while(usedBS[*add])
         *add = ((int)(ran()*bs));
      usedBS[*add] = true;
   }
   else{
      *add =-1;
      cout << "no add " << endl;
   }
   if(p>=(q_p/2)){
      *remove = ((int)(ran()*bs));
      if(RESTART_SOL){
         while(!usedBS[*remove])
            *remove = ((int)(ran()*bs));
      }
      else{
         while((!usedBS[*remove])||(bin_solution[*remove]))
            *remove = ((int)(ran()*bs));
```

```
      }
      usedBS[*remove] = false;
   }
   else{
      *remove = -1;
      cout << "no remove " << endl;
   }
}

//////////////////////////////////////  restore_prev  ////////////////////////////
///

void restore_prev(bool *usedBS, int *add, int *remove, double *obj, double *prev_obj){
   if(*add > -1)
      usedBS[*add] = false;
   if(*remove > -1)
      usedBS[*remove] = true;
  *obj = *prev_obj;
}

//////////////////////////////////////  p_screen  ////////////////////////////////
//

void p_screen(bool *global_sol, bool *usedEU, double *load_bs, struct bs_set_t *bs_set
, int bs, int eu, double *final_conn){
  int i, user = 0;
  cout << setw(5) << "bs"
       << setw(12) << "load_bs[i]"
       << setw(12) << "bs_set[i].x"
       << setw(12) << "bs_set[i].y"
       << setw(14) << "final_conn[i]"
       << endl;

  for (i=0;i<bs;i++)
    if(global_sol[i])
      cout << setw(5) << i
      << setw(12) << load_bs[i]
      << setw(12) << bs_set[i].x
      << setw(12) << bs_set[i].y
      << setw(12) << final_conn[i]
      << endl;

  for (i=0;i<eu;i++)
    if(!usedEU[i])
      user++;
  cout << setw(10) << user << " ikke forbundende brugere " << endl;
}


//////////////////////////////////////  Main  ////////////////////////////////
//

int main(){

  int i, global_count, local_count, accept, avr =0,t_count = 0, new_seed;
  int eu,bs, add, remove,out_loop;
  double obj = 0, global_obj = 1000000, prev_obj = 1000000, temperature;
  //bool flag;
  eu = counter(enduser);
  bs = counter(basestation);
  bool *usedEU       = new bool[eu];
  bool *usedBS       = new bool[bs];
  bool *global_sol   = new bool[bs];
  bool *bin_solution = new bool[bs];
  int *swap          = new int[bs];
  for (i=0;i<eu;i++){
```

```
      usedEU[i] = false;
      if(i<bs){
        usedBS[i]        = false;
        global_sol[i]    = false;
        bin_solution[i]  = false;
        swap[i]      = 0;
      }
    }
    for (i=0;i<ini;i++)
      bin_solution[int_solution[i]] = true;
    FILE *fp2;

    FILE *fp;
    double  *final_conn = new double[bs];
    double  *load_bs = new double[bs];
    for (i=0;i<bs;i++){
      load_bs[i] = 0;
      final_conn[i] = 0;
    }

    interpolate();
    struct eu_set_t *eu_set = indlaesEU(eu);
    struct bs_set_t *bs_set = indlaesBS(bs);
    sortBasestation_t **P_short_end_base = new sortBasestation_t *[eu];
    double  **dist = defDist(eu_set, bs_set, eu, bs);
    double  **distBB = defDistBB(bs_set, bs);
    sortBasestation_t **near_eu = sortBasestation(bs, eu, dist, bs_set);
    alter_dist(bs_set, bs);
    if(VAL_BY_DENSITY)
      defDensity(eu, eu_set);
    for(NUMBER_OF_BS = min_bs;NUMBER_OF_BS<=max_bs;NUMBER_OF_BS++){
    new_seed = seed;
    for(out_loop=0;out_loop<out_cnt;out_loop++){
      fp = fopen(output, "w");
      new_seed += (137*out_loop);
      srand(new_seed);
      for (i=0;i<bs;i++)
        usedBS[i]=false;
      new_startsol(usedBS, bs);
      find_nearest(P_short_end_base, near_eu, usedBS, eu);
      sort_end_base(P_short_end_base, eu);
      obj = conn_end(P_short_end_base, eu_set, distBB, final_conn, load_bs, usedBS, used
EU, eu, bs, near_eu, bs_set);
      new2global(usedBS, global_sol, bs, &obj, &global_obj);
      new2prev( &obj, &prev_obj);
      temperature = T_START;
      local_count = 0;
      global_count = 0;
      accept = 0;
      //flag = true;
      cout << setw(15) << "global_count"
      << setw(12) << "accept"
      << setw(12) << "global_obj"
      << setw(12) << "obj"
      << endl;
     while (global_count < GLOBAL_ITER){
        while ( (local_count < LOCAL_ITER)&&(accept < ACCEPTRATE)){
           t_count++;
           find_new_sol(usedBS, bs, &add, &remove, bin_solution);
// swap[add] +=1;
// swap[remove] +=1;
           obj = conn_end(P_short_end_base, eu_set, distBB, final_conn, load_bs, usedBS,
 usedEU, eu, bs, near_eu, bs_set);
           if(obj <= prev_obj){
              new2prev(&obj, &prev_obj);
              accept++;
```

```
            }
            else if(exp((prev_obj - obj)/temperature) > ran()){
               new2prev(&obj, &prev_obj);
               accept++;
            }
            else
               restore_prev(usedBS, &add, &remove, &obj, &prev_obj);

            if(obj < global_obj){
               new2global(usedBS, global_sol, bs, &obj, &global_obj);
               cout << " ***** New global_obj ***** "
                  << global_obj
                  << "   at count "
                  << t_count
                  << endl;
            }
            local_count++;
            fprintf(fp, "%10f %10f %10f\n", obj, global_obj, temperature);
         }
         global_count++;
         temperature *= COOL_RATE;
         avr += accept;
         local_count = 0;
         cout << setw(15) << global_count
            << setw(12) << accept
            << setw(12) << global_obj
            << setw(12) << obj
            << endl;
         accept = 0;
      }
      fclose(fp);
      cout << "average accept " << (avr/GLOBAL_ITER) << endl;
      fp2 = fopen(solution, "w");
      for (i=0;i<bs;i++)
         if(global_sol[i]){
            fprintf(fp2, "%d\n", (i+1) );
         }
         fclose(fp2);
         udskriv(dist, global_sol, usedEU, final_conn, near_eu, bs, eu, eu_set, P_short_e
nd_base, load_bs, distBB, bs_set, global_obj,out_loop);
         p_screen(global_sol, usedEU, load_bs, bs_set, bs, eu, final_conn);
         t_count = 0;
      }  // end outerloop
    }  //end base station for loop
    system("pause");
}
```