OMNIBUS TEST FOR CHANGE DETECTION IN A TIME SEQUENCE OF POLARIMETRIC SAR DATA

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ABSTRACT

Based on an omnibus likelihood ratio test statistic for the equality of several variance-covariance matrices following the complex Wishart distribution with an associated p-value and a factorization of this test statistic, change analysis in a (short) time series of multilook, polarimetric SAR data in the covariance matrix representation is carried out. The omnibus test statistic and its factorization detect if and when change(s) occur. The technique is demonstrated on airborne EMISAR C-band data but may be applied to ALOS, COSMO-SkyMed, RadarSat-2, Sentinel-1, TerraSAR-X, and Yoagan or other dual- and quad/full-pol data also.

1. INTRODUCTION

In earlier publications we have described a test statistic for the equality of two variance-covariance matrices following the complex Wishart distribution with an associated p-value [1]. We showed their application to bitemporal change detection and to edge detection [2] in multilook, polarimetric synthetic aperture radar (SAR) data in the covariance matrix representation. The test statistic and the associated p-value is described in [3] also. In [4] we focused on the block-diagonal case, we elaborated on some computer implementation issues, and we gave examples on the application to change detection in both full and dual polarization bitemporal, bifrequency, multilook SAR data.

In [5] we described an omnibus test statistic Q for the equality of $k \geq 2$ variance-covariance matrices following the complex Wishart distribution. We also described a factorization of $Q = \prod_{j=2}^{k} R_j$ where Q and R_j determine if and when a difference occurs. Additionally, we gave p-values for Q and R_j. Finally, we demonstrated the use of Q, R_j and the p-values to change detection in truly multitemporal, full polarization SAR data. For more references to change detection in polarimetric SAR data, see [5].

In [5] we applied the methods to a series of EMISAR [8,9] L-band data. In this paper we apply the methods to EMISAR C-band data. The methods may be applied to other polarimetric SAR data also such as data from ALOS, COSMO-SkyMed, RadarSat-2, Sentinel-1, TerraSAR-X, and Yoagan.

2. TEST STATISTICS AND THEIR DISTRIBUTIONS

This section gives the main results from [5]. The average covariance matrix for multilook polarimetric SAR is defined as [6]

$$\langle \boldsymbol{C} \rangle = \begin{bmatrix} \langle S_{hh}S_{hh}^* \rangle & \langle S_{hh}S_{hv}^* \rangle & \langle S_{hh}S_{vv}^* \rangle \\ \langle S_{hv}S_{hh}^* \rangle & \langle S_{hv}S_{hv}^* \rangle & \langle S_{hv}S_{vv}^* \rangle \\ \langle S_{vv}S_{hh}^* \rangle & \langle S_{vv}S_{hv}^* \rangle & \langle S_{vv}S_{vv}^* \rangle \end{bmatrix} \quad (1)$$

where $\langle \cdot \rangle$ denotes ensemble averaging and * denotes complex conjugation. S_{rt} denotes the complex scattering amplitude for receive and transmit polarization ($r, t \in \{h, v\}$ for horizontal and vertical polarization).

2.1. Test for equality of several complex covariance matrices

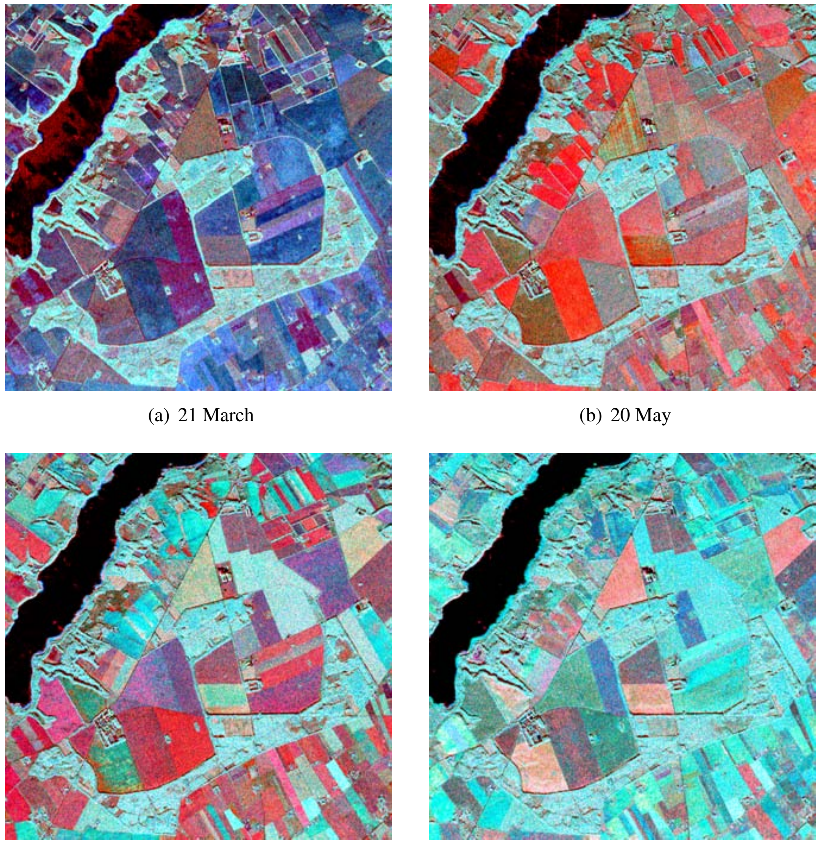
To test whether a series of $k \geq 2$ complex variance-covariance matrices $\boldsymbol{\Sigma}_i$ are equal, i.e., to test the null hypothesis

$$H_0: \ \boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_2 = \cdots = \boldsymbol{\Sigma}_k$$

against all alternatives, we use the following omnibus test statistic (for the real case see [7]; for the case with two complex matrices see [1,2]; $|\cdot|$ denotes the determinant)

$$Q = \left\{ k^{pk} \frac{\prod_{i=1}^{k} |\boldsymbol{X}_i|}{|\boldsymbol{X}|^k} \right\}^n. \quad (2)$$

Here the $\boldsymbol{\Sigma}_i$ (and the $\boldsymbol{X}_i$) are p by p ($p = 3$ for full pol data, $p = 2$ for dual pol data, and $p = 1$ for single channel power data), and the $\boldsymbol{X}_i = n\hat{\boldsymbol{\Sigma}}_i = n\langle \boldsymbol{C} \rangle_i$ follow the complex Wishart distribution, i.e., $\boldsymbol{X}_i \sim W_C(p, n, \boldsymbol{\Sigma}_i)$. n is the equivalent number of looks. Further, $\boldsymbol{X} = \sum_{i=1}^{k} \boldsymbol{X}_i \sim$



(a) 21 March (b) 20 May

(c) 16 June (d) 15 July

Fig. 1. C-band EMISAR data in Pauli representation; same stretching applied to all four images.

Table 1. Part of the change analysis structure for an example with data from four time points.

	$t_1 = \cdots = t_4$	$t_2 = \cdots = t_4$	$t_3 = t_4$
Omnibus	$Q^{(1)}$: $P\{Q^{(1)} < q^{(1)}_{\text{obs}}\}$	$Q^{(2)}$: $P\{Q^{(2)} < q^{(2)}_{\text{obs}}\}$	$Q^{(3)}$: $P\{Q^{(3)} < q^{(3)}_{\text{obs}}\}$
$t_1 = t_2$	$R^{(1)}_2$: $P\{R^{(1)}_2 < z^{(1)}_{2,\text{obs}}\}$		
$t_2 = t_3$	$R^{(1)}_3$: $P\{R^{(1)}_3 < z^{(1)}_{3,\text{obs}}\}$	$R^{(2)}_2$: $P\{R^{(2)}_2 < z^{(2)}_{2,\text{obs}}\}$	
$t_3 = t_4$	$R^{(1)}_4$: $P\{R^{(1)}_4 < z^{(1)}_{4,\text{obs}}\}$	$R^{(2)}_3$: $P\{R^{(2)}_3 < z^{(2)}_{3,\text{obs}}\}$	$R^{(3)}_2$: $P\{R^{(3)}_2 < z^{(3)}_{2,\text{obs}}\}$

$W_C(p, nk, \boldsymbol{\Sigma})$. If the hypothesis is true ("under H_0" in statistical parlance), $\hat{\boldsymbol{\Sigma}} = \boldsymbol{X}/(kn)$. $Q \in [0,1]$ with $Q = 1$ for equality.

For the logarithm of the test statistic we get

$$\ln Q = n\left\{pk\ln k + \sum_{i=1}^{k}\ln|\boldsymbol{X}_i| - k\ln|\boldsymbol{X}|\right\}. \quad (3)$$

Setting

$$\begin{aligned} f &= (k-1)p^2 \\ \rho &= 1 - \frac{(2p^2-1)}{6(k-1)p}\left(\frac{k}{n} - \frac{1}{nk}\right) \\ \omega_2 &= \frac{p^2(p^2-1)}{24\rho^2}\left(\frac{k}{n^2} - \frac{1}{(nk)^2}\right) - \frac{p^2(k-1)}{4}\left(1 - \frac{1}{\rho}\right)^2 \end{aligned}$$

the probability of finding a smaller value of $-2\rho\ln Q$ is ($z = -2\rho\ln q_{\rm obs}$)

$$\begin{aligned} P\{-2\rho\ln Q \le z\} \;\simeq\; & P\{\chi^2(f) \le z\} \quad (4) \\ & + \omega_2[P\{\chi^2(f+4) \le z\} - P\{\chi^2(f) \le z\}]. \end{aligned}$$

$P\{-2\rho\ln Q \le -2\rho\ln q_{\rm obs}\} = P\{Q \ge q_{\rm obs}\}$ is the change probability, $1 - P\{-2\rho\ln Q \le -2\rho\ln q_{\rm obs}\} = P\{Q < q_{\rm obs}\}$ is the no-change probability.

2.2. Test for equality of first $j \le k$ complex covariance matrices

If the above test shows that we cannot reject the hypothesis of equality, no change has occurred over the time span covered by the data. If we can reject the hypothesis, change has occurred at some time point. To test whether the first j complex variance-covariance matrices $\boldsymbol{\Sigma}_i$ are equal, i.e., given that

$$\boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_2 = \cdots = \boldsymbol{\Sigma}_{j-1}$$

then the likelihood ratio test statistic R_j for testing the hypothesis

$$H_{0,j} : \boldsymbol{\Sigma}_j = \boldsymbol{\Sigma}_1 \;\text{ against }\; H_{1,j} : \boldsymbol{\Sigma}_j \ne \boldsymbol{\Sigma}_1$$

is

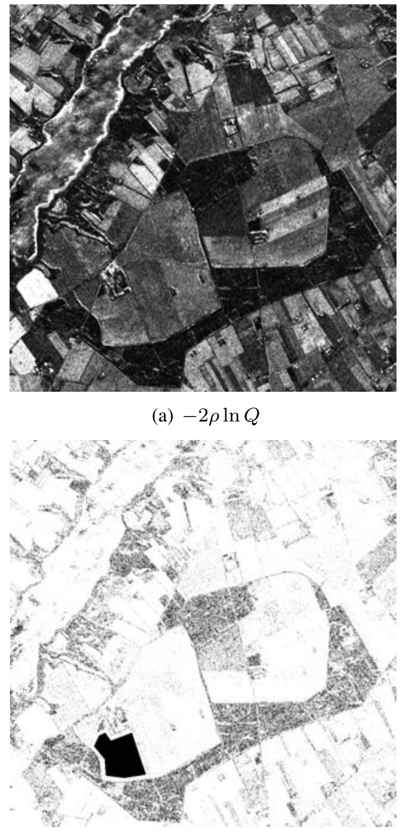
$$R_j = \left\{\frac{j^{jp}}{(j-1)^{(j-1)p}}\frac{|\boldsymbol{X}_1 + \cdots + \boldsymbol{X}_{j-1}|^{(j-1)}|\boldsymbol{X}_j|}{|\boldsymbol{X}_1 + \cdots + \boldsymbol{X}_j|^j}\right\}^n$$

or

$$\begin{aligned} \ln R_j = \; & n\{p(j\ln j - (j-1)\ln(j-1)) \\ & + (j-1)\ln|\sum_{i=1}^{j-1}\boldsymbol{X}_i| + \ln|\boldsymbol{X}_j| - j\ln|\sum_{i=1}^{j}\boldsymbol{X}_i|\}. \end{aligned}$$

Furthermore, the R_j constitute a factorization of Q

$$Q = \prod_{j=2}^{k} R_j$$



(a) $-2\rho\ln Q$

(b) p-value

Fig. 2. Test statistic (a) and p-value with grass field marked as black (b). Dark areas are no-change. p is approximately 1 in the grass field.

or $\ln Q = \sum_{j=2}^{k}\ln R_j$. If H_0 is true the R_j are independent. Finally, letting

$$\begin{aligned} f &= p^2 \\ \rho_j &= 1 - \frac{2p^2-1}{6pn}\left(1 + \frac{1}{j(j-1)}\right) \\ \omega_{2j} &= -\frac{p^2}{4}\left(1 - \frac{1}{\rho_j}\right)^2 \\ & \quad + \frac{1}{24n^2}p^2(p^2-1)\left(1 + \frac{2j-1}{j^2(j-1)^2}\right)\frac{1}{\rho_j^2} \end{aligned}$$

we get ($z_j = -2\rho_j\ln r_{j,\rm obs}$)

$$\begin{aligned} P\{-2\rho_j\ln R_j \le z_j\} \;\simeq\; & P\{\chi^2(f) \le z_j\} \\ & + \omega_{2j}[P\{\chi^2(f+4) \le z_j\} - P\{\chi^2(f) \le z_j\}]. \end{aligned}$$

3. CHANGE VISUALIZATION EXAMPLES

To illustrate the above we use full polarimetry EMISAR [8,9] C-band data acquired in 1998 over a Danish agricultural test

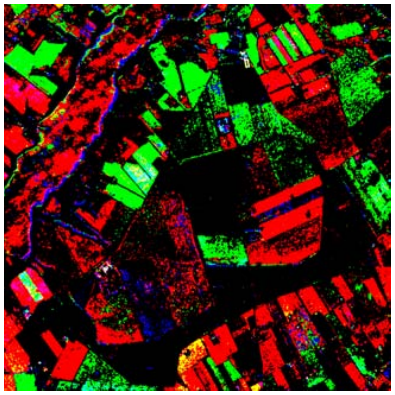


Fig. 3. Shows changes from t_1 to t_2 as blue, from t_2 to t_3 as green, from t_3 to t_4 as red (after application of a 3 by 3 mode filter); change probability significance level is 99.99%.

site on $t_1 = 21$ March, $t_2 = 20$ May, $t_3 = 16$ June and $t_4 = 15$ July. Figure 1 shows the diagonal elements of the covariance matrix in the Pauli representation where red shows single- or odd-bounce scattering, green shows volume scattering, and blue shows double or even-bounce scattering.

Table 1 shows (some of) the change structure built (for each pixel) for an example with data from four time points. The first column indicates which tests are performed for the row in question. The second column shows $Q^{(1)}$ and $P\{Q^{(1)} < q_{\text{obs}}\}$ ("Omnibus" row), or $R_j^{(1)}$ and $P\{R_j^{(1)} < r_{j,\text{obs}}\}$, $j = 2, \ldots, 4$ for all time points t_1 through t_4. The third column shows $Q^{(2)}$ and $P\{Q^{(2)} < q_{\text{obs}}\}$ ("Omnibus" row), or $R_j^{(2)}$ and $P\{R_j^{(2)} < r_{j,\text{obs}}\}$, $j = 2, 3$ for time points t_2 through t_4. The fourth column shows $Q^{(3)}$ and $P\{Q^{(3)} < q_{\text{obs}}\}$ ("Omnibus" row), or $R_j^{(3)}$ and $P\{R_2^{(3)} \leq r_{2,\text{obs}}\}$ for time points t_3 to t_4. Remember, that for a test for $R_j^{(\ell)}$ to be valid, all previous tests for $R_i^{(\ell)}$, $i = 2, \ldots, j-1$ must show equality, see hypothesis $H_{0,j}$ in Section 2.2.

Note, that $R_2^{(\ell)}$ are the (marginal, non-omnibus) pairwise tests for equality.

3.1. Per pixel change visualization

As examples of per pixel change visualization, Figure 2 shows the quantity $-2\rho \ln Q$ and the corresponding p-value, i.e., the change probability. Figure 3 shows changes from t_1 to t_2 as blue, from t_2 to t_3 as green, and from t_3 to t_4 as red after applying a 3 by 3 mode filter. Black areas have not changed.

3.2. Per field change visualization

Table 2 shows the average no-change probabilities for the grass field shown in Figure 2. Table 2 shows that the pairwise

Table 2. Average no-change probabilities for the grass field.

	$t_1 = \cdots = t_4$	$t_2 = \cdots = t_4$	$t_3 = t_4$
Omnibus	0.0049	0.0076	0.1213
$t_1 = t_2$	0.2883		
$t_2 = t_3$	0.1372	0.1500	
$t_3 = t_4$	0.0248	0.0378	0.1213

tests reveal no change over time for the grass field (p-values are 0.2883, 0.1500, and 0.1213, respectively). The omnibus test statistic Q indicates change at some time point between 21 March and 15 July ($P\{Q^{(1)} < q_{\text{obs}}^{(1)}\} = 0.0049$), and the R_j show that the change for this field occurs between 16 June and 15 July ($P\{R_4^{(1)} \leq r_{4,\text{obs}}^{(1)}\} = 0.0248$).

4. REFERENCES

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