CHANGE DETECTION IN A SHORT TIME SEQUENCE OF POLARIMETRIC C-BAND SAR DATA

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ABSTRACT

Based on an omnibus likelihood ratio test statistic for the equality of several variance-covariance matrices following the complex Wishart distribution and a factorization of this test statistic with associated \(p\)-values, change analysis in a time series of multilook, polarimetric SAR data in the covariance matrix representation is carried out. The omnibus test statistic and its factorization detect if and when change(s) occur. The technique is demonstrated on airborne EMISAR C-band data but may be applied to ALOS, COSMO-SkyMed, RadarSat-2 Sentinel-1, TerraSAR-X, and Yaogan data also.

2. TEST STATISTICS AND THEIR DISTRIBUTIONS

This section gives the main results from [5]. The average covariance matrix for multilook polarimetric SAR is defined as [8]

\[
\langle C \rangle = \begin{pmatrix}
\langle S_{hh}S_{hh}^* \rangle & \langle S_{hh}S_{hv}^* \rangle & \langle S_{hh}S_{vv}^* \rangle \\
\langle S_{hv}S_{hh}^* \rangle & \langle S_{hv}S_{hv}^* \rangle & \langle S_{hv}S_{vv}^* \rangle \\
\langle S_{vv}S_{hh}^* \rangle & \langle S_{vv}S_{hv}^* \rangle & \langle S_{vv}S_{vv}^* \rangle
\end{pmatrix}
\]

(1)

where \(\langle \cdot \rangle\) denotes ensemble averaging and \(^*\) denotes complex conjugation. \(S_{rt}\) denotes the complex scattering amplitude for receive and transmit polarization \((r, t \in \{h, v\})\) for horizontal and vertical polarization.

2.1. Test for equality of several complex covariance matrices

To test whether a series of \(k \geq 2\) complex variance-covariance matrices \(\Sigma_i\) are equal, i.e., to test the null hypothesis \(H_0\)

\[H_0: \Sigma_1 = \Sigma_2 = \cdots = \Sigma_k\]

against all alternatives, we use the following omnibus test statistic (for the real case see [9]; for the case with two complex matrices see [1, 2]; \(|\cdot|\) denotes the determinant)

\[Q = \left\{ k^{p/2} \prod_{i=1}^{k} |X_i|/|X|^k \right\}^n. \]

(2)

Here the \(\Sigma_i\) (and the \(X_i\)) are \(p \times p\) for \(p = 3\) for dual channel power data, and \(X_i = n \Sigma_i = n \langle C \rangle\), follow the complex Wishart distribution, i.e., \(X_i \sim W_C(p, n, \Sigma_i)\). Further, \(X = \sum_{i=1}^{k} X_i \sim W_C(p, nk, \Sigma)\). If the hypothesis is true ("under
For the logarithm of the test statistic we get

\[ Q = \ln Q_{\text{obs}} - \ln Q = n \left( pk \ln k + \sum_{i=1}^{k} \ln |X_i| - k \ln |X| \right). \]  \hspace{1cm} (3)

Setting

\[
\begin{align*}
    f &= (k-1)p^2 \\
    \rho &= 1 - \frac{(2p^2-1)}{6(k-1)p} \left( \frac{k}{n} - \frac{1}{nk} \right) \\
    \omega_2 &= \frac{p^2(p^2-1)}{24\rho^2} \left( \frac{k}{n} - \frac{1}{(nk)^2} \right) - \frac{p^2(k-1)}{4} \left( 1 - \frac{1}{\rho} \right)^2
\end{align*}
\]

the probability of finding a smaller value of \(-2\rho \ln Q\) is \((z = -2\rho \ln q_{\text{obs}})\), where \(q_{\text{obs}}\) is the observed value of \(Q\)

\[ P\{-2\rho \ln Q \leq z\} \approx P\{\chi^2(f) \leq z\} \]

and

\[ P\{-2\rho \ln Q \leq -2\rho \ln q_{\text{obs}}\} = P\{Q \geq q_{\text{obs}}\} \]

is the change probability. Hence, \(-P\{-2\rho \ln Q \leq -2\rho \ln q_{\text{obs}}\} = P\{Q < q_{\text{obs}}\}\) is the no-change probability.

### 2.2. Test for equality of first \(j \leq k\) complex covariance matrices

If the above test shows that we cannot reject the hypothesis of equality, no change has occurred over the time span covered by the data. If we can reject the hypothesis, change has occurred at some time point. To test whether the first \(j\) complex variance-covariance matrices \(\Sigma_i\) are equal, i.e., given that

\[ \Sigma_1 = \Sigma_2 = \cdots = \Sigma_{j-1} \]

then the likelihood ratio test statistic \(R_j\) for testing the hypothesis

\(H_{0,j} : \Sigma_j = \Sigma_1\) against \(H_{1,j} : \Sigma_j \neq \Sigma_1\)

is

\[
R_j = \left\{ \frac{j^p}{(j-1)^{j-1}} \frac{|X_1 + \cdots + X_{j-1}|^{|j-1|} |X_j|}{|X_1 + \cdots + X_j|^p} \right\}^n
\]

or

\[
\ln R_j = n \left( p(j \ln j - (j-1) \ln (j-1)) \right) + (j-1) \ln |\sum_{i=1}^{j-1} X_i| + \ln |X_j| - j \ln |\sum_{i=1}^{j} X_i|.
\]

Furthermore, the \(R_j\) constitute a factorization of \(Q\)

\[ Q = \prod_{j=2}^{k} R_j \]

or \(Q = \sum_{j=2}^{k} \ln R_j\). If \(H_0\) is true the \(R_j\) are independent. Finally, letting

\[
\begin{align*}
    f &= p^2 \\
    \rho_j &= 1 - \frac{2p^2-1}{6pn} \left( 1 + \frac{1}{j(j-1)} \right) \\
    \omega_{2j} &= \frac{p^2}{4} \left( 1 - \frac{1}{\rho_j} \right)^2
\end{align*}
\]

we get (\(z_j = -2\rho_j \ln r_{j,\text{obs}}\), where \(r_{j,\text{obs}}\) is the observed value of \(R_j\))

\[ P\{-2\rho_j \ln R_j \leq z_j\} \approx P\{\chi^2(f) \leq z_j\} \]

and

\[ + \omega_{2j}[P\{\chi^2(f+4) \leq z_j\} - P\{\chi^2(f) \leq z_j\}].\]
Fig. 1. C-band EMISAR data in Pauli representation; same stretching applied to all four images.

Fig. 2. C-band EMISAR data in Pauli representation; same stretching applied to all four images.
Fig. 3. Test statistic (a) and $p$-value with grass field marked as black (b); $p$ is approximately 1 in the grass field; in both (a) and (b) dark areas are no-change.

Fig. 4. Shows changes from $t_1$ to $t_2$ as blue, from $t_2$ to $t_3$ as green, from $t_3$ to $t_4$ as red; change probability significance level is 99.99%.

Fig. 5. For the grass field this figure shows average omnibus test based no-change probabilities tabulated in Table 2.
3. CHANGE VISUALIZATION EXAMPLES

To illustrate the above we use full polarimetry EMISAR [6,7] C-band data acquired in 1998 over a Danish agricultural test site on 21 March ($t_1$), 20 May ($t_2$), 16 June ($t_3$) and 15 July ($t_4$). Figures 1 and 2 show the diagonal elements of the covariance matrix in the Pauli representation where red shows single- or odd-bounce scattering, green shows volume scattering, and blue shows double or even-bounce scattering.

Table 1 shows the change structure built (for each pixel) for an example with data from four time points. The first column indicates which tests are performed for the row in question. The second column shows $Q^{(1)}$ and $P\{Q^{(1)} < q_{obs}\}$ (“Omnibus” row), or $R_j^{(1)}$ and $P\{R_j^{(1)} < r_{j,obs}\}$, $j = 2, \ldots, 4$ for all time points $t_1$ through $t_4$. The third column shows $Q^{(2)}$ and $P\{Q^{(2)} < q_{obs}\}$ (“Omnibus” row), or $R_j^{(2)}$ and $P\{R_j^{(2)} < r_{j,obs}\}$, $j = 2, 3$ for time points $t_2$ through $t_4$. The fourth column shows $Q^{(3)}$ and $P\{Q^{(3)} < q_{obs}\}$ (“Omnibus” row), or $R_j^{(3)}$ and $P\{R_j^{(3)} \leq r_{2,obs}\}$ for time points $t_3$ to $t_4$. Remember, that for a test for $R_j^{(ℓ)}$ to be valid, all previous tests for $R_i^{(ℓ)}$, $i = 2, \ldots, j - 1$ must show equality, see hypothesis $H_{0,j}$ in Section 2.2.

Note, that $R_j^{(ℓ)}$ are the (marginal, non-omnibus) pairwise tests for equality.

3.1. Per pixel change visualization

Figure 3 shows the quantity $-2p\ln Q$ and the corresponding $p$-value, i.e., the change probability.

As an example of per pixel change visualization Figure 4 shows changes from $t_1$ to $t_2$ as blue, from $t_2$ to $t_3$ as green, and from $t_3$ to $t_4$ as red. Black areas have not changed.

3.2. Per field change visualization

Table 2 shows the average no-change probabilities for the grass field shown in Figure 3. Figure 5 shows the same probabilities as graphs, one line per column in Table 2.

As an example of visualization of per field change detection Figure 6 shows mean values over the grass field of the backscatter coefficients and the span (first plot), the determinant of the covariance matrix (second plot), the magnitude of the correlation between HH and VV (third plot), and the phase difference between HH and VV for all four acquisitions (fourth plot). The last plot shows the average pairwise and omnibus based no-change probabilities.

These plots are meant to assist the analyst in determining what caused a possible change, for example an increase in...
backscatter as opposed to a decrease in phase angle. We expect that plots like these will be even more useful for interpreting change in longer time series covering for example several years where possible annual oscillations will be important and potentially conspicuous.

Both Table 2, Figures 5 and 6 show that the pairwise tests show no change over time for the grass field. The omnibus test shows change \( P(Q^{(1)} < q^{(1)}_{\text{obs}}) = 0.0049 \) and the change occurs between June and July \( P(r^{(4)}_{4,\text{obs}} \leq r^{(1)}_{4,\text{obs}}) = 0.0248 \).

4. SOFTWARE

We plan to publish Matlab code to perform the analysis described. This will include updating of the change analysis with new data and generation of some of the tables and figures shown above. Also Python code will be made available.

5. CONCLUSIONS

The new omnibus test statistic and its factorization with their \( p \)-values show if and when change(s) occur in a time series of multilook, polarimetric SAR data. Contrary to consecutive pairwise comparisons the omnibus test statistic in the example given shows change in C-band SAR data between June and July acquisitions for a grass field.

This type of analysis will become increasingly important and interesting as the databases with relevant SAR data collected routinely and globally keep growing.

6. REFERENCES


