ANALYSIS OF THE NON LINEAR DYNAMICS OF A 2–AXLE FREIGHT WAGON IN CURVES

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Abstract

The paper analyses the nonlinear dynamics in curves of 2–axle freight wagons. Both one isolated wagon or an assembly of three wagons are considered. The dependence on both curve radius and vehicle speed is pointed out and it is shown that, considering a single 2–axle freight car, the carbody experiences periodic oscillations with large amplitude (up to 12 mm) at relatively high speed values, which still lie in the operating speed range. The interaction between adjacent vehicles is also investigated, showing that the forces exchanged trough the coupling elements significantly affect the dynamics of the 2–axle freight wagons.

1 INTRODUCTION

Railway represents nowadays the most efficient transport mean in terms of energy usage and conveyance of goods, however, the increased competition from air travel and trucks continuously requires to increase the capacity on modern freight wagons. In order to reach this objective two options are available, increasing the total hauled mass or raising the speed of the wagons. The first option would require a radical redesign of the infrastructure to withstand the increase of the maximum axle–load, therefore, it is necessary to explore the possibility of increasing the maximum speed allowed for freight wagons. Hence a deep understanding of their dynamical behaviour in both straight and curved track is required. The 2–axle freight wagons represent highly nonlinear systems due to their cheap construction. Usually energy dissipation in the suspensions is obtained by means of dry friction elements which also makes these systems non–smooth, due to stick/slip transitions. From a scientific point of view the analysis of the running properties of these wagons represents a challenge since the modelling of non–smooth systems is not consolidated in the railway community. In recent years the dynamics of 2–axle freight wagons in straight track has been extensively analysed in [1, 2], whereas less attention has been paid to the curving behaviour of the same vehicles. Moreover, considering a complete trainset, it has been demonstrated [3] that forces generated in the coupling elements have a strong influence on lateral dynamics of freight wagons, increasing the possibility of derailment. Aim of this work is to study the nonlinear dynamics in curves of single 2–axle freight cars , considering both the use of one isolated wagon and of an assembly of wagons exchanging the forces exchanged through the buffers and the draw gear. The innovative aspects of this paper consist of the nonlinear analysis of the running dynamics of the vehicles in curves, which was to a large extent left unexplored by previous research studies, and of analysing effect of the interacting forces due to the coupling elements when the case of more than one wagon is considered.

The following steps have been taken in order to achieve the aims of the work:

- multibody modelling of the wagon including the nonlinear/non–smooth description of the behaviour of the UIC standard suspensions and the nonlinear/non–smooth aspects of wheel–rail contact;
- modelling of a group of 2–axle freight wagons using existing models able to reproduce with sufficient accuracy the forces provided by the buffers and the draw gear; in this work, a group of three wagons has been considered;
- nonlinear analysis of the running properties in curves; two tools have been used: first a map of the steady–state solution reached after the negotiation of curve transition is presented as a function of both the curve radius and the vehicle speed; moreover, bifurcations are identified for some particular values of the curve radius by means of the ramping method.

The paper is organised as follows: after a brief description of the mathematical model used in this work, the results for the single vehicle and the composition of three wagons are presented. Finally, some concluding remarks are provided.

2 MATHEMATICAL MODEL

The numerical model of 2–axle freight wagon was built by means of a software named A.D.Tre.S (which is the Italian acronym for “dynamical analysis of the interaction between train and structure”), which was developed in recent years by the railway dynamics research group established at the Department of Mechanical Engineering, Politecnico di Milano [4]. The model is based on a multi–body schematisation of the trainset, allowing to analyse the non–stationary running behaviour in straight and curved track of a single rail vehicle or of a complete trainset and its interaction with the infrastructure. The mathematical model of train–track interaction is made up of four parts: a multi–body model of the rail vehicle, a simplified/complete model of the track, a wheel–rail contact model and a model of the coupling elements between wagons, such as the traction gear and the buffers. For the sake of this work, which is mainly focused on the analysis of the dynamics of the vehicle rather than on track vibration, the assumption of infinitely rigid track is made.

The vehicle model is divided into elementary modules of the following types:

1) carbody, modelled as a single rigid or flexible body;
2) bogie assembly, modelled as a rigid bogie frame, connected by primary suspensions to two (or more) wheelsets, modelled either as rigid or deformable bodies;
3) wheelset, modelled either as rigid or deformable body;
4) suspension, used to connect each other modules of type 1, 2 and 3, modelled as a combination of linear and nonlinear lumped parameter visco–elastic elements, possibly including specific models (with internal state variables) to reproduce the frequency dependent behaviour of special suspension components.

By combining the above listed elementary units, a wide range of models can be derived, from single vehicle up to a complete articulated trainset. With regard to 2–axle freight wagon, the vehicle is modelled connecting the carbody (module type 1) to two wheelsets (module type 3) by means of four suspensions (module type 4) whose characteristics are highly nonlinear and non–smooth. The independent coordinates used to describe the motion of the *i*-th body are the vertical (x_i) and lateral displacements (y_i) of the body centre of gravity with respect to a moving reference system, whose origin is located on the track centreline at the same longitudinal position of the centre of gravity of the considered body, and the Cardan-angles (σ_i, β_i, ρ_i). No longitudinal degree of freedom is assigned to the body since the motion is assumed to be at constant forward speed.

The equations of motion for the trainset take the general form of :

$$\left[M\right]\underline{\ddot{x}} = \underline{Q}\left(\underline{x}, \underline{\dot{x}}, t\right) \tag{1}$$

where:

- $\underline{x}$ represents the vector of the independent coordinates of the system;
- $\left[M\right]$ is the mass matrix;
- $\underline{Q}$ is the vector of the generalized forces containing the contributions due to:
  - nonlinear elements in the suspensions;
  - nonlinear inertial terms;
  - wheel–rail contact forces;
  - forces due to coupling elements;
  - additional forces (aerodynamic forces, etc.).

The 2–axle freight wagons are equipped with the UIC standard suspension. The suspension is made up of two parts: a leaf spring and a link system. The stiffness in the vertical direction is obtained by means of the deflection of the leaves whereas energy dissipation is introduced into the system by dry friction forces generated in between the leaves. The link system works as a pendular suspension system for the horizontal motion, yielding stiffness both in lateral and longitudinal directions. Dry friction in the joints of the link elements provides the necessary damping for the horizontal motion and represents the only damping mechanism in the UIC suspension.

The model used in this paper is the one presented by Hoffman [1], based on the work by Fancher [5]. Basically it consists of a general model that can be used for both the trapezoidal leaf spring and two–stage parabolic leaf spring, governing the specific type features by means of the model parameters. The restoring force from the leaf spring is expressed by the following differential equation:

$$\frac{\partial F_x}{\partial \delta} = \frac{F_{env} - F_x}{\beta} \tag{2}$$

where F_x is the restoring force from the leaf spring, F_{env} is an envelope function, δ is the spring deflection and β is a decay constant.

The UIC double–links represent the second element composing the UIC standard suspension. The behaviour of the UIC double–link connection can be analysed considering it as a system composed by technical pendulums. The technical pendulum distinguishes itself from the mathematical one in that rolling and sliding in the joints are taken into account. A proper model for the technical pendulum is presented by Piotrowski in [6]: the model is composed by a linear spring in parallel with an elastic element with dry friction obeying Coulomb's friction law.

Based on this simple model of the technical pendulum, a model for the UIC links with (nominal) cylindrical geometry was developed by Piotrowski [6] and also used by Hoffman [1]. The model for the lateral direction is shown in Figure 1 (left), whereas in Figure 1 (right) the model for the longitudinal direction is shown. Both models consist of a linear spring in parallel with elastic elements with dry friction. In the lateral direction it is also modelled the interaction with the suspension bracket, in fact, when the clearance of 10 mm between the lower link and the suspension bracket is exceeded, the stiffness of the element increases, being the pendulum length practically halved. This effect is taken into account in the mathematical model introducing a linear spring with a dead band.

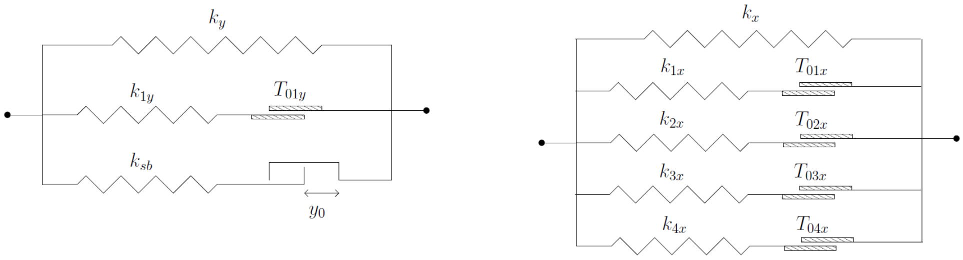

Figure 1 Model of the UIC links for the lateral (left) and longitudinal (right) direction.

Based on the work by Melzi et al. [3, 7], a mathematical model of both the buffers and the draw gear has been developed and introduced into the multi-body model of the trainset. In order to describe the dynamical behaviour of the buffers the introduction of additional degrees of freedom is needed. In particular, for each couple of buffers, a further degree of freedom ($\xi_{r,i}$), associated to the displacement of each rear buffer of the *i*-th vehicle, is considered. The model, described in details in [7], is based on laboratory experiments both in quasi-static and dynamic conditions. The parameters of the model are derived in order to obtain a best fit of the measured buffer forces.

The traction gear considered in this work is made up by two hooks and a chain. The dynamical behaviour of this component is reproduced by means of the nonlinear model proposed by Melzi [8]. Broadly speaking it consists of a nonlinear visco–elastic element, which takes into account also the initial load given to the chain by means of the releasing screw, described by the following equation:

$$F_g = \left(k_4 \Delta l^4 + k_3 \Delta l^3 + k_2 \Delta l^2 + k \Delta l_1 + k_0\right) \Delta l + \left(r_1 \Delta l + r_0\right) \dot{\Delta} l + F_s \quad (3)$$

where F_s is the force due to the deformation on the elastic elements of the hook after the preload is applied by means of the screw. In many countries, with regard to freight trains, the authorities prescribe not to tighten the screw coupling, therefore the force F_s is set to a zero value.

3 SINGLE VEHICLE ANALYSIS

The running properties of 2–axle freight wagons are not always satisfactory on straight track due to the well known and unwanted hunting motion [1, 2]. Using the model developed in this work, it is possible to investigate also the dynamics of these vehicles during curve negotiation. Time domain simulations and bifurcation diagrams provide the basic approach of analysis because they allow to account for the nonlinear effects which are demonstrated to have a very important role in the hunting of a railway vehicle. Steady–state solutions of curve negotiation belong to a space of codimension–3, the three parameters which span this vectorial space are the curve radius, the vehicle speed and the track cant. The track cant deficiency can be obtained by combining in a suitable way the parameters forming the basis of the space. In order to reduce the problem dimension the track cant has been considered fixed to the usual value of 150 mm, hence only dependencies on the curve radius and vehicle speed have been investigated.

In this work two kinds of analysis are carried out, first of all time domain simulations imposing different curve radius values and different vehicle speed values have been performed and steady-state solutions are found. No arbitrary initial condition is imposed to the vehicle in curve, but it is the result of curve transition negotiation. In fact the track is designed considering three different sections: 100 m of straight track, 100 m of curve transition and an infinitely long fully developed curve. Based on the results of the first analysis it is possible to identify some particular conditions corresponding to a change in the steady–state solution, either in the amplitude of the motion or in its shape. In order to find bifurcations in the codimension–2 space spanned by the curve radius and the vehicle speed, simulations in time domain are performed using the so-called ramping method, where the curve radius is fixed to a constant value and the vehicle speed varies linearly with time after steady–state conditions are reached. In this way it is possible to identify the bifurcation type and also to determine the vehicle speed corresponding to the bifurcation point.

All results presented here are obtained considering one specific 2–axle freight wagon, i.e. the Hbbills 311 which is a wagon equipped with sliding sidewalls, in tare configuration. The wheels are shaped to the standard S1002 wheel profile and, on the other side, the European UIC60 rail profile is used. In this work the effect of rail inclination on the nonlinear solution of the problem is not investigated as in [1], since only the 1/20 rail inclination is considered. Being interested in finding the steady–state solution no stochastic irregularity on neither the wheels nor the rails is considered.

In order to analyse the steady–state behaviour of the Hbbills 311 freight wagon different simulations varying curve radius and vehicle speed are performed. The amplitude of the lateral motion A_y, defined as follows, is used to characterize the solutions of the nonlinear problem:

$$A_y = \frac{\max(y) - \min(y)}{2} \tag{4}$$

where y represents the lateral displacement of each body. Essentially A_y is half the peak-to-peak amplitude, so, if the steady-state solution is stationary, A_y is equal to zero, whereas, if it is simple periodic, then A_y is the amplitude of the oscillation.

Curve radius varying from 250 m to 2000 m are considered, with a step of 50 m. For each curve radius simulations with different speeds are performed, in particular the minimum value corresponds to -0.8 m/s^2 cant deficiency and the maximum one corresponds to a value of 0.8 m/s^2. The maximum value here simulated exceeds the maximum one allowed for this type of train, since, at least in Italy, freight trains are not allowed to negotiate curves obtaining a track cant deficiency larger than 0.6 m/s^2. For each simulation, the amplitude of the steady-state motion A_y is calculated to define the dependence on both the curve radius and the vehicle speed.

In Figure 2 a contour plot for the lateral motion amplitude of the carbody is reported. The colours range from blue, corresponding to the smallest motion amplitude, to red, corresponding to the largest motion amplitude. In particular, the minimum value is equal to zero, that is a stationary solution is found. Analysing the results reported in Figure 2 it is observed that considering small values of curve radius, ranging from 250 m to approximately 800 m, the trend with regard to speed is practically the same, the larger the speed the larger the amplitude of the motion. On the contrary, as far as larger curve radii are concerned, the maximum amplitude of motion is obtained for a speed value lying approximately in the middle of the considered range.

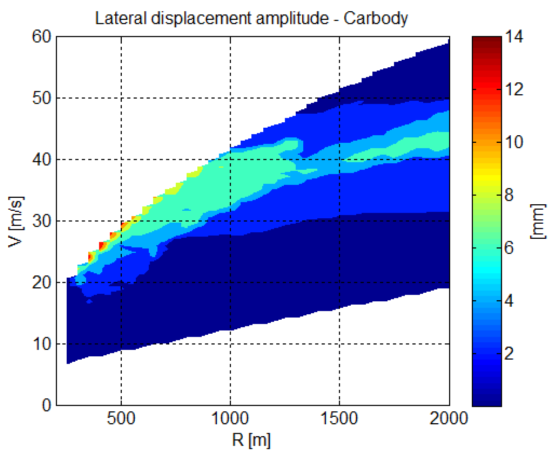

Figure 2 Map of the lateral motion amplitude of the carbody.

Focusing the attention on the 300 m radius curve it is possible to go into more depth analysing simulations performed with the ramping method. Essentially the vehicle speed is varied linearly with time, starting from an initial value and considering a constant acceleration of 1 m/s^2 or deceleration of -1 m/s^2. Figures 3 and 4 show the lateral displacement of the carbody (on the top) and the vehicle speed (on the bottom), considering respectively increasing and decreasing speed.

With regard to Figure 3 it is observed that the carbody, starting from an off-centred lateral position, moves outwards, due to the increased lateral acceleration, and then begins to oscillate when the speed is approximately equal to 22 m/s. On the contrary, considering the simulation with decreasing speed (Figure 4), it is observed that the carbody starts from a steady-state periodic motion which, when the speed is decreased, decreases its amplitude until a stationary solution is reached when the speed is approximately 12 m/s.

Taking into account that the ramping method leads to an overestimate or an underestimate of the bifurcation speed depending on whether the speed is increased or decreased, it is possible to state that the periodic solution is created by means of a tangent bifurcation, whereas the stationary solution loses its stability due to a Hopf subcritical bifurcation. In the range between 12 m/s and 22 m/s two attractors coexist. This information can be summarised in the bifurcation diagram shown in Figure 5, where all the solutions are plotted as a function of the speed. The stable branches are plotted in continuous line, whereas the unstable branches are represented by means of dashed line. The diagram obtained for the curve negotiation has the same shape of the one obtained by Hoffmann [1] considering straight track conditions, but obviously the speed values corresponding to the bifurcation points are different, in particular both the tangent bifurcation and the subcritical Hopf bifurcation take place at lower speeds in curves.

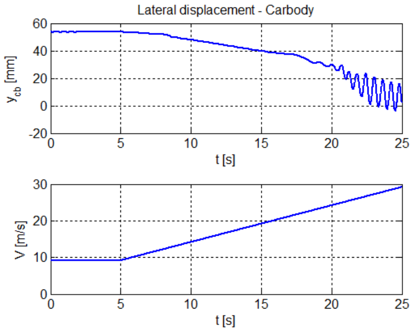

Figure 3 Lateral displacement of the carbody (R=300 m, increasing speed).

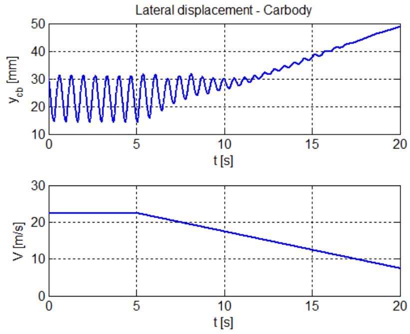

Figure 4 Lateral displacement of the carbody (R=300 m, decreasing speed).

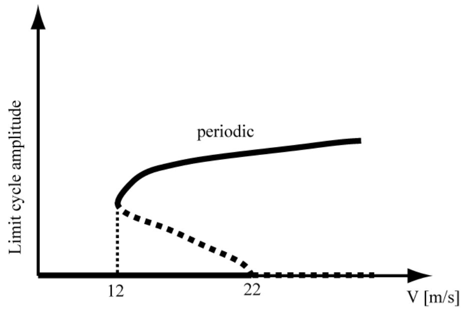

Figure 5 Carbody attractors (R=300 m).

4 VEHICLE COMPOSITION ANALYSIS

Generally speaking, the interaction between adjacent vehicles is usually not taken into account as far as dynamics in straight track is concerned. On the contrary, considering curve negotiation, the side buffers mounted on each carbody are compressed due to track curvature, thus generating forces which can significantly affect the dynamics of the vehicle [3].

In this work a composition of three freight wagons is analysed. Each freight wagon in the composition is the 2–axle wagon with sliding walls Hbbills 311. If the longitudinal dynamics during braking manoeuvres is not taken into account, as in this work, a composition of three wagons can be considered as sufficient to investigate the dynamics of a generic vehicle present in the composition. The first vehicle in the composition analysed does not represent any real condition, since in normal running conditions the first vehicle of the trainset is the locomotive, whereas the second vehicle is representative of any vehicle placed in the middle of the composition, being coupled by means of the buffers and the hook and chain coupler to the front and rear vehicles. The third vehicle, finally, represents the last vehicle of the trainset, since it is coupled to the rest of the trainset only by means of the front buffers and draw gear. All the vehicles in the composition analysed are in tare configuration. The procedure used to analyse the nonlinear behaviour of all the vehicles in the composition is the same applied in section 3, therefore simulations with different curve radii and speed values are performed in order to obtain the maps of the lateral motion amplitude. The cant of track is set to the usual value of 150 mm. For the sake of brevity only results for the wagon placed in the middle of the composition (which will be referred to as wagon 2 in the following) are reported.

In Figure 6 a contour plot of the lateral motion amplitude of the carbody is reported. Analogously to the map shown in section 3, the colours range from blue, corresponding to the smallest motion amplitude, to red, corresponding to the largest motion amplitude and, therefore, the scale of colours differs from the one used in Figure 2. Analysing the results reported in Figure 6 it is observed that considering large curve radii, in the range from 800 m to 2000 m, the carbody settles on the stationary solution, without oscillations; whereas, considering smaller curve radii, from 350 m to 800 m, the steady-state solution is oscillatory, even if its amplitude is relatively small (not exceeding 3 mm). Comparing these results to the ones obtained for the single vehicle (shown in section 3), it is possible to state that the effect of the buffers and the draw gear is very relevant. The periodic oscillations with large amplitudes observed for the single vehicle are suppressed and only small oscillations take place when the vehicle is coupled to the others in a composition.

Essentially, the coupling system works as a constraint between the wagons, increasing in a significant way the total stiffness generalized to the carbody degrees of freedom. In fact the UIC standard suspensions are relatively soft and the elastic elements of both the buffers and the draw gear can be considered as having comparable stiffness with regard to the yaw motion of the carbody.

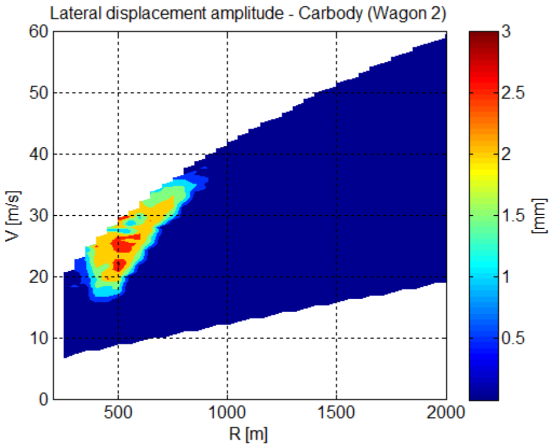

Figure 6 Map of the lateral motion amplitude of the carbody (wagon 2).

5 CONCLUSIONS

A mathematical model of the dynamics of a 2–axle freight wagon was presented in this work and used to investigate the running properties of these wagons on curved track. The forces provided by the UIC standard suspensions are non-smooth, being a consequence of dry friction but also the forces at wheel-rail interface are non-smooth due to the geometry of the contact between wheel and rail. Additionally the model accounts for the forces due to the coupling elements between the cars, so that both the dynamics of a single vehicle and of a group of three vehicles can be investigated. The wagon analysed in this work is the Hbbills 311, a 2–axle freight wagon with a long wheelbase (10 m), wheel and rail are shaped to the theoretical S1002 – UIC60 1/20 profiles.
With regard to the running properties of the single vehicle in curves tare condition was analysed. It was demonstrated that the running properties of the 2–axle freight wagons cannot be considered satisfactory when narrow curves are negotiated at relatively high speed values, which still lie in the operating speed range. Under these conditions the carbody settles into a periodic attractor with large amplitude (up to 12 mm). It was shown that the periodic attractor is generated by a tangent bifurcation and the stationary solution loses its stability in a subcritical Hopf bifurcation. The analysis performed on the group of three vehicles in tare condition showed that the effect of the coupling forces on the dynamics of 2–axle freight wagons is important, since the amplitudes of motion for the carbody of the second vehicle, which is representative of any wagon in the middle of the composition, are significantly reduced at higher speeds compared to the ones obtained for the single vehicle.

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