ABSTRACT

Change over time between two 512 by 512 (25 m by 25 m pixels) multispectral Landsat Thematic Mapper images dated 6 June 1986 and 27 June 1988 respectively covering a forested region in northern Sweden, is here detected by means of the iteratively reweighted multivariate alteration detection (IR-MAD) method followed by post-processing by means of kernel maximum autocorrelation factor (kMAF) analysis. The IR-MAD method builds on an iterated version of an established method in multivariate statistics, namely canonical correlation analysis (CCA). It finds orthogonal (i.e., uncorrelated) linear combinations of the multivariate data at two time points that have maximal correlation. These linear combinations are called the canonical variates (CV) and the corresponding correlations are called the canonical correlations. There is one set of CVs for each time point. The difference between the two set of CVs represent the change between the two time points and are called the MAD variates or the MADs for short. The MAD variates are invariant to linear and affine transformations of the original data.

The sum of the squared MAD variates (properly normed to unit variance) gives us change variables that will ideally follow a so-called $\chi^2$ (chi-squared) distribution with $p$ degrees of freedom for the no-change pixels; $p$ is the number of spectral bands in the image data. Here $p=6$, the thermal band is excluded from the analyses. The $\chi^2$ image is the basis for calculating an image of probability for no-change, i.e., the probability for finding a higher value of the $\chi^2$ statistic than the one actually found. This image is the weight image in the iteration scheme mentioned above. Iterations stop when the canonical correlations stop changing.

Principal component analysis (PCA) finds orthogonal (i.e., uncorrelated) linear combinations of the multivariate data that have maximal variance. A kernel version of PCA is based on a dual formulation also termed Q-mode analysis in which the data enter into the analysis via inner products in the so-called Gram matrix only. In the kernel version the inner products are replaced by inner products between nonlinear mappings into higher dimensional feature space of the original data. Via kernel substitution also known as the kernel trick these inner products between the mappings are in turn replaced by a kernel.
function and all quantities needed in the analysis are expressed in terms of this kernel function. This kernel version may be thought of as a nonlinear version of PCA.

Maximum autocorrelation factor (MAF) analysis finds orthogonal (i.e., uncorrelated) linear combinations of the multivariate data that have maximal autocorrelation. This type of analysis can be kernelized in a fashion similar to kernel PCA.

In both simple difference images, IR-MAD images and kernel MAF images grayish colours indicate no change, saturated colours indicate change. The kMAF transformation focuses on extreme observations, here the change pixels, and adapt to a varying multivariate background, here the no-change pixels.

**Keywords:** Orthogonal transformations, kernel methods.

**REFERENCES**


**Figur 1.** Landsat TM 6 June 1986 bands 5, 4 and 3 as RGB.

**Figur 2.** Landsat TM 27 June 1988 bands 5, 4 and 3 as RGB.
Figur 3. Landsat TM simple differences (1988 minus 1986) of bands 5, 4 and 3 as RGB (stretched linearly over 12 standard deviations). Simple differences make sense only when the data are properly normalized or calibrated (at least to the same zero and scale).

Figur 4. Landsat 1986/1988 IR-MAD variates 6, 5 and 4 as RGB (stretched linearly over 12 standard deviations). The IR-MAD variates may be considered as generalized differences which are insensitive to linear and affine transformations of the original data.
**Figure 5.** $\chi^2$ image (stretched linearly from min to max). Where the $\chi^2$ statistic has high values (i.e., where the image is bright) the probability of change is high.

**Figure 6.** $\chi^2$ image (stretched linearly from 0 to 1000).
Figur 7. kMAF variates 1, 2 and 3 of all simple differences as RGB.

Figur 8. kMAF variates 1, 2 and 3 of all IR-MAD variates as RGB.
Figur 9. Histograms and scatterplots for kMAF training data, simple differences. The no-change pixels are located near the origin of the scatterplots.

Figur 10. Histograms and scatterplots for kMAF all data, simple differences.
Figur 11. Histograms and scatterplots for kMAF training data, IR-MADs.

Figur 12. Histograms and scatterplots for kMAF all data, IR-MADs.