Extended Abstract:
Optimal Reinsertion of Cancelled Train Line

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1 Introduction to DSB S-tog

DSB S-tog (S-tog) is the operator of the suburban rail of Copenhagen, Denmark. The suburban network covers approximately 170 km of double-track and 80 stations. Daily the operator transports approximately 30.000 passengers. S-tog is the only user of the tracks, which are controlled by the infrastructural owner BaneDanmark (BD).

The S-tog network consists of *train lines* covering the S-tog infrastructure by various compositions of routes depending on the timetable in use. The network can be thought of as consisting of 8 sections; A central section, 6 fingers running from the central section into the suburbs of Copenhagen, and a circular rail running around the city center. The train lines merge in the central section, and they split as they re-enter the fingers according to their schedule.

The structure of the S-tog network implies that almost all of lines uses the central section. The trains on each train line all run with a 20 minutes frequency. Given 10 lines intersecting the central section this means that within 20 minutes there is on average only 2 minutes between each train in the central section Such low headway implies that even small delays can have significant negative effects on a high number of trains.

Each train line in the S-tog network is covered by 4 to 10 trains depending on the duration of the line circuit. A *line circuit* is the time it takes for a train to drive from one terminal to the other terminal and back. Each train consists of one or more (physical) *train units*. The composition of a train varies during the day according to the expected passenger demand.

A *train number* is associated with each operating train. The train number is changed every time a train turns at a line terminal to run in the opposite direction. For each train there is hence a series of train numbers during the day that defines the tasks of that particular train during that day. This series of numbers is called the train's *train sequence* and there can be only one train sequence for each train. Also, a train numbers cannot occur in two different train sequences.

Several information can be extracted from a train number. Each train number identifies the train line, stopping pattern, direction (north/south) and the time of day for a train i.e. the train numbers within a train line are numbered through out the day so that e.g. the train number 55124 is scheduled to depart early than the train number 55125.

Rolling stock depots are placed at most of the line terminals and at the central station. The only crew depot is located at the central station, so for the first trip of the day the required crew must be transported from the crew depot to the given rolling stock depot or meet up there. Trains are inserted into and taken out of the network from the rolling stock depots.

2 The Reinsertion Problem

Often disturbances in rail operations require immediate recovery. At DSB S-tog different recovery strategies are applied. When larger disturbances occur in the S-tog network an often used countermeasure is to take out entire train lines i.e. all departures on one or more train lines are cancelled. By taking out train lines additional slack is created in the timetable, i.e. the headways are increased between time adjacent train lines. This creates increased buffer times in the timetable and more room for absorbing the delays.

A take-out is executed by shunting the rolling stock to depot tracks as the trains arrive at rolling stock depots. In the process of take-out it is not allowed to drive "backwards" in the network. Trains in a cancelled line circuit therefore ends up being distributed among the depots along the line according to where they were in the network when the decision of cancelling the line was made. It is crucial to realize that train units that are taken out at a depot are not necessarily used to cover the same trains when reinserted. Recall that a train is defined by its train sequence and not by the train units covering it.

When an adequate level of regularity has been re-established in the operation, the cancelled train line is reinserted according to schedule. The status of operation is evaluated by a train controller from BD. After the decision of initiating reinsertion has been made, the reinsertion should be carried out as quickly as possible under the restrictions that the order of trains must be kept.

When a train is reinserted it is transported as empty stock from the depot tracks to a platform. A train driver arrives on a train in regular service from the crew depot. The train to be reinserted departs according to a scheduled departure on the relevant train line.

The problem is to decide when the reinsertion shall start on each rolling stock depot. Each train can be reinserted only once and in each time slot in a depot only one train can be reinserted. Also, it is given for each depot exactly how many trains must be inserted.

The reinsertion must be made under two different considerations of order. Firstly, if reinsertion has begun from a given rolling stock depot, the remaining trains to be inserted from that depot must be inserted according to the frequency. For example, at S-tog the frequency is 20 minutes on all train lines. If 3 trains must be reinserted from Farum rolling stock depot and the first reinserted train departs at 15:18, then the remaining 2 trains must be reinserted and depart at respectively 15:38 and 15:58. Inserting the remaining two trains at 15:58 and 16:18 would mean an unassigned frequency interval at 15:38 i.e. order would not have been kept and that would be an illegal solution. Secondly, the order with respect to frequency must also be kept across rolling stock depots. That is, after the initiation of reinsertion, the time between two adjacent departures on any station in the network must always be the frequency of 20 minutes.

3 The solution approach

One of the impediments of generating a reinsertion in hand is that it is time consuming. In a recovery operation this is a serious issue.

Recent surveys on rail operation models are given by Cordeau 1998 [1], Huisman et al 2005 [2] and Törnquist in 2006 [3]. The reinsertion problem and models for solving it is not mentioned in either of these surveys. A thorough search has not produced any additional literature that resembles the problem of reinserting train lines. The problem seems to be specific to the Copenhagen suburban network.

If a tool is available for generating the reinsertion, it is possible to generate an optimal reinsertion plan immediately when the distribution of trains among depots is known after the take out. As the timetable is periodic the reinsertion scheme calculated will in principle be unique except for the train numbers that must be assigned. This might lead to some advantages with respect to coordinating the train driver schedules according to the reinsertion, thereby preventing reinsertion schedules being discarded because of the lack of drivers.

The problem has been formulated as a mixed integer programming model. The goal of the model is to decide which train, $i \in I$ should be inserted from which depot, $k \in K$, where I is the set of train that must be inserted and K are the set of depots they can be inserted from. Each originally scheduled train i (before take out) must be covered with train units and hence reinserted in operation according to schedule. Also it must be decided for each train in which time slot $j \in J$ the reinsertion will take place. J is the set of available slot for reinsertion.

The variables representing which train to be inserted from which depot and when are binary:

$$x_{ijk} = \begin{cases} 1 & \text{if train } i \text{ is inserted in time slot } j \text{ from depot } k \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Each train must be covered exactly once.

$$\textstyle\sum_{j,k} x_{ijk} = 1, \qquad \forall\ i \in I \tag{2}$$

Inequalities (3) are included so that no time slot for a depot or train can be covered more than once:

$$\textstyle\sum_{i} x_{i,j,k} \leq 1, \qquad \forall\ j \in J, k \in K \tag{3}$$

Recall that the number of trains to be inserted from each depot is known. Therefore, binding constraints exist for each depot. For normal depots, $k \in K_T$, we get;

$$\textstyle\sum_{i,j} x_{i,j,k} = D_k, \qquad \forall k \in K^T \tag{4}$$

Insertion from intermediate depots can be made in both directions. According to current practise the trains on each intermediate depot are inserted one half of them in one direction and the other half in the other direction. This is handled in the model by including two depots for each intermediate depot. The set of intermediate depots is denoted K^I. It is constructed by sets of two depots together denoting one intermediate depot where reinsertion can be carried out in l directions, $K^I = K^I_1 \cup ... \cup K^I_l$, where $l \in L$. L is the set of directions, which in the S-tog network for all depots is north or south. The total set of depots is $K = K^T \cup K^I$. Variables $D^I_k, k \in K^I$ has been added to the model to represent the number of trains inserted each direction.

The sum of trains inserted in both directions should equal the total number of trains to be inserted from the intermediate depot. Equations (5) ensure that the number of trains inserted in each direction is the total number of trains to be inserted divided by 2. If an odd number of trains is to be inserted, the result is rounded up or down to nearest integer depending on which is more favorable to the model. See equations (6) and (7).

$$\sum_{i,j,k} x_{i,j,k} = \sum_k D_k, \qquad \forall \qquad l \in L, k \in K_l^I \tag{5}$$

$$\sum_{i,j} x_{i,j,k} = D_k^I, \qquad \forall \qquad k \in K^I \tag{6}$$

$$D_k^I \geq \lfloor \tfrac{D_k}{2} \rfloor \qquad D_k^I \leq \lceil \tfrac{D_k}{2} \rceil \tag{7}$$

It is crucial that certain orders are kept as the trains are inserted. As mentioned in Section 2 order should be kept within depots and between depots. Also, reinsertion must not begin on a depot before a train driver can arrive from the crew depot to drive the train to be reinserted.

To ensure that each train is inserted only once, it is necessary to take into consideration the train sequences of each train describing in which time slot each train is at the different depots. To handle this a constant is introduced, $in_{i,j,k}$, which is one if train i may depart from depot k in time slot j.

It is not possible to insert a train from a depot, if it is not there at that specific time slot, which is ensured by;

$$x_{i,j,k} \leq in_{i,j,k}, \qquad \forall\ i \in I, j \in J, k \in K \tag{8}$$

To model the order within stations we introduce two sets of integer variables, $start_k$ and end_k. Also, we introduce equations (9) to (12). Equations (9) connect the start and end variables. Equations (10) assure that reinsertion is not begun before the first driver can arrive at the depot. The constant, C_k, indicates how many trains has been scheduled at depot k from the time of the decision of reinsertion until drivers are able to reach the depot. Equations (11) and (12) ensure that when a reinsertion has begun on depot, it is carried out continuously in adjacent time slots.

$$start_k + \sum_{i,j} x_{i,j,k} - 1 = end_k, \qquad \forall\ k \in K \tag{9}$$

$$start_k \geq C_k + 1, \qquad \forall\ k \in K \tag{10}$$

$$start_k \leq j + M \cdot \Big(1 - x_{i,j,k}\Big), \qquad \forall\ i \in I, j \in J, k \in K \tag{11}$$

$$end_k \geq j - M \cdot \Big(1 - x_{i,j,k}\Big), \qquad \forall\ i \in I, j \in J, k \in K \tag{12}$$

Train numbers indicate the time of day. The train number to be inserted when $x_{i,j,k}$ is 1 is calculated from an initial train number on a train able to carry train drivers to the depots and some constant describing the relationship between the train numbers on the driver-carrying line and the line to be reinserted. It is adjusted according to the time slot, j;

$$\begin{aligned} TrainNumber_{i,j,k} = \Big(&InitialTrain + TrainConst_k + j\Big) \cdot x_{i,j,k}, \\ &\forall\ i \in I, j \in J, k \in K \end{aligned} \tag{13}$$

The objective function minimizes the maximum inserted train number, $MaxTrainNumber$, and thereby assures a quick reinsertion.

$$Minimize \qquad MaxTrainNumber \tag{14}$$

$$MaxTrainNumber \geq TrainNumber_{i,j,k}, \qquad \forall\ i \in I, j \in J, k \in K \tag{15}$$

The model has been implemented in Gams and solved with Cplex 8.1.0. The solution time is approx. 0.5 CPU seconds on a Pentium M 1700 MHz.

The rolling stock dispatchers has verified the solutions. Reinsertions carried out in operation are today generated by the reinsertion model presented.

References

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