Statistical methods for decision making in mine action

Jan Larsen
Intelligent Signal Processing
Technical University of Denmark
jl@imm.dtu.dk, www.imm.dtu.dk/~jl

Why do we need statistical models?

- Mine action is influenced by many uncertain factors – statistical modeling is the principled framework to handle uncertainty
- The use of statistical modeling enables empirical, consistent and robust decisions with associated risk estimates from acquired data and prior knowledge
- Pitfalls and misuse of statistical methods sometimes wrongly leads to the conclusion that they are of little practical use

The elements of statistical decision theory

Data
- Sensor
- Calibration
- Post clearance
- External factors

Prior knowledge
- Physical knowledge
- Experience
- Environment

Statistical models

ss function

- Decision
- Risk assessment

Inference:

Assign probabilties to hypotheses

Bayes theorem

$$\text{posterior} = \frac{\text{likelihood} \cdot \text{prior}}{\text{probability of data}}$$

What are the requirements for mine action risk

- Tolerable risk for individuals comparable to other natural risks
-
- d

commercial etc.)

Facts

- 99.6% is not an unrealistic requirement
- But… today's methods achieve at most 90% and are hard to evaluate!!!

GICHD and FFI are currently working on such methods [Håvard Bach, Ove Dullum NDRF SC2006]

Outline

- Statistical modeling
- What are the requirements for mine detection?
- The design and evaluation of mine equipment
- Improving performance by statistical learning and information fusion
- The advantage of using combined method

A simple inference model – assigning probabilities to data

- The detection system provides the probability of detection a mine in a specific area: Prob(detect)
- The land area usage be... probability of encounte...

Prob(casualty)=(1-Prob(d... ...r)

For discussion of assumptions and involved factors see

"Risk Assessment of Minefields in HMA – a Bayesian Approach"

PhD Thesis, IMM/DTU 2005 by Jan Vistisen

A simple loss/risk model

- Minimize the number of casualties
- Under mild assumptions this equivalent to minimizing the probability of casualty

Requirements on detection probability

Prob(causality)=(1-Prob(detection))*Prob(encounter)

Prob(detection)=1-Prob(causality)/Prob(encounter)

- Prob(encounter)= ρ*a
  - ρ : homogeneous mine density (mines/m^2), a: yearly footprint area (m^2)
- Prob(causality)=10^{-5} per year

Maximum yearly footprint area in m^2

P(detection)	ρ : mine density (mines/km²)				
	0.1	1	10	100	1000
0.996	25000	2500	250	25	2.5
0.9	1000	100	10	1	0.1

Reference: Bjarne Haugstad, FFI

Outline

- Statistical modeling
- What are the requirements for mine detection?
- The design and evaluation of mine equipment
- Improving performance by statistical learning and information fusion
- The advantage of using combined method

Optimizing the MA operation

System design phase

Changing environment

- Mine types, placement
- Soil and physical properties
- Unmodeled confounds

Overfitting

- Insufficient coverage of data
- Unmodeled confounding factors
- Insufficient model fusion and selection

Designing a mine clearance system

Methods

sensor

Prior knowledge

informal

Confou

param

target

operatio

environm

d test

Statistical learning is a principled framework for combining information and achieving optimal decisions

Evaluation and testing

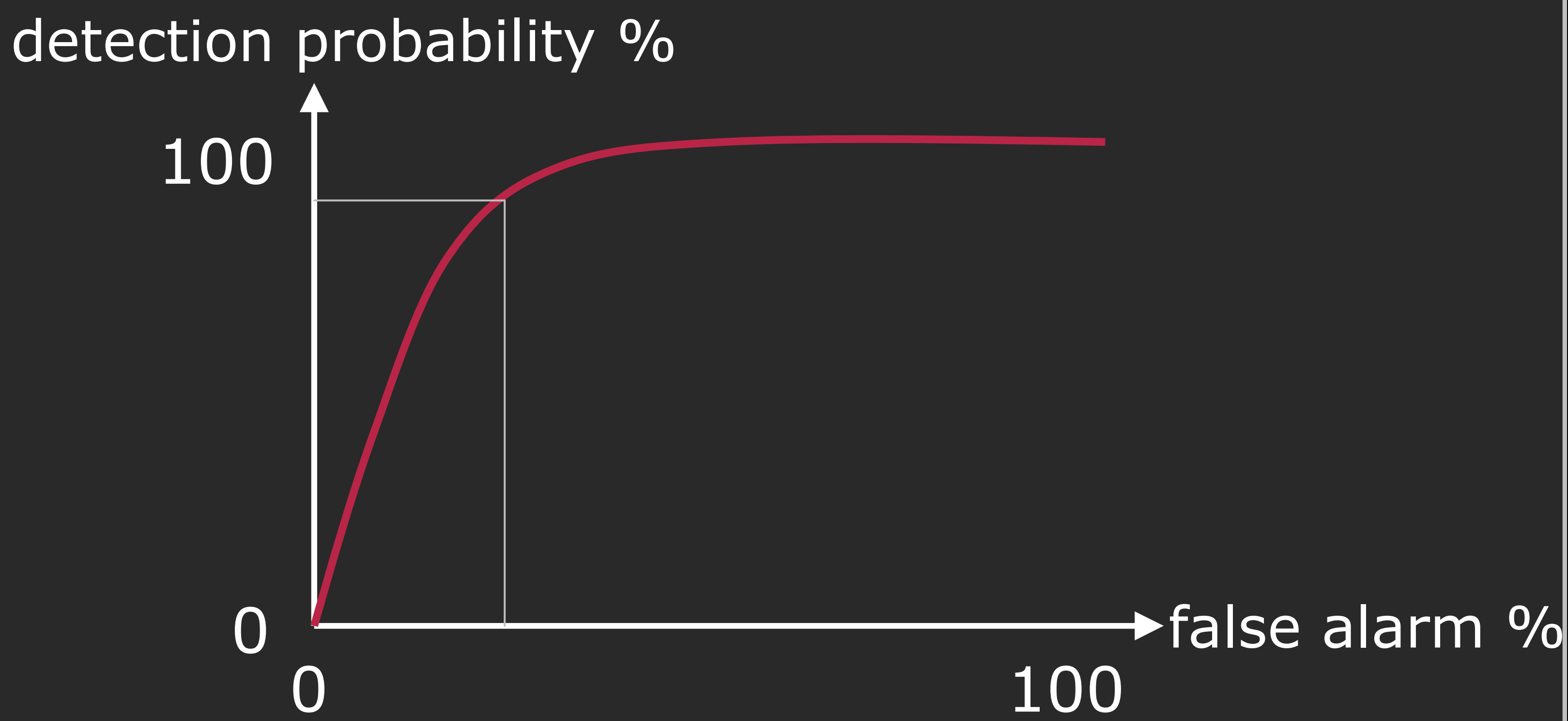
- How do we assess the performance/detection probability?
- What is the confidence?

Confusion matrix

		True	
		yes	no
Estimated	yes	a	b
	no	c	d

- Detection probability (sensitivity): a/(a+c)
- False alarm: b/(a+b)

Receiver operations curve (ROC)



Inferring the detection probability

- N independent mine areas for evaluation
- y detections observed
- true detection probability θ

$$P(y \mid \theta) \sim \text{Binom}(\theta \mid N) = \binom{N}{y} \theta^{y} \theta^{N-y}$$

Posterior probability via Bayes formula

$$P(\theta \mid y) = \frac{\ldots)\,p(\theta)}{P(y)}$$

prior

Prior probability of θ

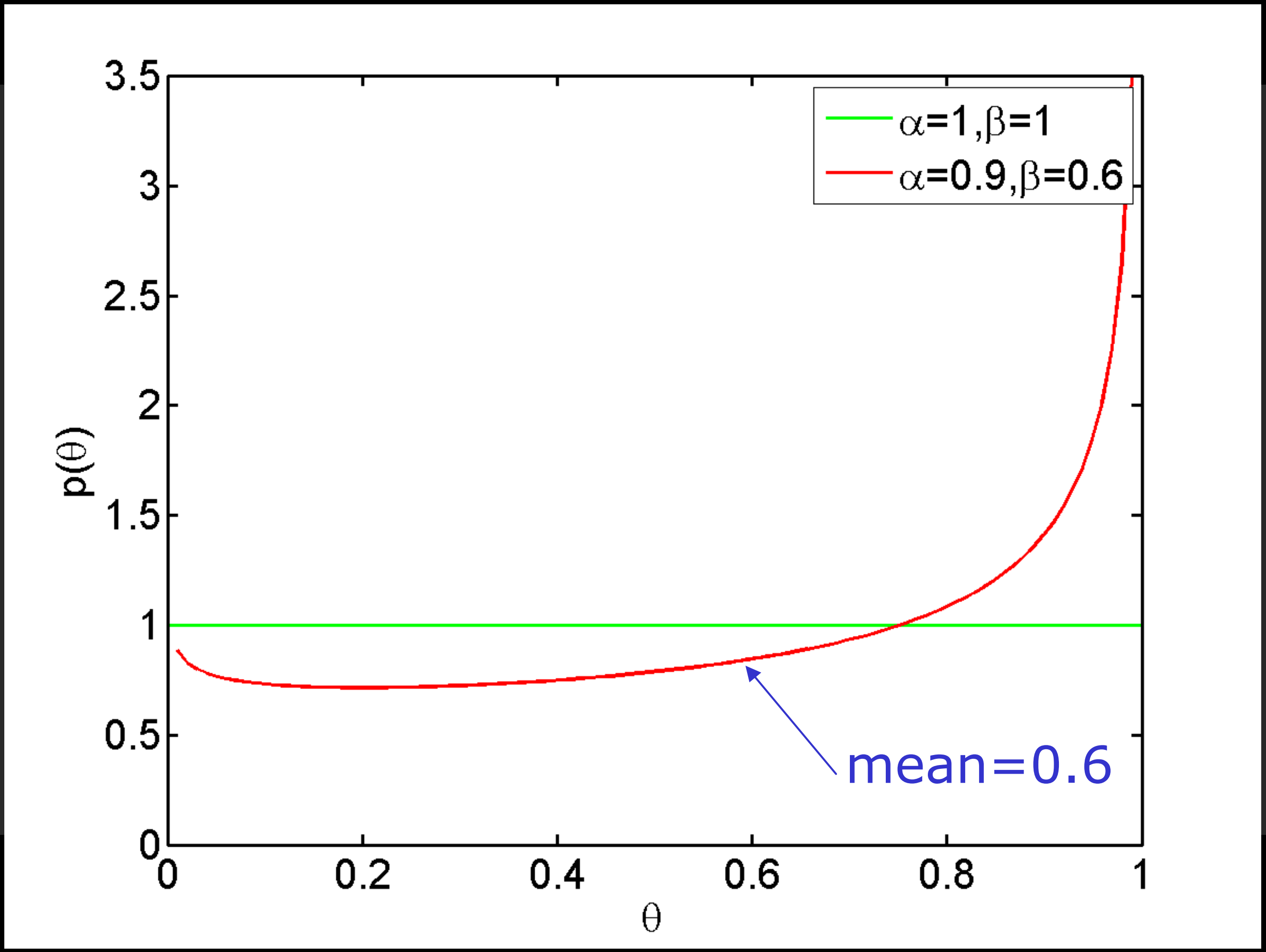
- No prior
- Non-informative prior

$$p(\theta) = Uniform(\theta \mid 0,1)$$

- Informative prior

$$p(\theta) = Beta(\theta \mid \alpha, \beta)$$

Prior distribution

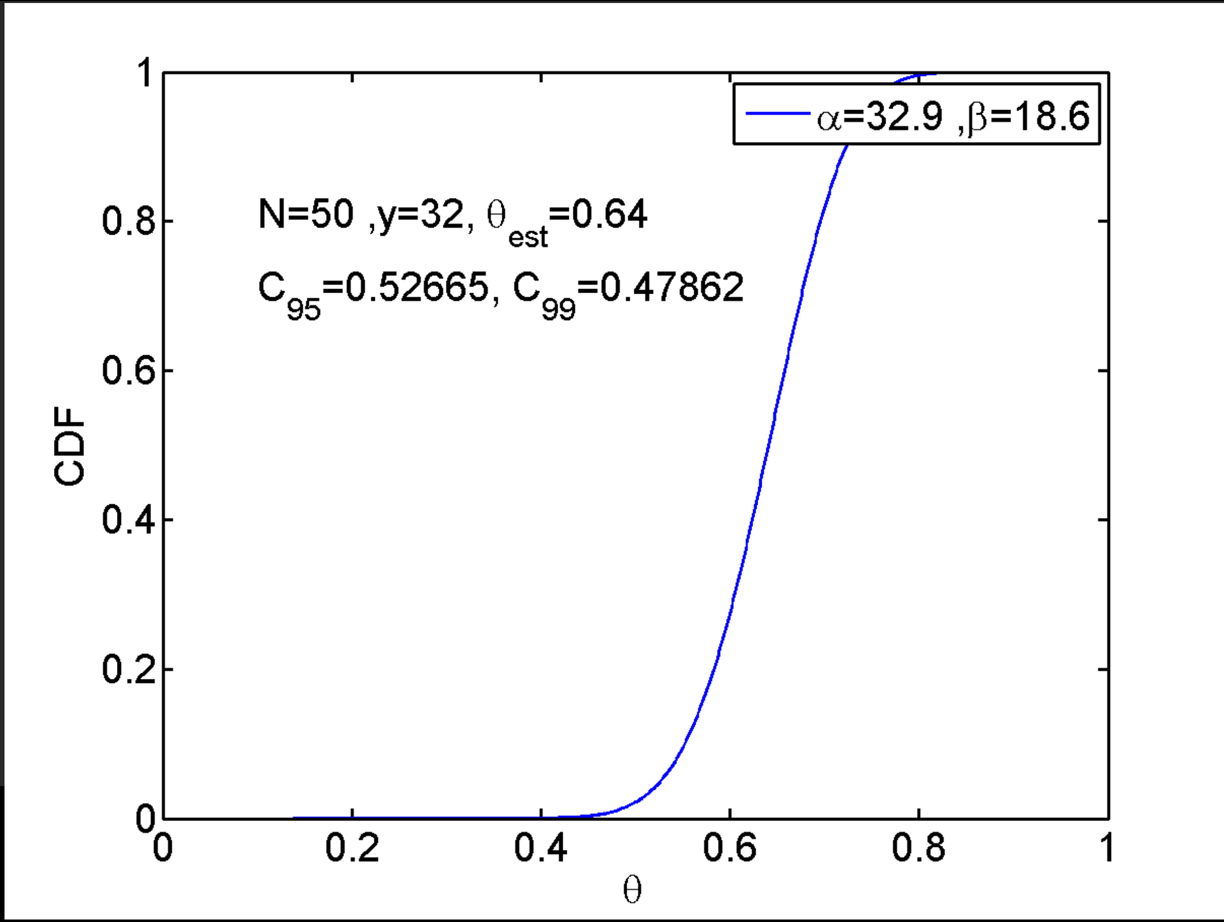


Posterior probability is also Beta

$$P(\theta \mid y) = Beta(\theta \mid y+\alpha, \beta+n-y) \sim \theta^{y+\alpha}\theta^{n-y+\beta}$$

HPD credible sets – the Bayesian confidence interval

$$C_{1-\varepsilon} = \{\theta : P(\theta \mid y) \geq k(\varepsilon)\},\ P(C \mid y) > 1 - \varepsilon$$



The required number of samples *N*

- We need to be confident about the estimated detection probability

$$\text{Prob}(\theta > 99.6\%) = C_{1-\varepsilon}$$

	$C_{95\%}$	$C_{99\%}$
$\theta_{est} = 99.7\%$	9303	18994
$\theta_{est} = 99.8\%$	2285	3995

Uniform prior

	$C_{95\%}$	$C_{99\%}$
$\theta_{est} = 99.7\%$	8317	18301
$\theta_{est} = 99.8\%$	2147	3493

Informative prior

α=0.9, β=0.6

Credible sets when detecting 100%

Minimum number of samples *N*

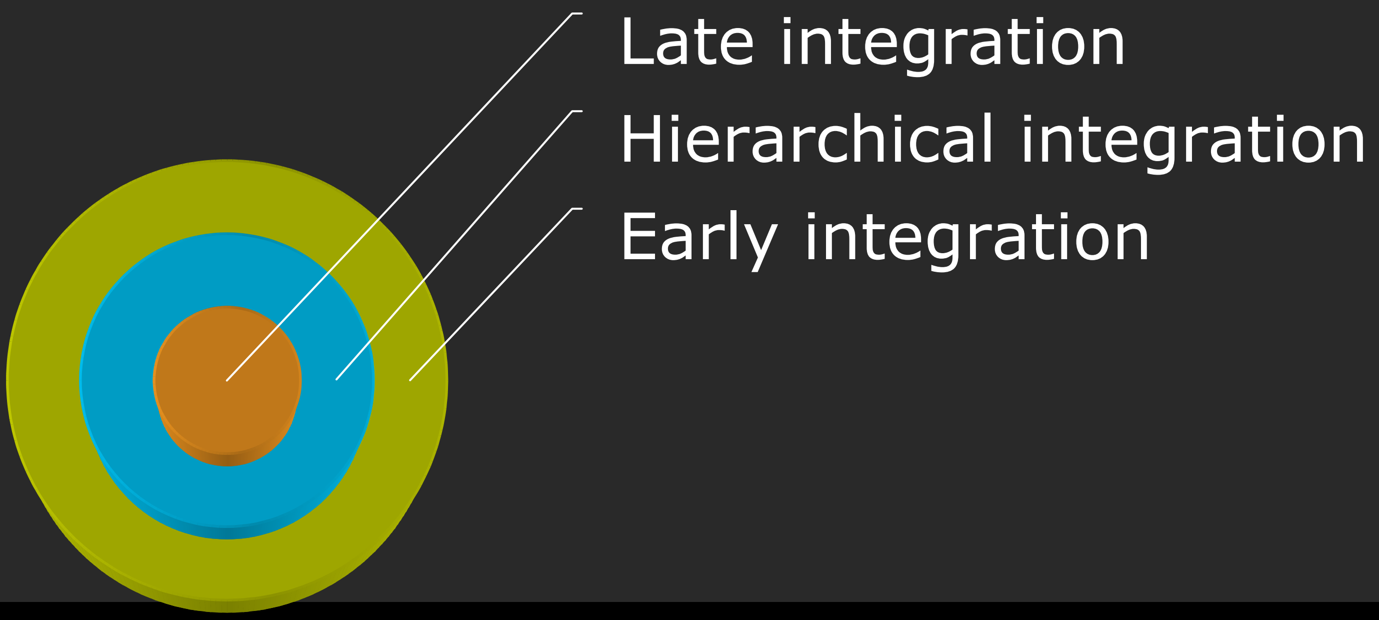
	Prob($\theta >$ 80%)	Prob($\theta >$ 99.6%)	Prob($\theta >$ 99.9%)
$C_{95\%}$	13	747	2994
$C_{99\%}$	20	1148	4602

Outline

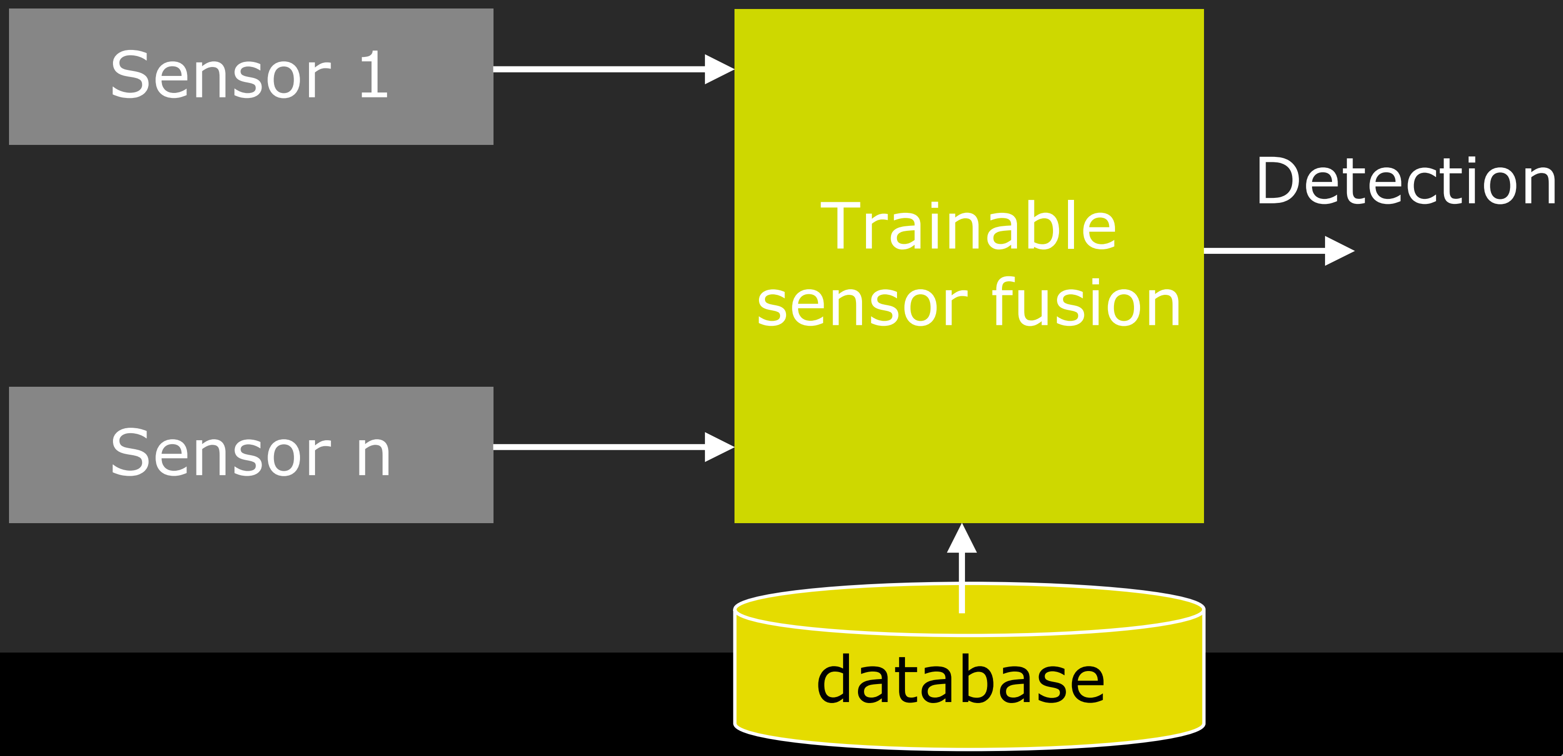
- Statistical modeling
- What are the requirements for mine detection?
- The design and evaluation of mine equipment
- Improving performance by statistical learning and information fusion
- The advantage of using combined method

Improving performance by fusion of methods

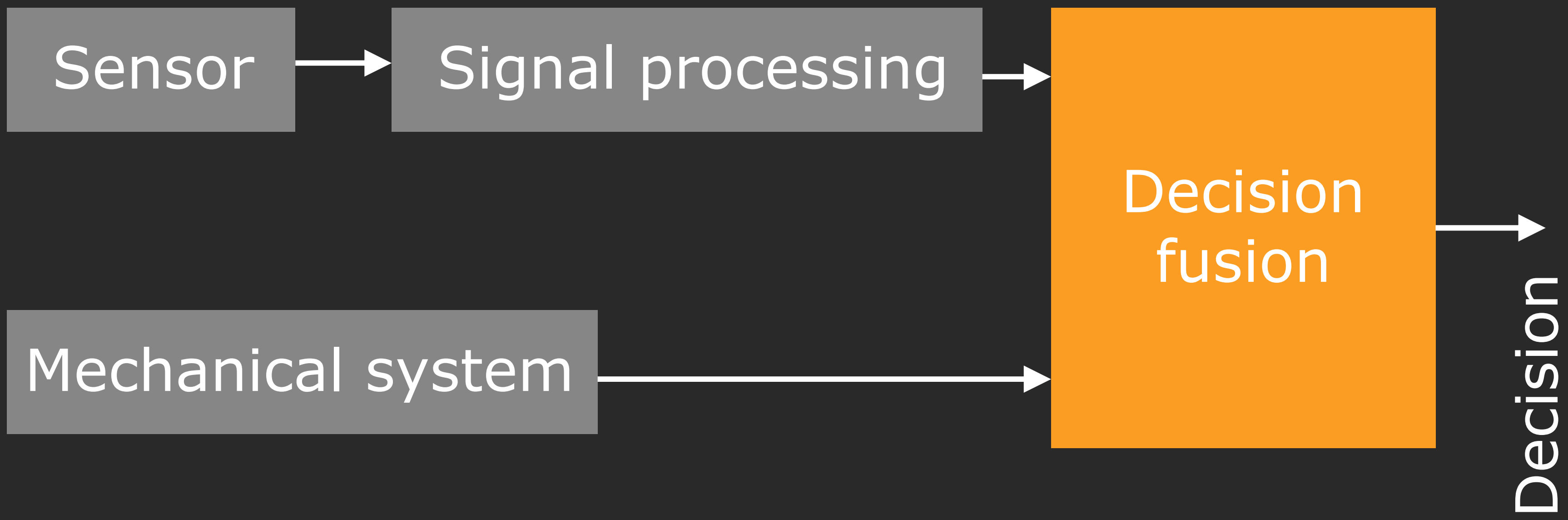
- Methods (sensors, mechanical etc.) supplement each other by exploiting different aspect of physical environment



Early integration – sensor fusion



Late integration – decision fusion



Suggestion

Apply binary (mine/no mine) decision fusion to existing detection equipment

Advantages

- Combination leads to a possible exponential increase in detection performance
- Combination leads to better robustness against changes in environmental conditions

Challenges

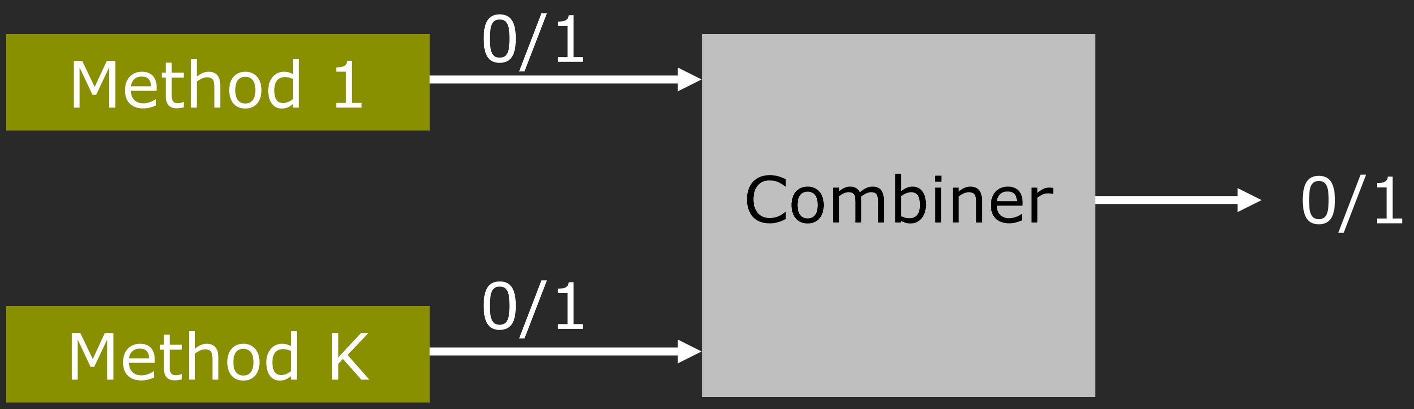
- Need for certification procedure of equipment under well-specified conditions (ala ISO)
- Need for new procedures which estimate statistical dependences between existing methods
- Need for new procedures for statistically optimal combination

Dependencies between methods

Contingency tables

Mine present		Method j	
		yes	no
Method i	yes	c11	c10
	no	c01	c00

Optimal combination



Optimal combiner depends on contingency tables

Optimal combiner

Method		Combiner						
1	**2**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0
0	1	0	0	0	1	[illegible]	1	1
1	0	0	1	1	0	0	1	1
1	1	1	0	1	0	1	0	1

OR rule is optimal for independent methods

$$2^{2^{K-1}} - 1$$ possible combiners

OR rule is optimal for independent methods

Method 1:	1	0	0	1	0	0	1	0	1	0
Method 2:	0	1	0	0	1	0	1	1	1	0
Combined:	1	1	0	1	1	0	1	1	1	0

$$P_d(OR) = P(\hat{y}_1 \vee \hat{y}_2 = 1 \mid y = 1)$$

$$= 1 - P(\hat{y}_1 = 0 \wedge \hat{y}_2 = 0 \mid y = 1)$$

$$= 1 - P(\hat{y}_1 = 0 \mid y = 1) \cdot P(\hat{y}_2 = 0 \mid y = 1)$$

$$= 1 - (1 - P_{d1}) \cdot (1 - P_{d2})$$

independence

False alarm follows a similar rule

$$
\begin{aligned}
&P_{fa}(OR) = \\
&P(\hat{y}_1 \vee \hat{y}_2 = 1 \mid y = 0) \\
&= 1 - P(\hat{y}_1 = 0 \wedge \hat{y}_2 = 0 \mid y = 0) \\
&= 1 - P(\hat{y}_1 = 0 \mid y = 0) \cdot P(\hat{y}_2 = 0 \mid y = 0) \\
&= 1 - (1 - P_{fa1}) \cdot (1 - P_{fa2})
\end{aligned}
$$

Example

$$p_{d1} = 0.8, p_{fa1} = 0.1 \quad p_{d2} = 0.7, p_{fa2} = 0.1$$

$$p_d = 1 - (1 - 0.8) \cdot (1 - 0.7) = 0.94$$

$$p_{fa} = 1 - (1 - 0.1) \cdot (1 - 0.1) = 0.19$$

Exponential increase in detection rate
Linear increase in false alarm rate

Joint discussions with: Bjarne Haugstad

Testing independence – Fisher's exact test

		Method j	
		yes	no
Method i	yes	c11	c10
	no	c01	c00

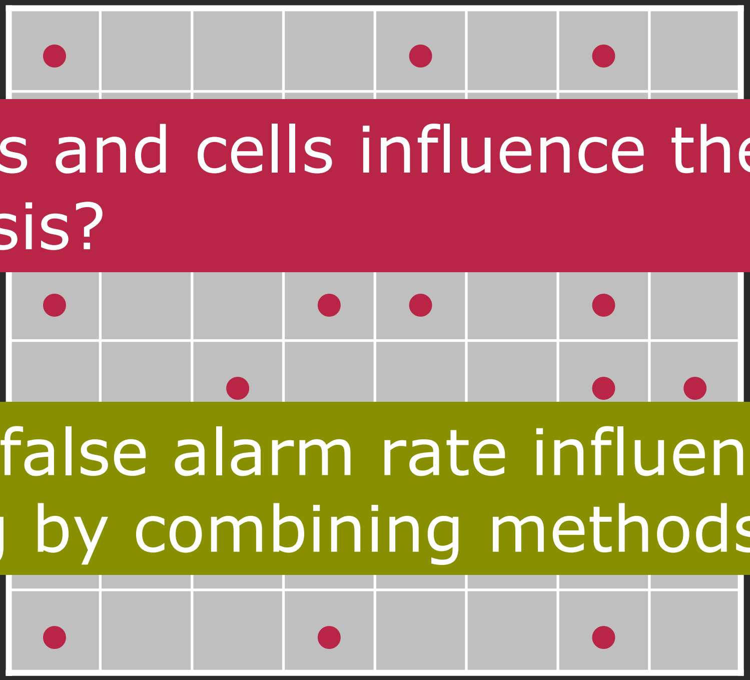
- Hypothesis: Method i and j are independent
- Alternatives: Dependent or positively (negatively) correlated

$$\mathrm{H}: P(\hat{y}_i = 0, \hat{y}_j = 0) = P(\hat{y}_i = 0) \cdot P(\hat{y}_j = 0)$$

$$\mathrm{A}: P(\hat{y}_i = 0, \hat{y}_j = 0) > P(\hat{y}_i = 0) \cdot P(\hat{y}_j = 0)$$

Artificial example

- N=23 mines
- Method 1: P(detection)=0.8,
- P(false alarm)=0.1
- Resolution: 64 cells



How does number of mines and cells influence the analysis?

How does detection and false alarm rate influence the possibility of gaining by combining methods?

Resolution

High

- Many cells provide possibility accurate spatial localization of mines
- Good estimation of false alarm rate
- Poor detection rate

Low

- Increased possibility of reliably estimating

  P(no mines in area)
- Poor spatial localization

Confusion matrix for method 1

		True	
		yes	no
Estimated	yes	19	5
	no	4	36

Confidence of estimated detection rate

- With *N*=23 mines 95%-credible intervals for detection rates are extremely large!!!!

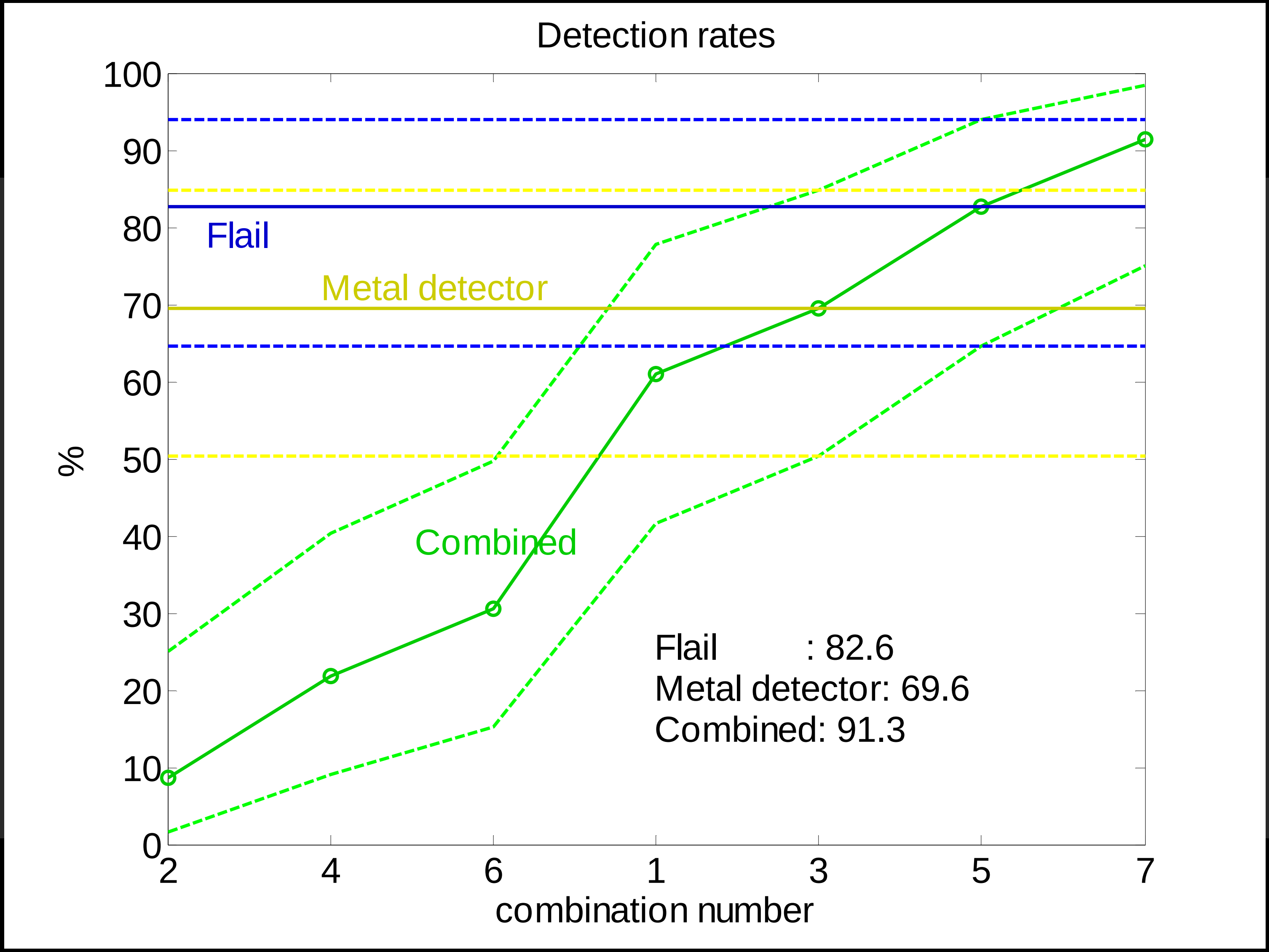
Method1 (flail): [64.5% 82.6% 93.8%]

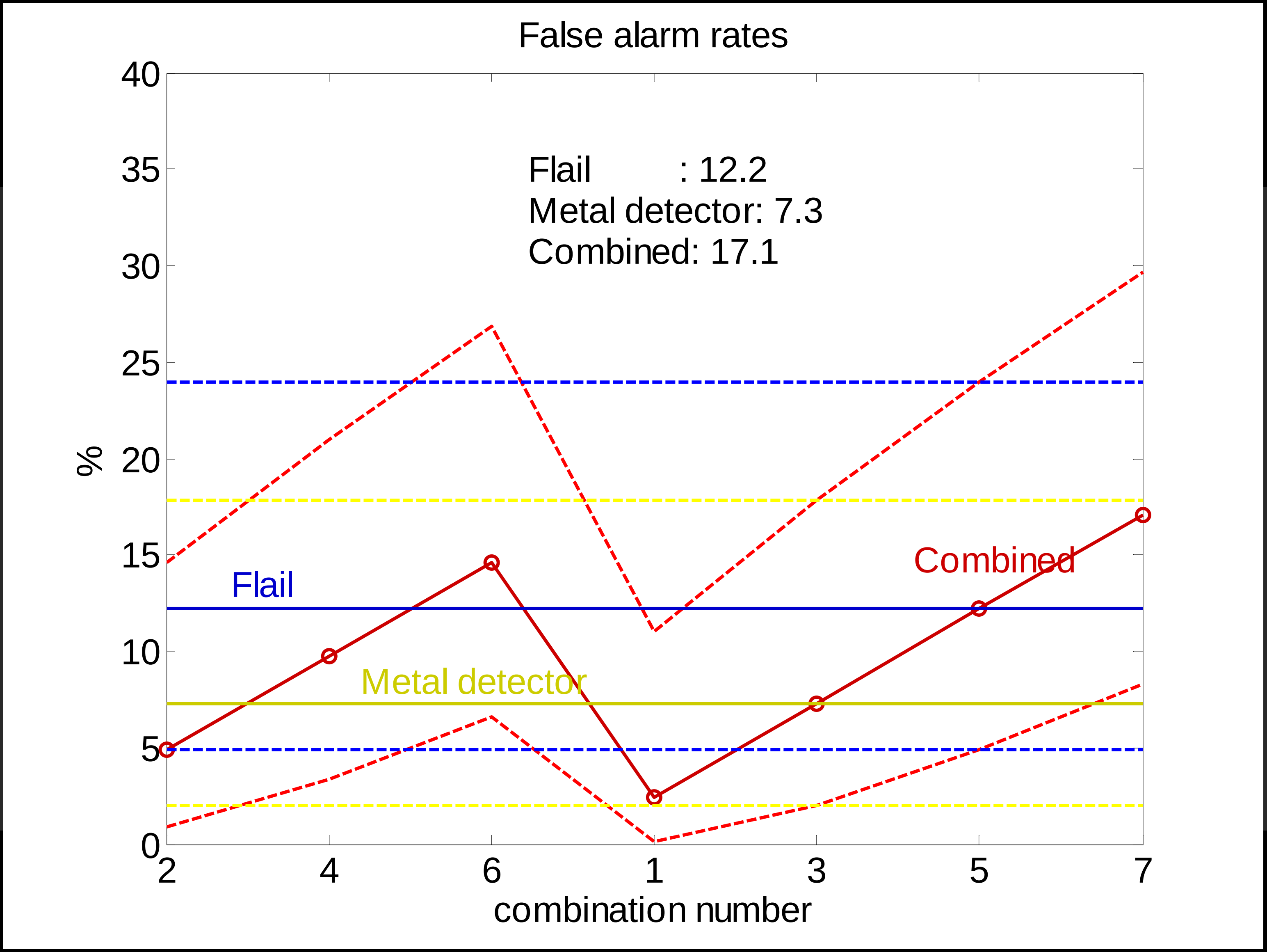
Method2 (MD): [50.4% 69.6% 84.8%]

Confidence for false alarm rates

- Determined by deployed resolution
- Large resolution - many cells gives many possibilities to evaluate false alarm.
- In present case: 64-23=41 non-mine cells

Method1 (flail): $[4.9\% \quad 12.2\% \quad 24.0\%]$


Detection rates
100
90
80
70
60
50
40
30
20
10
0
%
Flail
Metal detector
Combined
2
4
6
1
3
5
7
combination number
Flail : 82.6
Metal detector: 69.6
Combined: 91.3


False alarm rates
40
35
30
25
20
15
10
5
0
%
Flail : 12.2
Metal detector: 7.3
Combined: 17.1
Flail
Combined
Metal detector
2
4
6
1
3
5
7
combination number

Comparing methods

- Is the combined method better than any of the two orginal?
- Since methods are evaluated on same data a paired statistical McNemar with improved power is useful

Method1 (flail): 82.6% < 91.3% Combined

Method2 (MD): 69.6% < 91.3% Combined

They keys to a successful mine clearance system

- Use statistical learning which combines all available information in an optimal way
  - informal knowledge
  - data from design test phase
  - confounding parameters (environment, target, operational)
- Combine many different methods using statistical fusion

MineHunt System and HOSA concepts have been presented at NDRF summer conferences (98,99,01)

Outline

- Statistical modeling
- What are the requirements for mine detection?
- The design and evaluation of mine equipment
- Improving performance by statistical learning and information fusion
- The advantage of using combined method

How do we proceed?

NDRF has decided to take a leading role in initiating a project COLUMBINE to carry out suggested research

Project proposal

- Combine existing techniques: mechanical flail, dogs, metal detector, ground penetrating radar
- The methods use very different physical properties of mines and environment, hence the error patterns are likely to be independent
- Combining a few 60-90% methods will reach the goal
- Cost of 60%-90% systems are lower and requires fewer samples to evaluate and certify reliably
- Full efficiency and economic advantages has to include quality and management aspects

Project work packages

- *Current technologies*
  - identification a number of available and techniques
  - aim is to clarify how information about the methods and their operation can be extracted and stored in an efficient way
- *Physical properties of current technologies*
  - the aim is to get knowledge about how independent the methods are from a physical perspective
  - suggest a list of promising method combination schemes under various environment conditions
- *Controlled test of combined methods*
  - deployment of different methods and multiple runs on the same test lanes.
  - objective is to clarify the degree of statistical dependence among methods under specific mine objects, environments, and equipment conditions.

Project work packages

- *Procedures for the use of complementary methods*
  - development of a mathematical modeling framework for combination of methods
  - practical procedures for deploying complementary methods
  - modeling will be based on prior information and data from test sites
  - the belief function framework is a principled way to incorporate prior knowledge about the environment, mine density, informal knowledge (such as interviews with local people) etc.
  - prior information will be combined with test data using a statistical decision theoretic framework
  - sensor-based methods offer information integration at various levels: early integration of sensor signals via the Dempster-Shafer belief framework to late statistical based integration of object detections from single sensors
  - very heterogeneous methods such as e.g. dogs and metal detector can only be combined at the decision level

Project work packages

- *Validation of proposed procedures*
  - test and validated on test sites in close cooperation with end-users
  - suggest practical procedures with optimal cost-benefit tradeoff - requires significant engagement of end-users needs and views
- *Mine action information management system*
  - All information about individual methods, the procedures, prior knowledge, environment etc. will be integrated in an information management system
  - aim of providing a Total Quality Management of the mine action

Are today’s methods good enough?

- some operators believe that we already have sufficient clearance efficiency
- no single method achieve more than 90% efficiency
- clearance efficiency is perceived to be higher since many mine suspected areas actually have very few mines or a very uneven mine density
- today’s post clearance control requires an unrealistically high number of sample to get statistically reliable results

Are combined methods not already the common practice?

- today's combined schemes are ad hoc practices with limited scientific support and qualification
- believe that the full advantage of combined methods and procedures has not yet been achieved

Does the project require a lot of new development?

- No basic research or development is required
- start from today’s best practice and increase knowledge about the optimal use of the existing ”toolbox”

Is it realistic to design optimal strategies under highly variable operational conditions?

- it is already very hard to adapt existing methods to work with constantly high and proven efficiency under variable operational conditions
- proposed combined framework sets lower demand on clearance efficiency of the individual method and hence less sensitivity to environmental changes
- the uncertainty about clearance efficiency will be much less important when combining methods
- overall system will have an improved robustness to changing operational conditions

Conclusions

- Statistical decision theory and modeling is essential for optimally using prior information and empirical evidence
- It is very hard to assess the necessary high performance which is required to have a tolerable risk of casualty
- Combined method are promising to overcome current problems