Statistical methods for decision making in mine action

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Why do we need statistical models?

- Mine action is influenced by many uncertain factors – statistical modeling is the principled framework to handle uncertainty.
- The use of statistical modeling enables empirical, consistent and robust decisions with associated risk estimates from acquired data and prior knowledge.
- Pitfalls and misuse of statistical methods sometimes wrongly leads to the conclusion that they are of little practical use.
The elements of statistical decision theory

Data
- Sensor
- Calibration
- Post clearance
- External factors

Prior knowledge
- Physical knowledge
- Experience
- Environment

Statistical models

Loss function
- Decision
- Risk assessment

Inference:
Assign probabilities to hypotheses
Bayes theorem

\[
\text{posterior} = \frac{\text{likelihood} \cdot \text{prior}}{\text{probability of data}}
\]
What are the requirements for mine action risk

- Tolerable risk for individuals comparable to other natural risks

- **Facts**
  - 99.6% is not an unrealistic requirement
  - But... today’s methods achieve at most 90% and are hard to evaluate!!!

GICHD and FFI are currently working on such methods [Håvard Bach, Ove Dullum NDRF SC2006]
Outline

- Statistical modeling
- What are the requirements for mine detection?
- The design and evaluation of mine equipment
- Improving performance by statistical learning and information fusion
- The advantage of using combined method
A simple inference model – assigning probabilities to data

- The detection system provides the probability of detection a mine in a specific area: Prob(detect)
- The land area usage behavior pattern provides the probability of encounter

\[ \text{Prob(casualty)} = (1 - \text{Prob(detect)}) \times \text{Prob(mine encounter)} \]

For discussion of assumptions and involved factors see

“Risk Assessment of Minefields in HMA – a Bayesian Approach”
A simple loss/risk model

- Minimize the number of casualties
- Under mild assumptions this equivalent to minimizing the probability of casualty
Requirements on detection probability

\[ \text{Prob}(\text{causality}) = (1 - \text{Prob}(\text{detection})) \times \text{Prob}(\text{encounter}) \]

\[ \text{Prob}(\text{detection}) = 1 - \frac{\text{Prob}(\text{causality})}{\text{Prob}(\text{encounter})} \]

- \( \text{Prob}(\text{encounter}) = \rho \times a \)
  - \( \rho \) : homogeneous mine density (mines/m²), \( a \) : yearly footprint area (m²)
- \( \text{Prob}(\text{causality}) = 10^{-5} \) per year
## Maximum yearly footprint area in m²

<table>
<thead>
<tr>
<th>P(detection)</th>
<th>$\rho$ : mine density (mines/km²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.996</td>
<td>25000, 2500, 250, 25, 2.5</td>
</tr>
<tr>
<td>0.9</td>
<td>1000, 100, 10, 1, 0.1</td>
</tr>
</tbody>
</table>

Reference: Bjarne Haugstad, FFI
Outline

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Optimizing the MA operation

System design phase

Changing environment
- Mine types, placement
- Soil and physical properties
- Unmodeled confounds

Overfitting
- Insufficient coverage of data
- Unmodeled confounding factors
- Insufficient model fusion and selection
Designing a mine clearance system

Methods
- sensors
- dog
- mechanical

Prior knowledge
- expert
- informal

Confounding parameters
target
operation
environment

Statistical learning is a principled framework for combining information and achieving optimal decisions
Evaluation and testing

- How do we assess the performance/detection probability?
- What is the confidence?
## Confusion matrix

<table>
<thead>
<tr>
<th>Estimated</th>
<th>True</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>a</td>
</tr>
<tr>
<td>no</td>
<td>c</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>no</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>b</td>
</tr>
<tr>
<td>no</td>
<td>d</td>
</tr>
</tbody>
</table>

- **Detection probability (sensitivity):** $\frac{a}{a+c}$
- **False alarm:** $\frac{b}{a+b}$
Receiver operations curve (ROC)

detection probability %

false alarm %

Jan Larsen
Inferring the detection probability

- $N$ independent mine areas for evaluation
- $y$ detections observed
- true detection probability $\theta$

$$P(y \mid \theta) \sim \text{Binom}(\theta \mid N) = \binom{N}{y} \theta^y \theta^{N-y}$$
Posterior probability via Bayes formula

\[ P(\theta | y) = \frac{P(y | \theta)p(\theta)}{P(y)} \]
Prior probability of $\theta$

- No prior
- Non-informative prior

\[ p(\theta) = \text{Uniform}(\theta \mid 0, 1) \]

- Informative prior

\[ p(\theta) = \text{Beta}(\theta \mid \alpha, \beta) \]
Prior distribution

\[ p(\theta) \]

- Green line: \( \alpha=1, \beta=1 \)
- Red line: \( \alpha=0.9, \beta=0.6 \)

Mean = 0.6
Posterior probability is also Beta

\[ P(\theta \mid y) = \text{Beta}(\theta \mid y + \alpha, \beta + n - y) \sim \theta^{y+\alpha} \theta^{n-y+\beta} \]
HPD credible sets – the Bayesian confidence interval

$$C_{1-\varepsilon} = \{ \theta : P(\theta \mid y) \geq k(\varepsilon) \}, \quad P(C \mid y) > 1 - \varepsilon$$

- $N=50$, $y=32$, $\theta_{\text{est}}=0.64$
- $C_{95}=0.52665$, $C_{99}=0.47862$
The required number of samples $N$

- We need to be confident about the estimated detection probability

$$\text{Prob}(\theta > 99.6\%) = C_{1-\varepsilon}$$

<table>
<thead>
<tr>
<th>$\theta_{\text{est}}$</th>
<th>$C_{95%}$</th>
<th>$C_{99%}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.7%</td>
<td>9303</td>
<td>18994</td>
</tr>
<tr>
<td>99.8%</td>
<td>2285</td>
<td>3995</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\theta_{\text{est}}$</th>
<th>$C_{95%}$</th>
<th>$C_{99%}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.7%</td>
<td>8317</td>
<td>18301</td>
</tr>
<tr>
<td>99.8%</td>
<td>2147</td>
<td>3493</td>
</tr>
</tbody>
</table>

Uniform prior

Informative prior

$\alpha = 0.9, \beta = 0.6$
Credible sets when detecting 100%

Minimum number of samples $N$

<table>
<thead>
<tr>
<th></th>
<th>Prob($\theta &gt; 80%$)</th>
<th>Prob($\theta &gt; 99.6%$)</th>
<th>Prob($\theta &gt; 99.9%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{95%}$</td>
<td>13</td>
<td>747</td>
<td>2994</td>
</tr>
<tr>
<td>$C_{99%}$</td>
<td>20</td>
<td>1148</td>
<td>4602</td>
</tr>
</tbody>
</table>
Outline

- Statistical modeling
- What are the requirements for mine detection?
- The design and evaluation of mine equipment
- Improving performance by statistical learning and information fusion
- The advantage of using combined method
Improving performance by fusion of methods

- Methods (sensors, mechanical etc.) supplement each other by exploiting different aspect of physical environment

  - Late integration
  - Hierarchical integration
  - Early integration
Early integration – sensor fusion

Sensor 1

Sensor n

Trainable sensor fusion

Detection

database
Late integration – decision fusion

Sensor → Signal processing → Decision fusion

Mechanical system → Decision
Suggestion

Apply binary (mine/no mine) decision fusion to existing detection equipment
Advantages

- Combination leads to a possible exponential increase in detection performance
- Combination leads to better robustness against changes in environmental conditions
Challenges

- Need for certification procedure of equipment under well-specified conditions (ala ISO)
- Need for new procedures which estimate statistical dependences between existing methods
- Need for new procedures for statistically optimal combination
Dependencies between methods

Contingency tables

<table>
<thead>
<tr>
<th>Mine present</th>
<th>Method j</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>yes</td>
<td>c11</td>
</tr>
<tr>
<td>no</td>
<td>c01</td>
</tr>
</tbody>
</table>
Optimal combination

Optimal combiner depends on contingency tables
## Optimal combiner

<table>
<thead>
<tr>
<th>Method</th>
<th>Combiner</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 2 3 4 5 6 7</td>
</tr>
<tr>
<td>0 0</td>
<td>0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>0 1</td>
<td>0 0 0 1 1 1 1</td>
</tr>
<tr>
<td>1 0</td>
<td>0 1 1 0 0 1 1</td>
</tr>
<tr>
<td>1 1</td>
<td>1 0 1 0 1 0 1</td>
</tr>
</tbody>
</table>

**OR rule is optimal for independent methods**

\[ 2^{2^{K-1}} - 1 \] possible combiners
OR rule is optimal for independent methods

Method 1: 1 0 0 1 0 0 1 0 1 0
Method 2: 0 1 0 0 1 0 1 1 1 0
Combined: 1 1 0 1 1 0 1 1 1 0

\[ P_d(OR) = P(\hat{y}_1 \lor \hat{y}_2 = 1 \mid y = 1) \]
\[ = 1 - P(\hat{y}_1 = 0 \land \hat{y}_2 = 0 \mid y = 1) \]
\[ = 1 - P(\hat{y}_1 = 0 \mid y = 1) \cdot P(\hat{y}_2 = 0 \mid y = 1) \]
\[ = 1 - (1 - P_{d1}) \cdot (1 - P_{d2}) \]
False alarm follows a similar rule

\[ P_{fa}(OR) = \]
\[ P(\hat{y}_1 \lor \hat{y}_2 = 1 \mid y = 0) \]
\[ = 1 - P(\hat{y}_1 = 0 \land \hat{y}_2 = 0 \mid y = 0) \]
\[ = 1 - P(\hat{y}_1 = 0 \mid y = 0) \cdot P(\hat{y}_2 = 0 \mid y = 0) \]
\[ = 1 - (1 - P_{fa1}) \cdot (1 - P_{fa2}) \]
Example

\[ p_{d1} = 0.8, p_{fa1} = 0.1 \quad p_{d2} = 0.7, p_{fa2} = 0.1 \]

\[ p_d = 1 - (1 - 0.8) \cdot (1 - 0.7) = 0.94 \]
\[ p_{fa} = 1 - (1 - 0.1) \cdot (1 - 0.1) = 0.19 \]

Exponential increase in detection rate
Linear increase in false alarm rate

Joint discussions with: Bjarne Haugstad
Testing independence – Fisher’s exact test

Hypothesis: Method i and j are independent

Alternatives: Dependent or positively (negatively) correlated

<table>
<thead>
<tr>
<th>Method j</th>
<th>yes</th>
<th>no</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>c11</td>
<td>c10</td>
</tr>
<tr>
<td>no</td>
<td>c01</td>
<td>c00</td>
</tr>
</tbody>
</table>

\[
H : P(\hat{y}_i = 0, \hat{y}_j = 0) = P(\hat{y}_i = 0) \cdot P(\hat{y}_j = 0)
\]

\[
A : P(\hat{y}_i = 0, \hat{y}_j = 0) > P(\hat{y}_i = 0) \cdot P(\hat{y}_j = 0)
\]
Artificial example

- N=23 mines
- Method 1: \( P(\text{detection}) = 0.8 \), \( P(\text{false alarm}) = 0.1 \)
- Resolution: 64 cells

How does number of mines and cells influence the analysis?

How does detection and false alarm rate influence the possibility of gaining by combining methods?
Resolution

- Many cells provide possibility of accurate spatial localization of mines
- Good estimation of false alarm rate
- Poor detection rate

- Increased possibility of reliably estimating $P(\text{no mines in area})$
- Poor spatial localization
## Confusion matrix for method 1

<table>
<thead>
<tr>
<th>Estimated</th>
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</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>19</td>
</tr>
<tr>
<td>no</td>
<td>4</td>
</tr>
<tr>
<td>no</td>
<td>36</td>
</tr>
<tr>
<td>no</td>
<td>5</td>
</tr>
</tbody>
</table>
Confidence of estimated detection rate

- With $N=23$ mines 95%-credible intervals for detection rates are extremely large!!!

Method 1 (flail): [64.5% 82.6% 93.8%]

Method 2 (MD): [50.4% 69.6% 84.8%]
Confidence for false alarm rates

- Determined by deployed resolution
- Large resolution - many cells gives many possibilities to evaluate false alarm.
- In present case: 64-23=41 non-mine cells

Method1 (flail): [4.9% 12.2% 24.0%]
Detection rates

Flail: 82.6%
Metal detector: 69.6%
Combined: 91.3%
False alarm rates

- Flail: 12.2%
- Metal detector: 7.3%
- Combined: 17.1%
Comparing methods

- Is the combined method better than any of the two original?
- Since methods are evaluated on same data a paired statistical McNemar with improved power is useful

Method 1 (flail): 82.6% < 91.3% Combined

Method 2 (MD): 69.6% < 91.3% Combined
They keys to a successful mine clearance system

- Use statistical learning which combines all available information in an optimal way
  - informal knowledge
  - data from design test phase
  - confounding parameters (environment, target, operational)

- Combine many different methods using statistical fusion

MineHunt System and HOSA concepts have been presented at NDRF summer conferences (98, 99, 01)
Outline

- Statistical modeling
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How do we proceed?

NDRF has decided to take a leading role in initiating a project COLUMBINE to carry out suggested research.
Project proposal

- Combine existing techniques: mechanical flail, dogs, metal detector, ground penetrating radar
- The methods use very different physical properties of mines and environment, hence the error patterns are likely to be independent
- Combining a few 60-90% methods will reach the goal
- Cost of 60%-90% systems are lower and requires fewer samples to evaluate and certify reliably
- Full efficiency and economic advantages has to include quality and management aspects
Project work packages

- **Current technologies**
  - identification a number of available and techniques
  - aim is to clarify how information about the methods and their operation can be extracted and stored in an efficient way

- **Physical properties of current technologies**
  - the aim is to get knowledge about how independent the methods are from a physical perspective
  - suggest a list of promising method combination schemes under various environment conditions

- **Controlled test of combined methods**
  - deployment of different methods and multiple runs on the same test lanes.
  - objective is to clarify the degree of statistical dependence among methods under specific mine objects, environments, and equipment conditions.
Project work packages

- Procedures for the use of complementary methods
  - development of a mathematical modeling framework for combination of methods
  - practical procedures for deploying complementary methods
  - modeling will be based on prior information and data from test sites
  - the belief function framework is a principled way to incorporate prior knowledge about the environment, mine density, informal knowledge (such as interviews with local people) etc.
  - prior information will be combined with test data using a statistical decision theoretic framework
  - sensor-based methods offer information integration at various levels: early integration of sensor signals via the Dempster-Shafer belief framework to late statistical based integration of object detections from single sensors
  - very heterogeneous methods such as e.g. dogs and metal detector can only be combined at the decision level
Project work packages

- **Validation of proposed procedures**
  - test and validated on test sites in close cooperation with end-users
  - suggest practical procedures with optimal cost-benefit tradeoff - requires significant engagement of end-users needs and views

- **Mine action information management system**
  - All information about individual methods, the procedures, prior knowledge, environment etc. will be integrated in an information management system
  - aim of providing a Total Quality Management of the mine action
Are today’s methods good enough?

- Some operators believe that we already have sufficient clearance efficiency.
- No single method achieves more than 90% efficiency.
- Clearance efficiency is perceived to be higher since many mine suspected areas actually have very few mines or a very uneven mine density.
- Today’s post clearance control requires an unrealistically high number of samples to get statistically reliable results.
Are combined methods not already the common practice?

- today’s combined schemes are ad hoc practices with limited scientific support and qualification
- believe that the full advantage of combined methods and procedures has not yet been achieved
Does the project require a lot of new development?

- No basic research or development is required
- start from today’s best practice and increase knowledge about the optimal use of the existing “toolbox”
Is it realistic to design optimal strategies under highly variable operational conditions?

- It is already very hard to adapt existing methods to work with constantly high and proven efficiency under variable operational conditions.
- The proposed combined framework sets lower demand on clearance efficiency of the individual method and hence less sensitivity to environmental changes.
- The uncertainty about clearance efficiency will be much less important when combining methods.
- Overall system will have an improved robustness to changing operational conditions.
Conclusions

- Statistical decision theory and modeling is essential for optimally using prior information and empirical evidence.
- It is very hard to assess the necessary high performance which is required to have a tolerable risk of casualty.
- Combined method are promising to overcome current problems.