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# Modelling Danish local CHP on market conditions

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*In Denmark, the development of local combined heat and power (CHP) plants has been characterised by large growth throughout the nineties, based in part on government subsidies in the form of feed-in tariffs. Simultaneously, there has been a significant growth of wind power, particularly in the Western Danish system. As both the power produced by the local CHPs and the wind power are prioritised, the production of these types of power is occasionally sufficient to meet the total demand in the system, causing the market price to drop dramatically, sometimes even to zero-level.*

*In line with the liberalisation process of the energy sectors of the EU countries, it is however anticipated that Danish local CHP are to begin operating on market conditions within the year 2005. This means that the income that the local CHPs previously gained from selling electricity at the feed-in tariff is replaced in part by income gained from selling electricity on the Nordic spot market, Nord Pool. Thus, the production quantities of the local CHPs will depend on the market price.*

*This paper analyses the new situation. This is done by creating a model for the supply function of a local CHP, which takes into account the local heat demand as well as technical factors such as heat storage facilities and production unit characteristics. Based on an adaptive prognosis for electricity spot prices, bids for the spot market are made in accordance with the rules of the Nord Pool 24-hour cycle.*

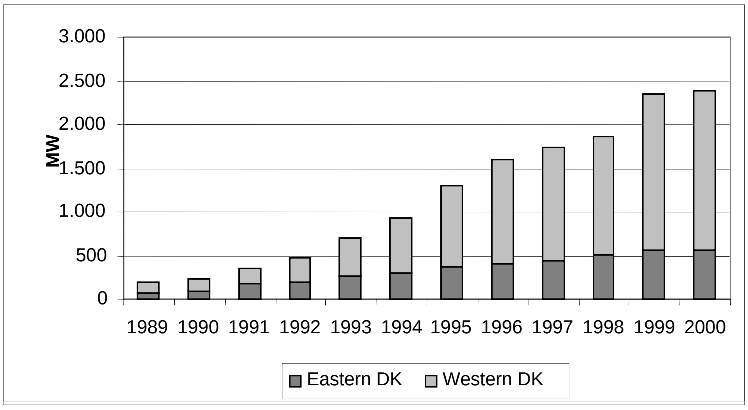
*The paper will discuss the consequences of acting in a liberalised market for a given CHP plant, based on the abovementioned bottom-up model. The key assumption determining the bottom line is the electricity spot price. The formation of the spot price in the Nordic area depends heavily upon the state of the water reservoirs in Norway and Sweden. For this reason, the analysis is undertaken as a parametric study of the electricity spot price.*

Keywords: *local CHP, liberalisation process, market conditions, Denmark, spot market.*

## 1. INTRODUCTION

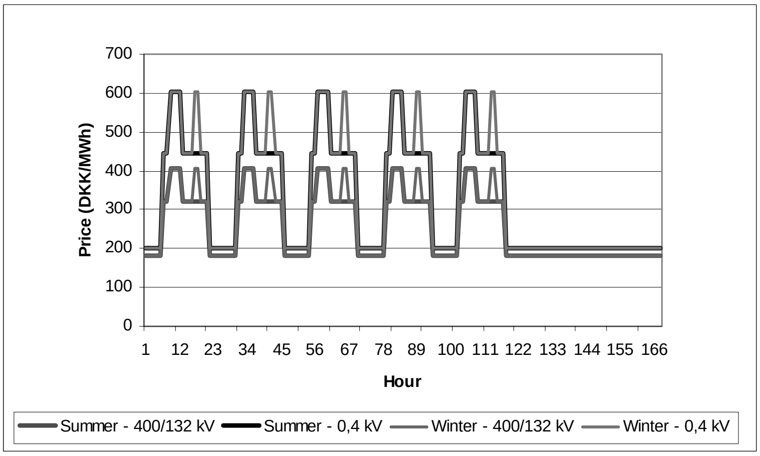
In Denmark, the development of local combined heat and power (CHP) plants has been characterised by large growth throughout the nineties (cf. Figure 1), based in part on government subsidies in the form of feed-in tariffs. In 2003, local CHP production in Western Denmark constituted almost a quarter of the total production [3]. Simultaneously, there has been a significant growth of wind power, particularly in the Western Danish system. As both the power produced by the local CHPs and the wind power are prioritised, the production of these types of power is occasionally sufficient to meet the total demand in the system, causing the market price to drop dramatically, sometimes even to zero-level.

**Figure 1. Local CHP growth during the nineties.**




At present, Danish local CHP plants trade electricity based on a three stage feed-in tariff given by a high price in peak load hours, a slightly lower price during high load hours, and a low price during low load hours (see Figure 2).

**Figure 2. The three stage tariff, according to which the local CHP plants sell their electricity at present.**




Further, the plants are obligated to cover the heat demand by CHP production. Only if there is insufficient capacity on the CHP units, may boilers be employed to meet demand. The electricity produced on the CHP units is then sold according to the abovementioned three stage tariff.

According to new legislation [1], however, all plants larger than 10 MW must operate completely on market conditions by January 1, 2005; all plants between 5 and 10 MW have an additional two year respite before they also must operate on market conditions; and plants smaller than 5 MW have a five year respite before their situation is to be re-evaluated.

The most significant difference regarding production planning on the local CHP plants with the new legislation, compared to the three stage tariff, is that prices are no longer known in advance. Thus, the local CHP must make their planning decisions under uncertainty. In addition, it seems reasonable to expect that the requirement mentioned above regarding the fulfilment of the heat demand primarily using the CHP unit will be slackened.

The mathematical models presented in this paper are aimed towards that particular problem, i.e., given a certain expectancy of the prices during the coming day, a production plan (which translates into a bid) is conceived that minimises production cost while meeting the given heat demand.

## 2. MODELLING A LOCAL CHP PLANT

### 2.1. The CHP plant

In the following, consider the situation where a local CHP plant supplies heat to the consumers and electricity to the electricity net. The plant consists of a CHP unit (typically a gas-fuelled motor), which produces both heat and power, and a boiler, which only produces heat. Furthermore, the plant is equipped with a heat storage facility. The electricity produced at the plant is sold on the Nord Pool spot market [4]. Electricity production bids must be submitted to this market, at a time where the spot prices are not yet determined.

The purpose of the models developed here is to achieve the least possible expected costs of supplying heat to the heat consumers. The most important element of this is determining how the local CHP should submit bids to Nord Pool.

The electricity-to-heat ratio (in the sequel termed the back pressure value) on the CHP unit is denoted $c_m$, i.e., 1 MWh heat produced corresponds to $c_m$ MWh power produced, and the maximal heat production is denoted $K$. Further, $V_{\max}$ denotes the limit on the heat storage facility. The production expenses on the plant are $c^{kv}$ (DKK/MWh$_{heat}$) for production on the CHP unit (note, that both heat and power are produced), and $c^k$ (DKK/MWh$_{heat}$) for heat from the boiler. Because of the interconnection between heat and power production on the CHP unit, one cannot be priced without considering the other.

### 2.2. The spot market

The procedure regarding the spot market is as follows. Early in the day prices and volumes are bid to Nord Pool, which then determines the spot price for every hour in the following 24 hour day (from midnight to midnight) based on a joint evaluation of the supply from the electricity producers and the demand of the consumers. In this case, it is assumed that the price which the CHP plant receives for the sale of electricity does not depend on the price or amount of the electricity bid to the market. Thus, the spot price is unknown at the time when the decision of the bidding price of the electricity produced at the CHP plant is made.

However, based on previous observations of the development of the spot price during a normal day at the given time of year, it is possible to make a qualified guess as to what the spot price might be. These guesses are denoted $\pi_t^s$, where $t = 1,\ldots,24$ indicates the time of day and $s = 1,\ldots,S$ indicates a suitable set of possible developments of the varying circumstances that influence the spot price (such as changes in weather, equipment failure, etc.). A reasonable set of instances of the spot price and associated probabilities $\phi^s$ may be attained by considering data from Nord Pool.

### 2.3. Models

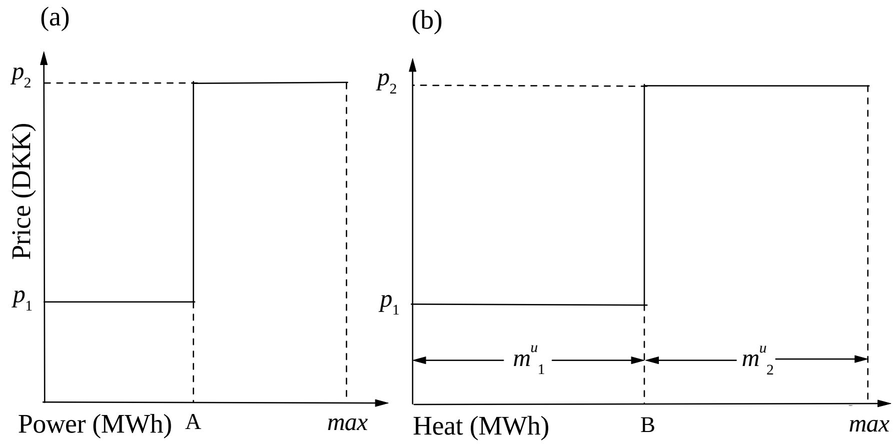
The objective is to minimise the expected net cost by optimising the hourly bid to Nord Pool while meeting heat demand. For a given hour, two possible situations may arise:

1. *The heat produced can be utilised*. The heat could alternately have been produced on the boiler at the price $c^k$ DKK/MWh$_{heat}$. The surplus costs of the CHP unit's power production is therefore $p_1 = (c^{kv} - c^k)/c_m$ DKK/MWh$_{power}$.

2. *The heat produced cannot be utilised*. The cost of power production is $p_2 = c^{kv}/c_m$ DKK/MWh$_{power}$.

Clearly, $p_1 < p_2$. A sketch of the supply curve of the CHP plant for a given hour may be seen in Figure 3, where $m_1^u$ resp. $m_2^u$ is the heat volume corresponding to the power volume produced when the price is $p_1$ resp. $p_2$.

**Figure 3. Supply curve for power production on the CHP unit, (a). (b) shows the corresponding heat amount.**




In Figure 3 (a) the bid price is given as a function of the power bid to the market, and in Figure 3 (b) the bid price is given as a function of the heat ($m^u$) (i.e., the heat volume corresponding to the power volume bid to the market). The volume of power produced and sold depends on the spot price. As there is a direct connection between power and heat volumes via the back pressure value, the corresponding heat volume used ($m^{used}$) may also be said to depend on the spot price. Thus, the (heat) volumes used are

$$m_1{}^{used} = 0 \text{ and } m_2{}^{used} = 0 \qquad \text{if } \pi^s < p_1,$$

$$m_1{}^{used} = m_1{}^u \text{ and } m_2{}^{used} = 0 \qquad \text{if } p_1 < \pi^s < p_2,$$

$$m_1{}^{used} = m_1{}^u \text{ and } m_2{}^{used} = m_2{}^u \quad \text{if } p_2 < \pi^s,$$

where $m_1{}^u$ and $m_2{}^u$ are the heat volumes corresponding to the power volumes bid at price $p_1$ and $p_2$, respectively. Any missing heat is produced on the boiler. As a help variable, define the binary indicator $\delta$ as

$$\delta_{1t}^s = \begin{cases} 1, & p_1 < \pi^s \\ 0, & \pi^s < p_1 \end{cases} \qquad \text{and} \qquad \delta_{2t}^s = \begin{cases} 1, & p_2 < \pi^s \\ 0, & \pi^s < p_2 \end{cases}.$$

See section 7 for a list of symbols.

### 2.3.1. Model with simple storage

The heat demand at hour $t$, $d_t$, is assumed deterministic. A simple storage facility is considered in this model. Here, 'simple' indicates that neither the maximal nor the minimal capacity limits of the heat storage are taken into account. There are no costs associated with retrieving heat from the heat storage or depositing heat in it. The problem is considered for single day (i.e., $T$ = 24) and a mathematical model of the problem may be written as follows:

$$\min \sum_{t=1}^{T}\sum_{s=1}^{S} \phi^s \left( \left( m_{1t}^u \delta_{1t}^s + m_{2t}^u \delta_{2t}^s \right) \cdot c^{kv} + m_{kt}^s c^k - \pi_t^s c_m \left( m_{1t}^u \delta_{1t}^s + m_{2t}^u \delta_{2t}^s \right) \right) \qquad \text{(CHP)}$$

$$\text{s.t.} \quad m_{1t}^u + m_{2t}^u \le K, \quad \forall t \qquad (1)$$

$$\sum_{t=1}^{T} m_{1t}^u \delta_{1t}^s + m_{2t}^u \delta_{2t}^s + m_{kt}^s = \sum_{t=1}^{T} d_t, \quad \forall s \qquad (2)$$

$$m_{1t}^u, m_{2t}^u, m_{kt}^s \ge 0, \quad \forall s, t \qquad (3)$$

The objective is to minimise the expected cost (which is compensated by sale of electricity) subject to the constraints that the total amount of heat corresponding to the power bid does not exceed capacity, (1); that the total heat production during each hour satisfies the heat demand during that hour, (2); and volumes cannot be negative, (3). Note that the models in the paper use the heat production as variables. Electricity production may be derived from the heat volumes using the electricity-to-heat ratio.

Exchanging the equality sign in constraint (2) for an inequality ($\leq$) allows heat cooling, i.e., surplus heat may be cooled off at no extra expenses. In this case, when the expected price is particularly high, it is possible to produce additional electricity for sale although the heat demand is already met.

Note, that this model presupposes that it is permissible to avoid production on the CHP unit when prices are low and cover the heat demand solely by boiler production (cf. section 1).

#### 2.3.2. Model with specified storage

The constraint (2) expresses that the heat production over a certain period (1,…,*T*) must equal the heat demand during that period. This is intuitively correct and permits a shift in time between production and consumption. However, in practice the storage capacity is limited and this should be reflected in the model.

Letting $V_t^s$ denote the heat storage contents at the beginning of hour *t* under scenario *s*, a mathematical model of the problem may be given as follows:

$$\min \sum_{t=1}^{T}\sum_{s=1}^{S}\phi^s\left(\left(m_{1t}^u\delta_{1t}^s + m_{2t}^u\delta_{2t}^s\right)\cdot c^{kv} + m_{kt}^s c^k - \pi_t^s c_m\left(m_{1t}^u\delta_{1t}^s + m_{2t}^u\delta_{2t}^s\right)\right) \qquad (\text{CHP}_l)$$

$$\text{s.t.} \quad m_{1t}^u + m_{2t}^u \leq K, \quad \forall t \qquad (4)$$

$$V_{t+1}^s = V_t^s + m_{1t}^u\delta_{1t}^s + m_{2t}^u\delta_{2t}^s + m_{kt}^s - d_t, \quad \forall s, t = 1,\ldots,T-1 \qquad (5)$$

$$V_1^s = V_T^s + m_{1T}^u\delta_{1T}^s + m_{2T}^u\delta_{2T}^s + m_{kT}^s - d_T, \quad \forall s \qquad (6)$$

$$0 \leq V_t^s \leq V_{\max}, \quad \forall s, t \qquad (7)$$

$$m_{1t}^u, m_{2t}^u, m_{kt}^s \geq 0, \quad \forall s, t \qquad (8)$$

Once again, the equality signs in constraints (5) and (6) indicate that heat cooling is not allowed. However, the equality sign in constraint (6) may be replaced by $\leq$ if heat may be retained for the next 24-hour period. To allow cooling, the equality sign in constraint (5) must be replaced with by $\leq$.

## 3. ANALYSIS

### 3.1. Lagrangian relaxation

Two versions of the model (CHP) are considered in this section: without cooling (equality in constraint (2)), and with cooling (inequality in constraint (2)).

#### 3.1.1. With cooling

The dual Lagrangian problem can be written as

$$\begin{aligned} &\max \quad f\left(\lambda^s\right) \\ &\text{s.t.} \quad \lambda^s \geq 0 \end{aligned}$$

where the multiplier $\lambda^s$ is associated with constraint (2) and where

$$f\left(\lambda^s\right)=\lambda^s\sum_t d_t+\min_{m\geq 0}\left\{\sum_{s,t}\phi^s\left(\left(m_{1t}^u\delta_{1t}^s+m_{2t}^u\delta_{2t}^s\right)\cdot c^{kv}+m_{kt}^s c^k-\pi_t^s c_m\left(m_{1t}^u\delta_{1t}^s+m_{2t}^u\delta_{2t}^s\right)\right)\right.$$

$$\left.-\sum_s\lambda^s\left(\sum_t m_{1t}^u\delta_{1t}^s+m_{2t}^u\delta_{2t}^s+m_{kt}^s\right)\right\}. \tag{9}$$

Note, that this objective function is unchanged when constraint (2) has an equality sign (i.e., when cooling is not allowed). The first term of the objective function (9) is constant and therefore has no influence on the optimisation. The second term may be rewritten by merging the coefficients of the decision variables (i.e., the *m*'s):

$$\min_{m\geq 0}\left\{\sum_t m_{1t}^u\left[\sum_s\delta_{1t}^s\left(\phi^s\left(c^{kv}-\pi_t^s c_m\right)-\lambda^s\right)\right]+\sum_t m_{2t}^u\left[\sum_s\delta_{2t}^s\left(\phi^s\left(c^{kv}-\pi_t^s c_m\right)-\lambda^s\right)\right]+\sum_{s,t}m_{kt}^s\left(\phi^s c^k-\lambda^s\right)\right\}$$

(10)

Thus, the variables may be optimised separately, reforming the problem to:

- For each of the $T$ $m_{1t}^u$ : min $m_{1t}^u\left[\sum_s\delta_{1t}^s\left(\phi^s\left(c^{kv}-\pi_t^s c_m\right)-\lambda^s\right)\right]$. (11)
- For each of the $T$ $m_{2t}^u$ : min $m_{2t}^u\left[\sum_s\delta_{2t}^s\left(\phi^s\left(c^{kv}-\pi_t^s c_m\right)-\lambda^s\right)\right]$. (12)
- For each of the $T$ x $S$ $m_{kt}^s$ : min $m_{kt}^s\left(\phi^s c^k-\lambda^s\right)$. (13)

The coefficients of $m_{1t}^u$ and $m_{2t}^u$ are of particular interest. They may be interpreted in the following way: $\phi^s(c^{kv} - \pi_t^s c_m)$ is the expected profit from sale of power in hour $t$ - however, the $\delta$ with which the expression is multiplied indicates that power is not sold unless the spot price is acceptable. Furthermore, $\lambda^s$ may be interpreted as the profit from "sales" of heat to a heat storage facility in hour $t$ (or what a fictive manager of the storage facility is willing to pay for heat, cf. section 4.5).

When heat cooling is permitted, only the capacity of the CHP unit limits the production, which makes it a reasonable move to turn the inequality in (1) into an equality. This may be argued as follows:

When the spot price is high ($\pi_t^s > p_2$) for but one of the instances, full capacity must be bid, i.e., $m_{1t}^u + m_{2t}^u = K$; otherwise a possible profit would be wasted. Should the spot price be in or under the medium level ($\pi_t^s < p_2$) for all instances of the spot price, the value of $m_{2t}^u$ is of no consequence. Thus, $m_{2t}^u$ may be chosen so that $m_{1t}^u + m_{2t}^u = K$, regardless of the value of $m_{1t}^u$.

If all instances of the spot price are in the low level ($\pi_t^s < p_1$), nothing is sold, wherefore the values of both $m_{1t}^u$ and $m_{2t}^u$ are of no consequence. Here it is also permissible to demand $m_{1t}^u + m_{2t}^u = K$, without influencing the optimal solution decisively. Based on this argumentation the inequality sign in (1) is replaced by an equality sign in model (CHP) from hereon in this section.

Based on this argumentation, constraint (1) makes it possible to write $m_{2t}^u$ as an expression of $m_{1t}^u$ and $K$: $m_{2t}^u = K - m_{1t}^u$. Thus, the two terms containing $m_{1t}^u$ and $m_{2t}^u$ may be written jointly as

$$m_{1t}^u\left\{\sum_s\left(\delta_{1t}^s-\delta_{2t}^s\right)\left[\phi^s\left(c^{kv}-\pi_t^s c_m\right)-\lambda^s\right]\right\}, \tag{14}$$

where the term concerning $K$ may be disregarded, as it becomes constant and therefore has no influence on the optimisation.

Let $\alpha_{1t}$ denote the coefficient for $m_{1t}^u$ in (11). As the expression is to be minimised, it is clear how $m_{1t}^u$ should be determined depending on the value of $\alpha_{1t}$. In the case of high or low spot price ($\pi_t^s > p_2$ and $\pi_t^s < p_1$ respectively), $\delta_{1t}^s$ - $\delta_{2t}^s$ becomes 0 (and consequently, $\alpha_{1t} = 0$). Therefore, these instances have no influence on the determination of $\alpha_{1t}$. This corresponds with the fact that in these cases the result is already given regardless of the value of $m_{1t}^u$ : at the high price everything is sold and at the low price nothing is sold.

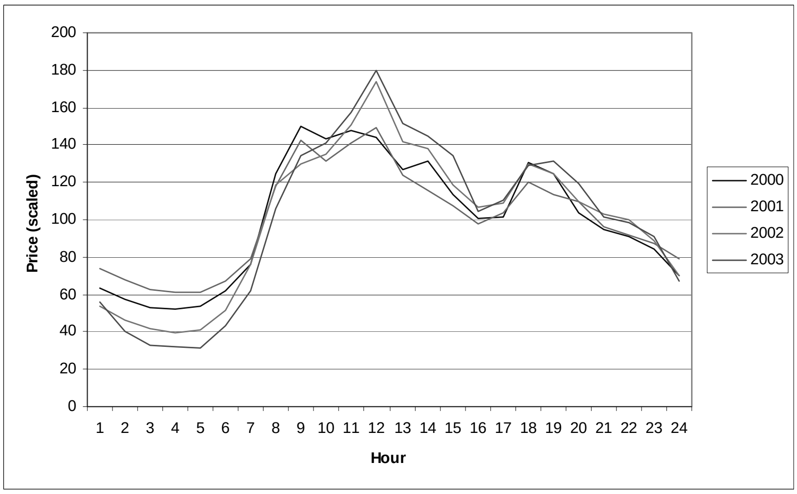
From the above it may be noted that when cooling is allowed it does not matter how much above the highest or below the lowest level the spot price is, only the medium level is interesting. At the medium price (i.e., $p_1 < \pi_t^s < p_2$), $\delta_{1t}^s$ - $\delta_{2t}^s$ becomes 1 and so these are the cases which truly determine the value of $\alpha_{1t}$. Three possibilities exist:

1. $\alpha_{1t} > 0 \Rightarrow m_{1t}^u = 0$,
2. $\alpha_{1t} < 0 \Rightarrow m_{1t}^u = K$,
3. $\alpha_{1t} = 0 \Rightarrow$ all values of $\alpha_{1t}$ between 0 and $K$ minimise the Lagrangian function.

The coefficient $\alpha$ indicates that for given $\lambda^s$'s the solution is such that the hours with negative $\alpha$ values bid full capacity at the low price. Thus it is, indirectly, the values of the $\lambda^s$'s that determine how many hours in which to bid full capacity to the market.

Considering historic average daily spot prices, as illustrated in Figure 4, an obvious daily pattern emerges.

**Figure 4. Average daily spot prices for Western Denmark 2000-2003.**




The hypothesis may be postulated that only the shape of the curve has influence on the optimal bidding, and that the general level of the curve has none. However, it may be shown with a simple example that when working with the models defined in this paper, the general price level is not without importance.

Consider two consecutive hours, let $S = 2$, and assume that both scenarios occur with equal probability, i.e., $\phi^1 = \phi^2 = 0.5$. Letting the costs be given as $c^{kv} = 150$ DKK/MWh and $c^k = 105$ DKK/MWh, and defining $c_m = 0.5$, yields $p_1 = 90$ DKK/MWh and $p_2 = 300$ DKK/MWh. Let the spot prices be for hour 1: $\pi_1^1 = 70$, $\pi_1^2 = 110$; and for hour 2: $\pi_2^1 = 130$, $\pi_2^2 = 40$. For the sake of simplicity, it is assumed that only 1 MWh heat is produced during the entire period. Considering the two hours separately yields:

*Hour 1*: In this case, $\pi_1^1 < p_1$ which means $\delta_{11}^1 = \delta_{21}^1 = 0$, and $p_1 < \pi_1^2 < p_2$ which means $\delta_{11}^2 = 1$ and $\delta_{21}^2 = 0$. The expected cost of producing in hour 1 is thus

$$0.5 \cdot (0 \cdot 150 + 1 \cdot 150 - 70 \cdot 0.5 \cdot 0) + 0.5 \cdot (1 \cdot 150 + 0 \cdot 150 - 110 \cdot 0.5 \cdot 1) = 100 \text{ DKK/MWh.}$$

*Hour 2*: In this case, $p_1 < \pi_2^1 < p_2$ which means $\delta_{12}^1 = 1$ and $\delta_{22}^1 = 0$, and $\pi_2^2 < p_1$ which means $\delta_{12}^2 = \delta_{22}^2 = 0$. The expected cost of producing in hour 2 is thus

$$0.5 \cdot (1 \cdot 150 + 0 \cdot 150 - 130 \cdot 0.5 \cdot 0) + 0.5 \cdot (0 \cdot 150 + 1 \cdot 150 - 40 \cdot 0.5 \cdot 1) = 95 \text{ DKK/MWh.}$$

It is desirable to place production in the hour with least cost, i.e., hour 2. However, when adjusting the level of the spot prices upwards with 100 DKK/MWh, the situation changes. Once again considering the two hours separately yields:

*Hour 1*: In this case, $p_1 < \pi_1^1 < p_2$ which means $\delta_{11}^1 = 1$ and $\delta_{21}^1 = 0$, and $p_1 < \pi_1^2 < p_2$ which means $\delta_{11}^2 = 1$ and $\delta_{21}^2 = 0$. The expected cost of producing in hour 1 is thus

$$0.5 \cdot (1 \cdot 150 + 0 \cdot 150 - 170 \cdot 0.5 \cdot 1) + 0.5 \cdot (1 \cdot 150 + 0 \cdot 150 - 210 \cdot 0.5 \cdot 1) = 55 \text{ DKK/MWh.}$$

*Hour 2*: In this case, $p_1 < \pi_2^1 < p_2$ which means $\delta_{12}^1 = 1$ and $\delta_{22}^1 = 0$, and $p_1 < \pi_2^2 < p_2$ which means $\delta_{12}^2 = 1$ and $\delta_{22}^2 = 0$. The expected cost of producing in hour 2 is thus

$$0.5 \cdot (1 \cdot 150 + 0 \cdot 150 - 230 \cdot 0.5 \cdot 1) + 0.5 \cdot (1 \cdot 150 + 0 \cdot 150 - 140 \cdot 0.5 \cdot 1) = 57.5 \text{ DKK/MWh.}$$

These results lead to the conclusion that production should be placed in the first hour, which is a direct contradiction to the conclusion at the original spot price level.

### 3.1.2. Without cooling

In this case, constraint (2) stands as is, i.e. as an equality. Contrarily, the capacity constraint for the CHP unit, (1), cannot be an equality, as it is only permissible to produce the heat which is demanded and thus not to cool off any excess heat. Therefore, this constraint should be included in the Lagrangian function. Associating the multiplier $\mu_t$ with constraint (1), the dual Lagrangian problem becomes

$$\begin{array}{ll} \max & f(\lambda^s, \mu_t) \\ \text{s.t.} & \lambda^s \text{ unlimited} \\ & \mu_t \le 0 \end{array}$$

where

$$f(\lambda^s) = \mu_t K + \lambda^s \sum_t d_t + \min_{m \ge 0} \Bigg\{ \sum_{s,t} \phi^s \left( \left( m_{1t}^u \delta_{1t}^s + m_{2t}^u \delta_{2t}^s \right) \cdot c^{kv} + m_{kt}^s c^k - \pi_t^s c_m \left( m_{1t}^u \delta_{1t}^s + m_{2t}^u \delta_{2t}^s \right) \right)$$
$$- \sum_t \mu_t \left( m_{1t}^u + m_{2t}^u \right) - \sum_s \lambda^s \left( \sum_t m_{1t}^u \delta_{1t}^s + m_{2t}^u \delta_{2t}^s + m_{kt}^s \right) \Bigg\}. \tag{15}$$

Only the minimising term of (15) has any influence on the optimisation, as the two first terms are constant. Once again, the coefficients of the decision variables are merged and then become:

- For each of the $T$ $m_{1t}^u$ : min $m_{1t}^u \left[ -\mu_t + \sum_s \delta_{1t}^s \left( \phi^s \left( c^{kv} - \pi_t^s c_m \right) - \lambda^s \right) \right]$. (16)
- For each of the $T$ $m_{2t}^u$ : min $m_{2t}^u \left[ -\mu_t + \sum_s \delta_{2t}^s \left( \phi^s \left( c^{kv} - \pi_t^s c_m \right) - \lambda^s \right) \right]$. (17)
- For each of the $T$ x $S$ $m_{kt}^s$ : min $m_{kt}^s \left( \phi^s c^k - \lambda^s \right)$. (18)

In contrast to the case where cooling is permitted, the coefficients of the two $m^u$'s cannot be merged. Let $\alpha_{1t}$ denote the coefficient for $m_{1t}^u$ and $\alpha_{2t}$ the coefficient for $m_{2t}^u$. For both coefficients the expression to the right of the summation may be interpreted as the expected value of the profit (as the heat is assumed to be 'sold' at the price $\lambda^s$). Note, that contrary to the case where cooling is allowed, in this case also high spot prices (i.e., higher than $p_2$) will influence the determination of the $\alpha$'s. It should also be noted that it could happen that full capacity would not be bid to the market, as $\alpha_{1t}$ and $\alpha_{2t}$ may have opposite signs. In the case of opposite signs there is no way of telling which $\alpha$ is positive and which is negative.

Also, note that it becomes more difficult to find a good solution – or even to guarantee a feasible one – as equality constraints (cooling not allowed) impose limitations on the model that are far less of an issue with inequality constraints (cooling allowed).

### 3.2. Shadow prices of the heat storage

Shadow prices may be interpreted as the marginal cost of production. When explaining why the shadow price has a certain value, one must ascertain which time period this marginal production takes place when specified heat storage is included, i.e., model ($CHP_l$). The marginal costs in those periods define the shadow price. The shadow prices can be related to the production patterns and costs. They may sometimes be deduced analytically, or calculated in an optimisation algorithm as a part of the optimisation.

Note, that the heat shadow prices, which may be calculated when the bid to the spot market is made, are not the same as the shadow prices that may be calculated when the sale to the spot market is known. There is a shadow price of the former kind for every hour, but one for every spot price scenario of the latter kind. The difference is, that the shadow prices calculated at the time of bidding relate to the expected marginal cost, whereas the other shadow prices relate to a specific spot price scenario (and thus to a specific instance of production).

The relation between these two sets of shadow prices is that the shadow prices calculated at the time of bidding are equal to the expected value (the weighted average) of the shadow prices calculated when the sales are known. Therefore, the numerical values of the latter shadow prices are relatively small. To obtain values comparable with the production costs, the values associated with a given instance of the spot price may be divided by the probability of the instance.

-When cooling is allowed, once the heat shadow price relating to constraint (2) is known, the optimal bid is given trivially by (11)-(13). However, in practice the heat shadow prices are not deduced until the bid is already made.

### 3.3. Derivation of optimal bidding

Based on the previous analysis some conclusions may be made relative to the nature of the optimal bids for the spot market.

Consider the first model (CHP) with simple storage. Only the case with cooling is considered here, the case with cooling may be treated similarly. Assume for the moment that the heat shadow price is known. Then (11)-(13) specify the optimal solution expressed in heat production terms (save for the indeterminate case also described in section 3.1.1), and from this the bidding may in turn be derived essentially as follows: the electricity volume corresponding to the heat volume $m_{1t}^u$ is bid at the price $p_1$, and the electricity volume corresponding to the heat volume $m_{2t}^u$ is bid at the price $p_2$ as described in section 2.3. Further details are given in section 3.1.1. Intuitively, the bidding should be concentrated on the hours with the highest expected spot prices, and presumably this may make good sense in practice. However, as shown by the counterexample in section 3.1.1 this bidding strategy is not truly optimal.

For the case with a specified storage the essentially same observations may be made. The primary difference from the case with simple storage is that now the heat shadow prices may differ between the hours. Such difference indicates that one of the storage bounds is reached or may be expected to be reached with a certain probability.

In practice the heat shadow prices are not known, and must be found by a numerical method. The models have been implemented in the modelling language GAMS [2] and the results are given in the sequel.

## 4. NUMERICAL EXAMPLES

To illustrate the models some numerical examples are presented in this section. For a single day, four different situations are considered (with and without cooling, with and without limited storage) associated with four different variations, totally, of the models (CHP) and ($CHP_l$).

In the following, let A denote (CHP); let B denote ($CHP_l$); let C denote (CHP) with cooling; and let D denote ($CHP_l$) with cooling. Five scenarios are considered represented by the spot price in Western Denmark January 1-5, 2003, each with equal probability ($\phi^s$ = 0.2 for $s$ = 1,…,5). A maximal capacity of 4 MW for the boiler is introduced. Furthermore, $c_m$ = 0.5, $c^{kv}$ = 150 DKK/MWh, $c^k$ = 105 DKK/MWh, $K$ = 5 MWh/h and the time index $t$ runs from 1 through 24. In cases B and D, where storage is limited, the storage limit is $V_{\max}$ = 15 MW, and the initial storage reserve is $V_1$ = 10 MW. The heat demand $d_t$ is given as 1.5 MW for hours 1-8 and 24, and 2.5 MW for hours 9-23. Running the models with these data yields the optimal values given in the table below.

| Model | A | B | C | D |
|---|---|---|---|---|
| Optimal value | - 4761 | - 4340 | - 6209 | - 6006 |

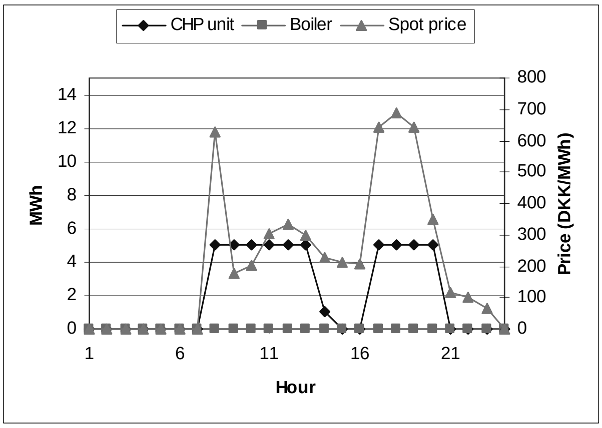
It comes as no great surprise, that better values are attained for the models with cooling (C and D) than the ones without cooling (A and B). Also, the model with unlimited storage is more effective than the one with limited storage both when cooling is allowed and when it is not. These result makes good sense mathematically, as the freedom granted by inequality constraints (cooling allowed) enables the models to attain better solutions than with equality constraints (cooling not allowed).

In the following, the four cases are considered more closely for scenarios 2 and 5.

### 4.1. Case A

This is the case of unlimited storage and no cooling (i.e., model (CHP)). Cooling is not allowed, and so no more heat may be produced than is demanded. As there are no storage limitations, the production may be scheduled where the best result is attained.

**Figure 5. Scenarios 2 (left) and 5 (right) for case A.**



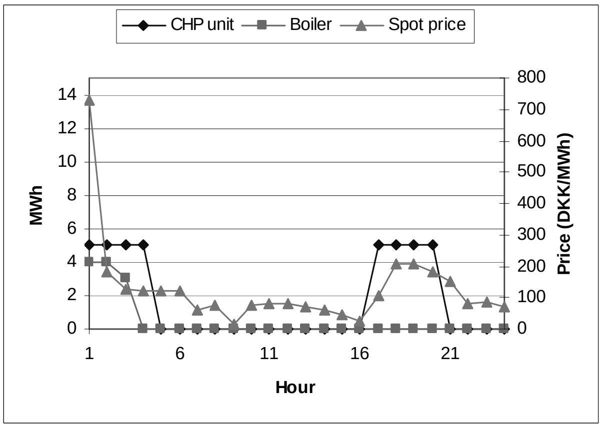
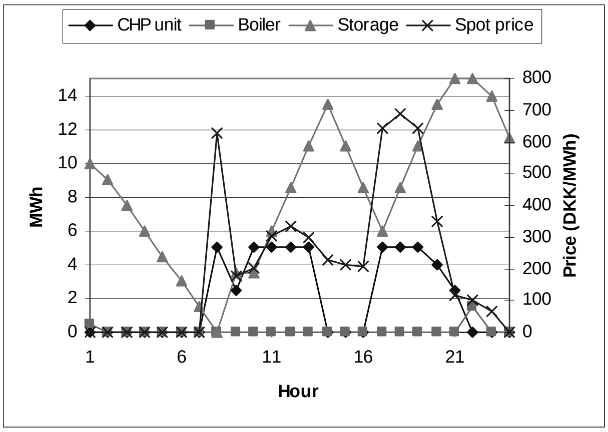



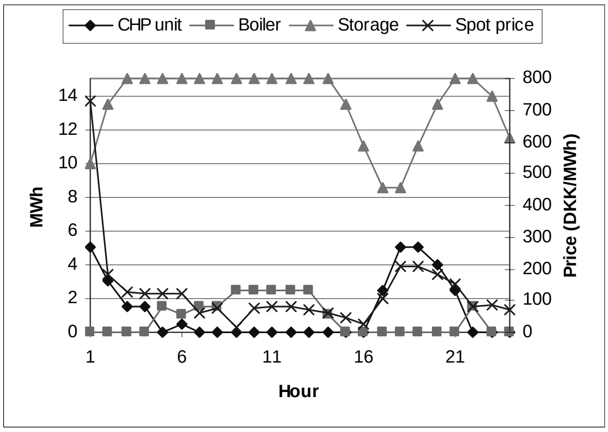

Figure 5 (left) shows, for each of the 24 hours, the heat production on the CHP unit and the boiler under price scenario 2. Additionally, the spot price is depicted. Primarily full capacity is bid except in period 14; here the bid is structured in such a way as to ensure that the total heat production equals the total heat consumption. In comparison, Figure 5 (right) depicts the same for price scenario 5. Here, the pattern is clear: when the price is high, full capacity is bid, and nothing otherwise.

### 4.2. Case B

In this case there are storage limitations but no cooling allowed (i.e., model ($CHP_l$)), which limits the optimal production pattern. Because of the storage limitations, the bid pattern is more complicated than in case A (see Figure 6) both in scenario 2 (left) and 5 (right).

**Figure 6. Scenarios 2 (left) and 5 (right) for case B.**



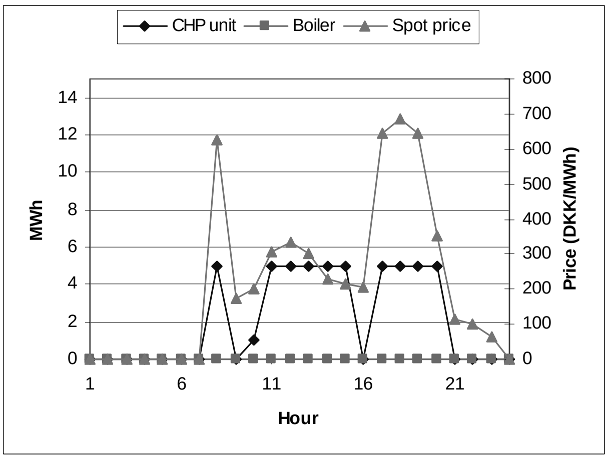



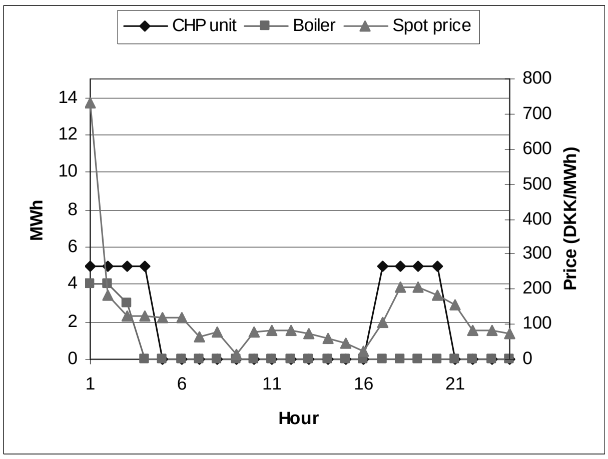

In scenario 2, with the limitations imposed, there is now boiler production both in the beginning and at the end of the day. Furthermore, it is not possible to take advantage of the high price in hour 20, as the storage is filled to capacity and cooling the heat is not permissible. In scenario 5, the pattern is also more complicated than in case A and the production on the CHP unit is constrained both by the fact that cooling is not allowed and by the storage limitations. The latter phenomenon is clearly seen in the way the storage limit is reached in hours 3-14 and again in hours 21-22.

### 4.3. Case C

Case C is the model (CHP) with cooling, i.e. inequality in constraint (2). In scenario 2, Figure 7 (left) shows that as much as possible, i.e., up to the maximal capacity on the CHP unit, is produced at the high prices. This is possible, as there are no storage limitations, which therefore allows production during any hour in the period. Furthermore, the lack of storage limitations means that boiler production may be avoided.

**Figure 7. Scenarios 2 (left) and 5 (right) for case C.**



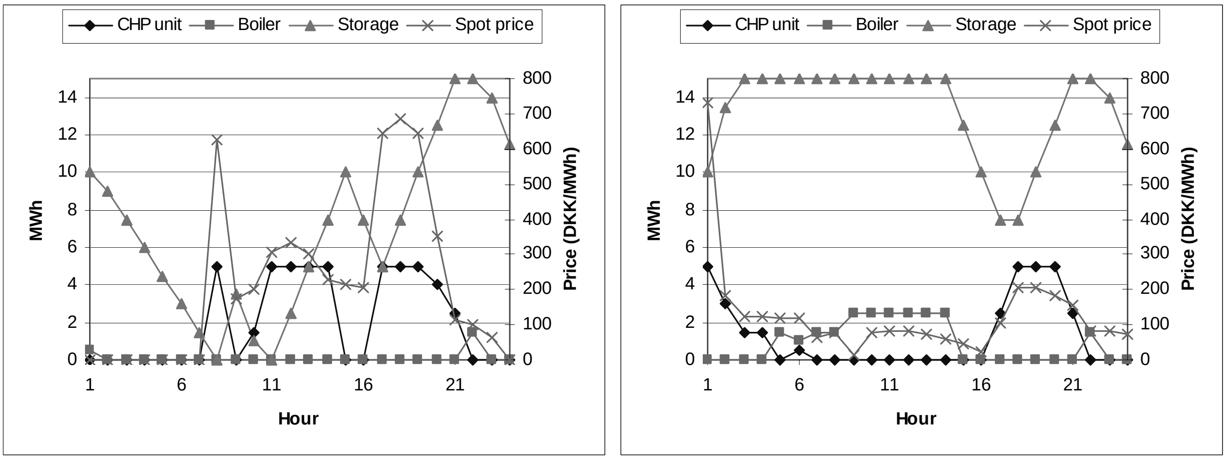

Note, that in scenario 2 0.5 MWh is produced in hour 10, even though it seems more reasonable to produce in hour 16, as the spot price in those two hours is 200.02 and 206.79 DKK/MWh, respectively. However, the production in hours 10 and 16 is not merely based on the spot price of the relevant scenario but rather on the jointly weighted spot price (cf. $\alpha_{1t}$ in section 4.1.1). The difference between the production pattern in this case and case A (see Figure 5) is due to the fact that the bid with regard to all five scenarios differs, where cooling is a deciding factor as to when the production is bid.

In scenario 5 (Figure 7, left), apart from the maximal CHP production, there is also boiler production in the beginning of the day, due to the extremely high spot price. There is also maximal CHP production later in the period but here the spot price is not high enough to warrant boiler production as well.

### 4.4. Case D

Once again limited storage, but now cooling is permitted. The limits to the storage facility incur, that production is no longer merely scheduled when prices are high. This explains the boiler production both in the beginning and the end of the period in scenario 2, in order to meet demand (see Figure 8 (left)).

**Figure 8. Scenarios 2 (left) and 5 (right) for case D.**




Further, there is production despite a very low price in hour 21, as the storage limit has already been reached and producing earlier in order to save production for later hours is no longer possible. The situation in scenario 5 (Figure 8 (right)) is basically the same as in case B, except that maximal capacity is bid in one extra hour (hour 20), which cooling permits.

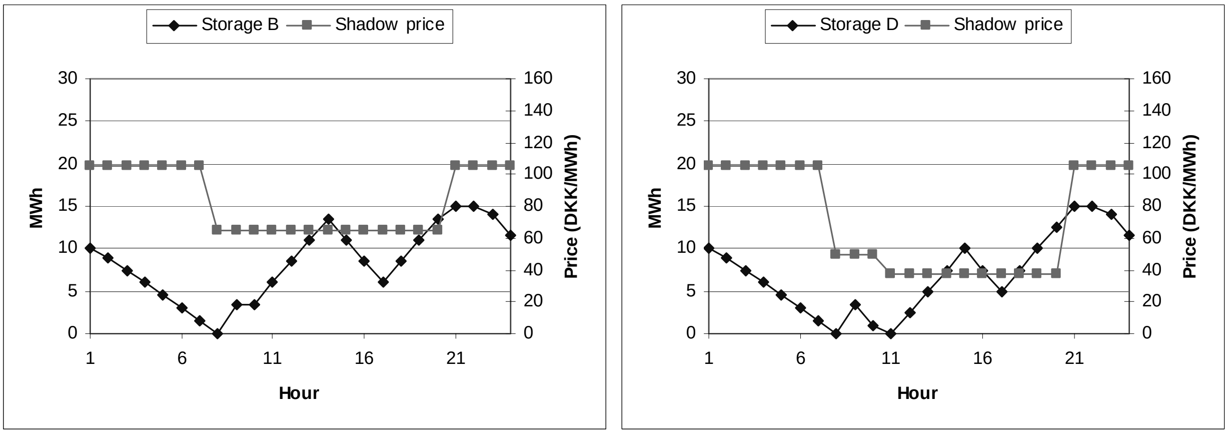
### 4.5. Shadow prices for the heat storage

In case A, considering scenario 2, the CHP unit is the marginal production unit in hour 14 (see Figure 5 (left)), but due to the 'unlimited' storage it is also marginal in all other hours. The marginal production price equals profits of power sales subtracted from the production costs: $150 - 0.5 \cdot 226 = 37$ DKK/MWh.

The same applies to case B (see Figure 9 (left)), though it should be noted that the shadow prices of the storage facility are generally higher than in case D (cf. Figure 9 (left) and (right), respectively). The explanation is that in case B cooling is not allowed, wherefore production is more expensive.

In case D, Figure 8 (left) shows that there is boiler production in hour 1. This means that boiler production is marginal in this hour. However, due to the storage facility, the boiler is also the marginal unit in hours 1-7, as the storage facility may save production in this time period. Because the storage limit is reached at the beginning of hour 8, production from hours 1 through 7 cannot be saved for later. The heat shadow prices in hours 1 through 7 are all 105 DKK/MWh, which is precisely the given production cost on the boiler.

**Figure 9. Shadow price on the heat storage and storage reserves for cases B (left) and D (right).**




The storage reaches its lower limit again in the beginning of period 11 (cf. Figure 9 (right)). The shadow price in periods 8 through 10 is 50 DKK/MWh, as seen in Figure 9 (right). Figure 8 (left) shows that the marginal unit in period 10 is the CHP unit (it produces between minimum and maximum levels). The electricity spot price in this hour is 200 DKK/MWh. The marginal cost of production in period 10 is then $150 - 0.5 \cdot 200 = 50$ DKK/MWh, which is precisely what may be read on the Figure 9 (right) as the heat shadow price in hours 8-10. Similarly, the heat shadow price in hours 22-24 is 105 DKK/MWh, which is once again the boiler production cost, and Figure 8 (left) shows that the boiler is indeed the marginal unit in period 22. The overall pattern is that when the storage reaches its lower limit, the heat shadow prices fall, and when the storage reaches its upper limit, the heat shadow prices rise.

For hours 11-20 there is an extra level to the interpretation. Recall that the shadow price may be interpreted as the marginal cost of production: if, e.g., 1 Wh extra must be produced and the cost of such a production is $x$ DKK, then the shadow price is $x$ DKK/Wh, or $10^6 \cdot x$ DKK/MWh. Similarly, if 1 Wh less must be produced, lowering the costs with $y$ DKK, then the shadow price is $y$ DKK/Wh, or $10^6 \cdot y$ DKK/MWh. However, it is not a given that $x$ and $y$ are the same value, e.g. if the marginal unit is not well defined.

In such cases it may therefore hold that one unit is marginal if more is produced while another unit, with different cost, is marginal if less is produced. An example of this may be found in hours 11-20. If less needs to be produced in an hour between 11 and 20, it will be in hour 14; here, the marginal costs are $150 - 0.5 \cdot 226 = 37$ DKK/MWh. If more needs to be produced in an hour between 11 and 20, it cannot be done on the CHP unit period 20, as it is already producing its maximum in that hour, whereas it is possible in hour 15, in which the marginal costs are $150 - 0.5 \cdot 214 = 43$ DKK/MWh. Figure 9 (right) shows the heat shadow price 37 DKK/MWh, but as argued it might as well have shown 43 DKK/MWh.

## 5. SIMULATION OF AN ENTIRE YEAR

The simulations in the following section are all based on the model with specified heat storage and no cooling, ($CHP_l$).

A simple spot price prognosis is utilised for the model. The scenarios used in the simulation of a given day are based on historical data, which are divided into two types: weekdays and weekend days. The reasoning behind this division is that when a prognosis is made for a given day, only (historical) days of the same type are considered. The prognosis (i.e., the scenarios to be used in the model) is constructed in the following way.

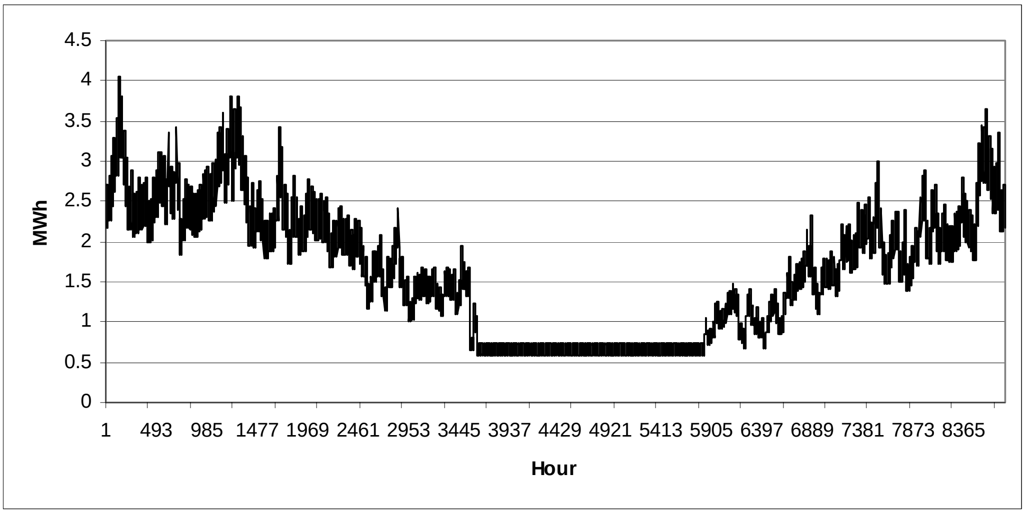
First, the number of scenarios ($N$) is chosen. Then, all but one of the scenarios are defined as the $N - 1$ previous days of the same type as the given day. Finally, an $N$'th scenario is constructed which resembles the daily spot price profile of the year (cf. Figure 3) only shifted upwards so the spot price in all hours is above $p_2$, ensuring that a bid is made even if all other scenarios are below $p_1$.

For the purpose of the simulations, an actual local CHP plant has been used as point of reference. The data is given in the table below.

| $c_m$ | $c^{kv}$ | $c^k$ | $V_{\max}$ | $p_1$ | $p_2$ |
|---|---|---|---|---|---|
| 0.848 | 345 DKK/MWh | 488 DKK/MWh | 23.2 MWh/h | 168.24 DKK | 575.12 DKK |

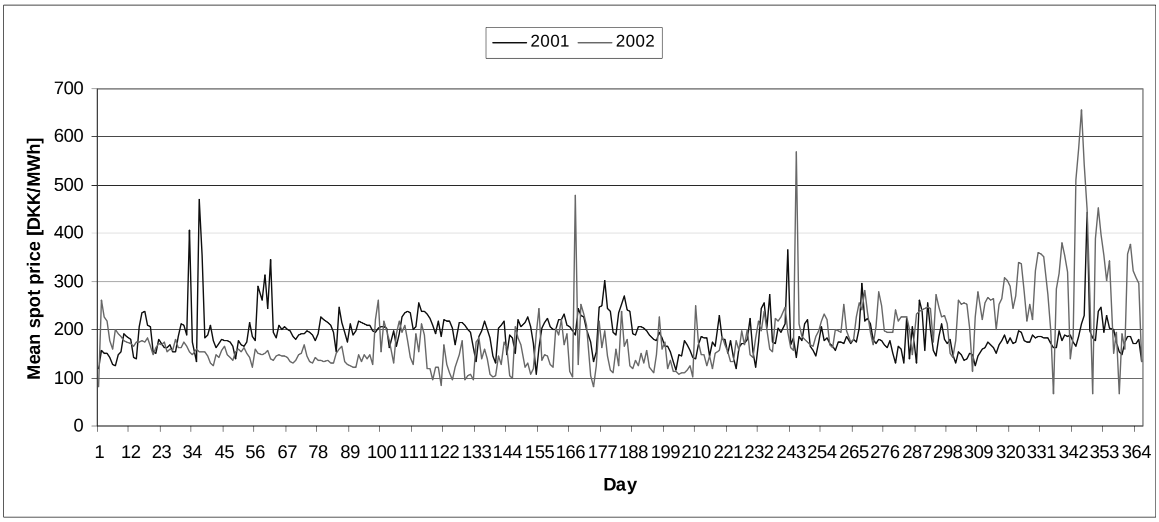
The heat demand for an entire year is illustrated in Figure 10.

**Figure 10. Heat demand for a year.**




The model is analysed using the Western Danish spot prices from 2001-2002, the daily means of which are depicted in Figure 11.

**Figure 11. Daily means of the spot price in Western Denmark, 2001-2002.**



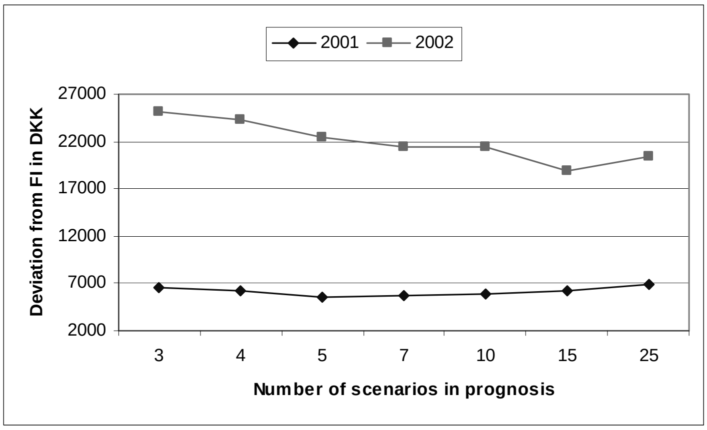

As a base of comparison, the optimal bids are calculated given that the prices are known in advance and the cost of these optimal bids is calculated. This base is termed full information (FI). It is then examined how the expected cost diverges from FI while varying the number of spot price scenarios included in the prognosis as well as the general level of the spot prices.

The structure of the simulations where the spot price is not known in advance is as follows. First, the prognosis is constructed as described above. Once the scenarios have been determined, the model is run for the given day and the results are noted. Then, the ultimate heat storage contents are set as the initial heat storage content for the next day and the process is repeated until an entire year has been simulated.

### 5.1. Number of scenarios in the prognosis

In this section it is analysed what the effect is of including different numbers of scenarios in the prognoses. Using, as mentioned, the Western Danish spot price for 2001 and 2002 yields the results depicted in Figure 12.

**Figure 12. Deviation from FI as a consequence of the number of scenarios in the prognosis.**




Initially, there is for both years a falling trend in the deviation from FI the more scenarios are included in the prognosis. However, the trend actually reverses when a large number of scenarios are included. Furthermore, the increase in run time is significant as the amount of scenarios becomes large, so it is clearly imperative not to include unnecessarily many scenarios in the prognosis. In contrast, one must include sufficiently many scenarios to get a reasonable representation of the future.

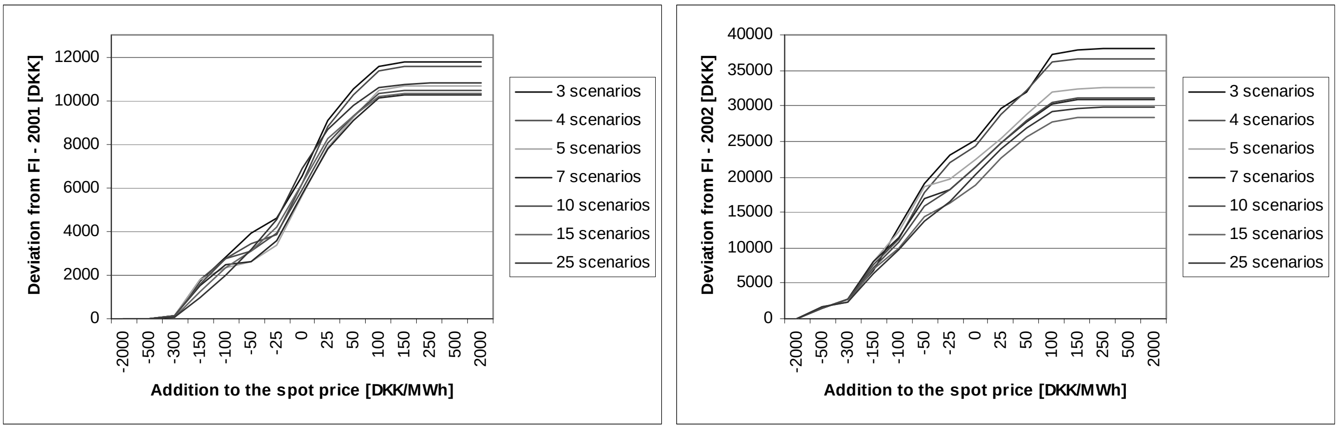
For 2001, the number of scenarios in the prognosis has no great influence on the deviation from FI (see Figure 12). The explanation for this phenomenon is probably that the level of the spot prices was relatively constant in 2001 (see Figure 11) and therefore there is not much gained by looking back three or fifteen days when constructing a prognosis.

For 2002, the deviation is generally larger. This may be explained by the fact that the prices towards the end of that year are quite volatile in their daily variation, which influences the quality of the prognosis. This trend also explains the increase in the deviation when including 25 scenarios in the prognosis, as the increasing trend during the final months is not 'caught' by the prognosis when too much history is included.

### 5.2. Level of the spot price

In section 3.3 an example was given which showed that the level of the spot price does influence the results. In this section it is shown how different spot price levels influence the deviation from FI for 2001 and 2002. The different levels are obtained by adding a given price (positive or negative) to every hour of the historical spot prices, thus shifting the level up or down in accordance with the addition. The deviation from the FI case for both 2001 and 2002 is shown in Figure 13.

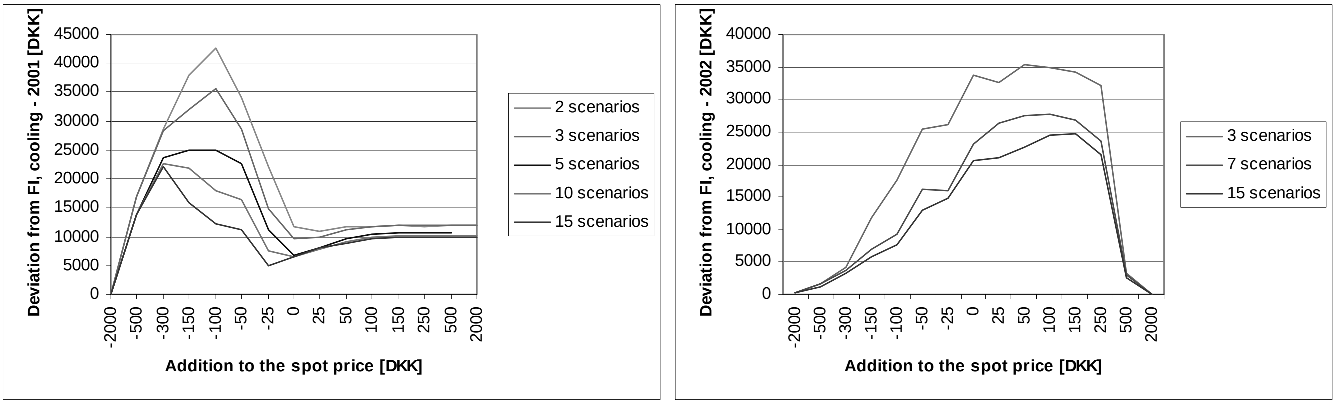
**Figure 13. Deviation from FI when cooling is not allowed.**




When the prices are very low there is actually no deviation from the FI case, as the heat demand (which is known in advance) is covered by boiler production. The constant shape of the curves at very high spot price levels is due to the fact that once the level is reached where all prices are higher than $p_1$, the same strategy is employed regardless of how high the prices are above $p_1$ and thus the error is the same for all higher levels. Once again, the deviation is greater for 2002 than 2001 for all scenarios but the explanation given in section 5.1 still holds, i.e., the large variations in the spot price in the last months of 2002 (cf. Figure 11) influence the quality of the prognosis. As a curiosity it may be noted that the total cost of fulfilling the heat demand in the 2002 FI case is 4439848.25 DKK and the maximal deviation is 38018.68 DKK, corresponding only to almost 1% of the total cost.

A study has also been made of the case where cooling is allowed (see Figure 14).

**Figure 14. Deviation from FI when cooling is allowed.**




The influence of the uncertain spot prices is clearly different than when cooling is not allowed (cf. Figure 13). In particular, the large uncertainty for very high spot price levels that was evident when cooling is not allowed all but disappears when cooling is permitted. The explanation is simple: once the prices reach a sufficiently high level, it is desirable to produce as much as possible in order to take advantage of this fact, as any excess heat produced as a consequence of this may simply be cooled off.

## 6. CONCLUDING REMARKS

In this paper, mathematical models to handle day-to-day bidding to in the spot market have been defined. This has been done from the point of view of a local CHP plant with heat storage. The analysis of these models included an examination of the shadow prices on the heat storage. Numerical examples to illustrate that the functionality of the models was reasonable were given for four variations: with and without heat cooling allowed, and with and without a specified heat storage.

A simple type of prognosis was constructed and the model with specified storage and no cooling was run consecutively for two separate years (2001 and 2002) using the actual spot prices from these years as basis for the prognoses. The influence of the number of scenarios included in the prognosis as well as the level of the spot prices was examined both with and without cooling.

Without cooling, the number of scenarios in the prognosis mostly affected the results for 2002, due to the fact that the prices during the final months of that year exhibited much larger variation as well as an increasing trend compared to the prices in 2001.

When spot price levels were low, the deviation from the full information (FI) case was quite small, as most production was supplied by the boiler. At very high levels, the deviation evened out, for once the prices reach a certain level the bidding strategy is unchanged for all levels above it.

With cooling, the shape of the deviation curves changed drastically, both for 2001 and 2002. The large uncertainty for high spot price levels all but disappeared, due to the fact that once the prices reach a sufficiently high level, it is desirable to produce as much as possible in order to take advantage of this fact, as any excess heat produced as a consequence of this may simply be cooled off without extra cost.

There are several avenues open for expansion of the models presented in this paper. One of the more evident is incorporating start-stop characteristics. However, certain numerical problems are associated with this, as the model evolves from a standard linear programming model to an integer programming model with all the difficulties inherit therein. Another avenue is the inclusion of the regulating market which in practice offers the plants the opportunity of additional income by selling or offering to withhold production to compensate for general imbalances in the system. Further, the model could be expanded to include additional CHP units and boilers.

The simulations may also be expanded by examining the influence of the heat demand. One might expect that if it is small and cooling is not allowed, fewer hours of production may be bid to the spot market, which means that it becomes of greater importance to place the bids in the 'right' hours, i.e., when the spot price turns out to be high. This may be taken one step further by modelling the heat demand as stochastic.

Further, it is not always expedient to have a daily circular constraint on the heat storage, i.e., that the storage contents at the end of a day should equal the storage contents at the beginning of the same day. At times, typically over a weekend where the spot prices generally are lower than on regular weekdays, it would be useful to expand the constraint to run over e.g. 96 hours rather than just 24. This enables the plant to utilise the heat storage to the fullest extent over a weekend, postponing production on the CHP unit till prices once again reached their weekday level.

These issues are currently being investigated in ongoing projects.

## ACKNOWLEDGEMENT

This project from which this paper has arisen was supported by Eltra [3] under the PSO programme.

## 7. LIST OF SYMBOLS

| **Index** | **Interval** | **Interpretation** |
|---|---|---|
| $t$ | $\{1,\ldots,T\}$ | Time (hours) |
| $s$ | $\{1,\ldots,S\}$ | Stochastic instance (scenario) |

| **Symbol** | **Unit** | **Interpretation** |
|---|---|---|
| $c_m$ | | Back pressure value |
| $c^{kv}$, $c^k$ | DKK/MWh$_{heat}$ | Production costs for CHP unit and boiler, respectively |
| $\pi_t^s$ | DKK/MWh$_{power}$ | Spot price at time $t$ for scenario $s$ |
| $p_1$, $p_2$ | DKK/MWh$_{power}$ | Level 1 and 2 prices, respectively |
| $m_{1t}{}^u$, $m_{2t}{}^u$ | MWh$_{heat}$ | Heat produced on the CHP unit at price $p_1$ resp. $p_2$ at time $t$ |
| $m_{kt}{}^s$ | MWh$_{heat}$ | Heat produced on the boiler at time $t$ for scenario $s$ |
| $\phi^s$ | $\in [0,1]$ | Probability of scenario $s$ occurring ($\sum_s \phi^s = 1$) |
| $\delta_{1t}{}^s$, $\delta_{2t}{}^s$ | $\in \{0,1\}$ | Indicate whether the price is above, between, or under $p_1$ and $p_2$ |
| $K$ | MW | Maximal capacity on the CHP unit |
| $d_t$ | MW | Heat demand at time $t$ |
| $V_t$ | MWh | Heat storage reserve at the beginning of hour $t$ |
| $V_{\max}$ | MWh | Heat storage limit |

## 8. REFERENCES

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[2] Brooke *et al.*, *GAMS - A User's Guide*, GAMS Development Corporation, http://www.gams.com (1998).

[3] Eltra, the Western Danish ISO, http://www.eltra.dk.

[4] The Nordic electricity exchange, Nord Pool (http://www.nordpool.no).