ING IN HYBRID DYNAMIC BAYESIAN NE

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The **EKF** is a minimum mean-square-error (MMSE) estimator based on the Taylor series expansion of the non-linear functions **f** and **h** around the estimates $\bar{\mathbf{x}}_{t|t-1}$ of the states $\mathbf{x}_t$, e.g.

$$\mathbf{f}(\mathbf{x}_t,\mathbf{v}_t) = \mathbf{f}(\bar{\mathbf{x}}_{t|t-1},\bar{\mathbf{v}}_{t|t-1}) + \frac{\partial \mathbf{f}(\mathbf{x}_t,\mathbf{v}_t)}{\partial \mathbf{x}_t}\Big|_{(\mathbf{x}_t=\bar{\mathbf{x}}_{t|t-1})}(\mathbf{x}_t-\bar{\mathbf{x}}_{t|t-1}) + \frac{\partial \mathbf{f}(\mathbf{x}_t,\mathbf{v}_t)}{\partial \mathbf{v}_t}\Big|_{(\mathbf{v}_t=\bar{\mathbf{v}}_{t|t-1})}(\mathbf{v}_t-\bar{\mathbf{v}}_{t|t-1}) + \ldots \tag{4}$$

The **UKF** is a recursive MMSE estimator that does not approximate the non-linear process and measurement models, but makes a Gaussian approximation of the distribution of the state random variable. When this variable is propagated through the true non-linear system, it captures the true mean and covariance to the second order for any non-linearity. **PF** is a generalization of Monte Carlo methods for a dynamic process. Particles are weighted recursively using importance weights

$$\omega_t = \omega_{t-1}\frac{p(\mathbf{y}_t|\mathbf{x}_t)\,p(\mathbf{x}_t|\mathbf{x}_{t-1})}{q(\mathbf{x}_t|\mathbf{x}_{0:t-1},\mathbf{y}_{1:t})} \tag{5}$$

The generic PF uses the transition prior as proposal distribution:

$$q(\mathbf{x}_t|\mathbf{x}_{0:t-1},\mathbf{y}_{1:t}) = p(\mathbf{x}_t|\mathbf{x}_{t-1}) \tag{6}$$

Hybrid models (PFEKF/PFUKF) generate and propagate a Gaussian proposal distribution for each particle

$$q(\mathbf{x}_t^{(i)}|\mathbf{x}_{0:t-1}^{(i)},\mathbf{y}_{1:t}) \doteq \mathcal{N}\left(\bar{\mathbf{x}}_t^{(i)},\hat{\mathbf{P}}_t^{(i)}\right), i = 1\ldots N \tag{7}$$

i.e. at time $t-1$ the mean and covariance of the importance distribution for each particle are computed using the EKF/UKF equations and the new observation.

Results

In EKF, the Kalman gain is partly based on the Jacobian of the measurement model. Even though pressure and conductance are correlated with the flow, the Kalman gain only influences the estimates of the hidden nodes directly connected to the observations. Figure 3 illustrates how EKF is making poor conductance and pressure estimates whereas the flow estimates are very accurate for all time steps in terms of RMSE.

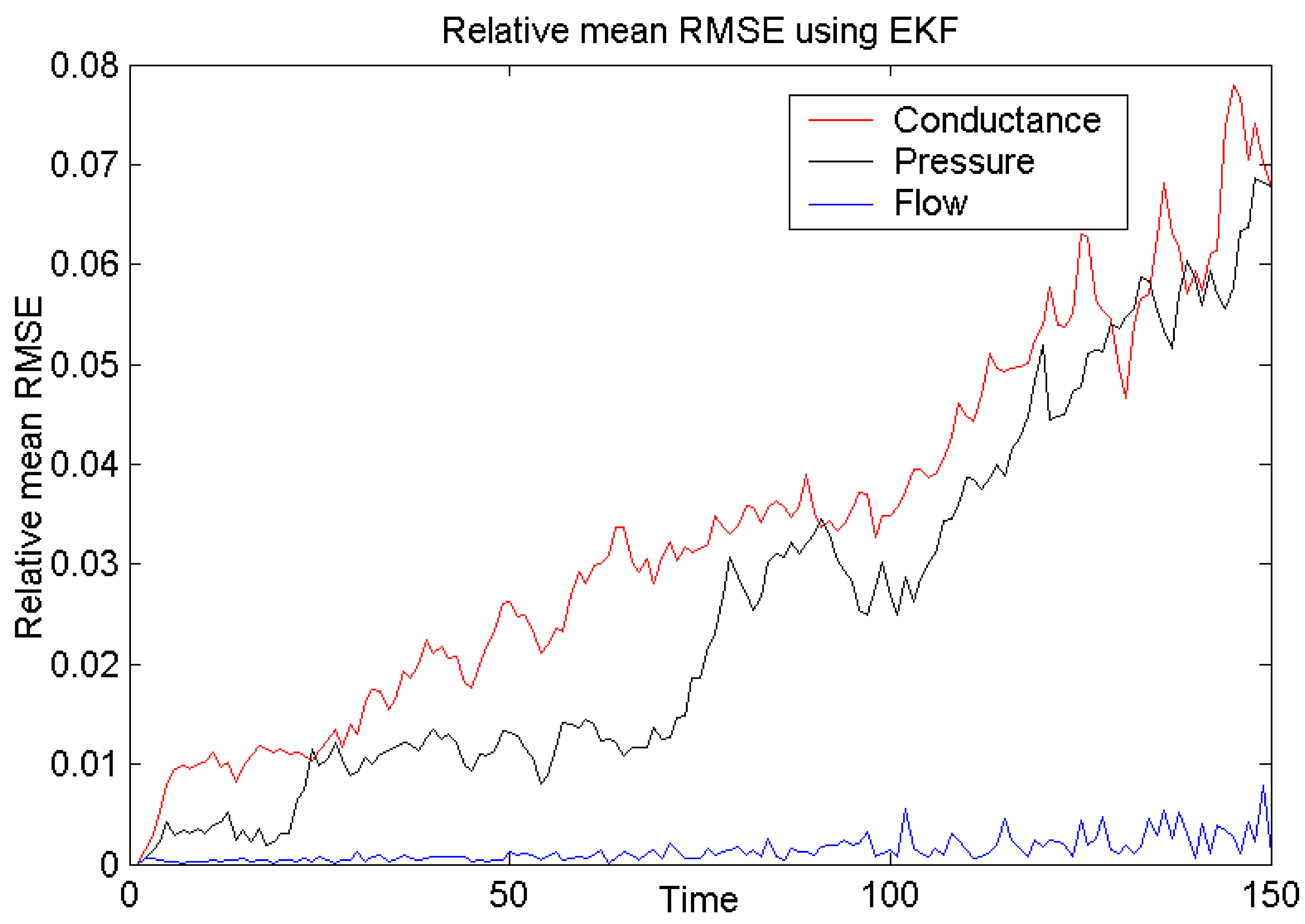

Figure 3

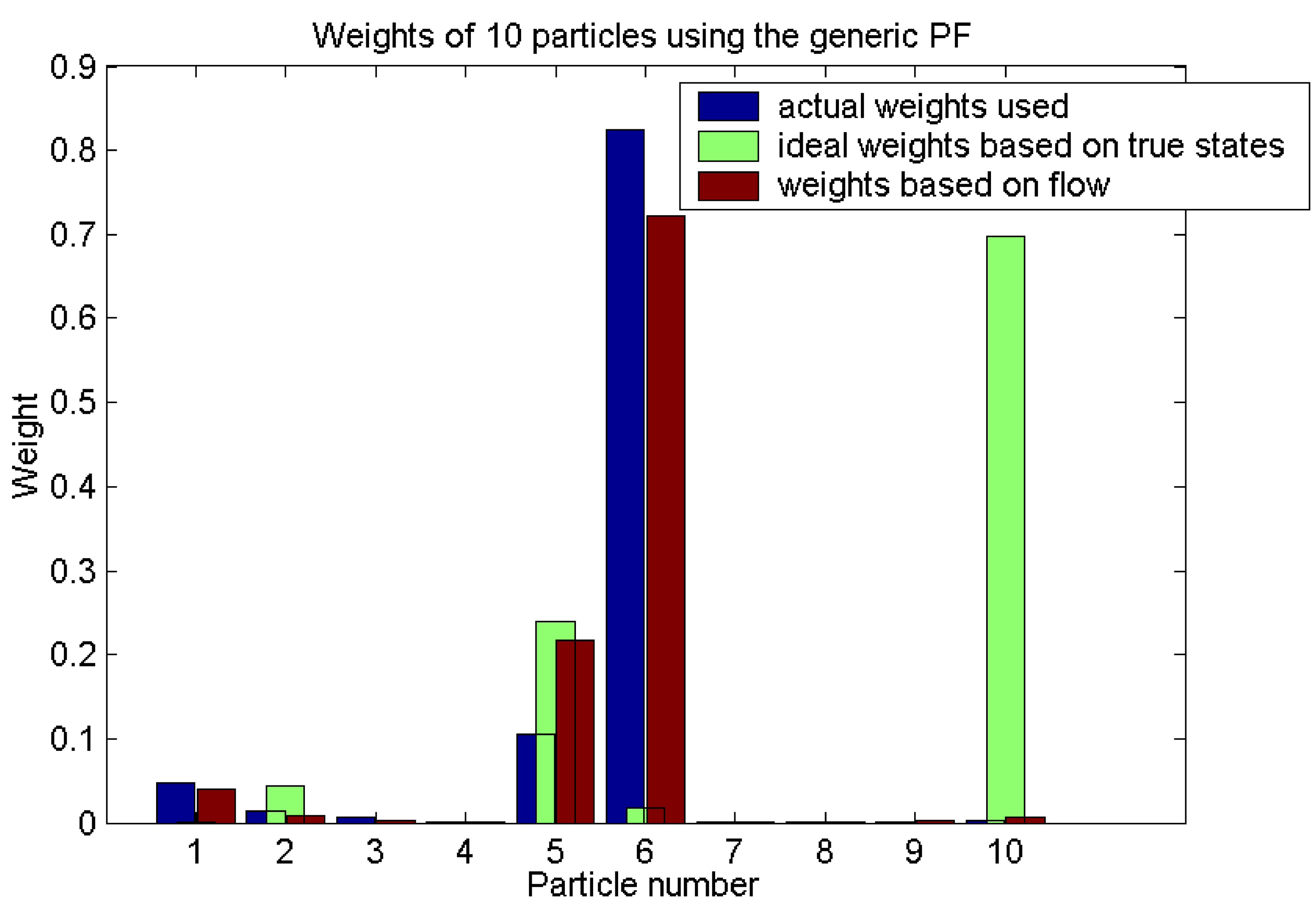

Figure 4

In the generic PF all particles are weighted according likelihood, i.e. to the difference between the true and predicted values of the observations. Hence, a particle with accurate flow values will give a high likelihood regardless of the conductance or pressure estimates. Figure 4 shows three different weighings of 10 particles. The actual weights used follow the weights based on the flow values and not the optimal weights based on the distance to the true states.