The Loewner Order and Direction of Detected Change in Sentinel-1 and Radarsat-2 Data

Allan A. Nielsen, Henning Skriver, *Member, IEEE*, and Knut Conradsen

***Abstract*—When the covariance matrix formulation is used for multi-look polarimetric synthetic aperture radar (SAR) data, the complex Wishart distribution can be used for change detection between acquisitions at two or more time points. Here we are concerned with the analysis of change between two time points and the "direction" of change: does the radar response increase, decrease or does it change structure/nature between the two time points? This is done by post-/co-processing the detected change with the Loewner order which calculates the definiteness of the difference of the covariance matrices at the two time points. We briefly describe the theory. Two case studies illustrate the technique on Sentinel-1 data covering the international Frankfurt Airport, Germany, and on Radarsat-2 data covering Bonn, Germany, and surroundings. We successfully demonstrate our "direction" of change approach to detected change areas.**

***Index Terms*—Hermitian matrices, Complex covariance matrices, the Complex Wishart Distribution, polarimetric SAR.**

I. Introduction

This contribution presents an important extension used for post- or co-processing of results from our previously published method for change detection in bi-temporal, multi-look, polarimetric synthetic aperture radar (SAR) data in the covariance matrix representation [1]. Many authors have worked with this change detection aspect, see for example [2]–[7]. In [4], [6] we deal not only with bi-temporal but truly multi-temporal polarimetric SAR data.

The extension introduced here consists of using the Loewner order [8] to look into whether radar response increases, decreases or changes structure/nature from time point one to time point two. The Loewner order calculates whether the difference of the covariance matrices at the two time points is positive semidefinite, negative semidefinite, or indefinite. In that sense we look into the "direction" of change. The method is specifically well suited for situations where man-made targets appear or disappear against a natural background, an important application of change detection in satellite data.

II. Theory

Obviously, for scalar quantities it is easy to establish whether one quantity is larger than another, for example, we could check whether the difference between them is positive, negative or zero. For matrices this is another and more intricate matter.

A. A. Nielsen and K. Conradsen are with DTU Compute, the Department of Applied Mathematics and Computer Science, H. Skriver is with DTU Space, the National Space Institute. Both departments are with the Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark. AAN: e-mail alan@dtu.dk, homepage https://people.compute.dtu.dk/alan.

Let C_p denote the set of $p \times p$ complex matrices, H_p the subset of C_p of Hermitian matrices, and $H_p^{\geq}$ and $H_p^{>}$ the subsets of positive semidefinite (or nonnegative definite) and positive definite Hermitian matrices, respectively. For matrices $\boldsymbol{X}, \boldsymbol{Y} \in C_p$ we define the Löwner (or Loewner) ordering [8] by

$$\boldsymbol{Y} \leq_L \boldsymbol{X} \Leftrightarrow \boldsymbol{X} - \boldsymbol{Y} \in H_p^{\geq},$$

i.e., $\boldsymbol{Y}$ is below $\boldsymbol{X}$ in the Loewner ordering if and only if $\boldsymbol{X} - \boldsymbol{Y}$ is positive semidefinite. This implies that

$$\lambda_i(\boldsymbol{Y}) \leq \lambda_i(\boldsymbol{X}),\ i = 1, \ldots, p,$$

where $\lambda_1(\boldsymbol{Z}) \geq \cdots \geq \lambda_p(\boldsymbol{Z})$ are ordered (real) eigenvalues of the matrix $\boldsymbol{Z}$. We say that $\boldsymbol{Y}$ is strictly below $\boldsymbol{X}$ ($\boldsymbol{Y} <_L \boldsymbol{X} \Leftrightarrow \boldsymbol{X} - \boldsymbol{Y} \in H_p^{>}$) if $\boldsymbol{X} - \boldsymbol{Y}$ is positive definite.

The relation $\leq_L$ is a *partial ordering*, i.e., it is

- *reflexive* ($\boldsymbol{X} \leq_L \boldsymbol{X}$ for all $\boldsymbol{X}$),
- *antisymmetric* ($\boldsymbol{X} \leq_L \boldsymbol{Y}$ and $\boldsymbol{Y} \leq_L \boldsymbol{X}$ implies $\boldsymbol{X} = \boldsymbol{Y}$),
- *transitive* ($\boldsymbol{X} \leq_L \boldsymbol{Y}$ and $\boldsymbol{Y} \leq_L \boldsymbol{Z}$ implies $\boldsymbol{X} \leq_L \boldsymbol{Z}$).

It is, however, not a *total ordering*, i.e., there exist matrices $\boldsymbol{X}, \boldsymbol{Y} \in H_p^{\geq}$ for which neither $\boldsymbol{X} \leq_L \boldsymbol{Y}$ nor $\boldsymbol{Y} \leq_L \boldsymbol{X}$ is true. In this case, $\boldsymbol{X} - \boldsymbol{Y}$ will be indefinite.

An important property is that for any $\boldsymbol{X}, \boldsymbol{Y} \in H_p^{\geq}$ we have

$$\boldsymbol{Y} \leq_L \boldsymbol{X} \Leftrightarrow \boldsymbol{X}^{-1} \leq_L \boldsymbol{Y}^{-1}$$

i.e., the Loewner ordering is *antitonic* with respect to matrix inversion.

When we work with multi-look polarimetric SAR data, we have a 3×3 Hermitian covariance (or alternatively a coherency) matrix for each pixel and not just a scalar variable [9]. In some cases we have dual polarization, i.e., a 2×2 matrix only. Sometimes we use the diagonal elements only.

Consider two independent, Hermitian, positive definite complex Wishart distributed variance-covariance matrices $\boldsymbol{X} \sim W_C(p, m, \boldsymbol{\Sigma}_x)$ and $\boldsymbol{Y} \sim W_C(p, n, \boldsymbol{\Sigma}_y)$ representing geometrically co-registered multi-looked covariance SAR data at two time points, t_1 and t_2, where m is the number of looks for $\boldsymbol{X} = m\langle \boldsymbol{C} \rangle_{t_1}$, and n is the number of looks for $\boldsymbol{Y} = n\langle \boldsymbol{C} \rangle_{t_2}$.

In [1], [3] we gave a test statistic ($|\cdot|$ is the determinant)

$$Q = \frac{(m+n)^{p(m+n)}}{m^{pm} n^{pn}} \frac{|\boldsymbol{X}|^m |\boldsymbol{Y}|^n}{|\boldsymbol{X} + \boldsymbol{Y}|^{m+n}}$$

with an associated cumulative distribution function for $-2\rho \ln Q$ (ρ is an auxiliary variable) to determine whether change occurred or not. High values of $-2\rho \ln Q$ indicate change. We did not look into whether radar response in some sense increased, decreased or changed structure. This is done

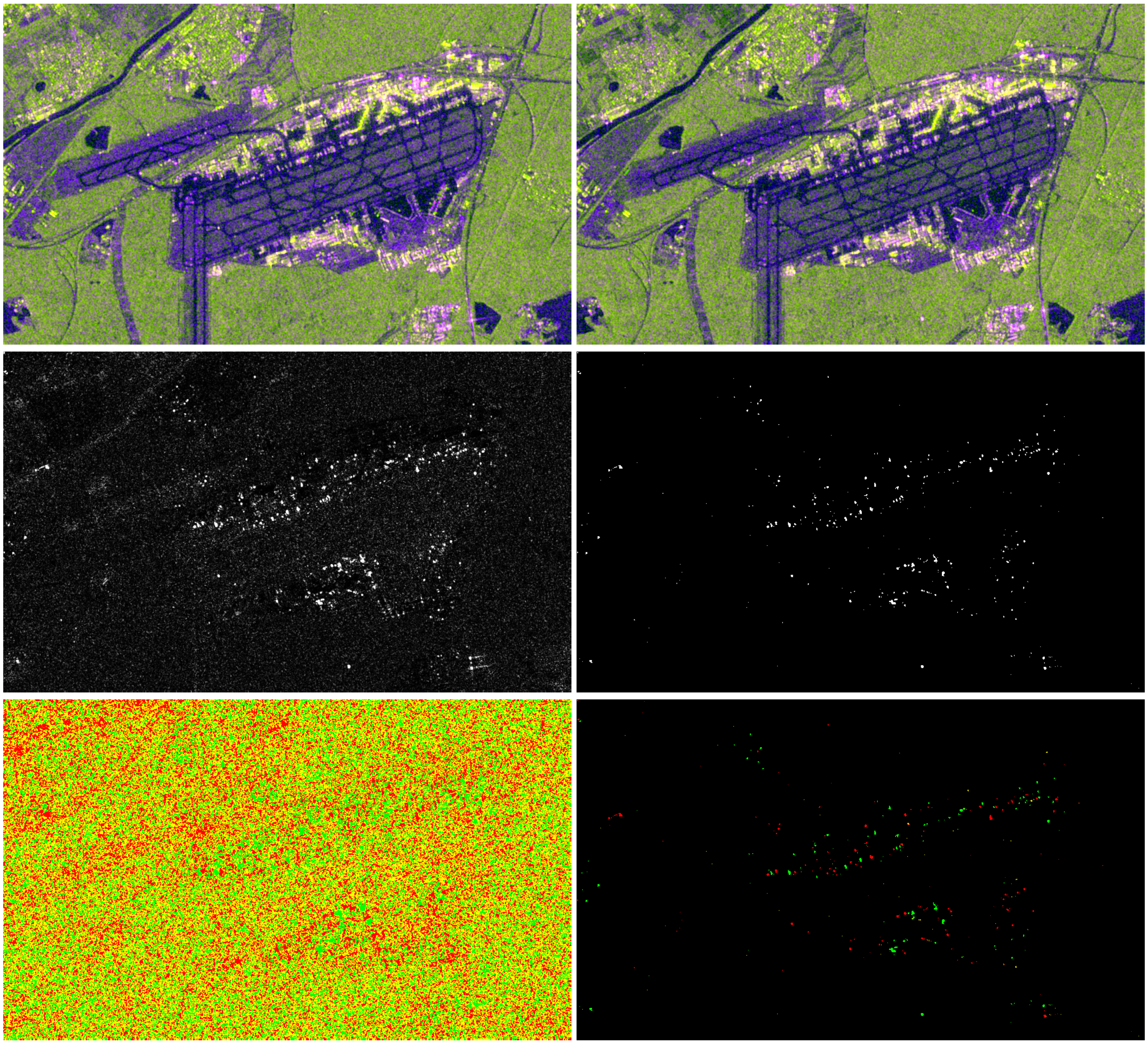

Fig. 1. Sentinel-1 $S_{vv}S_{vv}^*$, $S_{vh}S_{vh}^*$ and the ratio $S_{vv}S_{vv}^*/S_{vh}S_{vh}^*$ for the two time points as RGB (top row), the test statistic ($-2\rho \ln Q$, high values, i.e., bright pixels indicate change) for change between $\boldsymbol{X}$ and $\boldsymbol{Y}$ at the two time points as described in [1] (middle row left), the associated p-value thresholded at 99.99% (middle row right), positive definite matrix difference in red (i.e., $\boldsymbol{Y} <_L \boldsymbol{X}$), negative definite matrix difference in green (i.e., $\boldsymbol{X} <_L \boldsymbol{Y}$), indefinite matrix difference in yellow (bottom row left), and the same combination where the p-values are larger than 99.99% (bottom row right).

here by means of the Loewner order which calculates the definiteness of $\boldsymbol{X}-\boldsymbol{Y}$. If this difference is positive definite, $\boldsymbol{Y}$ is less than (or strictly below) $\boldsymbol{X}$, $\boldsymbol{Y} <_L \boldsymbol{X}$. If the difference is negative definite, $\boldsymbol{X}$ is less than (or strictly below) $\boldsymbol{Y}$, $\boldsymbol{X} <_L \boldsymbol{Y}$. If the difference is indefinite, the Loewner order cannot determine which is smaller or greater.

To determine the definiteness, we calculate the eigenvalues of the difference $\boldsymbol{X}-\boldsymbol{Y}$. If they are all positive, the difference is positive definite, if they are all negative, the difference is negative definite, if some are positive and some are negative, the difference is indefinite. Thus the Loewner order gives a multivariate statistics approach to the characterization of difference or change.

III. Case Studies

We show two examples, one based on dual polarization (VV and VH) Sentinel-1 C-band data over the international Frankfurt Airport, Germany, and one based on polarimetric Radarsat-2 C-band data over Bonn, Germany, and surroundings.

A. Sentinel-1 Data

The 4.4-look images used are 600 rows by 1000 columns 10 m pixel spacing over the Frankfurt Airport, Germany, acquired in IW mode on 29 March and 10 April 2016. The data (VV and VH only, no off-diagonal elements in the covariance matrix) are obtained from and preprocessed by the Google Earth Engine [10].

Figure 1 shows $S_{vv}S_{vv}^*$, $S_{vh}S_{vh}^*$ and the ratio $S_{vv}S_{vv}^*/S_{vh}S_{vh}^*$ for the two time points as RGB (top row), the test statistic ($-2\rho \ln Q$) for change between the two time points as described in [1] (middle row left), the associated p-value thresholded at 99.99% (middle row right), positive definite matrix difference in red (i.e., $\boldsymbol{Y} <_L \boldsymbol{X}$),

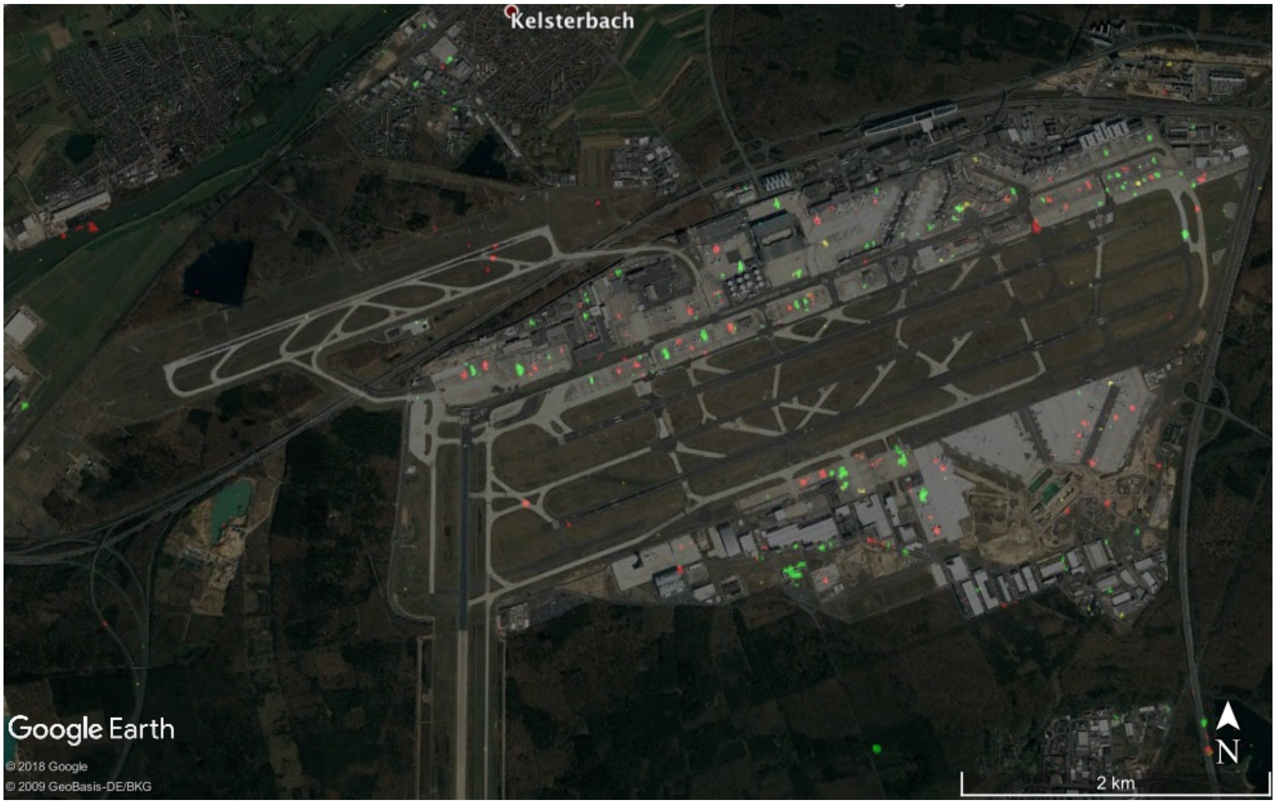

Fig. 2. Part of Figure 1 bottom row right overlaid in Google Earth. We see significant changes based on the Wishart change detector combined with direction of change based on the Loewner order occurring where aircraft at gates, on aprons, taxiways and runways, cars in parking lots and on the motorways, and ships on the River Main come and go. Overlaid colours are interpreted as: present at time point one and not at time point two (red), present at time point two and not at time point one (green), significant change but the nature of the change is such that the Loewner order cannot decide the direction (yellow).

negative definite matrix difference in green (i.e., $\boldsymbol{X} <_L \boldsymbol{Y}$), indefinite matrix difference in yellow (bottom row left), and the same combination only where the p-values are larger than 99.99% (bottom row right).

The point-like changes detected, indicate that we see mostly aircraft at gates, on aprons, taxiways and runways, cars in parking lots and on the motorways, and ships on the River Main coming and going. Our interpretation of the colouring of the figure in the bottom right is: where we have red pixels "something" (e.g., aircraft, cars) is present at time point one and not at time point two; where we have green pixels "something" is present at time point two and not at time point one; and where we have (a few) yellow pixels "something" has changed significantly but the change has a nature such that the Loewner order cannot decide which matrix is bigger.

Figure 2 shows significant change as detected by the complex Wishart distribution based method described in [1] combined with the Loewner order from a part of Figure 1 bottom row right overlaid in Google Earth. This figure clearly supports the observations made in the previous paragraph.

In this case where we have the two diagonal elements of the covariance matrix only, the eigenvalues are simply the diagonal elements themselves. Therefore positive definite matrix differences $\boldsymbol{X} - \boldsymbol{Y}$ occur where both $S_{vv}S_{vv}^*$ and $S_{vh}S_{vh}^*$ decrease from time point one to time point two, negative definite matrix differences $\boldsymbol{X} - \boldsymbol{Y}$ occur where both $S_{vv}S_{vv}^*$ and $S_{vh}S_{vh}^*$ increase from time point one to time point two, and indefinite matrix differences $\boldsymbol{X} - \boldsymbol{Y}$ occur where one of the two increases and the other decreases from time point one to time point two.

We have looked at a few examples of the three cases mentioned above. For the negative definite case, both VV- and VH-backscatter are very low at the first image indicating very smooth surface scattering probably from the paved apron, and in the second image both backscatter coefficients increase with more than 20 dB and 15 dB, respectively, indicating some kind of corner reflections, maybe tilted dihedral corner reflections or combinations of several reflections. For the positive definite case the situation is reversed. These changes clearly indicate a change from an empty apron to for example a parked airplane, or vice versa. For the indefinite cases, none of the images shows backscattering corresponding to smooth surface scattering, hence in these cases the change is probably not to/from empty from/to non-empty apron, but from non-empty to non-empty with two different objects in the two cases, and hence two different scattering compositions. Obviously, this analysis will be more complicated when we have off-diagonal elements also, and when we have fully polarimetric data.

B. Radarsat-2 Data

The 12-look polarimetric images over Bonn, Germany, and surroundings are 500 rows by 650 columns 20.5 m pixels acquired on 29 August and 16 October 2009.

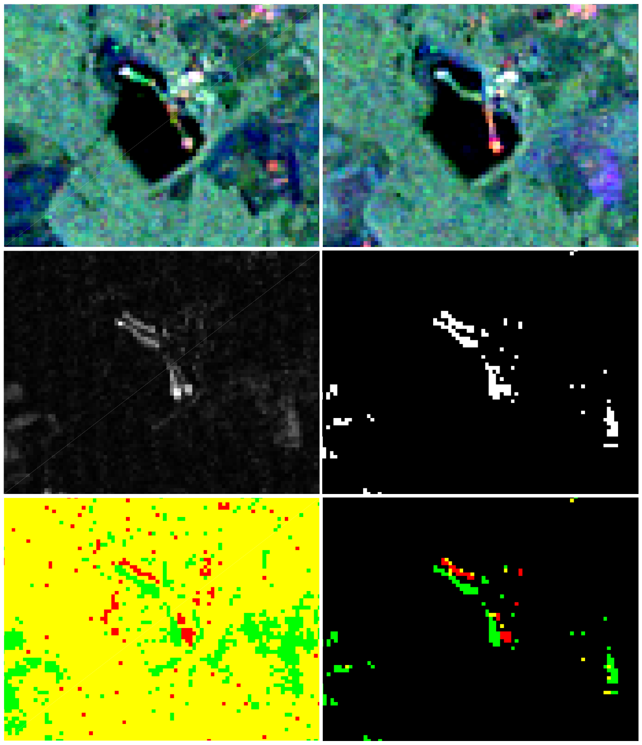

Fig. 3. Polarimetric Radarsat-2 data $\boldsymbol{X}$ and $\boldsymbol{Y}$ for the two time points as RGB (Pauli representation, top row), the test statistic ($-2\rho \ln Q$, high values, i.e., bright pixels indicate change) for change detected between the two time points as described in [1] (middle row left), the associated p-value thresholded at 99.9999% (middle row right), positive definite matrix difference in red (i.e., $\boldsymbol{Y} <_L \boldsymbol{X}$), negative definite matrix difference in green (i.e., $\boldsymbol{X} <_L \boldsymbol{Y}$), indefinite matrix difference in yellow (bottom row left), and the same combination where the p-values in the Wishart based test are larger than 99.9999% (bottom row right). Zoom on two moving dredging arms in the lake at the works Quarzwerke Witterschlick, see also Figure 5.

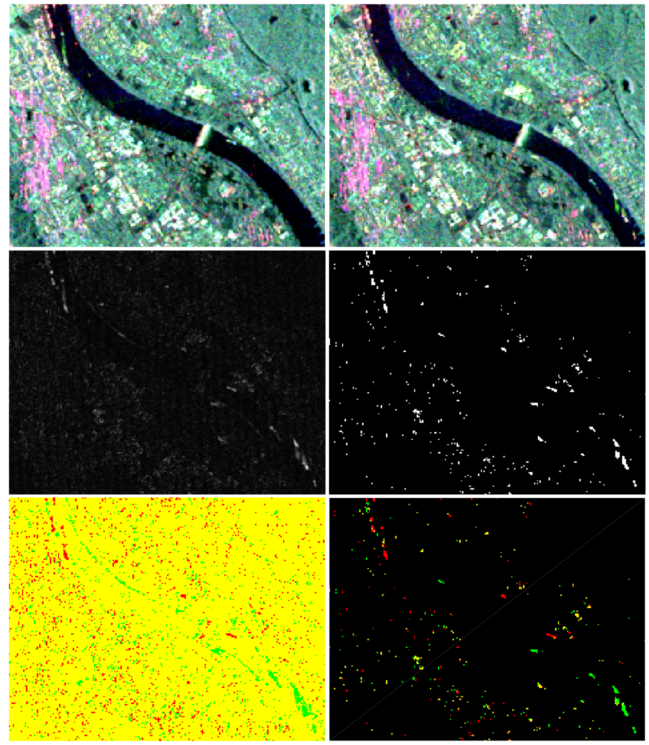

Fig. 4. Polarimetric Radarsat-2 data $\boldsymbol{X}$ and $\boldsymbol{Y}$ for the two time points as RGB (Pauli representation, top row), the test statistic ($-2\rho \ln Q$, high values, i.e., bright pixels indicate change) for change detected between the two time points as described in [1] (middle row left), the associated p-value thresholded at 99.9999% (middle row right), positive definite matrix difference in red (i.e., $\boldsymbol{Y} <_L \boldsymbol{X}$), negative definite matrix difference in green (i.e., $\boldsymbol{X} <_L \boldsymbol{Y}$), indefinite matrix difference in yellow (bottom row left), and the same combination where the p-values in the Wishart based test are larger than 99.9999% (bottom row right). Zoom on ships coming and going on the River Rhein south and south-east of central Bonn, see also Figure 5.

Figures 3 and 4 show two different zooms (see Figure 5 for larger area of the result) of the polarimetric data in the Pauli representation (red is $|S_{hh} - S_{vv}|^2$ indicating double bounce, green is $|S_{hv} + S_{vh}|^2$ (or rather $|2S_{vh}|^2$) indicating volume scattering, and blue is $|S_{hh} + S_{vv}|^2$ indicating surface scattering or single bounce) for the two time points (top row), the test statistic ($-2\rho \ln Q$) for change detected between the two time points as described in [1] (middle row left), the associated p-value thresholded at 99.9999% (middle row right), positive definite matrix difference in red (i.e., $\boldsymbol{Y} <_L \boldsymbol{X}$), negative definite matrix difference in green (i.e., $\boldsymbol{X} <_L \boldsymbol{Y}$), indefinite matrix difference in yellow (bottom row left), and the same combination only where the p-values are larger than 99.9999% (bottom row right).

Apart from the single-pixel/point-like or near single-pixel changes detected, we see two moving dredging arms in the lake at the works Quarzwerke Witterschlick (Figure 3). Also, ships coming and going on the River Rhein near the center of the city (Figure 4) are nicely detected. Again, our interpretation of the colouring of the figures: where we have red pixels "something" (e.g., ships, dredging arms) is present at time point one and not at time point two; where we have green pixels "something" is present at time point two and not at time point one; and where we have (a few) yellow pixels "something" has changed significantly but the change has a nature such that the Loewner order cannot decide which matrix is bigger. For the above-mentioned dredging arms we find a clear change from surface scattering to diplane scattering for the negative definite case, and vice versa for the positive definite case. The interpretation of indefinite cases is less clear than for the Sentinel-1 data.

Figure 5 shows significant change as detected by the complex Wishart distribution based method described in [1] combined with the Loewner order from a larger area overlaid in Google Earth. The geocoding in this example is not very accurate.

IV. Conclusions

The methods for change detection in polarimetric SAR data in the covariance matrix representation published earlier by the authors give no direction of change. This contribution gives an extension that can track the direction of change in bi-temporal data: it determines whether the radar response decreases, increases or changes structure by calculating whether the difference of covariance matrices at the two time points is positive semidefinite, negative semidefinite, or indefinite. The Loewner order can be used as an extension to other matrix based data representation change detection schemes also.

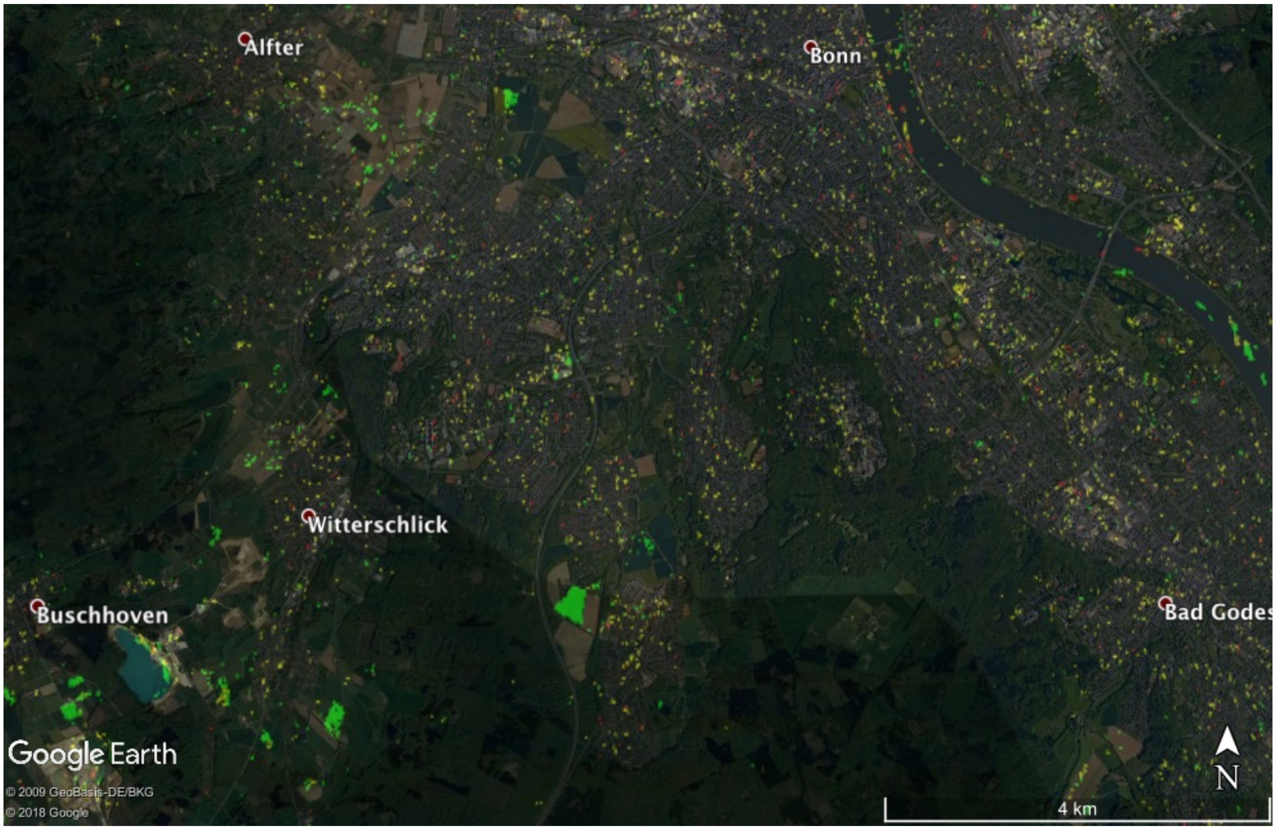

Fig. 5. Larger area overlaid in Google Earth. We see significant changes based on the Wishart change detector combined with direction of change based on the Loewner order occurring in a few agricultural fields, at two moving dredging arms in the lake at the works Quarzwerke Witterschlick (to the southwest in the image by Buschhoven), and where ships on the River Rhein come and go (southeast of central Bonn). Overlaid colours are interpreted as: present at time point one and not at time point two (red), present at time point two and not at time point one (green), significant change but the nature of the change is such that the Loewner order cannot decide the direction (yellow). The geocoding in this example is not very accurate.

In two examples with Sentinel-1 dual polarization data and Radarsat-2 polarimetric data, significant change and direction of change are measured by the test statistic in the complex Wishart distribution and the Loewner order of the difference of the Hermitian matrices from the two time points.

For both the Sentinel-1 and the Radarsat-2 data, the negative and the positive definite cases are found to involve a change between a pure surface scattering and corner reflection by for example a diplane. The indefinite cases are more complicated but do not contain surface scattering in the cases analysed.

Matlab code `loewner_order.m` to perform the analysis and to be used together with already published software to do the Wishart based change detection [1], [3], [4], [6], is available on Allan Nielsen's homepage.

REFERENCES

[1] K. Conradsen, A. A. Nielsen, J. Schou, and H. Skriver, "A test statistic in the complex Wishart distribution and its application to change detection in polarimetric SAR data," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 41, no. 1, pp. 4–19, Jan. 2003, https://doi.org/10.1109/TGRS.2002.808066 and http://www.imm.dtu.dk/pubdb/p.php?1219.

[2] M. J. Canty, *Image Analysis, Classification and Change Detection in Remote Sensing, with Algorithms for Python*, Taylor & Francis, CRC Press, fourth revised edition, 2019.

[3] A. A. Nielsen, K. Conradsen, and H. Skriver, "Change detection in full and dual polarization, single- and multi-frequency SAR data," *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, vol. 8, no. 8, pp. 4041–4048, Aug. 2015, https://doi.org/10.1109/JSTARS.2015.2416434 and http://www.imm.dtu.dk/pubdb/p.php?6827.

[4] K. Conradsen, A. A. Nielsen, and H. Skriver, "Determining the points of change in time series of polarimetric SAR data," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 54, no. 5, pp. 3007–3024, May 2016, https://doi.org/10.1109/TGRS.2015.2510160 and http://www.imm.dtu.dk/pubdb/p.php?6825.

[5] V. Akbari, S. N. Anfinsen, A. P. Doulgeris, T. Eltoft, G. Moser, and S. B. Serpico, "Polarimetric SAR change detection with the complex Hotelling-Lawley trace statistic," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 54, no. 7, pp. 3953–3966, Jul. 2016, https://doi.org/10.1109/TGRS.2016.2532320.

[6] A. A. Nielsen, K. Conradsen, H. Skriver, and M. J. Canty, "Visualization of and software for omnibus test based change detected in a time series of polarimetric SAR data," *Canadian Journal of Remote Sensing*, vol. 43, no. 6, pp. 582–592, 2017, https://doi.org/10.1080/07038992.2017.1394182 and http://www.imm.dtu.dk/pubdb/p.php?7027.

[7] A. D. C. Nascimento, A. C. Frery, and R. J. Cintra, "Detecting changes in fully polarimetric SAR imagery with statistical information theory," *IEEE Transactions on Geoscience and Remote Sensing (Early Access)*, 2018, https://doi.org/10.1109/TGRS.2018.2866367.

[8] S. K. Mitra, P. Bhimasankaram, and S. B. Malik, *Matrix Partial Orders, Shorted Operators and Applications. Series in Algebra, vol. 10*, World Scientific, Singapore, 2010.

[9] J. J. van Zyl and F. T. Ulaby, "Scattering matrix representation for simple targets," in *Radar Polarimetry for Geoscience Applications*, F. T. Ulaby and C. Elachi, Eds. Artech, Norwood, MA, 1990.

[10] N. Gorelick, M. Hancher, M. Dixon, S. Ilyushchenko, D. Tau, and R. Moore, "Google Earth Engine: Planetary-scale geospatial analysis for everyone," *Remote Sensing of Environment*, vol. 202, pp. 18–27, 2017, https://doi.org/10.1016/j.rse.2017.06.031.