DONG energy

Technical University of Denmark

DTU

# Unit Commitment and Economic Model Predictive Control for Optimal Operation of Power Systems

Peter Juhler Dinesen, s093053

M.Sc. Thesis, February 2015

**DTU Compute**
**Department of Applied Mathematics and Computer Science**
**Technical University of Denmark**

Matematiktorvet
Building 303B
DK-2800 Kongens Lyngby, Denmark
Phone +45 4525 3031
compute@compute.dtu.dk
www.compute.dtu.dk

# Abstract

This thesis focuses on combining the Unit Commitment (UC) optimization problem and the economic Model Predictive Control (MPC) problem for optimal operation of power systems. The growing uncertainty associated with the increasing share of intermittent renewable energy sources in the power supply has presented new challenges for optimal operation of power systems. Motivated by these challenges, we present a novel control strategy that shows capability of managing uncertainty with flexibility.

The proposed hierarchy control structure consists of two-levels:

- Apply UC to determine which power plants are running as well as the main distribution of power production.
- Apply economic MPC to repeatedly reoptimize the production in a receding horizon manner while considering updated and more reliable forecasts of power supply from renewable energy sources.

We mathematically formalize the UC as a mixed integer linear programming problem and the control problem as a soft constrained linear economic MPC optimization problem. Deterministic and stochastic formulations are provided, as well as disturbance modeling for offset free MPC.

The developed control strategy is tested on a power system consisting of a portfolio of controllable power plants and non-controllable farms of wind turbines. The results of the simulations successfully show that the novel control strategy appears to provide a feasible and a promising solution to overcome some of the important challenges. Furthermore, it show that the economic MPC method play an important role in the control of optimal power system operations. We demonstrate significant savings in imbalance cost and potential reduction in the need of the expensive spinning reserve.

Additionally, results indicate that the coarse discretization and the input parameterization for the UC have a cost impact on the solution. Solving the UC problem with high resolution yields the optimal production plan. Comparing to the optimal production plan, the UC solution with coarse discretization obtain 2.63% imbalance power while the economic MPC solution coincide with the optimal production plan. Simultaneous, the runtime for the economic MPC is 65x faster than solving the UC with high resolution.

# Resumé


Denne afhandling fokuserer på at sammenkoble Unit Commitment (UC) optimeringsproblem og økonomisk model prædiktiv regulering (MPC) for optimal styring af energisystemer. Energiforsyning fra vedvarende energikilder er varierende. Dermed opstår der nye udfordringer for at opretholde optimal drift og styring af energisystemer, nå disse energikilder udgør en større andel i det samlede forsyningsnet. Motiveret af disse udfordringer, præsenterer vi en innovativ kontrolstrategi.

Den forslåede kontrolstrategi består af to niveauer:

- Anvende UC til at bestemme, hvilke kraftværker der skal være tændt samt fordeling af energiproduktionen på disse.
- Anvende økonomisk MPC for gentagne gange at optimere produktionen, realtidsoptimering, med rullende horisont. Her tages opdateret og mere pålidelige prognoser for strømforsyning fra vedvarende energikilder i betragtning.

Vi formulerer matematisk UC som et blandet heltal lineært programmeringsproblem og reguleringsproblemet som et som et blødt begrænsede lineært økonomisk MPC optimeringsproblem. Vi præsenterer deterministiske og stokastiske formuleringer, samt modellere forstyrrelser for at opnå offset-free MPC.

Den udviklede kontrolstrategien testes på et energisystem bestående af en portefølje af styrbare kraftværker og ikke-styrbare vindmølle farm. Resultaterne af simuleringerne indikere at kontrolstrategien er en yderst lovende løsning til nogle af de vigtige udfordringer. Vi ser endvidere, at økonomisk MPC spiller en vigtig rolle i planlægning og realtidsoptimering til styring af energisystemer. Vi demonstrerer væsentlige besparelser i ubalanceomkostninger og potentiel reduktion i behovet for dyre reserver.

Derudover viser resultaterne, at den grove diskretisering og input parametrisering for UC har en omkostning på den opnåelige løsning. Den optimale produktionsplan opnås ved løsning af UC på fin tidsskala. Sammenlignet med den optimale produktionsplan, resultere UC løsningen på grov tidsskala 2,63% ubalance imens den økonomiske MPC løsning følger den optimale produktionsplan. Samtidigt finder økonomisk MPC løsningen 65 gange hurtigere end at løse UC på fin tidsskala.

# Preface

This thesis is submitted to the Technical University of Denmark (DTU) in partial fulfillment of the requirements for acquiring the Master of Science (M.Sc.) Elite degree in Industrial Mathematics. The elite program is an honors program for high performing students. The work reported in this thesis is conducted at the Department of Applied Mathematics and Computer Science (DTU Compute) at the Technical University of Denmark with Associate Professor John Bagterp Jørgensen as supervisor. The study and reporting is conducted in the period July 2014 to February 2015, having a workload of 30 ECTS points.

This thesis investigates the opportunity of combining the unit commitment optimization problem and the economic model predictive control problem for optimal operation of power systems. An intelligent control strategy that can manage the uncertainty associated with the increasing share of intermittent renewable energy sources in the power supply has presented new challenges for optimal operation of power systems. The thesis is accomplished in close collaboration with DONG Energy and the Technical University of Denmark. Our contributions, value creation, and experiences are relevant to both industry and academia.

We chose this project motivated by its topicality and its potential to be a very challenging and ambitious project. The project proved to be very challenging and far more comprehensive than initial expected. I am very curious by nature and find non-trivial problem highly interesting, thus, this thesis turned out to be exactly what I had hoped for.

The thesis consists of this report and a source code booklet, where the developed implementations are listed.

Kgs. Lyngby, February 2015

Peter Juhler Dinesen
s093053

# Acknowledgements

First and foremost, I would like to express my gratitude to my supervisor Associate Professor John Bagterp Jørgensen to arouse my curiosity to numerical optimization, optimal control, and to the model predictive control discipline. The excellent mentoring and fruitful discussions made this experience extremely satisfying.

I would also like to thank Industrial Ph.D. student Leo Emil Sokoler for excellent guidance and for being at disposal when questions occurred.

I am utmost grateful for this.

# Contents

**Abstract** i

**Resumé** iii

**Preface** v

**Acknowledgements** vii

**Contents** ix

**List of Figures** xiii

**List of Tables** xvii

**I Introduction and Background** 1

**1 Introduction** 3
- 1.1 Global energy challenges 3
- 1.2 Power production planning 4
- 1.3 Unit commitment 5
- 1.4 Model predictive control 6
- 1.5 Thesis statement 7
- 1.6 Thesis contributions 7
- 1.7 Previous work 8
- 1.8 Thesis structure 8

**2 Power Systems** 11
- 2.1 Power grid 11
- 2.2 Renewable energy sources 13
- 2.3 Control hierarchy 14
- 2.4 Electricity market 15
  - 2.4.1 Day-ahead market 16
  - 2.4.2 Intraday market 16

**3 Software** 17
3.1 IBM® ILOG® CPLEX® Optimization Studio . . . . . 17
3.2 Matlab® MathWorks® . . . . . 17

**II Theory** 19

**4 Unit Commitment** 21
4.1 Introduction . . . . . 21
4.2 Mathematical problem formulation . . . . . 22
4.2.1 Objective function . . . . . 22
4.2.2 Constraints . . . . . 23
4.3 The UC optimization problem . . . . . 26
4.4 Implementation . . . . . 27
4.5 Solution methods . . . . . 28
4.6 Case study . . . . . 29
4.6.1 3-unit power system . . . . . 29
4.6.2 10-unit power system . . . . . 30
4.7 Summary . . . . . 35

**5 Models for Predictive Control** 37
5.1 Modeling dynamical systems . . . . . 37
5.2 Modeling power systems . . . . . 39
5.2.1 Power system dynamics . . . . . 39
5.2.2 Discrete-time state-space model formulation . . . . . 41
5.2.3 Distributed independent power system . . . . . 42
5.3 Finite impulse response . . . . . 43
5.4 Kalman filtering and prediction . . . . . 45
5.5 Disturbance modeling for offset-free MPC . . . . . 47
5.6 Summary . . . . . 48

**6 Economic Model Predictive Control** 49
6.1 Introduction . . . . . 49
6.2 Mathematical problem formulation . . . . . 51
6.3 The economic MPC formulation . . . . . 52
6.3.1 Stability . . . . . 53
6.4 Solving the economic MPC problem . . . . . 55
6.4.1 Economic MPC formulated as LP problem . . . . . 55
6.4.2 Solvers . . . . . 57
6.4.3 Optimality conditions . . . . . 57
6.5 Implementation . . . . . 57
6.6 Case study . . . . . 60
6.6.1 2-unit power system . . . . . 60
6.7 Summary . . . . . 61

III Unit Commitment and Economic Model Predicetive Control for Power Systems 65

7 Introduction 67
7.1 Developed control strategy . . . 67
7.2 Considerations for combining UC and economic MPC . . . 68
7.3 Background for the simulations . . . 70
7.3.1 Demand load . . . 70
7.3.2 Operational parameters . . . 72

8 Discretization and Parameterization 75
8.1 Discretization . . . 75
8.2 Parameterization . . . 78
8.3 Key findings . . . 78

9 Deterministic Simulations 79
9.1 MISO simulations . . . 79
9.2 MIMO simulations . . . 85
9.2.1 System power output limits as trajectory . . . 85
9.2.2 Individual production plans as trajectory . . . 88
9.3 Key findings . . . 91

10 Stochastic Simulations 93
10.1 Modeling forecasts of wind power supply . . . 93
10.2 Step wind power . . . 94
10.2.1 Case 1 . . . 94
10.2.2 Case 2 . . . 96
10.3 Fluctuating wind power . . . 98
10.3.1 Case 1 . . . 99
10.3.2 Case 2 . . . 101
10.3.3 Case 3 . . . 104
10.3.4 Case 4 . . . 104
10.4 Key findings . . . 105

IV Conclusions and Perspectives 109

11 Conclusions and Perspectives 111
11.1 UC . . . 112
11.2 Economic MPC . . . 112
11.3 UC and economic MPC . . . 112
11.4 Perspectives and further research . . . 114

V Appendices 115

A Background Material 117
A.1 Linearization and discretization . . . 117
A.1.1 Continuous-time state-space model . . . 117
A.1.2 Continuous-time transfer function . . . 118
A.1.3 Discrete-time state-space model . . . 119
A.2 List of used theorems . . . 121
A.2.1 Propositional logic . . . 121
A.2.2 Laplace transform . . . 121
A.2.3 Z-transform . . . 121

B System Data 123

C The GRANI Program 127

Nomenclature 129

Bibliography 133

# List of Figures


1.1 Control strategy of combining the UC and the economic MPC. . . . . . . 5

2.1 Europe Brent Crude Oil Spot Price FOB in Dollars per Barrel; source U.S. Energy Information Administration (EIA) [EIA14]. . . . . . . . . . . . . . . 12
2.2 Distribution of electricity consumption by source of energy in 2010 and 2020 [MD12]. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 14
2.3 Control hierarchy. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 15
2.4 The considered wind power production forecasts on the tow levels: days-hours level and minute level. . . . . . . . . . . . . . . . . . . . . . . . . . 16

3.1 Simplified illustration of IBM ILOG CPLEX Optimization Studio. . . . . 18

4.4 24-hour demand load for the 10-unit power system [MW]. . . . . . . . . . 30
4.5 The optimal power production plan for each plants (blue line) with each plants minimum and maximum power output (red dashed line). . . . . . . 31
4.6 The optimal power production plan for plant 1 with its minimum and maximum power output limits. . . . . . . . . . . . . . . . . . . . . . . . . 32
4.7 The total production plan (dotted yellow) satisfy demand load (solid blue); coincide throughout planning horizon. The actual spinning reserve (dotted cyan) obey the required spinning reserve (dotted red). . . . . . . . . . . 32
4.8 Change in power output for each time step (solid blue) with each plant ramping limits (dotted red). . . . . . . . . . . . . . . . . . . . . . . . . . . 33
4.9 The total production cost (solid black) over the planning horizon. The variable cost (dotted blue), the fixed cost (dotted red), and startup and shutdown cost (dotted cyan). . . . . . . . . . . . . . . . . . . . . . . . . . 33
4.10 Distribution of the four cost components in the objective function. . . . . 34

5.1 A generic stochastic input-output model. . . . . . . . . . . . . . . . . . . . 38
5.2 State-space model realization of linear time-invariant models. . . . . . . . 38
5.3 Deterministic step responses of the transfer functions (5.5) and (5.6) discretized using zero-order hold with sampling time of $T_s = 20$ seconds. . . 40
5.4 Power grid with two controllable conventional power plants and one non-controllable predictable power generator, farms of wind turbines. . . . . . 44

6.1 Control principle of MPC scheme - moving horizon estimation [Wik]. . . . 50
6.2 A MPC system [PJ08]. . . . . . . . . . . . . . . . . . . . . . . . . . . . 50
6.3 Flowchart of developed MATLAB implementation. . . . . . . . . . . . . . . 59
6.5 Open-loop simulation of a power system without regularization term for excessive movement of the input. $T_s = 1$. . . . . . . . . . . . . . . . . . . 62
6.6 Open-loop simulation of a power system with regularization term for excessive movement of the input. $T_s = 1$. . . . . . . . . . . . . . . . . . . . . 63
6.7 Closed-loop economic MPC simulation of a power system. Prediction horizon is $N = 50$ time step with regularization term. $T_s = 1$. . . . . . . . 64

7.1 Control strategy of combining the UC and the economic MPC. . . . . . . 69
7.2 Power system with two controllable conventional power plants and a non-controllable predictable power generator, farms of wind turbines. . . . . . 70
7.3 24-hour demand load [MW]; see Table B.1(a) in Appendix B for the numerical representation. Spinning reserve is 10% of demand load for each time period. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 71

8.1 Power production obtained from $\mathrm{UC_{th}}$, $\mathrm{UC_{tc}}$, and $\mathrm{EMPC_{th}}$. . . . . . . . . 76

9.1 24-hour MISO closed-loop simulation applying the busy demand load as trajectory. UC production profile for power plants are unknown while committed plants are known for the economic MPC. . . . . . . . . . . . . 81
9.2 24-hour MISO closed-loop simulation applying the busy demand load. Performed inputs to the system and the rate of movement together with their limits. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 82
9.3 24-hour MISO closed-loop simulation applying the idle demand load as trajectory. UC production profile for power plants are unknown while committed plants are known for the economic MPC. . . . . . . . . . . . . 83
9.4 24-hour MISO closed-loop simulation applying the idle demand load. Performed inputs to the system and the rate of movement together with their limits. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 84
9.5 24-hour MIMO closed-loop simulation applying the busy demand load and system power output limits as trajectories. UC production profile for power plants are unknown while committed plants are known for the economic MPC. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 86
9.6 24-hour MIMO closed-loop simulation applying the idle demand load and system power output limits as trajectories. UC production profile for power plants are unknown while committed plants are known for the economic MPC. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 87
9.7 24-hour MIMO closed-loop simulation applying the busy demand load. The economic MPC is to obey obtained production plan from solving the UC problem within a defined range (trajectories). . . . . . . . . . . . . . . 89

9.8 24-hour MIMO closed-loop simulation applying the idle demand load. The economic MPC is to obey obtained production plan from solving the UC problem within a defined range (trajectories). . . . . . . . . . . . . . . . . . 90

10.1 6-hour closed-loop simulation when 150 MW wind power entering the system at hour 3. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 95
10.2 6-hour closed-loop simulation with the busy demand load. Figure 10.2(a) shows the result of offset MPC simulation and Figure 10.2(b) shows the result of offset free MPC. Both with same simulation setup. . . . . . . . . 97
10.3 24-hour demand load [MW]. Coarse grid demand load (tc) applied in the UC problem (solid blue) and high resolution demand load (th) applied in the economic MPC (solid cyan). Spinning reserve is 10% of demand load$_{tc}$ for each time period. . . . . . . . . . . . . . . . . . . . . . . . . . . . 98
10.4 Example illustration of (10.1) applying (10.2). . . . . . . . . . . . . . . . 99
10.6 6-hour closed-loop simulation. *Top:* Total power production from the UC and the economic MPC. Required power generated by plants (solid red). *Bottom:* Wind power production defined as (10.1). . . . . . . . . . . . . . . 100
10.7 Imbalance as function of time for simulation Figure 10.6. Calculated as the differences between the optimal required power production by plants and the two derived solutions: UC solution and economic MPC solution. . 101
10.9 24-hour closed-loop simulation with two different amplitude values. *Top:* Total power production from the UC and the economic MPC. Required power generated by plants (solid red). *Bottom:* Wind power production defined as (10.1). . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 103
10.11 6-hour closed-loop simulation. Fixed amplitude and vary frequency. Wind power modeled by (10.2) using parameters listed in Table 10.10. . . . . . 106
10.12 Same simulation as Figure 10.11(b) with the change of power output range to be ±0.03% of the demand load. . . . . . . . . . . . . . . . . . . . . . . . 107
10.13 6-hour closed-loop simulation. Consider the stochastic model with stochastic process noise and measurements noise distributed as (10.3). . . . . . 108

A.1 Input-ouput relation describing the transfer functions. . . . . . . . . . . . 118

# List of Tables


4.1 3-hour demand load for the 3-unit power system [MW]. . . . . . . . . . . 29
4.2 Operational parameters for the 3-unit power system. . . . . . . . . . . . . 29
4.3 Optimal production plan for the 3-unit power system [MW]. . . . . . . . . 30

6.4 Operational parameters. . . . . . . . . . . . . . . . . . . . . . . . . . . 60

7.4 Operational parameters to the UC problem. . . . . . . . . . . . . . . . . . 72
7.5 Operational parameters to the economic MPC. Penalty $\rho_{i,k} = [\rho_{1,k},\rho_{2,k},\rho_{T,k}] = [10,10,100]$, where $i = 1,2,\ldots,n_u$ and $\rho_{T,k}$ is the penalty associated to the overall demand load. . . . . . . . . . . . . . . . . . . . . . . . . . . 73

8.2 Results of 12-hour closed-loop simulation. Total power production [MWh] by the tree methods. Imbalance [MWh] is the absolute imbalance between $\mathrm{UC}_{\mathrm{th}}$ and the obtained production plan. . . . . . . . . . . . . . . . . . . . 77

10.5 Applied parameters in Figure 10.6 to (10.2). . . . . . . . . . . . . . . . . . 100
10.8 Results of 24-hour closed-loop with four different amplitudes. Imbalance is the absolute imbalance between the optimal production and the obtained production plan. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 101
10.10 Applied parameters in Figure 10.11(a) and Figure 10.11(b) to (10.2). . . . 104

B.1 24-hour demand load [MW] applied in simulations. Spinning reserve is 10% of demand load for each time period. . . . . . . . . . . . . . . . . . . 124
B.2 Operational parameters for the 10-unit power system. . . . . . . . . . . . 125

Part I

# Introduction and Background

CHAPTER 1

# Introduction

In this chapter, we bring the project into a context by outlining the challenges and directions from the global energy systems that motivate our work. We address the need for a new control strategy when introducing large amounts of intermittent renewable energy sources into the power grid. The used methods are briefly described. Additionally, we describe the thesis statement, the contributions of our work, and previous work. Lastly, an outline for the remainder of this thesis is given.

## 1.1 Global energy challenges

Energy is of paramount importance for a modern society. It has a great impact on everything we do like water delivery, food, internet, computer systems, communication systems, etc. Major breakdowns in power systems are a fundamental concern, since it would lead to an almost complete chaos in the Western countries. Simultaneously, we are in a global race for energy sources. Fossil fuels continue to dominate the world's energy supplies, counting for more than 80% of energy demand [EIAa; Eur13; Off13]. As we know, this energy supply is unsustainable and causing potentially catastrophic climate change, and horrendous pollution. The world is facing global energy challenge of

- satisfy the increasing energy demands,
- ensure adequate energy sources, and
- reduce climate changes and pollution.

In the global race for energy sources and for meeting the global energy challenges, renewable energy sources has come to occupy a dominant place on the agenda of governments in most industrialized countries. Renewable energy sources such as solar energy, hydro energy, and wind energy promise to be a feasible solution to the global energy challenge. However, large penetration of renewable energy sources involves gigantic challenges in managing the fluctuating and stochastic power supply that is inherent in its nature for most renewable energy sources. The power generation fluctuates independently from demand and is simply non-controllable as opposed to the traditional highly controllable fossil fueled power plants. Furthermore, forecasts of power supply from intermittent renewable energy sources are embedded with uncertainties, as the weather may change during the day. Along with the high requirement for power system reliability, it has become of increasing importance to be able to effectively control and manage the energy production in a flexible and proactive way. Thus, we require much more of our optimization and control methods, and the software we apply. We elaborate more on this in Chapter 2.

## 1.2 Power production planning

Planning the power production to match the demand load is an important optimizing task in daily operational planning of power systems for energy producing companies like DONG Energy. Unfortunately, determining the optimal production plan with a financial and environmental perspective is nontrivial. Consider a portfolio of controllable power plants. Then, a planning problem is basically twofold:

1. determine which power plants are running at each time step and
2. determine the production level for the running plants in a cost effective way.

This optimization problem may be solved by the Unit Commitment (UC) problem. Mathematically, UC is an NP-complete problem. For system with practical size (large-scale power systems), the UC problem quickly becomes very complex and extremely difficult solve within a limited time.

Introducing intermittent renewable energy sources into the power grid, reinforce the need to reoptimize the production during the day of operation in order to avoid shortage or surplus of power. It is impossible to solve the UC problem with a high frequency, e.g., every 2-4 minutes. Therefore, at this stage, spinning reserve capacity is used to balance the production. Spinning reserves is unutilized production capability that can be used when needed. Unfortunately, spinning reserve is very expensive to have and to utilize. Thus, to account for the variations in power supply from the renewable energy sources and to reduce the undesirable power imbalance, we introduce the economic Model Predictive Control (MPC) method. Based on updated and more reliable forecasts of power supply from renewable energy sources, the economic MPC reoptimize in real-time the optimal production in a receding horizon manner.

Consider the hierarchical control structure depicted in Figure 1.1. We solve the UC problem at the high-level. Here, we decide which power plants are running and the main distribution of power production on the running plants. To account for fluctuations, a low-level controller, economic MPC, is applied. Here, we reoptimizes the production plan and perform corrections.

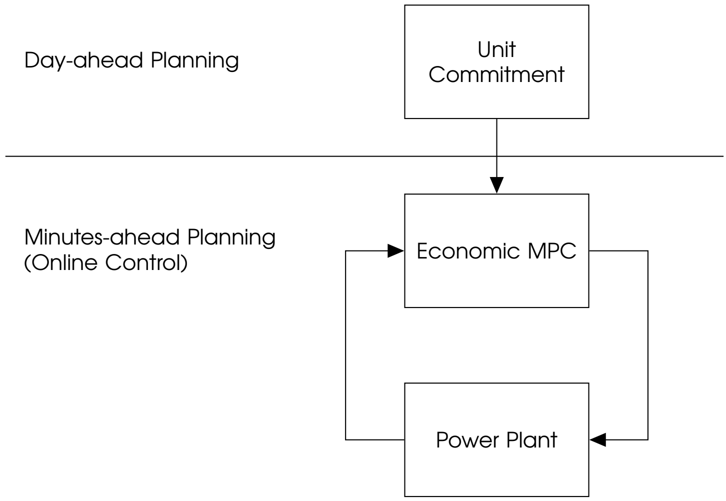



**Figure 1.1:** Control strategy of combining the UC and the economic MPC.

## 1.3 Unit commitment

The purpose of Unit Commitment (UC) is to schedule on a daily and hourly basis, the most cost effective dispatch subject to various requirements like power demand load, spinning reserves, physical limits of equipment, power system operating limits, etc. In this thesis, the UC problem is formulated with affine objective function and constrains. The problem involves both discrete and continuous variables, thus, we obtain a binary Mixed Integer Linear Programming (MILP) problem. Following is an example of a mathematical formulation of the UC problem with an objective cost function including fixed and variable operating cost, startup, and shutdown cost subject to satisfying the demand load for each timer period:

$$\begin{aligned} \text{minimize} \qquad & \text{Cost} = \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} [a_i u_{i,t} + b_i p_{i,t} + SU_i y_{i,t} + SD_i z_{i,t}] \\ \text{subject to} \qquad & \sum_{i \in \mathcal{I}} p_{i,t} \geq D_t, \qquad t \in \mathcal{T}. \end{aligned}$$

with $\mathcal{I} := \{1,2,\ldots,I\}$ defining the set of power plants and a specified time-varying demand over $\mathcal{T} := \{1,2,\ldots,T\}$ time periods defining the planning time horizon. The decision variables are $p_{i,t} \in \mathbb{R}_{\geq 0}$ and $u_{i,t}$, $y_{i,t}$, $z_{i,t} \in \mathbb{Z}_2$. We elaborate more on this in Chapter 4. Further literature and related research in UC problems includes, e.g., [WW12; OAV12; Cas+11; Pad04; NKF09; ZGH10; RG91; MNG14].

## 1.4 Model predictive control

Model Predictive Control (MPC) is a control methodology for optimal operation and control of dynamic systems and processes. This control methodology has been very successful in the process industries like chemical plants and oil refineries. MPC computes an optimal action based on a mathematical model of a dynamical system and its predicted future evolution. An advantage of MPC is the fact that it is mathematically formulated as a real-time optimization problem that repeatedly computes the control actions.

Traditionally, MPC is designed to follow a predefined set-point or trajectory subject to constraints. Our main goal is to minimize the operating costs. MPC based on economic performance function is known as economic MPC. Economic MPC provides the property of controlling a system over a time horizon subject to constraints while minimizing the cost of operations. We formulate the economic MPC as a discrete-time, constrained linear system of the form

$$\begin{aligned} x_{k+1} &= f(x_k,u_k) \\ y_k &= g(x_k,d_k) \\ z_k &= h(x_k,d_k), \end{aligned}$$

with $k \in \{0,1,\ldots,N\}$. $x$ is the dynamical states of the system, $u$ is the manipulated variables, and $d$ is a predictable disturbances. The system dynamics and constraints are considered linear. Consequently, the constrained optimal control problem may be formulated as the linear programming problem

$$\underset{x}{\text{minimize}} \qquad \phi = g^T x \tag{1.1a}$$

$$\text{subject to} \qquad Ax \geq b, \tag{1.1b}$$

where $g \in \mathbb{R}^n$, $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$, and $x \in \mathbb{R}^n$. We elaborate more on this in Chapter 5 present and Chapter 6. Further literature in MPC includes, e.g., [Jør05; Mac02; PJ08; QB03; CM87; Hal+14; Hov13].

## 1.5 Thesis statement

This thesis has the purpose to develop and investigate the novel coupling of UC and economic MPC for optimal operation of power systems. The aim is to develop a control strategy that intelligently can manage uncertainty with flexibility. The focus will be to minimize operational cost and reduce power imbalance subject to obey the overall demand load and various system requirements. Thus, optimal operations of power system is maintained and opens the possibility to reduce the need for the expensive reserve capacity.

The thesis primary objective is formulated into following hypothesis:

> *By combining the unit commitment optimization problem and the economic model predictive control problem, it is possible to obtain an intelligent control strategy that can overcome some of the important challenges associated with the increasing share of intermittent renewable energy sources in the power supply. This novel coupling will operate the power systems in a cost efficient manner while satisfying the overall demand load and various system requirements.*

In order to develop and investigate the control strategy, we

- examine and comprehend the theory for the UC problem and the economic MPC problem,
- demonstrate the two methods on a conceptual power systems to gain experiences and investigates their behavior, and
- combine the two methods and perform simulations and analyzes the results.

We apply the state-of-the-art algorithms and software to solve the numerical optimization problems and the optimal control problems; see Chapter 3 for description of the applied software.

## 1.6 Thesis contributions

The thesis is accomplished in close collaboration with DONG Energy and the Technical University of Denmark. Our contributions, value creation, and experiences are relevant to both industry and academia.

The contributions to DONG Energy are mainly the learning from the chosen control strategy. The strategy of combining the UC problem and economic MPC problem is particular interesting for DONG Energy's ongoing joint venture on the Faroe Islands through the program GRANI. This indeed shows the topicality of this thesis. In Appendix C, we describe the GRANI program further.

The contributions to the university are mainly the learning which can be applied to the research program in Smart Energy Systems at Department of Applied Mathematics and Computer Sciences, Technical University of Denmark [Jør]. The achieved results may bring ideas and further research to this topic.

## 1.7 Previous work

A deep literature review has been conducted to form the basis for this thesis. We have chosen to refer to relevant literature and previous work throughout the thesis; therefore, consult the appropriate chapters for literature review.

Energy and global energy challenges are of great interest worldwide, thus, a comprehensive literature exists on this topic. To the best of our knowledge, this is the initial research and proposal of combing the UC optimization problem and the economic MPC to account for fluctuations inherent in the increasing penetration of intermittent renewable energy sources into the power grid. [Con+11] present a computational framework for integrating weather prediction in a UC setting. However, the framework does not consider more detailed issue such as intraday rescheduling and effects of updating wind power forecast at higher frequency and higher resolutions. [XI09] present potential benefits of applying MPC to solving economic and environmental dispatch problem in electric power systems with many intermittent resources based on a short look-ahead approach (e.g., 5 minutes). In this thesis, we consider a bi-level framework for which both the day-ahead 24 hours power scheduling as well as rescheduling the day of operations.

## 1.8 Thesis structure

The thesis is divided into five parts. The first part provides an introduction and background of the thesis. The second part describes and formalizes needed theory. The third presents simulations and results of the developed control strategy. The four collects key findings and discuss perspectives. The five presents the appendices. The contents of each chapter are outlined in the following:

**Chapter 2** outlines the global energy challenge for the current power grid, explain the advantages and disadvantages with renewable energy sources, and briefly describe the control hierarchy for power systems and the electricity markets.

**Chapter 3** presents informally the software applied in this thesis.

**Chapter 4** describes and formalizes mathematically the UC optimization problem. The complexity of solving the UC problem is outlined. Syntax comparison is showed between implementing in IBM ILOG CPLEX Optimization Studio and MATLAB and a demonstration of the formulated UC problem is applied on conceptual power system setups.

**Chapter 5** outlines the basic of modeling dynamical systems for predictive control. The mathematical model applied for modeling the dynamics of power systems in our simulations is conducted. In addition, the model is extended to achieve offset free MPC. Lastly, finite impulse response model and stationary Kalman filtering and prediction is presented.

**Chapter 6** describes and formalizes mathematically the soft constrained linear economic MPC problem. It is presented how the optimization problem is solved as well as the developed control framework implementation. Lastly, a demonstration of the formulated economic MPC problem is applied on conceptual power system setups.

**Chapter 7** provides an overview of the simulations that follows and a description of the developed control strategy. Furthermore, presents the considered power system, operational parameters, and other background information for the simulations that follows.

**Chapter 8** provides a study on the impact the discretization and input parameterization of the UC problem in terms of imbalance and costs.

**Chapter 9** presents simulations of combining the UC problem and the economic MPC problem without power supply from renewable energy sources in the power system.

**Chapter 10** presents simulations of combining the UC problem and the economic MPC problem with power supply from renewable energy sources in the power system.

**Chapter 11** summarize key finding and provide concluding remarks of the work and results. Lastly, possible extensions and directions for further research are addressed.

**Appendices.** Appendix A presents the basic concepts of how a linear time-invariant continuous-time model may be linearized and discretized to obtain a linear time-invariant discrete-time state-space model, as well as list of used theorems. Appendix B presents data used in the thesis. Appendix C describes the GRANI program. Lastly, the nomenclature used in the thesis is presented

CHAPTER 2

# Power Systems

This chapter outlines the global energy challenge for the current power grid and explains the advantages and disadvantages with penetration of renewable energy sources. Additionally, we briefly describe the control hierarchy for power systems and the electricity markets.

## 2.1 Power grid

Energy is of paramount importance for a modern society. Today's power grid is a very stable and reliable system in most western countries. However, the global energy challenge for the current power grid is at least threefold:

- satisfy the increasing energy demands,
- ensure adequate energy sources, and
- reduce climate changes and pollution.

The world energy consumption has increased nearly 180% from 1980 to 2010 and is expected to increase at a rate of about 2% per year [EIAa; EIAb]. In Europe, the energy production is far from covering our own demand. Consequently, energy sources are imported from third countries. Europe's energy import dependence has increased over the years and will continue to increase. Import of the utmost energy sources, fossil fuel, is set to increase more than 80% by 2035 [Eur13]. This expose Europe to the bargaining power of the few suppliers, exposed to the market power, and the risk for excessively high prices. The spot price movement on crude oil over the years, depicted in Figure 2.1, indicates that crude oil has more than tripled the last 10 years. The trend may continue as fossil fuel supplies diminish. In the meantime, it is commonly known that using fossil fuel to energy production has a negative impact on the carbon footprint and the environment.

The supply chain for electricity differs compared to most other products in terms of inventory and storage. The possibilities of effective storage of electricity are limited and imply relative high costs. Therefore, balancing the equation of producing accurately enough electricity to meet the consumption is of great importance. With the majority of conventional fossil fueled power generators on the grid, the task of

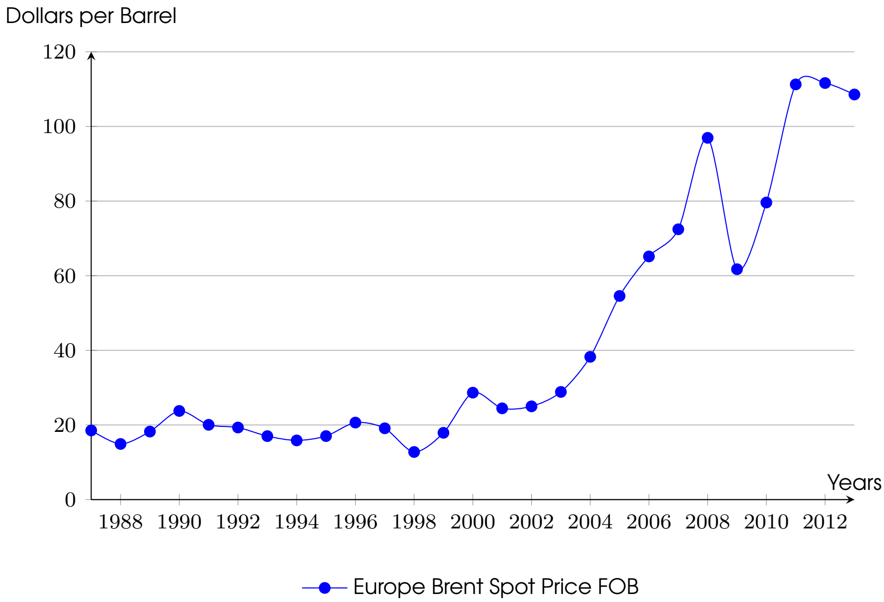



**Figure 2.1:** Europe Brent Crude Oil Spot Price FOB in Dollars per Barrel; source U.S. Energy Information Administration (EIA) [EIA14].

balancing the production is manageable since these power generators are rather controllable. To obey the global energy challenge, an increasing penetration of renewable energy sources is introduced into the power grid. This involves challenges in managing the balancing with production and consumption due to the fluctuating power supply that is inherent in its nature for most renewable energy sources. We lose a lot of the traditional flexibility and controllability, in which the current power grid are relying on. Therefore, the energy system as we know it today is changing from a highly predictable system in which production matches consumption at all times to a fluctuating energy system in which intermittent renewable energy sources contribute to unwanted imbalances in the power system.

In general, power imbalance in power systems is unwanted and have an adversely impact. The consequences of imbalance may differ dependent on the power system, but include inefficient production, additional costs, stability issues, etc. Following example illustrates the concept for Danish power producer. Consider two player of power producer at hour 1. Player 1 has shortages of 100 MW cf. the plan. Player 2 has 100 MW in surplus cf. the plan. Player 1 buy the 100 MW by Energinet.dk[1]. Player 2 sells the 100 MW to Energinet.dk. Depending on the particular day, the

[1] Energinet.dk are a non-profit enterprise owned by the Danish Climate and Energy Ministry. Energinet.dk is responsible for supplying Denmark with electricity and natural gas, ensuring fair competition and promoting green energy solutions.

buy price and sell price vary. In general, the price for buying (player 1 situation) is higher that the spot price, while the price for selling (player 2 situation) is lower than spot price. Thus, power producer loss money in both cases. In fact, Energinet.dk gain on an annual basis approximately DKK 20 M for the balance transaction (source Energinet.dk, Henning Parbo). As a consequence, we need to investigate solutions such that we efficiently can control and manage the energy production in a flexible and proactive manner.

Tomorrow's green, flexible and intelligent power grid, which takes up these challenges, will change the current power grid with a consolidated collection of power generation system to be an intelligent network of many independent power producers and consumers. This, we refer to as the Smart Grid [Ene14; Jen; Jør]. Smart Grid change the power system, as we know it today, to a system that integrates the behavior of producers and consumers.

## 2.2 Renewable energy sources

Intermittent renewable energy sources such as solar energy, hydro energy, and wind energy promise to be a feasible solution to the global energy challenge. These energy sources are clean compared to fossil fuel, sustainable, plentiful, and the resources are available over widely geographical areas, in contrast to fossil fuel there are concentrated in few countries. Additionally, the energy payback is small meaning they all "produce" more energy than they "consume" [Wei+13; San]. The government in Denmark has established an ambitious and politically broad green energy agreement to point towards the goal of full conversion to renewable energy in 2050. One initiative is to increase the share of wind power to 50% of the electricity consumption by 2020 as depicted in Figure 2.2 [MD12]. In 2014, a major step towards meeting these goals were taken, since 39% of the Danish energy consumption were supplied by wind turbines [Eneb].

Intermittent renewable energy sources are stochastic in nature and fluctuates independently from demand. This introduce, e.g., undesirable power imbalance and stability issues. To offset the unavoidable imbalance that will appear during realtime operation of the power systems, different opportunities has been researched. Currently, spinning reserve is allocated in advance as a buffer to cover unexpected shortages of energy supply in real time. That is, determine the production plan such that the power system operates at less than its full capacity. Unfortunately, spinning reserve is very expensive to have. Power generators that quickly can cover shortages are normally embedded with high startup cost and running cost. Furthermore, one can dissemble that a better production plan may exist if available power plants were permitted to operate at full capacity. Thus, reducing the need for reserve capacity will imply a cost reduction. Another way to facilitate the imbalance is by large-scale storage capabilities. However, [PSH09] indicates that this field needs further research and development before it become a reality.

Besides the fluctuating energy supply, a large collection of renewable energy

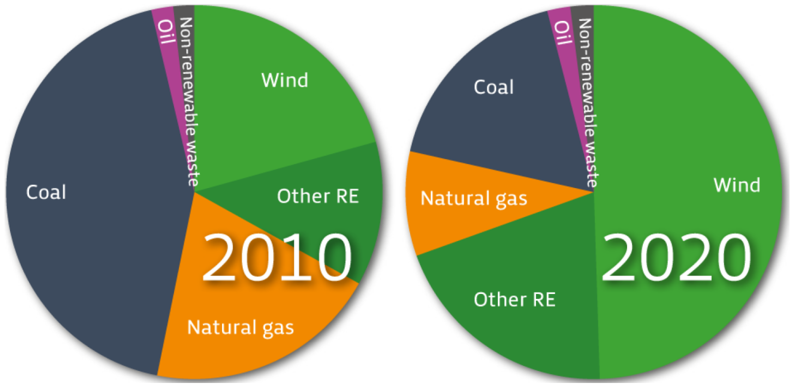



**Figure 2.2:** Distribution of electricity consumption by source of energy in 2010 and 2020 [MD12].

sources are required before it have an impact on the power grid. Since, so far, not one renewable energy source is powerful enough to solve the energy challenges. Thus, the already complex power grid becomes even bigger as more energy sources are connected to the grid. This return into, e.g., challenge of how to control large collection of power generators and interconnection issues when moving the electricity where needed.

## 2.3 Control hierarchy

We briefly describe the control hierarchy for power systems; see Figure 2.3. We refer to [SM71; Hol+09] for further details about the control hierarchy in power systems.

Due to economic, political, social constrains, and consideration of reliability some of the hierarchical decomposition of the power system to achieve decentralized control is almost mandatory [SM71]. There are many types of decomposition in hierarchical theory, mostly, depending on the system and the problem of interest. Two types of decomposition could be level and time decomposition, where the former usually is geographically related and the latter naturally arises due to response time in a power system.

At the high level, we find long-term planning, adequacy of grid and power, and maintenance. Usually, control functions at a higher level imply slower time scale.

At the next level, energy management schedules the needed demand load on a daily and hourly basis; the scheduling dispatch in the day-ahead market. This level introduces day- or hour-ahead predictions of the future demand load and weather forecasts. The prediction uncertainty increases significantly when large amount of fluctuating renewable energy supply are introduced into the power grid. Additionally, during the day of operations, the weather is likely to change from forecasts conducted the day before. Hence, an intelligent control of spinning reserves to handle

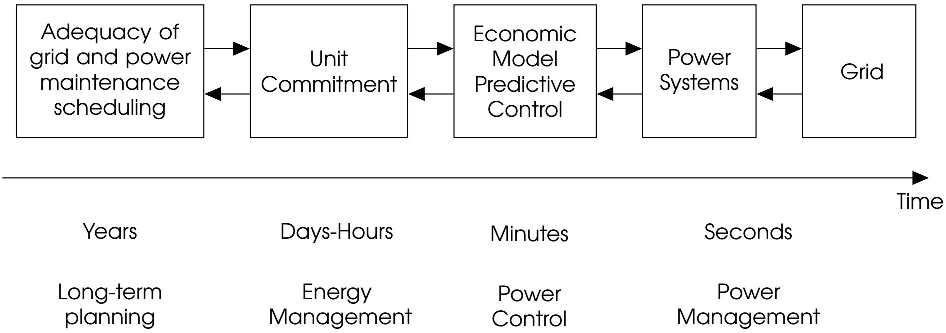



**Figure 2.3:** Control hierarchy.

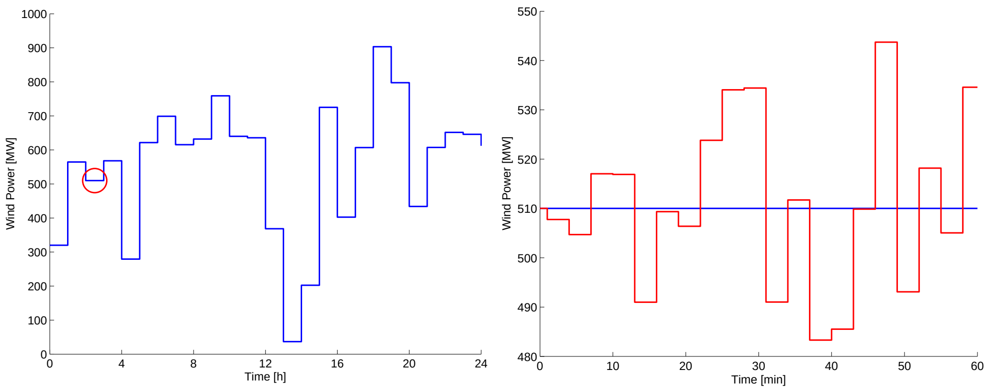
the uncertainties are needed [Mor+14]. The UC model is usually used to find the dispatch that satisfy a forecasted demand load at a minimum cost. As we know, these models can result in high computational complexity due to be generally NP-complete.

At the next level, we apply economic MPC to handle the uncertainty and manage the prediction errors of the renewable power supply at a high frequency level, which cannot be addressed in the UC level because of the optimization solvers execution time. The economic MPC is applied in a rolling horizon manner, thus, updated and more reliable forecasts are used. Figure 2.4 illustrates the idea in the two above-mentioned levels. Energy management plan the hourly power production based on day-ahead forecasts of, e.g., wind power production; see Figure 2.4(a). However, the weather is likely to change during the day of operations and the power supply from renewable energy sources is unlikely constant within an hour. E.g., a closer look at hour 3 may show a wind power production develop as Figure 2.4(b). This variation may result in imbalance and inefficient power production.

Lastly, the power management levels relates to system stability like stabilizing voltage and frequency before the electricity is distributed out to the power grid and finally to the consumers.

## 2.4 Electricity market

Electricity is a commodity production there can be bought, sold, and traded. The electricity market is a market where contracts are made between seller and buyer for the delivery of power. Nord Pool Spot is the main arena for trading power in the Northern Europe and Baltic region. Nord Pool Spot facilitate the day-ahead market and the intraday marked [Enea; Nor; Mor+14]. These two trading categories are interesting in context of this thesis.




**(a)** 24-hours wind power production forecast.

**(b)** Wind power production variation at hour 3.

**Figure 2.4:** The considered wind power production forecasts on the tow levels: days-hours level and minute level.

### 2.4.1 Day-ahead market

The day-ahead market takes place the day before energy delivery. A buyer estimates needed volume of energy to meet demand the following day and the price willing to pay for this volume, hour by hour. A seller decides the volume of energy which can be delivered and at what price, hour by hour. Deadline for submitting bids is 12:00 CET. Elspot setting the price, hour by hour, using the supply and demand principles and closing the deals taking into account the limitations of the power grid. The power is physically delivered at 00:00 CET according to the contracts agreed. The majority of the volume handled by Nord Pool Spot is traded on the day-ahead market.

### 2.4.2 Intraday market

The intraday market takes place during the day of operation when the day-ahead market is closed. Elbas contributes to balance production and consumption in the power market for Northern Europe. Elbas is a continuous market where trading take place every day until one hour before delivery. Prices are set on a first-come, first-served principle; the best prices come first.

This market is progressively relevant as more renewable energy sources such as wind energy and solar energy are integrating into the power grid. These types of energy sources are intermittent and stochastic in nature. Therefore, it is a difficult task to provide accurate predictions of the amount produced prior to the closing of the day-ahead market. Hence, imbalances between day-ahead contracts and produced volume may need to be equalized.

CHAPTER 3

# Software

In this chapter, we informally present the software used in this thesis.

## 3.1 IBM® ILOG® CPLEX® Optimization Studio

We model the mathematical UC optimization problems with IBM ILOG CPLEX Optimization Studio V12.6 by IBM® [IBM14]. This software consolidates an Integrated Development Environment (IDE) with the Optimization Programming Language (OPL) and the state-of-the-art ILOG CPLEX and CP Optimizer solution engines. The motivation for applying this optimization software product is primarily due to following two reasons:

- The modeling language OPL provides built-in tools and a syntax that is very close to the mathematical formulation. This makes an easier transition from a mathematically written model to a model that is solvable by a computer.
- Permit a clearer separation between model and input data. So, with a little effort, the same model can be solved with different input data. The software facilities integration of external data sources like databases and spreadsheets from Microsoft Excel by read and write references.

Figure 3.1 illustrates a simplified overview of IBM ILOG CPLEX Optimization Studio. Comprehensive documentation and release notes for V12.6 are available in [IBM14; IBM11].

## 3.2 Matlab® MathWorks®

The control framework is modeled and implemented in MATLAB® release R2014a by MathWorks® [Mat]. MATLAB stands for MATrix LABoratory and is a high-level language and interactive environment. Section 6.5 provide a description of the control framework and a flowchart of the function calls.

For a better integration with the UC optimization problem and the economic MPC problem, UC is also implemented in MATLAB and solved using IBM ILOG CPLEX Optimizer V12.6 MATLAB interface. The control framework is implemented to supports three solvers, which easily can be selected by a flag. The solvers are CPLEX

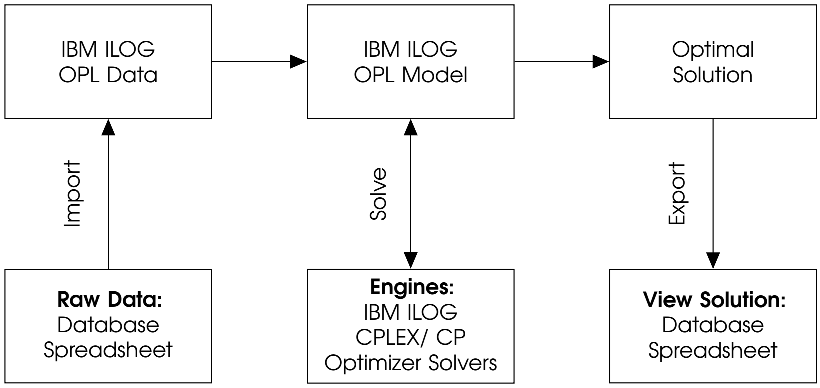



**Figure 3.1:** Simplified illustration of IBM ILOG CPLEX Optimization Studio.

Optimizer V12.6 developed by ILOG IBM Software [IBM14], MOSEK V7.0 developed by MOSEK ApS [MOS14], and Gurobi Optimizer V5.6 developed by Gurobi Optimization, Inc [Gur14]. These are commercial solvers; however, for academic use the solvers can be provided for free.

# Part II

# Theory

CHAPTER 4

# Unit Commitment

In this chapter, we give an introduction to the UC optimization problem. We describe and formalize mathematically the components of the UC problem in Section 4.2. Section 4.3 present the UC optimization problem as a MILP problem. Section 4.4 discuss implementation and a syntax comparison between IBM ILOG CPLEX Optimization Studio and MATLAB, and Section 4.5 informally address the complexity of solving the UC problem. In Section 4.6, we demonstrate and apply the formulated UC optimization problems in the application of two conceptual power systems.

## 4.1 Introduction

Planning the power production to match demand load and reserve capacity with a financial and environmental perspective is nontrivial. Conceptually, consider a portfolio of controllable power generating plants. Then, a planning problem is basically twofold:

1. determine which power plants are running each time step and
2. determine the production level for the running plants in a cost effective way.

We say that power plants are committed when turned on and decommitted when turned off. Finding the optimal cost effective power production plan of the committed plants while satisfying various requirements denotes economic dispatch. This planning problem may be solved by formulating the Unit Commitment (UC) optimization problem. As we see, the problem consist of discrete and continuous decisions, thus, the general UC is a Mixed Integer Programming (MIP) problem. The presences of integer variables yields into a computational complex problem there is not straightforward to solve. In Section 4.5, we address this further.

In the following, we mathematical formulate the UC optimization problem used in this thesis. It should be noted that other formulation exist, since the problem is very system dependent. Therefore, for further literature and related research on this topic includes, e.g., [WW12; OAV12; Cas+11; Pad04; NKF09; ZGH10; RG91; MNG14]. Furthermore, we notice that the notation used in UC may be confused with the notation used in economic MPC. It is decided to apply same notation as in the literature in both fields for not to mislead the reader. However, in context, the notation should not be misunderstood.

## 4.2 Mathematical problem formulation

In this section, we present verbally and formulate mathematically the affine objective function and the constraints for the UC optimization problem. The discrete part of the problem will be formulated with three binary variables. Thus, we formulate a binary mixed integer linear programming problem.

Consider a set $\mathcal{I} := \{1,2,\ldots,I\}$ of power generating plants and a specified time-varying demand over $\mathcal{T} := \{1,2,\ldots,T\}$ time periods defining the planning time horizon. The mathematical programming model involves $I \times T$ continuous nonnegative real variables, $p_{i,t} \in \mathbb{R}_{\geq 0}$; and three $I \times T$ binary variables: $u_{i,t}$, $y_{i,t}$, $z_{i,t} \in \mathbb{Z}_2$.

### 4.2.1 Objective function

The objective cost function is formulated to minimize the total operating power production cost. The operating cost consists of running cost, startup cost, and shutdown cost.

The running cost is model by fixed and variable cost. The fixed cost is expressed as

$$a_i u_{i,t}, \tag{4.1}$$

where $a_i$ is the fixed cost of plant $i$ and $u_{i,t}$ is a binary variable that is equal to one if plant $i$ is committed during time period $t$ and zero otherwise. The variable cost is expressed as proportional to the plant power output:

$$b_i p_{i,t}, \tag{4.2}$$

where $b_i$ is the variable cost of plant $i$ and $p_{i,t}$ is the nonnegative real variable that is the power output of plant $i$ during time period $i$.

The startup and shutdown cost is considered constant. Every time a plant is started up, its startup cost is added. Similar, every time a plant is shut down, its shutdown cost is added. Thus, we obtain

$$SU_i y_{i,t} + SD_i z_{i,t}, \tag{4.3}$$

where $SU_i$ and $SD_i$ are the startup and shutdown cost of plant $i$, respectively. $y_{i,t}$ is a binary variable that is equal to one if plant $i$ is started up at the beginning of time period $i$ and zero otherwise and $z_{i,t}$ is a binary variable that is equal to one if plant $i$ is shut down at the beginning of time period $i$ and zero otherwise.

The function to be minimized is obtained by combining (4.1)–(4.3), thus, the objective function of the UC problem is

$$\phi = \sum_{i\in\mathcal{I}} \sum_{t\in\mathcal{T}} \left[a_i u_{i,t} + b_i p_{i,t} + SU_i y_{i,t} + SD_i z_{i,t}\right]. \tag{4.4}$$

### 4.2.2 Constraints

Several constraints may be placed to the UC problem, as many different requirements can be given; e.g., individual requirements on power demand, reliability, physical limits of equipment, power system operating limits, etc. The list presented here is by no means exhaustive; these are the ones being considered and implement in this thesis. To deduce some of the constraints, we use logical equivalences; see Appendix A.2.1.

#### 4.2.2.1 Power demand load balance

The system power demand should be satisfied for each time period:

$$\sum_{i \in \mathcal{I}} p_{i,t} \geq D_t, \qquad t \in \mathcal{T}, \tag{4.5}$$

where $D_t$ is the power demand in time period $t$. Power contribution from non-controllable renewable power sources will change the need for producing power from the conventional power plants. $PW_t$ represent forecasted power production from renewable power sources at time period $t$. Thus, (4.5) i extended to

$$\sum_{i \in \mathcal{I}} p_{i,t} \geq D_t - PW_t, \qquad t \in \mathcal{T}. \tag{4.6}$$

#### 4.2.2.2 Spinning reserve

In order to ensure reliability in term of enough resources available during the real-time operation of the power system, the system operator allocates reserve capacity to cover unexpected shortages of energy supply in real-time.

The required spinning reserve should be guaranteed to be available by the committed plants:

$$\sum_{i \in \mathcal{I}} PU_i u_{i,t} \geq D_t + R_t, \qquad t \in \mathcal{T}. \tag{4.7}$$

where $PU_i$ is the maximum power output generation of plant $i$ and $R_t$ is the required spinning reserve at time period $t$. Like the demand load balance constraint above, we introduce $PW_t$ in the event of contribution from renewable power sources, thus,

$$\sum_{i \in \mathcal{I}} PU_i u_{i,t} \geq D_t + R_t - PW_t, \qquad t \in \mathcal{T}. \tag{4.8}$$

#### 4.2.2.3 Power output limitations

The power plants are limited within an operating range, i.e., if a plant is committed, the power output is to be within its minimum and maximum power output generation. This may be expressed as

$$PL_i u_{i,t} \leq p_{i,t} \leq PU_i u_{i,t}, \qquad i \in \mathcal{I}, t \in \mathcal{T}, \tag{4.9}$$

where $PL_i$ and $PU_i$ are the minimum and maximum power output generation of plant $i$, respectively. We see, if plant $i$ at time period $t$ is committed, $u_{i,t} = 1$, the power output, $p_{i,t}$, is to be within limits, whereas if plant $i$ at time period $t$ is decommitted, $u_{i,t} = 0$, the preceding constraint forces $p_{i,t} = 0$.

#### 4.2.2.4 Ramping rate limitations

The power plants ability to increase and decrease to higher and lower power output from time period $k$ to $k+1$ is limited. The so called ramp rate limits may be expressed by

$$p_{i,t} - p_{i,t-1} \leq RU_i, \qquad i \in \mathcal{I},\ t \in \mathcal{T}, \tag{4.10}$$

$$p_{i,t-1} - p_{i,t} \leq RD_i, \qquad i \in \mathcal{I},\ t \in \mathcal{T}. \tag{4.11}$$

For the time period $t = 1$, $p_{i,0}$ is given by the initial output power of plant $i$. $RU_i$ and $RD_i$ are the maximum ramp-up and ramp-down limit of plant $i$, respectively.

#### 4.2.2.5 Startup and shutdown

Any committed plants can be shut down but and not started up, and analogously, any decommitted plants can be started up but not shut down. This can be expressed by logic constraints with startup and shutdown cost term added, respectively:

$$u_{i,t} - u_{i,t-1} \leq y_{i,t}, \qquad i \in \mathcal{I},\ t \in \mathcal{T}, \tag{4.12}$$

$$u_{i,t-1} - u_{i,t} \leq z_{i,t}, \qquad i \in \mathcal{I},\ t \in \mathcal{T}. \tag{4.13}$$

For the time period $t = 1$, $u_{i,0}$ is given by the plants status preceding the first period of the planning horizon. By considering the possible scenarios, the logic constraints (4.12) and (4.13) gives intuitively sense. E.g., consider the lefthand side of (4.12). The only scenario this yields to one is when $u_{i,t} = 1$ and $u_{i,t-1} = 0$, thus, startup cost should be added to the objective function. The expressions, however, may be derived from logic conditions [RG91]. Consider the startup scenario. Startup cost should be added to the objective function if $u_{i,t} = 1$ and $u_{i,t-1} = 0$. Let $P_A$ denote a committed plant $i$ at time $t$, $\neg P_B$ denote a decommitted plant $i$ at time $t-1$, and $P_C = y_{i,t}$ denote whether startup cost is add, $y_{i,t} = 1$, or not, $y_{i,t} = 0$. Then, we have

$$P_A \wedge \neg P_B \;\Rightarrow\; P_C$$

By (A.11), we can remove the implication, thus

$$\neg(P_A \wedge \neg P_B) \vee P_C.$$

By applying (A.12) (De Morgan's theorem), we have

$$\neg P_A \vee P_B \vee P_C.$$

With the implication from above and that, e.g., $\neg P_A = 1 - u_{i,t}$, the conjunction form can be translated into its equivalent mathematical linear form:

$$\begin{aligned} 1 - u_{i,t} + u_{i,t-1} + y_{i,t} \geq 1 \qquad &\Rightarrow \\ u_{i,t} - u_{i,t-1} \leq 1, \end{aligned}$$

which is equivalent to (4.12). Likewise, (4.13) can be deduced by same approach.

#### 4.2.2.6 Minimum up- and downtime

Due to physical characteristics, power plants may not immediately be able to startup and shutdown and vice versa. Let $TU_i$ denote the minimum uptime for plant $i$, once it has started up. If $y_{i,t} = 1$, then $u_{i,t+1} = 1$, $u_{i,t+2} = 1$, ..., $u_{i,t+TU_i}$; thus, we write the logic expression

$$y_{i,t} \Rightarrow \bigwedge_{j \in \mathcal{U}_i} u_{i,t+j}, \tag{4.14}$$

where $\mathcal{U}_i := \{1,2,\dots,TU_i\}$. By (A.11), (4.14) can be rewritten as

$$\neg y_{i,t} \vee u_{i,t+j} \tag{4.15}$$

Translating (4.15) conjunction expression into its equivalent mathematical linear form gives the minimum uptime constraint:

$$\begin{aligned} 1 - y_{i,t} + u_{i,t+j} \geq 1 \qquad &\Rightarrow \\ u_{i,t+j} \geq y_{i,t}, \qquad &j \in \mathcal{U}_i. \end{aligned} \tag{4.16}$$

Similar, we derive the minimum downtime. Let $TD_i$ denote the minimum downtime for plant $i$, once it has been shutdown. If $z_{i,t} = 1$, then $u_{i,t+1} = 0$, $u_{i,t+2} = 0$, ..., $u_{i,t+TD_i}$, which leads to the logic expression

$$z_{i,t} \Rightarrow \bigwedge_{j \in \mathcal{D}_i} \neg u_{i,t+j}, \tag{4.17}$$

where $\mathcal{D}_i := \{1,2,\dots,TD_i\}$. Hence, its equivalent mathematical linear form gives the minimum downtime constraint:

$$\begin{aligned} 1 - z_{i,t} + 1 - u_{i,t+j} \geq 1 \qquad &\Rightarrow \\ z_{i,t} + u_{i,t+j} \leq 1, \qquad &j \in \mathcal{D}_i. \end{aligned} \tag{4.18}$$

#### 4.2.2.7 Restricting carbon dioxide emission

There may be restricting on carbon dioxide emission when generating power. This may be expressed as

$$\sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} EC_i p_{i,t} \leq EU, \tag{4.19}$$

where $EC_i$ is the $CO_2$ emission rate for plant $i$ and $EU$ denote the maximum $CO_2$ emission allowed.

## 4.3 The UC optimization problem

In Section 4.2.1, we present the objective cost function which is to be minimized while satisfying the constraints provided in Section 4.2.2. Two formulations of the UC optimization problem will be considered; see demonstration of the two in Section 4.6. They differ in terms of whether or not including the minimum up- and downtime constraints and the restricting carbon dioxide emission constraint. First, we formulate the UC optimization problem as the following MILP:

$$\begin{aligned}
\text{minimize} \quad & \phi = \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} [a_i u_{i,t} + b_i p_{i,t} + SU_i y_{i,t} + SD_i z_{i,t}] && && (4.20a)\\
\text{subject to} \quad & \sum_{i\in\mathcal{I}} p_{i,t} \geq D_t - PW_t && t\in\mathcal{T} && (4.20b)\\
& \sum_{i\in\mathcal{I}} PU_i u_{i,t} \geq D_t + R_t - PW_t && t\in\mathcal{T} && (4.20c)\\
& PL_i u_{i,t} \leq p_{i,t} \leq PU_i u_{i,t} && i\in\mathcal{I},\, t\in\mathcal{T} && (4.20d)\\
& p_{i,t} - p_{i,t-1} \leq RU_i && i\in\mathcal{I},\, t\in\mathcal{T} && (4.20e)\\
& p_{i,t-1} - p_{i,t} \leq RD_i && i\in\mathcal{I},\, t\in\mathcal{T} && (4.20f)\\
& u_{i,t} - u_{i,t-1} \leq y_{i,t} && i\in\mathcal{I},\, t\in\mathcal{T} && (4.20g)\\
& u_{i,t-1} - u_{i,t} \leq z_{i,t} && i\in\mathcal{I},\, t\in\mathcal{T} && (4.20h)\\
& u_{i,t},\, y_{i,t},\, z_{i,t} \in \mathbb{Z}_2 && && (4.20i)\\
& p_{i,t} \in \mathbb{R}_{\geq 0} && && (4.20j)
\end{aligned}$$

with $\mathcal{I} := \{1,2,\ldots,I\}$ thermal generating plants and $\mathcal{T} := \{1,2,\ldots,T\}$ time periods defining the planning time horizon.

Including the minimum up- and downtime constraints and the carbon dioxide emission restricting yields into the following:

$$\begin{aligned}
\text{minimize} \quad & \phi = \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} [a_i u_{i,t} + b_i p_{i,t} + SU_i y_{i,t} + SD_i z_{i,t}] && && (4.21a)\\
\text{subject to} \quad & (4.20b) - (4.20h) && && (4.21b)\\
& u_{i,t+j} \geq y_{i,t} && j\in\mathcal{U}_i && (4.21c)\\
& z_{i,t} + u_{i,t+j} \leq 1 && j\in\mathcal{D}_i && (4.21d)\\
& \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} EC_i p_{i,t} \leq EU && && (4.21e)\\
& u_{i,t},\, y_{i,t},\, z_{i,t} \in \mathbb{Z}_2 && && (4.21f)\\
& p_{i,t} \in \mathbb{R}_{\geq 0} && && (4.21g)
\end{aligned}$$

with $\mathcal{I} := \{1,2,\ldots,I\}$ thermal generating plants, $\mathcal{T} := \{1,2,\ldots,T\}$ time periods defining the planning time horizon, $\mathcal{U}_i := \{1,2,\ldots,TU_i\}$, and $\mathcal{D}_i := \{1,2,\ldots,TD_i\}$.

The difficulties for solving the UC optimization problem (4.20) or (4.21) are due to the presences of binary variables, since the objective function and all the constraints are linear.

## 4.4 Implementation

In this section, we show how the UC problem may be implemented in IBM® ILOG® CPLEX® Optimization Studio V12.6 and in MATLAB by MathWorks®. A comparison of the two languages is provided. The MATLAB version is implemented such that CPLEX MATLAB interface is used. We present the code for implementing the objective function (4.4) and power demand load balance constraint (4.6). The full implementations are provided in Source Code Booklet Section 1.1 and Section 2.1, respectively.

**Listing 4.1:** Syntax for IBM ILOG CPLEX Optimization Studio.

```
// Objective function
minimize
  sum(t in T, i in I) (a[i]*u[i,t] + b[i]*p[i,t]
                      + SU[i]*y[i,t] + SD[i]*z[i,t]);
// Constraints
subject to {

  // Demand load balance
  forall(t in T) {
    sum(i in I) p[i,t] >= D[t] - PW[t];
  }
};
```

**Listing 4.2:** Syntax for MATLAB.

```
% Objective function
coeffpvar = repmat(b,Horizon,1);
coeffuvar = repmat(a,Horizon,1);
coeffyvar = repmat(SU,Horizon,1);
coeffzvar = repmat(SD,Horizon,1);
cplex.Model.obj = [coeffpvar; coeffuvar; coeffyvar; coeffzvar];

% Demand load balance
cplex.Model.A = [kron(eye(Horizon),ones(1,Nunits)) zeros(Horizon,Nvar)];
cplex.Model.lhs = D - PW;
cplex.Model.rhs = Inf(Horizon,1);
```

Instantly, we see that the MATLAB implementation Listing 4.2 is much more complex and not as simple and intuitive as IBM ILOG CPLEX Optimization Studio Listing 4.1. Some of the reasons for this are outlined in the following. We are considering four two-dimensional decision variables. In MATLAB, these decision variables are to be stacked into one one-dimensional vector in the following manner

$$x = \begin{bmatrix} p & u & y & z \end{bmatrix}^T.$$

Each two-dimensional decision variable are again stacked like this

$$p = \begin{bmatrix} p_{1,1} & p_{2,1} & \dots & p_{N,1} & p_{1,2} & p_{2,2} & \dots & p_{N,2} & \dots & p_{N,M} \end{bmatrix}^T,$$

where $N$ is the number of power plants and $M$ the number of planning time periods. Furthermore, each row in the system matrix $A$ represent one constraint for one specific plant for one time period and each column in $A$ represent one decision variable for one specific plant for one time period. Consequently, a lot of bookkeeping are needed in the MATLAB implementation. Comparing this with the modeling language OPL syntax in Listing 4.1 truly shows its benefits. The syntax is very close to the mathematical formulation. It is light and one can literally just write out the equations. OPL syntax provide an easy way to write an optimization problem for which is solvable by a computer.

## 4.5 Solution methods

The general UC problem is nontrivial to solve. The UC problem quickly becomes very complex and extremely difficult to obtain the optimal solution for system with practical size (large-scale power systems). In fact, the industry is in some situations compelled to manually stop the optimization software and simply use the current iterative obtained solution due to limited period of time. Mathematically, UC is an NP-complete MIP problem, which make it impossible to develop an algorithm with polynomial computation time; i.e., in worst case, the solution time grows exponentially with the problem size [AH04; BMM99; GZP03].

To illustrate the complexity, we consider the brute force methods. Let $N$ denote the number of power plants and $M$ denote the number of planning time periods. Then, the number of combinations to be evaluated each time period is $2^N - 1$. For the total planning horizon, the maximum number of possible combinations is $(2^N - 1)^M$, which scales badly and quickly return into an enormous number. E.g., consider $N = \{10,20\}$ power plants each hour over a 24-hour planning horizon, this result in

$$\begin{aligned} N &= 10, \qquad (2^{10} - 1)^{24} = 1.73 \cdot 10^{72} \\ N &= 20, \qquad (2^{20} - 1)^{24} = 3.12 \cdot 10^{144}. \end{aligned}$$

Thus, the number of combinations immediately becomes very big. Of course, not all combinations are feasible solutions sets, nevertheless, all need to be checked.

There exist many optimization techniques for solving the UC problem, e.g.,

- Priority list schemes
- Dynamical programming
- Lagrangian relation
- Branch-and-bound method

Solution methods for UC problems are an important topic; however, a study to cover them is outside the scope of this thesis. Therefore, for further information of the various methods available for solving the UC problem see, e.g., [SK98; WW12].

## 4.6 Case study

In this section, we establish two conceptual power systems to demonstrate the formulated UC optimization problem. Firstly, we apply (4.20) to a 3-unit power system. Secondly, we apply (4.21) to a complex 10-unit power system. Besides complexity, they differ in terms of specific system requirements. Renewable power contribution is fixed to zero: $PW_t = 0, \forall t$.

The case studies are implemented in IBM ILOG CPLEX Optimization Studio using CPLEX 12.6 and are provided in Source Code Booklet Chapter 1. [Cas+11; ZGH10] inspired the two case studies.

### 4.6.1 3-unit power system

We consider three power plants over a three hour planning horizon. Table 4.1 lists the demand load to be satisfied for each time period and Table 4.2 lists the operational parameters of the power plants. Let all plants be decommitted at time period $t = 0$.

The UC optimization problem (4.20) presented mathematically in Section 4.3 is solved for global optimality. The MILP problem for the 3-unit system contains 60 constraints and 37 variables, where 27 are binary variables. The obtained optimal power production plan, reported in Table 4.3, result in a minimum cost of $191.

We see that the demand load, the spinning reserve, and the system requirements are satisfied. Even though this case study is small a trend can be deduced. The heavier generators are scheduled first (close to their maximum capacity), while the

**Table 4.1:** 3-hour demand load for the 3-unit power system [MW].

| Hour | $D_t$ | $R_t$ |
|---|---|---|
| 1 | 150 | 15 |
| 2 | 500 | 50 |
| 3 | 400 | 40 |

**Table 4.2:** Operational parameters for the 3-unit power system.

| Unit | $a_i$ [$/h] | $b_i$ [$/MWh] | $SU_i$ [$/h] | $SD_i$ [$/h] | $PL_i$ [MW] | $PU_i$ [MW] | $RD_i$ [MW/h] | $RU_i$ [MW/h] |
|---|---|---|---|---|---|---|---|---|
| 1 | 5 | .100 | 20 | 5 | 50 | 350 | 300 | 200 |
| 2 | 7 | .125 | 18 | 3 | 80 | 200 | 150 | 100 |
| 3 | 6 | .150 | 5 | 1 | 40 | 140 | 100 | 100 |

**Table 4.3:** Optimal production plan for the 3-unit power system [MW].

| Plant | Hour | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| 1 | 150 | 350 | 320 |
| 2 | 0 | 100 | 80 |
| 3 | 0 | 50 | 0 |

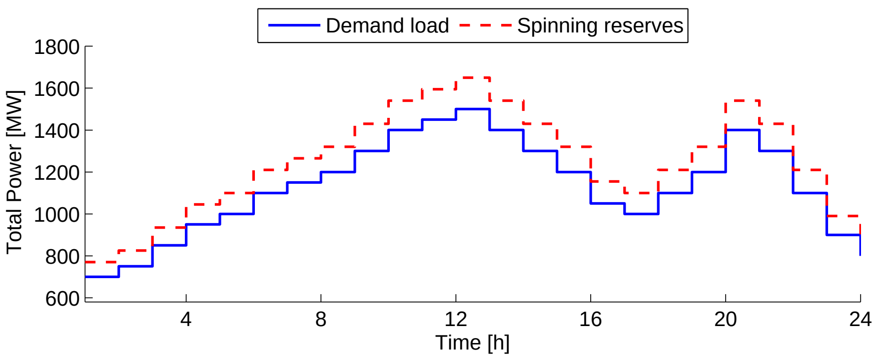



**Figure 4.4:** 24-hour demand load for the 10-unit power system [MW].

smaller generators are used to deal with the fluctuations in the demand load. This corresponds to a commonly used scheduling strategy called Priority List (PL) scheme [SK98; WW12].

### 4.6.2 10-unit power system

We consider ten power plants over a 24-hour planning horizon. Figure 4.4 represents the demand load to be satisfied for each time period; see Appendix B Table B.1(a) for numerical representation. The spinning reserve is set as a 10% of the demand load for each time period. The power plants operational parameters are listed in Appendix B Table B.2. Let all plants be decommitted at time period $t = 0$ and let the maximum $CO_2$ emission be fixed to 15,500 g.

The UC optimization problem (4.21) presented mathematically in Section 4.3 is solved for global optimality. The MILP problem contains 7,335 constraints and 961 variables, where 720 are binary variables. The obtained optimal power production plan, reported in Figure 4.5, result in a minimum cost of $572,395. Figure 4.6 illustrates the optimal power production plan for plant 1.

In the following, we show that the obtained solution complies with the system requirements. Figure 4.7 shows that the obtained production plan satisfy the demand load (production exactly equals the demand load) and the spinning reserve require-

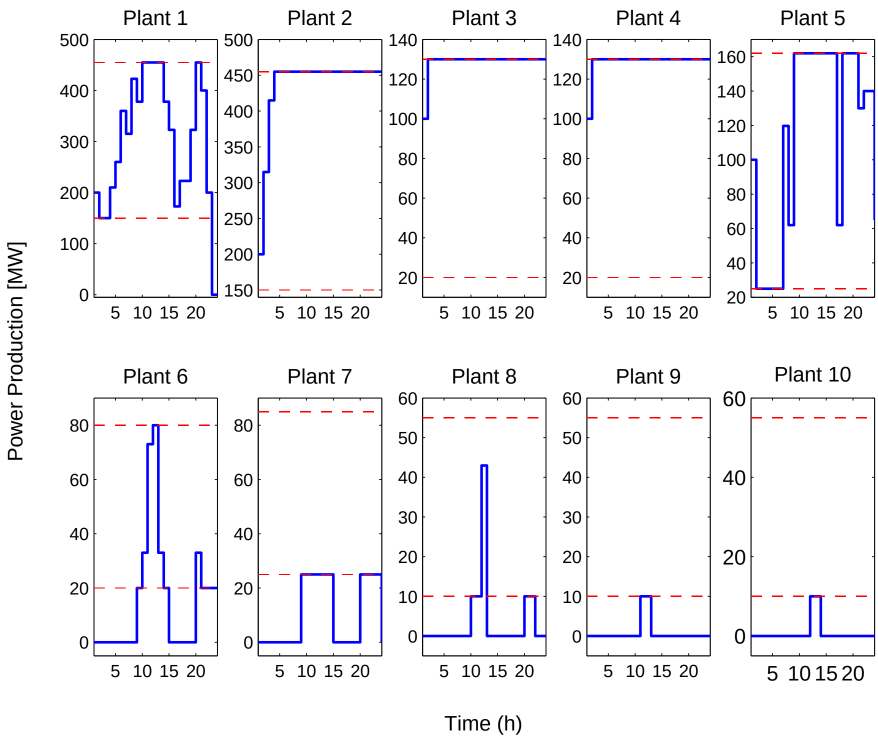



**Figure 4.5:** The optimal power production plan for each plants (blue line) with each plants minimum and maximum power output (red dashed line).

ment at each time period. Figure 4.5 and Figure 4.8 show that the plants minimum and maximum power output and ramping limits are fulfilled at each time period, respectively. Lastly, the $CO_2$ emission is exactly below the maximum. Consequently, the formulated constraints (4.21) are satisfied.

Figure 4.9 and Figure 4.10 illustrates the distribution of the four cost components in the objective function. The total production cost dominates by the variable cost accounting for ∼82% of the total production cost. Next is the fixed cost with ∼18%, while startup and shutdown only account for <1%. Consequently, the total production cost follows more or less the demand load.

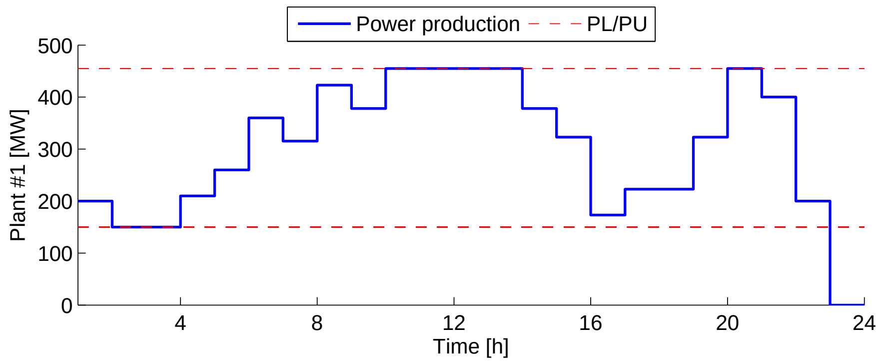



**Figure 4.6:** The optimal power production plan for plant 1 with its minimum and maximum power output limits.

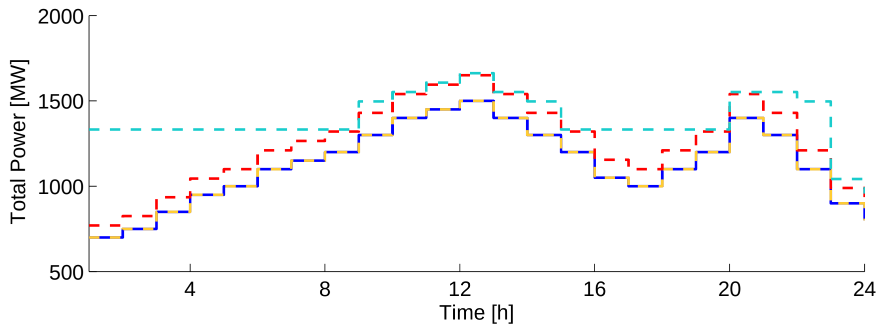



**Figure 4.7:** The total production plan (dotted yellow) satisfy demand load (solid blue); coincide throughout planning horizon. The actual spinning reserve (dotted cyan) obey the required spinning reserve (dotted red).

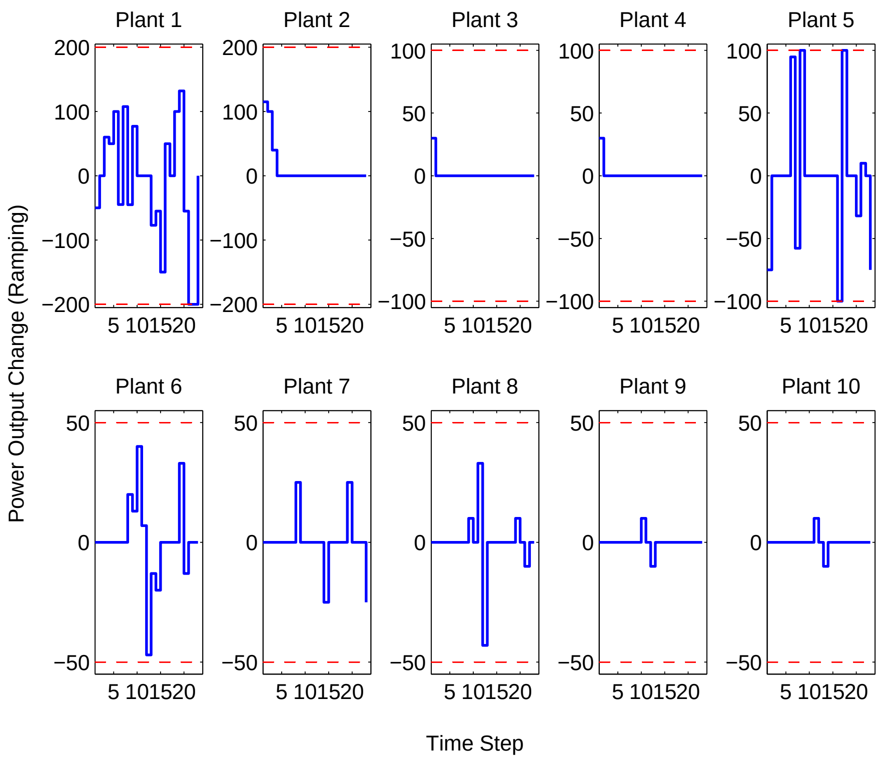



**Figure 4.8:** Change in power output for each time step (solid blue) with each plant ramping limits (dotted red).

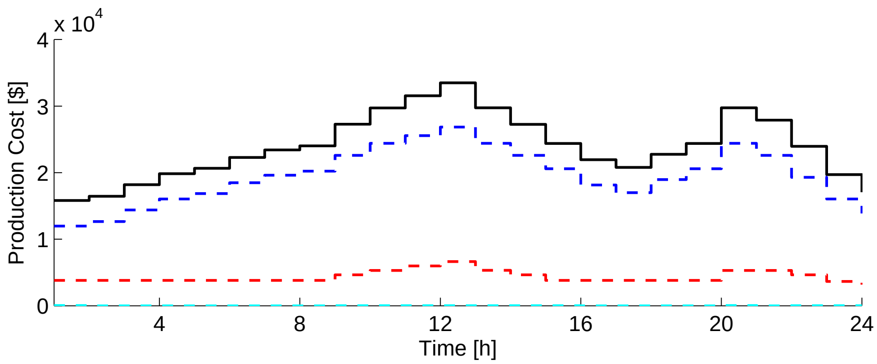



**Figure 4.9:** The total production cost (solid black) over the planning horizon. The variable cost (dotted blue), the fixed cost (dotted red), and startup and shutdown cost (dotted cyan).

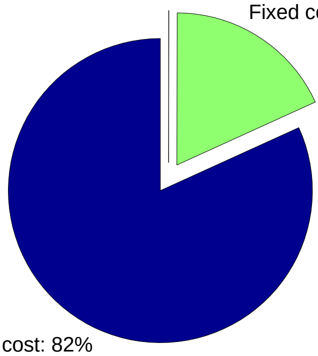



**Figure 4.10:** Distribution of the four cost components in the objective function.

## 4.7 Summary

This chapter formulates mathematically the UC optimization problem as a MILP problem. We show that the OPL syntax provided in IBM ILOG CPLEX Optimization Studio is very close to the mathematical formulation compared to the MATLAB syntax and informally present optimization techniques for solving the UC problem. Lastly, we apply the formulated UC optimization problems at two conceptual power systems. The case studies demonstrate the UC and that the implementation behaves as expected.

CHAPTER 5

# Models for Predictive Control

This chapter present key parts in development of our economic MPC framework in Chapter 6. We outline the basic idea of modeling dynamical systems for predictive control. In Section 5.2, we formulate the mathematical model applied for modeling the dynamics of a power system (portfolio of power generating plants). Section 5.3 define a finite impulse response model and Section 5.4 present a Kalman filtering and prediction for the linear time-invariant discrete-time state-space models formulated in Section 5.2. Additionally, in the presence of unmeasured nonzero disturbance, Section 5.5 present how to achieve offset free tracking performance for MPC.

## 5.1 Modeling dynamical systems

A dynamical system describes the behavior of a physical system evolution over time. The variables describing the state of the mathematical system, at any given time, are characterized by its state variables. The state variables are stacked in a time-varying state vector $x(t)$, $t \in \mathbb{R}$. The set of all the possible values of the state variables is the state-space. As the process evolve in time, the state variables change from its initial state $x(t_0) = x_0$ by the underlying dynamical processes. A dynamical system may, e.g., be modeled by systems of ordinary differential equations in the form

$$\frac{d}{dx}x(t) = \dot{x}(t) = f(x(t), u(t), d(t), w(t)), \qquad x(t_0) = x_0, \tag{5.1}$$

where $x(t)$ is the state vector, $u(t)$ is the manipulated input variables, $d(t)$ is the disturbance, and $w(t)$ is the stochastic process noise. The measured outputs, $y(t)$, and the controlled outputs, $z(t)$, may be modeled by

$$y(t) = g(x(t), v(t)) \tag{5.2a}$$

$$z(t) = h(x(t)), \tag{5.2b}$$

where $v(t)$ is the stochastic measurement noise. The deterministic formulation is obtained without measurement and process noise. Let $n_x$ denote the number of coupled nonlinear differential equations, which also is the number of variables in $x$.

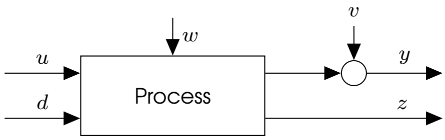



**Figure 5.1:** A generic stochastic input-output model.

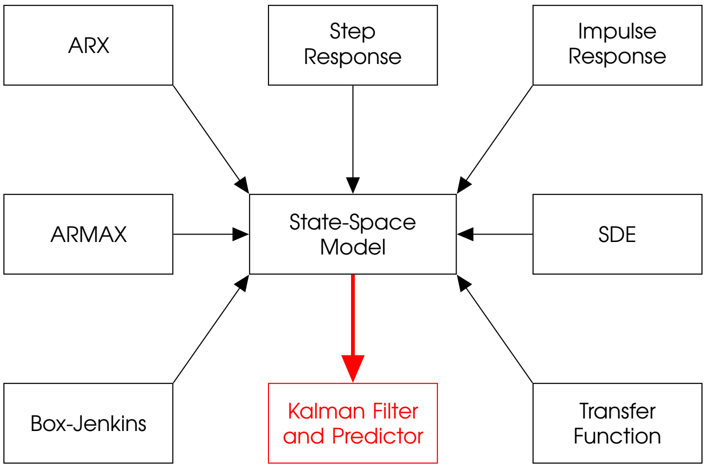



**Figure 5.2:** State-space model realization of linear time-invariant models.

Following that notation, we label $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$, $y \in \mathbb{R}^{n_y}$, and $z \in \mathbb{R}^{n_z}$. In this thesis, linear systems of finite dimension will be considered, thus, the functions $f$, $g$, and $h$ are linear.

The generic input-output model relations consist of a process model of the system (5.1), the sensor function (5.2a), and the output function (5.2b). The block diagram in Figure 5.1 illustrates the above stochastic formulation.

There exist several methods for achieving mathematical models in which describe a dynamical system. Many time-invariant mathematical models can be formulated as a discrete-time state-space model as depicted in Figure 5.2. This formulation is beneficial in the control framework presented in Chapter 6. Each of the realization models depicted in Figure 5.2 will not be addressed, since it would be outside the scope of the thesis. Appendix A.1 provide background material of the basic concepts of how a linear time-invariant continuous-time model may be linearized and discretized to obtain a linear time-invariant discrete-time state-space model, for which is suitable for control problems. For further literature to this subject, consult, e.g., [Jør11; Jør05; Mac02; Hal+14; Dat04].

## 5.2 Modeling power systems

In this section, we present the system models applied for modeling a power system. We formulate both a MIMO system and a MISO system. We consider a linear distributed system with independent power generators there must satisfy their own objective while collaborate to satisfy a common objective, the overall demand load.

### 5.2.1 Power system dynamics

The power system consists of a portfolio of $n_u$ controllable power generators (e.g. thermal power plants) and a portfolio of $n_d$ non-controllable predictable power generators representing the power production from the renewable energy sources. We assume that the non-controllable power generators are farms of wind turbines.

Each individual power generators are independent systems and modeled separately, as the activity in one generator does not directly affect the others. However, they are coupled in order to collaborate to satisfy the overall power demand load. We represent the controllable and non-controllable power generators by following linear systems

$$Z_{u,i}(s) = G_i(s)U_i(s), \qquad i = 1,2,\dots,n_u, \tag{5.3a}$$

$$Z_{d,j}(s) = D_j(s), \qquad j = 1,2,\dots,n_d. \tag{5.3b}$$

$U_i(s)$ is the manipulable variables describing the unit of fuel supplied to plant $i$, $D_j(s)$ is the known forecast from wind turbine $j$, $Z_{u,i}(s)$ is the power produced by plant $i$, and $Z_{d,j}(s)$ is the power produced by wind turbine $j$. The transfer functions $G_i(s)$ describe system dynamics for plant $i$. The total power production is obtained by

$$\begin{aligned} Z_T(s) &= \sum_{i=1}^{n_u} Z_{u,i}(s) + \sum_{j=1}^{n_d} Z_{d,j}(s) \\ &= \sum_{i=1}^{n_u} G_i(s)U_i(s) + \sum_{j=1}^{n_d} D_j(s). \end{aligned} \tag{5.4}$$

We model the dynamics of the controllable power plants, $G_i(s)$, with following third order model

$$G_i(s) = \frac{1}{(\tau_i s + 1)^3}. \tag{5.5}$$

This third order model has been provided and validated against experimental data at DONG Energy, Denmark [EMB09]. Let the non-controllable power generators be a wind turbine. The dynamics of a wind turbine may be model by following first order model [EMB09; Hov13]

$$H_j(s) = \frac{1}{\tau_j s + 1}, \tag{5.6}$$

with a time constant, $\tau_j$, less than 5 seconds. Wind turbines have fast dynamics and react very quickly to set-point changes. In fact, the dynamics will be drowned

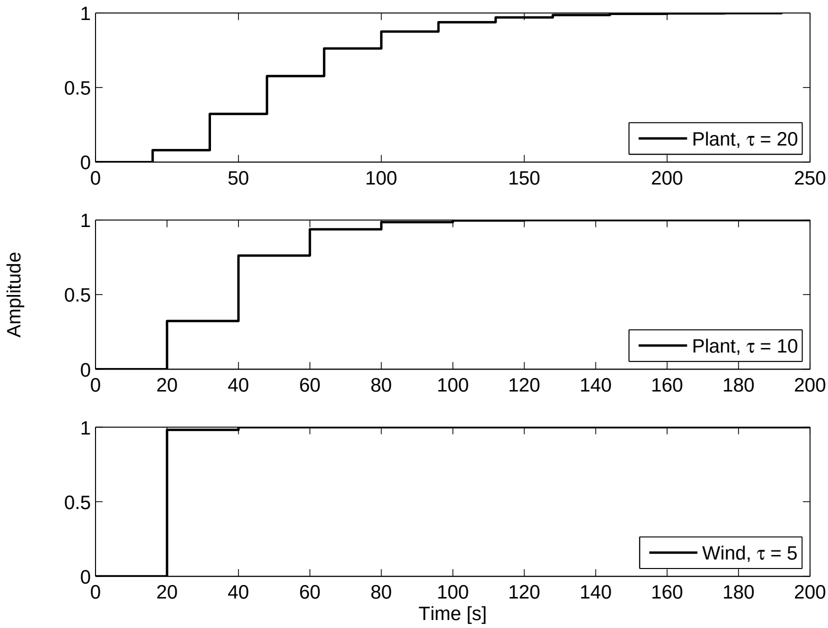



**Figure 5.3:** Deterministic step responses of the transfer functions (5.5) and (5.6) discretized using zero-order hold with sampling time of $T_s = 20$ seconds.

by the power plant dynamics and discretization, and thereby have no influence in our simulations. Therefore, we disregard the first order transfer function and replace $H_j(s) = 1\ \forall j$. The forecasted power production from the renewable energy sources are assumed to be in power [MW]. If the controller would get, e.g., the wind speed of wind turbines one could incorporate this into the model.

#### 5.2.1.1 Step response

We perform a deterministic step response of the two transfer functions (5.5) and (5.6) in discrete-time with time constants $\tau = \{20, 10\}$ and $\tau = 5$, respectively. Figure 5.3 illustrates the system reaction on a step.

For the power plant systems, we obtain an instantly positive response and a gain $K = 1$. These observations are consistent with the given transfer functions (5.5). As expected, the dynamic of the wind turbine is drowned, which verify our assumption above. We see that sampling time of $T_s = 20$ seconds is adequate enough to capture the system dynamics while not been too detailed. Hence, this sampling time or lower will be used throughout the economic MPC simulations.

### 5.2.2 Discrete-time state-space model formulation

The stochastic linear system in continuous-time used to describe the dynamics of the power system (see Section 5.2.1) is represented as

$$Z(s) = G(s)\,(U(s) + W(s)) + H(s)D(s) \tag{5.7a}$$

$$= G(s)\,(U(s) + W(s)) + D(s) \tag{5.7b}$$

with the measurements

$$Y(s) = Z(s) + V(s). \tag{5.8}$$

$U(s)$ is the manipulable variables, $D(s)$ is the known forecast, $W(s)$ is stochastic process noise, $V(s)$ is stochastic measurement noise, $Z(s)$ is the output, and $Y(s)$ is the measurements. The deterministic linear system in continuous-time is obtained with $W(s) = V(s) = 0$, i.e.,

$$Z(s) = G(s)U(s) + D(s) \tag{5.9a}$$

$$Y(s) = Z(s). \tag{5.9b}$$

$G(s)$ and $H(s)$ are transfer function matrices of compatible size in order to obtain the desired system. Cf. Section 5.2.1, $H(s) = 1$. Using a zero-order hold discretization of the continuous-time variables, assumed to be piecewise constant, (5.7) and (5.8) may be represented as the stochastic discrete-time state-space model

$$x_{k+1} = Ax_k + Bu_k + Gw_k \tag{5.10a}$$

$$y_k = Cx_k + d_k + v_k \tag{5.10b}$$

$$z_k = C_z x_k + d_k, \tag{5.10c}$$

where $k \in \mathcal{N}_0 := \{0,1,\dots,N\}$, with $N$ being the predictive horizon, and $G = B$. Assume that the model and the true system are identical. Then, the uncertainties in the state prediction arise from the stochastic process noise and measurement noise. Hence, the optimal filter and predictor is the Kalman filter and predictor [Hal+14]. We presented the Kalman filtering and prediction in Section 5.4.

By similar approach, (5.9) may be represented as the deterministic discrete-time state-space model

$$x_{k+1} = Ax_k + Bu_k \tag{5.11a}$$

$$y_k = Cx_k + d_k \tag{5.11b}$$

$$z_k = C_z x_k + d_k, \tag{5.11c}$$

with $k \in \mathcal{N}_0$. In our simulations, we apply both (5.10) and (5.11).

### 5.2.3 Distributed independent power system

We consider a distributed independent system. The dynamically independently power generators must satisfy their own objective while collaborate to satisfy a common objective, the overall demand load. To obtain these properties, we formulate a Multiple-Input and Multiple-Output (MIMO) system. A special case of the MIMO system, the Multiple-Input and Single-Output (MISO) system, is also formulated for the purpose of only satisfying the overall demand load. Both formulations are used in our simulations. In the following, we present the MIMO system the MISO system in that order.

Applying (5.3) and (5.4). Then, this representation may be related to (5.9) by

$$
\begin{aligned}
Z(s) &= \begin{bmatrix} Z_{u,1}(s) & Z_{u,2}(s) & \cdots & Z_{u,n_u}(s) & Z_T(s) \end{bmatrix}^T \\
U(s) &= \begin{bmatrix} U_1(s) & U_2(s) & \cdots & U_{n_u}(s) \end{bmatrix}^T \\
D(s) &= Z_{d,1}(s) + Z_{d,2}(s) + \cdots + Z_{d,n_d}(s).
\end{aligned}
$$

The transfer function $G(s)$ is represented as

$$
\begin{bmatrix}
G_1(s) & & & \\
& G_2(s) & & \\
& & \ddots & \\
& & & G_{n_u}(s) \\
G_1(s) & G_2(s) & \cdots & G_{n_u}(s)
\end{bmatrix}.
$$

Let $x_{i,k}$ denote the states variables for plant $i$, $u_{i,k}$ denote the manipulable variable for plant $i$, $d_{j,k}$ denote the know forecast for wind turbine $j$, $z_{i,k}$ denote power output for plant $i$, and $z_{T,k}$ denote the total power output. Then, (5.11) may be stated by the block-angular structure

$$
\begin{aligned}
\begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \\ \vdots \\ x_{n_u,k+1} \end{bmatrix} &=
\begin{bmatrix} A_1 & & & \\ & A_2 & & \\ & & \ddots & \\ & & & A_{n_u} \end{bmatrix}
\begin{bmatrix} x_{1,k} \\ x_{2,k} \\ \vdots \\ x_{n_u,k} \end{bmatrix} \\
&+ \begin{bmatrix} B_1 & & & \\ & B_2 & & \\ & & \ddots & \\ & & & B_{n_u} \end{bmatrix}
\begin{bmatrix} u_{1,k} \\ u_{2,k} \\ \vdots \\ u_{n_u,k} \end{bmatrix}
\end{aligned}
$$

$$\begin{bmatrix} z_{1,k} \\ z_{2,k} \\ \vdots \\ z_{n_u,k} \\ z_{T,k} \end{bmatrix} = \begin{bmatrix} C_1 & & & \\ & C_2 & & \\ & & \ddots & \\ & & & C_{n_u} \\ C_1 & C_2 & \cdots & C_{n_u} \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \\ \vdots \\ x_{n_u,k} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ d_{1,k} + d_{2,k} + \cdots + d_{n_d,k} \end{bmatrix}.$$

The known forecast, $D(s)$, only have effect on the total power output.

We present the MISO system by consider a power system with two power plants, $n_u = 2$, and a wind farm, $n_d = 1$; see Figure 5.4. Above assumptions are still valid. The total power production is given by

$$Z_T(s) = G_1(s)U_1(s) + G_2(s)U_2(s) + D_1(s). \tag{5.12}$$

The first two terms are the output from plant 1, $Z_1(s)$, and plant 2, $Z_2(s)$, while the last term is the output from wind farm, $Z_3(s)$. We represent $Z_1(s)$ and $Z_2(s)$ as the discrete-time state-space model

$$Z_i(s) = G_i(s)U_i(s) \Rightarrow \begin{cases} x_{i,k+1} &= A_i x_{i,k} + B_i u_{i,k} \\ z_{i,k} &= C_i x_{i,k}, \end{cases} \qquad i = 1,2. \tag{5.13}$$

The discrete-time state-space model for $Z_3(s)$:

$$Z_3(s) = D_1(s) \Rightarrow \left\{ z_{3,k} = d_k. \right. \tag{5.14}$$

Rewriting into matrix form yields

$$\begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} u_{1,k} \\ u_{2,k} \end{bmatrix} \tag{5.15a}$$

$$z_k = z_{1,k} + z_{2,k} + z_{3,k} = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} + d_k. \tag{5.15b}$$

## 5.3 Finite impulse response

We introduce a Finite Impulse Response (FIR) model for representation of dynamics for the discrete-time state-space model (5.11) [PJ08; ESJ09]. This is possible for stable processes. Given the current state, we can predict the expected future state evolution by combining (5.11a)–(5.11c). The state, $x_k$, may be represented in terms of the initial state, $x_0$, and the past inputs, $\{u_i\}_{i=0}^{k-1}$,

$$x_k = A^k x_0 + \sum_{i=0}^{k-1} A^{k-1-i} B u_i. \tag{5.16}$$

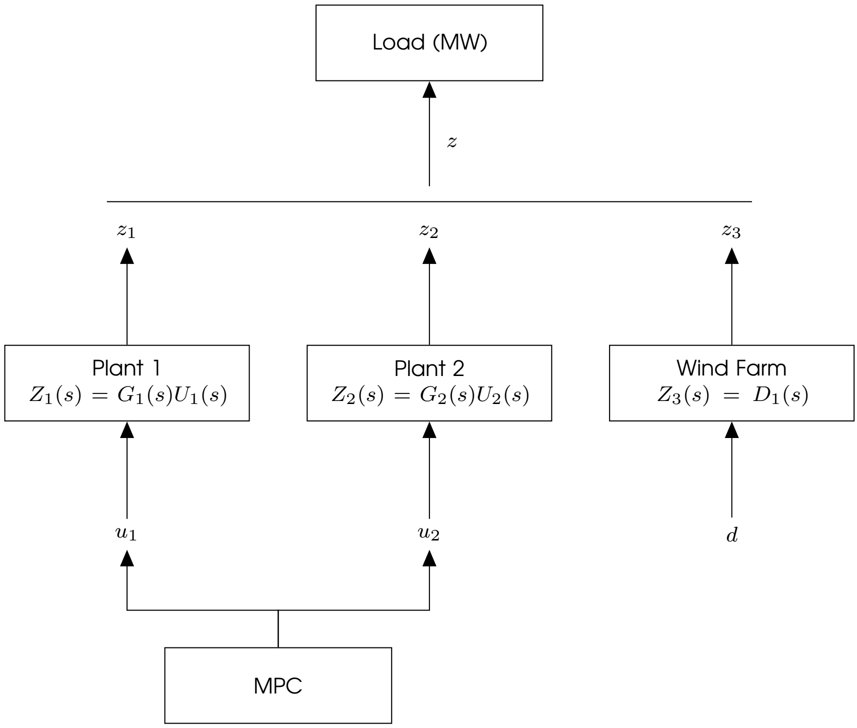



**Figure 5.4:** Power grid with two controllable conventional power plants and one non-controllable predictable power generator, farms of wind turbines.

This expression for the states, the measured output, $y_k$, and the control output $z_k$, may be related to the initial state, $x_0$, and the past inputs, $\{u_i\}_{i=0}^{k-1}$, by

$$\begin{aligned} y_k &= Cx_k + d_k \\ &= C\left(A^k x_0 + \sum_{i=0}^{k-1} A^{k-1-i} B u_i\right) + d_k \\ &= CA^k x_0 + \sum_{i=0}^{k-1} CA^{k-1-i} B u_i + d_k \\ &= CA^k x_0 + \sum_{i=0}^{k-1} H^y_{u,k-i} u_i + d_k, \end{aligned} \tag{5.17}$$

$$\begin{aligned} z_k &= C_z x_k + d_k \\ &= C_z A^k x_0 + \sum_{i=0}^{k-1} H^z_{u,k-i} u_i + d_k, \end{aligned} \tag{5.18}$$

with $k = 1,2,\ldots,N$ and the impulse response coefficient (Markov parameters) defined by

$$H^y_{u,i} = CA^{i-1}B, \qquad i = 1,2,\ldots,N, \tag{5.19a}$$
$$H^z_{u,i} = C_zA^{i-1}B, \qquad i = 1,2,\ldots,N. \tag{5.19b}$$

$N$ is the number of approximate time steps needed to represent the impulse response. The impulse response coefficients may be obtained by observing the output after applying a pulse input vector; i.e., let $u_0 \neq 0$ and $u_k = 0\ \forall k > 0$.

The dynamics in (5.17) and (5.18) can be represented by the linear relation [ESJ09],

$$Y = \Phi_y x_0 + \Gamma_{yu}U + D$$
$$Z = \Phi_z x_0 + \Gamma_{zu}U + D,$$

where we define the vectors,

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}, \quad Z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_N \end{bmatrix}, \quad U = \begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_{N-1} \end{bmatrix}, \quad D = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_N \end{bmatrix},$$

and the matrices

$$\Phi_y = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N-1} \end{bmatrix}, \Phi_z = \begin{bmatrix} C_zA \\ C_zA^2 \\ \vdots \\ C_zA^{N-1} \end{bmatrix}, \Gamma_u = \begin{bmatrix} H^\beta_{u,1} & 0 & \cdots & 0 \\ H^\beta_{u,2} & H^\beta_{u,1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H^\beta_{u,N} & H^\beta_{u,N-1} & \cdots & H^\beta_{u,1} \end{bmatrix},$$

with $\beta \in \{y,z\}$.

## 5.4 Kalman filtering and prediction

We develop the filter and predictor for the stochastic linear time-invariant discrete-time system (5.10) with process noise, $w_k$, and measurements noise, $v_k$, assumed to be identically independently normally distributed (iid) as

$$\begin{bmatrix} w_k \\ v_k \end{bmatrix} \sim N_{iid}\left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} R_{ww} & R_{wv} \\ R_{vw} & R_{vv} \end{bmatrix} \right). \tag{5.20}$$

Let the process noise, $w_k$, and the measurement noise, $v_k$, be correlated, i.e., $R_{wv} = R^T_{vw} \neq 0$. Let the initial state, $x_0 \sim N(\hat{x}_{0|-1}, P_{0|-1})$, be independent of $\{w_k\}$ and $\{v_k\}$.

We assume that the system is stochastic stationary, i.e., $P_{0|-1} = P$. $P$ is computed by the solution of the discrete algebraic Riccati equation (DARE) [JHR11]:

$$\begin{aligned} P &= APA^T + GR_{ww}G^T \\ &\quad - (APC^T + GR_{wv})(R_{vv} + CPC^T)^{-1}(APC^T + GR_{wv})^T. \end{aligned}$$

In case of above assumption is not valid, one can easily obtain the recursive Kalman filter by letting the Kalman filter gain and covariance matrix, $P$, be time dependent. Consequently, be calculated in each time step. Our implementation offers this opportunity too.

According to the theory of linear estimation [KSH00; JHR11; Sta+12], the optimal predictor of a system governed by (5.10) is based on the past measurements, $\{y_j\}_{j=0}^{k}$, and past control inputs, $\{u_j\}_{j=0}^{k-1}$. The filter estimator for the measurement update is slightly different than the standard Kalman filter due to the introduction of $d_k$. Given a new measurement, $y_k$, and the conditional prediction of the disturbance, $\hat{d}_{k|k-1}$, the optimal filtered state estimate, $\hat{x}_{k|k}$, is obtained by first computing the innovation

$$\hat{y}_{k|k-1} = C\hat{x}_{k|k-1} + \hat{d}_{k|k-1} \tag{5.21a}$$
$$e_k = y_k - \hat{y}_{k|k-1} \tag{5.21b}$$

and subsequently compute the innovation covariance and the Kalman filter gain matrices used by the filter and the one-step-ahead predictor by

$$R_{fe} = CPC^T + R_{vv} \tag{5.22a}$$
$$K_{fx} = PC^T R_{fe}^{-1} \tag{5.22b}$$
$$K_{fw} = R_{wv} R_{fe}^{-1}. \tag{5.22c}$$

The current filtered state estimate, $\hat{x}_{k|k}$, and process noise estimate, $\hat{w}_{k|k}$, are obtained by

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_{fx} e_k \tag{5.23a}$$
$$\hat{w}_{k|k} = K_{fw} e_k. \tag{5.23b}$$

The one-step-ahead prediction of the states, $\hat{x}_{k+1|k}$, the $(j+1)$-step-ahead predictions of the states, $\hat{x}_{k+1+j|k}$, as well as the output are obtained by

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + B\hat{u}_{k|k} + G\hat{w}_{k|k} \tag{5.24a}$$
$$\hat{x}_{k+1+j|k} = A\hat{x}_{k+j|k} + B\hat{u}_{k+j|k}, \qquad j = 1,2,\ldots,N-1 \tag{5.24b}$$
$$\hat{z}_{k+j+1|k} = C_z\hat{x}_{k+1+j|k} + \hat{d}_{k+j|k}, \qquad j = 0,1,\ldots,N-1 \tag{5.24c}$$

where $N$ is the predictive horizon and $\forall k \geq 0$. In case of uncorrelated process and measurement noise, $R_{wv} = R_{vw}^T = 0$. Then, $K_{fw} = 0$ implying $\hat{w}_{k|k} = 0$, thus, the term $K\hat{w}_{k|k}$ is zero and drops out.

## 5.5 Disturbance modeling for offset-free MPC

In closed-loop systems, the controller using the linear process model (5.10) described in Section 5.2.2 may never reach the desire control trajectory if unmeasured nonzero disturbances enters the system or if model error is present. To achieve offset free tracking performance for MPC, in the presence of unmeasured nonzero disturbance, the traditional approach is to incorporating disturbance states into the process model. Thus, designs a control system for which can remove asymptotically constant, nonzero disturbances [Huu+10; MB02].

Let $p_k \in \mathbb{R}^{n_p}$ denote the unmeasured disturbance, $n_p$ is the number of augmented output disturbance states. Introduce $p_k$ into (5.10) yields

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + B_p p_k + Gw_k \\ y_k &= Cx_k + C_p p_k + d_k + v_k. \end{aligned}$$

Assume that the disturbance evolves as

$$p_{k+1} = p_k + \xi_k$$

and that the noise in the system is distributed as

$$\begin{bmatrix} w_k \\ \xi_k \\ v_k \end{bmatrix} \sim N_{iid}\left( \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} R_{ww} & 0 & R_{wv} \\ 0 & R_\xi & 0 \\ R_{vw} & 0 & R_{vv} \end{bmatrix} \right).$$

Then, we construct an output disturbance model using the following augmented state-space model

$$\begin{bmatrix} x_{k+1} \\ p_{k+1} \end{bmatrix} = \begin{bmatrix} A & B_p \\ 0 & I \end{bmatrix} \begin{bmatrix} x_k \\ p_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} G & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} w_k \\ \xi_k \end{bmatrix} \tag{5.25a}$$

$$y_k = \begin{bmatrix} C & C_p \end{bmatrix} \begin{bmatrix} x_k \\ p_k \end{bmatrix} + d_k + v_k. \tag{5.25b}$$

Let $B_d = 0$ and $C_d = I$ and we obtain the output disturbance model.

We present the modification for the Kalman filtering and prediction when applying the disturbance model. The output disturbance is estimated using the augmented state-space model (5.25). Now, the one-step-ahead prediction equations are defined as

$$\begin{aligned} \hat{x}_{k+1|k} &= A\hat{x}_{k|k} + B\hat{u}_{k|k} + G\hat{w}_{k|k} \\ \hat{p}_{k+1|k} &= \hat{p}_{k|k}. \end{aligned}$$

The innovation is computed as

$$\hat{y}_{k|k-1} = \begin{bmatrix} C & C_p \end{bmatrix} \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{p}_{k|k-1} \end{bmatrix} + \hat{d}_{k|k-1} \tag{5.26a}$$

$$e_k = y_k - \hat{y}_{k|k-1}. \tag{5.26b}$$

The optimal filtered state estimates and process noise estimates are obtained by

$$\begin{bmatrix} \hat{x}_{k|k} \\ \hat{p}_{k|k} \end{bmatrix} = \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{p}_{k|k-1} \end{bmatrix} + \begin{bmatrix} K_{fx} \\ K_{fp} \end{bmatrix} e_k \tag{5.27a}$$

$$\hat{w}_{k|k} = K_{fw} e_k. \tag{5.27b}$$

By an appropriate design of the Kalman filter gain matrices $K_{fx}$, $K_{fp}$, and $K_{fx}$. Consequently, the stationary Kalman filter are represented by (5.26) and (5.27). The state estimator can estimate the unmeasured nonzero disturbance and render the controller capable of offset free MPC.

## 5.6 Summary

This chapter present key parts in models for predictive control. We formulate the mathematical model applied for modeling the dynamics of a power system (portfolio of power generating plants). We define finite impulse response model present a Kalman filtering and prediction for the linear time-invariant discrete-time state-space model. In addition, we extend the model to achieve offset free MPC in the presence of unmeasured nonzero disturbance.

CHAPTER 6

# Economic Model Predictive Control

This chapter motivates the choice of economic MPC. We formulate a soft constrained linear economic MPC problem and show how it can be formulated as a LP problem. The finite impulse response model and the Kalman filtering and prediction defined in Chapter 5 will be used. Section 6.3 provides the controller algorithm and Section 6.5 present the developed MATLAB implementation. In Section 6.6, we demonstrate and apply the formulated and implemented control framework to a power system.

## 6.1 Introduction

Model Predictive Control (MPC) is a control methodology for optimal operation and control of dynamical systems and processes [Jør05; Mac02; PJ08; QB03; CM87; Hal+14; Hov13]. In the process industries like chemical plants and oil refineries, MPC has been very popular due to its natural and explicit handling of multivariable constrained optimal control problems. Besides the feature of incorporating constraints in the controller, MPC has the capability to integrate predictions and forecasts of the considered system. I.e., MPC can anticipate future events and can take control actions accordingly, which, e.g., PID controller do not support.

MPC computes an optimal action based on a dynamical system, which may be a empirical model obtained by system identification, and its predicted future evolution. The objective may be related to forcing the system to follow a predefined trajectory or related to a cost function, e.g., maximizing profit or minimizing costs. An advantage of MPC is the fact that it is mathematically formulated as a real-time optimization problem that repeatedly computes the control actions. That is, for each time step a finite time horizon optimization problem is solved. The optimization yields an optimal input sequence for the entire horizon, in which only the inputs associated with that time step are implemented in the physical plant [Mac02]. New measurements are obtained from the physical plant and the real-time optimization approach repeats. This principle is referred to as receding horizon control and is depicted in Figure 6.1. By repeating this optimization scheme, we obtain closed-loop feedback. This allow the possibly to react to model uncertainties and external disturbances.

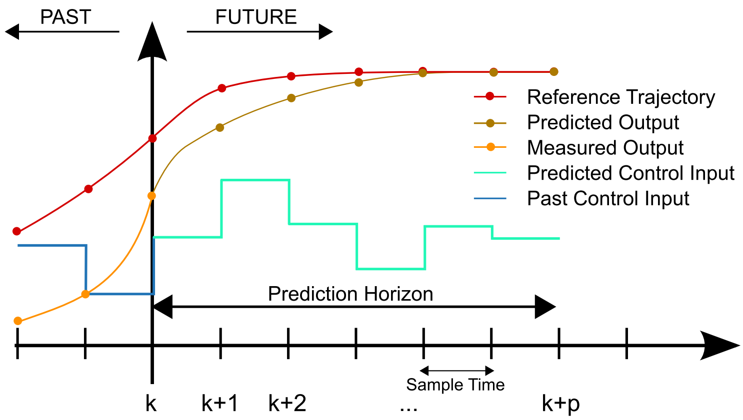



**Figure 6.1:** Control principle of MPC scheme - moving horizon estimation [Wik].

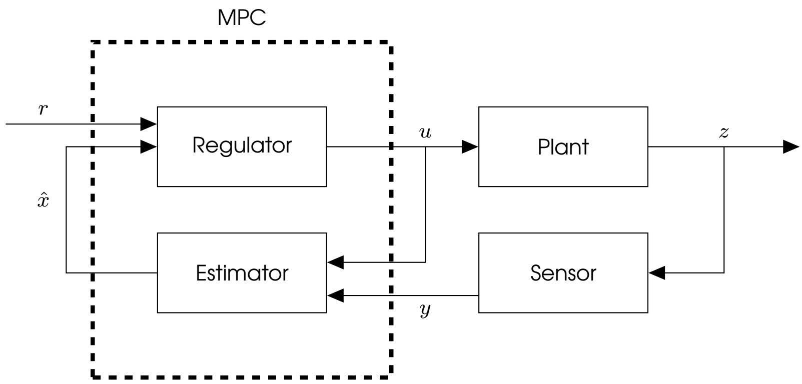



**Figure 6.2:** A MPC system [PJ08].

Figure 6.2 illustrates a general control system. A MPC system consists of an estimator and a regulator. Inputs to the MPC is the target trajectory, $r$, and the estimated state, $\hat{x}$, that may be obtained by a Kalman filter using historical data. Then, MPC return manipulated inputs, $u$, to the physical plant, such that the predicted output, $z$, obey the target as well as possible. The aim in our work is to minimize the operating costs associated with operation of power systems rather than tracking a predefined trajectory as in the traditional MPC framework. Economic MPC provides a MPC framework that minimizes the cost of operations subject to constraints.

We notice that the notation used in UC may be confused with the notation used in economic MPC. It is decided to apply same notation as in the literature in both fields for not to mislead the reader. However, in context, the notation should not be misunderstood.

## 6.2 Mathematical problem formulation

We formulate a soft constrained economic MPC optimization problem. The objective of the economic MPC is to implement a feedback strategy, such that the system remains feasible while the production cost

$$\phi = \sum_{k=0}^{N-1} \left( c_k^T u_k + \rho_{k+1}^T s_{k+1} \right) + \|\Delta u_k\|_{1,\alpha_k} , \tag{6.1}$$

over the control and prediction horizon, $N$, is minimized. The objective cost function (6.1) is linear in $u_k$ and $s_{k+1}$. In the following, an interpretation of (6.1) is given in the setting of considering one power plant.

The term, $c_k^T u_k$, define the cost of producing power, i.e., the cost of fuel, taxes, emission, etc. $c_k$ is the cost vector there may be time-varying and $u_k$ is the computed control inputs. The manipulable inputs are subject to input constraints; the allowable operating range and the rate of movement, respectively,

$$\underline{u}_k \leq u_k \leq \overline{u}_k \tag{6.2a}$$

$$\Delta\underline{u}_k \leq \Delta u_k \leq \Delta\overline{u}_k, \tag{6.2b}$$

where $\underline{u}_k$ and $\overline{u}_k$ is the minimum and maximum value of the manipulable input variable, respectively. $\Delta\underline{u}_k$ and $\Delta\overline{u}_k$ is the maximum rate of movement (ramp-down and ramp-up), respectively, with $\Delta u_k = u_k - u_{k-1}$.

The system output, $z_k$, are limited to be within the interval

$$\underline{z}_k \leq z_k \leq \overline{z}_k,$$

where $\underline{z}_k$ and $\overline{z}_k$ is the minimum and maximum power output. The interval may represent the overall demand load or the production plan for the individual power plant provided by solving the UC problem. For some scenarios, it may be impossible or very expensive to meet the output constraints, which leads to infeasible optimization problems with no solution. Therefore, to ensure feasible production plan $\{u_k\}_{k=0}^{N}$, we soften the hard output constraints by introducing the slack variable $s_k$, $s_k \geq 0$, [ZJM10]. Thus,

$$\underline{z}_k - s_k \leq z_k \leq \overline{z}_k + s_k.$$

The slack variable is minimized and penalized with the weight $\rho_{k+1}$, hence, the term, $\rho_{k+1}^T s_{k+1}$. This penalty can be interpreted as, e.g., the price for selling to others or buying elsewhere to meet the demand load. The penalty is modeled such that one can penalize individually on the power plants as well as the total production. Furthermore, the penalty is selected sufficiently large such that demand load is met whenever possible; i.e., $s$ is only nonzero if the output is not within the interval.

The last term, $\|\Delta u_k\|_{1,\alpha_k}$, discourages disproportionate movement of the manipulable variables by penalizes excessive movement of the input with weight $\alpha_k$. This regularization term is important to obtain a realistic well behaved control action and helps reduce wear and tear of the systems.

## 6.3 The economic MPC formulation

In Section 5.2, we defined a stochastic linear time-invariant discrete-time state-space model (5.10) and in Section 5.4 established the optimal filtering and prediction for this system. We formulate the finite horizon soft constrained linear regulation problem and describe the economic MPC controller algorithm for computing the manipulated variables, $u_k$, for this system [Sta+12; Hal+14]. Algorithm 1 lists the economic MPC controller algorithm.

At any time step $k$, we solve a finite horizon open-loop optimal control problem by looking $N$ prediction steps ahead and using the current state as the initial state. The first control input from the optimal control sequence, obtained by the optimization, is applied to the plant. We define the power production range, $\mathcal{R}_k$, and the forecast of power supply from renewable energy sources, $\mathcal{D}_k$, as

$$\mathcal{R}_k = \left\{ \underline{z}_{k+j+1|k}, \overline{z}_{k+j+1|k} \right\}_{j=0}^{N-1}, \tag{6.3a}$$

$$\mathcal{D}_k = \left\{ \hat{d}_{k+j|k} \right\}_{j=0}^{N-1}. \tag{6.3b}$$

We define the objective cost function

$$\phi = \sum_{j \in \mathcal{N}_0} \left( c_{k+j|k}^T \hat{u}_{k+j|k} + \rho_{k+j+1|k}^T \hat{s}_{k+j+1|k} \right) + \left\| \Delta u_{k+j|k} \right\|_{1,\alpha_{k+j|k}}.$$

The optimal trajectory of the predicted manipulated variables and slack variables, $\left\{ \hat{u}_{k+j|k}, \hat{s}_{k+j+1|k} \right\}_{j=0}^{N-1}$, are obtained by the solution to the soft constrained linear economic MPC optimization problem

$$\begin{aligned}
\underset{\{\hat{u},\hat{s}\}}{\text{minimize}} \quad & \phi(\left\{ \hat{u}_{k+j|k}, \hat{s}_{k+j+1|k} \right\}_{j=0}^{N-1}) && && (6.4a)\\
\text{subject to} \quad & \hat{x}_{k+1|k} = A\hat{x}_{k|k} + B\hat{u}_{k|k} + G\hat{w}_{k|k} && && (6.4b)\\
& \hat{x}_{k+1+j|k} = A\hat{x}_{k+j|k} + B\hat{u}_{k+j|k} && j \in \mathcal{N}_1 && (6.4c)\\
& \hat{y}_{k+j+1|k} = C\hat{x}_{k+1+j|k} + \hat{d}_{k+j|k} && j \in \mathcal{N}_0 && (6.4d)\\
& \hat{z}_{k+j+1|k} = C_z\hat{x}_{k+1+j|k} + \hat{d}_{k+j|k} && j \in \mathcal{N}_0 && (6.4e)\\
& \underline{u}_{k+j+1|k} \leq \hat{u}_{k+j|k} \leq \overline{u}_{k+j+1|k} && j \in \mathcal{N}_0 && (6.4f)\\
& \Delta\underline{u}_{k+j+1|k} \leq \Delta\hat{u}_{k+j|k} \leq \Delta\overline{u}_{k+j+1|k} && j \in \mathcal{N}_0 && (6.4g)\\
& \hat{z}_{k+j+1|k} + \hat{s}_{k+j+1|k} \geq \underline{z}_{k+j+1|k} && j \in \mathcal{N}_0 && (6.4h)\\
& \hat{z}_{k+j+1|k} - \hat{s}_{k+j+1|k} \leq \overline{z}_{k+j+1|k} && j \in \mathcal{N}_0 && (6.4i)\\
& \hat{s}_{k+j+1|k} \geq 0 && j \in \mathcal{N}_0 && (6.4j)
\end{aligned}$$

where $j \in \mathcal{N}_i := \{0+i, 1+i, \ldots, N-1\}$, with $N$ being the predictive horizon. By including the possible non-zero filtered process noise, $\hat{w}_{k|k}$, in (6.4b), the regulator allows for possible cross-couple process and measurement noise in the stochastic linear state-space model describing the system.

The function that solves (6.4) and select $\hat{u}_{k|k}$ is denoted as

$$u_{k|k} = \hat{u}_{k|k} = \mu(\hat{x}_{k|k}, u_{k-1}, \mathcal{R}_k, \mathcal{D}_k). \tag{6.5}$$

Hence, at time step $k$ we compute, based on looking $N$ time step ahead, the optimal sequences $\left\{u_{k|k}\right\}_{k=0}^{N-1}$ in which the predicted output trajectory, $\{z_k\}_{k=0}^{N-1}$, obey the allowable power production range. Then, the first input, $\hat{u}_{k|k}$, from $\left\{u_{k|k}\right\}_{k=0}^{N-1}$ is applied. This recur in a receding horizon manner as new information becomes available [JHR11].

### 6.3.1 Stability

Stability of economic MPC framework is a very important theme, which certainty has its attention in the literature [AAR12; Raw+08; May+00]. In this thesis, however, we do not focus on stability issue. Instead we are aware of choosing sufficient long prediction and control horizon compared to system dynamics, and to apply linear stable systems to obtain stability properties according to [May+00].

**Algorithm 1** Economic MPC with external forecasts

**Require** $y_k$, $u_{k-1}$, $\hat{x}_{k-1|k-1}$, the external forecasts

$$\mathcal{R}_k = \left\{ \underline{z}_{k+j+1|k}, \overline{z}_{k+j+1|k} \right\}_{j=0}^{N-1}$$
$$\mathcal{D}_k = \left\{ \hat{d}_{k+j|k} \right\}_{j=0}^{N-1},$$

and the Kalman filter gain matrices

$$R_{fe} = CPC^T + R_{vv}$$
$$K_{fx} = PC^T R_{fe}^{-1}$$
$$K_{fw} = R_{wv} R_{fe}^{-1}$$

**Compute one-step-ahead predictor**
Compute the one-step-ahead predictor

$$\hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} + Bu_{k-1} + G\hat{w}_{k-1|k-1}$$
$$\hat{y}_{k|k-1} = C\hat{x}_{k|k-1} + \hat{d}_{k|k-1}$$

Compute innovation

$$e_k = y_k - \hat{y}_{k|k-1}$$

Compute the filtered state

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_{fx} e_k$$
$$\hat{w}_{k|k} = K_{fw} e_k$$

**Regulator**
Compute $u_{k|k} = \mu(\hat{x}_{k|k}, u_{k-1}, \mathcal{R}_k, \mathcal{D}_k)$ by solution of LP (6.4)
**Return** $u_k$, $\hat{x}_{k|k}$

## 6.4 Solving the economic MPC problem

In this section, we show that the formulated soft constraint linear economic MPC (6.4) can be expressed as a Linear Programming (LP) optimization problem. Furthermore, we outline available solvers and the optimality conditions for this type of problem.

### 6.4.1 Economic MPC formulated as LP problem

The soft constrained linear economic MPC (6.4) can be formulated as the LP

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \phi = g^T x && (6.6a) \\ \text{subject to} \quad & Ax \geq b, && (6.6b) \end{aligned}$$

for which efficient algorithms exist for solving LP problems. Using the finite impulse response model of the state-space parameterizations [PJ08; ESJ09]; see Section 5.3. The stacked output relation is

$$Z = \Phi x_0 + \Gamma_u U + D.$$

Then, (6.4) may be expressed as

$$\begin{aligned} \underset{U,S,V}{\text{minimize}} \quad & \phi = c^T U + \rho^T S + \alpha^T V && (6.7a) \\ \text{subject to} \quad & Z = \Phi x_0 + \Gamma_u U + D && (6.7b) \\ & \underline{U} \leq U \leq \overline{U} && (6.7c) \\ & \Delta \underline{U} \leq \Delta U \leq \Delta \overline{U} && (6.7d) \\ & Z + S \geq \underline{Z} && (6.7e) \\ & Z - S \leq \overline{Z} && (6.7f) \\ & S \geq 0 && (6.7g) \end{aligned}$$

where, additional to those defined in Section 5.3, we define

$$S = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_N \end{bmatrix}, \quad V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}, \quad \underline{Z} = \begin{bmatrix} \underline{z}_1 \\ \underline{z}_2 \\ \vdots \\ \underline{z}_N \end{bmatrix}, \quad \overline{Z} = \begin{bmatrix} \overline{z}_1 \\ \overline{z}_2 \\ \vdots \\ \overline{z}_N \end{bmatrix}.$$

$\underline{U}$ and $\overline{U}$ are simply $\underline{u}$ and $\overline{u}$ stacked $N$ times, respectively. The rate of movement constraint (6.7d) is as usually $\Delta u_k = u_k - u_{k-1}$. Thus, by introducing the matrices

(for the case $N = 5$)

$$I_0 = \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \Psi = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ -I & I & 0 & 0 & 0 \\ 0 & -I & I & 0 & 0 \\ 0 & 0 & -I & I & 0 \\ 0 & 0 & 0 & -I & I \end{bmatrix}$$

we obtain the following expression

$$\Delta U = \Psi U - I_0 u_{-1}.$$

Using matrix notation, we build (6.7) as the LP optimization problem (6.6) by

$$x = \begin{bmatrix} U \\ S \\ V \end{bmatrix}, \quad g = \begin{bmatrix} c \\ \rho \\ \alpha \end{bmatrix}, \quad A = \begin{bmatrix} I & 0 & 0 \\ -I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ \Psi & 0 & 0 \\ -\Psi & 0 & 0 \\ \Psi & 0 & I \\ -\Psi & 0 & I \\ \Gamma_u & I & 0 \\ -\Gamma_u & I & 0 \end{bmatrix}, \quad b = \begin{bmatrix} \underline{U} \\ -\overline{U} \\ 0 \\ 0 \\ b_l \\ -b_u \\ I_0 u_{-1} \\ -I_0 u_{-1} \\ Z_l \\ -Z_u \end{bmatrix}, \tag{6.8}$$

with the parameters

$$b_l = \Delta \underline{U} + I_0 u_{-1} \qquad Z_l = \underline{Z} - \Phi x_0 - D \tag{6.9a}$$

$$b_u = \Delta \overline{U} + I_0 u_{-1} \qquad Z_u = \overline{Z} - \Phi x_0 - D. \tag{6.9b}$$

Consequently, the solution to (6.4) is obtained by solving (6.6) expressed by (6.8) and (6.9).

The coefficient matrix $A$ is sparse and highly structured. It is composed of $\Psi$ and $\Gamma$ (which itself are structured), and the identity matrix, $I$. The structure is to be utilized for efficiently solving the constrained optimal control problem. Thus, the coefficient matrix $A$ is implemented as a sparse matrix.

### 6.4.2 Solvers

See Chapter 3 for applied solvers in our control framework. Even though it is significant, a study on solver performances is outside the scope of this thesis. There exist many efficient methods for solving linear programs obtained from MPC problems. Also methods utilizing the specific structure, e.g., decomposition methods like Dantzig-Wolfe decomposition returning the master problem into independently subproblems, which may be solved in parallel [ESJ09; Sok+12; Sok+13]. On a small note, Graphics Processing Unit (GPU) on graphics cards provides exceptional qualities in scientific computations and high-performance computing [DA12].

### 6.4.3 Optimality conditions

The optimality conditions for (6.6) will informally be introduced. The Lagrangian of (6.6) is

$$\mathcal{L}(x,\lambda) = g^T x - \lambda^T (Ax - b).$$

Hence, the first order necessary and sufficient optimality conditions may be stated as [NW06]

$$\nabla_x \mathcal{L}(x,\lambda) = g - A^T \lambda = 0 \tag{6.10a}$$

$$s - Ax + b = 0 \tag{6.10b}$$

$$S\Lambda \mathbf{1} = 0 \tag{6.10c}$$

$$(s,\lambda) \geq 0, \tag{6.10d}$$

where $S$ being the matrix with slack variable $s := Ax - b$ in the diagonal, similarly $\Lambda$ being the matrix with Lagrange multiplier in the diagonal, and $\mathbf{1}$ is a vector will all components one. (6.10c) is the complementarity conditions. The conditions (6.10) are known as the Karush–Kuhn–Tucker conditions.

## 6.5 Implementation

We outline the developed MATLAB implementations of the economic MPC in (6.4). The full implementations are provided in Source Code Booklet Chapter 2. A flowchart of the implementation is illustrated in Figure 6.3. Our implementation is developed as adaptive and generic as possible. Thus, it can be utilized on different simulation scenarios, different system setup (MISO, MIMO), different state-space realizations, etc. This is done to obtain better code quality and a better starting point for further research and development in this area.

The main script, `main_closedloop`, contain definition of simulation parameters (e.g., model sampling time steps, number of simulation hours, simulation horizon, and prediction horizon), initialization as well as function calls to developed MPC functions. All parameters can easily be modified to achieve the desired simulation. In the following, we describe the framework and the purpose of the developed functions.

- `setupSystem` set up system model parameters and operational parameters. In addition, the system model (e.g., a MIMO system) is defined by applying continuous-time transfer function.
- `setupModel` realize the model into a discrete-time state-space model.
- `setupScenario` generate the simulation scenario. Firstly, forecast of power supply from renewable energy sources (disturbance) is defined. Secondly, the individual power production as well as the total power production range (trajectory for the economic MPC) is determined. Based on the power system, the production plan for each power plant is derived by solving the UC optimization problem. The UC optimization problem presented mathematically in Section 4.3 by (4.20) is implemented in the function `ucSolver` and solved for global optimality using CPLEX. Lastly, variables for the Kalman filter are initialized.
- `mpcDesign` compute and build following: the impulse response matrices by calling the function `impulseResponse`; the input and input rate constraints; $A$ and $g$ of (6.8); the discrete algebraic Riccati equation; and the Kalman filter gains.

These four functions set up the power system and simulation parameters; the offline MPC. Following represent the online MPC.

- `mpcClosedloop` perform the closed-loop simulation. The function do following: simulate the system (output measurements from sensor and state simulation); execute the MPC controller algorithm; and store applicable values from the closed-loop simulation, which are returned. To execute the MPC controller algorithm, the function call two functions `mpcUpdate` and `mpcCompute`:
  - `mpcUpdate` update the current open-loop MPC matrices. Following are updated: current power production range (trajectory) for each power plant; current forecasted power production from renewable energy sources (disturbance); input and input rate constraints; and estimates by performing one-step predictor and Kalman filtering by calling the function `updateStateEstimate`.
  - `mpcCompute`, is the regulator, i.e., solve the open-loop MPC. It build the non-constant vectors and solve the soft constraint economic MPC problem as an linear programming problem by calling the `lpSolver`. The obtained solution is return and feed into the system for each simulation step. `lpSolver` is a interface for solving linear optimization problems. Three solvers are supported: CPLEX, MOSEK, and Gurobi; see Chapter 3, there is to be selected in the main script, `main_closedloop`.

The results, the historical values returned by `mpcClosedloop`, is then illustrated using various developed MPC plot functions.

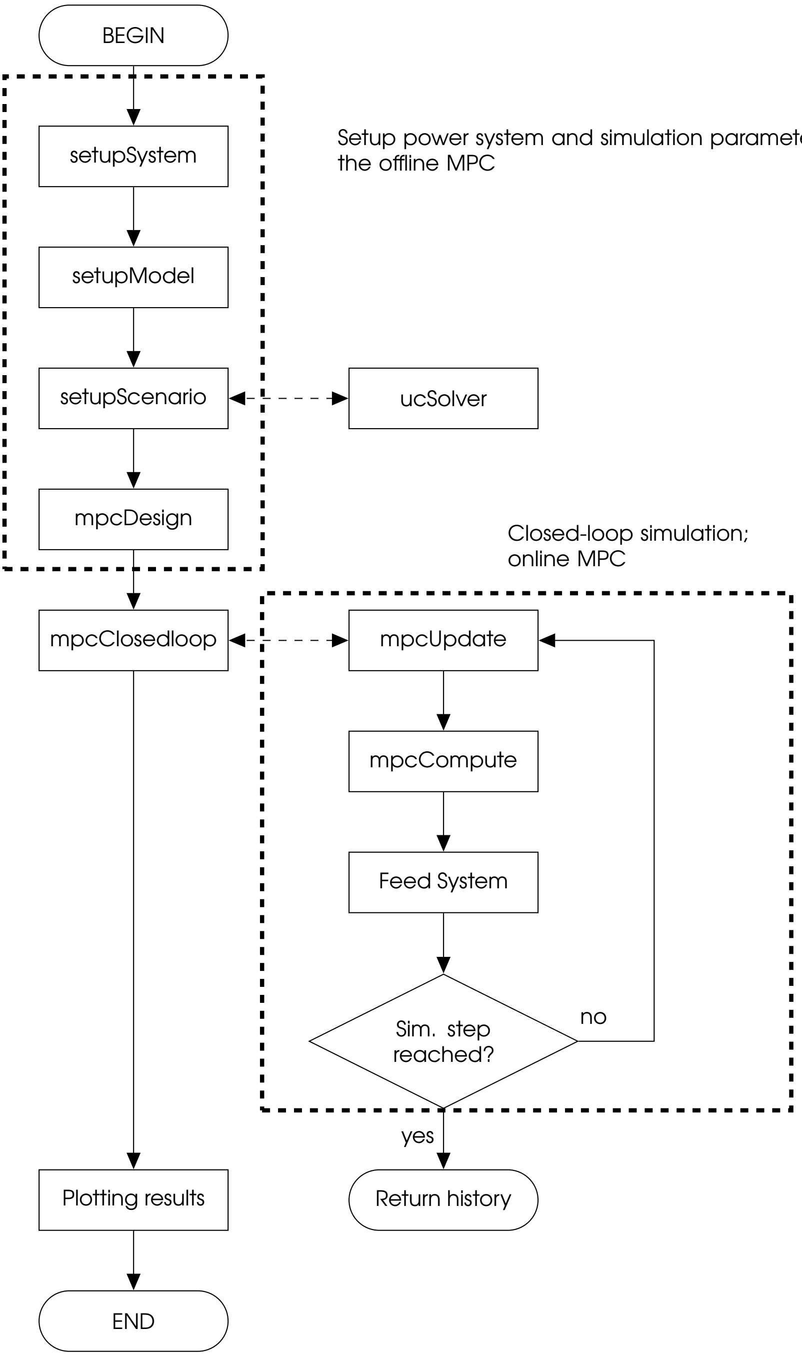



**Figure 6.3:** Flowchart of developed MATLAB implementation.

## 6.6 Case study

In this section, we establish a power system to demonstrate the formulated soft constrained economic MPC (6.4) with dynamics of a power plant modeled as presented in Section 5.2. Firstly, we present the case study and operational parameter of power plants. Secondly, open-loop and closed-loop simulations are performed. Source code is listed in Source Code Booklet Section 2.2 and Section 2.3. [HEJ10] inspired to the case study.

### 6.6.1 2-unit power system

Two power plants with different operational features are controlled. Table 6.4 lists the controller parameters. The parameters are chosen such that power plant 1 is cheap and slow, whereas power plant 2 is expensive and fast. We consider the MISO formulation of the power plants presented in Section 5.2. For convenience the model is given below

$$Z_i(s) = \frac{1}{(\tau_i s + 1)^3} U_i(s), \qquad i = 1{,}2. \tag{6.11}$$

The total production is

$$Z(s) = Z_1(s) + Z_2(s). \tag{6.12}$$

We perform simulations with and without the regularization term to show its impact to the solution. We apply the economic optimizing MPC (6.4), where the system is realized in a discrete-time state-space form with a sampling time of $T_s = 1$ seconds; thus, system dynamics are captured. The objective is to minimize operation cost subject to obey demand load and various operational requirements.

#### 6.6.1.1 Open-loop and closed-loop simulations

The results of an open-loop simulation without regularization term is illustrated in Figure 6.5. Figure 6.5(a) shows that the total power production satisfy the predefined demand load. The cheapest power plant, plant 1, produces the majority of the load, whereas the more expensive and fast power plant, plant 2, operates whenever faster dynamics are required. This behavior is expected considering the operational parameter of the power system. Figure 6.5(b) and Figure 6.5(c) show that the input constraints are satisfied and active at some time periods. We see excessive movement of input for particular plant 2, which may not be desirable due to wear and tear of

**Table 6.4:** Operational parameters.

| Unit | $\tau$ | $c$ | $\rho$ | $\alpha$ | $\underline{u}_k$ | $\overline{u}_k$ | $\Delta\underline{u}_k$ | $\Delta\overline{u}_k$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 20 | 1 | $1.0 \cdot 10^2$ | 0.5 | 0 | 10 | -1 | 1 |
| 2 | 10 | 2 | $1.0 \cdot 10^2$ | 1.0 | 0 | 10 | -3 | 3 |

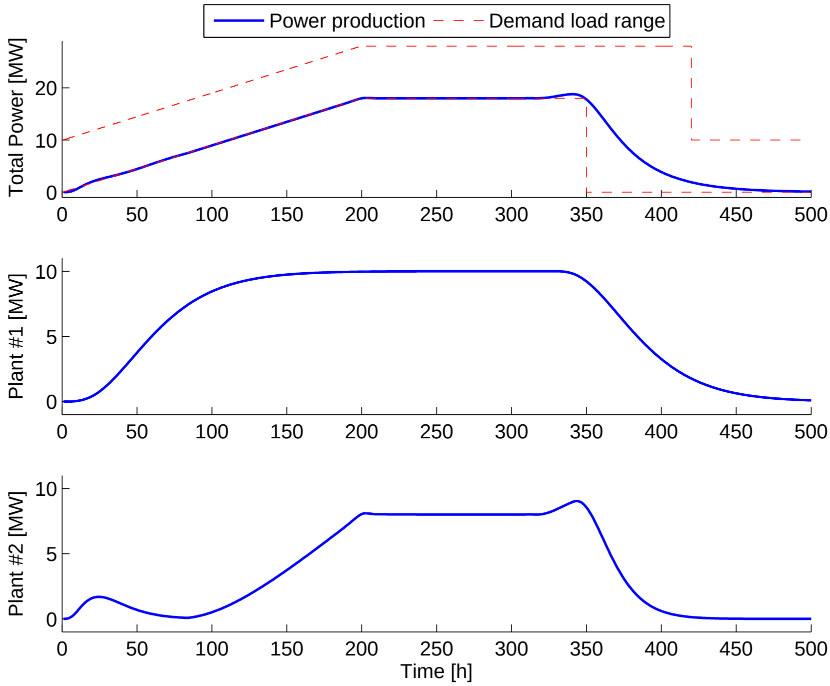
the systems. In order to manage these movements, the regularization term may be added to the objective function.

Figure 6.6 shows the results of an open-loop simulation with the same setup as before, but with the regularization term for excessive movement of the input added to the objective function. There are no significantly changes in the production plan; however, Figure 6.6(c) and Figure 6.6(c) show significant changes in inputs and rate of movement for plant 2. As desirable, we obtain nearly same solution but with less change in inputs. Henceforth, the regularization term is included.

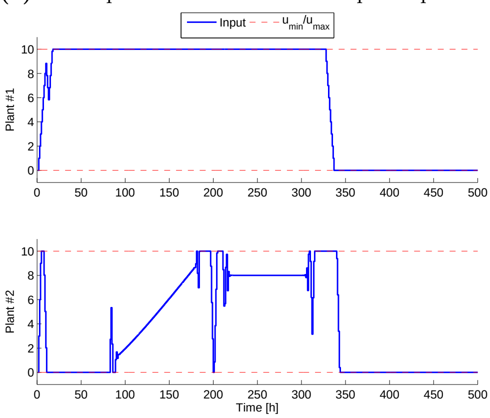
At the same setup as before, we execute a closed-loop simulation with prediction horizon $N = 50$ time step. The results of the simulation, reported in Figure 6.7, shows nearly same solution as obtained in the open-loop simulation Figure 6.6. However, Figure 6.7(b) and Figure 6.7(c) show that the inputs are not as smooth as in the open-loop. Increasing the prediction horizon will smooth this and in fact lead the solution closer to the open-loop simulation. This is expected as we do not utilize the quality of closed-loop simulation, since no valuable feedback information is obtained.

## 6.7 Summary

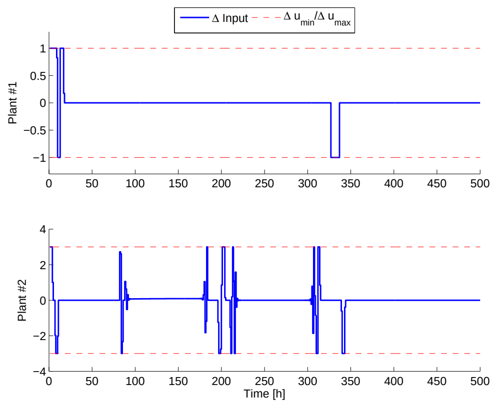
This chapter motivates the choice of economic MPC as control framework and gives an introduction to MPC. We formulate a soft constraint linear economic MPC. We outline the controller algorithm for an economic MPC with external forecasts and filtered state estimators computed by a Kalman filter. It is showed how to convert and solve the optimization problem as a LP optimization problem. We informally present literature on stability of economic MPC problems and solvers as well as stating the optimality conditions for a LP problem. The developed MATLAB implementation for our economic MPC control framework is presented. Lastly, we demonstrate the formulation to a power system. The case studies demonstrated open-loop and closed-loop simulations and that the implementation behaves as expected.




**(a)** Power productions from the two power plants and total production satisfying demand load range.

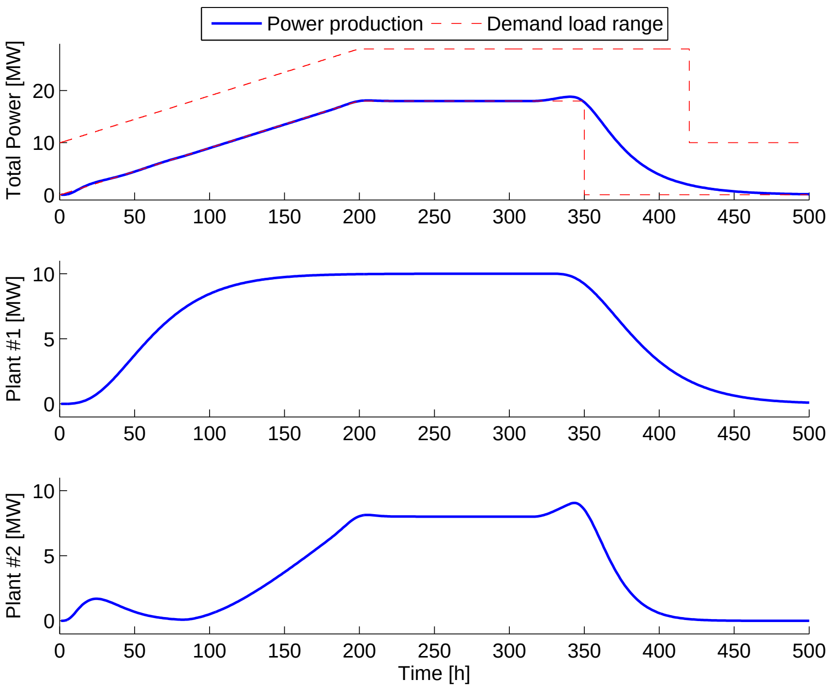



**(b)** System inputs with its limits.

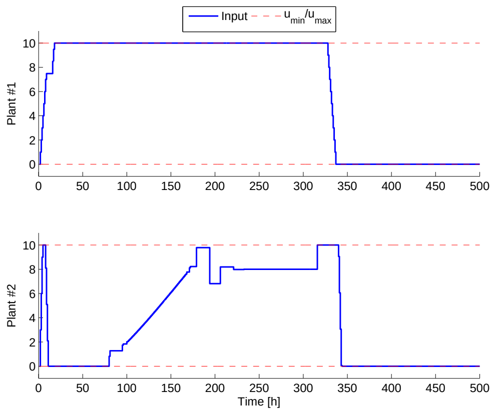



**(c)** Rate of movement for inputs with its limits.

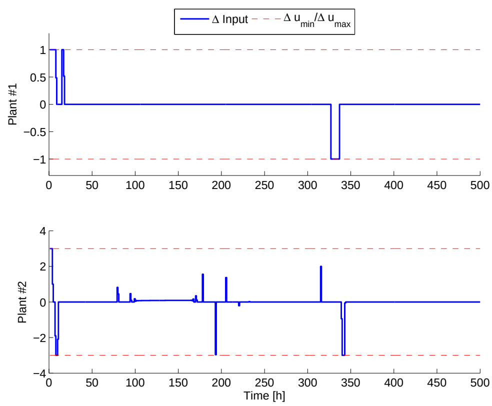
**Figure 6.5:** Open-loop simulation of a power system without regularization term for excessive movement of the input. $T_s = 1$.




**(a)** Power productions from the two power plants and total production satisfying demand load range.

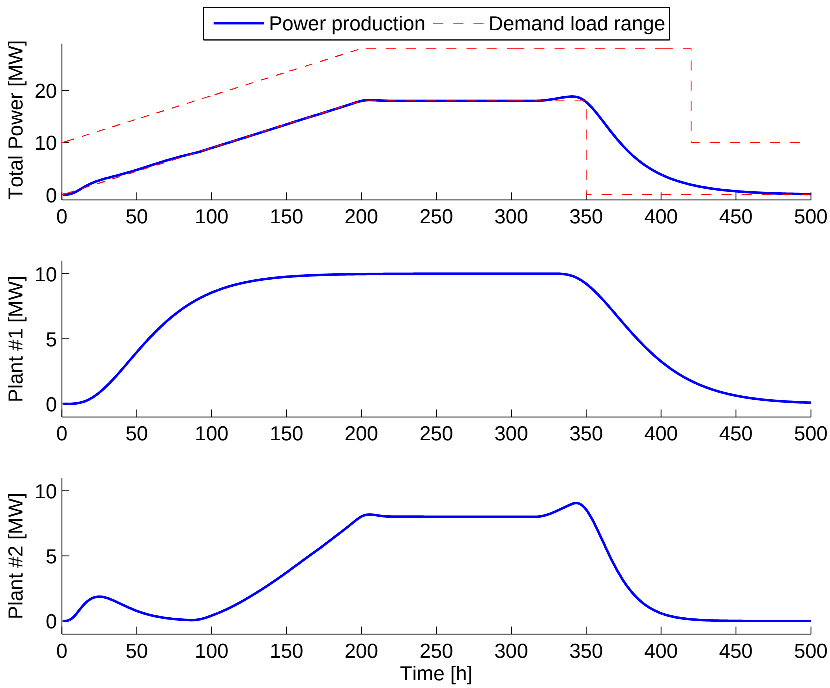



**(b)** System inputs with its limits.

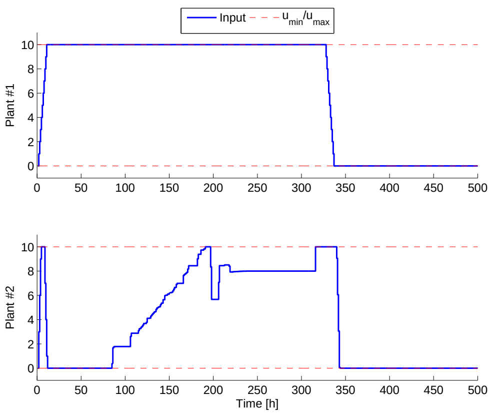



**(c)** Rate of movement with its limits.

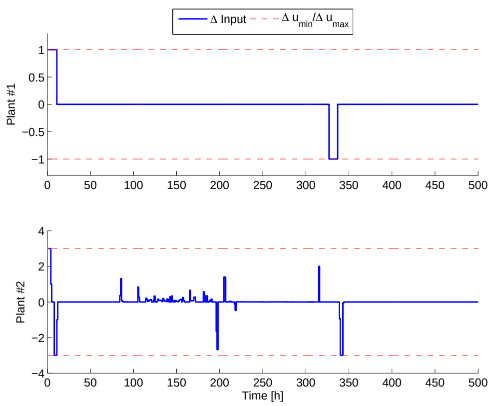
**Figure 6.6:** Open-loop simulation of a power system with regularization term for excessive movement of the input. $T_s = 1$.




**(a)** Power productions from the two power plants and total production satisfying demand load range.




**(b)** Inputs to the system with its limits.




**(c)** Rate of movement with its limits.

**Figure 6.7:** Closed-loop economic MPC simulation of a power system. Prediction horizon is $N = 50$ time step with regularization term. $T_s = 1$.

Part III

# Unit Commitment and Economic Model Predicetive Control for Power Systems

CHAPTER 7

# Introduction

This part present simulation of combining the UC optimization problem and the economic MPC problem in the application of optimal operation of power systems.

In this chapter, we provide an overview of the chapters that follows. In Section 7.1 and Section 7.2, we describe the developed control strategy and informally discuss reflection of combining the two methods. Section 7.3 provides the background material for the simulations that follows.

An outline of the individual chapters is as following:

**Chapter 8** provides a simulation of the impact discretization and input parameterization have on the achievable outcome in terms of power imbalance and costs.

**Chapter 9** provides simulations of combining the UC problem and the economic MPC problem without power supply from renewable energy sources in the power system.

**Chapter 10** provides simulations of combining the UC problem and the economic MPC problem with power supply from renewable energy sources in the power system.

Chapter 9 and Chapter 10 starts with an overview of the perform simulations and ends with a summary of key findings from the simulations.

## 7.1 Developed control strategy

The aim for the novel coupling of the UC problem and economic MPC problem is to develop a control strategy that intelligently can overcome some of the challenges for managing the growing uncertainty associated with increasing penetration of non-controllable renewable energy sources into the power grid. Chapter 1 and Chapter 2 motivates and discuss some of the challenges with renewable energy sources.

In the following, we present key features of the UC problem and economic MPC problem underpinning for the developed control strategy outlined subsequent.

**UC problem.** The UC problem is an intuitive and effective model for optimal power production planning. In order to intercept the variations in power supply from the renewable energy sources and thereby plan the power production accordingly, we want to solve the UC problem with a high frequency. However, for power systems with practical size (large-scale power systems), the high computational complexity makes it impossible to solve the UC problem with a high frequency.

**Economic MPC.** The economic MPC advantage is the real-time optimization, which can be solved with a high frequency. In addition, economic MPC can anticipate future events and take action accordingly. Consequently, the variations in power supply from renewable energy sources can be taking into account by reoptimizing the production plan.

**Combining UC and economic MPC.** With that in mind, the developed control strategy considers the two methods at two distinct levels, as depicted in Figure 7.1. The hierarchy structure is following:

- *Day-ahead planning:* Firstly, day-ahead planning is performed on a coarse time grid. Based on available forecasts of tomorrow demand load and power supply from renewable energy sources, the UC problem derive the production plan for the next 24 hours. The UC solution determine which power plants there are committed and decommitted (binary decisions) and the production level at the committed plants in an economic effective way (economic dispatch).
- *Minutes-ahead planning:* Secondly, online minutes-head planning is performed during the day of operations on a high resolution time grid. Based on updated and more reliable forecasts of power supply from renewable energy sources, the economic MPC reoptimize the production plan in an economic effective way. Thus, we obtain a balance controller there adjust to fluctuations and avoid the undesirable imbalance.

This strategy utilizes the benefits from each of the two methods. We have chosen to solve the UC problem once every 24 hour. The frequency of solving the UC problem, e.g., periodically every six hours, is a very interesting discussion, however, this will only be briefly touched upon in this thesis.

## 7.2 Considerations for combining UC and economic MPC

The UC problem and the economic MPC are not completely the same problem. Therefore, some considerations and decisions need to be made. In the following, we list some of the differences.

- System dynamics are considered and modeled in the economic MPC, not in the UC problem.

- The discretization is different for the two methods. In the literature, the UC problem is modeled and solved with a very coarse discretization, usually hour by hour. Whereas the economic MPC is modeled and solved with a high resolution discretization, sampling time $T_s$. Usually, the sampling time is chosen small in order to capture system dynamics.

- The input parameterization is different for the two methods. This opens a question of how we apply the plants operational parameters in the UC and economic MPC. In our simulations, we want the operational features to be identical for both methods. Therefore, we perform a direct conversion with the discretization in account.

- The decision of which power plants there are committed and decommitted for each time step is solely determined by the UC problem. The described economic MPC in Chapter 6 does not include binary decisions. Consequently, the controller only reoptimizes the production plan and do not decide whether a plant shall startup or shutdown in a time period.

The considerations and challenges evokes interesting discussion and questions, for which will be addressed in the following simulations.

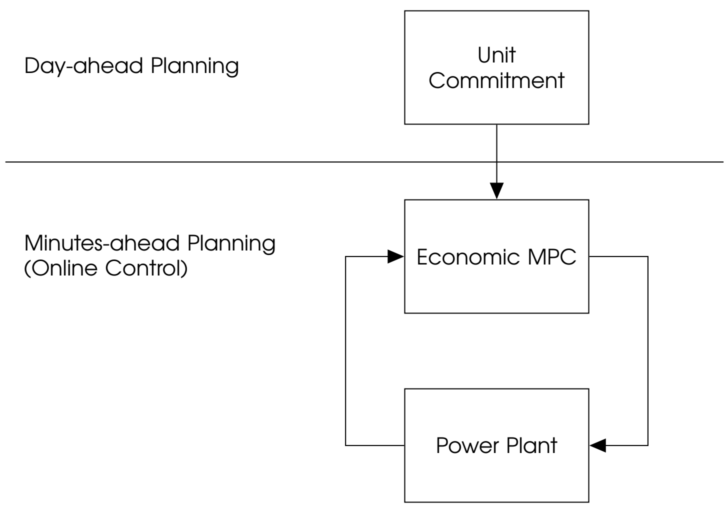



**Figure 7.1:** Control strategy of combining the UC and the economic MPC.

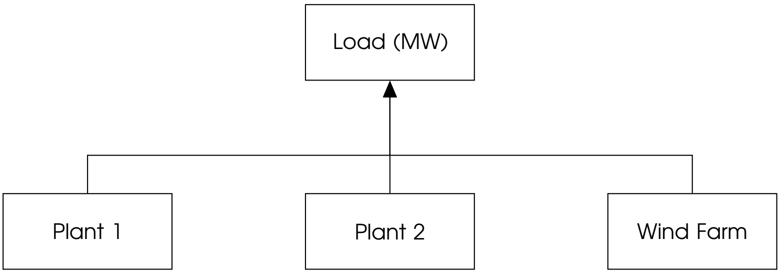



**Figure 7.2:** Power system with two controllable conventional power plants and a non-controllable predictable power generator, farms of wind turbines.

## 7.3 Background for the simulations

In this section, we present the considered power system, the considered demand loads, the operational parameters used in the UC problem and economic MPC, as well as the initialization.

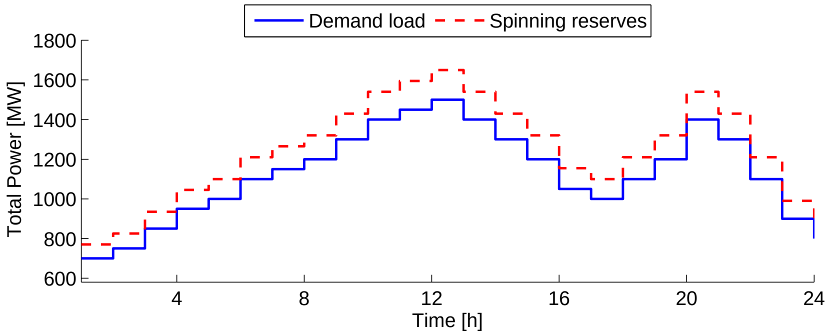
The power system is inspired from the UC and economic MPC case studies in Section 4.6 and Section 6.6, respectively. We consider the power grid illustrated in Figure 7.2; in details see Figure 5.4. The power grid includes two controllable conventional power plants and a non-controllable predictable power generator, farms of wind turbines. The wind farm represents the penetration of intermittent renewable energy sources into the power grid for which we cannot control.

As a remark, we will not consider capacity and distribution challenges in the power grid. Thus, we assume sufficient capacity and distribution possibility in terms of power plants and renewable energy sources.

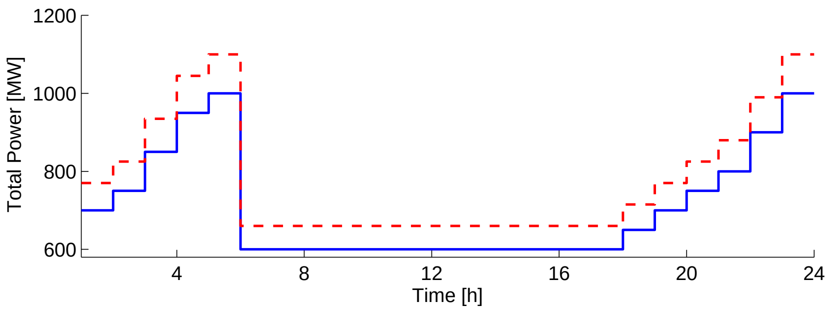
### 7.3.1 Demand load

We will consider two distinct demand loads in Chapter 9 and Chapter 10. Figure 7.3(a) represents a demand load requiring both power plants to be committed over the 24-hour planning horizon; we refer to *busy demand load*. Figure 7.3(b) represent a demand load where one power plant is decommitted in a time period; we refer to *idle demand load*. The idle demand load show how our implementation manage the situation when plants shutdown and startup within the 24-hour planning horizon. In Appendix B Table B.1(a) present both demand loads numerically. The spinning reserve is set as a 10% of the demand load for each time period.

Demand load is naturally continuous in time. As Figure 7.3 indicates, the demand load is discretized hour by hour. In Chapter 8, we discover that the discretization have an impact on the solution. We want to investigate various setups impact on performs. Therefore, in order to not account for discretization loss in all the simulations, the economic MPC receive the coarse demand load as the UC problem. In a practical setting, one should not apply a coarse discretized demand load to the controller,




**(a)** Busy demand load.




**(b)** Idle demand load.

**Figure 7.3:** 24-hour demand load [MW]; see Table B.1(a) in Appendix B for the numerical representation. Spinning reserve is 10% of demand load for each time period.

since it will imply unnecessary information loss. Furthermore, it is assumed that the demand load do not develop over time, i.e., the demand load is the same for the day-ahead planning as well as the day of operation. These choices eventually only have a negative effect on the economic MPC performances. Thus, if removed, the controller will offhand obtain a better solution, and hence a stronger argument for this strategy to be compelling.

If not otherwise stated, the controller is set to at least satisfy the demand load given to the UC optimization problem; let $\underline{z}_k$ denote the demand load at time step $k$, then

$$\underline{z}_k \leq z_{T,k} \leq \infty,$$

where $z_k$ is the total power output at time step $k$. $\infty$ denote an upper limit which is not reached. The reason for this decision is to achieve a fair basis of comparison when comparing the results obtained by the two methods, UC optimization problem and the economic MPC.

**Table 7.4:** Operational parameters to the UC problem.

| Unit | $a_i$ [$/h] | $b_i$ [$/MWh] | $SU_i$ [$/h] | $SD_i$ [$/h] | $PL_i$ [MW] | $PU_i$ [MW] | $RD_i$ [MW/h] | $RU_i$ [MW/h] |
|---|---|---|---|---|---|---|---|---|
| 1 | 1000 | 15 | 10 | 10 | 150 | 850 | 200 | 200 |
| 2 | 500 | 30 | 8 | 8 | 150 | 850 | 600 | 600 |

### 7.3.2 Operational parameters

The two conventional power plants are modeled to have different operational features. The parameters are chosen such that power plant 1 is cheap and slow, whereas power plant 2 is expensive and fast. We want the used operational features to be identical for both methods. We have chosen to perform a direct conversion of the units. E.g., the UC variable cost is given in [$/MWh], then, the economic MPC variable cost is determined by converting the data into [$/(MW·$T_s$)].

#### 7.3.2.1 UC parameters

The UC optimization problem presented mathematically in Section 4.3 by (4.20) is solved for global optimality in a open-loop manner. We use IBM ILOG CPLEX Optimizer V12.6 MATLAB interface for solving the UC optimization problem. Table 7.4 lists the operational parameters for the UC problem. The spinning reserve is set as a 10% of the demand load for each time period.

#### 7.3.2.2 Economic MPC parameters

The economic optimizing MPC presented mathematically in Section 6.3 by (6.4) is solved for global optimality in a closed-loop manner. We use Gurobi Optimizer V5.6 MATLAB interface for solving the LP optimization problem. The other implemented solvers have been tested to find the same solution. The controller considers the linear system presented in Section 5.2. Thus, the power plants dynamics are modeled by a third order model and the dynamics for the wind farms are neglected due to the quick dynamics of wind turbines. The system is realized in a discrete-time state-space form with a sampling time of $T_s = 20$ seconds. The sampling time is chosen such that system dynamics are captured. In closed-loop simulations are the prediction horizon $N = 100$ time step. Table 7.5 lists the operational parameters for the economic MPC.

We find it more important to satisfy the overall demand load than satisfying the individual plants production plan given by the UC, thus, we penalize more heavily on this parameter:

$$\rho_{i,k} = [\rho_{1,k}, \rho_{2,k}, \rho_{T,k}] = [10, 10, 100], \tag{7.1}$$

where $i = 1, 2, \ldots, n_u$ and $\rho_{T,k}$ is the penalty associated to the overall demand load. The listed penalties, $\rho$ and $\alpha$, are subject to change during the simulations; in that case, it will be explicitly given.

**Table 7.5:** Operational parameters to the economic MPC. Penalty $\rho_{i,k} = [\rho_{1,k},\rho_{2,k},\rho_{T,k}] = [10{,}10{,}100]$, where $i = 1,2,\dots,n_u$ and $\rho_{T,k}$ is the penalty associated to the overall demand load.

| Unit | $\tau$ | $c$ [\$/(MW·T$_s$)] | $\alpha$ [\$/T$_s$] | $\underline{u}_k$ [MW] | $\overline{u}_k$ [MW] | $\Delta\underline{u}_k$ [MW/T$_s$] | $\Delta\overline{u}_k$ [MW/T$_s$] |
|---|---|---|---|---|---|---|---|
| 1 | 20 | 0.0833 | 0.0417 | 150 | 850 | -1.1111 | 1.1111 |
| 2 | 10 | 0.1667 | 0.0833 | 150 | 850 | -3.3333 | 3.3333 |

#### 7.3.2.3 Initialization

The performed simulations are initialized in following manner. The day-ahead planning is determined by solving the UC problem. We assume no prior knowledge; consequently, the problem is initialized such that power output at time period $t = 0$ is zero,

$$p_{i,0} = 0 \qquad \forall i,$$

and the plants status preceding the first period is switch off,

$$u_{i,0} = 0 \qquad \forall i.$$

The controller is initialized by applying the obtained solution from the UC problem at time period $t = 0$. Thus, the UC problem determine which plants there are committed and decommitted.

CHAPTER 8

# Discretization and Parameterization

In the literature, the UC optimization problem is modeled and solved with a very coarse time discretization and without system dynamics. This evokes some questions as, e.g., has the discretization and parameterization an impact on the achievable outcome in terms of power imbalance and costs. In the following, we set up a test case to show the interesting aspects.

We consider a 2-unit power system with the operational parameters given in Section 7.3.2. Since this is a very computationally expensive task and due to limited memory, we perform a 12-hour simulations.

## 8.1 Discretization

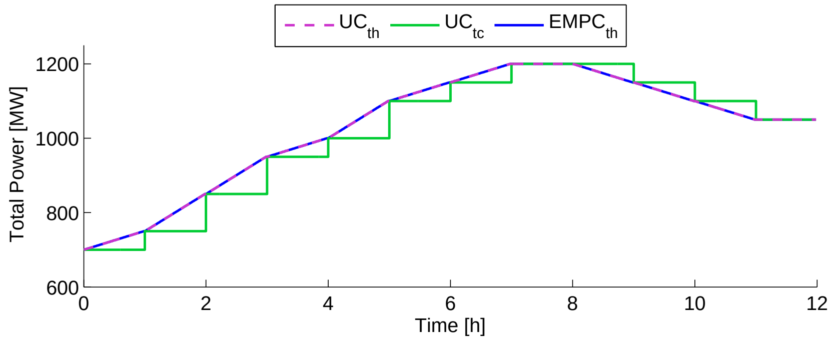
We define following two demand loads:

- $D_{th}$ with a high resolution grid, one value for each minutes.
- $D_{tc}$ with a coarse grid, one value for each hour.

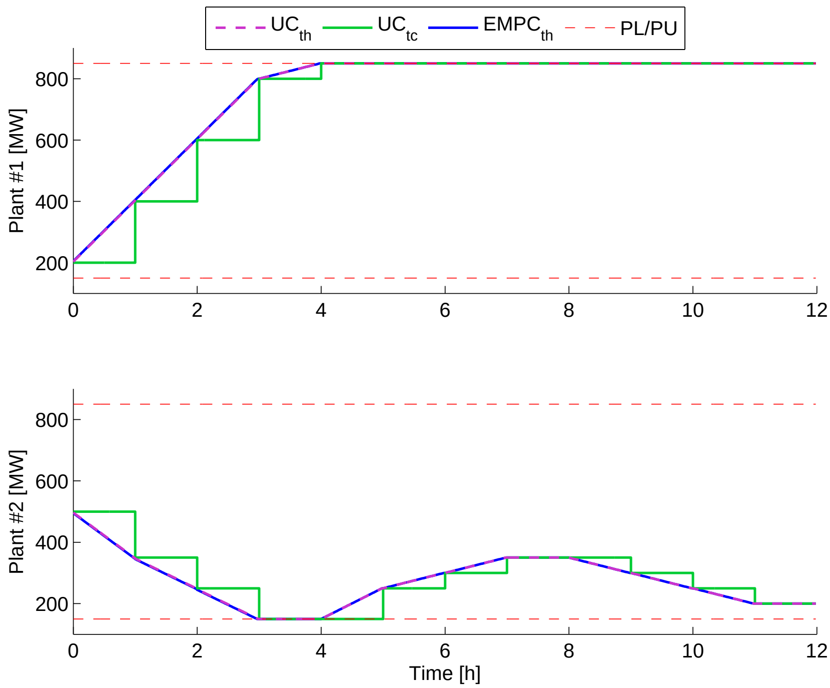
Let $D_{th}$ be the true demand load and $D_{tc}$ be the discretized version of $D_{th}$. Assume the production to be piecewise constant (ZOH approximation). We investigate the impact of discretization and input parameterization in the UC optimization problem by following approach:

1. Solve UC using $D_{th}$. Let $UC_{th}$ denote the solution. Simulate the solution $UC_{th}$ and derive the associated total amount of production.
2. Solve UC using $D_{tc}$. Let $UC_{tc}$ denote the solution. Simulate the solution $UC_{tc}$ on the high resolution grid and derive the associated total amount of production.
3. Solve economic MPC problem with rolling horizon on the high resolution grid using $D_{th}$ as trajectory. Let $EMPC_{th}$ denote the solution. Derive the associated total amount of production.

Figure 8.1 illustrates $UC_{tc}$, $UC_{th}$, and $EMPC_{th}$. We present some initial key findings




**(a)** Total power production.




**(b)** Plants power production.

**Figure 8.1:** Power production obtained from $\mathrm{UC_{th}}$, $\mathrm{UC_{tc}}$, and $\mathrm{EMPC_{th}}$.

- The two UC solutions, $\mathrm{UC_{th}}$ and $\mathrm{UC_{tc}}$, are exactly the demand loads, $\mathrm{D_{th}}$ and $\mathrm{D_{tc}}$, respectively.

- In Figure 8.1(b), we see that the cheapest power plant, plant 1, produces the majority of the load, whereas the more expensive and fast power plant, plant 2, operates whenever faster dynamics are required. This behavior is expected considering operational parameter of the power system.

- $UC_{th}$ and $EMPC_{th}$ obtain nearly identical solution, the two solutions coincide.
- The $UC_{tc}$ solution yields shortages of power production from hour 0-7 and surplus of power production from hour 8-11, thus, $UC_{tc}$ imply imbalance in the power grid.

As we know, power imbalance is undesirable. We derive the absolute imbalance with the true demand load, $D_{th}$, as reference. We choose to consider the amount of power produced by the three methods instead of production cost to avoid introducing uncertainties and assumptions regarding properly choice of cost prices, imbalance cost, profit margin, etc. However, the results are presented in a way such that reader easily can calculate the cost with own figures.

Table 8.2 lists the simulation results numeric. $UC_{tc}$ solution yields to an absolute imbalance of 325 MWh while being 175 MWh short of the true demand load. 2.63% of the power production is imbalance. $EMPC_{th}$ coincide with the optimal production plan, only creating 0.0187 MWh imbalance. These observations indicate that there indeed is a discretization loss in setting up the UC problem on a coarse grid. The discretization loss has a significantly impact on the imbalance, which may imply additional cost or unstable power systems. On the other hand, the result indicates that applying economic MPC may offset this discretization loss and thereby reduce cost or help stabilizing the power system. As a result of less imbalance, the need of spinning reserve will be reduced. This is desirable, as the spinning reserve is costly and implies unutilized production capability.

Table 8.2 also provides the runtime for solving the three methods. In order to present useful data, the runtime is determined by averaging 10 runs. $EMPC_{th}$ runtime is the execution time for one open-loop simulation. We see that the runtime for $UC_{th}$ is significantly higher than the other two methods. $UC_{tc}$ give a 22x speedup compared to $UC_{th}$; however, this also lead to a discretization loss and power imbalance. In contrast, $EMPC_{th}$ give a 65x speedup compared to $UC_{th}$ while obtaining the same solution. Consequently, economic MPC can indeed be solved with a higher frequency while obtaining a solution as good as solving the UC problem on a high resolution time grid.

**Table 8.2:** Results of 12-hour closed-loop simulation. Total power production [MWh] by the tree methods. Imbalance [MWh] is the absolute imbalance between $UC_{th}$ and the obtained production plan.

| Methods | Total Power | Imb. | %-deviation | Runtime [s] | Speedup |
|---|---|---|---|---|---|
| $UC_{th}$ | 12,375 | - | - | 2.60 | - |
| $UC_{tc}$ | 12,200 | 325 | 2.63% | 0.12 | 22x |
| $EMPC_{th}$ | 12,375 | 0.0187 | 0.00% | 0.04 | 65x |

## 8.2 Parameterization

Now, let us dwell on the operational parameters and the input parameterization of UC problem. Consider Figure 8.1(b). We see that the system increase power production at plant 1 as much as possible as fast as possible and the reverse at plant 2. Consequently, plant 1 ramp-up constraint are active until maximum power output, $PU_1 = 850$ MW, is attained. $UC_{tc}$ power production is constant between hours and when the power level change is it accomplished within 0 second. E.g., plant 1 increase the power level with $RU_1 = 200$ MW within 0 seconds at hour 1, 2, 3, and 4. In between, the production is constant. This evolution occurs because of the representation of the UC problem; however, how should it be interpreted? Consider following two cases:

1. the power level can increase reality fast (0-10 minutes); or
2. the power level cannot increase that fast and use nearly an hour to attain maximum ramp-up.

Case 1 indicates nonoptimal utilization of the power plants capabilities. If the ramp-up is fast, the power level may as well increase again within 1 hour instead of waiting until next hour. Case 2 indicates that the used parameterization in the UC problem is not appropriate compared to the physical plant. If the ramp-up is not instantly but rather as a slope, then, the parameterization should be so.

## 8.3 Key findings

The findings in this section show that the discretization and input parameterization have a cost impact on the solution. $EMPC_{th}$ solution coincide with the optimal production plan while $UC_{tc}$ solution yields 2.63% imbalance power of the total power production. Furthermore, $EMPC_{th}$ give a 65x speedup compared to $UC_{th}$ while obtaining the same solution. A deeper analysis and research may find hidden cost savings and unutilized resources. Maybe traditions have surpassed research.

An interesting perspective: the total production cost by $UC_{tc}$ solution is 2.63% subject to additional cost by imbalance cost. In Section 4.6.2, we considered a 10-unit power system. In this case study, the startup and shutdown cost where less than 1% of the total production cost. Consequently, if it is important to include startup and shutdown cost in the objective function when solving the UC optimization problem, then, it must likewise be interesting to look closely on discretization loss.

CHAPTER 9

# Deterministic Simulations

In this chapter, we present simulations of combining the UC problem and the economic MPC problem without power supply from renewable energy sources in the power system. We perform following simulations:

- Apply MISO formulation in the controller
- Apply MIMO formulation in the controller
  - Use system power output limits as trajectory
  - Use the UC solution as trajectory

Section 5.2 provides the MISO and MIMO formulation. In all cases, 24-hour closed-loop simulations are performed considering the two demand load provided in Section 7.3.1. The obtained production plan from solving the UC problem is included in the illustrations. Lastly, we present key findings from the performed simulations.

## 9.1 MISO simulations

In this section, the MISO system formulation is applied. The economic MPC objective is to minimize operation cost while satisfying the predefined demand load subject to system limits.

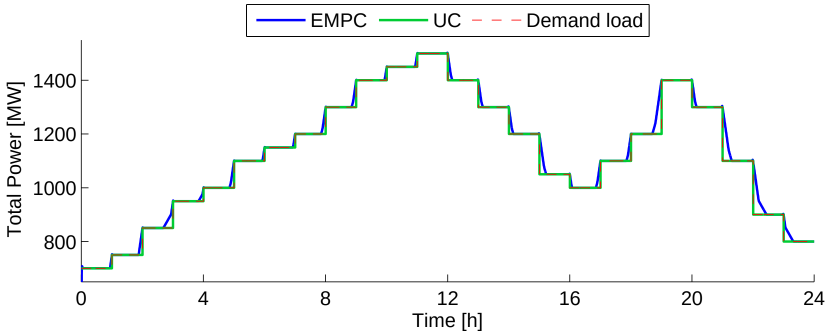
The results of the simulation with the busy demand load are reported in Figure 9.1. The production plan found by the economic MPC follow the UC solution with the exception of system dynamics included in the MPC framework.

Figure 9.1(b) shows that the cheapest power plant, plant 1, produces the majority of the load, whereas the more expensive and fast power plant, plant 2, operates whenever faster dynamics are required. This behavior is expected considering operational parameter of the power system. An interesting observation is that the economic MPC constantly increase power production on plant 1 until maximum power output on 850 MW is reached. This illustrate the discussion about used input parameterization in the UC problem; see Chapter 8. The small drops in EMPC production for plant 1 is due to the dynamics of the system when power level changes. If this is not desirable in practice, one can adjust control parameters accordingly to reduce them, e.g., by increase penalty of excessive movement of the input.

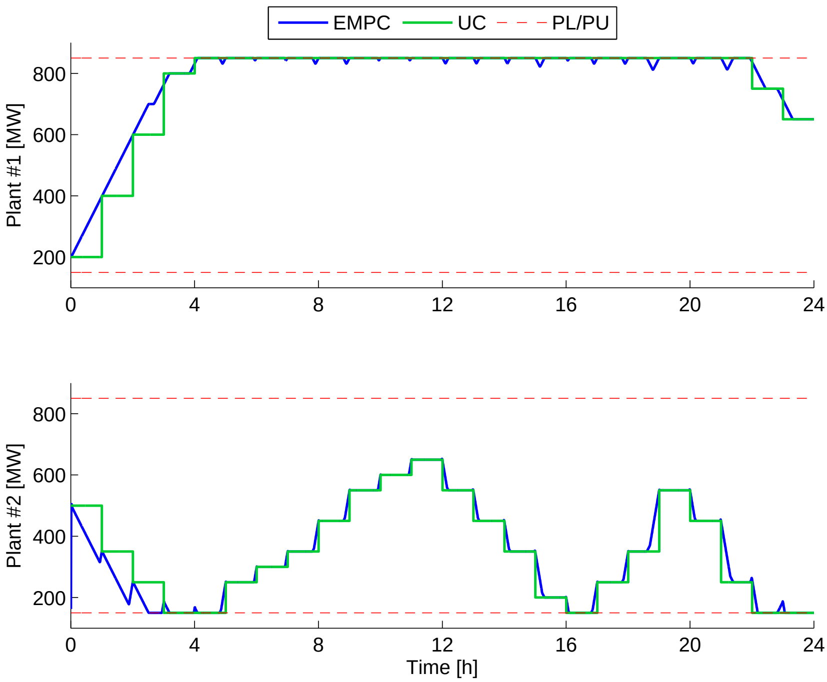
The performed inputs to the system and the rate of movement, reported in Figure 9.2, shows that the constraints are satisfied and are active at some time periods. Particularly, the power plants ability to ramp-up and ramp-down is a limit in the simulation.

The results of the simulation with the idle demand load are reported in Figure 9.3. Our implementation can handle the case when power plants change from committed to decommitted and vice versa. We set $\underline{u} = 0$ and $PL = 0$ at decommitted hours whereas $\overline{u} = 0$ and $PU = 0$ one hour after and before shutdown and startup such that the system is not penalties due to system dynamics. Around shutdown and startup the economic MPC solution differ from the UC solution; e.g., Figure 9.3(b) at hour 4 to 5. The UC increase power production on the expensive plant 2 since otherwise it will be impossible to decrease the power level to 600 MW at hour 6. In contrast, the EMPC increase the power production on the cheap plant 1. This is possible due to the system dynamics and parameterization of the economic MPC

The performed inputs to the system and the rate of movement, reported in Figure 9.4, shows that the constraints are satisfied and are active at some time periods. Again, the power plants ability to ramp-up and ramp-down is a limit in the simulation. Additionally, we see that there is no input to plant 2 when it is decommitted.

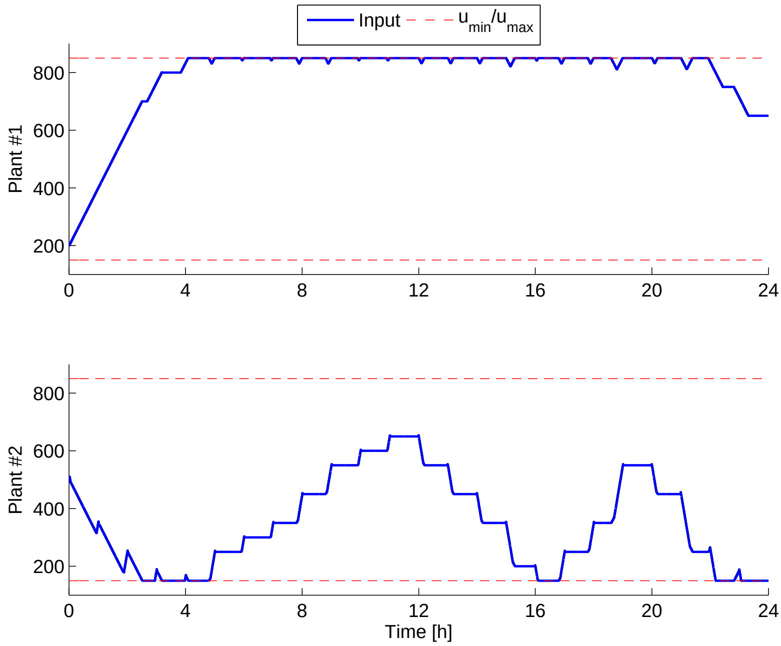



**(a)** Total power production.

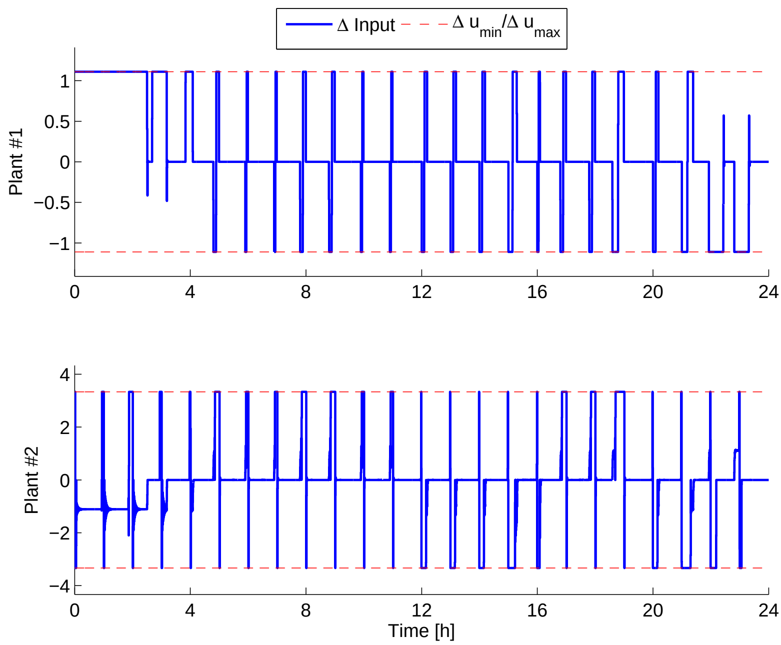



**(b)** Plants power production.

**Figure 9.1:** 24-hour MISO closed-loop simulation applying the busy demand load as trajectory. UC production profile for power plants are unknown while committed plants are known for the economic MPC.

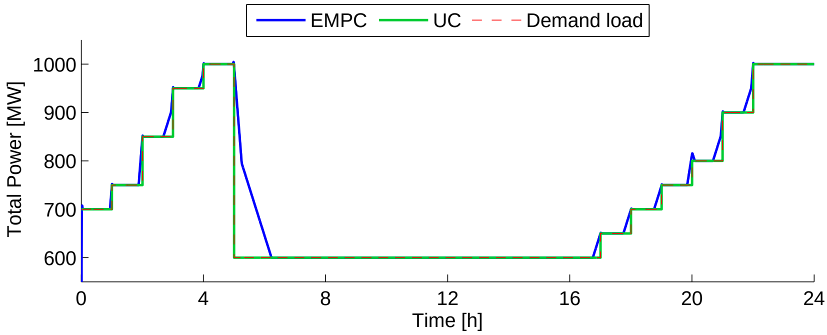



**(a)** System inputs with its limits.

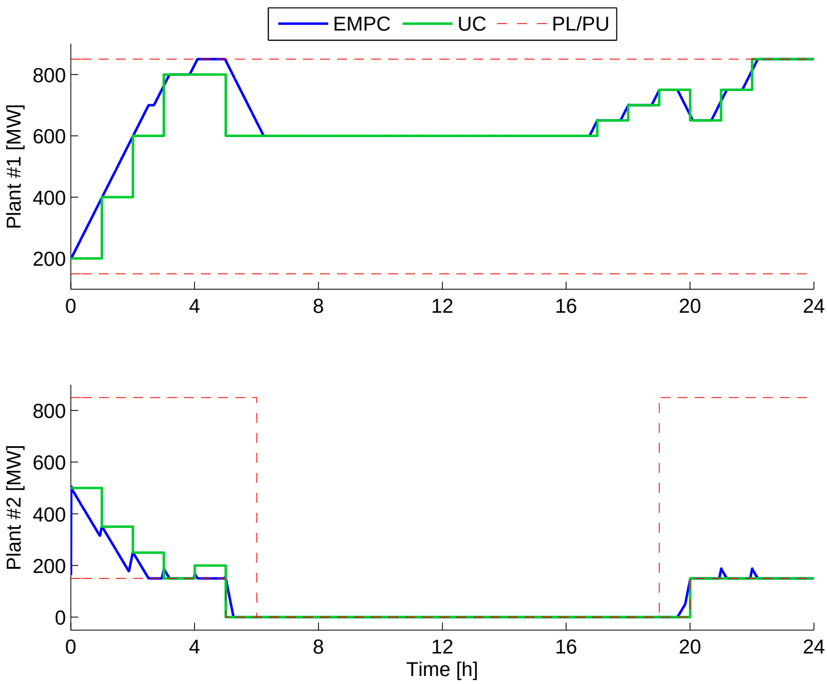



**(b)** Rate of movement for inputs with its limits.

**Figure 9.2:** 24-hour MISO closed-loop simulation applying the busy demand load. Performed inputs to the system and the rate of movement together with their limits.

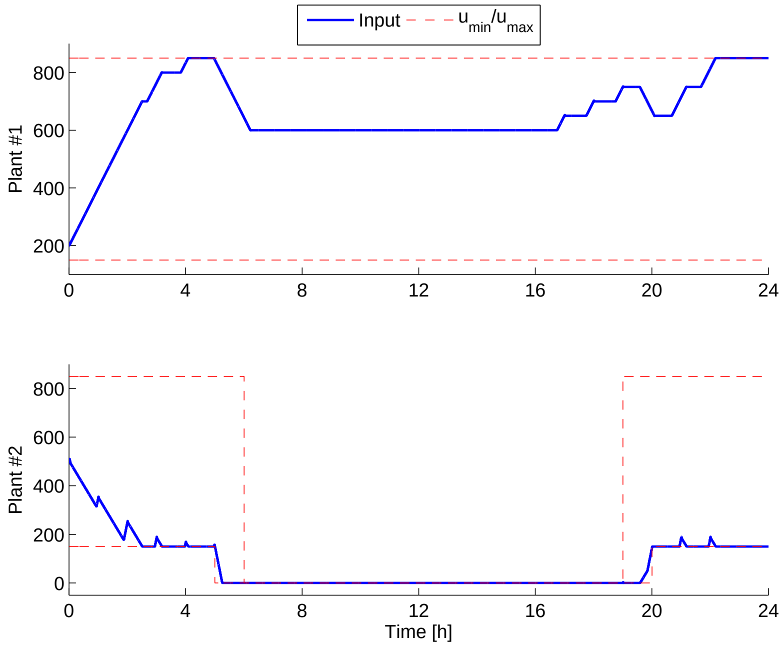



**(a)** Total power production.

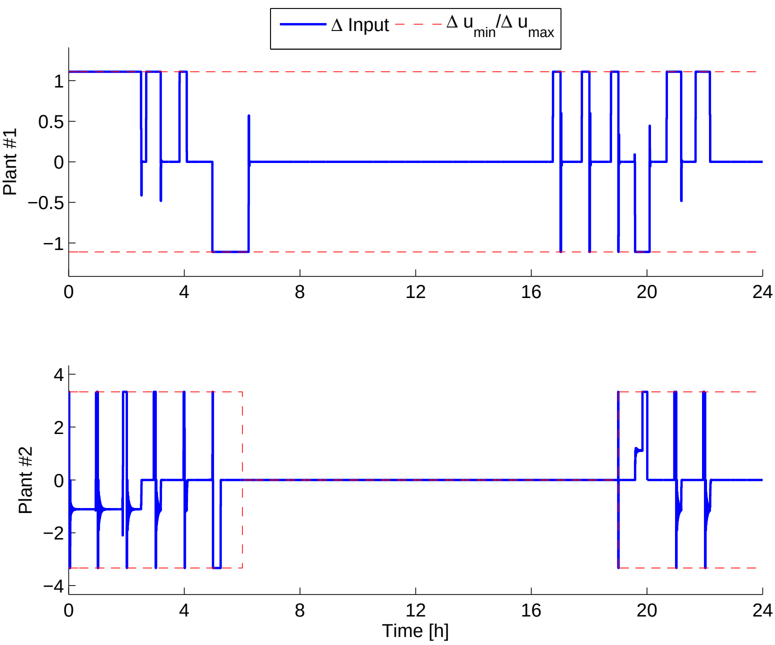



**(b)** Plants power production.

**Figure 9.3:** 24-hour MISO closed-loop simulation applying the idle demand load as trajectory. UC production profile for power plants are unknown while committed plants are known for the economic MPC.




**(a)** System inputs with its limits.




**(b)** Rate of movement for inputs with its limits.

**Figure 9.4:** 24-hour MISO closed-loop simulation applying the idle demand load. Performed inputs to the system and the rate of movement together with their limits.

## 9.2 MIMO simulations

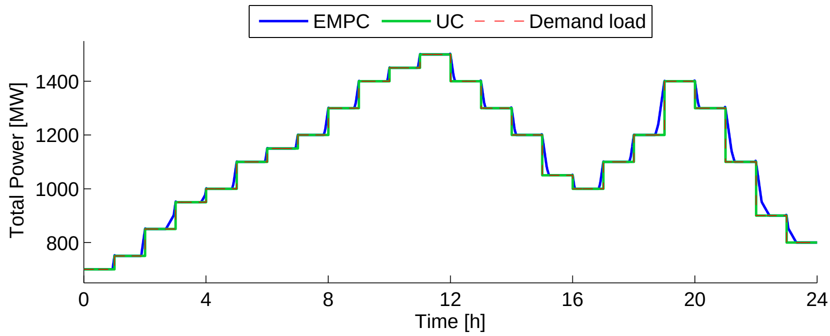
In this section, the MIMO system formulation is applied. The economic MPC objective is to minimize operation cost while satisfying the predefined demand load and power plants trajectory subject to system limits. Two different ways of modeling the trajectories are presented. In Section 9.2.1, we let the UC problem determine the committed and decommitted plants while the production plan (economic dispatch) is entirely controlled by the economic MPC. In Section 9.2.2, we let the UC problem determine the committed and decommitted plants as well as the production plan for the individual power plants, which the controller is set to obey.

### 9.2.1 System power output limits as trajectory

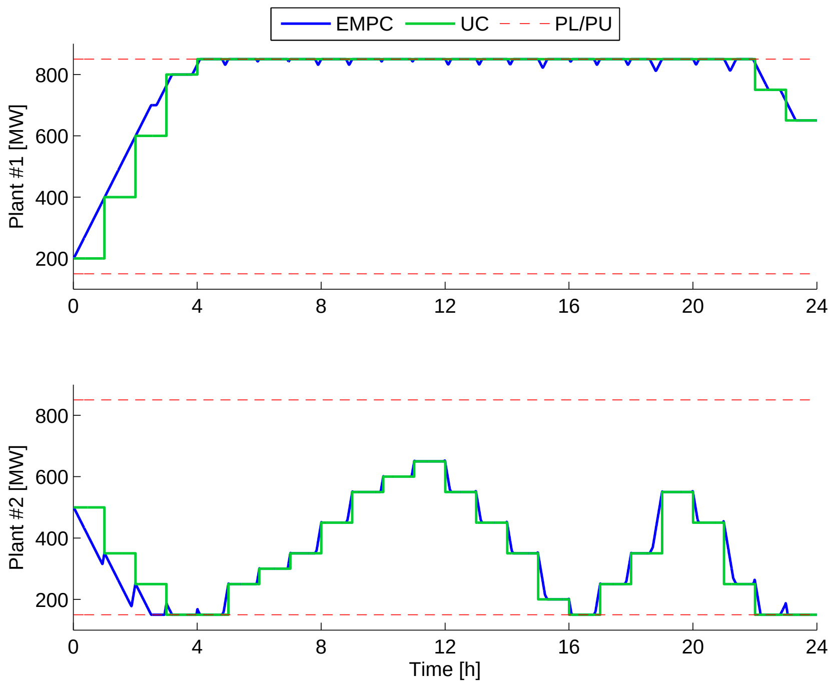
This simulation uses the UC problem to determine the committed and decommitted plants. The production plan is entirely controlled by the economic MPC to be within system power output range, $PL = 150$ MW and $PU = 850$ MW. Thus, the optimal production plan obtained by the UC is unknown for the economic MPC; however, for comparing purpose are the solutions shown.

Firstly, we consider the busy demand load. Figure 9.5 illustrates the total power production and the power production on each power plant. The obtained results are comparable to previous simulations in Section 9.1. Thus, same observations are applicable.

Secondly, we consider the idle demand load. This simulation differs from the one in Section 9.1 in terms of setup. The MIMO formulation provide the ability to control the power plants independently, which underlies the reason for applying this formulation, as described in Section 5.2. In the case of decommitted plants, the manipulable input range is zero as well as the plants individual trajectory. We set $\underline{u} = 0$ at decommitted hours whereas $\overline{u} = 0$ one hour after and before shutdown and startup. The plants trajectory is set to zero, i.e., $PL = PU = 0$, in all decommitted hours. Thus, penalty is added if power is produced in decommitted hours. The results of the simulation, reported in Figure 9.6, show faster response when plant 2 startup at hour 21. This leads to following: shorter production time on the expensive plant 2; longer production time on the cheap plant 1; a higher overshoot in the total power production; and less rate of input movement for plant 2 (see Figure 9.4(b)). All findings are due to the economic MPC ability to set the trajectories and penalties to push the solution towards the desired behavior.

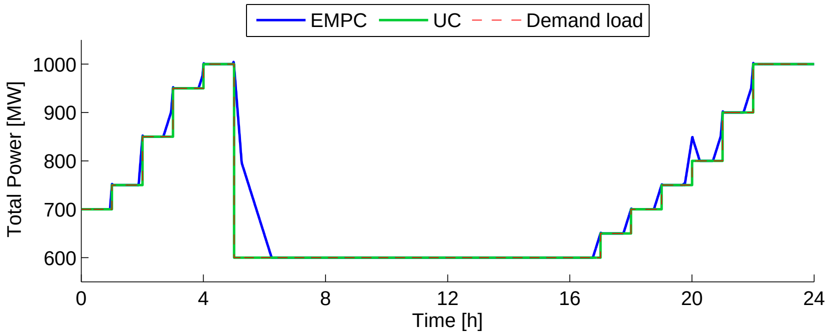



**(a)** Total power production.

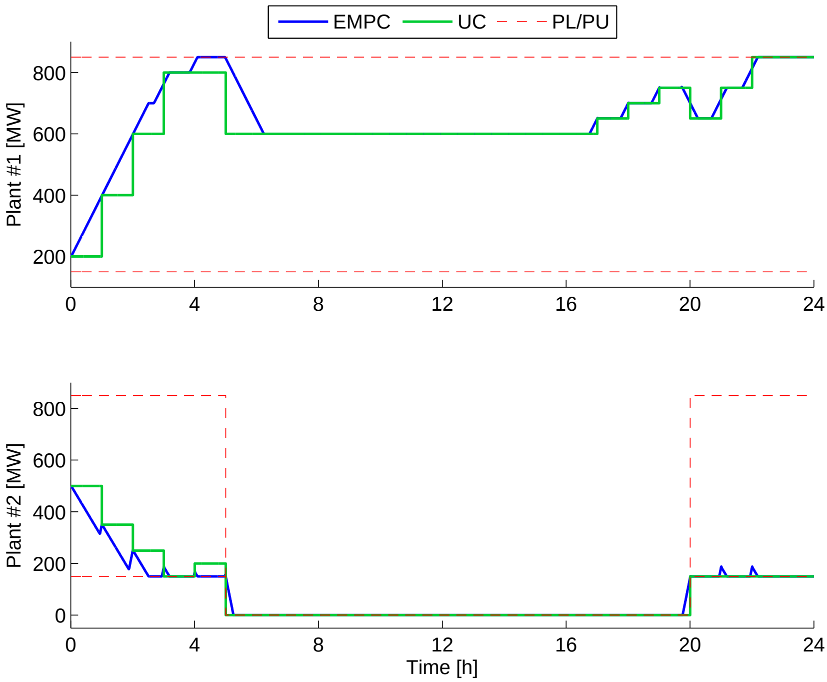



**(b)** Plants power production.

**Figure 9.5:** 24-hour MIMO closed-loop simulation applying the busy demand load and system power output limits as trajectories. UC production profile for power plants are unknown while committed plants are known for the economic MPC.




**(a)** Total power production.




**(b)** Plants power production.

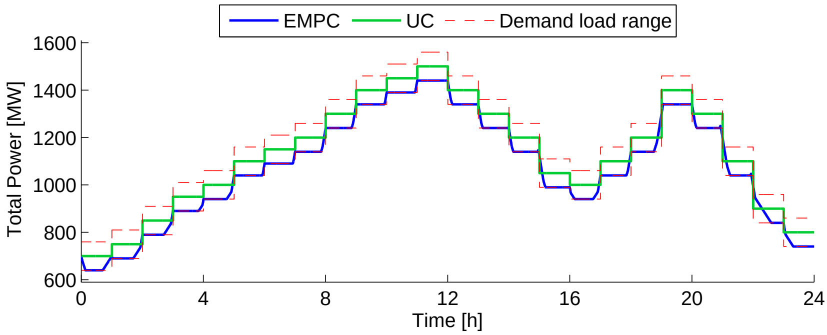
**Figure 9.6:** 24-hour MIMO closed-loop simulation applying the idle demand load and system power output limits as trajectories. UC production profile for power plants are unknown while committed plants are known for the economic MPC.

### 9.2.2 Individual production plans as trajectory

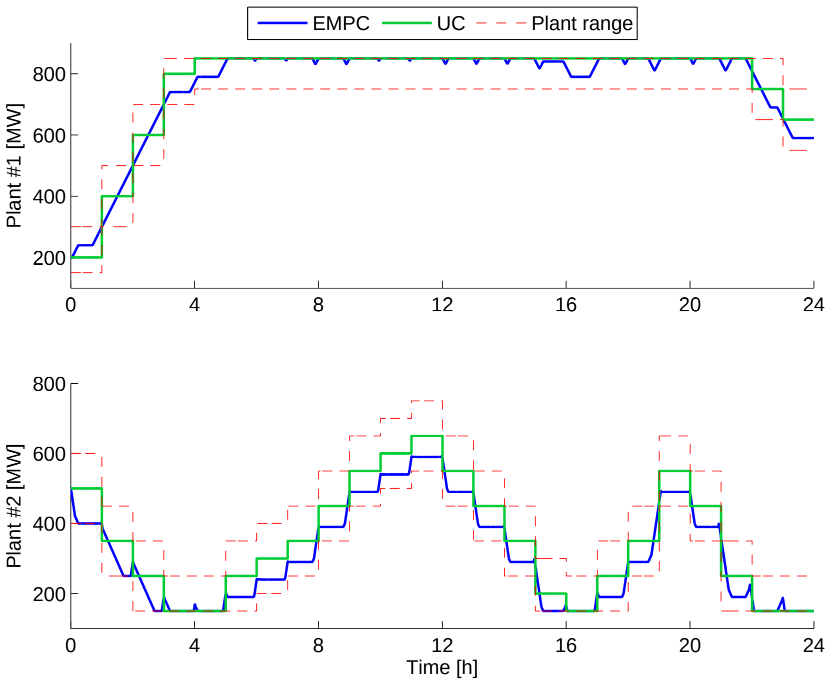
This simulation uses the UC problem to determine the committed and decommitted plants as well as the individual production plan for the power plants. The economic MPC is set to obey these production plans. This introduces a new way of controlling the power production. Now, the controller can be forced to be close to the demand load, close to the individual production plans for the power plants, or a combination of the two. Thus, many combinations are possible. We set a range around the plans by shifting the power level for each time periods either by a fixed amount [MW] or by a percents of the power level. The range for the power plants is set to $\pm 60$ MW and the demand load range is set to $\pm 100$ MW for each time period.

The results of the simulation with the busy demand load and the idle demand load are reported in Figure 9.7 and Figure 9.8, respectively. The economic MPC produce the minimum allowed power during the 24 hours. It still produced maximum power at plant 1 while the production is decreased at plant 2. This result is as expected.

Besides the lower power production, the result differs from Section 9.2.1 when the range is reached. E.g., consider the first few hours. Figure 9.7(b) shows that plant 1 and 2 have small plateau compared to Figure 9.5(b) where plant 1 have a steady power increase until maximum power output is attained and plant 2 have a more steep power decrease. This demonstrates that introducing individual ranges enables the possibility to manage the power production on each power plants as desired and thereby secondarily satisfy the overall demand load. Meanwhile, the controller ensures to find the minimum production cost subject to system limits.

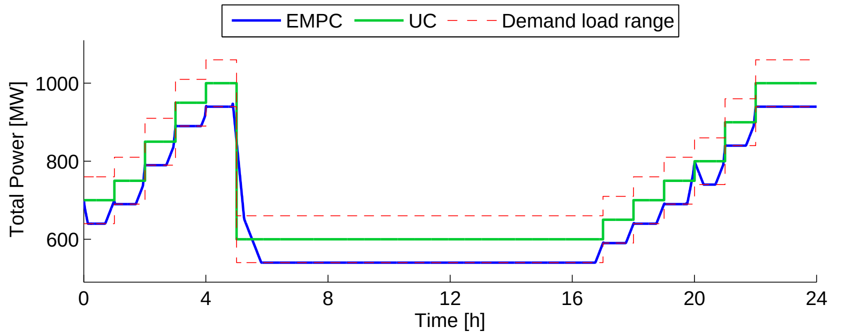



**(a)** Total power production.

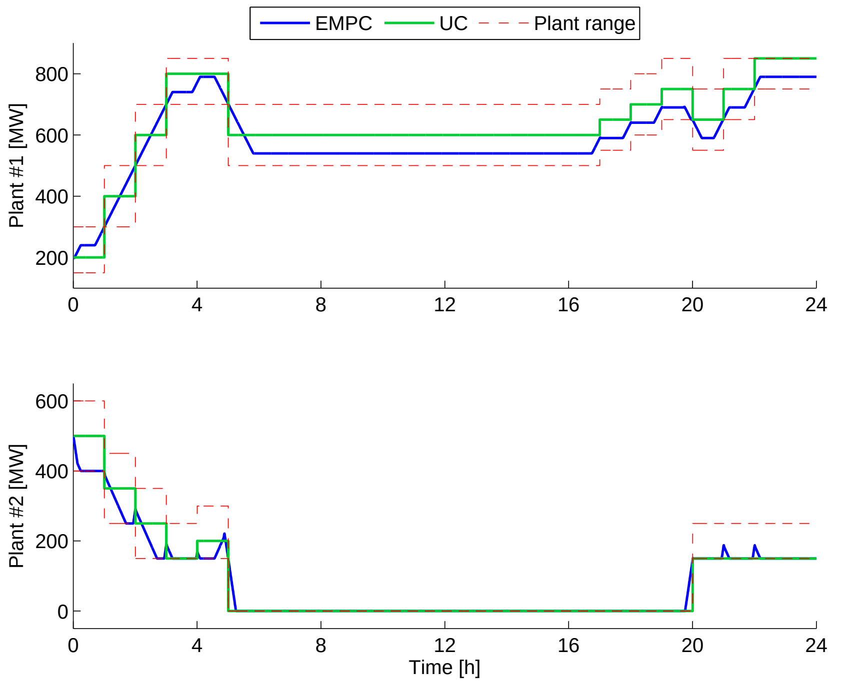



**(b)** Plants power production.

**Figure 9.7:** 24-hour MIMO closed-loop simulation applying the busy demand load. The economic MPC is to obey obtained production plan from solving the UC problem within a defined range (trajectories).




**(a)** Total power production.




**(b)** Plants power production.

**Figure 9.8:** 24-hour MIMO closed-loop simulation applying the idle demand load. The economic MPC is to obey obtained production plan from solving the UC problem within a defined range (trajectories).

## 9.3 Key findings

We perform simulations of combining the UC problem and the economic MPC problem without power supply from renewable energy sources in the power system. Both the MISO and MIMO formulation of the power system were applied in the controller. We demonstrate that our economic MPC implementation can follow the solution from the UC problem.

The MIMO formulation provides benefits in terms of the possibility for individual control of the power plants while collaborate to satisfy a common objective, the overall demand load. Section 9.2 present two different setups, which are useful depending on the situations and purpose. Introducing individual ranges around the UC solution enables endless possibilities to manage the power production. Consequently, the exact way of coupling the UC problem and economic MPC should be situation and purpose dependent.

CHAPTER 10

# Stochastic Simulations

In this chapter, we perform simulations of combining the UC problem and the economic MPC problem with power supply from renewable energy sources in the power system. The controller consider the MIMO system; see Section 5.2. The power supply from renewable energy sources are modeled as following:

- Wind power modeled as a step
- Wind power modeled as a fluctuating function, a sine wave

In our simulations, we let the UC determine the committed and decommitted plants while the production plan is entirely controlled by the economic MPC to be within system power output limits. Lastly, we present key findings from the performed simulations.

## 10.1 Modeling forecasts of wind power supply

We model the power production from the wind turbines as a composition of three values each representing the layer of knowledge and reliability of the forecasts. Let

- $\mathrm{dst}_1$ denote the day-ahead forecasts of wind power production;
- $\mathrm{dst}_2$ denote the adjusted forecasts of wind power production obtainable during the day of operations; and
- $\mathrm{dst}_3$ denote the necessary adjustments to obtain the true power wind power production the day of operations.

Then, we model

$$\mathrm{dst}_{\mathrm{UC}} = \mathrm{dst}_1 \tag{10.1a}$$

$$\mathrm{dst}_{\mathrm{EMPC}} = \mathrm{dst}_{\mathrm{UC}} + \mathrm{dst}_2 \tag{10.1b}$$

$$\mathrm{dst}_{\mathrm{Sim}} = \mathrm{dst}_{\mathrm{EMPC}} + \mathrm{dst}_3. \tag{10.1c}$$

$\mathrm{dst}_{\mathrm{UC}}$ is the forecast applied to the UC problem. $\mathrm{dst}_{\mathrm{EMPC}}$ is the forecast for the economic MPC in a receding horizon manner, and $\mathrm{dst}_{\mathrm{Sim}}$ is the disturbance measured by sensor. $\mathrm{dst}_{\mathrm{Sim}}$ is the true wind power production. This approach of modeling the wind production offers the opportunity to simulate the real-world forecasts. Since, as time goes, more information is available and forecasts will be more reliable. However, forecasts are embedded with some uncertainties, which means that some fluctuations will occur between the best wind power production forecast and the true wind power production, which is modeled by the unknown disturbances, $\mathrm{dst}_3$.

## 10.2 Step wind power

We investigate the system response when power supply from renewable energy sources instantly change from initial forecasted. We consider following two cases:

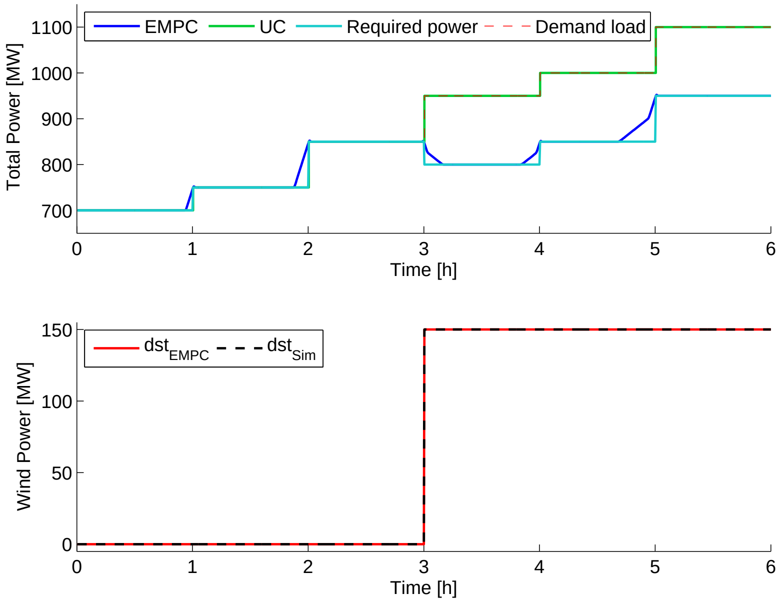
1. Wind contribution is known for the economic MPC though $\hat{d}_{k+j|k}$ in (6.4). This simulates the situation where updated and more reliable wind forecasts are provided to the economic MPC.
2. Wind contribution is unknown and modeled as a disturbance. Thus, the economic MPC identify the disturbance from the measurements. This simulates the situation of unexpected wind power production.

For both cases, the wind production is unknown at the time the UC is solved. 6-hour closed-loop simulations are performed considering the busy demand load. At hour 3, 150 MW wind power enter the system. As new information is available, the required production for the plants will be changed. Hopefully, the set-point change will be detected by the economic MPC due to the rolling horizon and feedback, and hence change the power production accordingly. In the opposite case where less wind power is received than initially forecasted, the system response will just be the reversed.

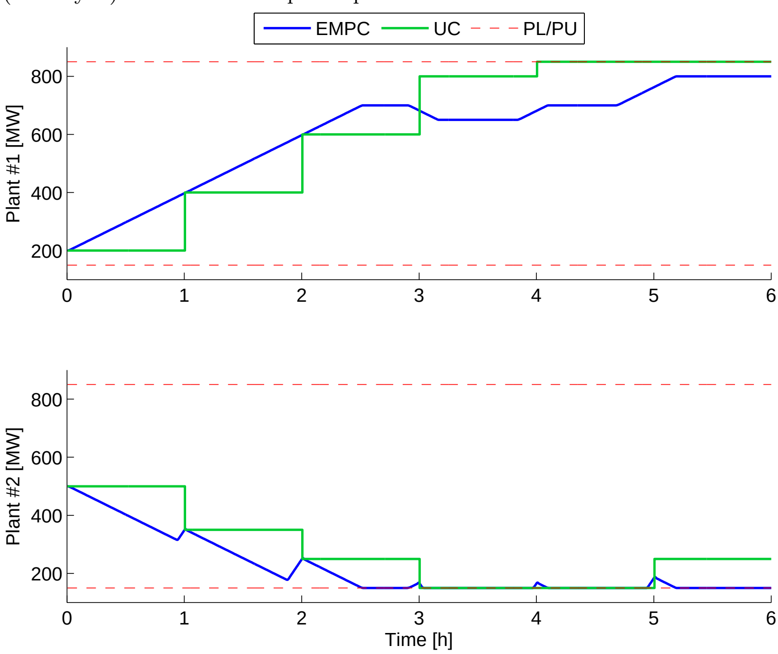
### 10.2.1 Case 1

The results of the 6-hour simulation, reported in Figure 10.1, show that the controller react to the set-point change at hour 3, whereas the production plan derived from the UC yields into unnecessary power production. The actual required power generated by the plants is represented by the solid cyan line. Figure 10.1(b) shows that the economic MPC reduce the power production at plant 1 when the wind power entering the system. This is to be expected, since the production at the expensive plant 2 cannot be reduced further due to the minimum power output limit.

We see the controllers ability to react to changes when new and more reliable information is available. In addition, we also see a challenge with the controller. The economic MPC do not include binary decisions, as the planning regarding which plants there are committed and decommitted is determined by the UC. Consequently, surplus of power may occur if following is true: 1) power supply from renewable energy sources change a lot compared to initial forecasts, and 2) the system cannot decrease the production further due to minimum power output limits. In that case, a shutdown of a power plant may be a good decision. This indicates that solving the UC problem ones the day-ahead may not be the optimal solution.




**(a)** *Top:* Total power production from the UC and the economic MPC. Required power generated by plants (solid cyan). *Bottom:* Wind power production.




**(b)** Plants power production.

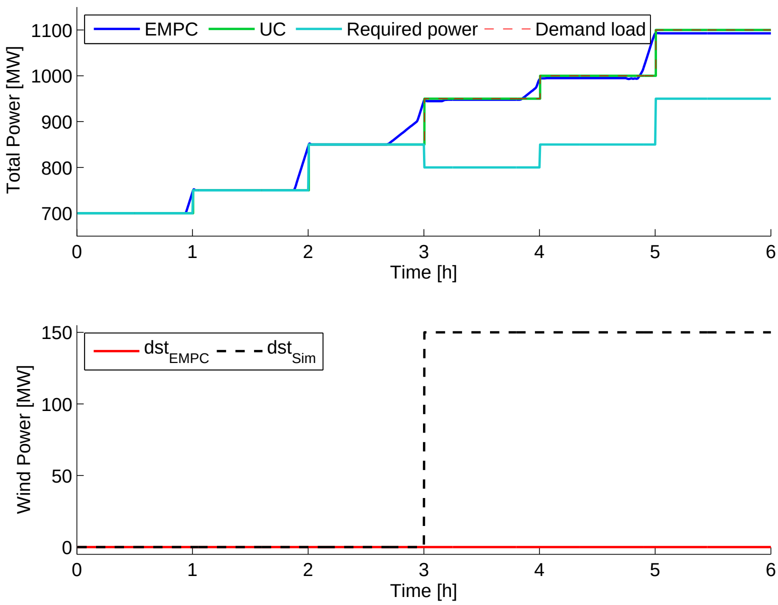
**Figure 10.1:** 6-hour closed-loop simulation when 150 MW wind power entering the system at hour 3.

### 10.2.2 Case 2

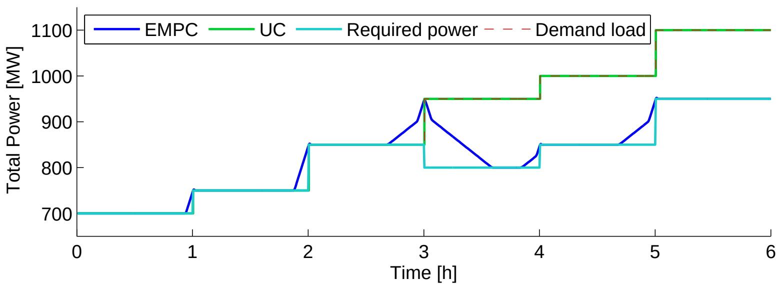
The results of 6-hour simulation, reported in Figure 10.2(a), show that the controller do not react in the same manner as before. The production is reduced but nowhere near enough. We obtain this phenomenon, since the disturbance is not modeled in our control framework. In order to remove offset and hence obtain offset free control, we model an unmeasured disturbance model by incorporating an output disturbance into the process model; see Section 5.5.

Figure 10.2(b) illustrates the same setup as before, but with offset free MPC. Now, a much better solution is obtained, as the controller reacts on the unexpected set-point change and decrease power production accordingly. We see interesting MPC characteristics around hour 3. As we know, the control framework is solved in a rolling horizon manner and in each time step look $N$ prediction steps ahead. Therefore, increases the power production towards hour 3 to prepare for the increasing demand load at hour 3. After hour 3, the required power production to satisfy the demand load deceases because of the sudden wind power supply. By measurements from the physical plant, the controller gets new information and adapt to the changed situation. Consequently, we obtain a peak at hour 3. Figure 10.1(a) show no peak at hour 3, because the controller gets the new update information prior the set-point change.

In this simulation, we identified MPC ability to anticipate future events and to take actions accordingly when disturbance entering the system by the closed-loop feedback. In the rest of our simulations, we apply offset free MPC.




**(a)** *Top:* Total power production from the UC and the economic MPC. Required power generated by plants (solid cyan). *Bottom:* Wind power production model as disturbance to the system.




**(b)** Offset free MPC simulation. Same setup and wind power disturbance profile as Figure 10.2(a).

**Figure 10.2:** 6-hour closed-loop simulation with the busy demand load. Figure 10.2(a) shows the result of offset MPC simulation and Figure 10.2(b) shows the result of offset free MPC. Both with same simulation setup.

## 10.3 Fluctuating wind power

We investigate the system response when power supply from renewable energy sources is modeled to be fluctuating. Based on our acquired experience from above, we apply the offset free MPC presented in Section 5.5. We consider following four cases:

1. First example of introducing fluctuating non-controllable predictable wind power production.
2. Examine the impact wind power have on power system imbalance by fixing the frequency, the fluctuations of the wind power, and vary the amplitude, the amount of wind power production.
3. The opposite as above: fixing the amplitude, the amount of wind power production, and vary the frequency, the fluctuations of the wind power.
4. Introduce stochastic process noise and measurements noise.

In following simulations, we apply the busy demand load from Figure 7.3(a). However, the discretization setup is changed for the economic MPC. Now, the controller receives the demand load represented on the high resolution time grid. Figure 10.3 depicted the demand load on the coarse grid and high resolution grid.

In order to test our implementations ability to adapt to a fluctuating trajectory is the forecast generated by the sine wave function (10.2). The sine wave function is chosen since this function gives a smooth repetitive oscillation. The sine wave is defined as a function of time, $t$, in the form:

$$f(t) = \alpha \sin\left(\frac{2\pi\omega t}{24 \cdot 60}\right), \tag{10.2}$$

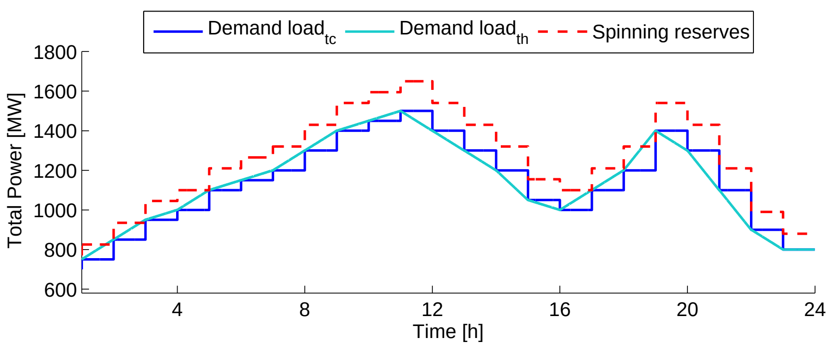



**Figure 10.3:** 24-hour demand load [MW]. Coarse grid demand load (tc) applied in the UC problem (solid blue) and high resolution demand load (th) applied in the economic MPC (solid cyan). Spinning reserve is 10% of demand load$_{tc}$ for each time period.

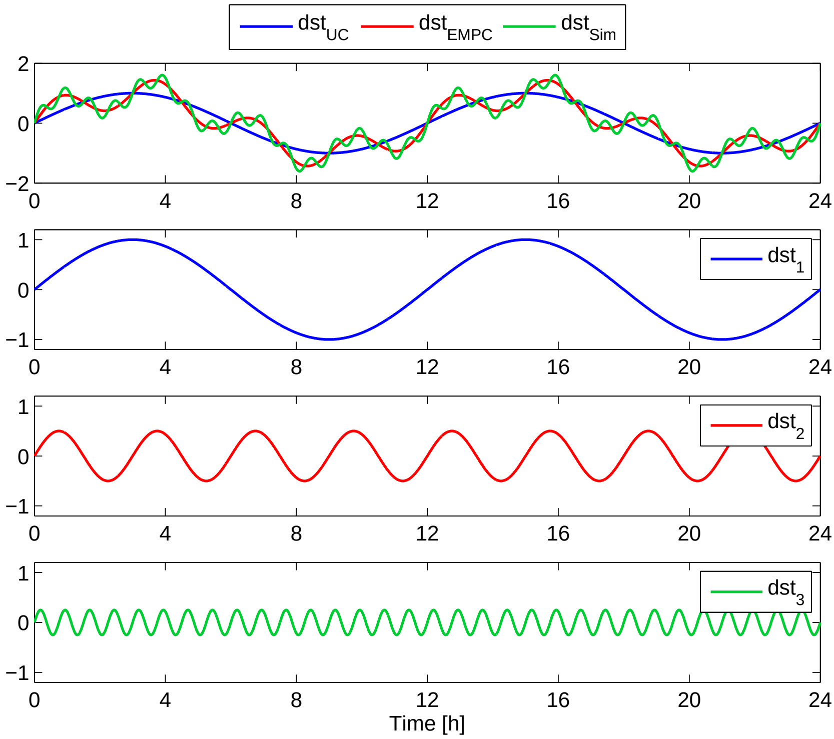



**Figure 10.4:** Example illustration of (10.1) applying (10.2).

where $\alpha$ is the amplitude and $\omega$ is the frequency. Figure 10.4 illustrates an example of the computed (10.1) applying (10.2). We chose to fix the ratio between $\text{dst}_1$, $\text{dst}_2$, and $\text{dst}_3$. It is preliminary clear that if the day-ahead wind power production forecast, $\text{dst}_1$, is embedded with more uncertainty, the performance difference between the UC problem and the economic MPC will be to the controllers advantage.

As we know, the discretization is different for the two methods. The applied wind power forecasts, $\text{dst}_{\text{UC}}$, $\text{dst}_{\text{EMPC}}$, and $\text{dst}_{\text{Sim}}$, are derived on high resolution time grid. Then, $\text{dst}_{\text{UC}}$ is discretized, hour by hour, to fit the discretization in the UC optimization problem. We calculate the discretization as the mean value for each hour; thus, at each hour the production level is shift up or down.

### 10.3.1 Case 1

The wind power supply is modeled by (10.2) using the parameters listed in Table 10.5.

The results of 6-hour simulation, reported in Figure 10.6, show many interesting aspects. As expected, the UC solution return into imbalance situation due to the discretization and the change in wind power production during the day of operations.

**Table 10.5:** Applied parameters in Figure 10.6 to (10.2).

| Forecasts | $\alpha$ | $\omega$ |
|---|---|---|
| $\mathrm{dst}_1$ | 50 | 0.1 |
| $\mathrm{dst}_2$ | 25 | 0.4 |
| $\mathrm{dst}_3$ | 12.5 | 0.8 |

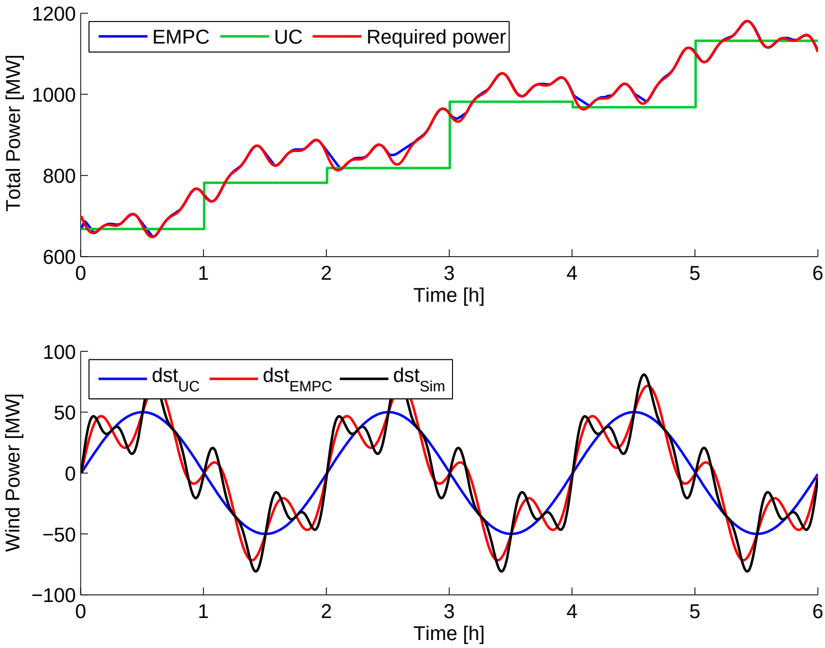



**Figure 10.6:** 6-hour closed-loop simulation. *Top:* Total power production from the UC and the economic MPC. Required power generated by plants (solid red). *Bottom:* Wind power production defined as (10.1).

Meanwhile, the economic MPC nicely follow the required power production by the two plants to satisfy the demand load. In some situations, the economic MPC cannot follow the fast changes; e.g., between hour 2 and 3. The imbalance as function of time, reported in Figure 10.7, show how good the economic MPC solution is compared to the UC solution.

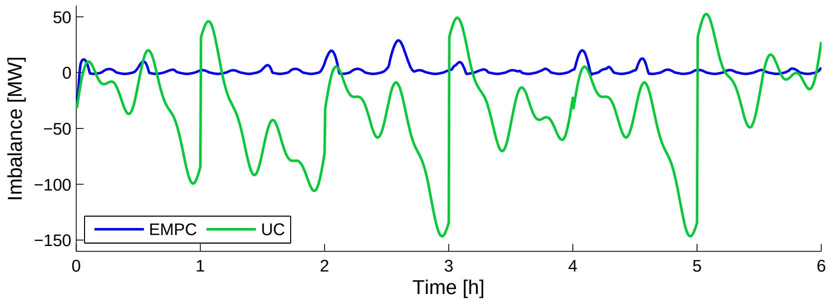



**Figure 10.7:** Imbalance as function of time for simulation Figure 10.6. Calculated as the differences between the optimal required power production by plants and the two derived solutions: UC solution and economic MPC solution.

### 10.3.2 Case 2

We fix the frequency, the fluctuations of the wind power, and vary the amplitude, the amount of wind power production. We want to consider the impact of wind power have on power system imbalance. We calculate the absolute imbalance between the optimal production and the derived. We choose to consider the amount of power produced instead of production cost to avoid introducing uncertainties and assumptions regarding properly choice of cost prices, imbalance cost, profit margin, etc. However, results will be presented in a way such that reader easily can calculate the cost with own figures.

**Table 10.8:** Results of 24-hour closed-loop with four different amplitudes. Imbalance is the absolute imbalance between the optimal production and the obtained production plan.

| $\alpha$ | Methods | Total Power [MWh] | Imb. [MWh] | %-deviation |
|---|---|---|---|---|
| - | Optimal | 27,150 | - | - |
| 0 | UC | 27,100 | 1,150 | 4.24% |
| | EMPC | 27,150 | 0.07 | 0.00% |
| 14 | UC | 27,100 | 1,169 | 4.31% |
| | EMPC | 27,155 | 10 | 0.04% |
| 35 | UC | 27,100 | 1,219 | 4.49% |
| | EMPC | 27,175 | 37 | 0.14% |
| 70 | UC | 27,100 | 1,356 | 5.00% |
| | EMPC | 27,267 | 152 | 0.56% |

24-hour closed-loop simulations are performed with four different amplitudes,

$$\alpha = \{0,14,35,70\}$$

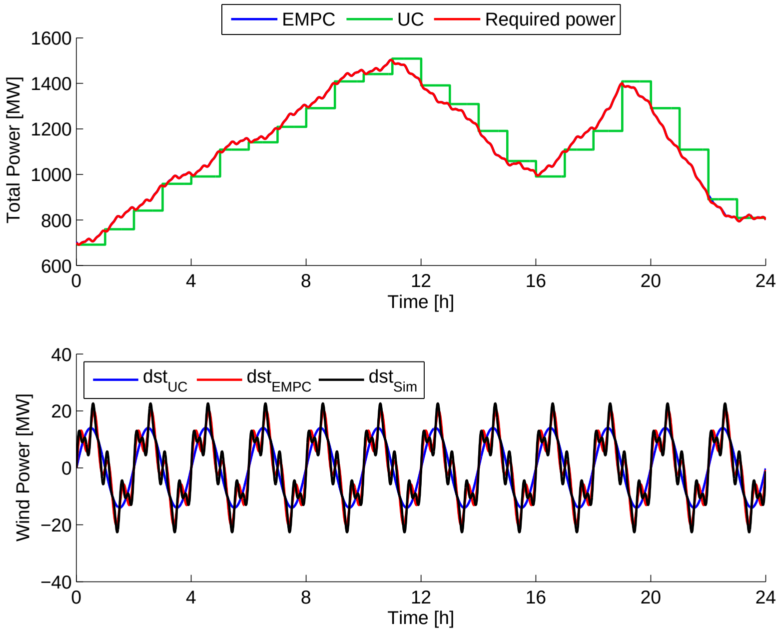
and the frequency,

$$\omega = \{\mathrm{dst}_1,\mathrm{dst}_2,\mathrm{dst}_3\} = \{0.1,0.4,0.8\}\,.$$

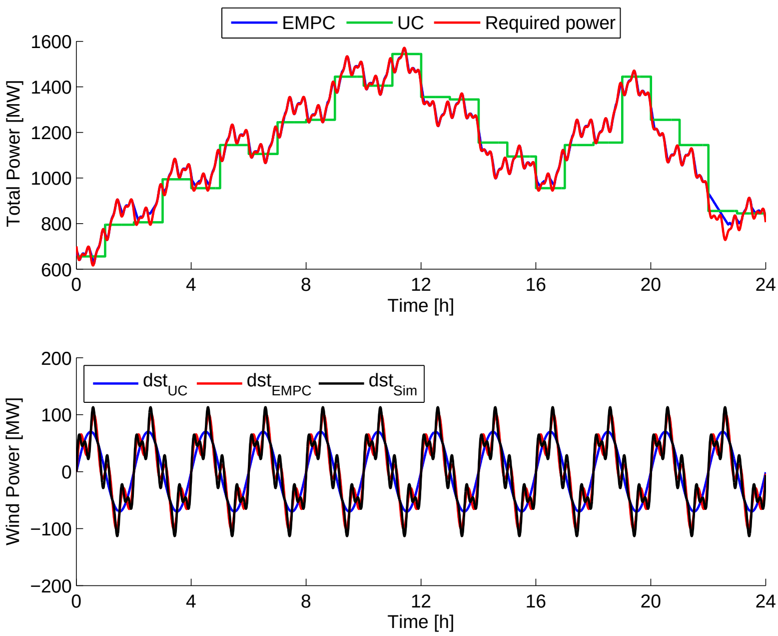
Because of the way we model wind power production, the total power production obtained by the optimal power production will numerical be the same in all four simulations. The same is true of the UC optimization problem. This does not necessarily apply for the absolute imbalance.

Table 10.8 lists the obtained results of the simulations. In all cases, the economic MPC obtain a better solution than the UC problem, nearly identical to the optimal. Without wind power production, $\alpha = 0$, the UC solution yields to 4.24% of the power production is imbalance while the comparable economic MPC solution coincide with the optimal. The only differences between the two methods are the discretization. Thus, in this example, the discretization loss is significant. As $\alpha$ increases, the imbalance increases for the two methods. In particular, we see that the controller begins to have difficulties to follow the very fluctuating wind power production. Further research of control parameters may optimize the results. In the hardest case, $\alpha = 70$, the amount of imbalance the controller produce only stands for 0.56% of the total optimal power production, whereas the solution from the UC optimization problem yields 5%.

The obtained solution with $\alpha = 14$ and $\alpha = 70$ are depicted in Figure 10.9. The illustrations confirm the data analyzed before. In Figure 10.9(b), we see that the controller has difficulties to follow in some cases when fluctuations are too large. We note that penalty is added if the controller produce less than demand load




**(a)** $\alpha = 14$.




**(b)** $\alpha = 70$.

**Figure 10.9:** 24-hour closed-loop simulation with two different amplitude values. *Top:* Total power production from the UC and the economic MPC. Required power generated by plants (solid red). *Bottom:* Wind power production defined as (10.1).

### 10.3.3 Case 3

We fix the amplitude, the amount of wind power production, and vary the frequency, the fluctuations of the wind power. Two 6-hour simulations are performed where the frequency is doubled. The wind power production is modeled by (10.2) using the parameters listed in Table 10.10.

The results of the simulations are reported in Figure 10.11. The economic MPC obtain a decent result. As the frequency increases, the controller has difficulties in some time periods to follow the optimal power production completely due to the integrated system dynamics and limits. The economic MPC solution follows the upper peaks, because penalty is added if production is below the demand load.

In the following, we set the power output range to be $\pm 0.03\%$ of the demand load. Figure 10.12 shows the result with $\omega_2$ (the same simulation as in Figure 10.11(b)). Now, the production plan is more in the middle and do not necessarily follow the upper (nor the lower) peaks. Consequently, control parameters may be adjusted in order to obtain the desired behavior.

### 10.3.4 Case 4

Now, we take a step back. We want to show an interesting challenge with uncertainties. The utilized data and forecasts are not incontestable and thus embedded with some uncertainties. E.g., forecasts of demand load, forecasts of renewable energy supply, forecasts of prices, etc. Additionally, in scientific computing where we try to simulate the real-world, model uncertainties are also present. If uncertainties become a significant part of provided data, time should be spent on investigating the emerging challenges.

As we know, one of the reasons to apply MPC for optimal control of systems is its great feature of getting close to system limits without exceeding. However, this may become a challenge if uncertainties are significant. We note that it is very expensive to exceeding system limits, as it may result in stability issue and possible breakdowns in the power grid. In present simulations, we have seen that the controller increase the power production at plant 1 to its maximum power output. In the following, we apply significant process and measurement noise into the system to show that this may be an issue. Let the noise in the system be uncorrelated and identically

**Table 10.10:** Applied parameters in Figure 10.11(a) and Figure 10.11(b) to (10.2).

| Forecasts | $\alpha$ | $\omega_1$ | $\omega_2$ |
|---|---|---|---|
| $\text{dst}_1$ | 50 | 0.2 | 0.4 |
| $\text{dst}_2$ | 25 | 0.8 | 1.6 |
| $\text{dst}_3$ | 12.5 | 1.6 | 3.2 |

independently normally distributed as

$$\begin{bmatrix} w_k \\ \xi_k \\ v_k \end{bmatrix} \sim N_{iid}\left( \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 8^2 & 0 & 0 \\ 0 & 8^2 & 0 \\ 0 & 0 & 20^2 \end{bmatrix} \right). \tag{10.3}$$

6-hour closed-loop simulation, reported in Figure 10.13, show that the demand load is not satisfied in all time steps. The economic MPC find a feasible solution, since the output constraints are soften, thus, penalty is added. Furthermore, we see that plant 1 exceed its maximum power output in the last hours.

The purpose of this simulation is to show that uncertainties may be an issue in practices. An investigation on these challenges is outside the scope of this thesis. However, we would like to mention a possible way to address this issue. The stochastic uncertainties may be modeled into the economic MPC model. The Mean-Variance economic MPC include the mean and variance of the data [Sok+14; Cap+15; Mor+14]. E.g., consider the production cost $c^T u$, where $c$ is the prices of producing power. Then, we may formulate the objective as

$$\underset{\{u\}}{\text{minimize}} \qquad \phi = \lambda \mathbb{E}[c^T u] + (1-\lambda)\mathbb{V}ar[c^T u], \qquad \lambda \in [0,1]. \tag{10.4}$$

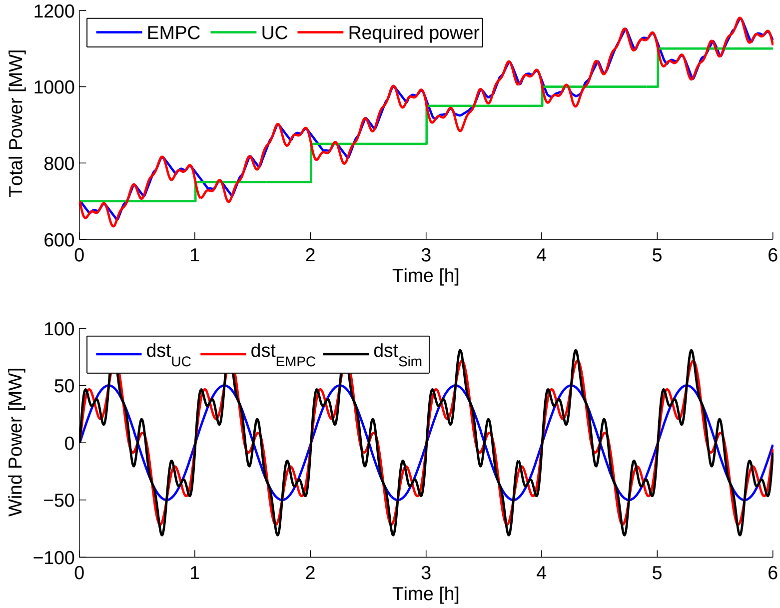
$\mathbb{E}[c^T u]$ is the expected value of the cost and $\mathbb{V}ar[c^T u]$ is the variance of the cost. $\lambda$ is a risk aversion parameter that determines the trade-off between the expected cost and the cost variance.

## 10.4 Key findings

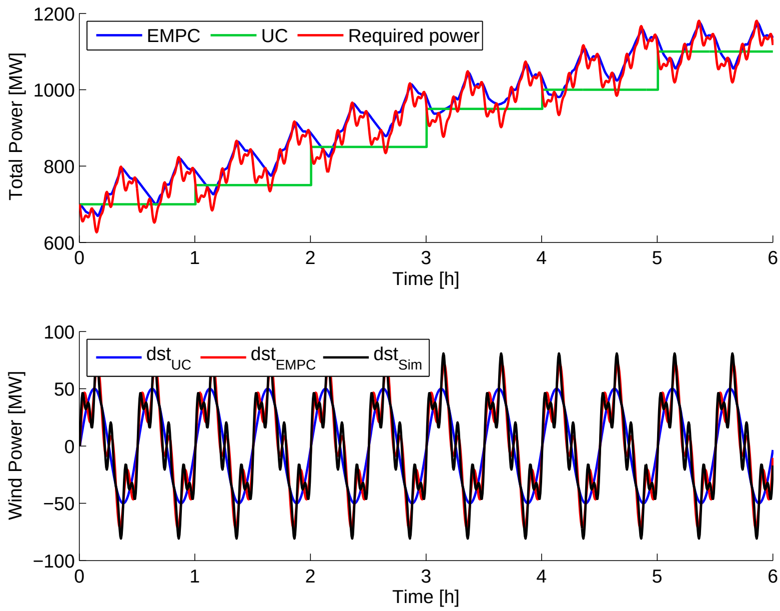
We perform simulations of combining the UC problem and the economic MPC problem with power supply from renewable energy sources in the power system. We identify MPC ability to anticipate future events and to take actions accordingly when disturbance entering the system by the closed-loop feedback. Thus, as new and more reliable information is available, the controller reoptimizes the production plan. We observe that modeling the unmeasured disturbance, by incorporating an output disturbance into the process model, yields nicely behavior and we obtain offset free MPC.

Simulations indicate that the solutions obtained by the economic MPC are better than the solution obtained by the UC optimization problem. Less power imbalance is created using economic MPC. As the fluctuations can be managed by the controller in a predictive manner, the need for reserve capacity is reduced. This implies potential cost reduction with a good impact on the environment.

Furthermore, we address some challenges with the controller when the renewable power supply is fluctuating too much too fast and when introducing uncertainties in data or model. These challenges are interesting, because too fluctuating power supply may result into undesirable imbalance and unstable power systems.




**(a)** Simulation with $\omega_1$ presented in Table 10.10.




**(b)** Simulation with $\omega_2$ presented in Table 10.10.

**Figure 10.11:** 6-hour closed-loop simulation. Fixed amplitude and vary frequency. Wind power modeled by (10.2) using parameters listed in Table 10.10.

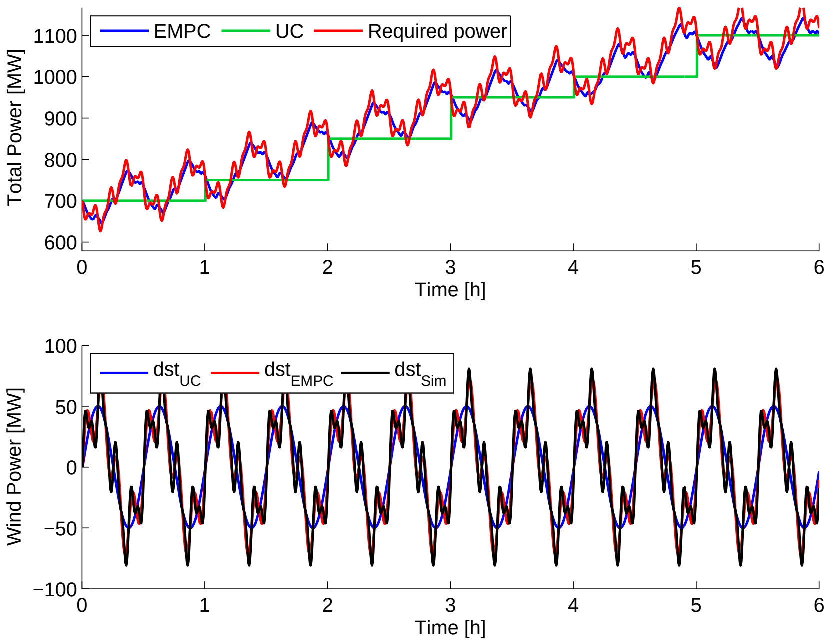
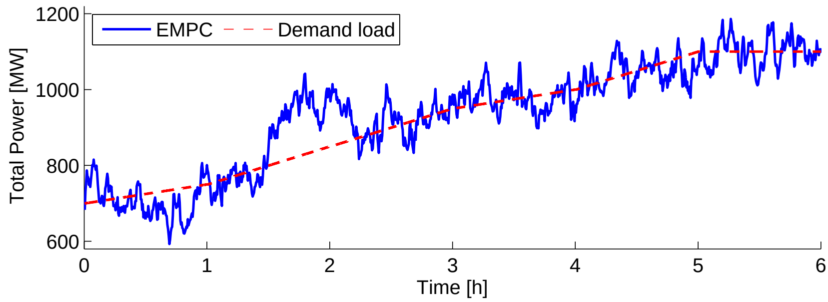
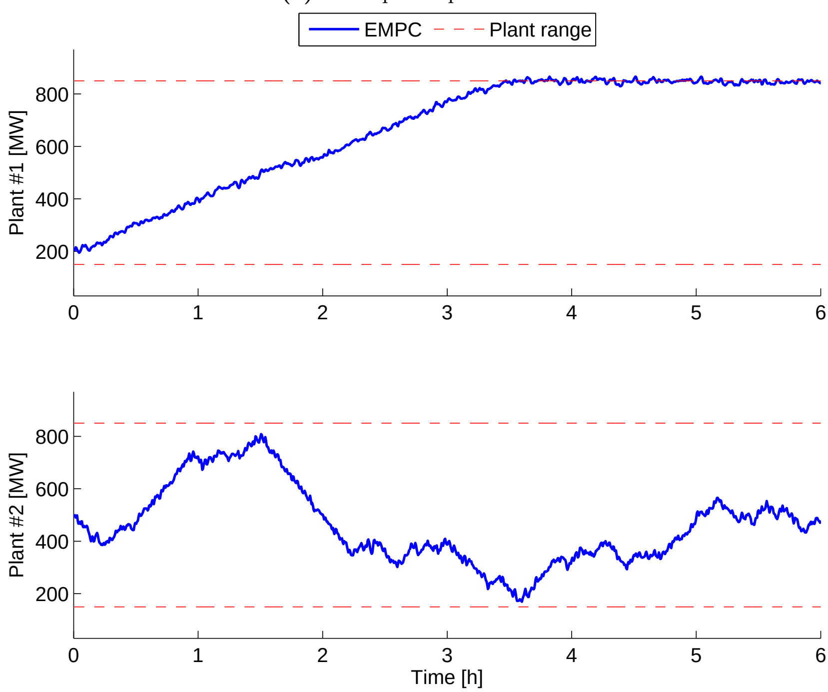



**Figure 10.12:** Same simulation as Figure 10.11(b) with the change of power output range to be ±0.03% of the demand load.




**(a)** Total power production.




**(b)** Plants power production.

**Figure 10.13:** 6-hour closed-loop simulation. Consider the stochastic model with stochastic process noise and measurements noise distributed as (10.3).

Part IV

# Conclusions and Perspectives

CHAPTER 11

# Conclusions and Perspectives

In this thesis, we have investigated the design and implementation of combing the Unit Commitment (UC) optimization problem and the economic Model Predictive Control (MPC) problem for optimal operation of power systems. This chapter will collect the key findings presented in the thesis and provide concluding remarks, as well as address possible extensions and directions for future research.

The thesis primary objective is repeated: *By combining the unit commitment optimization problem and the economic model predictive control problem, it is possible to obtain an intelligent control strategy that can overcome some of the important challenges associated with the increasing share of intermittent renewable energy sources in the power supply. This novel coupling will operate the power systems in a cost efficient manner while satisfying the overall demand load and various system requirements.*

We have shown that the developed novel control strategy appears to provide a feasible and a promising solution to overcome some of the important challenges. We provide an intelligent control strategy that shows properties of managing uncertainty with flexibility. We demonstrate a potential for significant savings in imbalance cost.

Reliable forecasts of power supply from renewable energy sources are of great importance, since the uncertainties in these forecasts have a significantly impact on the imbalance cost and stability issues. We demonstrate the importance of reoptimize the production plan during the day of operation in order to account for fluctuations and reducing the dependence on the correctness of forecasts. We show that economic MPC is indeed an appealing method to enable for this functionality. By real-time optimization with feedback, economic MPC successfully adapt, predict, and change the production plan according to the fluctuations inherent in renewable power supply. A reduction in imbalance will also lead to less need of the expensive spinning reserve, which again yields cost savings and have a good impact on the environment.

Conclusively, the provided valuable learning will without a doubt be interesting to the academia and industry like DONG Energy.

## 11.1 UC

In Chapter 4, we describe and formalize mathematically the UC optimization problem as a mixed integer linear programming problem. UC is an intuitive method to determine the optimal production plan for a giving portfolio of power generating plants. Scheduling with a financial and environmental perspective is nontrivial. The complexity is a key challenge for this type of problem. Generally, UC is NP-complete. Consequently, for power systems with practical size (large-scale power systems), the high computational complexity makes it impossible to solve the UC problem with a high frequency, in order to intercept the variations inherent in the nature of renewable energy sources.

## 11.2 Economic MPC

Chapter 5 introduce the principles of models for predictive control including realistic linear dynamical model of power plants in a power system. Furthermore, we introduce model for achieving offset free tracking performance in the presences of unmeasured nonzero disturbances; representation of dynamics by the Finite Impulse Response (FIR) model; as well as Kalman state estimation, filtering, and prediction subject to stochastic noise.

In Chapter 6, we motivate the choice of economic MPC as our control framework, as well as describe and formalize mathematically a soft constrained linear economic MPC framework into a linear optimization programming problem. In our work, we consider both a deterministic and a stochastic formulation.

MPC capability to integrate predictions and forecasts of dynamical systems, as well as anticipate future events and take control actions accordingly are received eagerly, since this is exactly the functionality we require. Compared to the UC optimization problem, the economic MPC is designed to be solved in real-time at a high frequency, with updated and more reliable forecast as input.

## 11.3 UC and economic MPC

We present a two-level control strategy: At the high-level, the day-ahead production plan is performed on a coarse time grid (e.g., hourly) by solving the UC problem. At the low-level, the minutes-ahead production plan, or the online control, is performed on a high resolution time grid (e.g., 20 sec) by applying the economic MPC.

In Chapter 8, a case study indicates that the coarse discretization for the UC have a cost impact on the solution. Compared to the optimal production plan, the UC solution with coarse discretization yields 2.63% imbalance power while the economic MPC solution coincide with the optimal production plan. Simultaneous, the economic MPC have a runtime of 0.04 sec. Solving the UC problem on a high resolution time grid yields a runtime of 2.60 sec. Thus, we obtain a 65x speedup. This supports the choice of using economic MPC as the method for reoptimizing the production plan.

In Chapter 9, we observe that our economic MPC implementation yields great tracking performance. The provided MIMO formulation enables the controller the opportunity to manage each power plant independently while tracking them to satisfy a common goal, e.g., obey the overall demand load. This formulation enables endless tuning and setup possibilities to manage the power production as desirable while minimizing the operating cost subject to system limitations.

In Chapter 10, we examine the system reaction when intermittent renewable energy sources enter the power system. The results show that the controller indeed have the ability to anticipate future events and to take actions accordingly, when disturbance enter the system. We identified the importance of formulating the economic MPC to achieve offset free tracking performance, when unmeasured nonzero disturbance entering the system.

We address some challenges for the controller when the renewable power supply is fluctuating too much too fast and when introducing uncertainties in the data or model. The latter is particular interesting. MPC great feature of operating the system to its limits may introduce challenges if data or forecasts are embedded with significant uncertainty. By simulations, we show that system limits may be exceeded in the event of significant process noise and measurement noise. Therefore, we suggest incorporating uncertainties into the control framework

## 11.4 Perspectives and further research

The subject of this thesis is far more extensive than the scope of the thesis. There are two paths to be followed after this project. One path is to test the control strategy suitability in a practical setting by an industrial implementation maturing. The other path is further academic development. In the following, we address possible extensions and directions for future research.

- Further development, design prototypes, and real-life testing will definitely prove the feasibility and verify the initial results of an economic and environmental potential.
- In Chapter 8, results show that the UC discretization and parameterization have a significant impact on the cost. Further investigation should be conducted to determine whether there exists hidden cost savings and unutilized resources.
- We solve the UC once at the day-ahead planning. Maybe changing the cycles yield a better performance, e.g., executing the UC problem in a closed-loop manner with rolling-shrinking horizon approach. Then, apply the economic MPC to balancing the production in between. Thus, the controller is reset by a new and updated UC solution.
- Initial results show that the developed control framework may have some challenges if data or model uncertainties are significant. Therefore, for the control strategy to be applicable one may look on how to handle these uncertainties appropriately. It might also be interesting to examine the formulation of the economic MPC objective. E.g., one could formulate

  $$\alpha \left\| u_k - \bar{u}_k \right\| + (1 - \alpha) \left\| \Delta u_k \right\|,$$

  where $u_k$ is the manipulated variables, $\bar{u}_k$ is the trajectory from solving the UC optimization problem, and $\alpha$ is a risk aversion parameter that determines the trade-off between following the UC solution and the cost of discourages disproportionate movement of the manipulable variables. In addition, we model the cost function as a linear cost. If the costs more correctly follow a nonliner trend, the change to nonlinear economic MPC may introduce new issues as solution time, stability issues, and robustness [JYB14; Luc+14].
- The electricity market does not really match to the current and future way of producing power. The market should maybe be modified to better fit the new way of power production. We see that the intraday market has come increasingly important as more renewable energy sources enter the power grid. Imbalance between day-ahead contracts and actually produced often need to be offset.

  By curious, the electricity is a commodity production there can be bought, sold, and traded. Rather comparable to the financial market where financial securities are traded. Thus, can we imagine a scenario where stocks and alike only were traded once a day?

# Part V

# Appendices

APPENDIX A

# Background Material

This appendix present mathematical background, results, and theorems applied in this thesis.

## A.1 Linearization and discretization

This section outlines the basic concepts of how a linear time-invariant continuous-time model may be linearized and discretized to obtain a linear time-invariant discrete-time state-space model. For simplification, an example of a deterministic nonlinear dynamical system will be considered.

### A.1.1 Continuous-time state-space model

The deterministic nonlinear dynamical system may be stated in terms of ordinary differential equations

$$\dot{x}(t) = f(x(t), u(t), d(t)), \qquad x(t_0) = x_0 \tag{A.1a}$$
$$y(t) = g(x(t)) \tag{A.1b}$$
$$z(t) = h(x(t)). \tag{A.1c}$$

Approximating (A.1) using Taylor expansion around the equilibrium point $(x_s,u_s,d_s,y_s,z_y)$ defined by

$$f(x_s,u_s,d_s) = 0$$
$$y_s(t) = g(x_s)$$
$$z_s(t) = h(x_s),$$

yields the corresponding linear time-invariant continuous-time state-space model

$$\dot{X}(t) = AX(t) + BU(t) + ED(t), \qquad X(t_0) = X_0 \tag{A.3a}$$
$$Y(t) = CX(t) \tag{A.3b}$$
$$Z(t) = C_z X(t), \tag{A.3c}$$

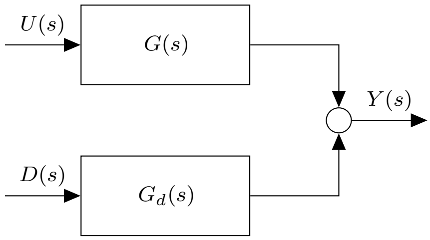



**Figure A.1:** Input-ouput relation describing the transfer functions.

where the linear system state matrices $A$, $B$, $C$, $E$, and $C_z$ are computed by

$$\begin{aligned} A &= \nabla_x f(x_s,u_s,d_s), & B &= \nabla_u f(x_s,u_s,d_s), & E &= \nabla_d f(x_s,u_s,d_s), \\ C &= \nabla_x g(x_s), & C_z &= \nabla_x f(x_s). \end{aligned}$$

The deviation variables $X(t)$, $U(t)$, $D(t)$, $Y(t)$, and $Z(t)$ are defined by

$$\begin{aligned} X(t) &= x(t) - x_s, & U(t) &= u(t) - u_s, & D(t) &= d(t) - d_s, \\ Y(t) &= y(t) - y_s, & Z(t) &= z(t) - z_s. \end{aligned}$$

Notice that the system state matrices can be time-varying. The stochastic continuous-time state-space representation of this is

$$\begin{aligned} \dot{X}(t) &= AX(t) + BU(t) + ED(t) + GW(t) \\ Y(t) &= CX(t) + V(t) \\ Z(t) &= C_z X(t). \end{aligned}$$

### A.1.2 Continuous-time transfer function

The linear time-invariant continuous-time system (A.3) with the initial condition $X(0) = 0$ and $Y \equiv Z$ can be represented as the input-output function in the Laplace domain

$$Y(s) = G(s)U(s) + G_d(s)D(s),$$

where $U(s)$ is the input function, $Y(s)$ the output function, $D(s)$ the disturbance function, and $G(s)$ and $G_d(s)$ are the transfer functions. Figure A.1 is a block diagram of the input-output relation. The transfer functions $G(s)$ and $G_d(s)$ describe the relation between input and output of the system, thus,

$$\begin{aligned} G(s) &= C(sI - A)^{-1}B \\ G_d(s) &= C(sI - A)^{-1}E, \end{aligned}$$

where $I$ is the identity matrix. Various continuous-time transfer functions exists. Two examples are given below:

$$G(s) = \frac{K}{(\tau s+1)^n}$$
$$G(s) = \frac{K(\beta s+1)}{(\tau_1 s+1)(\tau_2 s+1)}.$$

A system respond time to an input can be modeled by adding a time-delay term to the transfer function. We obtain the transfer function

$$G(s) = \frac{Y(s)}{U(s)} e^{-\tau_d s}.$$

Examples,

$$G(s) = \frac{K}{(\tau s+1)^n} e^{-\tau_d s}$$
$$G(s) = \frac{K(\beta s+1)}{(\tau_1 s+1)(\tau_2 s+1)} e^{-\tau_d s}.$$

Systems with multiple input and multiple output may be described by set of transfer functions. E.g., the transfer function for a system with two inputs and one output, a MISO system, is

$$Y(s) = \begin{bmatrix} G_1(s) & G_2(s) \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}.$$

### A.1.3 Discrete-time state-space model

The model describing the dynamical systems, independent of realization, can be discretized into a discrete-time state-space model. Let the input, $U(t)$, be piecewise constant. This is the case when the input is determined by a computer. We assume a zero-order hold (ZOH) discrete sampling. Let $T_s$ be the sampling time, such that the sampling instants, $t_k$, are

$$t_k = t_0 + kT_s, \qquad k = 0,1,2,\dots,$$

the zero-order hold of the inputs be characterized by

$$U(t) = U_k, \qquad t_k \le t < t_{k+1},$$

and the states at discrete times $X(t_k) = X_k$. Then, the linear time-invariant continuous-time system (A.3) is equivalent with the linear time-invariant discrete-time state-space model

$$X_{k+1} = A_d X_k + B_d U_k + E_d D_k, \qquad X(t_k) = X_k \tag{A.7a}$$
$$Y_k = C_d X_k \tag{A.7b}$$
$$Z_k = C_{z,d} X_k, \tag{A.7c}$$

with

$$A_d = e^{AT_s}, \qquad B_d = \int_0^{T_s} e^{A\tau} B d\tau, \qquad E_d = \int_0^{T_s} e^{A\tau} E d\tau,$$
$$C_d = C, \qquad C_{z,d} = C_z.$$

$A_d$, $B_d$, and $E_d$ are computed using the matrix exponential with sampling period $T_s$ by

$$\begin{bmatrix} A_d(T_s) & B_d(T_s) & E_d(T_s) \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} = \exp\left( \begin{bmatrix} A & B & E \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} T_s \right).$$

The discrete-time linear time-invariant system (A.7) can be represented as the input-output function in the Z-domain; see Appendix A.2.3.

The subscript $d$ to explicit indicate discrete-time state-space matrices will be skipped as we only consider discrete-time state-space systems. Consequently, we formulate the deterministic linear time-invariant discrete-time state-space model by

$$x_{k+1} = Ax_k + Bu_k + Ed_k \tag{A.9a}$$
$$y_k = Cx_k \tag{A.9b}$$
$$z_k = C_z x_k. \tag{A.9c}$$

and the stochastic linear time-invariant discrete-time state-space model by

$$x_{k+1} = Ax_k + Bu_k + Ed_k + Gw_k \tag{A.10a}$$
$$y_k = Cx_k + v_k \tag{A.10b}$$
$$z_k = C_z x_k, \tag{A.10c}$$

where $x_k$ denotes the states, $u_k$ the manipulated input variables, $y_k$ the measured outputs, $z_k$ the controlled outputs, $w_k$ the stochastic process noise, and $v_k$ the measurement noise. $d_k$ is the process disturbance that can be predicted by a prognosis and are predicted independently of the measurements $y$. Consequently, $d_k$ shall be considered as the non-controllable power contribution from, e.g., wind power production, which in some sense can be predicted by weather forecasts.

Additionally, models used by the predictive controller may be obtained by system identification, e.g., from experimental analysis; that is, fitting a transfer function to experimental observations obtained by performing a step or impulse response of the system. Although, system identification is a very important theme, it is beyond this thesis scope.

## A.2 List of used theorems

The following sections list used theorems.

### A.2.1 Propositional logic

[RG91] present following propositional logic, which is applied in the thesis.

$$P \Rightarrow Q \Leftrightarrow \neg P \wedge Q. \tag{A.11}$$

De Morgan's theorem

$$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q \tag{A.12a}$$

and

$$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q. \tag{A.12b}$$

### A.2.2 Laplace transform

The Laplace transform is a linear operator that transform a function $f(t)$, $t \in \mathbb{R}_{\geq 0}$, to a function $F(s)$, $s \in \mathbb{C}$, given by the integral [Str08]

$$F(s) = \int_0^\infty f(t)e^{-st}dt. \tag{A.13}$$

### A.2.3 Z-transform

Z-transform converts a discrete-time signal sequence $x[n]$ into a complex frequency domain representation $X[z]$ by

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}, \tag{A.14}$$

where $z = \mathrm{Re}(z) + j\mathrm{Im}(z)$.

APPENDIX B

# System Data

This appendix present data used in the thesis.

**Table B.1:** 24-hour demand load [MW] applied in simulations. Spinning reserve is 10% of demand load for each time period.

**(a)** Refered as the *busy demand load*.

| Hour | Demand load Busy |
|---|---|
| 1 | 700 |
| 2 | 750 |
| 3 | 850 |
| 4 | 950 |
| 5 | 1000 |
| 6 | 1100 |
| 7 | 1150 |
| 8 | 1200 |
| 9 | 1300 |
| 10 | 1400 |
| 11 | 1450 |
| 12 | 1500 |
| 13 | 1400 |
| 14 | 1300 |
| 15 | 1200 |
| 16 | 1050 |
| 17 | 1000 |
| 18 | 1100 |
| 19 | 1200 |
| 20 | 1400 |
| 21 | 1300 |
| 22 | 1100 |
| 23 | 900 |
| 24 | 800 |

**(b)** Refered as the *idle demand load*.

| Hour | Demand load Idle |
|---|---|
| 1 | 700 |
| 2 | 750 |
| 3 | 850 |
| 4 | 950 |
| 5 | 1000 |
| 6 | 600 |
| 7 | 600 |
| 8 | 600 |
| 9 | 600 |
| 10 | 600 |
| 11 | 600 |
| 12 | 600 |
| 13 | 600 |
| 14 | 600 |
| 15 | 600 |
| 16 | 600 |
| 17 | 600 |
| 18 | 650 |
| 19 | 700 |
| 20 | 750 |
| 21 | 800 |
| 22 | 900 |
| 23 | 1000 |
| 24 | 1000 |

**Table B.2:** Operational parameters for the 10-unit power system.

| Plant | $a_i$ [$/h] | $b_i$ [$/MWh] | $SU_i$ [$/h] | $SD_i$ [$/h] | $PL_i$ [MW] | $PU_i$ [MW] | $RD_i$ [MW/h] | $RU_i$ [MW/h] | $TU_i$ [h] | $TD_i$ [h] | $EC$ [g/kWh] |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1000 | 16.19 | 10 | 10 | 150 | 455 | 200 | 200 | 8 | 8 | 780 |
| 2 | 970 | 17.26 | 10 | 10 | 150 | 455 | 200 | 200 | 8 | 8 | 500 |
| 3 | 700 | 16.6 | 8 | 8 | 20 | 130 | 100 | 100 | 5 | 5 | 500 |
| 4 | 680 | 16.5 | 8 | 8 | 20 | 130 | 100 | 100 | 5 | 5 | 500 |
| 5 | 450 | 19.7 | 8 | 8 | 25 | 162 | 100 | 100 | 6 | 6 | 500 |
| 6 | 370 | 22.26 | 10 | 10 | 20 | 80 | 50 | 50 | 3 | 3 | 500 |
| 7 | 480 | 27.74 | 10 | 10 | 25 | 85 | 50 | 50 | 3 | 3 | 500 |
| 8 | 660 | 25.92 | 8 | 8 | 10 | 55 | 50 | 50 | 1 | 1 | 500 |
| 9 | 665 | 27.27 | 8 | 8 | 10 | 55 | 50 | 50 | 1 | 1 | 500 |
| 10 | 670 | 27.79 | 8 | 8 | 10 | 55 | 50 | 50 | 1 | 1 | 500 |

APPENDIX C

# The GRANI Program

The GRANI program is a ground-breaking Smart Grid technology on the Faroe Islands launched on November 2012. GRANI is a strategic joint venture mainly between SEV[1] and DONG Energy[2] with a budget of approximately DKK 4 million. The program is part of the European Union, Seventh Framework Programme (FP7), Twenties Project, and DONG Energy Power Hub Technology. Following quote present the background for the GRANI program:

> *"Develop and test new technologies for the integration of fluctuating renewable energy in the isolated electricity network located in the Faroe Islands"*.[ND09, §2.1].

The scale of the GRANI program is endless. The achieved experience and knowledge will be applied in larger power systems such as Denmark for then to introduce into even larger systems. The two central goals of the GRANI program are [ND09]:

1. Integration of more renewable energy into the energy system and to serve as a large-scale test facility helping implementation of the EU 2020 vision, while solving the world's energy and climate problems.

2. Opportunity to demonstrate, tests, and develop new solutions for integrating fluctuating renewable energy in an isolated power grid.

Faroe Island produce 14-45 MW power; 60% is produced by expensive heavy fuel oil, 35% by hydro power, and only 5% is produced by wind power [NB13]. According to Figure 2.1, has the oil price more than triple the last 10 years. To reduce the dependency on oil and to reduce the carbon footprint, Faroe Island goal is that 75% of the power production in 2020 will come from renewable energy sources [NB13].

Faroe Island is unique as a demo-test case. It is unique in terms of size (50.000 inhabitants), location, and power production facilities. The size makes the island to an isolated big city in Denmark. The isolated location in the Atlantic Ocean provides the island some of the world's best and worst wind resources; harsh weather conditions

[1] SEV is Faroe Islands power company owned by the municipalities.
[2] DONG Energy is one of the leading energy groups in Northern Europe there procuring, producing, distributing, and trading energy products.

with frequent storms and very hard to forecast. Furthermore, due to a small power system and no interconnections to other counties, the power system is exposed to a high number of blackouts when comparing to continental Europe [BND12]. Despite these challenges the goal is to increase penetration of fluctuating renewable energy, which calls for developing a system that can provide stability, is of great importance.

# Nomenclature

## List of abbreviations

| | |
|---|---|
| EIA | U.S. Energy Information Administration |
| EMPC | Economic Model Predictive Control |
| FIR | Finite Impulse Response |
| FOB | Free On Board; a legal trade term |
| KKT | Karush–Kuhn–Tucker (conditions) |
| LTI | Linear Time-Invariant |
| MILP | Mixed Integer Linear Programming |
| MIMO | Multiple Input Multiple Output |
| MIP | Mixed Integer Programming |
| MISO | Multiple Input Single Output |
| MPC | Model Predictive Control |
| PL | Priority List method |
| SIMO | Single Input Multiple Output |
| SISO | Single Input Single Output |
| UC | Unit Commitment |
| ZOH | Zero-Order Hold |

## List of symbols

| | |
|---|---|
| $\forall$ | Logical sign, meaning "for all" |
| $\mathbb{R}$ | The real numbers |
| $\mathbb{C}$ | The complex numbers |
| $\mathbb{Z}_2$ | The binary number: $\{0, 1\}$ |
| $\mathbb{R}_{\geq 0}$ | The nonnegative real numbers: $\{x \in \mathbb{R} \mid x \geq 0\}$ |
| $\underline{x}$ | The minimum/lower value of $x$ |
| $\overline{x}$ | The maximum/upper value of $x$ |
| $\mathbf{1}$ | A vector with all components one |
| $x^T$ | The transpose of a vector or matrix |
| $\nabla$ | The nabla operator. In the Cartesian coordinate system $\mathbb{R}^n$ with coordinates $(x_1, x_2, \dots, x_n)$, the nabla operator is defined in terms of partial derivative operators as $\nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \dots, \frac{\partial}{\partial x_n}\right)$ |
| $\|x\|_1$ | $\ell_1$-norm of a vector $x$, i.e., $\|x\|_1 = \sum_{k=1}^{n} \lvert x_k \rvert$, $x = (x_1, x_2, \dots, x_n)$ |

## Symbols used in UC

**Indexes:**

| | |
|---|---|
| $i$ | Plant index |
| $t$ | Time period index |

**Constants:**

| | |
|---|---|
| $I$ | Total number of power generating plants |
| $T$ | Length of the planning horizon |
| $D_t$ | System power load demand for time period $t$ |
| $R_t$ | Spinning reserve required at time period $t$ |
| $PW_t$ | Forecasted power production from renewable power sources at time period $t$ |
| $a_i$, $b_i$ | Coefficients of the production cost function of plant $i$ |
| $SU_i$ | Startup cost of plant $i$ |
| $SD_i$ | Shutdown cost of plant $i$ |
| $PL_i$ | Minimum power output generation of plant $i$ |
| $PU_i$ | Maximum power output generation of plant $i$ |

| | |
|---|---|
| $RD_i$ | Maximum ramp-down of plant $i$ |
| $RU_i$ | Maximum ramp-up of plant $i$ |
| $EC_i$ | $CO_2$ emission rate for plant $i$ |
| $EU$ | Maximum $CO_2$ emission allowed |

**Variables:**

| | |
|---|---|
| $u_{i,t}$ | Binary variable; 1 if plant $i$ is committed in time period $t$ and 0 otherwise |
| $y_{i,t}$ | Binary variable; 1 if plant $i$ is started up at the beginning of time period $t$ and 0 otherwise |
| $z_{i,t}$ | Binary variable; 1 if plant $i$ is shutdown at the beginning of time period $t$ and 0 otherwise |
| $p_{i,t}$ | Nonnegative real variable; power output of plant $i$ in time period $t$ |

## Symbols used in economic MPC

| | |
|---|---|
| $k$ | Time period index |
| $N$ | Prediction horizon |
| $T_s$ | Time sample |
| $K$ | Variable gain for transfer function |
| $x_k$ | State vector |
| $u_k$ | Manipulable input variable |
| $d_k$ | Disturbance |
| $y_k$ | Measured output |
| $z_k$ | Controlled outputs |
| $p_k$ | Unmeasured disturbance |
| $w_k$ | Stochastic process noise |
| $v_k$ | Stochastic measurement noise |
| $\xi_k$ | Stochastic unmeasured disturbance noise |
| $A$, $B$, $C$, $C_z$, $F$, $F_z$, $G$ | Linear system state matrices |
| $B_p$, $C_p$ | Linear system state matrices for modeling unmeasured disturbance, $B_p = 0$ and $C_p = I$ |
| $\Phi_y$, $\Gamma_{yu}$, $\Phi_z$, $\Gamma_{zu}$ | Linear representation of dynamics |
| $c_k$ | Cost of producing power |
| $s_k$ | Slack variable for penalizing term for satisfy demand load |
| $\rho_k$ | Weight for penalizing term for satisfy demand load |

| | |
|---|---|
| $\alpha$ | Weight for penalizing term for discourages disproportionate movement of the manipulable variables |
| $\mathcal{R}_k$ | Power output range |
| $\mathcal{D}_k$ | Power production forecasts from renewable energy sources |
| $\mu$ | Function solve the soft constrained linear economic MPC optimization problem |
| $\underline{z_k}$ | Minimum value of power output |
| $\overline{z_k}$ | Maximum value of power output |
| $\underline{u_k}$ | Minimum value of manipulable input |
| $\overline{u_k}$ | Maximum value of manipulable input |
| $\Delta u_k$ | Rate of movement for manipulable input |
| $\Delta \underline{u_k}$ | Maximum ramp-down rate of movement |
| $\Delta \overline{u_k}$ | Maximum ramp-up rate of movement |

**Kalman filter:**

| | |
|---|---|
| $N_{iid}$ | The independent and identically normal distribution |
| $R_{ww},\ R_{wv},\ R_{vw},\ R_{\xi}$ | Variances for process noise, measurement noise, and disturbance noise |
| $P$ | Discrete algebraic Riccati equation (DARE) |
| $R_{fe},\ K_{fx},\ K_{fw},\ K_{fp}$ | Innovation covariance and Kalman filter gain and |
| $e_k$ | Innovation |

# Bibliography


[AAR12] D. Angeli, R. Amrit, and J. B. Rawlings. *On Average Performance and Stability of Economic Model Predictive Control*. In: *Automatic Control, IEEE Transactions on* 57.7 (July 2012), pages 1615–1626. ISSN: 0018-9286.

[AC13] O. Adeodu and D. J. Chmielewski. *Control of electric power transmission networks with massive energy storage using economic MPC*. In: *American Control Conference (ACC), 2013*. June 2013, pages 5839–5844.

[AH04] D. Axehill and A. Hansson. *A preprocessing algorithm for MIQP solvers with applications to MPC*. In: *Decision and Control, 2004. CDC. 43rd IEEE Conference on*. Volume 3. December 2004, 2497–2502 Vol.3.

[And04] G. Andersson. *Modelling and analysis of electric power systems*. In: *EEH-Power Systems Laboratory, Swiss Federal Institute of Technology (ETH), Zürich, Switzerland* (2004).

[And12] G. Andersson. *Dynamics and control of electric power systems*. In: *Lecture notes* (2012), pages 227–0528.

[BMM99] A. Bemporad, D. Mignone, and M. Morari. *An Efficient Branch and Bound Algorithm for State Estimation and Control of Hybrid Systems*. In: *European Control Conference*. Karlsruhe, Germany, August 1999.

[BND12] E. den Boer, C. Nielsen, and H. Djurhuus. *Launch of ground breaking smart grid technology on the Faroe Islands*. Invitation by SEV and DONG. Online; Visited AUG 11, 2014. November 2012. URL: http://www.twenties-project.eu/system/files/Launch%20of%20ground%20breaking%20smart%20grid%20technology%20in%20the%20Faroese%20Islands.pdf.

[Cap+15] A. Capolei, E. Suwartadi, B. Foss, and J. B. Jørgensen. *A mean variance objective for robust production optimization in uncertain geological scenarios*. In: *Journal of Petroleum Science and Engineering* 125 (2015), pages 23–37. ISSN: 0920-4105.

[Cas+11] E. Castillo, A. J. Conejo, P. Pedregal, R. García, and N. Alguacil. *Building and Solving Mathematical Programming Models in Engineering and Science*. Pure and Applied Mathematics: A Wiley Series of Texts, Monographs and Tracts. Wiley, 2011. ISBN: 9780471461654.

[CM87] P. J. Campo and M. Morari. *Robust Model Predictive Control*. In: *American Control Conference, 1987*. June 1987, pages 1021–1026.

[Con+11] E. M. Constantinescu, V. M. Zavala, M. Rocklin, S. Lee, and M. Anitescu. *A Computational Framework for Uncertainty Quantification and Stochastic Optimization in Unit Commitment With Wind Power Generation*. In: *Power Systems, IEEE Transactions on* 26.1 (February 2011), pages 431–441. ISSN: 0885-8950.

[DA12] P. J. Dinesen and P. E. Aackermann. *Development of a GPU-accelerated MIKE 21 Solver for Water Wave Dynamics*. Supervised by Associate Professor Allan P. Engsig-Karup, apek@imm.dtu.dk, DTU Informatics. Asmussens Alle, Building 305, DK-2800 Kgs. Lyngby, Denmark, 2012. URL: http://www2.imm.dtu.dk/pubdb/views/publication_details.php?id=6367.

[Dat04] B. N. Datta. *Numerical Methods for Linear Control Systems: Design and Analysis*. Numerical Methods for Linear Control Systems: Design and Analysis vb. 1. Elsevier Academic Press, 2004. ISBN: 9780122035906.

[Din10] A. L. Diniz. *Test cases for unit commitment and hydrothermal scheduling problems*. In: *Power and Energy Society General Meeting, 2010 IEEE*. July 2010, pages 1–8.

[DTU] DTU - Technical University of Denmark. *Study Handbook: MSc in Mathematical Modelling and Computation at DTU 2014/2015*. URL: http://sdb.dtu.dk/2014/5/59.

[EIAa] EIA U.S. Energy Information Administration. *International Energy Outlook 2013; Release Date: July 25, 2013*. Visited AUG 25, 2014. URL: http://www.eia.gov/forecasts/ieo/world.cfm.

[EIAb] EIA U.S. Energy Information Administration. *International Energy Statistics*. Visited AUG 25, 2014. URL: http://www.eia.gov/cfapps/ipdbproject/IEDIndex3.cfm.

[EIA14] EIA U.S. Energy Information Administration. *Spot prices of crude oil and petroleum products by selected U.S. and international areas*. Visited AUG 13, 2014. August 2014. URL: http://www.eia.gov/dnav/pet/PET_PRI_SPT_S1_A.htm.

[EMB09] K. Edlund, T. Mølbak, and J. D. Bendtsen. *Simple models for model-based portfolio load balancing controller synthesis*. In: *Power Plants and Power Systems Control*. Volume 1. 1. 2009, pages 173–178.

[Enea] Energinet.dk. *About the wholesale market*. Non-profit enterprise owned by the Danish Climate and Energy Ministry. Visited DEC 3, 2014. URL: http://www.energinet.dk/EN/El/Engrosmarked/Viden-om-engrosmarkedet/Sider/Viden-om-engrosmarkedet.aspx.

[Eneb] Energinet.dk. *Vindmøller slog rekord i 2014 (Danish)*. Non-profit enterprise owned by the Danish Climate and Energy Ministry. Visited JAN 9, 2015. URL: http://www.energinet.dk/DA/El/Nyheder/Sider/Vindmoeller-slog-rekord-i-2014.aspx.

[Ene14] Energinet.dk. *Smart Grid in Denmark 2.0*. Technical Report, Energinet.dk, The Danish TSO owned by the Danish Climate and Energy Ministry. Visited DEC 9, 2014. September 2014. URL: http://www.energinet.dk/EN/FORSKNING/Energinet-dks-forskning-og-udvikling/Smart-Grid/Sider/default.aspx.

[ESJ09] K. Edlund, L. E. Sokoler, and J. B. Jørgensen. *A Primal-Dual Interior-Point Linear Programming Algorithm for MPC*. In: *Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on*. December 2009, pages 351–356.

[Eur03] European Commission. *The Energy Challenge of the 21st Century: The role of nuclear energy*. 2003. URL: ftp://ftp.cordis.europa.eu/pub/fp6/docs/euratom_challenge_21stcentury.pdf.

[Eur13] European Commission. *Energy challenges and policy*. May 2013. URL: http://ec.europa.eu/europe2020/pdf/energy2_en.pdf.

[Gur14] Gurobi Optimization, Inc. *Gurobi Optimizer*. Visited SEP 20, 2014. 2014. URL: http://www.gurobi.com/products/gurobi-optimizer/gurobi-overview.

[GZP03] X. Guan, Q. Zhai, and A. Papalexopoulos. *Optimization based methods for unit commitment: Lagrangian relaxation versus general mixed integer programming*. In: *Power Engineering Society General Meeting, 2003, IEEE*. Volume 2. IEEE. July 2003.

[Hal+14] R. Halvgaard, J. B. Jørgensen, N. K. Poulsen, and H. Madsen. *Model Predictive Control for Smart Energy Systems*. PhD Thesis. Technical University of Denmark, 2014.

[HEJ10] T. G. Hovgaard, K. Edlund, and J. B. Jørgensen. *The potential of Economic MPC for power management*. In: *Decision and Control (CDC), 2010 49th IEEE Conference on*. December 2010, pages 7533–7538.

[Hol+09] H. Holttinen, P. Meibom, A. Orths, F. van Hulle, B. Lange, M. O'Malley, J. Pierik, B. Ummels, J. Tande, A. Estanqueiro, M. Matos, E. Gomez, L. S'oder, G. Strbac, A. Shakoor, J. Ricardo, J. Smith, M. Milligan, and E. Ela. *Design and operation of power systems with large amounts of wind power: Final report, IEA WIND Task 25, Phase one 2006-2008*. VTT Tiedotteita - Research Notes 2493. VTT Technical Research Centre of Finland, 2009. ISBN: 978-951-38-7308-0.

[Hov13] T. G. Hovgaard. *Power Management for Energy Systems*. PhD Thesis. Technical University of Denmark, February 2013.

[Huu+10] J. K. Huusom, N. K. Poulsen, S. B. Jørgensen, and J. B. Jørgensen. *Tuning of methods for offset free MPC based on ARX model representations*. In: *American Control Conference (ACC), 2010*. June 2010, pages 2355–2360.

[IBM11] IBM Corporation Software. *Modeling Optimization Problems with IBM ILOG Cplex Optimization Studio*. Visited AUG 28, 2014. 2011. URL: https://www-950.ibm.com/events/wwe/grp/grp004.nsf/vLookupPDFs/1_ILOG_Modeling_Examples/$file/1_ILOG_Modeling_Examples.pdf.

[IBM14] IBM Corporation Software. *IBM ILOG CPLEX Optimization Studio*. Visited AUG 05, 2014. 2014. URL: http://www-03.ibm.com/software/products/en/ibmilogcpleoptistud/.

[Jen] M. Jensen. *DONG Energy - Power Hub*. Asset Manager. Visited DEC 3, 2014. URL: https://powerhub.dk/.

[JHR11] J. B. Jørgensen, J. K. Huusom, and J. B. Rawlings. *Finite Horizon MPC for Systems in Innovation Form*. In: *Proceeding of the 50th IEEE Conference on Decision and Control and European Control Conference*. IEEE, 2011, pages 1896–1903. ISBN: 978-1-61284-799-3.

[Jør] J. B. Jørgensen. *Smart Energy Systems*. Visited DEC 3, 2014. URL: http://www.imm.dtu.dk/~jbjo/smartenergy.html.

[Jør05] J. B. Jørgensen. *Moving Horizon Estimation and Control*. PhD thesis. Department of Chemical Engineering, Technical University of Denmark, 2005.

[Jør11] J. B. Jørgensen. *Constrained Predictive Control: A Computational Approach*. Springer, 2011.

[JYB14] J. J schke, X. Yang, and L. T. Biegler. *Fast economic model predictive control based on NLP-sensitivities*. In: *Journal of Process Control* 24.8 (2014). Economic nonlinear model predictive control, pages 1260–1272. ISSN: 0959-1524.

[KBP96] S. A. Kazarlis, A. G. Bakirtzis, and V. Petridis. *A genetic algorithm solution to the unit commitment problem*. In: *Power Systems, IEEE Transactions on* 11.1 (February 1996), pages 83–92. ISSN: 0885-8950.

[KSH00] T. Kailath, A. H. Sayed, and B. Hassibi. *Linear Estimation*. Prentice-Hall information and system sciences series. Prentice Hall, 2000. ISBN: 9780130224644.

[Luc+14] S. Lucia, J. A. E. Andersson, H. Brandt, M. Diehl, and S. Engell. *Handling uncertainty in economic nonlinear model predictive control: A comparative case study*. In: *Journal of Process Control* 24.8 (2014). Economic nonlinear model predictive control, pages 1247–1259. ISSN: 0959-1524.

[Mac02] J. M. Maciejowski. *Predictive Control with Constraints*. Prentice Hall, 2002. ISBN: 9780201398236.

[Mat] MathWorks. *MATLAB: Mathematical Computing Software for Engineers and Scientists*. URL: http://www.mathworks.com/.

[May+00] D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. M. Scokaert. *Constrained model predictive control: Stability and optimality*. In: *Automatica* 36.6 (2000), pages 789–814. ISSN: 0005-1098.

[MB02] K. R. Muske and T. A. Badgwell. *Disturbance modeling for offset-free linear model predictive control*. In: *Journal of Process Control* 12.5 (2002), pages 617–632. ISSN: 0959-1524.

[MD12] E. Ministry of Climate and B. to the Danish Parliament. *Energy policy report 2012*. Technical report. May 2012.

[MNG14] M. G. Marcovecchio, A. Q. Novais, and I. E. Grossmann. *Deterministic optimization of the thermal Unit Commitment problem: A Branch and Cut search*. In: *Computers & Chemical Engineering* 67 (2014), pages 53–68. ISSN: 0098-1354.

[Mor+14] J. M. Morales, A. J. Conejo, H. Madsen, P. Pinson, and M. Zugno. *Integrating Renewables in Electricity Markets: Operational Problems*. International Series in Operations Research and Management Science. Springer, 2014. ISBN: 978-1-4614-9410-2.

[MOS14] MOSEK ApS. *MOSEK Optimization Software*. Visited SEP 20, 2014. 2014. URL: http://www.mosek.com/products/mosek.

[NB13] T. Nielsen and A. Birke. *Solutions for small power systems*. Presentation by SEV and DONG. Visited AUG 11, 2014. January 2013. URL: http://www.twenties-project.eu/system/files/13%20Power%20Hub%20Demonstration%20day%20-%20Solutions%20for%20small%20power%20systems%20-%20Terji%20Nielsen,%20SEV%20%26%20Anders%20Birke,%20DONG%20Energy.pdf.

[ND09] C. Nielsen and H. Djurhuus. *SEV and Dong Energy put the Faroes on the world map in electrical energy*. SEV and DONG Energy joint venture agreement. Visited AUG 11, 2014. June 2009. URL: http://sev.net.dynamicweb.dk/Default.aspx?ID=1767.

[NKF09] T. Niknam, A. Khodaei, and F. Fallahi. *A new decomposition approach for the thermal unit commitment problem*. In: *Applied Energy* 86.9 (2009), pages 1667–1674. ISSN: 0306-2619.

[Nor] Nord Pool Spot. *The Power Market*. Visited DEC 3, 2014. URL: http://www.nordpoolspot.com/How-does-it-work/.

[NW06] J. Nocedal and S. J. Wright. *Numerical Optimization*. Springer Series in Operations Research and Financial Engineering. Springer New York, 2006. ISBN: 9780387303031.

[OAV12] J. Ostrowski, M. F. Anjos, and A. Vannelli. *Tight mixed integer linear programming formulations for the unit commitment problem*. In: *IEEE Transactions on Power Systems* 27.1 (2012), page 39.

[Off13] Officers of the World Energy Council. *World Energy Resources*. 2013. URL: http://www.worldenergy.org/wp-content/uploads/2013/09/Complete_WER_2013_Survey.pdf.

[Pad04] N. P. Padhy. *Unit commitment-a bibliographical survey*. In: *Power Systems, IEEE Transactions on* 19.2 (May 2004), pages 1196–1205. ISSN: 0885-8950.

[PJ08] G. Prasath and J. B. Jørgensen. *Model predictive control based on finite impulse response models*. In: *American Control Conference, 2008*. June 2008, pages 441–446.

[PSH09] W. F. Pickard, A. Q. Shen, and N. J. Hansing. *Parking the power: Strategies and physical limitations for bulk energy storage in supply demand matching on a grid whose input power is provided by intermittent sources*. In: *Renewable and Sustainable Energy Reviews* 13.8 (2009), pages 1934–1945. ISSN: 1364-0321.

[QB03] S. J. Qin and T. A. Badgwell. *A survey of industrial model predictive control technology*. In: *Control Engineering Practice* 11.7 (2003), pages 733–764. ISSN: 0967-0661.

[Raw+08] J. B. Rawlings, D. Bonn , J. B. Jørgensen, A. N. Venkat, and S. B. Jørgensen. *Unreachable Setpoints in Model Predictive Control*. In: *Ieee Transactions on Automatic Control* 53.9 (2008), pages 2209–2215. ISSN: 15582523, 00189286.

[RG91] R. Raman and I. E. Grossmann. *Relation between MILP modelling and logical inference for chemical process synthesis*. In: *Computers & Chemical Engineering* 15.2 (1991), pages 73–84. ISSN: 0098-1354.

[San] I. Sanne Wittrup. *6 MW wind turbines payback energy 33 times*. Danish article. Visited DEC 9, 2014. URL: http://ing.dk/artikel/6-mw-vindmoelle-betaler-sig-energimaessigt-tilbage-33-gange-172542.

[SK98] S. Sen and D. P. Kothari. *Optimal thermal generating unit commitment: a review*. In: *International Journal of Electrical Power & Energy Systems* 20.7 (1998), pages 443–451. ISSN: 0142-0615.

[SM71] F. C. Schweppe and S. K. Mitter. *Hierarchical System Theory and Electric Power Systems. Real-time Control of Electric Power Systems: Proceedings of the Symposium on Real-Time Control of Electric Power Systems*. Brown, Boveri & Company Limited, Baden Switzerland. 1971.

[Sok+12] L. E. Sokoler, K. Edlund, G. Frison, A. Skajaa, and J. B. Jørgensen. *Real-Time Optimization for Economic Model Predictive Control*. In: *10th European Workshop on Advanced Control and Diagnosis*. 2012.

[Sok+13] L. E. Sokoler, G. Frison, A. Skajaa, R. Halvgaard, and J. B. Jørgensen. *A Homogeneous and Self-Dual Interior-Point Linear Programming Algorithm for Economic Model Predictive Control*. In: *IEEE Transactions on Automatic Control* (2013).

[Sok+14] L. E. Sokoler, B. Dammann, H. Madsen, and J. B. Jørgensen. *A decomposition algorithm for Mean-Variance Economic Model Predictive Control of stochastic linear systems*. In: *Intelligent Control (ISIC), 2014 IEEE International Symposium on*. October 2014, pages 1086–1093.

[Sta+12] L. Standardi, K. Edlund, N. K. Poulsen, and J. B. Jørgensen. *A Dantzig-Wolfe Decomposition Algorithm for Linear Economic MPC of a Power Plant Portfolio*. In: *Proceedings of the 17th Nordic Process Control Workshop*. Technical University of Denmark, 2012, page 141. ISBN: 978-87-643-0946-1.

[Str08] W. A. Strauss. *Partial Differential Equations: An Introduction*. Second editions. Wiley, 2008. ISBN: 9780470054567.

[Wee+12] B. M. Weedy, B. J. Cory, N. Jenkins, J. Ekanayake, and G. Strbac. *Electric power systems*. John Wiley & Sons, 2012.

[Wei+13] D. Weißbach, G. Ruprecht, A. Huke, K. Czerski, S. Gottlieb, and A. Hussein. *Energy intensities, {EROIs} (energy returned on invested), and energy payback times of electricity generating power plants*. In: *Energy* 52 (2013), pages 210–221. ISSN: 0360-5442.

[Wik] Wikipedia. *Model predictive control*. Visited NOV 11, 2014. URL: http://en.wikipedia.org/wiki/Model_predictive_control.

[WW12] A. J. Wood and B. F. Wollenberg. *Power Generation, Operation, and Control*. Wiley, 2012. ISBN: 9781118585955.

[XI09] L. Xie and M. D. Ilić. *Model predictive economic/environmental dispatch of power systems with intermittent resources*. In: *Power Energy Society General Meeting, 2009. PES '09. IEEE*. July 2009, pages 1–6.

[ZB07] M. Zima and M. Bockarjova. *Operation, monitoring and control technology of power systems*. In: *EEH Power Systems Laboratory, ETH Zurich,[Online]. Available: http://www. eeh. ee. ethz. ch, Accessed March* (2007).

[ZGH10] E. Zondervan, I. E. Grossmann, and A. B. de Haan. *Energy optimization in the process industries: Unit Commitment at systems level*. In: *20th European Symposium on Computer Aided Process Engineering*. Edited by S. Pierucci and G. B. Ferraris. Volume 28. Computer Aided Chemical Engineering. Elsevier, 2010, pages 931–936.

[Zhu09] J. Zhu. *Optimization of Power System Operation*. IEEE Press Series on Power Engineering. Wiley, 2009. ISBN: 9780470466964.

[ZJM10] M. N. Zeilinger, C. N. Jones, and M. Morari. *Robust stability properties of soft constrained MPC*. In: *Decision and Control (CDC), 2010 49th IEEE Conference on*. December 2010, pages 5276–5282.