DTU Compute
Department of Applied Mathematics and Computer Science

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Master Thesis

Probabilistic forecasting and optimization in CHP systems

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Abstract

Denmark has committed towards increasing the wind power production to cover 50% of the power consumption by 2020. As the amount of wind by nature is uncertain, an integration of wind power into the current highly efficient combined heat and power (CHP) system, requires new flexible measures to reduce forced heat production in periods of high wind. Heat pumps (HP) and electric immersion boilers (EB) show excellent potential to increase flexibility and utilize excess power. The HP is more efficient but requires higher investments while not being as flexible as the EB.

As a consequence of decreasing taxes for electricity based heat production, HPs and EBs start to appear in district heating systems around Denmark. However, the operational strategy for these units is still unexplored, which has instigated the search for a structured operational strategy. As heat dispatch occurs before electricity prices are known, uncertainty is present. This impacts the operational costs for the HP and EB which both depend highly on the electricity prices.

This master thesis analyzes a CHP system in the Copenhagen district heating system in order to define an appropriate framework for integrating a HP and EB. An operational strategy for a HP and EB operating in a CHP system comprising a HP, EB CHP and storage is developed. This strategy is based on illustrative probabilistic forecasts of the heat demand and electricity price, used in a stochastic two-stage optimization model with recourse. Both the heat demand and electricity price are included as stochastic variables. Furthermore, it is assumed that a fixed amount of power is sold in the first stage decision. Thus, the second stage decision is used to adjust the production to meet the realized heat demand and power price in the most optimal manner. This constitutes a novel approach for the integration of HPs and EBs in a CHP system. Illustrative examples of the stochastic model and the deterministic equivalent confirm the working principles and appropriability of this approach to be used as an operational strategy.

Results from model simulations of four representative weeks during 2013 show a potential for economical benefits when a stochastic instead of a deterministic equivalent approach is used, especially during summer. This is due to the high degree of flexibility resulting from the HP, EB and storage. Decreasing the capacity of the HP and EB, the benefits of a stochastic approach increase.

Cases, analyzing the sensitivity to system changes and investment decisions, indicate a potential for substantial monetary benefits of HPs and EBs. In the event of decreasing electricity prices the impact of a HP and EB is found most significant. Moreover, increasing the efficiency of the HP leads to reduced heat costs while a reduction in HP and EB capacity yields significant additional costs.

This project thus successfully develops an operational strategy for a HP and EB in a CHP system, and results indicate substantial cost reduction resulting from the flexibility the HP and EB provide.

Preface

This master thesis is submitted as a partial fulfillment of the requirements for obtaining the Master of Science in Engineering degree in Management Engineering at the Technical University of Denmark (DTU). It corresponds to 30 ECTS point and has been carried out from February 3^{rd} to July 3^{rd} 2014 in the Dynamical Systems (DynSys) research group at DTU Compute - Department of Applied Mathematics and Computer Science.

Furthermore, this master thesis constitutes part of the work done in the Cities project [1,2] lead by Prof. Henrik Madsen.

The thesis is supervised by Assoc. Prof. Juan Miguel Morales González, Postdoc Marco Zugno and Prof Henrik Madsen (Head of Section), all from the DynSys group. Furthermore, Jørgen Boldt, HOFOR, Thomas Engberg, COWI, and Henrik Aalborg Nielsen, ENFOR, supervised the project as external supervisors.

I would like to thank all of my supervisors for their valuable inputs and guidance throughout the project. A special thanks to Juan Miguel and Marco for their dedication and support during the last few months, and to Henrik for keeping the overview in this multidisciplinary project.

Moreover, Jørgen Boldt, Jane G. Nielsen, Henrik Damgaard and the planning department at HOFOR deserves a special thanks for their hospitality and helpfull discussions. Also a thanks to Pierre-Julien Trombe, Dorthe Rosenbak Andersen, Lars Grønnegaard and Lene Sommer for their support.

Finally, a special thanks to my boyfriend for supporting my dedication to this project during the last five months, and for always being available for discussions concerning the project.

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________________________________ 3. July 2014

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Contents

Abbreviations and concepts

ACF Autocorrelation function

CHP Combined heat and power

COP Coefficient of performance

CTR District heating transmission operator in Greater Copenhagen

DONG Energy Combined heat and power production supplier

EB Electric immersion boiler

Elbas market Intra-day market for trading of electricity

Elspot market Day-ahead market for trading of electricity

Energinet.dk Transmission operator in Denmark

FDR Frequency controlled disturbance reserve

FNR Frequency controlled normal operation reserve

HOFOR District heating distribution company in Copenhagen

HOFOR Kraftvarme Combined heat and power production company, former Vattenfall

HP Heat pump

Nord Pool Spot Company managing the Nordic power market

PACF Partial autocorrelation function

Spot price Hourly electricity price resulting from the Elspot market

TSO Transmission system operator

Varmelast.dk Responsible of the daily heat dispatch in Copenhagen. Consists of one employee from VEKS, CTR and HOFOR

VEKS District heating transmission operator in Greater Copenhagen

Chapter 1

Introduction

1.1 Motivation

Denmark has, as part of an European agreement, committed to pursue a 100% supply of renewable energy by 2050. To fulfill this goal, it has been decided that the heat and power supply should be completely renewable by 2035. Furthermore, by 2020 50% of the consumed electricity should consist of wind power [3].

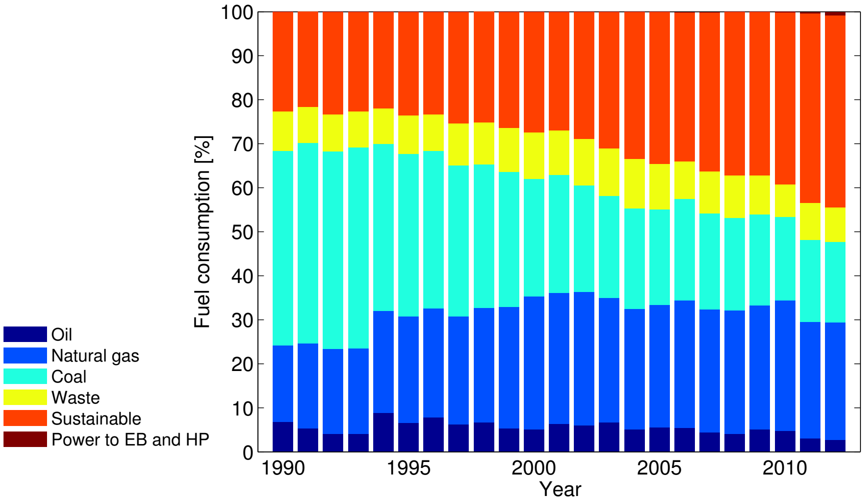


Figure 1.1 – Disitribution of fuels used for district heating in Denmark from 1990 to 2012.

Figure 1.1 shows historical development of distribution of fuels used for district heating in Denmark in the years from 1990 to 2012 [4]. Non-renewable energy sources, such as oil and coal currently have a significant share in the Danish heat and power production. Traditionally, these non-renewable sources have been widely used for generating heat and power, through the use of highly efficient combined heat and power (CHP) plants. However, an increasing number of plants are being converted to, the more sustainable alternative, biomass. However, due to the simultaneous production of heat and power as well as the operation restrictions, CHP plants are not very flexible.

The increasing share of wind power that is expected in the future will not provide additional flexibility. Contrarily, the unpredictable behavior of wind and the seasonal and daily variations, which inherently arise from using wind power, reduce the flexibility. Moreover, this means that wind power cannot satisfy power demand alone. Consequently, wind power requires integration into a flexible system that can supply power in the case of low wind. However, wind power has the advantage of having zero marginal costs as well as being a sustainable power production method.

Another challenge from using wind power is the excess production that can occur on days with high wind and low power demand. Currently, this results in curtailment or very low electricity prices in the Nordic electricity market [5]. The political decisions stated earlier will lead to a significant increase of wind power in the coming years, which will increase the occurrence of excess power production. The problem complicates even further when considering the heat demand[1] in periods of high wind power production. Traditionally CHP plants are used to satisfy the heat demand but if electricity is no longer needed, the economic gain of high efficiency co-production at CHP units vanish. A solution to this problem, contradicting the political goal for 2020, could be wind power curtailment to allow for CHP production. A second alternative is the use of traditional oil or coal fueled heat boilers with high emission and low efficiency. In addition to extensive increases in heat costs, this also contradict the political emission goals.

Clearly, there is a need of other and more sustainable ways of integrating wind power and decrease heat demand driven power production at CHP plants. Hence, heat pumps (HP) and electric immersion boilers (EB) become very interesting as they present a way of increasing flexibility.

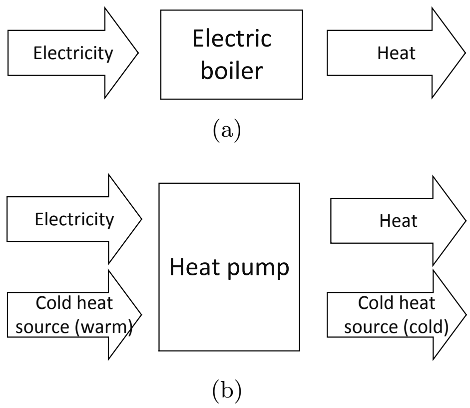


Figure 1.2 – Simple illustration of an EB (a) and a HP (b).

The basic principles of an EB and a HP are displayed in Figure 1.2. Electricity is used as input and is by the unit converted to heat. The HP also utilizes energy from an additional colder heat source such as waste water, sea or air, as outlined in Figure 1.2(b). These two units improve the system from two angles. First, heat production happens without a simultaneous power production and second, electricity is used as fuel such that excess, low cost, electricity is reduced.

Both HPs and EBs have started to appear during the last decade in district heating systems in Denmark, but due to high taxes the profitability has been limited [6]. In 2013 a significant

[1]The terms heat demand and heat load will be used interchangeably.

tax reduction was decided for this specific type of production technology, favoring especially the HP which is more efficient than the EB [7].

For HPs and EBs to provide the desired flexibility a number of issues must be addressed. The framework in which they should be integrated must be defined based on a thorough analysis. This should be followed by the development of an operational strategy in order to secure the optimal operation of the unit. These are all issues addressed in this project.

1.2 Project objective

The main challenge in this project is to identify, analyze and evaluate a mathematical optimization model providing an operational strategy for a HP and EB in a CHP system supplying the Greater Copenhagen district heating system. Furthermore, this project wish to analyze the benefits of using probabilistic forecasting and stochastic optimization for the chosen system as well as assess the monetary benefits of HPs and EBs in a CHP system.

To be able to develop a realistic and appropriate model, the framework in which the HP and EB operate must be analyzed. The suitable organizational and physical location should be analyzed to find the most optimal configuration. The relevant markets and the corresponding decisions should be identified in order to analyze the consequences these might have for the operational strategy of a HP and EB. Based on these analyses, a relevant CHP system comprising the HP and EB can be modelled. Probabilistic forecasting allows for a stochastic optimization of the modelled system. This will provide the principles of an appropriate operational strategy for an EB and HP. Optimizing using both a deterministic and stochastic model set-up will allow for a comparison to illustrate the potential benefits of a stochastic approach.

1.3 Literature review

The field of combined heat and power production has increasingly received attention during the past decades. Multiple CHP plants have been constructed and due to the complexity of co-generating heat and power, which subsequently are sold in different markets, a need for mathematical optimization models arose. Examples of such studies are found in [8] and [9] but multiple others exist. These models generally tend to be deterministic. With the increasing focus on integrating wind power, which in nature is highly uncertain, a number of papers start to introduce stochastic optimization for the planning of CHP production, heat dispatch, and bidding in the electricity market [10–12]. As an example *Zugno et al.* [13], use robust optimization to model a CHP system that treat both the day-ahead and real-time heat dispatch.

Related to industrial HPs and EBs the research available is very sparse. However, a few specific instances are modelled. In [14], *Blarke et al.*, model a system comprising a HP that utilizes flue gas from a CHP. Both a cold and a hot storage tank are used for storage of flue gas and heat, respectively. This increases the flexibility such that the HP can operate concurrent with the CHP. The model is linear and deterministic, which means that the heat demand and electricity prices are considered known.

A recent study [15], analyzes the potential for HP to utilize waste heat from industrial facilities. Furthermore a study on how to introduce HPs is also presented in [16], however, with a focus on the thermodynamic properties of HPs.

Most relevant to this project is the work by *Meibom et al.* [17]. A stochastic set-up is here used in modelling a system comprising wind power, HP and EB production. The paper investigates and compares the impact of HPs and EBs for wind power integration in different configurations. Only the wind power is considered stochastic. It was here found beneficial to introduce HPs and EBs to decrease curtailed wind power and costs for regulating power. Especially, in the case of the marginal heat production costs being high, such as when using oil or gas fueled boilers, good results were obtained. The analysis was only carried out for a specific short period in February, where the wind power production usually fluctuates much and thus a high benefit from introducing a HP and EB would be expected in this period.

Finally, a number of internal documents and analysis has been made by HOFOR, estimating the investment potential and the different options for choice of HPs [18] [19]. The deterministic analysis tool, Balmorel, which models the entire Greater Copenhagen district heating system including the Nordic power market, is generally used for investment analysis as it provides long-term information on an aggregated level [20]. Simulations including HPs have been modelled, deterministically using Balmorel but merely for investigating future scenarios and the economic impact of including HPs.

Generally, none of the above presented research provide decision support for the daily operation of a HP and an EB in a CHP system. Neither is stochastic approaches found for models optimizing the daily operation.

1.4 Research contribution

In relation to the above section this work aims to model and optimize the daily operation of a system comprising both CHP production, storage, a HP and an EB. This has not previously been reported in the literature. Furthermore, a stochastic optimization model approach is developed, using probabilistic forecasts to represent a stochastic spot price and heat demand. This constitutes, to the best of my knowledge, a novel approach for the optimization of systems including HPs and EBs.

1.5 Thesis Outline

The following list provides an overview of the entire thesis, in short, describing the contents of each chapter:

Chapter 2 presents the general principles of a CHP system as well as characteristics and functionalities of a HP and EB including the heat costs for different production units. The Nordic power market and the Copenhagen district heating system is furthermore outlined.

Chapter 3 presents an analysis of the framework and management issues relevant for the introduction of HPs and EBs in the Copenhagen district heating system. Furthermore, the layout for an operational strategy is presented and used to decide how the decision making process can be modelled.

Chapter 4 presents the developed deterministic and stochastic optimization models for the operation of an EB and HP in the CHP system.

Chapter 5 presents a method for probabilistic forecasting of the electricity price and heat demand to be used as an input to the deterministic and the stochastic model.

Chapter 6 presents illustrative results from solving both the deterministic and the stochastic model and additionally analyze the model sensitivity to several parameters.

Chapter 7 presents estimates on yearly monetary benefits from the stochastic modelling approach compared to the deterministic equivalent. Results from three case studies are presented and the impact of introducing an EB and HP discussed.

Chapter 8 will conclude on the thesis as well as give a number of suggestions for interesting studies for the future.

Chapter 2

Combined heat and power systems

This chapter presents the components of a CHP system. These include heat and power production technologies with focus on CHP plants as well as HPs and EBs. A comparison of the heat costs and their dependence of the spot price is presented, as well as the influence of taxes and fees. Moreover, the relevant electricity markets in Eastern Denmark will be outlined. These are in short; the Nordic power market, which constitutes a powerful platform for trading of power at variable prices; the regulating market for balancing production and consumption; ancillary services bought by Energinet.dk to ensure adequate capacity for frequency deviations and disruptions. Subsequently, the Copenhagen district heating system is outlined. The procedure for the daily heat dispatch is presented together with the corresponding decision making process for both the heat supplier and distributor.

2.1 Heat and power production

Several options exist for producing heat, both in terms of technology and fuel. Among the most common in Denmark are waste incinerators, CHP plants and heat only boilers. However, other technologies such as EBs and HPs are emerging and have attracted more attention during the last years. This is, among other, due to an increased focus on sustainable production of heat, as well as the uncertain future for prices and taxes on fuel and electricity.

The following sections outline the most common methods for heat production in Greater Copenhagen. The CHP plant at Amagerværket is used as an example when describing the CHP units. As these are all well known technologies, only the general operating principles will be outlined. The main focus is instead on the operation of HPs and EBs and their mutual differences.

2.1.1 Heat only boiler

Ordinary heat boilers only produce heat. This is either in form of hot water or steam, as there are still areas in Copenhagen supplied by steam. Heat boiler are usually are fueled with oil or gas. They do not have the advantage of co-generating heat and power and are consequently less efficient overall. A low efficiency and high tax usually make boilers the least favorable choice for heat production, and often they are only used as backup or during peak load periods during the winter. In relation to combined heat and power production, the heat boiler is simple as operating costs are independent of electricity prices. Due to the high costs of the heat boiler and the unfavorable production, this production unit will not be given further attention in this project.

2.1.2 CHP production

In Denmark, centralized heat production is based on CHP plants. By producing combined heat and power, very high total energy efficiencies are obtained which generally makes CHP production the preferred and most widely used option for heat production in centralized areas such as Greater Copenhagen. In addition to waste incinerators one mainly distinguish between two types of CHP production namely back-pressure CHP and extraction CHP production. Each production type have specific production characteristics elaborated in the following.

Back-pressure unit

The operating principle for a back-pressure CHP is illustrated in Figure 2.1. In the boiler, water is heated to steam which is sent through the turbine. The turbine runs a generator which allows for electricity production. Not all energy in the turbine is utilized and the output from the turbine can be used for district heating.

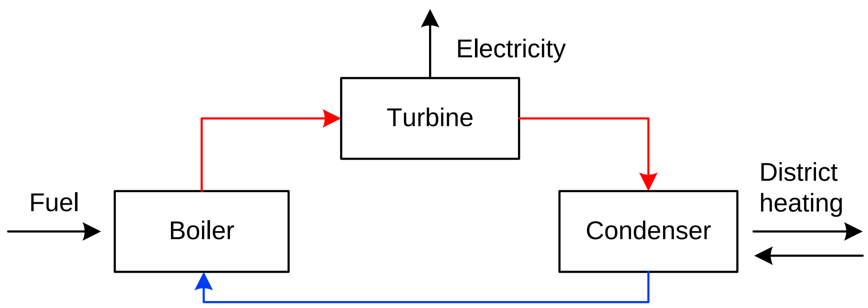


Figure 2.1 – Operating principles for a back-pressure unit.

The back-pressure unit operate with a fixed power to heat ratio, c_b as displayed in Figure 2.2. This decreases the flexibility and in the case of heat production from this unit, there will unavoidably be a power production.

Extraction unit

Figure 2.3 outlines the operating principles for an extraction CHP. Similar to the back-pressure unit a boiler heats water to steam which is transported through a multi-stage turbine. This allows for utilization of steam of lower pressure in which the resulting heat is too cold for district heating. However, steam for district heating can be extracted in the turbine. The extraction unit is therefore more flexible by allowing a variable heat to power ratio. The relationship between heat and power production can approximately be

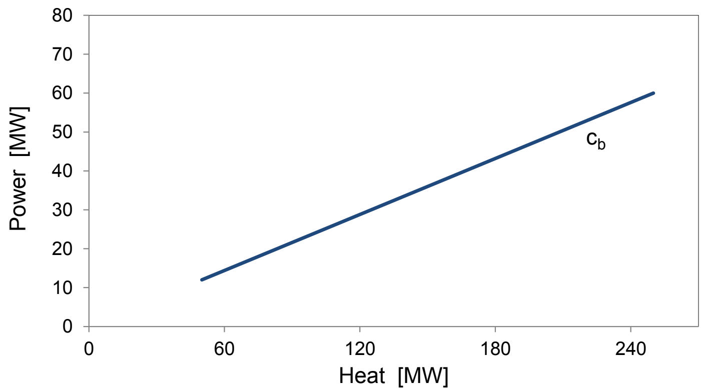


Figure 2.2 – Heat and power production ratio for a back-pressure CHP unit. A fixed ratio, c_b, applies together with a minimum production.

characterized by the operating lines in Figure 2.4. c_b is the power to heat ratio in back-pressure operation whereas c_v is the reduction in power production corresponding to a unit increase in heat.

Furthermore, the figure shows the fuel consumption along different production strategies. Each of the dashed lines represents a constant fuel consumption. This means that using a fixed amount of fuel, the CHP can produce e.g. 250 MWh electricity and no heat, or 211 MWh electricity and 330 MWh heat. This clearly shows that the most efficient production is at the right most point of the line corresponding to the chosen fuel consumption. However, this unit provides the opportunity of solely producing electricity even though this totally results in a less efficient production configuration.

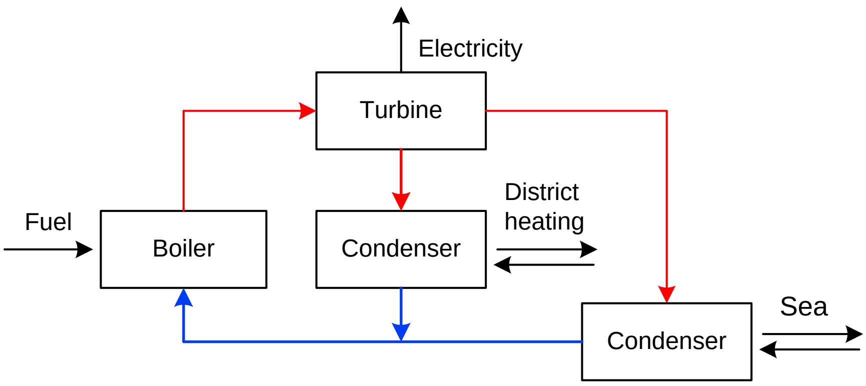


Figure 2.3 – Operating principles for an extraction unit.

Different types of fuel is used for CHP production. Since the oil crisis in the 1970's, coal has generally been the most widely used option due to the low and stable price [21]. However, taxes on coal are increasing drastically while other more sustainable alternatives, such as biomass, have been excluded from taxes to give an incentive to increased production using this type of fuel. Some units, waste incinerators, use waste e.g household waste as fuel. In the Greater Copenhagen district heating system these units are given priority for heat production which makes the unit less interesting for optimization purposes. These will therefore not be addressed further.

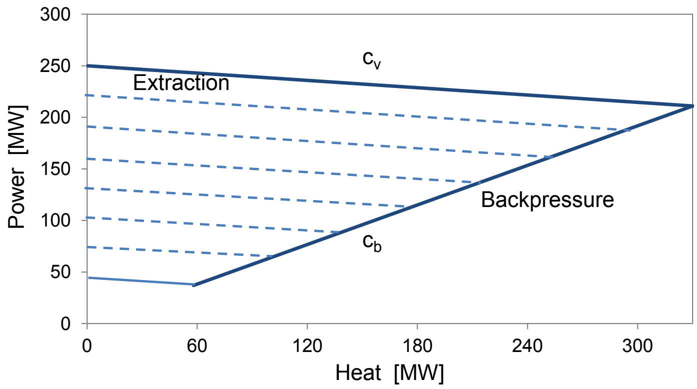


Figure 2.4 – Production of heat and power from an extraction CHP. All combinations within the solid lines are valid. Each declining blue line is comprised of operating points with a constant fuel consumption. The optimal production point is hence the right-most point.

2.1.3 Heat from electricity

The process of generating heat from electricity is expected to have a significant impact in the coming years' energy supply [22]. This is due to the expected increase in wind power production that will result in an increasing number of hours of excess power production, and thus low electricity prices. Even though the methods for producing heat from electricity have previously been considered less economical due to the general price and tax level for electricity, it allows for a separate production of heat without co-production of electricity. Another reason is the uncertain future for biomass fueled CHP production, especially if biomass become a scarce resource for sustainable heat and power production.

Two known methods to produce heat from electricity using an EB and a HP. Both have different advantages and disadvantages, potentially making them suitable in different situations. The following will give a brief overview of the two methods, including the mutual differences and the integration potential.

Electric heat boiler

The EB is a simple technology that converts electrical power into thermal power with an efficiency of approximately 1. The principle is illustrated in Figure 2.5(b) and the corresponding electrical diagram is shown in Figure 2.5(a).

EBs have the advantage of being very flexible. The unit is capable of starting up in a few seconds and up and down regulate the production with similar speed only with marginal loses in efficiency. No fuel feeding system or stack is required as electricity is the only source. Furthermore, EBs are based on a well developed and tested technology involving no complex components [23]. This makes it extremely reliable and easy to maintain. Already existing EBs typically have capacities spanning 1-25 MW, while larger capacities are obtained by coupling of units. EBs are commercially available and they are considered a cheap investment with prices around 0.15 mio € per MW for small EBs and decreasing unit costs for larger EBs [23, 24]. However, EBs have the disadvantage of being completely dependent on electricity and thus the electricity prices. The operational costs therefore vary with the variable electricity prices which together wit taxes generally has been too high for the EB

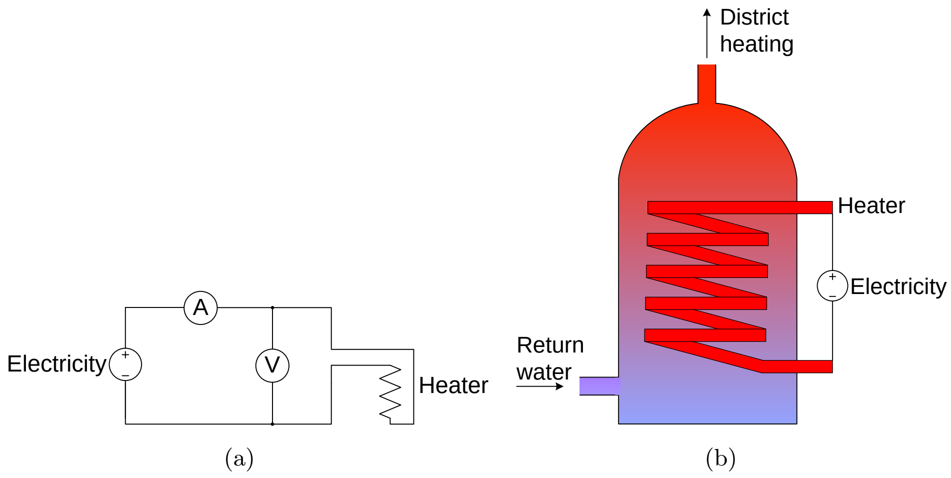


Figure 2.5 – (a) Circuit diagram for an EB. (b) Illustrative example of an EB providing heat for district heating.

to be very profitable.

Heat pump

Heat flows naturally from a higher to a lower temperature. However, HPs are able to force the heat flow in the other direction, using a relatively small amount of drive energy such as electricity, fuel, or high-temperature waste heat. The focus will here be on electricity driven HPs.

The principle of a HP is identical to that of a reverse refrigerator. For HPs, the heat that is extracted from the "refrigerator" is the interesting part. Figure 2.6 illustrates the working principle.

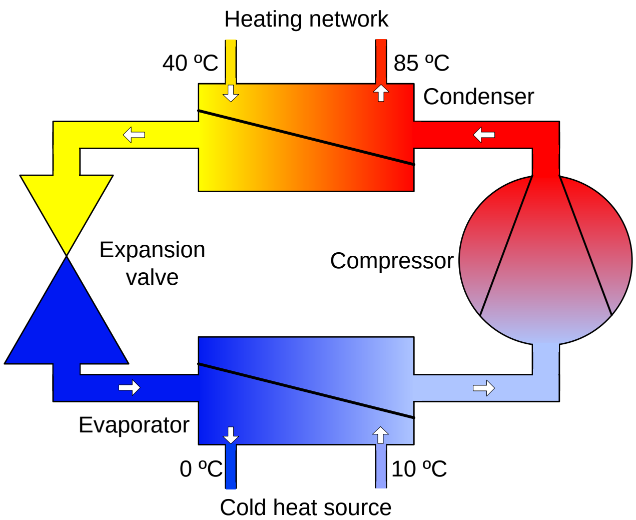


Figure 2.6 – Diagram of a HP. In the compressor the temperature of the refrigerant is increased by compression which is subsequently exhanged with water to be heated in the condenser. An expansion valve descrease the pressure and the cycle continues.

Energy from the cold source is transported to the heating network by a refrigerant, which has specific thermodynamic properties. At the evaporator the refrigerant absorbs heat and vaporizes. Subsequently, the refrigerant is compressed to increase the temperature. The compressor is driven by an electrical motor which is the main part to consume electricity. In the condenser the refrigerant is cooled such that it condenses and release heat to the heating network (district heating). Finally, the expansion valve lowers the pressure and the cycle starts again.

Several options exists for the cold heat source: Air, sea water, waste water and geothermal energy are examples of some of the most frequently used. The choice of cold heat source reflects the stability and performance of the HP. If air is used, and the air temperature varies significantly during the year, the performance will vary accordingly and possibly lead to an unstable system [24]. This argues for use of geothermal heat, sea water or waste water as less variation is found for these sources.

The most commonly applied refrigerant is currently ammonia (NH_3). However CO_2 is also starting to be applied due to superior abilities to extract heat from cold sources below $\approx$ 20 °C and its ability to provide high condensing temperatures.

The efficiency of the HP varies depending on the temperature requirements. The coefficient of performance (COP) describe the ratio between heat output and electricity input. The theoretical COP for a HP is calculated based on the Carnot efficiency [25]:

$$COP_{carnot} = \frac{T_h}{T_h - T_l}$$

where T_h is the supply temperature and T_l is the temperature of the cold medium both in K. If a waste water temperature of approximately 10 °C (283 K) and an output water temperature of 85 °C (358 K) is assumed, the resulting Carnot COP is:

$$COP_{Carnot} = \frac{358\text{K}}{358\text{K} - 283\text{K}} = 4.8$$

If geothermal water is used instead, the cold medium temperature would be around 50 °C (323 K) [18], resulting in a much higher efficiency of:

$$COP_{Carnot} = \frac{358\text{K}}{358\text{K} - 323\text{K}} = 10.2$$

It should be emphasized that these are theoretical maximum efficiencies. In reality it has been found that the efficiency is approximately 50-70% of the Carnot efficiency [18]. The COP for HPs are therefore in reality typically between 2 and 5, even though higher values can be obtained. In addition to the temperature, several other factors such as the compressor efficiency and choice of refrigerants also affects the COP.

Just as the EB, the HP is a flexible solution for separating heat and power production. It has a high efficiency and is comparably less dependent on the electricity price. The HP can utilize heat from otherwise wasted sources such as waste water, sea water or geothermal heat. However, due to the complex structure of HPs they require extensive investments with long pay back times. The prices are approximately 0.5 mio € per MW output, and furthermore, maintenance costs should also included [23].

Compared to the EB, the HP is not as flexible in terms of ramping during start-up and shut-down. Figure 2.7 illustrates this issue simply. For the EB, start-up occurs almost instantaneously in a matter of seconds. The HP is slower, when starting up compared to the EB. A CHP unit is generally less flexible and slow compared to both the HP and EB, as illustrated in Figure 2.7. Furthermore, a slightly higher production often occurs depending on the engineer operator the unit[1].

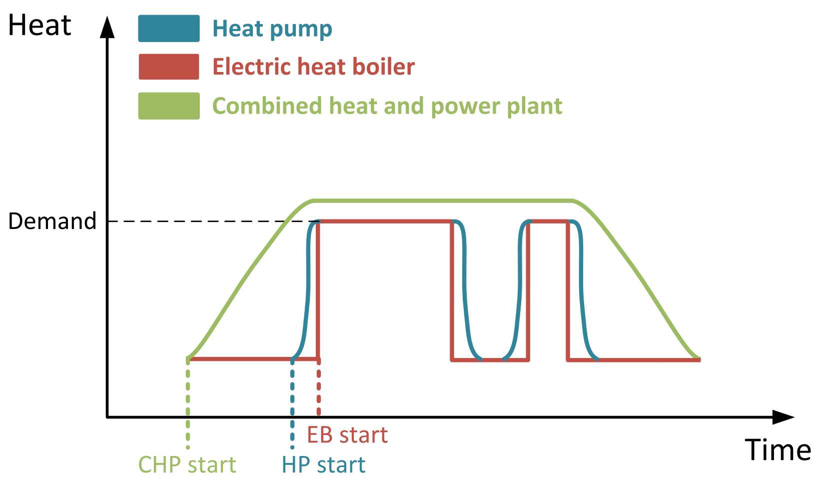


Figure 2.7 – Ramping principles for a HP, EB and a CHP illustrating the difference between the three units.

The operation of CHPs, HPs and EBs have now been outlined allowing for the operational costs for heat production on such units to be presented. This is the subject of the next section. This will provide an intuitive understanding of the impact of electricity prices on the optimal choice of heat production unit. Taxes on heat production induce significant changes to the marginal heat costs and consequently the next section will present the relevant taxes and fees imposed on heat produced by CHP, HP and EB units.

2.2 Heat cost comparison

Marginal heat production costs can be calculated for both the EB, HP and the two types of CHP units, based on the knowledge obtained in Section 2.1. In the following the back-pressure CHP will be denoted "CHP", and the extraction CHP, "CHP2". It is assumed that the back-pressure unit (CHP) is biomass fueled and that the extraction unit (CHP2) is fueled with coal as this resembles the production at one of the large CHP plants in Copenhagen, Amagerværket.

2.2.1 Marginal heat production costs

Initially, the heat production costs are calculated without the addition of taxes and fees. Subsequently, taxes and fees that apply will be outlined, and the changes it induce will be illustrated.

The EB is very simple as is only consumes electricity. As the price of electricity varies the heat costs as a function of the electricity price is found. Thus, the marginal heat cost, c_t^{EB},

[1]Oral conversation with H. Damgaard, Energy Planner, HOFOR.

is found as:

$$c_t^{EB} = p_t^{spot} \tag{2.1}$$

where p_t^{spot} is the electricity price at time t. The marginal cost for the HP, c_t^{HP}, is calculated similarly, including the COP, COP^{HP}.

$$c_t^{HP} = \frac{1}{COP^{HP}} p_t^{spot} \tag{2.2}$$

The two CHP units have an electricity production which is sold. This is reflected in the marginal heat costs. The back-pressure CHP has a marginal cost, c_t^{CHP}, determined by:

$$c_t^{CHP} = \frac{1}{\eta^{CHP}} \left(1 + cb^{CHP}\right) c^{f,bio} - cb^{CHP} p_t^{spot} \tag{2.3}$$

Here, η^{CHP} is the total efficiency of the unit, $c^{f,bio}$ is the cost of biomass, and cb^{CHP} is the power to heat ratio corresponding to the slope in Figure 2.2.

The extraction CHP2 has two operational possibilities. First, is the operation in back-pressure mode (see Figure 2.4) with heat cost, $c_{t,back-pres.}^{CHP2}$, of:

$$c_{t,back-pres.}^{CHP2} = \frac{1}{\eta^{CHP2}} \left(1 + cb^{CHP2}\right) c^{f,coal} - cb^{CHP2} p_t^{spot} \tag{2.4}$$

where $c^{f,coal}$ is the cost of coal, cb^{HP} is the power to heat ratio and η^{CHP2} is the total efficiency of the CHP2. Thus, the first term represents additional fuel costs while the second subtracts the turnover from selling power. Alternatively, it can operate in extraction mode and increase the heat production while decreasing the power production at a rate, cv^{CHP2}. The costs, $c_{t,extrac.}^{CHP2}$, are here a matter of the opportunity cost for lost power sales.

$$c_{t,extrac.}^{CHP2} = cv^{CHP2} p_t^{spot} \tag{2.5}$$

Using the values presented in Table 2.1, the marginal heat costs are calculated and displayed in Figure 2.8, illustrating the unit heat costs as a function of the power price.

Parameter	**Value**	**Explanation**
$c^{f,bio}$	40 DKK/GJ	Fuel costs for biomass
$c^{f,coal}$	20 DKK/GJ	Fuel costs for coal
cb^{CHP}	0.24	Power to heat ratio for CHP
cb^{CHP2}	0.64	Power to heat ratio for CHP2 in back-pressure
cv^{CHP2}	-0.12	Power to heat ratio for CHP2 in extraction
η^{CHP}	1.1	Total fuel efficiency for the CHP unit[2]
η^{CHP2}	0.9	Total fuel efficiency for the CHP2 unit
η^{HP}	3	COP of HP

Table 2.1 – Parameters for a HP, a back-pressure and an extraction unit.

[2] A fuel efficiency above 1 is reached due to flue gas condensations which is not officially included in the energy contents of the fuel.

The behavior of the extraction CHP2 is illustrated with two different functions for two different regimes. The first represents the costs in back-pressure operation mode described by (2.4) which will occur when the power prices are low as the electricity production is comparably low in this mode. At the intersection of the back-pressure and extraction mode cost functions in (2.4) and (2.5), the prices are high enough for CHP2 to optimally operate in extraction mode. Thus, for this price and upwards the heat costs are based on the extraction mode.

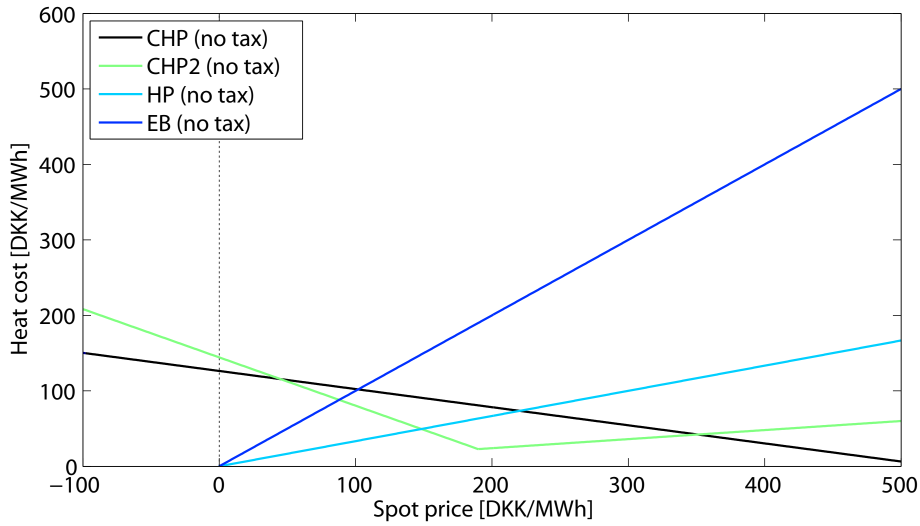


Figure 2.8 – Marginal heat costs, excluding taxes and fees, as a function of the electricity price for different production units.

When taxes are not included, the HP is generally favorable for electricity price below 150 DKK/MWh compared to the other units. At this point the extraction CHP2 becomes more economic. Only at high electricity prices, above 350 DKK/MWh, the back-pressure CHP is favorable. The EB generally has higher marginal heat cost compared to the HP and can only compete with the two CHP units when the electricity price is lower than ≈100 DKK/MWh.

The following section will introduce the taxes and fees that apply to HPs, EBs and CHPs in order to provide a realistic view of the heat costs and dependency of taxes and fees.

2.2.2 Taxes and fees on heat production

The tax system for heat and power production and consumption is complex and suffers from constant changes and amendments. These are made to accommodate changes in environmental goals, technology development, resource availability etc. The increased focus on sustainable production of heat and power has resulted in increasing taxes on coal compared to biomass, which is currently exempted from most taxes. Even though coal as a resource only is half the price of biomass, biomass production is significantly less costly compared to coal production when taxes are included. In a similar way, taxes have recently started to favor electricity based heat production, such as heat from EBs and HPs. In addition to regular taxes there are also a number of fees for consumption of electricity. Taxes and fees can account for more than 50% of the production costs which makes it important to address them properly.

Heat production at CHP plants

Electricity production does generally not impose any taxes. Due to the liberal nature of the Nordic power market, tax applies at the consumer level. Heat production is, on the contrary, taxed at the production stage. For CHP production, only the fuel corresponding to the heat production is taxed. This is not a one-to-one relationship as heat is produced more efficiently on a CHP compared to power. Depending on the production, the fuel to be taxed is calculated from either the electricity or heat production. For the two CHP units that are analyzed here, the tax method based on heat production is used, and the taxed heat production is found as [26]:

$$y^{tax} = \frac{y^{prod}}{1.2}$$

where y^{prod} is the heat production. Depending on the fuel used to produce heat, the size of the tax vary. The tax on coal is specified in Kulafgiftsloven and for 2014 it is [27]:

$$c^{tax,coal} = 258.5\,\mathrm{DKK/MWh}$$

As mentioned, biomass used for CHP produced heat is currently not taxed with regular fuel tax. Power produced at CHPs fueled with biomass receives a 150 DKK/MWh supplement to promote this form of production even further [28].

In addition to the regular fuel tax, tax for emission of carbon dioxide (CO_2), nitrogen oxide (NO_x) and sulphur oxides (SO_x) exists. However, carbon dioxide tax is not imposed on biomass production as opposed to coal based production. The price is typically [29]:

$$c^{tax,CO2} = 57\,\mathrm{DKK/MWh}$$

Finally, nitrogen oxide tax is also included for heat produced by a coal fueled CHP; however, being less significant [30]:

$$c^{tax,NOx} = 9\,\mathrm{DKK/MWh}$$

The sulphur oxide tax is below 1 DKK/MWh and is therefore considered negligible in these studies.

Heat production at HPs and EBs

HPs and EBs generally have both taxes and fees, some of which only applies in certain situations. In addition to the electricity price the EB and HP generally have costs for

1. Transmission and distribution [219 DKK/MWh power]
2. PSO[3] [190 DKK/MWh]
3. Tax

[3]PSO (public service obligation) is a tax paid to support environmentally friendly power production such as wind power production.

Parameter	Value	Explanation
$c^{tax,coal}$	258.5 DKK/MWh fuel	Tax on coal
$p^{bio,sup}$	150 DKK/MWh power	Supplement for biomass produced power
$c^{tax,el}$	412 DKK/MWh power	Tax on electricity consumption by an EB or HP
$c^{tax,heat}$	263 DKK/MWh heat	Tax on heat production for units covered by Elpatronloven [31] (E.A.4.2.9)
$c^{tariff,net}$	219 DKK/MWh heat	Tax/fee for electricity distribution
$c^{tax,CO2}$	57 DKK/MWh heat	Carbon dioxide tax
$c^{tax,NOx}$	9 DKK/MWh heat	Nitrogen oxide tax

Table 2.2 – Current taxes on heat production from CHPs, HPs and EBs.

However, in Elforsyningsloven §9a, it is stated that a company producing district heating is not obliged to pay PSO.

Producers using HPs and EBs can under specific circumstances choose between paying either tax of the electricity consumption or the heat production. The electricity tax can always be applied, and for tax registered companies there is a newly introduced reduction of this tax for electricity driven heat production such as with EBs and HPs. Previously, this tax amounts to 833 DKK/MWh. However, the reduction decreases this to 412 DKK/MWh.

Under certain conditions a HP and EB can be considered under the law for EBs (Elpatronloven) [31] (E.A.4.2.9). This requires the HP and EB to be part of a CHP system or owned by a heat or CHP producing company. In this case tax is only paid for the heat production, which amounts to 263 DKK/MWh heat, comparable to the taxes for a heat only boiler. However, for production units having a high electricity to heat ratio this is not favorable. Comparing to the electricity tax of 412 DKK/MWh, a unit with a COP higher than $412/263 = 1.6$, Elpatronloven should not be applied. Instead the regular electricity tax (412 DKK/MWh) should be used as this will be economically most favorable. Generally, this means that EBs, which have a COP of 1, if possible should follow Elpatronloven, and pay tax based on heat output. HPs, with a COP higher than 1.6, should on the contrary choose to pay the electricity tax instead.

The tax can in certain situations also be removed completely. According to [32] the HP and EB production is not taxed if the units are directly connected to, and supplied by, a CHP unit. The connection should be internal, such that it could be considered internal consumption. Finally, it can also be assumed that the transmission and distribution tariff does not apply if the EB or HP unit is located and internally connected to the CHP from which it receives electricity[4].

The taxes, just explained including the current value are summarized in Table 2.2.

Heat costs including tax

With the addition of taxes outlined in the previous paragraphs, the heat costs for the production units, found in (2.1)-(2.5) are now updated such to include taxes and fees.

[4]Oral discussion with T. Engberg, Chief Project and Market Manager, COWI.

Assuming that the EB is not connected directly to a power producing unit, it has to pay both the net tariff as well as heat tax:

$$c_t^{EB} = p_t^{spot} + c^{tax,h} + c^{tariff,net}, \tag{2.6}$$

where $c^{tax,h}$ is the tax on the heat production for units covered by Elpatronloven and $c^{tariff,net}$ is the electricity distribution tariff.

The HP is assumed to pay the electricity tax just described. This leads to the cost being described by:

$$c_t^{HP} = \frac{1}{COP^{HP}} p_t^{spot} + \frac{1}{COP^{HP}} \left(c^{tax,el} + c^{tariff,net} \right) \tag{2.7}$$

where $c^{tax,el}$ is the electricity tax that applies to the electricity consumption.

The back-pressure CHP costs, including taxes and supplements are described by:

$$c_t^{CHP} = \frac{1}{\eta^{CHP}} \left(1 + cb^{CHP}\right) \left(c^{f,bio} + c^{tax,NOx} \right) - cb^{CHP} \left(p_t^{spot} + p^{bio,sup} \right) \tag{2.8}$$

For the extraction CHP the cost, including taxes, when operating in back-pressure mode becomes:

$$c_{t,back-pres.}^{CHP2} = \left(1 + cb^{CHP2}\right) c^{f,coal} - cb^{CHP2} p_t^{spot} + \frac{1}{1.2} \left(c^{tax,coal} + c^{tax,CO2} + c^{tax,NOx} \right) \tag{2.9}$$

Taxes imposed on extraction mode operation result in the costs:

$$c_{t,extract.}^{CHP2} = cv^{CHP2} p_t^{spot} + \frac{1}{1.2} \left(c^{tax,coal} + c^{tax,CO2} + c^{tax,NOx} \right) \tag{2.10}$$

Figure 2.9 shows the heat costs when the taxes listed in Table 2.2 are applied.

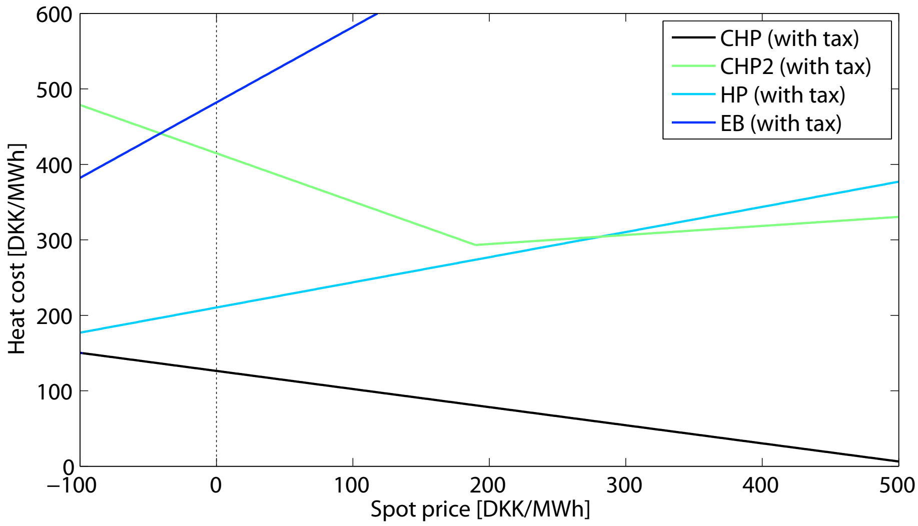


Figure 2.9 – Marginal heat costs as a function of spot price for different production units when taxes are applied.

The biomass back-pressure CHP is now consistently the cheapest unit due to the tax exemption for biomass production along with the supplement received for power production from biomass. Only for electricity prices below -100 DKK/MWh the HP is competitive. It should also be noted that the EB now have the highest marginal heat costs as seen from Figure 2.9, as opposed to the situation without tax, where it was among the most competitive.

2.3 Electricity markets

This section presents the electricity markets relevant for CHP, HP and EB production. Both the market for buying and selling power as well as ancillary services will be presented. The market structures vary between regions and countries around the world. The system for Eastern Denmark will be used as reference here.

2.3.1 Nord Pool Spot

In Denmark and the Nordic countries energy can be traded on several liberalized markets run by Nord Pool Spot. Nord Pool Spot is owned by the Nordic and Baltic transmission operators; for Denmark this is Energinet.dk. There is 370 members generally consisting of power producers, suppliers and traders as well as large end-users. 84% of all power in the Nordic and Baltic regions was traded on Nord Pool Spot in 2013, which makes it the worlds largest market for buying and selling power [33]. Two complementary markets exists; The one day-ahead market, Elspot, and the intra-day market, Elbas. These will be outlined in the following.

Elspot

The day-ahead market, Elspot, is most widely used as 71% of the total amount of traded capacity is traded here [33]. Before noon, orders are placed hour by hour, for delivery on the next day. Prices are calculated based on supply, demand and transmission capacity.

First, power producers provide a price curve reflecting the price required for different quantities. This supply curve is usually very influenced by the production method and includes a certain amount of uncertainty, as power from intermittent sources such as wind power cannot be predicted with certainty.

Power demand bids are placed in a similar manner. The demand curves are generally inelastic as consumers are not very sensitive to price changes.

Aggregating the supply and demand curves results in a situation similar to the one in Figure 2.10. The supply curve shows a step-wise behavior which roughly corresponds to the marginal costs of the production method. The cheapest is wind power but nuclear power and hydro power have very low marginal costs as well. On the contrary, oil and gas turbines that have high marginal costs due to high fuel costs and taxes and low efficiencies, lies in the top. Furthermore, Figure 2.10 also illustrates the impact an increase or decrease in wind power production have on the spot price. Due to the inelastic demand curve, a small horizontal displacement of the supply curve can change the spot price significantly.

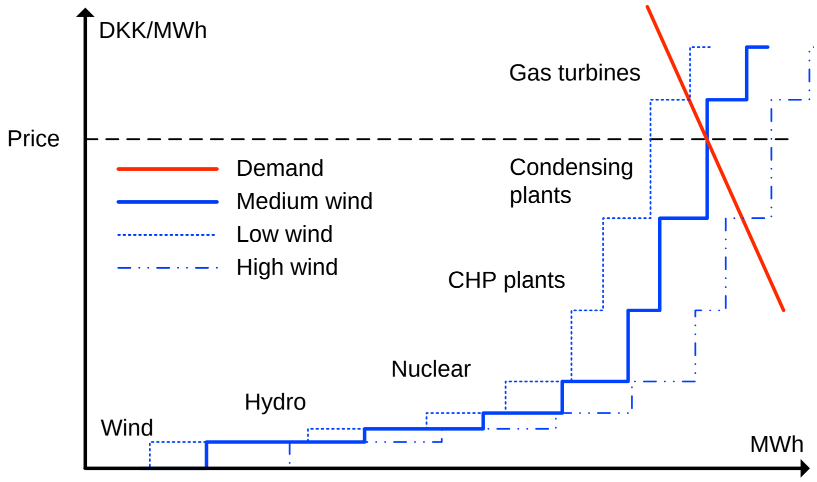


Figure 2.10 – Supply and demand curve for power determining the power price. If the wind production changes the entire supply curve shifts horizontally, which results in large changes in the power price. Plotted with inspiration from [12].

Based on the submitted power bid and offers of power, the electricity price (spot price)[5] is calculated to balance supply and demand taking into account possible limitation of the transmission capacity [33].

Elbas

As most energy production, especially wind power production, is not known exactly one day-ahead the intraday-market, Elbas, is used to help balance the realized production to the realized demand.

After the spot price is announced the capacity available for the Elbas market is published at 14.00. Elbas is a continuous market where trading happens until one hour before delivery. Prices are based on a first-come first-serve principle. This means that highest buy price and lowest sell price comes first [34]. This market is generally not used very much. This could be due to the existence of the regulating market described in the following section. The Elbas market will, due to its small impact, not be considered in this project.

2.3.2 Ancillary services

Deviations in production and consumption as well as disturbances at production facilities impact the system balance, and cause frequency deviations in the grid. Minor imbalances can cause unstable system operation, and consequently Energinet.dk buys ancillary services to ensure that they are always able to balance the frequency.

In addition to these, a joint Nordic market for regulating power exists to balance realized production as consumption. This market will be outlined in the next section followed by a description of the ancillary services bought by Energinet.dk.

[5]In the this report the electricity price, power price and spot price will be used interchangeably and refer to the spot price determined in the Elspot market.

Regulating power

Regulating power is production capacity or consumption offered by the market players to Energinet.dk during the actual day of operation. The purpose is to neutralize imbalances occurring during the day. Flexible units, able to increase or decrease their production, forward bids for upward and downward regulation, stating the volume offered and the price of activating the power. An offer of upward regulation corresponds to the ability to increase the power production (or decrease the consumption) and similarly a downwards regulation offer is a decrease in power production (or increase in consumption). Based on the offers, and the need for up or down regulating power, the marginal offer that is activated determines the regulating prices for all activated offers. However, the price for up regulation can never exceed the spot price, just as the down regulation price can never be lower than the spot price. Basically, a better price is obtained at the regulating market compared to the Elspot market, but only in the event that the bid is activated.

Reserve power

The following ancillary services are bought by Energinet.dk for Eastern Denmark:

1. Frequency-controlled disturbance reserve (FDR)
2. Frequency-controlled normal operation reserve (FNR)
3. Manual reserves
4. Short-circuit power, reactive reserves and voltage control

In the FDR market HPs and EBs are not accepted and it is not considered further. Manual reserves must be activated within 15 minutes which also makes it suitable for CHP units. This means that a HP and EB would compete against CHP units for this market.

The focus is here on the FNR market, which is very appropriate for fast regulating units such as a HP and EB. For this type of operation ordinary CHP units are not fast enough.

The FNR is meant for small frequency deviations of $\pm$ 0.1 Hz. The power should be activated automatically and be delivered within 150 seconds [35]. The offer should also be symmetric, meaning that an offer of 2 MW requires the ability to regulate both up and down by 2 MW. In Denmark only 23 MW is bought daily, which makes this a small market. An availability price is submitted either one or two days before. A pay-as-bid[6] concept is used for the availability price. The actual production and consumption resulting from the activation is paid according to the up and down regulating prices descrbied in the previous section. It is very difficult to predict the prices in this market, as only the average of the trade together with Sweden is available. The price is highly influenced by the Swedish water reservoirs that can provide both up and down regulation at almost zero cost when they are already running. This is only during the day and the FNR prices are, thus, usually higher at night. It has been estimated that the Danish price is approximately 50% higher than the average prices[7].

[6] The supplier receives the price that was stated in the bid. This is generally the alternative to marginal pricing where all accepted bids receive the same marginal price.

[7] Oral discussion with H. Damgaard, Energy Planer, HOFOR

2.4 District heating in Greater Copenhagen

This section introduces the district heating system for Greater Copenhagen. Compared to other district heating systems nationally and world wide, this is considered both to be a large and complex system.

2.4.1 Heat distribution

Heat is not easily transported longer distances as opposed to electricity that can be transported hundreds of kilometres with minor losses. However, heat is restricted to the specific area in which it is produced, as transport losses are high. This limits the heat distribution to a relatively confined area depending on the available temperatures and the design, characteristics and quality of the pipes. Figure 2.11 illustrates the district heating network of Greater Copenhagen including the production units and the different distribution areas.

In the Greater Copenhagen area, there are two transmission operators VEKS and CTR, and one main distributor, HOFOR, exists. Each operate within different areas of Greater Copenhagen, see Figure 2.11. However, heat can be transported through the area of another company, if necessary. Two producers provide heat, namely DONG Energy and HOFOR Kraftvarme (former Vattenfall). DONG Energy owns and operates Avedøreværket, Svanemølleværket and H.C. Ørstedsværket and HOFOR Kraftvarme Amagerværket.

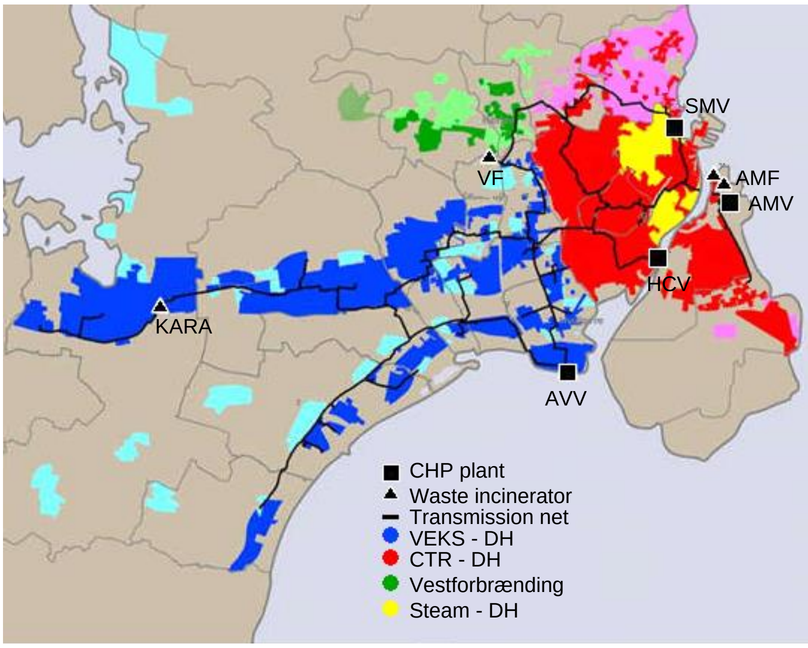


Figure 2.11 – Overview of the Greater Copenhagen district heating network [36]. DH refers to district heating areas.

The heating network in Greater Copenhagen includes both a transmission and a distribution network. The transmission network, visualised in Figure 2.11, is a high pressure (25 bar) network meant for transporting heat longer distances. Heat is either distributed as hot water and steam depending on the area. However, this project only considers heat production in the form of hot water as this simplifies the operation. Furthermore, a project converting the steam based distribution to water based is currently ongoing.

The supply temperature for the transmission network varies between 100 °C in summer and 120 °C in the winter, generally increasing with lower outside temperature and higher heat demand[8].

The distribution network is connected to the transmission network through large heat exchangers. It supplies buildings with heat at 60 °C. The loss in the distribution network is significantly higher than in the transmission network, and depending on the distance the heat has to travel it accumulates to approximately 20% [37]. The supply temperature in the distribution network is typically around 60-95 °C depending on the position in the distribution network, the outside temperature and the heat demand. The distribution network is only a 6 bar network which increases the temperature requirements for the production.

2.4.2 Heat dispatch in Copenhagen

This section presents Varmelast.dk which is responsible for the daily heat dispatch in Copenhagen. The procedure for heat dispatch is subsequently outlined.

Varmelast.dk

Varmelast.dk is a company consisting of one employee from each of the three companies VEKS, HOFOR and CTR, in Greater Copenhagen. While VEKS and CTR are transmission companies supplying many local distribution companies around Copenhagen, HOFOR is the distributor of district heating in Copenhagen. The purpose of Varmelast.dk is to provide the overall most optimal and feasible heat dispatch between the two suppliers of district heating, DONG Energy and HOFOR Kraftvarme. As a part of this, Varmelast.dk wish to induce a degree of competition between the two suppliers.

Completely separated from Varmelast.dk, contracts are made between each distributors/ transmission operator (VEKS, CTR, HOFOR) and suppliers (DONG Energy and HOFOR Kraftvarme) determining the monthly price to be paid for heat. Contracts include variable costs depending on the amount supplied as well as a fixed part of the investment for the production units.

Day-ahead heat dispatch

The process for daily heat dispatch made by Varmelast.dk is outlined in Figure 2.12.

At 07:45 Varmelast.dk sends a forecast of the expected heat demand for the upcoming day to the producers, DONG Energy and HOFOR Kraftvarme (arrow 1). Based on this forecast, each of the producers create a number of supply points. One point contains the production costs for a given quantity of water and a quantity of steam. As the calculation of these points is time-consuming and cumbersome only a limited number of points are provided (approximately five from HOFOR Kraftvarme and 20 from DONG Energy). The points are send to Varmelast.dk at approximately 8:45 (arrow 2). Varmelast.dk assumes a linear

[8]Oral discussion with H. Damgaard, Energy Planer, HOFOR.

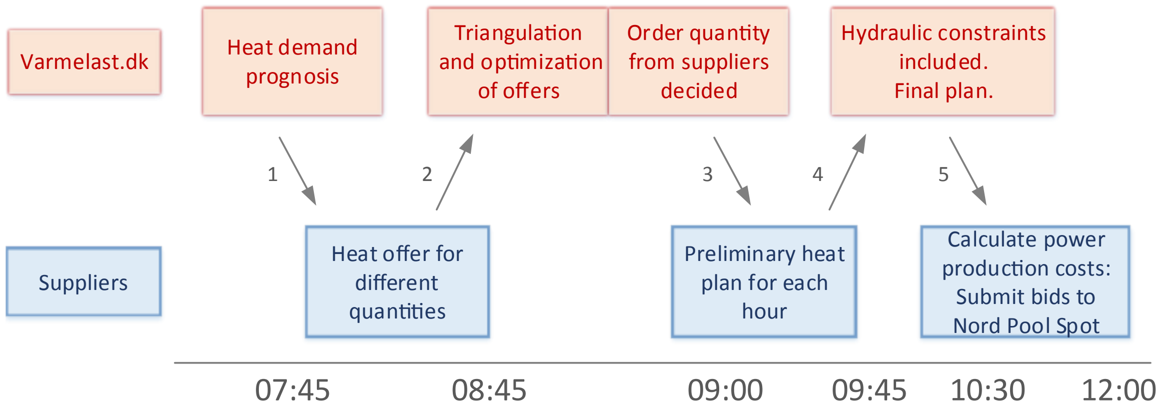


Figure 2.12 – Time line for Varmelast.dk heat dispatch process with inspiration from [38].

relationships between the points and a supply curve/plane is constructed. Varmelast.dk determines the quantity from each suppliers that minimizes the total costs and sends back the quantity of steam and water that is required from each of the producers (arrow 3). Based on the amount requested from Varmelast.dk, each supplier now make an hourly preliminary plan on how and where to produce. This plan is sent to Varmelast.dk at 09:45 (arrow 4). The producers neither take the physical limitations of the system into account, nor do they know the specific production plan of the competitor. Varmelast.dk therefore has to take both preliminary plans and run them through a flow model, that contains the physical constraints in the network. If the plans are feasible nothing is changed. However, if this is not the case, the model returns the feasible solution with the fewest changes in volume taking into account the marginal costs that the suppliers provide. At 10:30 the final plan is sent back to the producers (arrow 5). The amount of electricity they will produce is now determined and bids are submitted to the Nord Pool Elspot market before noon.

Follow-up and intraday

Three times during the day; 15:30, 22:00 and 08:00 a follow up is made. Changes in heat demand is included and production is changed according to a least-cost principle using the marginal costs provided by the producers.

2.5 Chapter summary

This chapter presented the principles of CHP production on a back-pressure and an extraction plant. Furthermore, the operational principles for an EB and HP were outlined. Heat costs for these production units were derived and the significant impact of taxes and fees illustrated. The Nordic electricity market was outlined together with the ancillary services, which are bought to secure grid stability. Finally, the district heating system of Greater Copenhagen was presented and the procedures for heat dispatch outlined. This allows for an assessment of the operational possibilities a HP and EB have if integrated in a CHP system. Moreover, it allows for an analysis of the framework on which an operational strategy can be developed. This will be the subject of the following chapter.

Chapter 3

Operational framework and strategy

This chapter constitutes the foundation for the subsequent modelling and analysis chapters. The previous chapter outlined and described the heat and power markets including the relevant actors. Combined heat and power production was described as well as the working principle of HPs and EBs. With this as the basis, this chapter analyzes and sets up a framework for integrating a HP and EB in the Copenhagen district heating network and Nordic power market. The most essential issues to be discussed include the physical and organizational location of the HP and EB, as well as operational considerations concerning the relevant markets. Addressing these issues properly, allows for an assessment of the operational strategy, based on which, an appropriate modelling set-up can be defined.

3.1 Organizational location and information access

If an EB and HP are to be introduced into the system several possibilities exist for the organizational location within the district heating system in Greater Copenhagen. Both operational as well as political issues affected by the organizational location should be considered before making this decision.

The heat dispatch in Greater Copenhagen, described in Section 2.4, is based on defined procedures intended to provide an optimal heat dispatch and to ensure a degree of competition between the two suppliers. An EB and HP will therefore have to obey the principles set-up in this agreement and enter the market in a way that is satisfactory for all parties. As a consequence, detailed production information from both suppliers cannot be assumed to be accessible simultaneously.

Three distinctive organizational locations are identified for the system in Greater Copenhagen:

1. Stand alone: Owned by third-party
2. Owned by Varmelast.dk
3. Owned by supplier: DONG Energy or HOFOR Kraftvarme

These options have been selected after discussing the possibilities with both Varmelast.dk and HOFOR, and are believed to constitute realistic options. The political as well as operational consequences for each of these options are assessed in the following paragraphs and summarized in Table 3.1.

In the first option the EB and HP operate as individual units. In this way the units are competing equally against CHP units from HOFOR Kraftvarme and DONG Energy. In relation to the level of information available, this will be limited to the day-ahead electricity price forecasts and the daily heat demand forecast for the upcoming day. Thus, the operation will have to be based on forecasts with large margins, to lower the risk of uneconomical situations, as no rescheduling is possible. Furthermore, the HP and EB will not be able to produce if the power price, unexpectedly, becomes very low and no heat dispatch is given based on the forecasts for the electricity price.

The second option is to let Varmelast.dk operate the EB and HP, and profit to be divided between the involved parts (DONG Energy, HOFOR Kraftvarme, HOFOR, VEKS and CTR). This allows for the dispatch to be decided knowing the bids from the other suppliers. However, such a set-up is politically complex and the production companies might not accept this construction due to the risk of loosing profit. However, Varmelast.dk do not have full access to supplier information and hence the flexibility of the HP and EB will not be fully explored in this option.

Finally, in the third option the HP and EB are a part of a bigger system including additional ways of producing heat, such as CHP plants. This portfolio of production options allows for a better optimization of the overall operation. Knowing information concerning other production units, the heat and power bidding can take place on an aggregated level with more flexibility to reschedule internally in the event of unexpected electricity prices. Furthermore, access to storage is usually available in connection to large CHP units which provide additional flexibility. This will result in cheaper bids to Varmelast.dk and thus more heat dispatch.

Following, it seems most beneficial to opt for the third option and let the HP and EB be part of a CHP system, owned by one supplier. This will not lead to any political conflicts and allows for the flexibility of the HP and EB to be fully utilized.

Organizational location	**Political**	**Operational**
Stand alone	Simple solution. Operate and compete equally against competitors	No information, no flexibility and high uncertainty
Varmelast.dk	Complex as it might favor the HP and EB. Producers might object	Overall production knowledge available. Low flexibility. Back-up functionality
Supplier	Simple but requires internal integration	Operational information available. Optimize as part of CHP system. High flexibility

Table 3.1 – Summary of political and operational consequences of three distinctive organizational locations.

3.2 Strategic market operation

This section analyzes the different markets in which a HP and EB might be beneficial to operate. Section 2.3 outlined the markets for buying and selling electricity. These are, important as the driving force of the EB and HP considered here is electricity. The markets for selling heat and reserve capacity, described in Section 2.4 and 2.3, is also of interest. Even though Varmelast.dk carries out an overall optimization based on the incoming bids, an internal optimization still occurs after dispatch. With the introduction of a HP and EB it thus needs to be considered, how they most optimally can operate in both the heat and electricity market.

3.2.1 Heat dispatch and production planning

Heat dispatch at Varmelast.dk

Heat offers made by the suppliers to Varmelast.dk should always be based on production costs. For CHP system with and without HPs and EBs, the costs are variable and uncertain as the spot price is not yet known at the time the heat offer is made. Including HPs and EBs a more competitive offer should be possible when the spot price is expected to be low. However, the volatility of the spot prices and the uncertainty in the heat demand prediction should be accounted for. Offering a price that ensures with e.g. 90% confidence that the realized price will still result in a profitable situation could be a simple, yet good, strategy.

In the case where the supplier solely owns an EB or HP, the offers that are made will be binding. In other words, if for instance the supplier offer and accepts to produce 50 MW, the supplier is obligated to do so, no matter the costs. Consequently, a price margin should be added to the bid to account for the volatility in the power prices. On the contrary, if the supplier have a portfolio of production units (CHPs, EBs, HPs, etc.), the supplier could in case of high power prices, produce the promised heat on a different and more economical unit given the realized power price.

Heat and power schedule

Based on the heat dispatch provided one day ahead in the morning, the decision is now how to produce the dispatched heat and how much to offer in the Elspot market. If the HP and EB operate alone they simply have to buy the required amount at any price and be turned on at the given time.

If a portfolio of different production units are available it becomes more complicated but opens up for several new options. The HP and EB can only consume power, but if a back-pressure and an extraction CHP is available, they can produce power and heat in different ratios and with different efficiencies. Adding a storage opens up for even further possibilities. In this situation it becomes difficult to come up with a simple operational strategy that can plan the HP and EB operation in the complex system.

3.2.2 The regulating market

Both the EB and HP could operate in the regulation market, utilizing their flexibility, by selling up or down regulation hour by hour. As explained in Section 2.3.2 up or down regulation is a service Energinet.dk buys based on offers provided at latest 45 minutes before the hour in question. For an EB or HP this means that in order to up regulate it has shut down and thus consume less electricity. Down regulation corresponds to increasing the power consumption by turning on the HP or EB.

For downwards regulation, the offered quantity has to be available if activated. This means that if an EB offer 5 MW down regulating, and its capacity is also 5 MW, it cannot operate in any other market or supply heat. It has to be turned off and wait for the offer to be activated. If activated, an amount of heat will be produced. This either has to be stored in a storage tank, in the network, or used for supplying the district heating network. When operating in the up-regulating market the EB or HP has to consume electricity in order to be able to up-regulate. This means that if offering 5 MW up-regulation, the EB needs to have a planed consumption of 5 MW such that it can turn off the power consumption if activated. This means it will require heat dispatch when in this market. If activated, the heat has to be supplied from another unit or from storage. However, if the unit is only activated in a short amount of time the district heating network might be able to absorb the change. An assessment of this possibility will be outlined for the reserve market in Section 3.2.3, but the same principles apply to this market. Generally, the EB and HP have to be integrated with a CHP, boiler, storage or another device that can supply heat if operating in the regulating market.

3.2.3 Reserve operation

The FNR reserve market is as explained in Section 2.3.2, both one and two days ahead. In the following, the first paragraph analyzes some of the challenges of a bidding strategy, whereas the second paragraph analyzes the possibility of utilizing the network to absorb changes in heat production resulting from this market.

Bidding strategy

At the time of the two days ahead market, the electricity prices are very uncertain. The reserve market clearing is difficult to forecast as only the average prices are public. If a HP or EB were to offer reserve capacity in the two days ahead market, the bid would be based on a forecast for the spot price with a high uncertainty. Also the marginal cost for heat dispatch is not known and must be forecast. If the forecast for marginal heat cost is denoted, $\hat{p}_t^{heat}$, the electricity price forecast, $\hat{p}_t^{spot}$, and the expected minimum acceptance cost for the reserve market, $\hat{p}_t^{res}$, three different scenarios will be of specific interest for the simple example of an EB considering to operate in the reserve market:

1. $\hat{p}_t^{res} + \hat{p}_t^{heat} \geq \hat{p}_t^{spot}$ and $\hat{p}_t^{heat} \geq \hat{p}_t^{spot}$
2. $\hat{p}_t^{spot}$ is low
3. $\hat{p}_t^{res} \geq \hat{p}_t^{spot} - \hat{p}_t^{heat}$ and $\hat{p}_t^{spot} \geq \hat{p}_t^{heat}$

In the first situation it might be profitable to bid in the reserve market. However, if the supplier places an offer to the reserve market, only 50% of the capacity can be be sold at the heat market. If not accepted in the reserve market a potential profit is lost by not supplying 100% capacity to the heat market.

In the second situation it might still be profitable to bid in the reserve market. However, if the electricity price turns out very low or even negative, the benefits from operating in the heat market with full capacity (as opposed to half the capacity) would be much higher.

Finally, in the last situation the expected spot price is higher than the expected heat market clearing price. This means, that under normal circumstances the EB would not get any heat dispatch as the operating costs would be too high. However, in the situation that the reserve market can provide additional profit, such that the sum of reserve and heat profit exceeds the electricity price, it would be reasonable to consider to offer in the reserve market. Clearly, this market is very uncertain and operating here could easily lead to uneconomical situations.

Complicating the issue even further, the actual use of the EB is paid the regulating price for reserve market operation. This is not considered in the scenarios described above. Generally, offers in the reserve market should be provided with very high margins due to the uncertainty.

The one day-ahead market, which happens at 19:00, has the advantage of known power prices. This means that a situation as the one described in bullet (2) above would not occur. However, the offer to the heat market now occurs before the reserve market is decided. If the heat price is expected to be lower than the electricity price, the supplier needs to decide between a full load (heat market only) or a 50% load (heat and reserve). The decision will be affected by the risk-willingness, as operating only in the heat market has significantly less associated risk. However, all uncertainties are generally reduced in the one day-ahead market, hence this market is clearly preferred if the supplier was to chose.

Finally, there is also a challenge of political nature in relation to this. Currently, the costs provided to Varmelast.dk in the daily heat dispatch must not be influenced by the benefits obtained in any reserve market. For CHP plants this is only an insignificant part of their total production. However, for an EB, operating on the reserve market will in some situations, like scenario 3 described above, be crucial for its competitiveness on the heat market.

Heat from FNR operation

A relevant issue when considering into introduce HPs and EBs to the reserve market is the consequences on the heat market. If frequency reserve is required the heating network has to be flexible and absorb the resulting fluctuating heat production. Otherwise a storage, or other heat producing units, needs to be connected to the unit. The impact on the network depends on the volume, the power of the HP and EB and the running time.

The temperature difference, ΔT, due to an increased energy supply is found as [39]:

$$\Delta T = \frac{Q}{m \cdot C_p} \tag{3.1}$$

where Q is the energy added, m is the mass and C_p is the heat capacity of the heating medium (here water).

Calculations are carried out for Amagerland and Tingbjerg which are the largest and smallest distribution networks in Copenhagen, respectively[1].

For an EB supplying 1 MW for 1 hour in Amagerland which has a volume of 8500 m^3 this gives:

$$\Delta T = \frac{3.6 \times 10^6 \text{kJ}}{8.5 \times 10^6 \text{kg} \cdot 4.18 \frac{\text{kJ}}{\text{kg} \cdot {}^\circ\text{C}}} = 0.1\,^\circ\text{C} \tag{3.2}$$

For the smallest network Tingbjerg with only 62 m^3 the corresponding calculation results in:

$$\Delta T = 13.9\,^\circ\text{C} \tag{3.3}$$

These calculations emphasize the impact of the location of the unit if this market should be considered for an EB without a storage available. Normally, activation in the FNR market corresponds to continuous small deviations to be counteracted continuously. However, there is no limit for the maximum duration of an activated reserve capacity, and thus, the specific district heating network should be considered before deciding to enter this market. The network would not be as suitable for absorbing regulating power, as described in Section 3.2.2, due to the long activation times that more often occur in this market.

3.3 Physical location of the EB and HP

The physical location of an EB and HP can have a large impact on the associated benefits as was already illustrated in the last section. The different options along with their advantages and disadvantages are outlined in this section.

3.3.1 Distribution or transmission network

It is important to decide whether the HP and EB should be connected to the transmission network or the distribution network. Deciding between one or the other will have both benefits and drawbacks.

Currently, HPs are only designed to produce heat with a supply temperature of up to 85 °C[2]. This is mainly due the refrigerants that is used and the corresponding high pressure that is required for making the high temperature. However, as the technology matures the available temperatures might increase. In order to supply the transmission network the temperature has to be significantly higher, ∼ 100 °C. This means that the location of the HP currently is limited to the distribution network. Thus, the HP will be limited to production for only one distribution area. In situations of a bottleneck in the distribution network, in can be an advantage to be situated in the distribution network. A bottleneck

[1]Information on the two distribution networks was provided by D. Lindblom, HOFOR.
[2]Discussion with T. Engberg, Chief Project and Market Manager, COWI.

usually requires an increase in the supply temperature or the start-up of an oil or gas boiler which can be extremely costly. If an HP was located on the right side of the critical point it would be favorable to supply this area and the temperature would not have to be increased. However, the monetary benefits for this type of operation is difficult to assess and requires the optimal location to be found. In addition production can not be planned in advance and thus, the unit cannot buy power on the Elspot market. This will increase the operating cost of the unit, but since it replaces more expensive gas or oil boilers it might still be profitable. Furthermore, the HP has an additional restriction concerning the location as it requires access to a cold source of energy. In the case of sea water, the HP is limited to a location in a distribution network adjacent to the sea.

For an EB the situation is similar concerning the beneficial location when there are bottlenecks. If the area after the bottleneck is small, the EB will be more suitable than the HP since the EB is a small investment and does not require many operating hours to be profitable compared to a HP. Furthermore, EBs do not have the restrictions in temperature and can produce at any temperature. However, EBs are still not as efficient when it comes to production and the operational costs are high due to the high use of electricity and comparable high taxes and fees. In order to reduce the marginal costs, it should be considered to locate the EB at the CHP plants as described below.

3.3.2 Locate the EB at a CHP plant

If an EB is located at a CHP plant connected to the transmission network, it is not able to explore the benefits of getting dispatch due to a favorable location. However, it can easily be connected to existing transformers and it will often have access to storage facilities. This keeps the investment costs even lower. However, the main incentive to locate the EB at a CHP plant is, to be exempted from the heat tax and distribution fee, according to the tax rules described in Section 2.2.2.

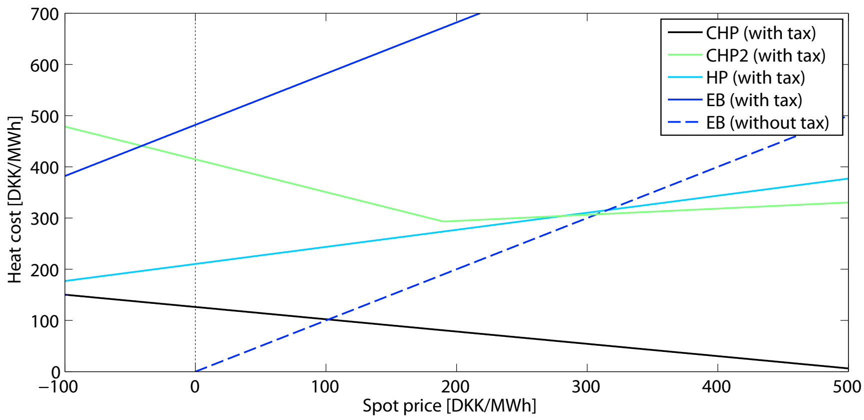


Figure 3.1 – Marginal heat costs for production units including both the marginal cost for an EB with and without tax.

Figure 3.1 shows the marginal heat costs presented in Section 2.2 with the addition of marginal costs for the EB without tax and distribution fee. The impact of locating the EB at a CHP is huge and significantly increase the competitiveness of the EB. If tax is included it is generally the least favored unit. Excluding the tax makes it competitive with both

CHP units as well as the HP.

The following section summarize the operational possibilities for the HP and EB, and the corresponding time line for the decision making process.

3.4 Modelling an operational strategy

A full operational strategy for an EB and HP, including all the relevant markets analyzed in the previous sections, would result in multiple interlinked decisions to be taken at different times. As was realized from the previous sections, different markets have different structure and time horizons, which makes such operational strategy highly complex. Figure 3.2 displays a time line of decisions to be taken, including both, the ordinary heat market, the Elspot market, the one day-ahead reserve market, the regulating market and finally the real-time changes based on the realized heat demand. In the vertical axis additional information on the decision is outlined. "Basis" highlights the information available at the time of the decision and other previous constraining decisions. "Characteristics" indicates the complexity of the given market/decision and the relative importance of an operational strategy.

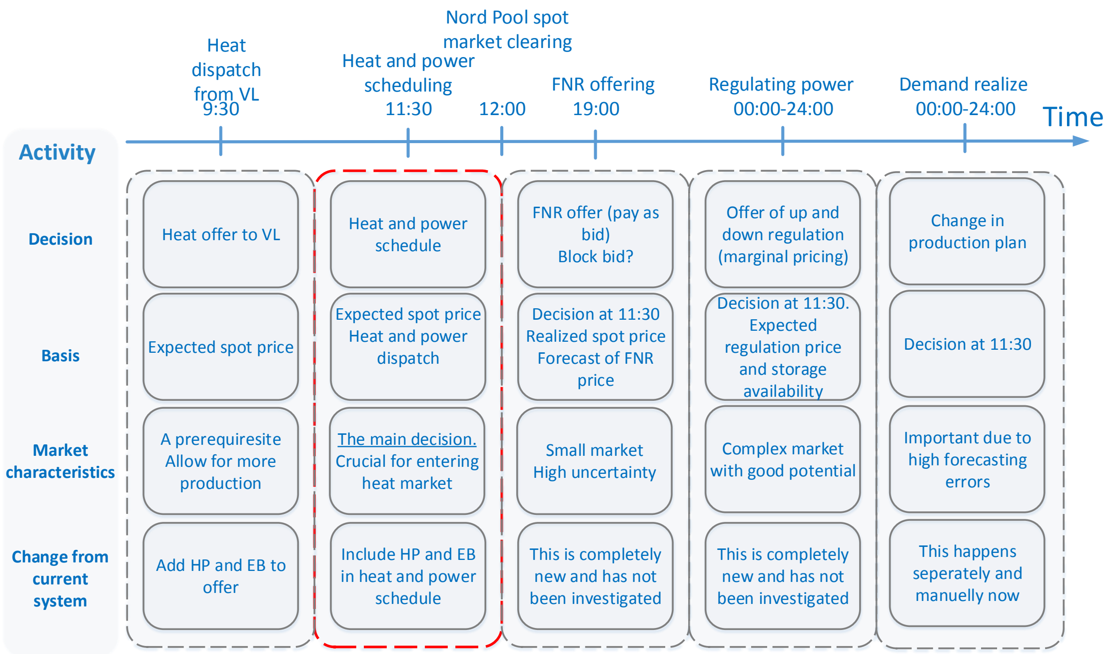


Figure 3.2 – Outline for an operational stategy for an EB and HP including both heat and electricity markets. Markets are ordered according to the time at which decisions are made.

As this system is both complex and interlinked, all decisions should optimally be included into one model describing the full system. This could act as a decision support tool in all the decision phases outlined in Figure 3.2. However, such a model requires extensive modelling and might be computationally intractable, due to the complexity of the system and the excessive amount of data that would serve as an input. Instead, the most important decision(s) must be identified and used as a starting point for the development of an operational strategy.

Even though the heat dispatch, see column 1 in Figure 3.2, is very important, it does not provide information on how to operate the HP and EB, but merely how much these optimally should produce. Naturally, this is very important if the aim is to ensure the maximum dispatch, but less significant here, where the goal is an operational strategy.

The FNR market, outlined in column 3, is very small and competition has decreased the profitability from this market. The high uncertainty in this market, described in Section 2.3.2, makes the risk of uneconomical situations higher if a specific low-risk strategy is not developed. In addition, the current heat dispatch procedure does not allow for bids to be affected by this market.

The regulating market, summarized in column 4, is an interesting market to participate in. A CHP unit could benefit from this market, as power production might have to be adjusted to meet the realized heat demand. The EB and HP could be included by offering flexibility during period they are not competitive in the heat market. However, if a system comprising both CHP, HP and EB units is available, the flexibility of the HP and EB could make this market less important.

Essentially, the development of an operational strategy for an HP and EB starts by defining how to integrate these units into the ordinary heat and power scheduling that occurs before the spot prices are known, but after the heat dispatch has taken place, see column 2. The other markets and services are secondary options to increase the profit, while the first step is to model the integration of a HP and EB in the heat and power production planning in a CHP system.

3.5 Chapter summary

This chapter presented a framework for developing an operational strategy for a HP and EB. The potential of relevant markets were analyzed, and a time line for an operational strategy comprising all relevant markets was presented. The challenge and importance of each decision was stressed, and it was concluded that the main challenge is to find the operational strategy, which includes the HP and EB in the ordinary heat and power scheduling and subsequently offer power to the Nordic spot market.

In order to make a heat and power schedule for a CHP system, a mathematical model seems necessary as the complexity of the system and decision process makes it difficult to choose optimally. This will be the scope of following chapters.

Chapter 4

Operation models for a CHP system

Based on the analysis of the framework carried out in the previous chapter, this chapter introduces a model describing the heat and power operation in a CHP system including a HP and EB supplying a district heating network. The CHP system comprise two CHP units, a HP, an EB, a small local and a large heat accumulator (storage), $s1$ and s, respectively. The small local storage is only connected to the HP. Electricity from the CHP units is used to supply the EB directly. The remaining electricity is sold on the Nordic day-ahead Elspot market. This system, without the HP and EB resembles a realistic integrated heat and power production system in Copenhagen. The EB and HP constitute two additional units to increase flexibility and take advantage of low electricity prices.

4.1 System framework

Figure 4.1 shows a simple overview of the system. Red dashed lines represent heat transfer from normal production; black arrows represent electricity inputs or outputs. The two CHP units, one extraction unit and one back-pressure unit, produce directly to the transmission network and to a large heat accumulator.

The transmission network supplies several local distribution networks of which only two are shown here. The HP is, due to temperature limitations, only connected to a local distribution network and a small local heat accumulator. The HP operates as a negative load for the total system, meaning that it is assumed that the size and heat demand of the distribution network is large enough for the HP to produce at any time. Both the HP and EB consume electricity and offer heat.

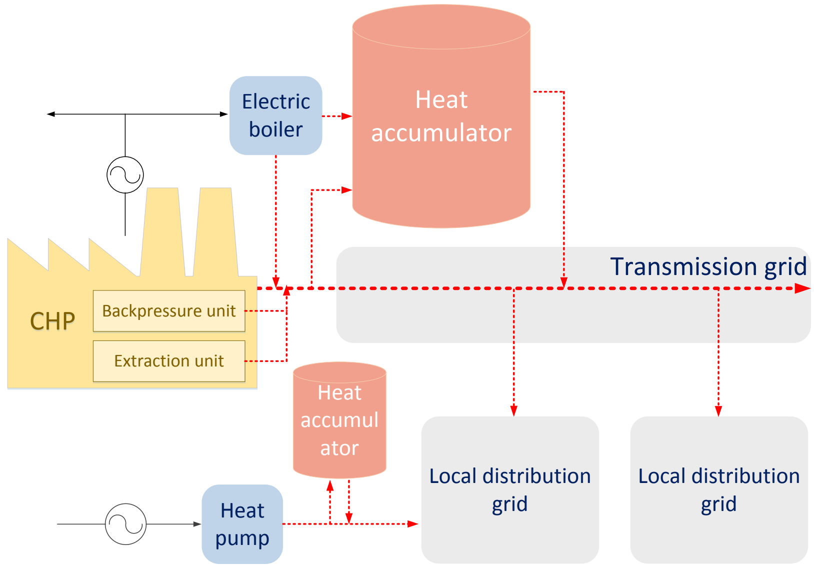


Figure 4.1 – Overview of a district heating system comprising two CHP units, an EB, a HP, a large and a local heat accumulator.

4.2 Deterministic model for a CHP system

4.2.1 Parameters

Table 4.1 and 4.2 list the time dependent and constant parameters, respectively. Generally, "C" denotes heat capacities, "c" costs, "P" power limitations and "H" heat limitations.

Parameter	Unit	Explanation
$\hat{p}_t^{spot}$	DKK/MWh	Forecast spot price at time t
$\hat{d}_t$	MWh	Forecast heat load at time t

Table 4.1 – Time dependent parameters.

4.2.2 Variables

Table 4.3 lists the variables used in the model along with the type and definition. All variables are in greek letters with subscript indicating an index, such as time and superscript denoting the units or specifying the variable. As an example "d" and "s" is used to specify a production to (heat) demand and storage, respectively. When referring to the two CHP units, the back-pressure CHP is denoted as "CHP" and the extraction CHP as "CHP2". Generally, variables denoted by χ, ρ, α and β represent heat transfer, power production, storage level and binary variables, respectively. Variable $\chi_t^{d,CHP}$ is hence, the heat production from the back-pressure CHP to cover the heat demand at time t.

Parameter	**Value**	**Explanation**
C^s	3000 MWh	Storage capacity
C^{s1}	300 MWh	HP storage capacity
C^{HP}	75 MWh	Heat production capacity for HP
C^{EB}	75 MWh	Heat production capacity for EB
C^{CHP}	250 MWh	Heat production capacity for back-pressure CHP
C^{CHP2}	330 MWh	Heat production capacity for extraction CHP2
$c^{f,CHP}$	144 DKK/MWh	Fuel cost for CHP (biomass)
$c^{f,CHP2}$	72 DKK/MWh	Fuel cost for extraction CHP2 (coal)
COP^{HP}	3	COP for HP
η^{CHP}	1.1	Total efficiency for back-pressure CHP
η^{CHP2}	0.35	Power efficiency for extraction CHP2
cb^{CHP}	0.24	Power to heat relationship for back-pressure CHP
R^{CHP}	50 MW	Max ramp up and down rate for back-pressure CHP
R^{CHP2}	40 MW	Max ramp up and down rate for extraction CHP2
S^{flow}	300 MW	Maximum flow to and from storage
S^{loss}	1.05 MW	Loss when using heat from storage
cv^{CHP2}	-0.12	Power to heat ratio for CHP2 in extraction operation
cb^{CHP2}	0.64	Power to heat ratio for CHP2 in back-pressure operation
$P^{min,CHP}$	12 MWh	Minimum power production from CHP
$P^{max,CHP2}$	250 MWh	Maximum power production from CHP2
$P^{min,CHP2}$	40 MWh	Minimum power production from CHP2
$H^{min,HP}$	10 MWh	Minimum heat production from HP
c^{su}	125000 DKK	Start up cost for CHP and CHP2
c^{sd}	125000 DKK	Shut-down cost for CHP and CHP2
$c^{su,HP}$	2500 DKK	Start up cost for HP
$c^{tax,coal}$	258.5 DKK/MWh	Tax for production from coal
$c^{tariff,net}$	219 DKK/MWh	Distribution fee for electricity consumption
$c^{tax,el}$	412 DKK/MWh	Electricity consumption tax
c^{CO2}	57 DKK/MWh	CO_2 tax for heat production from fossil fuels
c^{NOx}	9 DKK/MWh	NO_x tax for heat production from fossil fuels
$c^{bio,sup}$	150 DKK/MWh	Supplement for power produced by biomass CHP
c^{inf}	1000 DKK/MWh	Penalty for the heat demand not being satisfied
$r^{tax,f}$	1.2	Ratio between heat production and fuel to be taxed

Table 4.2 – Constant parameter values used for the stochastic and the deterministic model

Variable	Type	Explanation
ν_t^{prod}	$\in \mathbb{R}^+$	Total electricity production at time t
ν_t^{con}	$\in \mathbb{R}^+$	Total electricity consumption at time t
$\nu_t^{CHP,EB}$	$\in \mathbb{R}^+$	Amount of CHP power production used by EB at time t
$\chi_t^{s,CHP}$	$\in \mathbb{R}^+$	Production from CHP to storage, s, at time t
$\chi_t^{d,CHP}$	$\in \mathbb{R}^+$	Production from CHP to demand at time t
$\chi_t^{s,CHP2}$	$\in \mathbb{R}^+$	Heat production to storage s from CHP2 at time t
$\chi_t^{d,CHP2}$	$\in \mathbb{R}^+$	Heat production to demand from CHP2 at time t
$\chi_t^{s1,HP}$	$\in \mathbb{R}^+$	Production from HP to storage $s1$ at time t
$\chi_t^{d,HP}$	$\in \mathbb{R}^+$	Production from HP to demand at time t
$\chi_t^{s,EB}$	$\in \mathbb{R}^+$	Production from EB to storage s at time t
$\chi_t^{d,EB}$	$\in \mathbb{R}^+$	Production from EB to demand at time t
$\chi_t^{d,s}$	$\in \mathbb{R}^+$	Amount taken from storage s at time t
$\chi_t^{d,s1}$	$\in \mathbb{R}^+$	Amount taken from HP storage s1 at time t
χ_t^{inf}	$\in \mathbb{R}^+$	Heat demand not covered at time t
α_t^{s}	$\in \mathbb{R}^+$	Amount in storage s at time t
α_t^{s1}	$\in \mathbb{R}^+$	Amount in HP storage $s1$ at time t
$\rho_t^{f,CHP2}$	$\in \mathbb{R}^+$	Max power corresponding to constant fuel use for extraction CHP2 at time t
γ_t^{CHP2}	$\in \mathbb{R}^+$	Fuel consumption from CHP2 at time t
γ_t^{CHP}	$\in \mathbb{R}^+$	Fuel consumption from CHP at time t
ρ_t^{CHP2}	$\in \mathbb{R}^+$	Power production from CHP2 at time t
β_t^{CHP}	$\in \mathbb{B}$	Is one if CHP is producing at time t
$\beta_t^{su,CHP}$	$\in \mathbb{B}$	Is one if CHP is in start-up at time t
$\beta_t^{sd,CHP}$	$\in \mathbb{B}$	Is one if CHP is in shut-down at time t
β_t^{CHP2}	$\in \mathbb{B}$	Is one if CHP2 is producing at time t
$\beta_t^{su,CHP2}$	$\in \mathbb{B}$	Is one if CHP2 is in start-up at time t
$\beta_t^{sd,CHP2}$	$\in \mathbb{B}$	Is one if CHP2 is in shut-down at time t
β_t^{HP}	$\in \mathbb{B}$	Is one if HP is producing at time t
$\beta_t^{su,HP}$	$\in \mathbb{B}$	Is one if HP is in start-up at time t
z	$\in \mathbb{R}$	Total profit

Table 4.3 – Variables used in the deterministic optimization model for a CHP system.

4.2.3 Objective function

The objective function states the overall objective of the optimization. For this model, it is chosen to minimize costs. This is stated in (4.1)-(4.3):

$$\text{min.} \sum_t \Bigg(\hat{p}_t^{spot} \left(\nu_t^{con} - \nu_t^{prod} \right) + \pi_t^{tax,HP} + \pi_t^{tax,CHP2} + \pi_t^{tax,CHP} + \tag{4.1}$$

$$\left(\rho_t^{CHP} - \nu_t^{CHP,EB} \right) c^{bio,sup} + c^{fCHP} \gamma_t^{CHP} + c^{fCHP2} \gamma_t^{CHP2} + c^{inf} \chi_t^{inf} \tag{4.2}$$

$$+ c^{su} \left(\beta_t^{su,CHP} + \beta_t^{su,CHP2} \right) + c^{sd} \left(\beta_t^{sd,CHP} + \beta_t^{sd,CHP2} \right) + c^{su,HP} \beta_t^{su,HP} \Bigg) \tag{4.3}$$

where the tax costs included in (4.1) are different for each unit and calculated as:

$$\pi_t^{tax,HP} = \frac{1}{COP^{HP}} \left(\chi_t^{d,HP} + \chi_t^{s,HP} \right) \left(c^{tax,el} + c^{tariff,net} \right) \quad , \forall t \tag{4.4}$$

$$\pi_t^{tax,CHP} = \frac{1}{r^{tax,f}} \left(\chi_t^{d,CHP} + \chi_t^{s,CHP} \right) c^{NOx} \quad , \forall t \tag{4.5}$$

$$\pi_t^{tax,CHP2} = \frac{1}{r^{tax,f}} \left(\chi_t^{d,CHP2} + \chi_t^{s,CHP2} \right) \left(c^{tax,coal} + c^{CO2} + c^{NOx} \right) \quad , \forall t \tag{4.6}$$

The first part of the objective function, described by (4.1), includes the cost from power consumption by the EB and HP, the expected turnover from the power traded on the Elspot market as well a the total tax costs for the HP, CHP and CHP2 outlined in Section 2.2. These costs are defined in (4.4)-(4.6). The EB does not pay tax as it is connected directly to the CHP units. The second part of the objective function, described by (4.2), includes the fuel costs for the two CHP units, the supplement given to power produced by the biomass (the back-pressure CHP) and a cost to not fulfill the heat demand. $\nu_t^{CHP,EB}$ corresponds to the amount of the power production from the back-pressure CHP that was used for the EB, as this share should not receive the supplement. Finally, (4.3) allows for start-up costs for both CHP units and the HP and shut down costs for the CHPs.

4.2.4 Constraints

A number of constraints is defined to constrain the production, which ensure that results resemble the reality to a certain extent. The following outlines each of the constraints ordered according to their function and not by units.

Heat and power production

The heat demand, d_t, is satisfied through the equality constraint in (4.7). However, the variable χ_t^{inf} is included to allow for a solution even if the heat demand cannot be met. Use of this variable is penalized in the objective function. The terms in (4.7) represent the heat production from the two CHP units, the HP and EB and the amount of heat extracted from the two storages[1]. On the right hand side is the heat demand that is not covered:

$$\chi_t^{d,CHP} + \chi_t^{d,HP} + \chi_t^{d,EB} + \chi_t^{d,s} + \chi_t^{d,s1} + \chi_t^{d,CHP2} = d_t - \chi_t^{inf} \quad , \forall t \tag{4.7}$$

The planned production of electricity, ν_t^{prod}, is defined in by:

$$\nu_t^{prod} = \rho_t^{CHP} + \rho_t^{CHP2} \ , \ \forall t \tag{4.8}$$

For the back-pressure CHP unit there is a fixed relationship, cb^{CHP}, between power and heat production, and thus the power production is calculated as in (4.12). The power production from the extraction CHP2, ρ_t^{CHP2}, is defined through equations (4.14)-(4.17).

The consumption of electricity from the EB and HP, ν_t^{con}, is defined in (4.9):

$$\nu_t^{con} = \chi_t^{s,EB} + \chi_t^{d,EB} + \frac{1}{COP^{HP}} \left(\chi_t^{d,HP} + \chi_t^{s,HP} \right) \ , \ \forall t \tag{4.9}$$

In order for the EB to avoid tax, the power consumption always have to be supplied by either one of the CHP units and thus it is required that:

$$\chi_t^{s,EB} + \chi_t^{d,EB} \leq \rho_t^{CHP} + \rho_t^{CHP2} \ , \ \forall t \tag{4.10}$$

The amount of electricity used by the EB and HP, not supplied by the CHP2, is found as:

$$\nu_t^{CHP,EB} \geq \nu_t^{con} - \rho_t^{CHP2} \ , \ \forall t \tag{4.11}$$

Power production and fuel consumption of CHP units

The power production and fuel usage from the back-pressure CHP is calculated in (4.12) and (4.13). The power production and the power efficiency, η^{CHP}, is used to find the fuel consumption:

$$\rho_t^{CHP} = \left(\chi_t^{s,CHP} + \chi_t^{d,CHP} \right) cb^{CHP} \quad , \ \forall t \tag{4.12}$$

$$\gamma_t^{CHP} = \frac{1}{\eta^{CHP}} \left(\rho_t^{CHP} + \chi_t^{s,CHP} + \chi_t^{d,CHP} \right) \quad , \ \forall t \tag{4.13}$$

Due to the operational possibilities of an extraction CHP, displayed previously in Figure 2.4, it does not have a fixed relationship between heat and power. Thus, it requires a few additional constraints. These constraints are presented in (4.14)-(4.17). The first two limits the power production, based on the heat production, to be within the feasible area:

$$\rho_t^{CHP2} \leq cv^{CHP2} \left(\chi_t^{d,CHP2} + \chi_t^{s,CHP2} \right) + P^{max,CHP2} \ , \ \forall t \tag{4.14}$$

$$\rho_t^{CHP2} \geq cb^{CHP2} \left(\chi_t^{d,CHP2} + \chi_t^{s,CHP2} \right) + P^{min,CHP2} \ , \ \forall t \tag{4.15}$$

The constraints in (4.16) and (4.17) define the power production, ρ_t^{CHP2}, and the corresponding fuel consumption, γ_t^{CHP2}, such that the fuel usage follows the principle outlined

[1]It should be noticed that this assumes that the HP and the small storage unit will never deliver more than the local distribution network requires, as they are not able to deliver to the transmission network.

in Figure 2.4. This ensures that the right-most point in Figure 2.4 also appears optimal in the model:

$$\rho_t^{CHP2} = cv^{CHP2}\left(\chi_t^{d,CHP2} + \chi_t^{s,CHP2}\right) + \rho_t^{f,CHP2} \qquad , \forall t \tag{4.16}$$

$$\gamma_t^{CHP2} = \frac{1}{\eta^{CHP2}}\rho_t^{f,CHP2} \qquad , \forall t \tag{4.17}$$

Ramping

Ramping constraints are also introduced for the back-pressure and extraction CHPs, see (4.18)-(4.21). These are necessary due to the physical limitations of the CHP units. This means it is only possible to increase or decrease the production by a limited amount, R^{CHP}, from one hour to the next:

$$\left(\chi_t^{d,CHP} + \chi_t^{s,CHP}\right) - \left(\chi_{t-1}^{d,CHP} + \chi_{t-1}^{s,CHP}\right) \leq R^{CHP} \qquad , \forall t \tag{4.18}$$

$$\left(\chi_t^{d,CHP} + \chi_t^{s,CHP}\right) - \left(\chi_{t-1}^{d,CHP} + \chi_{t-1}^{s,CHP}\right) \geq -R^{CHP} \qquad , \forall t \tag{4.19}$$

$$\left(\chi_t^{d,CHP2} + \chi_t^{s,CHP2}\right) - \left(\chi_{t-1}^{d,CHP2} + \chi_{t-1}^{s,CHP2}\right) \leq R^{CHP2} \qquad , \forall t \tag{4.20}$$

$$\left(\chi_t^{d,CHP2} + \chi_t^{s,CHP2}\right) - \left(\chi_{t-1}^{d,CHP2} + \chi_{t-1}^{s,CHP2}\right) \geq -R^{CHP2} \qquad , \forall t \tag{4.21}$$

These constraints do not apply to the HP and EB. As long as the capacity of the HP is not very large, relative to the CHP units, it is assumed that ramping is not as important for the HP. The EB can go to full load in a matter of minutes or seconds, and thus ramping constraints are omitted here as well.

Start-up and shut-down

Start-up and shut-down costs are also introduced for both the back-pressure and the extraction CHP. The HP does also have start-up costs, but no shut-down cost is implied to allow for a more flexible production. The constraints defining the start ups are identically constructed for the three units.

In (4.22)-(4.24) the binary variables, β_t^{CHP}, β_t^{CHP2} and β_t^{HP} are one if the respective unit have a heat or power production different from zero:

$$\chi_t^{s,CHP} + \chi_t^{d,CHP} \leq \beta_t^{CHP} C^{CHP} \qquad , \forall t \tag{4.22}$$

$$\rho_t^{CHP2} + \chi_t^{s,CHP2} + \chi_t^{d,CHP2} \leq \beta_t^{CHP2}\left(P^{max,CHP2} + C^{CHP2}\right) \qquad , \forall t \tag{4.23}$$

$$\chi_t^{s,HP} + \chi_t^{d,HP} \leq \beta_t^{HP} C^{HP} \qquad , \forall t \tag{4.24}$$

The constraint concerning the extraction CHP2, (4.23), also includes the power production, as this unit, as opposed to the back-pressure unit, can produce power without simultaneously producing heat.

Naturally, there are costs associated with a start-up of CHP units. The constraints in (4.25)-(4.27) defines the start-up of the units. If no production occurred at the previous time step a production in the current time step will require a start up and thus the binary variables, $\beta_t^{su,CHP}$, $\beta_t^{su,CHP2}$ and $\beta_t^{su,HP}$ are required to be one:

$$\beta_t^{su,CHP} \geq \beta_t^{CHP} - \beta_{t-1}^{CHP} \quad , \forall t \tag{4.25}$$
$$\beta_t^{su,CHP2} \geq \beta_t^{CHP2} - \beta_{t-1}^{CHP2} \quad , \forall t \tag{4.26}$$
$$\beta_t^{su,HP} \geq c^{HP,su} \left(\beta_t^{HP} - \beta_{t-1}^{HP}\right) \quad , \forall t \tag{4.27}$$

Similar to the start-ups, shut-down constraints for the back-pressure CHP and the extraction CHP2 are presented in (4.28) and (4.29):

$$\beta_t^{sd,CHP} \geq \beta_{t-1}^{CHP} - \beta_t^{CHP} \quad , \forall t \tag{4.28}$$
$$\beta_t^{sd,CHP2} \geq \beta_{t-1}^{CHP2} - \beta_t^{CHP2} \quad , \forall t \tag{4.29}$$

Minimum load

A minimum load constraint is also formulated since, in reality, a minimum production is required to get the turbines running:

$$\rho_t^{CHP} \geq \beta_t^{CHP} P^{min,CHP} \quad , \forall t \tag{4.30}$$
$$\rho_t^{f,CHP2} \geq \beta_t^{CHP2} P^{min,CHP2} \quad , \forall t \tag{4.31}$$
$$\chi_t^{s,HP} + \chi_t^{d,HP} \geq \beta_t^{HP} H^{min,HP} \quad , \forall t \tag{4.32}$$

Storage operation

Both storages need state transition equations to describe the heat level at any time. These are formulated in (4.33) and (4.34), as the previous heat level plus the net production in time t. The net production is the production to storage subtracted the consumption from the storage:

$$\alpha_t^s = \alpha_{t-1}^s + \chi_t^{s,CHP} + \chi_t^{s,CHP2} + \chi_t^{s,EB} - s^{loss} \chi_t^{d,s} \quad , \forall t \tag{4.33}$$
$$\alpha_t^s 1 = \alpha_{t-1}^{s1} + \chi_t^{s,HP} - s^{loss} \chi_t^{d,s1} \quad , \forall t \tag{4.34}$$

where s^{loss} is introduced to incur a loss when using the storage and to ensure that the storage is not favorable compared to direct delivery.

There is a physical limit on the amount of heat that can be delivered to and from the storage for each hour. This constraint is modelled in (4.35) and (4.36):

$$\chi_t^{d,s} \leq S^{flow} \quad , \forall t \tag{4.35}$$
$$\chi_t^{s,CHP} + \chi_t^{s,EB} + \chi_t^{s,CHP2} \leq S^{flow} \quad , \forall t \tag{4.36}$$

It is assumed that the small storage does not have the same flow constraint as the one just described for the large storage.

Capacity constraints

A number of capacity constraints are required to limit the production from the CHP2, EB, HP and the storages. These are presented in (4.37)-(4.39):

$$\alpha_t^s \leq C^s \quad , \forall t \tag{4.37}$$

$$\alpha_t^{s1} \leq C^{s1} \quad , \forall t \tag{4.38}$$

$$\chi_t^{s,EB} + \chi_t^{d,EB} \leq C^{EB} \quad , \forall t \tag{4.39}$$

The capacity for the back-pressure CHP and the HP was ensured in the definition of the binary variables in (4.22) and (4.24), respectively.

The objective function described by (4.1) and (4.3), together with the constraints in (4.7)-(4.39) constitute the deterministic operation model used for the subsequent modelling and analysis.

The model described in this section was implemented in GAMS and the script can be found in Appendix A.

4.3 Stochastic model for a CHP system

In the analysis of the framework in Chapter 3 it became apparent that uncertainty has a big impact on the decisions to make in heat and power markets. Both the electricity prices as well as the heat demand is uncertain, and both are based on point forecasts in the deterministic model. Furthermore, all decisions are made before the spot price and heat demand is realized with no option of adjusting production to the realization. In reality the supplier would have to adjust the production to the realized heat demand. Including both the uncertain nature of the heat demand and electricity prices together with the adjustment of the heat demand calls for a stochastic two-stage model with recourse. The general principles of this type of model will be outlined in the following section and followed by a formulation of such a model.

4.3.1 Stochastic optimization

The general formulation of a linear stochastic optimization problem can be stated as [40]:

$$\begin{aligned} \min\ & z = c^T x + E_\xi[Q(x,\xi)] \\ \text{s.t. } & Ax = b, \\ & x \geq 0 \end{aligned}$$

where c and b are vectors of parameters, A is a matrix of parameters and x is a vector containing the decision variables.

The recourse function $Q(x,\xi)$ is the optimal value of the second stage problem:

$$\begin{aligned} \min\ & q(\xi)^T y(\xi) \\ & T(\xi)x + Wy(\xi) = h(\xi) \\ & y(\xi) \geq 0 \end{aligned}$$

where W is a matrix of parameters, $q(\xi)$ and $h(\xi)$ are parameters dependent on the stochastic variable and y is the second stage decision variable. The principle of a two-stage program is that a first stage "here-and-now" decision, x, is taken before the realization of the uncertain information, ξ, is known. For a minimization problem the costs should be minimized. However, the expectation of the second stage problem is included. This resembles the optimal behavior given the realization of the uncertain data. It is also referred to as the recourse cost, i.e. the cost of compensating for any difference between the first stage decision and the realization of uncertainty.

In order to solve a two-stage problem numerically, it can be useful to assume that the random vector, ξ, has a finite number of realizations. These can be considered scenarios, each with an assigned probability. In this case the expectation of the recourse function can be formulated as a sum:

$$E[Q(x,\xi)] = \sum_{k=1}^{K} p_k Q(x,\xi_k) \tag{4.40}$$

where $k = 1...K$ are the scenarios, and p_k is the probability of scenario k.

This approach will be the subject of the following section. A more thorough introduction to stochastic programming can be found in [40].

4.3.2 Two-stage stochastic model with recourse

In order to model this system as a two-stage problem a number of first and second stage variables should be defined. A general overview of this two-stage problem is shown in Figure 4.2. The first stage decision variables are as described in the Section 4.2. The second stage variables are displayed in Table 4.4.

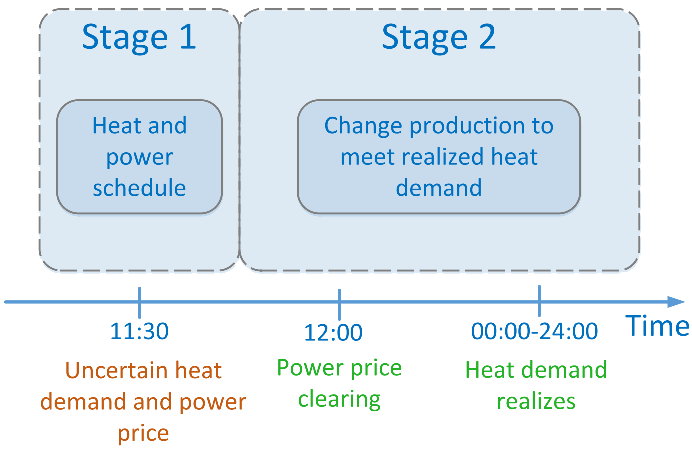


Figure 4.2 – Overview of the two-stage problem. In the first stage a production plan for heat and power is made and a fixed amount of power is sold on the Elspot market. In second stage, the heat demand and spot price is known and adjustments to satisfy the realized heat demand are made accordingly, without changing the net power production.

All parameters are as described in Table 4.5 and 4.2. However, a number of scenarios, indexed by ξ, are introduced to solve the problem numerically. Thus, $d_{t,\xi}$ and $p^{spot}_{t,\xi}$, respectively, represent the realized heat demand and electricity price for scenario ξ and time t.

4.3.3 Objective function

The formulation of the objective function follows the principles outlined in Section 4.3. In (4.41) - (4.43) the first stage decision is formulated with the addition of the expectation of the second stage in (4.44):

$$\text{min.} \sum_t \Bigg(\hat{p}_t^{spot} \left(\nu_t^{con} - \nu_t^{prod} \right) + \pi_t^{HP,tax} + \pi_t^{CHP2,tax} + \pi_t^{CHP,tax} \tag{4.41}$$

$$+ \left(\rho_t^{CHP} - \nu_t^{CHP,EB} \right) c^{bios} + c^{fCHP} \gamma_t^{CHP} + c^{fCHP2} \gamma_t^{CHP2} + c^{inf} \chi_t^{inf} \tag{4.42}$$

$$+ c^{su} \left(\beta_t^{su,CHP} + \beta_t^{su,CHP2} \right) + c^{sd} \left(\beta_t^{sd,CHP} + \beta_t^{sd,CHP2} \right) + c^{su,HP} \beta_t^{su,HP} \Bigg) \tag{4.43}$$

$$+ E[Q(x,\xi)] \tag{4.44}$$

where $E[Q(x,\xi)]$ is the expected value of the recourse function $Q(x,\xi)$ which is defined as:

$$Q(x,\xi) = \text{min.} \sum_t \Bigg(p_{t,\xi}^{spot} \left(\Delta\nu_{t,\xi}^{con} - \Delta\nu_{t,\xi}^{prod} \right) + \tag{4.45}$$

$$\Delta\pi_{t,\xi}^{HP,tax} + \Delta\pi_{t,\xi}^{CHP2,tax} + \Delta\pi_{t,\xi}^{CHP,tax} + (\Delta\rho_{t,\xi}^{CHP} - \Delta\nu_{t,\xi}^{CHP,EB}) c^{bio,sup} + \tag{4.46}$$

$$c^{fCHP} \Delta\gamma_{t,\xi}^{CHP} + c^{fCHP2} \Delta\gamma_{t,\xi}^{CHP2} + c^{inf} \Delta\chi_{t,\xi}^{inf} + \tag{4.47}$$

$$c^{su} \left(\Delta\beta_{t,\xi}^{su,CHP} + \Delta\beta_{t,\xi}^{su,CHP2} \right) + c^{sd} \left(\Delta\beta_{t,\xi}^{sd,CHP} + \Delta\beta_{t,\xi}^{sd,CHP2} \right) + c^{su,HP} \Delta\beta_{t,\xi}^{su,HP} \Bigg) \tag{4.48}$$

The first part described by (4.45) includes the change in power production and consumption. Next, the change in tax costs as well as the supplement for biomass produced power is considered in (4.46). The change in fuel consumption and cost of uncovered heat demand is stated in (4.47), and finally the change in start-ups and shut-downs is introduced in (4.48).

4.3.4 Constraints

A number of first and second-stage constraints are required to limit production and obtain a practically feasible solution. The first stage constraints are identical to (4.7)-(4.39) presented in Section 4.2. This means that the model should also be feasible in the first stage solution. In addition to those constraints, a number of new constraints are necessary to represent the changes from the second stage decision. Many of the constraints are similar to those presented in Section 4.2 but with the addition of second stage variables. Those will only be described briefly as a thorough explanation can be found in Section 4.2. However, one very important equation is introduced to constrain the net power production. It is therefore presented first.

Variable	Type	Explanation
$\Delta\nu_{t,\xi}^{prod}$	$\in \mathbb{R}$	Change in electricity production †
$\Delta\nu_{t,\xi}^{con}$	$\in \mathbb{R}$	Change in electricity consumption †
$\Delta\nu_{t,\xi}^{CHP,EB}$	$\in \mathbb{R}$	Change in CHP power production used by EB †
$\Delta\chi_{t,\xi}^{s,CHP}$	$\in \mathbb{R}$	Change in heat production from CHP to storage s †
$\Delta\chi_{t,\xi}^{d,CHP}$	$\in \mathbb{R}$	Change in heat production from CHP to demand †
$\Delta\chi_{t,\xi}^{s,CHP2}$	$\in \mathbb{R}$	Change in heat production to storage from CHP2 †
$\Delta\chi_{t,\xi}^{d,CHP2}$	$\in \mathbb{R}$	Change in heat production to demand from CHP2 †
$\Delta\chi_{t,\xi}^{s1,HP}$	$\in \mathbb{R}$	Change in production from HP to storage s1 †
$\Delta\chi_{t,\xi}^{d,HP}$	$\in \mathbb{R}$	Change in production from HP to demand †
$\Delta\chi_{t,\xi}^{s,EB}$	$\in \mathbb{R}$	Change in production from EB to storage †
$\Delta\chi_{t,\xi}^{d,EB}$	$\in \mathbb{R}$	Change in production from EB to demand †
$\Delta\chi_{t,\xi}^{d,s}$	$\in \mathbb{R}$	Additional amount taken from storage s †
$\Delta\chi_{t,\xi}^{d,s1}$	$\in \mathbb{R}$	Additional amount taken from storage s1 †
$\Delta\chi_{t,\xi}^{inf}$	$\in \mathbb{R}$	Change in demand not satisfied †
$\Delta\alpha_{t,\xi}^{s}$	$\in \mathbb{R}$	Level change in large storage s †
$\Delta\alpha_{t,\xi}^{s1}$	$\in \mathbb{R}$	Level change in small storage s1 †
$\Delta\rho_{t,\xi}^{f,CHP2}$	$\in \mathbb{R}$	Power corresponding to constant fuel use for extraction CHP2 with additional production †
$\Delta\gamma_{t,\xi}^{CHP2}$	$\in \mathbb{R}$	Additional fuel consumption from CHP2 †
$\Delta\gamma_{t,\xi}^{CHP}$	$\in \mathbb{R}$	Change in fuel consumption from CHP †
$\Delta\rho_{t,\xi}^{CHP2}$	$\in \mathbb{R}$	Change in power production from CHP2 †
$\beta_{t,\xi}^{CHP}$	$\in \mathbb{B}$	Is one if CHP is producing including changes †
$\beta_{t,\xi}^{su,CHP}$	$\in \mathbb{B}$	Is one if CHP is in start-up †
$\beta_{t,\xi}^{sd,CHP}$	$\in \mathbb{B}$	Is one if CHP is in shut-down †
$\beta_{t,\xi}^{CHP2}$	$\in \mathbb{B}$	Is one if CHP2 is producing †
$\beta_{t,\xi}^{su,CHP2}$	$\in \mathbb{B}$	Is one if CHP2 is in start-up †
$\beta_{t,\xi}^{sd,CHP2}$	$\in \mathbb{B}$	Is one if CHP2 is in shut-down †
$\beta_{t,\xi}^{HP}$	$\in \mathbb{B}$	Is one if HP is producing †
$\beta_{t,\xi}^{su,HP}$	$\in \mathbb{B}$	Is one if HP is in start-up †
$\Delta\beta_{t,\xi}^{su,HP}$	$\in \mathbb{R}$	The difference between HP start-up in first and second stage †
$\Delta\beta_{t,\xi}^{su,CHP}$	$\in \mathbb{R}$	The difference between CHP start-up in first and second stage †
$\Delta\beta_{t,\xi}^{su,CHP2}$	$\in \mathbb{R}$	The difference between CHP2 start-up in first and second stage †
$\Delta\beta_{t,\xi}^{sd,CHP}$	$\in \mathbb{R}$	The difference between CHP shut-down in first and second stage †
$\Delta\beta_{t,\xi}^{sd,CHP2}$	$\in \mathbb{R}$	The difference between CHP2 shut-down in first and second stage †
z	$\in \mathbb{R}$	Total profit

Table 4.4 – Variables used in the stochastic model for the operation on a HP and EB in a CHP system. †, at time t and scenario ξ.

Parameter	**Unit**	**Explanation**
$p_{t,\xi}^{spot}$	DKK/MWh	Realized spot price at time t and scenario ξ
$d_{t,\xi}$	MWh	Realized heat load at time t and scenario ξ

Table 4.5 – Time dependent parameters for the stochastic optimization model.

Net power production

In the first stage, the amount of power sold on the Elspot market is decided. This model does not allow for this amount to be changed in the second stage[2]. The net power production in the second stage must therefore be equal to zero. This leads to the constraint:

$$\Delta\nu_{t,\xi}^{prod} - \Delta\nu_{t,\xi}^{con} = 0 \ , \ \forall t, \xi \tag{4.49}$$

This constraint limits the second stage decisions such that it cannot always adjust to the most desirable production. In the scenario where the heat demand is higher than expected a CHP is required to produce additional heat, and thus also power, meaning that a corresponding power consumption from the EB or HP should occur. If this is not considered, as in a deterministic set-up, it can result in the heat demand not being satisfied. The stochastic set-up includes these situations and the first stage decision is made such that it allows for second-stage decisions to meet the realizations of the stochastic variables, heat demand and spot price most optimally.

Demand

The realized heat demand, $d_{t,\xi}$, should be satisfied when introducing the second stage variables. This is ensured through:

$$\chi_t^{d,CHP} + \chi_t^{d,HP} + \chi_t^{d,EB} + \chi_t^{d,s} + \chi_t^{d,s1} + \chi_t^{d,CHP2} \tag{4.50}$$
$$+\Delta\chi_{t,\xi}^{d,CHP} + \Delta\chi_{t,\xi}^{d,HP} + \Delta\chi_{t,\xi}^{d,EB} + \Delta\chi_{t,\xi}^{d,s} + \Delta\chi_{t,\xi}^{d,s1} + \Delta\chi_{t,\xi}^{d,CHP2} \tag{4.51}$$
$$=d_{t,\xi} - \Delta\chi_t^{inf} \qquad , \ \forall t, \xi \tag{4.52}$$

Tax costs

Changes in tax costs as a result of the second stage are defined by:

$$\Delta\pi_t^{tax,HP} = \frac{1}{COP^{HP}} \left(\Delta\chi_t^{d,HP} + \Delta\chi_t^{s,HP}\right) \left(c^{tax,el} + c^{tariff,net}\right) \qquad , \ \forall t, \xi \tag{4.53}$$
$$\Delta\pi_t^{tax,CHP2} = \frac{1}{r^{tax,f}} \left(\Delta\chi_t^{d,CHP2} + \Delta\chi_t^{s,CHP2}\right) \left(c^{tax,coal} + c^{CO2} + c^{NOx}\right) \qquad , \ \forall t, \xi \tag{4.54}$$
$$\Delta\pi_t^{tax,CHP} = \frac{1}{r^{tax,f}} \left(\Delta\chi_t^{d,CHP} + \Delta\chi_t^{s,CHP}\right) c^{NOx} \qquad , \ \forall t, \xi \tag{4.55}$$

[2] In reality excess power can be traded on the regulating market at less favorable prices.

Power production and consumption

Similar to the deterministic equations (4.8) and (4.9), the second stage equations (4.56) and (4.57) describe the change in power production and consumption by:

$$\Delta\nu_{t,\xi}^{prod} = \Delta\rho_{r,\xi}^{CHP} + \Delta\rho_{t,\xi}^{CHP2} \quad , \forall t,\xi \qquad (4.56)$$

$$\Delta\nu_{t,\xi}^{con} = \Delta\chi_{t,\xi}^{d,EB} + \Delta\chi_{t,\xi}^{s,EB} + \frac{1}{COP^{HP}}\left(\Delta\chi_{t,\xi}^{d,HP} + \Delta\chi_{t,\xi}^{s,HP}\right) \quad , \forall t,\xi \qquad (4.57)$$

Power production and fuel consumption

The power production and fuel consumption constraints for the back-pressure CHP, with the addition of the stochastic variables similar to the equivalent deterministic equations (4.13)-(4.17):

$$\Delta\rho_{t,\xi}^{CHP} = \left(\Delta\chi_{t,\xi}^{s,CHP} + \Delta\chi_{t,\xi}^{d,CHP}\right) cb^{CHP} \quad , \forall t,\xi \qquad (4.58)$$

$$\Delta\gamma_{t,\xi}^{CHP} = \frac{1}{\eta^{CHP}}\Delta\rho_{t,\xi}^{CHP} \quad , \forall t,\xi \qquad (4.59)$$

where $\Delta\rho_{t,\xi}^{CHP}$ is the change in power production from the back-pressure CHP and $\Delta\gamma_{t,\xi}^{CHP}$ is the change in fuel consumption from the back-pressure CHP.

As in the deterministic case, the extraction CHP2 has a production constrained by:

$$\begin{aligned}&\rho_t^{CHP2} + \Delta\rho_{t,\xi}^{CHP2} \leq \\ &cv^{CHP2}\left(\chi_t^{d,CHP2} + \chi_t^{s,CHP2} + \Delta\chi_{t,\xi}^{d,CHP2} + \Delta\chi_{t,\xi}^{s,CHP2}\right) + P^{max,CHP2} \quad , \forall t,\xi \qquad (4.60)\end{aligned}$$

$$\begin{aligned}&\rho_t^{CHP2} + \Delta\rho_{t,\xi}^{CHP2} \geq \\ &cb^{CHP2}\left(\chi_t^{d,CHP2} + \chi_t^{s,CHP2} + \Delta\chi_{t,\xi}^{d,CHP2} + \Delta\chi_{t,\xi}^{s,CHP2}\right) + P^{min,CHP2} \quad , \forall t,\xi \qquad (4.61)\end{aligned}$$

and the fuel consumption including the second stage variables is found similar to (4.16) and (4.17) yielding:

$$\Delta\rho_{t,\xi}^{CHP2} = cv^{CHP2}\left(\Delta\chi_{t,\xi}^{d,CHP2} + \Delta\chi_{t,\xi}^{s,CHP2}\right) + \Delta\rho_{t,\xi}^{f,CHP2} \quad , \forall t,\xi \qquad (4.62)$$

$$\Delta\gamma_{t,\xi}^{CHP2} = \frac{1}{\eta^{CHP2}}\Delta\rho_{t,\xi}^{f,CHP2} \quad , \forall t,\xi \qquad (4.63)$$

Ramping

The ramping constraints in the deterministic model, (4.18)- (4.21) also apply in the stochastic model with addition of the second stage variables. For the back-pressure CHP these are formulated as:

$$\left(\chi_t^{d,CHP} + \chi_t^{s,CHP} + \Delta\chi_{t,\xi}^{s,CHP} + \Delta\chi_{t,\xi}^{d,CHP}\right) - \left(\chi_{t-1}^{d,CHP} + \chi_{t-1}^{s,CHP} + \Delta\chi_{t-1,\xi}^{s,CHP} + \Delta\chi_{t-1,\xi}^{d,CHP}\right) \leq R^{CHP} \quad , \forall t, \xi \tag{4.64}$$

$$\left(\chi_t^{d,CHP} + \chi_t^{s,CHP} + \Delta\chi_{t,\xi}^{s,CHP} + \Delta\chi_{t,\xi}^{d,CHP}\right) - \left(\chi_{t-1}^{d,CHP} + \chi_{t-1}^{s,CHP} + \Delta\chi_{t-1,\xi}^{s,CHP} + \Delta\chi_{t-1,\xi}^{d,CHP}\right) \geq -R^{CHP} \quad , \forall t, \xi \tag{4.65}$$

while the extraction CHP2 has identical ramping constraints:

$$\left(\chi_t^{d,CHP2} + \chi_t^{s,CHP2} + \Delta\chi_{t,\xi}^{s,CHP2} + \Delta\chi_{t,\xi}^{d,CHP2}\right) - \left(\chi_{t-1}^{d,CHP2} + \chi_{t-1}^{s,CHP2} + \Delta\chi_{t-1,\xi}^{s,CHP2} + \Delta\chi_{t-1,\xi}^{d,CHP2}\right) \leq R^{CHP2} \quad , \forall t, \xi \tag{4.66}$$

$$\left(\chi_t^{d,CHP2} + \chi_t^{s,CHP2} + \Delta\chi_{t,\xi}^{s,CHP2} + \Delta\chi_{t,\xi}^{d,CHP2}\right) - \left(\chi_{t-1}^{d,CHP2} + \chi_{t-1}^{s,CHP2} + \Delta\chi_{t-1,\xi}^{s,CHP2} + \Delta\chi_{t-1,\xi}^{d,CHP2}\right) \geq -R^{CHP2} \quad , \forall t, \xi \tag{4.67}$$

Start-up and shut-down

The second stage binary variables for start-up are defined as the first stage equivalent in (4.22)-(4.24). This means that they include all start-ups after the second stage realization.

The constraints in (4.68)-(4.73) defines when the unit has a non-zero production after the addition of the second stage realization:

$$\chi_t^{s,CHP} + \chi_t^{d,CHP} + \Delta\chi_{t,\xi}^{s,CHP} + \Delta\chi_{t,\xi}^{d,CHP} \leq \beta_{t,\xi}^{CHP} C^{CHP} \quad , \forall t, \xi \tag{4.68}$$

$$\rho_t^{CHP2} + \Delta\rho_{t,\xi}^{CHP2} \leq \beta_{t,\xi}^{CHP2} \left(P^{max,CHP2} + C^{CHP2}\right) \quad , \forall t, \xi \tag{4.69}$$

$$\chi_t^{s,HP} + \chi_t^{d,HP} + \Delta\chi_{t,\xi}^{s,HP} + \Delta\chi_{t,\xi}^{d,HP} \leq \beta_{t,\xi}^{HP} C^{HP} \quad , \forall t, \xi \tag{4.70}$$

the binary start-up variables $\beta_{t,\xi}^{su,CHP}$, $\beta_{t,\xi}^{su,CHP2}$ and $\beta_{t,\xi}^{su,HP}$ are one of the respective unit have a heat or power production different from zero:

$$\beta_{t,\xi}^{su,CHP} \geq \beta_{t,\xi}^{CHP} - \beta_{t-1,\xi}^{CHP} \quad , \forall t, \xi \tag{4.71}$$

$$\beta_{t,\xi}^{su,CHP2} \geq \beta_{t,\xi}^{CHP2} - \beta_{t-1,\xi}^{CHP2} \quad , \forall t, \xi \tag{4.72}$$

$$\beta_{t,\xi}^{su,HP} \geq \beta_{t,\xi}^{HP} - \beta_{t-1,\xi}^{HP} \quad , \forall t, \xi \tag{4.73}$$

The change in start-ups are then defined by the variables $\Delta\beta_{t,\xi}^{CHP}$, $\Delta\beta_{t,\xi}^{CHP2}$ and $\Delta\beta_{t,\xi}^{HP}$ such that:

$$\Delta\beta_{t,\xi}^{su,CHP} = \beta_t^{CHP} - \beta_{t,\xi}^{CHP} \quad , \forall t, \xi \tag{4.74}$$

$$\Delta\beta_{t,\xi}^{su,CHP2} = \beta_t^{CHP2} - \beta_{t,\xi}^{CHP2} \quad , \forall t, \xi \tag{4.75}$$

$$\Delta\beta_{t,\xi}^{su,HP} = \beta_t^{HP} - \beta_{t,\xi}^{HP} \quad , \forall t, \xi \tag{4.76}$$

The second stage shut-down variables are modelled similar to the start-up constraints in (4.74)-(4.75):

$$\beta^{sd,CHP}_{t,\xi} \geq \beta^{CHP}_{t-1,\xi} - \beta^{CHP}_{t,\xi} \qquad , \forall t, \xi \tag{4.77}$$

$$\beta^{sd,CHP2}_{t,\xi} \geq \beta^{CHP2}_{t-1,\xi} - \beta^{CHP2}_{t,\xi} \qquad , \forall t, \xi \tag{4.78}$$

The difference in shut-downs are then defined by the variables $\Delta\beta^{sd,CHP}_{t,\xi}$, $\Delta\beta^{sd,CHP2}_{t,\xi}$ such that:

$$\Delta\beta^{sd,CHP}_{t,\xi} = \beta^{CHP}_{t} - \beta^{CHP}_{t,\xi} \qquad , \forall t, \xi \tag{4.79}$$

$$\Delta\beta^{sd,CHP2}_{t,\xi} = \beta^{CHP2}_{t} - \beta^{CHP2}_{t,\xi} \qquad , \forall t, \xi \tag{4.80}$$

Minimum load

The minimum load constraints from the deterministic model still apply and are formulated as:

$$\rho^{CHP}_t + \Delta\rho^{CHP}_{t,\xi} \geq \beta^{CHP}_{t,\xi} P^{min,CHP} \qquad , \forall t, \xi \tag{4.81}$$

$$\rho^{CHP2}_t + \Delta\rho^{CHP2}_{t,\xi} \geq \beta^{CHP2}_{t,\xi} P^{min,CHP2} \qquad , \forall t, \xi \tag{4.82}$$

$$\chi^{s,HP}_t + \chi^{d,HP}_t + \Delta\chi^{s,HP}_{t,\xi} + \Delta\chi^{d,HP}_{t,\xi} \geq \beta^{HP}_{t,\xi} H^{min,HP} \qquad , \forall t, \xi \tag{4.83}$$

Storage

The change in storage level resulting from the second stage are described by:

$$\Delta\alpha^{s}_{t,\xi} = \Delta\alpha^{s}_{t-1,\xi} + \Delta\chi^{s,CHP}_{t,\xi} + \Delta\chi^{s,CHP2}_{t,\xi} + \Delta\chi^{s,EB}_{t,\xi} - s^{loss}\Delta\chi^{d,s}_{t,\xi} \qquad , \forall t, \xi \tag{4.84}$$

$$\Delta\alpha^{s1}_{t,\xi} = \Delta\alpha^{s}_{t-1,\xi}1 + \Delta\chi^{s,HP}_{t,\xi} - s^{loss}\Delta\chi^{d,s1}_{t,\xi} \qquad , \forall t, \xi \tag{4.85}$$

The flow constraints to and from storage, presented in (4.35) and (4.36), should include additional production from the second-stage variables, and thus:

$$\chi^{d,s}_t + \Delta\chi^{d,s}_{t,\xi} \leq S^{flow} \qquad , \forall t, \xi \tag{4.86}$$

$$\chi^{s,CHP}_t + \chi^{s,EB}_t + \chi^{s,CHP2}_t + \Delta\chi^{s,CHP}_{t,\xi} + \Delta\chi^{s,EB}_{t,\xi} + \Delta\chi^{s,CHP2}_{t,\xi} \leq S^{flow} \qquad , \forall t, \xi \tag{4.87}$$

The capacity constraints also reappear but with the addition of the second stage variables such that the capacity is not exceeded:

$$\alpha^{s}_t + \Delta\alpha^{s}_{t,\xi} \leq C^{s} \qquad , \forall t, \xi \tag{4.88}$$

$$\alpha^{s1}_t + \Delta\alpha^{s1}_{t,\xi} \leq C^{s1} \qquad , \forall t, \xi \tag{4.89}$$

$$\chi^{s1,HP}_t + \chi^{d,HP}_t + \Delta\chi^{s1,HP}_{t,\xi} + \Delta\chi^{d,HP}_{t,\xi} \leq C^{HP} \qquad , \forall t, \xi \tag{4.90}$$

$$\chi^{s,CHP}_t + \chi^{d,CHP}_t + \Delta\chi^{s,CHP}_{t,\xi} + \Delta\chi^{d,CHP}_{t,\xi} \leq \beta^{CHP}_t C^{CHP} \qquad , \forall t, \xi \tag{4.91}$$

$$\chi^{s,EB}_t + \chi^{d,EB}_t + \Delta\chi^{s,EB}_{t,\xi} + \Delta\chi^{d,EB}_{t,\xi} \leq C^{EB} \qquad , \forall t, \xi \tag{4.92}$$

Finally, there is a limit on the amount that can be changed downwards since negative production cannot occur. This is relevant for all four production units as well as the storages:

$$-\Delta\chi_{t,\xi}^{s,CHP} \leq \chi_t^{s,CHP} \quad , \forall t, \xi \tag{4.93}$$

$$-\Delta\chi_{t,\xi}^{d,CHP} \leq \chi_t^{d,CHP} \quad , \forall t, \xi \tag{4.94}$$

$$-\Delta\chi_{t,\xi}^{s,CHP2} \leq \chi_t^{s,CHP2} \quad , \forall t, \xi \tag{4.95}$$

$$-\Delta\chi_{t,\xi}^{d,CHP2} \leq \chi_t^{d,CHP2} \quad , \forall t, \xi \tag{4.96}$$

$$-\Delta\rho_{t,\xi}^{CHP2} \leq \rho_t^{CHP2} \quad , \forall t, \xi \tag{4.97}$$

$$-\Delta\chi_{t,\xi}^{s,EB} \leq \chi_t^{s,EB} \quad , \forall t, \xi \tag{4.98}$$

$$-\Delta\chi_{t,\xi}^{d,EB} \leq \chi_t^{d,EB} \quad , \forall t, \xi \tag{4.99}$$

$$-\Delta\chi_{t,\xi}^{s,HP} \leq \chi_t^{s,HP} \quad , \forall t, \xi \tag{4.100}$$

$$-\Delta\chi_{t,\xi}^{d,HP} \leq \chi_t^{d,HP} \quad , \forall t, \xi \tag{4.101}$$

$$-\Delta\chi_{t,\xi}^{d,s} \leq \chi_t^{d,s} \quad , \forall t, \xi \tag{4.102}$$

$$-\Delta\chi_{t,\xi}^{d,s1} \leq \chi_t^{d,s1} \quad , \forall t, \xi \tag{4.103}$$

$$-\Delta\alpha_{t,\xi}^{s} \leq \alpha_t^{s} \quad , \forall t, \xi \tag{4.104}$$

$$-\Delta\alpha_{t,\xi}^{s1} \leq \alpha_t^{s1} \quad , \forall t, \xi \tag{4.105}$$

$$-\Delta\nu_{t,\xi}^{CHP,EB} \leq \nu_t^{CHP,EB} \quad , \forall t, \xi \tag{4.106}$$

The stochastic two-stage model therefore consists of the objective function in (4.42)-(4.47) and the first-stage constraints in (4.7)-(4.39) together with the second-stage constraints just presented in (4.50)-(4.106). This model was implemented in GAMS and the script can be found in Appendix B.

4.4 Chapter summary

This chapter presented the mathematical formulation for a deterministic and a stochastic optimization model. Both are developed to provide an operational strategy for an EB and HP in a CHP system. The objective of the optimization is cost minimization and operational constraints such as ramping, start-up, shut-down, minimum load and storage flow were included to resemble the reality of the system. The following chapter develops a model for probabilistic forecasting of electricity price and heat demand. This allows for numerical results for the deterministic and the stochastic optimization model, just presented, to be obtained and analyzed in Chapter 6 and 7.

Chapter 5

Forecasts and scenario generation

This chapter presents a method for probabilistic forecasting of the heat load and spot price. This is necessary for solving and comparing the deterministic and the stochastic optimization model presented in the previous chapter.

Time series models describing the heat load and spot price are identified and analyzed in order to provide 38 hour ahead forecasts of the heat load and spot price. Based on the forecasts, a scenario generation method to construct scenarios for the heat load and spot price is presented. Both the forecasts and the scenario generation could have been the single subject of this type of project. Forecasting the spot price is generally considered highly complex and is investigated in numerous papers [5, 12, 41, 42]. However, this thesis is focused on modelling and optimization, thus the forecasts and scenario generation only feature the main principles based on the sparse data available, in order to construct the necessary data. The forecasts presented in this chapter are therefore not meant to be state-of-the-art, but illustrate the methods and capture the main characteristics, such that they can work as a input to the deterministic and the stochastic model.

5.1 Forecasting heat load and spot price

The data provided for the heat load comprise the expected hourly heat demand for the full year of 2013. The heat demand covers the Greater Copenhagen district heating areas supplied by VEKS, HOFOR and CTR, see Chapter 2.4. The demand forecast is each morning provided to the heat suppliers (HOFOR Kraftvarme and DONG Energy) as three individual forecasts; One for each of the areas operated by CTR, VEKS and HOFOR. Each of these forecasts are provided by different suppliers with different forecasting methods, which means that e.g. not all include the expected wind speed. No uncertainty on the forecast is provided, but the monthly deviations are between 5% and 15%[1] being highest in the spring and fall where the weather is less predictable. Figure 5.1(a) and 5.1(b) shows the provided demand forecast for one year (2013) and one week, respectively. A clear seasonal trend is apparent from Figure 5.1(a), due to the strong difference in temperature between summer and winter and thus a different need for heating of living spaces. Furthermore, a daily seasonality is observed due to specific consumption patterns during the day.

[1]Information provided by D. R. Andersen, Varmelast.dk

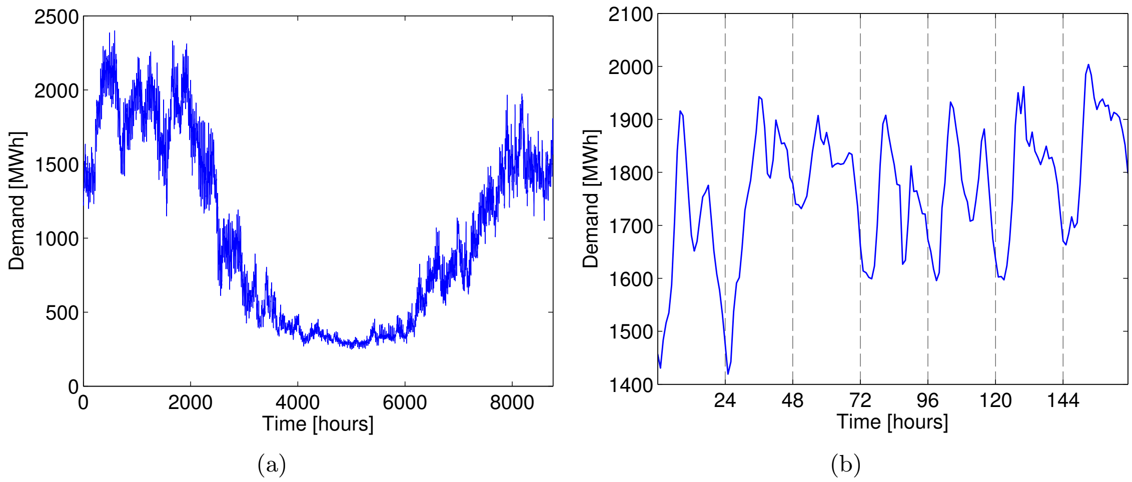


Figure 5.1 – Expected heat load for 2013 (a) and one week in 2013 (b).

Regarding the spot price, hourly realized spot price are publicly available through [43]. Figure 5.2(a) shows the hourly spot prices for 2013 and 5.2(b) shows the prices for one week in 2013. No increasing or decreasing trends are apparent on a yearly basis. Similar to the heat load, but less significant, the spot price shows a daily pattern with the price being lowest at night.

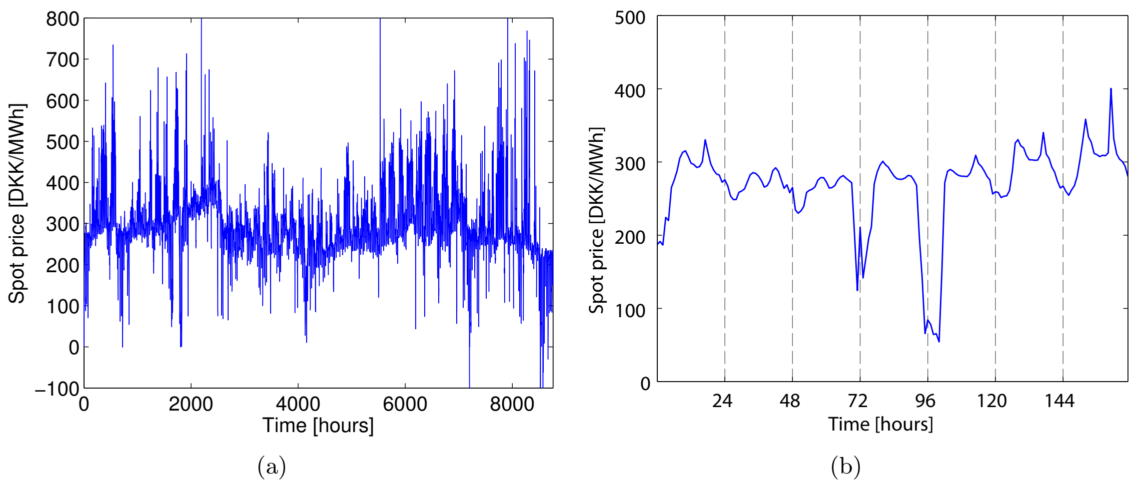


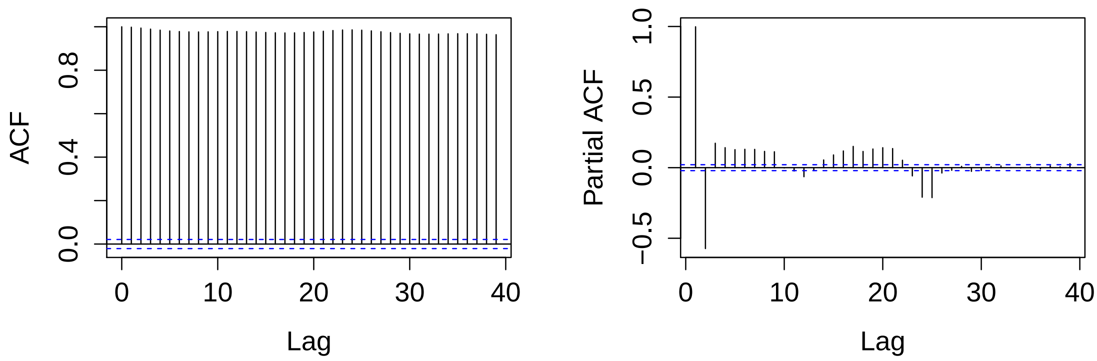
Figure 5.2 – (a) The realized spot prices for 2013. (b) Realized spot prices for one week in 2013.

5.1.1 Heat load forecast

In order to use stochastic optimization, the uncertainty on the forecast is of uttermost importance. The provided heat load forecast did not include the uncertainty of the predictions. To provide this, a new forecast is made for the heat load. This heat load forecast is based on the provided heat load forecast data, since the realized heat load was not available. Optimally, historic heat loads should be used in combination with external data inputs, such as expected outside temperature and wind speed. However, the approach presented here still captures the main idea.

No general increasing or decreasing trend is present when looking at the yearly data in Figure 5.1(a). Assuming the heat demand is a stationary stochastic process the variance is assumes to be a constant. A yearly seasonality seem to be present when looking at the yearly development in Figure 5.1(a), but in the absence of data for a second year it was not possible to account for this seasonality. From the weekly data in Figure 5.1(b) a daily pattern seems to appear.

In order to investigate this daily dependence along with other dependencies, the autocorrelation function (ACF) and the partial autocorrelation function (PACF) are calculated. A detailed explanation of the meaning of these can be found in [44].



(a) (b)

Figure 5.3 – (a) Autocorrelation function for the yearly time series of the heat load. (b) Partial autocorrelation function from which significant lags help determining the order of an autoregressive model.

Figure 5.3(a) and 5.3(a) shows the ACF and PACF of the heat demand data. The slowly decreasing ACF and significant lags in the PACF imply a seasonal autoregressive model.

In order to keep the model simple only the four most significant lags (1,2, 24 and 25) are included. This results in the following model to describe the heat load:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-24} + \phi_4 Y_{t-25} + \epsilon_t \tag{5.1}$$

where Y_t is the demand at time t, ϕ_1-ϕ_4 are parameters to be estimated and ϵ_t is a white noise process $\sim N(0, \sigma)$. The parameter estimation is done in the statistical software R using the prediction error method where the sum of squared residuals for the one-step ahead predictions are minimized [44]. Defining the one step ahead prediction error as a function of the four parameters $\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3, \phi_4)$:

$$\epsilon_t(\boldsymbol{\phi}) = Y_t - E[Y_t|\boldsymbol{\phi}, \mathbf{Y}_{t-1}) \tag{5.2}$$

where $\mathbf{Y}_{t-1}$ contains the heat load information up to $t-1$. The parameters are estimated as:

$$\hat{\boldsymbol{\phi}} = \arg\min_{\phi} S(\boldsymbol{\phi}) = \sum_{t=s+P}^{N} \epsilon_t^2(\boldsymbol{\phi})$$

where $N = 8760$ is the amount of hours in one year, s is the season and P is the number of seasonal terms included. Thus $s + P = 26$. The corresponding estimated variance is obtained as [44]:

$$\hat{\sigma}_\epsilon = \frac{S(\hat{\boldsymbol{\phi}})}{N - (p + P)}$$

where p is the order of the autoregressive model. Models of lower order, e.g. removing the Y_{t-2} or Y_{t-25} dependence, are subsequently tested using a likelihood ratio test to see if the model can be reduced further [44]. However, all of the conducted tests revealed that the dependence on Y_{t-2} or Y_{t-25} adds significant information to the model with a χ^2 value of 0.00.

The model used to forecast the heat load is hence described by (5.1) with the parameters estimated to:

$$(\phi_1, \phi_2, \phi_3, \phi_4) = (1.35, -0.40, 0.42, -0.37) \tag{5.3}$$

and the variance and standard deviation:

$$\sigma_\epsilon^2 = 728.32\,\mathrm{MWh}^2 \tag{5.4}$$

$$\mathrm{sd} = 26.66\,\mathrm{MWh} \tag{5.5}$$

A 38 hour ahead forecast is made to resemble the reality where day-ahead forecasts are provided at 10:00 on the day before. Only the last 24 hours of the forecasts are thus used for modelling.

The forecast is displayed for one week in Figure 5.4. It is observed that the model, to a certain extent, captures both of the two daily peaks as was expected when using this type of model for forecasting.

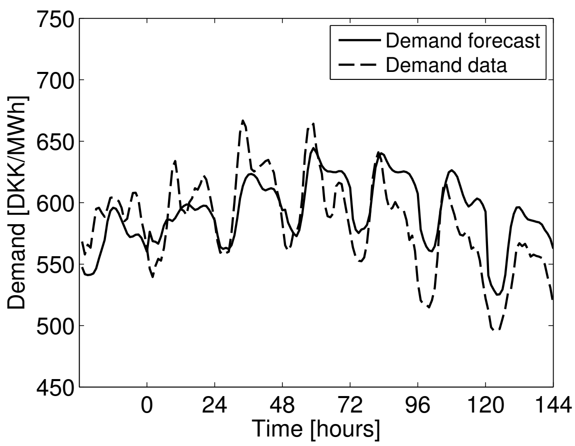


Figure 5.4 – Forecast for the heat demand together with the input data for one week.

5.1.2 Scaling the demand

The data provided for the expected heat demand is normally given by two suppliers. The system modelled in this thesis does not comprise the full production as it is modelled from the viewpoint of one supplier and. This also requires the demand to be scaled accordingly. Normally, the demand is dispatched each day as the first step in Varmelast.dk procedure, see Chapter 2.4. Optimally, the heat dispatch decision was modelled and optimized for the system including the HP and EB. This would expectedly lead to additional production dispatched as a results of increased flexibility and lower heat costs when electricity prices are low. However, this is an entire new modelling problem and not the main focus in this project. Alternatively, historical data for the dispatch of heat production could be used. However, these were not available. The heat dispatched to this system is hence found as a fraction of the total heat demand. This fraction is decided to be 30% since it results in a heat dispatch that is reasonable in relation to the real operation. Using 30% dispatch both CHP units are required during most of the winter while only the back-pressure CHP is needed in the summer. This is consistent with reality[2].

5.1.3 Spot price forecast

The power price is forecast based on the realized hourly spot prices for 2013 [45]. Figure 5.2(b), displaying the weekly power prices, indicates a daily pattern similar to that observed for the heat demand. Looking at the heat load and spot price for one week plotted together in Figure 5.5 a correlation is observed.

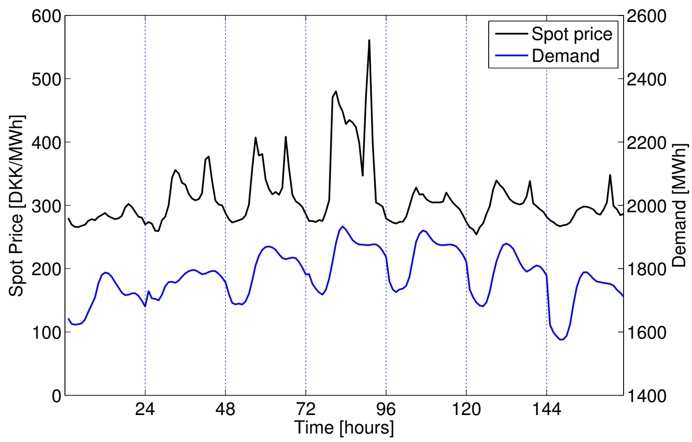


Figure 5.5 – Heat load and the spot prices for one week. A correlation between the two is observed.

The cross correlation is therefore explored and displayed in the cross correlogram in Figure 5.6. This supports the hypothesis of a correlation between the spot price and demand.

A regression model with the demand forecast as explanatory variable is therefore suggested to model the spot price. This approach is explained in detail for similar instances in [10].

Similar to the heat demand forecast, the ACF and PACF of the spot price indicates a seasonal second order autoregressive model according to the definitions in [44]. The parameter

[2]Oral conversation with J. G. Hansen, HOFOR.

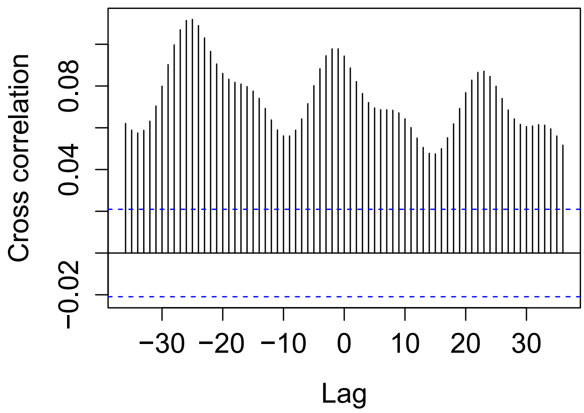


Figure 5.6 – Cross correlogram for the spot price and the demand forecast. The correlation appears to be strongest around lag 0 and for each 24 hour cycle.

estimation and check for lower model order is investigated just as for the heat demand. As no model reduction is possible, the spot price at time t, X_t, is described by:

$$X_t = \theta_1 X_{t-1} + \theta_2 X_{t-2} + \theta_3 X_{t-24} + \theta_4 Y_t \tag{5.6}$$

where the explanatory variable Y_t is the heat demand forecast at time t. The parameter values are estimated to:

$$(\theta_1, \theta_2, \theta_3, \theta_4) = (1.10, -0.26, 0.15, 0.002) \tag{5.7}$$

and the standard deviation:

$$sd = 34.2 \text{ MWh}$$

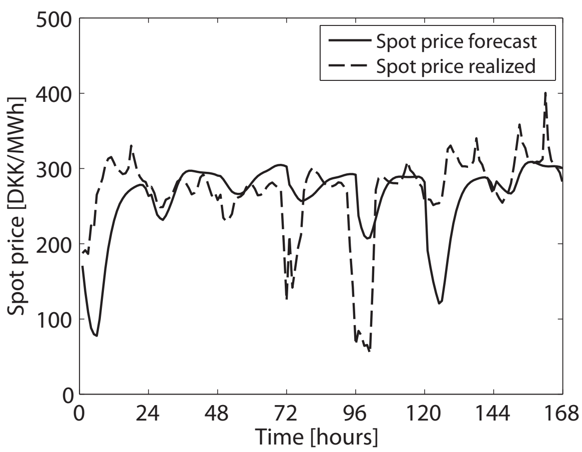


Figure 5.7 – Spot price forecast for one week on 2013. The general daily trend is seen to be captured with the model

Figure 5.7 shows the forecast together with the realized spot price. Unexpected spikes or are not captured by the model. Some of these could most likely be predicted using wind speed forecasts, since the wind power production is found to have a negative correlation with the spot prices [42].

5.2 Scenario generation

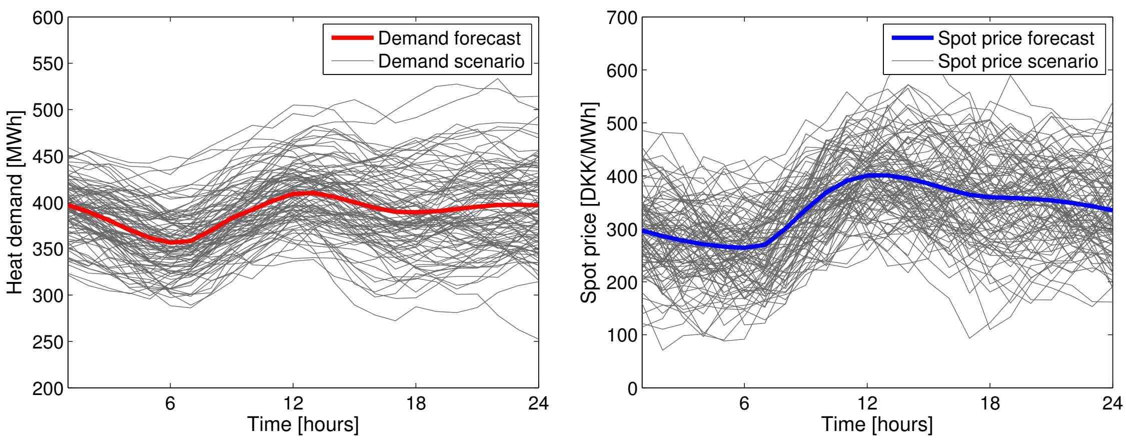
In order to be able to solve the stochastic optimization problem described in Section 4.3, the stochastic variables are represented by a number of scenarios. Each scenario corresponds to a possible realization of the stochastic process. Naturally, it is important to have sufficient scenarios such that the most plausible realizations of the stochastic process are included. However, a high number of scenarios can lead to computational intractability.

In order to generate scenarios a model describing the stochastic process is needed. These models were developed in the previous section for both the heat demand and the spot price.

The approach used to construct the scenarios is based on a number of simulations for the heat demand and spot price. The algorithm used is summarized [10]:

1. Initialize scenario counter, $s = 1$.
2. Generate error for the heat demand forecast, $\epsilon_t^{demand} \sim \mathrm{N}(0,\sigma)$.
3. Generate error for the spot price forecast, $\epsilon_t^{spot} \sim \mathrm{N}(0,\sigma)$.
4. Simulate the heat demand forecast based on model (5.1).
5. Simulate the spot price forecast based on model (5.6) and (5.7) with the heat demand simulated in step four as input.
6. Continue until all the desired scenarios have been issued.

Based on this algorithm, 100 scenarios were constructed. The point forecast is considered as the first scenario These are illustrated in Figure 5.8.



(a) (b)

Figure 5.8 – (a) Heat load in 100 simulations. The red line corresponds to the forecast. (b) 100 simulation of the spot price using the demand scenarios as input. The blue line is the spot price forecast.

5.3 Chapter summary

This chapter presented the methods used to make simple forecasts of the heat load and spot price. A seasonal time series model was identified for the heat load and the parameters were estimated using a one step ahead prediction method. A similar approach was utilized to forecast the spot price. Due to the correlation between spot price and heat load, the heat load forecast was used as an explanatory variable in the model describing the spot price. This resulted in 38 hour ahead forecasts to be used as data inputs in the deterministic and the stochastic model. Finally, a scenario generation method was developed and scenarios representing different realizations of the spot price and heat load was constructed.

Chapter 6

Model validation and analysis

This chapter is meant to provide an elaborate and illustrative analysis of the models developed in Chapter 4 and the corresponding results. The forecasts developed in the previous chapter serve as an input for the two optimization models.

The deterministic and the stochastic model are equivalent in terms of features and operational constraints. Consequently, these features are only illustrated in the first section for the deterministic model, as they can be directly transferred to the stochastic model. The section concerning the stochastic model will thus only focus on the differences between the deterministic and the stochastic approach.

6.1 The deterministic model

This section explains and illustrates the results from deterministic model. Results obtained from simple studies performed with the model are compared to analytical calculations, based on the marginal costs of the various production technologies, to emphasize the impact features such as start-up costs and ramping constraints. Furthermore, illustrative examples of demand satisfaction, power production and storage usage are provided to improve the understanding of the model.

The parameter values used for both the deterministic and the stochastic model are displayed in Table 4.3. The optimization considers a 24-hour cycle such that each day is optimized separately. However, the production and storage levels at the last hour of each day are used as a fixed input to the following day.

6.1.1 The simple model

To evaluate model results, the unit heat costs for the different production units, presented in equations (2.1), (2.2), (2.3) and (2.4) and illustrated in Figure 2.8, are re-printed in Figure 6.1. Note that the costs do not include the cost or value of factors such as start-up, shut-down, minimum load, ramping and storage. Vertical dashed lines are added in Figure 6.1 to indicate the spot price equal to zero and the spot price at the intersection between any two lines. At this intersection the most economical order of production units changes.

The critical spot prices are:

$$
\begin{aligned}
\text{CHP-EB} &: 101\,\text{DKK/MWh} \\
\text{CHP2-HP} &: 281\,\text{DKK/MWh} \\
\text{CHP2-EB} &: 307\,\text{DKK/MWh} \\
\text{HP-EB} &: 315\,\text{DKK/MWh}
\end{aligned}
$$

For instance, this means that when the spot price exceeds 101 DKK/MWh the CHP is more economical than the EB.

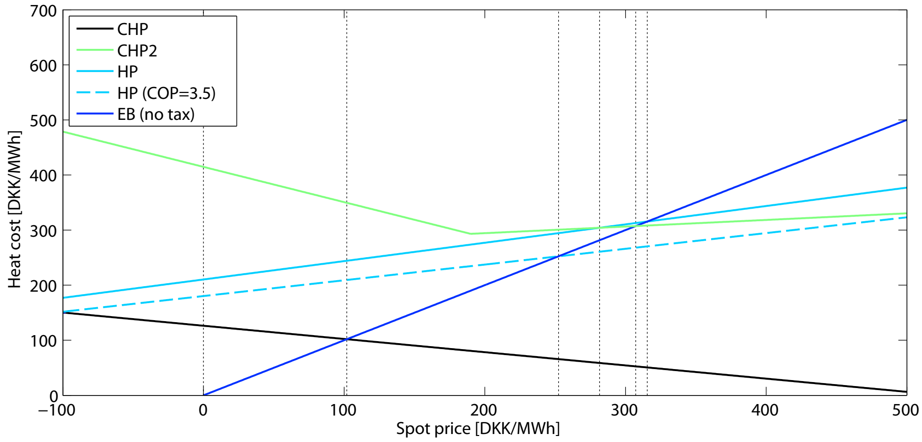


Figure 6.1 – Heat costs as a function of the spot price for different production units. Vertical dashed lines indicate the spot price for which one unit becomes superior to another in terms of costs.

As a preliminary validation of the deterministic model presented in Section 4.2, (4.18)-(4.21), (4.22)-(4.29), (4.30)-(4.32) is excluded from the model. This corresponds to ramping constraints for CHP units and storage and start-up and shut-down costs. In this way, results consistent with the above analysis based on the marginal heat production costs, should be obtained.

The production plan when solving the deterministic model for one week in February is visualised in Figure 6.2. This figure shows a stacked plot comprising all contributions to satisfy the heat demand; consequently, the heat demand corresponds to the dashed line in the top. The chosen week is a high demand week where the demand can not always be covered by the two CHP units alone. However, in February this would not be uncommon and expensive oil and gas boilers would instead cover peak loads. The spot price forecast for the same period is plotted as the black line on the right y-axis. This allows for an evaluation of the use of different production units in different spot price regimes.

Correspondence between the theoretical measures presented in Figure 6.1 and the solution shown in Figure 6.2 is clearly observed. The theoretical model suggests that the EB is cheaper than the CHP2 and HP for prices below 307 DKK/MWh and 315 DKK/MWh, respectively. In the first half of the week (hours 0 to 72) the spot price alternates between values above and below 300 DKK/MWh, and the EB is found only to run when the price is below 307 DKK/MWh. The HP does not run as frequent as the EB indicating that it requires lower prices to be economical compared to the EB and CHP2. This corresponds with the result from the theoretical calculation. These state that the HP should be superior to the CHP2 for spot prices below 281 DKK/MWh. In day two and three (hours 24-72) the

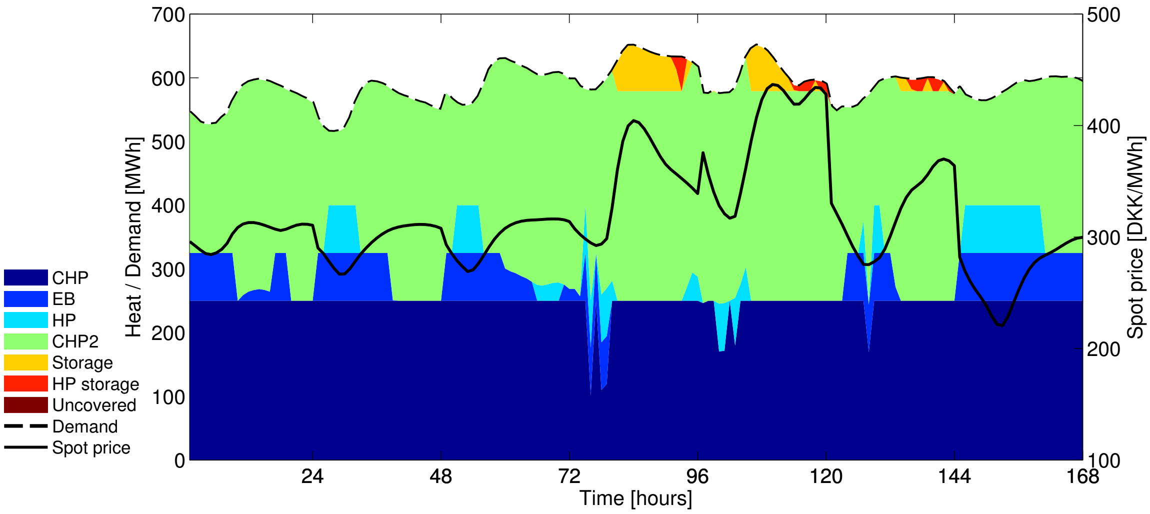


Figure 6.2 – The optimal production plan as found from solving the deterministic model for one week in February. This model is run without start-up costs, ramping constraints and minimum load requirements for comparison to results from Figure 6.1

spot price drops to this level for a number of hours and the production from the CHP2 is substituted by production from the HP. The remaining production is still covered by CHP2 due to the high demand that cannot be covered alone by the CHP, HP and EB.

At day four (hours 72-96) the spot price increases significantly and it is observed that CHP2 is producing at max capacity. This corresponds to the behavior expected from the theoretical model, where the CHP2 becomes more economical than the HP and EB at spot prices above 281 and 307 DKK/MWh, respectively.

The back-pressure CHP should, based on the theoretical results displayed in Figure 6.1, be prioritized for spot prices above 100 DKK/MWh. For the period considered here, the spot price does not fall below 100 DKK/MWh and the CHP is producing to cover the demand at full load almost constantly. The three drops in CHP production starting at the third day (hours 72-96) can be explained by investigating the production to storage in the same period, see Figure 6.3. When the CHP stops producing directly to cover the demand, it starts accumulating storage, as seen in Figure 6.2. Utilizing the storage happens for two reasons: First, the demand at this period cannot be covered by the two CHPs alone, which means that either the EB, HP or the storage should provide the remaining heat. Furthermore, the spot price is high, which makes the EB and HP expensive. The best solution is thus to use the HP and EB when the price is low in the beginning of the day and meanwhile let the CHP accumulate heat to storage. This allows the HP and EB to turn off when the spot price is high and the heat demand is lower than the total heat capacity for CHP and CHP2.

6.1.2 tart-up and shut-down costs

When start-up and shut-down costs are added, a minimum load must be introduced, such that the unit cannot produce an infinitely small amount to avoid a shut-down, and subsequently a start-up. The minimum load is fixed at 10 MW heat for the HP, and 40 MW

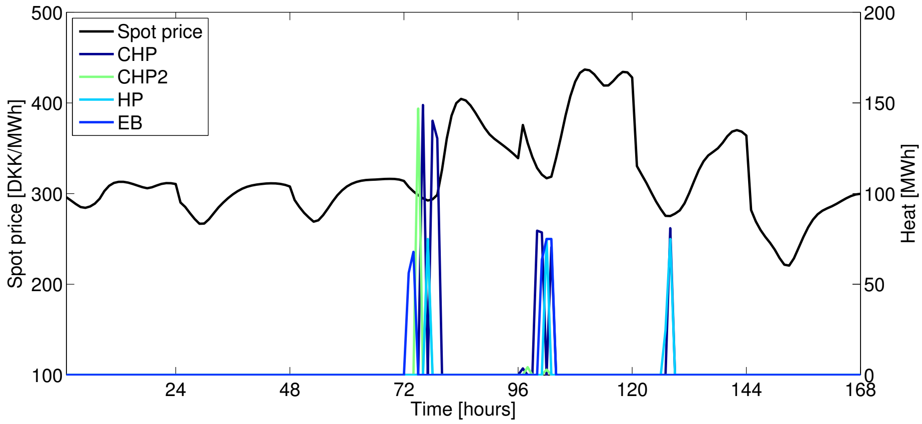


Figure 6.3 – Production from CHP, CHP2, HP and EB to storage for one week. The corresponding spot price is displayed on the left y-axis. It is observed that storage is accumulated when the spot price is at the lowest, within the day.

electricity only[1] for CHP2 and 12 MW electricity for the CHP (corresponding to the max heat ramp). The start-up and shut-down costs are as follows:

$$
\begin{aligned}
c^{su,HP} &= 2500\,\text{DKK} \\
c^{su,CHP} &= 125000\,\text{DKK} \\
c^{su,CHP2} &= 125000\,\text{DKK} \\
c^{sd,CHP} &= 125000\,\text{DKK} \\
c^{sd,CHP2} &= 125000\,\text{DKK}
\end{aligned}
$$

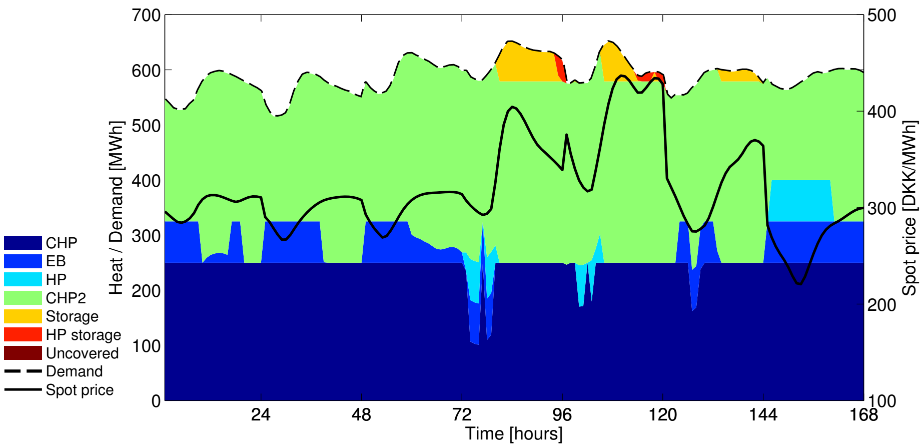


Figure 6.4 – Production to satisfy the heat demand for one week when start-up and shut-down costs are included together with a minimum load constraint. Significant changes are observed compared to Figure 6.2. Production from the HP has descreased due to the start-up costs.

The results for this model, for the same days as showed in Figure 6.2 are displayed in Figure 6.4. A few significant changes are evident. The HP does no longer produce in the first three days (hours 0-72). Also a few small HP production hours around hour 120 and

[1]Electricity production without heat production

160 have vanished. This is most likely due to the start-up costs for the HP which makes the CHP2 cheaper to use, since it is already running (and cannot be shut down due to the high demand). In order to support the idea that this follows from adding a start-up cost a theoretical calculation is performed. The missing HP production at hours 27-32 is used as a basis as the spot price here is slightly lower than during the other missing HP production hours. This should make this production the first to reappear if lowering the start-up cost to the right level.

The average spot price for hours 27-32 is calculated to:

$$p_{avg}^{spot} = \frac{1}{6}\sum_{t=27}^{32} p_t^{spot} = 272.5\,\mathrm{DKK/MWh}$$

The heat costs for the HP and CHP2 is calculated based on the theoretical costs in (2.2) and (2.4), and found to be:

$$c_{|p_{spot}=272.4}^{HP} = 301.09\,\mathrm{DKK/MWh}$$
$$c_{|p_{spot}=272.4}^{CHP2} = 303.13\,\mathrm{DKK/MWh}$$

Clearly, only a very small difference in price for the HP and CHP2 is obtained when the spot price is around 270 DKK/MWh. According to the theoretical heat costs presented earlier in Figure 6.1, the HP should be superior to the CHP if the start-up costs are reduced to:

$$c^{su,HP} = (303.13 - 301.09)75 \cdot 6 = 882\,\mathrm{DKK},$$

where 75 is the capacity of the HP and 6 is the number of hours it is on. The model was subsequently run with a HP start-up cost of 875 DKK (to account for decimal rounding). The resulting HP production is displayed in Figure 6.5 (red line) along with the results obtained with no start-up (blue area) and with the original start-up cost (blue dashed line). From this, it is clear that if the start-up costs are decreased to 875 DKK the HP gets the production at hours 27-32. However, the start-up cost is not low enough for the HP to get production at hours 49-53. In these hours the spot price is generally higher and thus, it requires a lower start-up cost for the HP to get production.

Similarly, both CHP units experience a change when adding start-up cost. However, this was not the case for the days illustrated previously. Looking instead at day 177 and 184 where the demand is significantly lower, Figure 6.6 shows a comparison of the on/off status for the CHP2 with no start-up cost (red dashed line) and with a start-up cost (blue line). It is clear that the majority of the start-up events that are present in the scenario with no start-up cost are avoided when the start-up cost is introduced. In the seven day period, a total of eight start-ups were reduced to one, highlighting the impact of the start-up and shut-down cost. The shut-down cost furthermore ensures that the unit does not shut-down just at the end of the 24 hour planning horizon and thereby induce a start-up for the next days production.

Generally the CHP2 is designed to either be turned on for a longer period or turned off for a longer period. It is only in certain transition periods the CHP2 has several shut downs and these are limited by the initial start-up cost.

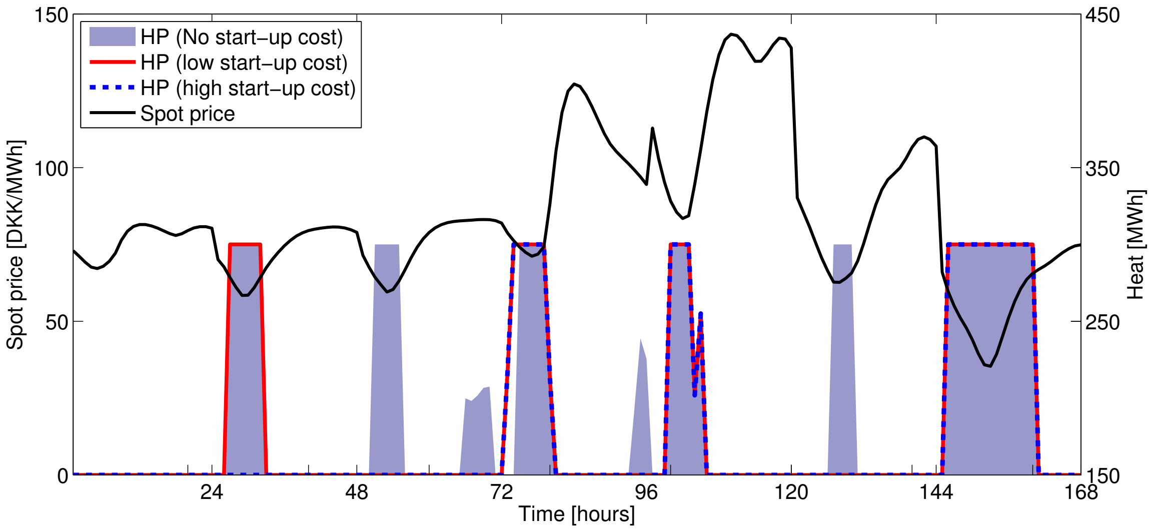


Figure 6.5 – HP production to cover demand with no startup cost, a start-up cost of 875 DKK and a start-up cost of 2500 DKK. The HP production decreases significantly when the high start-up cost is applied.

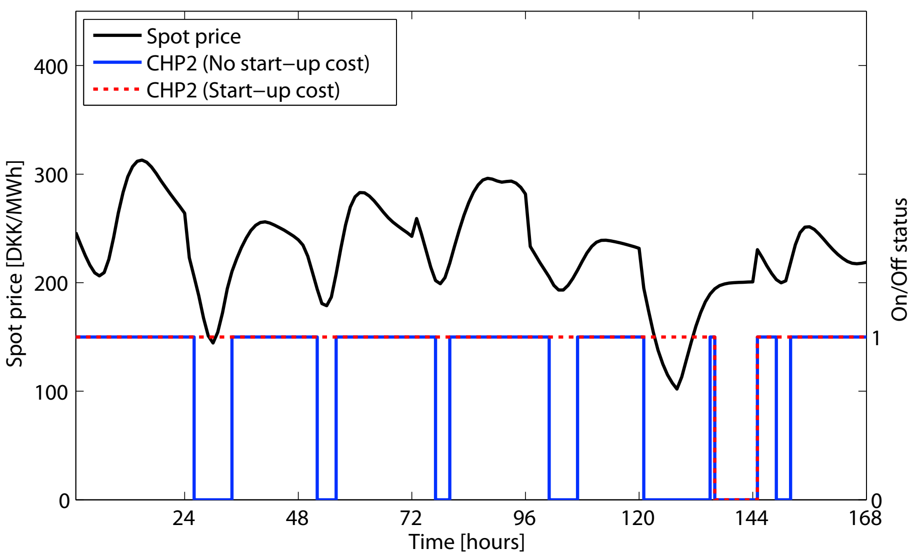


Figure 6.6 – CHP2 production at day 177 to 184 with and without CHP start-up costs of 125000 DKK. A significant reduction in the number of start-ups is observed when introducing the start-up cost.

6.1.3 The full model

Additional complexity is added to the model in the form of ramping constraints limiting the increase or decrease of the production between two consecutive hours. Due to the high capacity of both CHP units they cannot go from zero to full load in one hour. Generally the HP also has a limited ramping ramping, but since it has a significantly smaller capacity, it can still reach full capacity significantly faster than the CHP units. The ramping constraint is therefore not of the same importance for the HP and is left out.

Figure 6.7 shows the optimal heat production plan when the full model, including start-up, shut-down, storage flow and ramping is solved. For this specific week, the ramping and storage flow constraints do not have much impact. However, when looking at days in the summer, they have a significant influence. Figure 6.8 shows the power production from the CHP during one week in the summer (day 140-146). The ramping constraints are nearly always limiting indicating the importance of this constraint.

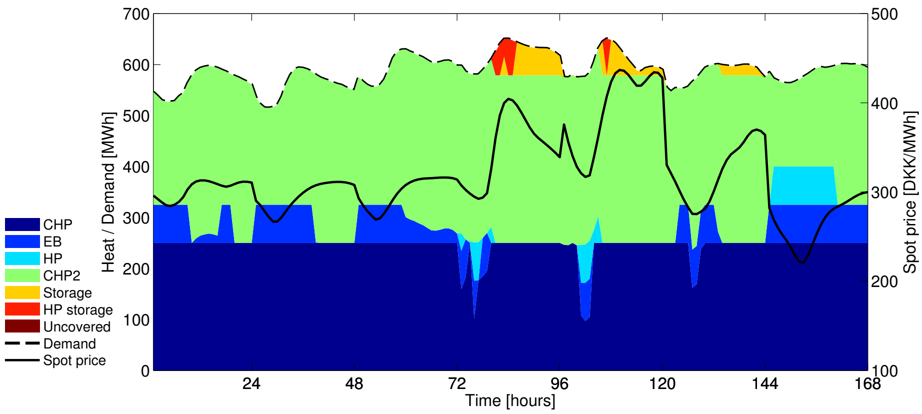


Figure 6.7 – Optimal production plan to satisfy the demand when solving the full deterministic model.

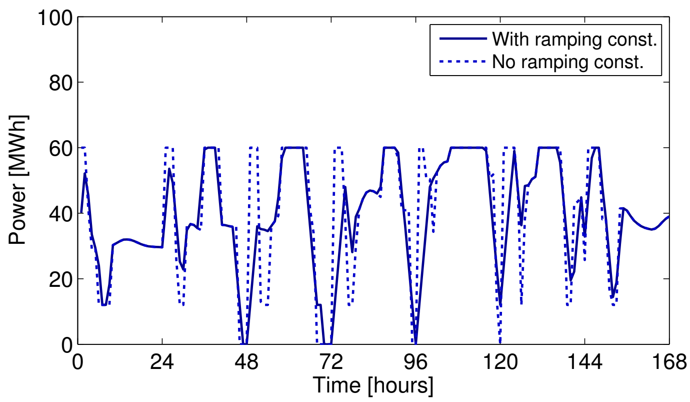


Figure 6.8 – Power production for the CHP during one summer week (day 140-146). The influence of the ramping constraint is clearly visible.

Finally, the power production from the CHP units as well as the power consumption by the HP and EB is displayed in Figure 6.9. The CHP2 is found to produce just around the maximum heat capacity. This is very reasonable due to the high heat demand and the high efficiency when running in this point. When the spot prices are high the CHP2 generally lowers the heat production slightly, even though it is more favorable to produce heat at this time. This is explained by simultaneous drops in power consumption by the EB. At high prices, the EB is very expensive and it is most economical to stop the EB and produce the remaining heat at the CHP2. The HP consumes power in accordance with the heat production displayed in Figure 6.7. Finally, the CHP has a constant power production of 60 MW, corresponding to the constant maximum heat production.

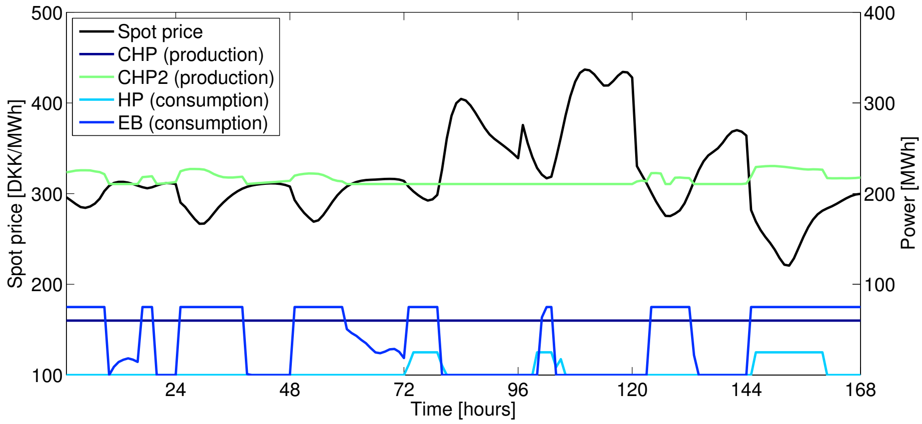


Figure 6.9 – Spot price, left y-axis, and power production from the two CHP unit as well as the power consumption by the HP and EB, right y-axis. A correlation between the spot price and the power production is observed.

6.1.4 Yearly heat production

Solving the deterministic model for the full year of 2013 results in the heat production schedule displayed in Figure 6.10. The months indicate the time of the year while an hourly resolution is used in the plot. The back-pressure CHP is generally used throughout the whole year. During the summer time this unit is generally the only to produce heat. The extraction CHP2 is mainly used during winter, early spring and late autumn. Both observations are consistent with the production patterns for HOFORs two CHP units at Amagerværket[2]. The HP and EB appear in the production schedule as well, mainly during winter, spring and autumn. Reviewing the yearly data for the spot price in Figure 5.2(a), it can be seen that winter suffers from higher volatility compared to the other seasons. This results in more frequent occurrences of a low spot price. Combined with the high heat demand in the winter, this explains the occurrence of the HP and EB production during these periods.

6.1.5 Increased COP for the HP

Throughout the previous examples, the COP of the HP was fixed at $COP^{HP} = 3.0$. Many HPs have a COP around 3, but different values can be obtained as outlined in section 2.1.3. In addition, the COP of a HP generally depends on the required forward temperature resulting from e.g. the change in outside temperature, which vary throughout the year. As a consequence, the required forward temperature of the produced water vary as well.

[2]Presentation at HOFOR

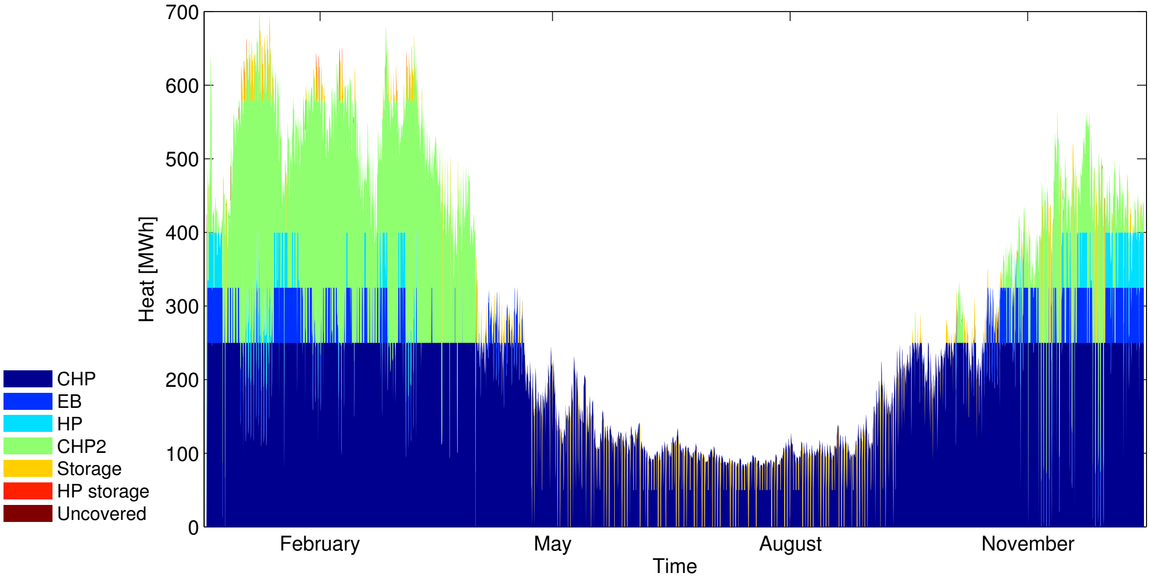


Figure 6.10 – Heat production schedule for 2013, resulting from the deterministic optimization model.

In addition to a decreased electricity consumption the resulting tax per MWh heat would decrease since the applied tax is per electricity input. Figure 6.1, which displays the theoretical heat costs, suggests that increasing the COP to 3.5 will have a significant impact on the production schedule. The HP will for spot prices up to 250 DKK/MWh and 500 DKK/MWh, be superior to the EB and CHP2, respectively, see Figure 6.1.

The production schedule after increasing the COP of the HP is displayed in Figure 6.11. Comparing to the results obtained with a COP of 3.0, see Figure 6.7, significant changes are observed. In accordance with the theoretically based expectations, the HP is now superior to the CHP2 during all seven days of the test period and is scheduled for full heat production. The clear correspondence between analytical calculations and simulation results concludes this section. The next section continues analyzing operational results obtained from the stochastic optimization model.

6.2 The stochastic model

This section presents and discuss simulation results from the stochastic model. Differences between the stochastic and the deterministic simulation results are highlighted to signify the impact of the stochastic modelling approach.

The stochastic model accounts for the uncertainty in the spot price and heat demand. Benefits from stochastic optimization appears when the objective function is asymmetric, and for this type of model considered here, start-up costs, minimum load requirements and ramping constraints are examples of features which contributes to an asymmetric objective function.

The features of the stochastic programming model, presented in Section 4.3, are illustrated through an analysis of the optimization results from two days in November. Based on the

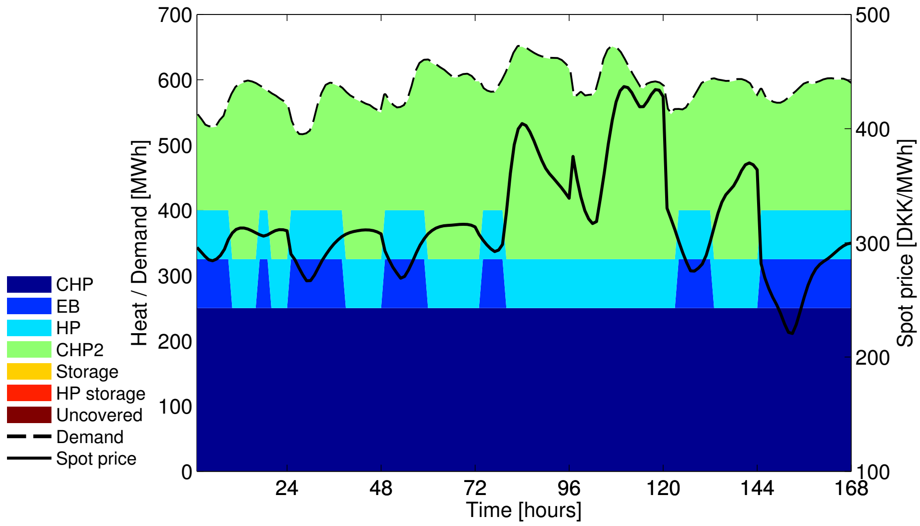


Figure 6.11 – Heat production plan when using a COP of 3.5 for the HP. Comparing to the case with a COP of 3, as displayed in Figure 6.7, the HP experience a significant increase in production. For the investigated days, it is superior to CHP2 at all times and produces at maximum capacity.

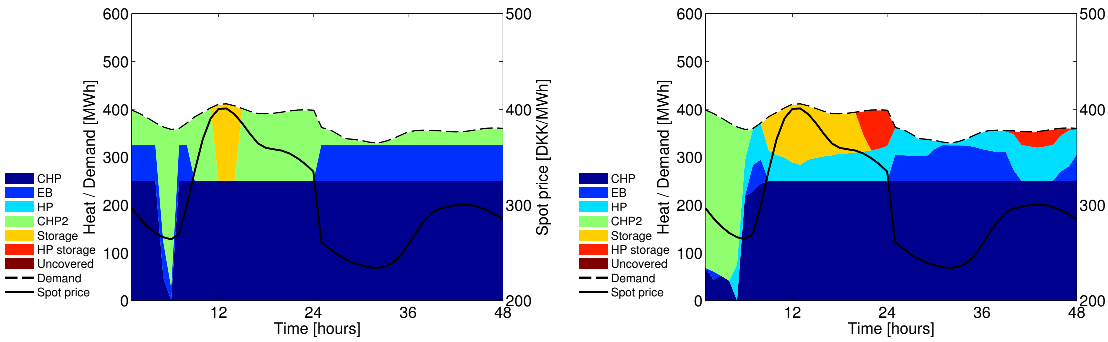
scenario generation method described in Section 5.2, 100 scenarios for the spot price and heat demand are generated and used in the stochastic programming model. To achieve results within a reasonable time frame of 5-15 minutes, the model was solved with an optimality gap equal to 0.5%.

6.2.1 Scheduled heat production

The parameter values for the deterministic model presented in Table 4.3, are used for the stochastic model as well. The second stage variables are fixed to zero for the scenario corresponding to the heat demand forecast (expected value). This corresponds to requiring the first stage solution to satisfy the expected value for the heat demand. Furthermore, it should be noted that the net production of electricity cannot change from the first to the second stage. If either of the CHP units are to produce more electricity than initially planned, the EB, HP or the other CHP is required to increase the production accordingly.

Figures 6.12(a) and 6.12(b) show the scheduled heat production when solving the deterministic and stochastic model, respectively, for two days in November. Significant differences are observed comparing the two figures. The deterministic model results, Figure 6.12(a) show a lower storage use compared to the stochastic model results. Generally, a storage provides more flexibility, which is central in the stochastic optimization model, as the system is optimized such as to adapt to different realizations of the demand and spot price. In addition, it is observed that the use of the storage in both the deterministic and stochastic production schedules occurs when the spot prices are high and the EB and HP consequently are more expensive to operate. Around hour 12 the deterministic solution does not schedule heat production to CHP2. This could be due to the high power price, which makes it more favorable to produce power.

The use of the storage in the stochastic simulations is investigated further to identify which and when units supply heat to the storage and the possible correlation between storage usage

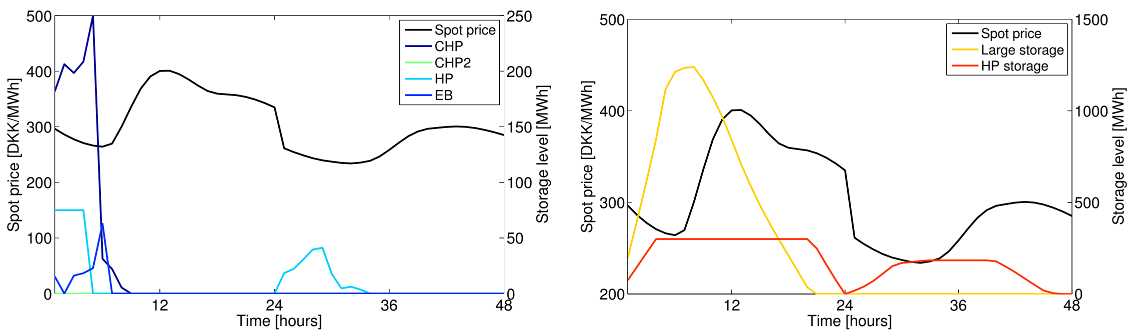


(a) (b)

Figure 6.12 – Planned heat production as obtained from the deterministic model (a) and the stochastic programming model (b) for two days in November. For the stochastic model this corresponds to the first-stage decision.

and spot price. Figure 6.13(a) shows the heat production from all units to the storages as well as the spot price forecast for the same hours. In the first hours, the CHP is used to supply the storage, which explains its absence from the first hours in Figure 6.12(b). It is moreover observed that production to storage occurs during hours of low electricity price, which coincide with the first hours of each day.

Figure 6.13(b) shows the development of the storage level for both storages. Both the HP storage and the large storage are utilized as expected from Figure 6.13(a). Furthermore, the storage level is found consistent with the storage use in Figure 6.12(b).

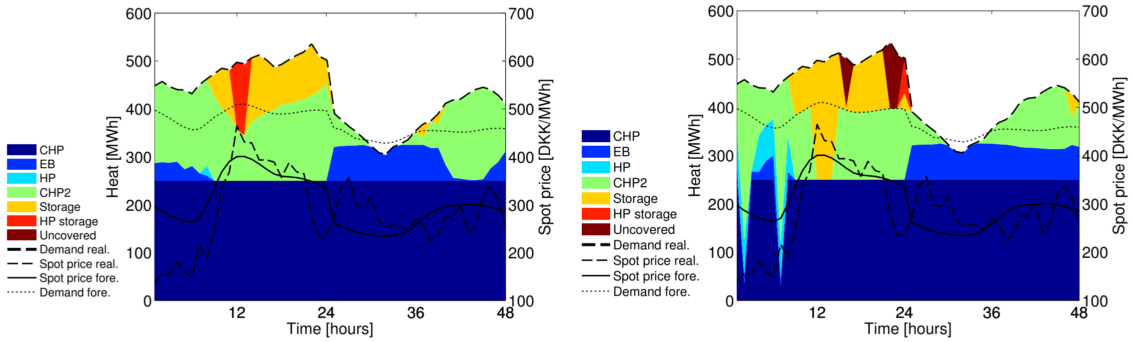


(a) (b)

Figure 6.13 – (a) Planned heat production to the two storage units based on stochastic optimization results. (b) Storage level for both the large and small HP storage units when solving the stochastic optimization model for two days. The large storage receives heat from both CHPs and the EB, whereas the HP storage only is supplied by the HP.

6.2.2 High-demand realization

Naturally, it is of interest to investigate how the deterministic and the stochastic model adjust the production to meet the realized demand. Figure 6.14(a) and 6.14(b) shows the final production plan from the stochastic and the deterministic model, respectively, to meet the heat demand in the realization of one scenario. For the stochastic model this corresponds to the addition of the second stage decision. The forecast for the heat demand and spot price are included in the figure together with the realization in the specific scenario. The scenario constitutes a high-demand scenario where the realization for the demand is higher than the expected value.



(a) (b)

Figure 6.14 – Heat production plan, resulting from the stochastic programming model (a) and deterministic in-sample (b), to meet the heat demand in one realization of heat demand and spot price.

A significant difference between the production plans resulting from the deterministic and the stochastic model, is the occurrence of uncovered demand in the deterministic model results, Figure 6.14(b). This is generally very undesirable, as it might lead to the start-up of expensive and non-sustainable gas and oil boilers. The reasoning behind this behavior is found when reviewing the initial heat production schedule presented in figure 6.12 as well as the corresponding sold and consumed power, displayed in Figure 6.15. The latter compares the scheduled accumulated power production and consumption resulting from the deterministic (Det) and stochastic (Stoch) models.

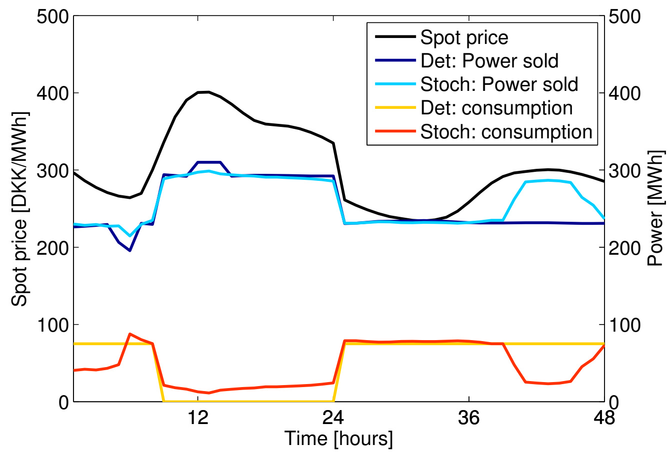


Figure 6.15 – Comparison of power production (to be sold) and power consumption in the deterministic and stochastic optimization results.

The limiting factor between the first and second stage is the requirement of a constant net power production. This means that any additional power production in the second stage should be matched by an increase in power consumption of equal size. In Figure

6.15 at hour 12, a difference between the stochastic and deterministic power production to be sold is observed. According to the deterministic optimization results, 310 MWh power (maximum total power production from CHP and CHP2) is sold at this time. On the contrary, the results from the stochastic model show a smaller amount, approximately 300 MWh. Furthermore, a planned power consumption around hour 12 is only apparent in the stochastic schedule.

Full power capacity is already sold from CHP2, which means it is not able to produce any heat. Furthermore, the HP and EB are limited as they require an additional power consumption, not available in this system. With the heat demand being realized at a higher value the CHP2 is not able to produce heat, as this would require a decrease in net power production. This explains the uncovered demand in the deterministic solution for the realization of a high demand scenario. Furthermore, this example illustrates the benefits of stochastic compared to deterministic optimization.

6.3 Chapter summary

This chapter analyzed and discussed results from the deterministic and the stochastic optimization model. Analytical calculations of marginal heat production costs as a function of the spot price were presented, and compared to a simplified version of the deterministic model. The comparison showed an agreement between the simplified model and analytical calculations.

Extending the model to its full form, i.e. taking into account start-up, ramping etc. allowed for comparisons and further analyses to be made. Based on the presented results it is concluded that the deterministic model provides adequate results, where ramping constraints and start-up costs are important features. This allows for the subsequent use of the model to provide numerical result presented in Chapter 7.

Additionally, the stochastic model was investigated for two days in November, using 100 scenarios for the heat demand and spot price as input. The heat production schedule was compared to the schedule obtained with the deterministic model. Especially, an increased use of storage was observed, consistent with the increased need for flexibility due to the presence of uncertainty. A high-demand scenario, representing one realization of the heat demand and electricity price, was analyzed. This resulted in uncovered heat demand in the deterministic optimization result, while the stochastic model was still able to meet the heat demand. Analyzing the corresponding power production, allowed for an explanation of the uncovered demand. This emphasized the difference between the stochastic and the deterministic model.

Both the deterministic and the stochastic optimization model were found to provide reasonable results consistent with intuitive, analytical and numerical expectations. The results from the models, thus provide an operational strategy for the introduction of a HP and EB in the Nordic power market and Copenhagen district heating system.

This allows for an analysis of the monetary benefits of stochastic optimization as opposed to deterministic which is presented in the following chapter together with numerical results for the impact of HPs and EBs.

Chapter 7

Numerical results

This chapter presents numerical results from solving the deterministic and the stochastic model developed in Chapter 4. The difference in heat production costs obtained from the stochastic and the deterministic model are analyzed and the influence of different production capacities for the HP and EB is investigated. Numerical results from three case studies, based on the stochastic model, are presented, analyzed and mutually compared to each other.

7.1 Computational performance

Optimally, the comparison between the stochastic and the deterministic model should be made considering a full year. However, the stochastic model is cumbersome to solve with just 100 scenarios. Consequently, instead of reducing the number of scenarios, the model is only solved for four different weeks, chosen to represent the yearly variation. This means that the first week of February, May, August and November is used. The computation time for solving a single day using the stochastic model vary from a few minutes to more than an hour. An optimality gap of maximum 0.5% is therefore accepted as well as a time limit of one hour for each day is used in the solver. This might lead to a small deviation from the optimal solution for the stochastic model, and a slightly suboptimal solution. This will favor the deterministic model slightly and reduce the benefits from the stochastic optimization model.

The solver, CPLEX, was used together with the interface program GAMS to solve the developed optimization models that in their nature are mixed integer linear programs (MILP). The computer used has an Intel® Core™ i5 processor at a clock-speed of 1.7 GHz, 4GB ram and Windows 8 64 bit.

7.2 Deterministic and stochastic comparison

In order to compare the stochastic and the deterministic model, an in-sample approach is used. This means that the deterministic solution is used as a fixed first stage solution in the stochastic model. Subsequently, this model is solved resulting in the optimal second stage behavior for the deterministic model, given the initial solution based on the expected value for the heat demand and power price. This allows for a comparison of the numerical results from the deterministic and the stochastic model.

7.2.1 Model comparison

The first case study concerns the deterministic and the stochastic model described in Chapter 4. The in-sample approach, described above, is here used to compare the two types.

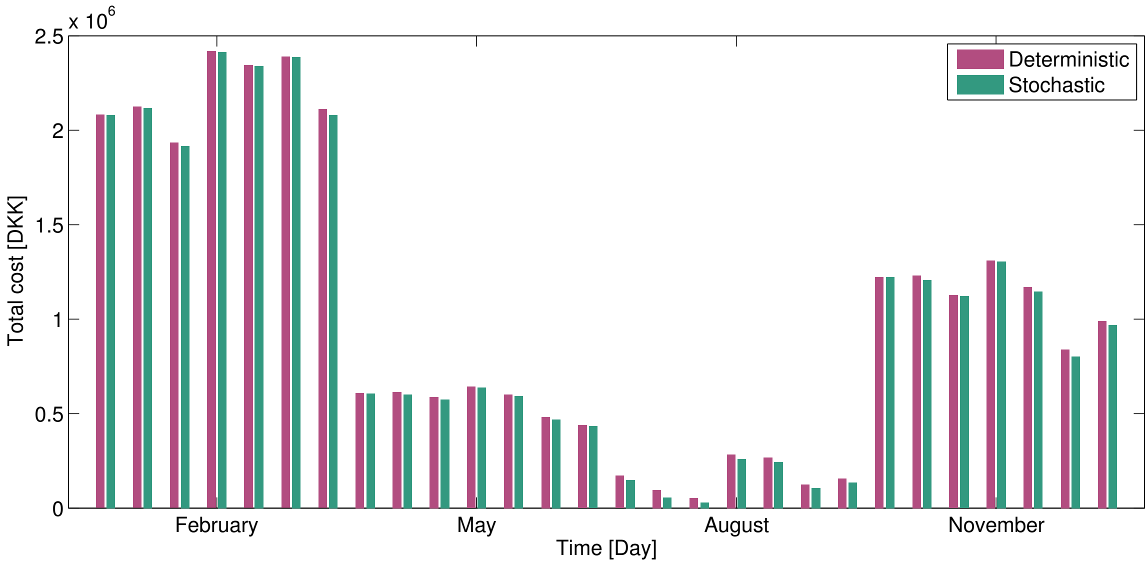


Figure 7.1 – Comparison of the stochastic model results and the deterministic in-sample results for the first week of February, May, August and November 2013. A small difference between the stochastic and deterministic model results are observed, most prominant during the days in August.

Figure 7.1 shows the total daily heat costs, for the first week of February, May, August and November for the deterministic in-sample and the stochastic model. The differences between the stochastic and the deterministic model costs are numerically small during all weeks. However, the relative differences are found higher in August compared to other weeks. Calculating the difference between the stochastic and the deterministic model costs for each of the weeks, as a percentage, results in:

$$w_{Feb} = 0.5\% \tag{7.1}$$

$$w_{May} = 1.4\% \tag{7.2}$$

$$w_{Aug} = 17.4\% \tag{7.3}$$

$$w_{Nov} = 1.5\% \tag{7.4}$$

The reason for these very small differences in February, May and November might be due to the high degree of flexibility present in the system. As long as the net power production is not changed, the production units can adjust production according to the demand and spot price realization. In Chapter 5 the standard deviation of the heat demand was found in (5.5), to $\sigma_\epsilon = 26.66$ MWh. As this standard deviation is small relative to the capacity of the HP and EB, the system might be flexible enough for the deterministic model to handle the deviations almost good as the stochastic optimization model. In May, the demand is very low, and the CHP is generally the only unit used for heat production, as was seen in Figure 6.10. In this situation the flexibility is limited and thus a higher benefit from using the stochastic model is experienced.

7.2.2 Capacity impact

This section carries out an analysis to investigate the impact of the HP and EB capacity on the appropriability of stochastic as opposed to deterministic optimization.

A number of relevant scenarios are selected and both the stochastic and the deterministic in-sample model are solved for each scenario. The three scenarios chosen are:

1. 100% capacity for both the HP and EB (reference case 0)
2. 50% capacity for both the HP and EB
3. 0% capacity for both the HP and EB

A comparison of the three scenarios is displayed in Figure 7.2. For each scenario and week, the difference between the deterministic and the stochastic model results are found as a percentage of the stochastic result. Thus a positive difference implies that the stochastic model provides lower heat costs compared to the deterministic in-sample.

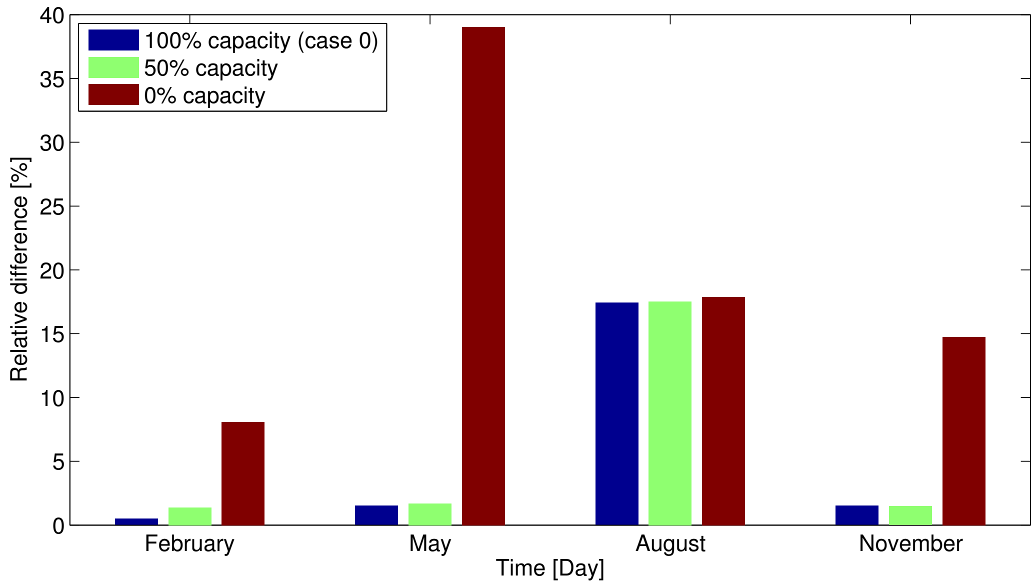


Figure 7.2 – Comparison of the stochastic results and the deterministic in-sample for the first week of February, May, August and November 2013. A large positive value signifies that the stochastic model results in lowered heat costs.

For January and November the same pattern appears. Production at full capacity on the HP and EB results in a small relative difference. However, these months experience the highest costs due to a high demand, see Figure 6.10. The high demand forces many production units to operate simultaneously, which increase the flexibility.

Decreasing the capacity of the HP and EB, the impact of the stochastic approach increases. Especially when the capacity for the EB and HP is fixed to zero, i.e. removing the HP and EB from the system, the stochastic solution is significantly better than the deterministic, which again reflects the influence of system flexibility.

In May and August the benefits from the stochastic model are generally higher and almost constant for August. This is likely due to the low heat demand during these months and the corresponding choice of production unit. Section 6.1 and 2.2 illustrated how the back-pressure CHP was most economical at almost any electricity price. During periods with low heat demand, such as May and August, the back-pressure CHP preferably satisfies the demand solely and the HP and EB do not get operating hours, see Figure 6.10. Therefore, a change the in capacity does not influence the results significantly compared to periods where the HP and EB are used frequently.

This analysis, thus, indicates a negative correlation between the capacity of the HP and EB, and the significance of stochastic optimization, where capacity reduction results in increased benefits from the stochastic model compared to the deterministic model. This should be taken into consideration when deciding specific HP and EB capacities for a system.

7.3 Case studies

This section presents a number of case studies and compares the monetary benefits obtained in each case. The comparison is made based on stochastic solutions only.

Case 0 denotes the reference case, which is the result of using the stochastic optimization model presented in Section 4.3 with the parameter values in Table 4.2.

In addition to the reference case, three additional case studies are carried out. These are primarily chosen such as to investigate the impact of the HP and EB - especially related to an investment decision and prospective changes resulting from increased wind power penetration. The case studies therefore are the following:

1. Change in capacity for the EB and HP: A 50% capacity is compared to the reference case of 100% capacity and the case of 0% capacity. This will show if the costs increase linearly with additional HP and EB capacity.
2. Change COP for the HP: The COP of the HP is changed to 2.5 and 3.5. In the previous chapter, the example in section 6.1.5 indicated an increase in production hours for the HP resulting from an increase in COP.
3. Decrease the electricity price. This is expected in the future as a result of increasing wind power penetration [41].

7.3.1 Case 1: Capacity reduction for HP and CHP

In the reference case 0, a heat capacity of 75 MW, is used for both the EB and HP. This size is based on an, although relatively large, yet realistic, achievable size[1]. However, it is not necessarily the most optimal size. It might be, that the system does not need this amount of

[1]Discussion with T. Engberg, Chief Project and Market Manager, COWI.

flexibility at this moment. As the investment depends strongly on the implemented capacity, it is important to find the most optimal capacity. This case study investigates the effect of reducing the HP and EB capacity to 50% and 0%, meaning that the capacity parameters takes the following values:

$$
\begin{aligned}
C^{HP}_{100\%} &= 75 \text{ MWh} \\
C^{EB}_{100\%} &= 75 \text{ MWh} \\
C^{HP}_{50\%} &= 37.5 \text{ MWh} \\
C^{EB}_{50\%} &= 37.5 \text{ MWh} \\
C^{HP}_{0\%} &= 0 \text{ MWh} \\
C^{HP}_{0\%} &= 0 \text{ MWh}
\end{aligned}
$$

Figure 7.3 shows a comparison of the results when solving the stochastic optimization model using the above parameters. For each of the weeks solved, the average daily total heat costs are displayed. The results indicate a non-linear relationship between HP and EB capacity, and the heat costs. There is a significant cost increase when the HP and EB are removed. The difference also appears to be largest in February and November, which is the period where the units generally are utilized the most, see figure 6.10.

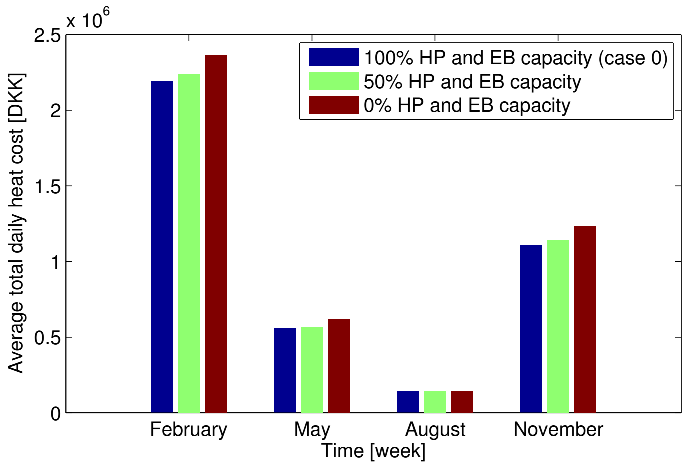


Figure 7.3 – Comparison of average daily heat costs for different cases of HP and EB capacity. A significant increase in the heat cost is observed when the HP and EB have 0% capacity, corresponding to being removed from the system.

The average daily monetary savings having 50% and 100% capacity as opposed to 0% is found based on the four investigated weeks:

$$
\begin{aligned}
r^{avg,day}_{50\%} &= 67.0 \times 10^3 \text{ DKK} \\
r^{avg,daily}_{100\%} &= 88.7 \times 10^3 \text{ DKK}
\end{aligned}
$$

This corresponds to cost reductions of 6.2% (50%) and 8.2% (100%) compared to the zero capacity case. The yearly benefit from 100% and 50% HP and EB capacity is estimated based on the four week sample that is assumed representative for the behavior in a year:

$$z^{avg,year}_{cap,50\%} = 24.4 \times 10^6 \text{ DKK}$$
$$z^{avg,year}_{cap,100\%} = 32.3 \times 10^6 \text{ DKK}$$

It should be noted that these results are based on the average cost for the 100 scenarios that are used. However, the economical value the HP and EB was found to provide is still in same order of magnitude as found in a a study made by HOFOR concerning the economical feasibility of HPs [18].

The benefit of doubling the capacity from 50% to 100% results in a cost reduction of:

$$32.3 \times 10^6 \text{ DKK} - 24.4 \times 10^6 \text{ DKK} = 7.9 \times 10^6 \text{DKK}.$$

It is therefore clear that the first increase in capacity from 0% to 50% is more significant than the additional capacity from 50% to 100%.

These results and estimates assume that the stochastic optimization is used to schedule the heat and power production. The monetary benefit are expected to be smaller when using the deterministic model, especially when the HP and EB capacity is reduced as illustrated in Section 7.2.

Furthermore, this suggests that the capacity maybe could be reduced, as the economic benefit decreases when the capacity increases. In order to evaluate this properly the investment costs should be considered as the unit capacity cost decrease for larger units [23]. A number of additional simulations for different capacities should be carried out and analyzed to find the optimum between investment cost and heat cost reduction.

7.3.2 Case 2: Change in COP for HP

In the reference study (case 0) the COP for the HP was set to $COP^{HP} = 3.0$. This constitutes a realistic value. However, both higher and lower values for the COP could occur depending on characteristics of the HP and the choice of the cold heat source.

In the following case the COP for the HP is one by one increased to 3.5 and decreased to 2.5, in order to investigate the impact on the heat costs. This allows for an assessment of the influence of a COP variation of 0.5. This could, in addition, correspond to the yearly deviation of the COP due to varying temperature requirements or variations in the cold source temperature [16]. Figure 7.4 shows a comparison of daily average heat cost with a COP of 2.5, 3.0 and 3.5.

The daily monetary benefits are here averaged to be:

$$z^{avg,day}_{COP=2.5} = 17.7 \times 10^3 \text{ DKK}$$
$$z^{avg,day}_{COP=3.5} = 27.1 \times 10^3 \text{ DKK}$$

while the yearly average estimates are:

$$z^{avg,year}_{COP=2.5} = 6.5 \times 10^6 \text{ DKK}$$
$$z^{avg,year}_{COP=3.5} = 9.9 \times 10^6 \text{ DKK}$$

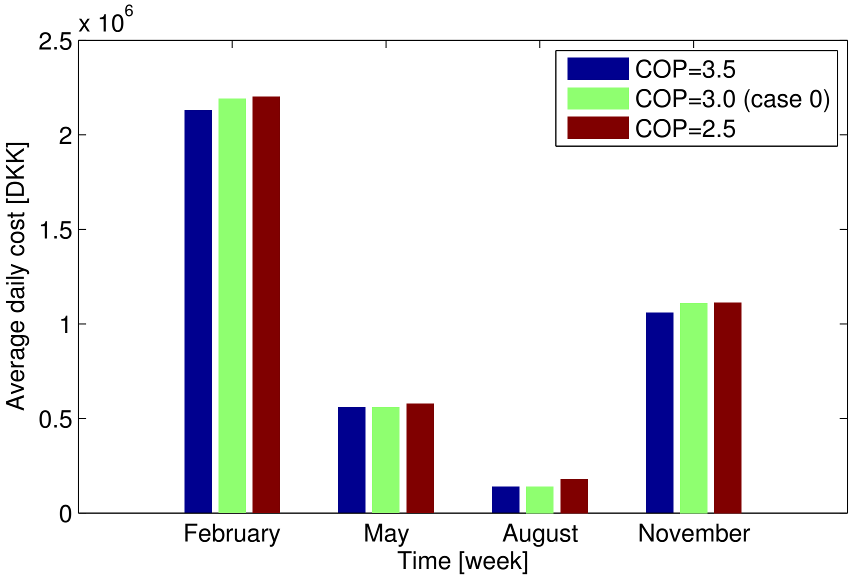


Figure 7.4 – Results from using a COP for the HP of 2.5, 3.0 and 3.5, respectively. The simulation has been carried out for four weeks in 2013 and the average daily cost are found for each week. A decrease in total costs is observed when a COP of 3.5 applies.

These results could potentially be used as input to the decision process concerning the choice of HP characteristics. If an increase in COP from 3.0 to 3.5 is achievable at costs similar to the yearly costs outlined above, the payback time for this additional investment is one year.

7.3.3 Case 3: Electricity price decrease

In this case, the effects of an increasing share of wind power and thus decreasing electricity prices are investigated. The low electricity price will in reality usually occur when there is high wind power penetration [5]. However, for this simple study it is assumed that all electricity prices are lowered by the same amount. This allows for a study of the electricity price impact on the economical benefits of HPs and EBs. Two scenarios, in addition to the reference, are investigated. In both scenarios the electricity price is lowered. However, one scenario does not include a HP and EB in the system. The two scenarios are characterized by the following parameters:

1. $p_{t,\xi}^{spot,red} = p_{t,\xi}^{spot} - 50\,\text{DKK/MWh}$
   $C^{HP} = C^{EB} = 75$ MW

2. $p_{t,\xi}^{spot,red} = p_{t,\xi}^{spot} - 50\,\text{DKK/MWh}$
   $C^{HP} = C^{EB} = 0$ MW

Figure 7.5 shows the average daily heat costs in the two scenarios compared to the reference case 0. If the electricity prices decrease, a significant increase in the total cost is observed. This can be explained from the high forced production of power at the CHP plants which is sold at a low price.

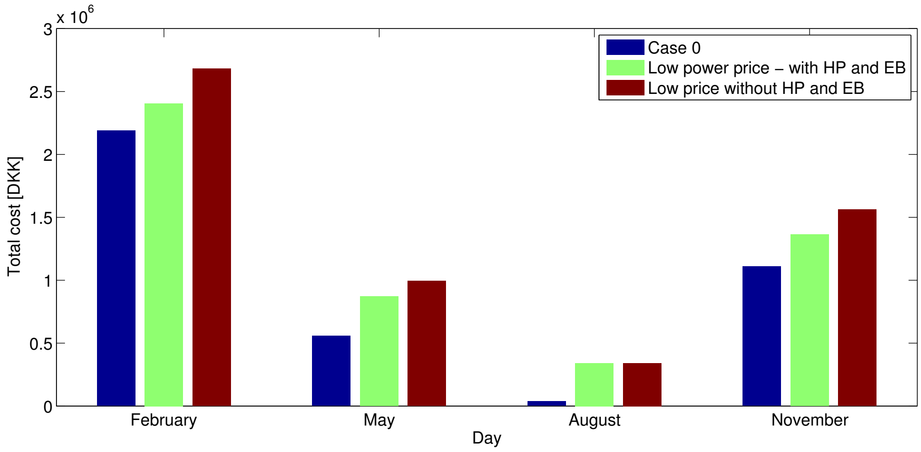


Figure 7.5 – A comparison of the average daily heat costs with and without a HP and EB, if the electricity prices are decreased by 50 DKK/MWh.

The average daily benefit of having an HP and EB, in the case that the spot price decreases by 50 DKK/MWh, is calculated to be:

$$z_{power}^{avg,day} = 148.9 \times 10^3 \text{ DKK}$$

while the yearly estimate is:

$$z_{power}^{avg,year} = 54.4 \times 10^6 \text{ DKK}$$

This clearly indicates a large economical potential for HPs and EBs in the event of decreasing power prices. Considering the investment of HPs and EBs this should be included as the power prices most likely will reach lower levels with the increasing wind power production. Additional simulations, where the spot price was decreased according to hours of increasing wind power production, could be useful in determining the full potential of HPs and EBs under these circumstances.

7.4 Case evaluation

This section compares the cases presented in the previous sections. Such comparison can be very useful to identify where the highest potential for increasing profit is found by comparing with the corresponding investments. Figure 7.6 shows the average daily cost reduction for the investigated cases.

Figure 7.6 shows the potential economical gain from all previous three cases. For case 2, concerning the capacity, this plot shows the cost reduction when including a HP and EB capacity of 37.5 MW and 75 MW, respectively, as opposed to the situation where no HP and EB are included. Generally, the economic potential is in the range of 0.5 mio DKK to 3 mio per week for each of the scenarios. The largest potential is obtained in the case where power prices are decreased and the HP and EB are included in the system as opposed to a system without these units. The benefit from increasing the COP of the HP from 3.0 to 3.5 provides slightly less economical gain compared to increasing the COP from 2.5 to 3.0.

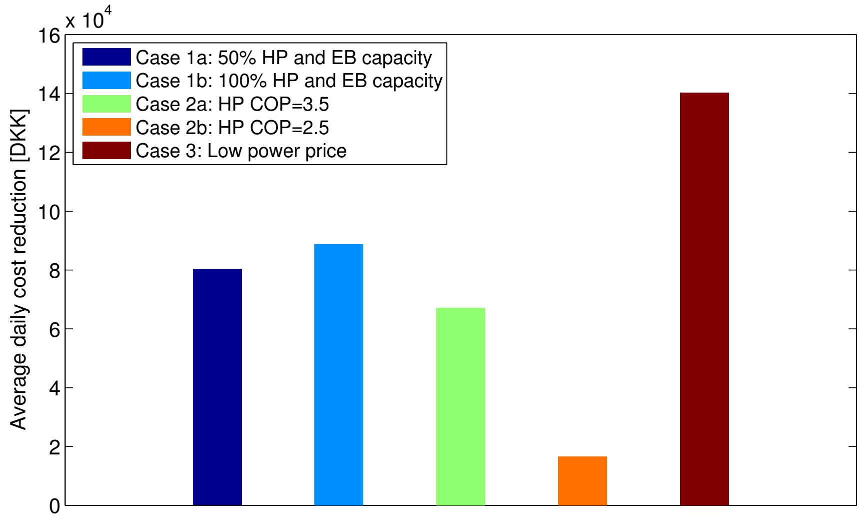


Figure 7.6 – Comparison of the average daily monetary benefits from each of the cases presented in this chapter.

7.5 Chapter summary

This chapter presented and analyzed numerical results from the deterministic and the stochastic optimization model. The results from the stochastic model was compared to the deterministic in-sample results. The impact of stochastic optimization was found to increase with decreasing HP and EB capacity, due to the flexibility they provide. The economical benefits of stochastic compared to deterministic optimization model are generally higher during summer, as the system is less flexible during this period. This is due to the back-pressure CHP being the only active heat production unit and consequently the system is less capable of adjusting the production to meet the realized heat demand and spot price.

Three case studies for the stochastic model were subsequently presented and analyzed. Changing the COP of the HP was found to have an impact on the heat costs. Furthermore, the monetary benefits connected to the HP and EB capacity showed a non-linear behavior. Here, an increase in capacity from 0 MW to 37.5 MW provided larger cost reductions compared to an increase of equal size from 37.5 MW to 75 MW. Finally, the impact of having a HP and EB, in the case of lower electricity prices, was investigated and compared to the previous cases. The comparison showed that the highest economical potential for the introduction of a HP and EB was obtained in the event of lower electricity prices.

Before making the decision concerning the implementation of these units, the economical benefits should be compared to the corresponding investments associated with integrating the HP and EB in the CHP system.

Chapter 8

Conclusion and future work

8.1 Conclusion

This project aimed at developing an operational strategy for an EB and a HP in a CHP system through the use probabilistic forecasting and stochastic optimization. This was intended to provide a foundation for integration of a HP and EB and improve flexibility, which should ultimately allow for a complete integration of intermittent power in a CHP system.

First, the basic concepts of CHP production were introduced, followed by an introduction to HPs and EBs including their mutual differences. Taxes applying to heat produced at CHPs, HPs and EBs were presented and analytical expressions for marginal heat costs for these units were introduced to illustrate the significant impact of taxes. In addition, this outlined the principal economical order of the production units. In addition, the Nordic electricity market as well as the reserve and regulating markets were outlined. Last, the Copenhagen district heating system was introduced and the daily heat dispatch system was presented.

This allowed for an analysis of the framework in which an HP and EB should operate. The benefits of different organizational and physical locations were assessed, and the relevance of the electricity markets discussed. It was found that the HP and EB should operate as part of a CHP system, such that the flexibility the units provide can be utilized fully. Furthermore, the EB would only be competitive in case the tax was excluded, which can only occur if the EB is located and connected directly to a power producing unit such as a CHP. However, the possibility of a more economical operation in district heating areas experiencing frequent bottlenecks cannot be rejected. The monetary benefits are nevertheless difficult to assess without detailed information about the heat dispatch in such areas.

The current state of the art technology does not allow HPs to reach temperatures high enough for it to be connected to the transmission network. As a consequence, the HP should be located in the distribution network with access to a cold medium source, e.g. sea water or waste water.

The potential of offering ancillary services and regulating power was also analyzed. While the FNR market was appropriate for units such as the EB, a high degree of uncertainty on the bidding is present. This could result in uneconomical situations or require a customized risk-averse bidding strategy.

The regulating market was generally found relevant for both HPs, EBs and CHP units. While the CHP unit should utilize the regulating market in case of deviating heat demands or outages, the HP and EB could offer flexible production. However, in a system comprising both a CHP, HP and EB unit the flexibility required by the CHP could be internally balanced by the HP and EB, making the regulating market less important.

The principles for a complex strategy involving both the heat market, the electricity market, frequency reserve, regulating power and intra day heat adjustments were outlined. All decisions showed a clear correlation to previous and future decisions. The importance and potential of the markets were assessed, and the successful operation of a HP and EB, in the heat market was found to be central to the operation in all other markets.

Based on the previous findings, an operational strategy for a CHP system comprising a HP, EB and two CHP units, operating in the heat and Nordic power market, was modelled. The main challenge for this set-up is the uncertainty of the electricity price at this point in time the schedule for district heating is made.

A deterministic optimization model was developed to provide a 24 hour operational strategy for the production of heat and power. The model was constructed such that it could follow the current time frame in the system, implying that the heat production was decided at approximately 10:00 on the day before delivery, and based on the available information at this point in time. Illustrative examples of the model results were presented and compared to simple analytical calculations. The significance of operational constraints were highlighted and their impact illustrated.

In order to account for uncertainty in the heat demand and spot price, the model was extended to a stochastic two-stage model. The first stage corresponds to the decision made at the time just after of the heat dispatch (before Elspot market clearing) and the second stage corresponds to the adjustment made after the spot price and demand realizes. As a consequence of the fixed offer to the Elspot market, the net production of power must be kept constant when the second stage adjustments are included. Probabilistic forecasts for the demand and spot price were developed through the use of time series analysis principles. A scenario approach was used to solve the stochastic optimization problem. The stochastic approach was shown to be only marginally better than the deterministic when a high HP and EB capacity was included. Decreasing the capacity, and thus the flexibility of the system, resulted in an increasing difference between the stochastic and deterministic results.

A number of case studies was then carried out for the stochastic model in order to evaluate the impact on the economical potential of HPs and EBs. The investigated cases include capacity reduction, change in the COP for the HP as well as a decrease in power prices. The benefit of the HP and EB was largest when the electricity prices were reduced.

This project, thus, developed and analyzed an operational strategy for a HP and EB in a CHP system. Results indicated substantial cost reduction resulting from the flexibility the HP and EB provide. However, a number of areas could benefit from additional research. These will be outlined in the following.

8.2 Future work

Stochastic optimization of CHP systems comprising HPs and EBs is generally very unexplored and can provide material for numerous projects in the future. In connection to this, a number of suggestions for future studies in continuation of the work done in this thesis are presented:

1. Extend the forecasting model for the spot price by including the expected power load and wind power production as explanatory variables. This would allow for better forecasts of the spot price. Especially if the forecast is able to predict very high and very low spot prices, this would be beneficial.
2. Extending the optimization model to include more stages. As was outlined in section 3.4 decisions are taken at different times and updates of the production schedule occur during the day when better forecasts for the heat load is available. One could also include the initial offer to Varmelast.dk such that the bidding curve/point was dependent on the stochastic spot price. Including these stages by making a multi-stage model would provide a more realistic view of the process.
3. The addition of new features, such as varying supply temperatures and thus COP for the HP, or other markets discussed previously could also provide interesting insight and allow for further analysis of the benefits of HPs and EBs.
4. An improvement of the model could also happen by allowing the optimization to include more than 24 hours. This could allow for a better use of the large storage unit which, under the current set-up is not fully utilized.

All of the above suggestions could provide valuable insight to the development of an operational strategy for a HP and EB in a CHP system.

Ultimately this will provide more flexible CHP systems allowing for increasing shares of intermittent renewables to be integrated.

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Appendix A

GAMS script for the deterministic model

```
*
$eolcom //
option iterlim=999999999;     // avoid limit on iterations
option reslim=900;            // timelimit for solver in sec.
option optcr=0.0;             // gap tolerance
option solprint=ON;           // include solution print in .lst file
option limrow=100;            // limit number of rows in .lst file
option limcol=100;            // limit number of columns in .lst file
 //

Sets
        tt   /tt1*tt24/
        s    /s1/
        d    /d1*d365/

   dd(d) ;

ALIAS(d1,d)  ;
sets ddd(d1)
;

*************************************************************************
*  PARAMETERS                                                           *
*************************************************************************

PARAMETER SP(d,tt,s);
$GDXIN 'C:\Users\Maria\Dropbox\Speciale2014\Master Thesis\M2G_SP_det.gdx';
$LOAD SP
$GDXIN

PARAMETER DM(d,tt,s);
$GDXIN 'C:\Users\Maria\Dropbox\Speciale2014\Master Thesis\M2G_DM_det.gdx';
$LOAD DM
$GDXIN

dd(d) = no;

parameter dm_s(d,tt,s);
```

```
dm_s(d,tt,s)=DM(d,tt,s)*0.3;

parameter p_spot_s(d,tt,s);
p_spot_s(d,tt,s)=SP(d,tt,s);

PARAMETER prob(s)  ;
         prob(s) =0.0;

PARAMETERS
         d_start                         /1/
         d_slut                          /365/
         st_cap                          /3000/
         st1_cap                         /300/
         HP_cop                          /3/
         eta_CHP                         /1.1/
         k                               /0.24/
         CHP_cap                         /250/
         CHP2_cap                        /330/
         CHP2_ex                         /211/
         HP_cap                          /75/
         HP_min                          /10/
         EB_cap                          /75/
         Ramp_chp 'MW '                  /12/
         Ramp_chp2 'MW '                 /40/
         flow_max 'Max from to/from storage'  /300/
         k_ex1 ''                        / 0.12/
         k_ex2 ''                        /0.64/
         k_fuel 'relationship between fuel and power lig eta' /0.35/
         pmax 'max power from chp2'      /250/
         pmin 'min power from chp2'      /0/
         c_start ''                      /125000/
         HP_start ''                     /2500/
         CHP2_min 'minimum power usage'  /40/
         CHP_min 'minimum power prod'    /12/
         nox                             /9/
         v_tax                           /263/
         tax 'kr per MWh'                /412/
         tax_net                         /219/
         c_fueltax 'tax on fossil fuels' /258.5/
         c_CHPfuel 'Pr MJ'               /144/
         c_CHP2fuel 'Pr MJ'              /72/
         co2                             /57/
         cinf  'Infeasibility cost'      /1000/
         cbio 'Biomass supplment'        /150/
         rtax                            /1.2/
         sloss                           /1.05/
 ;
parameter Rxs_chp(d,tt), Rxs_chp2(d,tt), Rxs_hp(d,tt), Rxs_eb(d,tt),
          Rxd_eb(d,tt), Rxd_chp(d,tt), Rxd_chp2(d,tt), Rxd_hp(d,tt);
Parameter Rp_chp2(d,tt), Rp_chp(d,tt), Rb_sup(d,tt)
Parameter Rst(d,tt), Rst1(d,tt), Rxd_st(d,tt);
Parameter R1xs_chp2(d,tt), R1xs_hp(d,tt), R1xs_eb(d,tt), R1xd_chp2(d,tt),
          R1xd_hp(d,tt), R1st(d,tt), R1st1(d,tt), R1xd_st(d,tt), Rxd_st1(d,tt),
          R1xs_chp(d,tt),R1xd_chp(d,tt), R1p_chp(d,tt) ;
Parameter R1b_chp(d,tt), R1b_chp_s(d,tt,s), R1b_hp(d,tt), R1b_chp2(d,tt),
          R1bs_chp(d,tt),  R1bs_chp2(d,tt);
Parameter HP_costs(d,tt), EB_costs(d,tt), CHP_costs(d,tt), CHP2_costs(d,tt);
Parameter HP_c(tt), EB_c(tt), CHP_c(tt), CHP2_c(tt);
```

```
Parameter p_spot_dd(tt), R1p_fuel(d,tt);
          R1b_chp(d,tt)$(ord(d)=d_start 1)=0;
          R1b_hp(d,tt)$(ord(d)=d_start 1)=0;
          R1b_chp2(d,tt)$(ord(d)=d_start 1)=0;
          R1xs_hp(d,tt)$(ord(d)=d_start 1)=0   ;
          R1xs_eb(d,tt)$(ord(d)=d_start 1)=0 ;
          R1xd_hp(d,tt)$(ord(d)=d_start 1)=0   ;
          R1st(d,tt)$(ord(d)=d_start 1)=0      ;
          R1st1(d,tt)$(ord(d)=d_start 1)=0     ;
          R1xd_st(d,tt)$(ord(d)=d_start 1)=200   ;
          Rxd_st1(d,tt)$(ord(d)=d_start 1)=0   ;
          R1xs_chp(d,tt)$(ord(d)=d_start 1)=0   ;
          R1xs_chp2(d,tt)$(ord(d)=d_start 1)=0   ;
          R1xd_chp(d,tt)$(ord(d)=d_start 1)=0   ;
          R1xd_chp2(d,tt)$(ord(d)=d_start 1)=0   ;
          R1p_fuel(d,tt)$(ord(d)=d_start 1)=0;
          R1p_chp(d,tt)$(ord(d)=d_start 1)=0;

Parameter Rin;
Parameter Rz(d);

*****************************************************************************
*         VARIABLES                                                         *
*****************************************************************************
Variables
          heat_chp
          xs_chp(tt) 'CHP heat for storage'
          xd_chp(tt) 'CHP heat to cover demand'
          xd_hp(tt)  'heat from HP to fulfill demand'
          xs_eb(tt)  'heat produced by EB to storage'
          xd_eb(tt)  'heat produced by EB to storage'
          xd_st(tt)  'heat taen from storage to cover demand'
          xs_hp(tt)  'From hp to storage'
          xd_st1(tt) 'From small storage to demand'
          st(tt)     'amount in storage'
          e(tt)      'Total amount of electricity bought at time t'
          y_chp(tt)  'amount of fuel used at time t'
          st1(tt)    'Storage in distribution net'
          p_fuel(tt) 'Power corresponding to fuel use'
          xs_chp2(tt)'Heat from CHP2 to storage'
          xd_chp2(tt)'Heat from CHP2 to demand'
          y_chp2(tt) 'fuel consumption from chp2'
          p_chp2(tt) 'Power prod from chp2'
          z          'profit'
          b_chp2(tt) 'Binary variable to be one if CHP2 1 is turned on'
          c_chp2(tt) 'Cost of start up '
          b_chp(tt)  'Binary variable to be one if CHP2 1 is turned on'
          c_chp(tt)  'cost of startup'
          b_hp(tt)   'Binary variable to be one if CHP2 1 is turned on'
          c_hp(tt)   'cost of start up '
          Q          'Second stage value'
          bs_chp2(tt)'Binary variable to be one if CHP2 1 is turned on'
          bs_chp(tt) 'Binary variable to be one if CHP2 1 is turned on'
          bs1_chp    'Binary variable to be one if CHP2 1 is turned on'
          bsd_chp2   'Binary variable to be one if CHP2 1 is turned on'
          bsd_chp    'Binary variable to be one if CHP2 1 is turned on'
          bs_hp(tt)  'Binary variable to be one if CHP2 1 is turned on'
          ec(tt)     'consumed power'
          p_chp(tt)  'power production at CHP'
```

```
         b_sup(tt)  'Amount of biomass production that does not receive sup'
;

Variable z;
positive variable heat_chp, heat_chp_s, in(tt), in_s(tt,s);
                  xs_chp, xd_chp, xd_hp, xs_hp, xs_eb,  xd_eb , xd_st;
                 y_chp, st, st1, xd_st1, p_fuel, xd_chp2, xs_chp2, y_chp2,
                 p_chp2, ec, e, p_chp, b_sup;
binary variable  b_chp2, bs_chp2, b_chp, bs_chp, b_hp, bs_hp, bs1_chp,
                 bs1_chp2, bs2_chp,bs21_chp, bs22_chp,  bs2_chp2, bs21_chp2,
                 bs22_chp2, b_s_chp2,  b_s_chp, b_s_hp, bs_s_chp, bs_s_chp2,
                 bs_s_hp, bs2_s_chp, b21_s_chp, bs22_s_chp,
                 bs2_s_chp2, bs1_s_chp;


********************************************************************************
*        EQUATION                                                              *
********************************************************************************

Equations
         cost              'define objective function'
         demand1(d,tt)    'Ensure demand fulfilled'
         eb_el(tt)
         elprod(tt)        'Define the required amount of el to be bought/sold'
         elcon(tt)
         Fuel_tax
         fuel(tt)          'Relationsip between fuel and heat production'
         chp2_upper(tt)
         chp2_lower(tt)
         chp2_prod(tt)
         chp2_fuel(tt)
         chp_power(tt)
         rh_up(tt)        'up ramp contraint for heat'
         rh_up1(d,tt)
         rh_do(tt)        'down ramp contraint for heat'
         rh_do1(d,tt)     'Constraints for t=1'
         rh_up_chp2(tt)   'up ramp contraint for heat'
         rh_up1_chp2(d,tt)
         rh_do_chp2(tt)    'down ramp contraint for heat'
         rh_do1_chp2(d,tt)
         chp_sdown1(tt)
         chp_sdown(tt)
         chp2_sdown1(tt)
         chp2_sdown(tt)
         chp_cost(tt)
         chp2_cost(tt)
         chp2_cost1(d,tt)
         chp_cost1(d,tt)
         hp_cost(tt)
         hp_cost1(d,tt)
         chp_on(tt)
         hp_on(tt)
         chp2_on(tt)
         chp2_minl(tt)
         chp_minl(tt)
         hp_minl(tt)
         Storageeq(tt)    'storage equilibrium '
         Storageeq1(d,tt)
         st1eq(tt)
```

```
          st1eq1(d,tt)
          st_flowOut(tt)  'Maximum to be delivered from storage'
          st_flowIn(tt)    'Maximum to be delivered in storage'
          EBcap(tt)       'capacity for EB'
          Stcap(tt)       'capacity for storage'
          chp2cap(tt)
          HPcap(tt)       'capacity for HP'
          St1cap(tt)
          bio_substract(tt)
          ;

// Objective function
cost .. z =e=    sum((tt), sum(dd,p_spot_s(dd,tt,'s1'))*( e(tt)+ec(tt)) +
                 (1/HP_cop)*(xd_hp(tt)+xs_hp(tt))*(tax+tax_net)
                 (p_chp(tt) b_sup(tt))*cbio  + (xd_chp(tt)+xs_chp(tt))1/rtax*nox
                 +c_CHPfuel*y_CHP(tt)+c_CHP2fuel*y_chp2(tt)+in(tt)*cinf )+
                 c_start*(sum(tt, bs_chp(tt) +bsd_chp2(tt)+bsd_chp(tt)+bs_chp2(tt)))
                 +sum(tt,bs_hp(tt))*hp_start +heat_chp ;

//Tax on CHP
Fuel_tax ..  heat_chp =g= (co2+nox+c_fueltax)*sum(tt,xd_chp2(tt)+xs_chp2(tt) )*(1/rtax) ;

//Total production and demand
demand1(dd,tt) .. xd_chp(tt)+xd_hp(tt)+xd_st(tt)+xd_st1(tt)+xd_chp2(tt)+xd_eb(tt) =e= dm_s(dd,tt,'s1') in(tt);

eb_el(tt) .. xs_eb(tt)+xd_eb(tt)=l=p_chp(tt)+ p_chp2(tt);
elprod(tt) .. e(tt) =e=   p_chp(tt)  + p_chp2(tt)  ;
elcon(tt)  .. ec(tt) =e= xs_eb(tt)+xd_eb(tt) +(1/HP_cop)*(xd_hp(tt)+xs_hp(tt));

//Production and fuel  equations
chp_power(tt) .. p_chp(tt)   =e= (xs_chp(tt)+xd_chp(tt))*k    ;
fuel(tt) .. y_CHP(tt) =e= (p_chp(tt)+xs_chp(tt)+xd_chp(tt))/eta_chp;

chp2_upper(tt) .. p_chp2(tt) =l= k_ex1*(xd_chp2(tt)+xs_chp2(tt))+pmax ;
chp2_lower(tt) .. p_chp2(tt) =g= k_ex2*(xd_chp2(tt)+xs_chp2(tt))+pmin  ;
chp2_prod(tt) .. p_chp2(tt) =e= k_ex1*(xd_chp2(tt)+xs_chp2(tt))+p_fuel(tt) ;
chp2_fuel(tt) .. y_chp2(tt) =e= (1/k_fuel)*p_fuel(tt);

//Ramping
rh_up(tt)$(ord(tt)>1).. (p_chp(tt)) (p_chp(tt 1)) =l=   Ramp_chp;
rh_up1(ddd,tt)$(ord(tt)=1).. (p_chp(tt)) (R1p_chp(ddd,'tt24')) =l=   Ramp_chp;

rh_do(tt)$(ord(tt)>1).. (p_chp(tt)) (p_chp(tt 1)) =g=    Ramp_chp;
rh_do1(ddd,tt)$(ord(tt)=1).. (p_chp(tt)) (R1p_chp(ddd,'tt24')) =g=    Ramp_chp;

rh_up_chp2(tt)$(ord(tt)>1).. (p_fuel(tt)) (p_fuel(tt 1)) =l=   Ramp_chp2;
rh_up1_chp2(ddd,tt)$(ord(tt)=1).. (p_fuel(tt)) (R1p_fuel(ddd,'tt24')) =l= Ramp_chp2;

rh_do_chp2(tt)$(ord(tt)>1).. (p_fuel(tt)) (p_fuel(tt 1)) =g=    Ramp_chp2;
rh_do1_chp2(ddd,tt)$(ord(tt)=1).. (p_fuel(tt)) (R1p_fuel(ddd,'tt24')) =g= Ramp_chp2;

//Startup
chp_on(tt) .. xs_chp(tt)+xd_chp(tt) =l= b_chp(tt)*chp_cap;
```

```
chp_sdown(tt)$(ord(tt)>1).. bsd_chp(tt)=g= b_chp(tt 1) b_chp(tt)  ;
chp_sdown1(tt)$(ord(tt)=1).. bsd_chp(tt)=g= b_chp('tt24') b_chp(tt)
                    ;

chp2_sdown(tt)$(ord(tt)>1).. bsd_chp2(tt)=g= b_chp2(tt 1) b_chp2(tt)  ;
chp2_sdown1(tt)$(ord(tt)=1).. bsd_chp2(tt)=g= b_chp2('tt24') b_chp2(tt)  ;

chp_cost(tt) $(ord(tt)>1).. bs_chp(tt)=g= b_chp(tt) b_chp(tt 1);
chp_cost1(dd,tt)$(ord(tt)=1).. bs_chp(tt) =g= b_chp(tt) R1b_chp(dd,'tt24');

chp2_on(tt) .. p_chp2(tt)+xs_chp2(tt)+xd_chp2(tt) =l= b_chp2(tt) *(CHP2_cap+
    CHP2_ex);

chp2_cost(tt)$(ord(tt)>1) .. bs_chp2(tt)=g= (b_chp2(tt) b_chp2(tt 1))   ;
chp2_cost1(dd,tt)$(ord(tt)=1) .. bs_chp2(tt)=g=(b_chp2(tt) R1b_chp2(dd,'tt24'))
      ;

hp_on(tt) .. xs_hp(tt)+xd_hp(tt) =l= b_hp(tt)*Hp_cap;

hp_cost(tt)$(ord(tt)>1) .. bs_hp(tt)=g=(b_hp(tt) b_hp(tt 1) ) ;
hp_cost1(dd,tt)$(ord(tt)=1) .. bs_hp(tt)=g= (b_hp(tt) R1b_hp(dd,'tt24') ) ;

//Minimum load

chp2_minl(tt)..  p_fuel(tt)=g=b_chp2(tt)*CHP2_min;
chp_minl(tt)..   p_chp(tt)=g=b_chp(tt)*CHP_min;
hp_minl(tt) ..   xs_hp(tt)+xd_hp(tt)=g=b_hp(tt)*Hp_min;

//Storage constraints
Storageeq(tt)$(ord(tt)>1) .. st(tt)=e= st(tt 1)+xs_chp(tt)+xs_chp2(tt)+xs_eb(tt)
      sloss*xd_st(tt)  ;
Storageeq1(dd,tt)$(ord(tt)=1) .. st(tt)=e= R1st(dd,'tt24')+xs_chp(tt)+xs_chp2(tt
    )+xs_eb(tt)  sloss*xd_st(tt)  ;

st1eq(tt)$(ord(tt)>1) .. st1(tt)=e=st1(tt 1) +xs_hp(tt) sloss*xd_st1(tt);
st1eq1(dd,tt)$(ord(tt)=1) .. st1(tt)=e=R1st1(dd,'tt24') +xs_hp(tt) sloss*xd_st1(
    tt);

st_flowOut(tt) .. xd_st(tt)=l=flow_max;
st_flowIn(tt) .. xs_chp(tt)+xs_eb(tt)+xs_chp2(tt)=l=flow_max;

//Capacity constraints
EBcap(tt) ..   xs_eb(tt)+xd_eb(tt)=l=eb_cap;
HPcap(tt) .. xd_hp(tt)+xs_hp(tt) =l= hp_cap;
chp2cap(tt).. xd_chp2(tt)+xs_chp2(tt)=l=CHP2_cap;
Stcap(tt) .. st(tt)=l= st_cap ;
St1cap(tt) .. st1(tt)=l= st1_cap;

//Define amount not to recieve bio sup
bio_substract(tt).. b_sup(tt) =g= ec(tt) p_chp2(tt);

Model det /all/ ;

loop(d$(ord(d)>d_start and ord(d)<d_slut)     ,
dd(d) = yes;

ddd(d1)$(ord(d1)=ord(d) 1) = yes;
```

```
display dd;
display ddd;

display dm_s;
Solve det using mip minimizing z ;
```

Appendix B

GAMS script for the stochastic model

```
*
$eolcom //
option iterlim=999999999;     // avoid limit on iterations
option reslim=3000;            // timelimit for solver in sec.
option optcr=0.05;             // gap tolerance
option solprint=ON;           // include solution print in .lst file
option limrow=100;            // limit number of rows in .lst file
option limcol=100;            // limit number of columns in .lst file
 //

Sets
        tt   /tt1*tt24/
        s    /s1*s100/
        d     /d1*d365/

   dd(d) ;

ALIAS(d1,d)  ;
sets ddd(d1)
;
*************************************************************************
*  PARAMETERS                                                           *
*************************************************************************

PARAMETER SP(d,tt,s);
$GDXIN 'C:\Users\Maria\Dropbox\Speciale2014\Master Thesis\M2G_SP.gdx';
$LOAD SP
$GDXIN

PARAMETER DM(d,tt,s);
$GDXIN 'C:\Users\Maria\Dropbox\Speciale2014\Master Thesis\M2G_DM.gdx';
$LOAD DM
$GDXIN

dd(d) = no;

parameter dm_s(d,tt,s);
         dm_s(d,tt,s)=DM(d,tt,s)*0.3;
```

```
parameter p_spot_s(d,tt,s);
         p_spot_s(d,tt,s)=SP(d,tt,s) 50;

parameter  prob(s)  ;
         prob(s) =0.01;


parameter
         d_start                         /1/
         d_slut                          /365/
         st_cap                          /3000/
         st1_cap                         /300/
         HP_cop                          /3/
         eta_CHP                         /1.1/
         k                               /0.24/
         CHP_cap                         /250/
         CHP2_cap                        /330/
         CHP2_ex                         /211/
         HP_cap                          /75/
         HP_min                          /10/
         EB_cap                          /75/
         Ramp_chp 'MW '                  /12/
         Ramp_chp2 'MW '                 /40/
         flow_max 'Max from to/from storage'  /300/
         k_ex1 ''                        / 0.12/
         k_ex2 ''                        /0.64/
         k_fuel 'relationship between fuel and power lig eta' /0.35/
         pmax 'max power from chp2'      /250/
         pmin 'min power from chp2'      /0/
         c_start ''                      /125000/
         HP_start ''                     /2500/
         CHP2_min 'minimum power usage'  /40/
         CHP_min 'minimum power prod'    /12/
         nox                             /9/
         v_tax                           /263/
         tax 'kr per MWh'                /412/
         tax_net                         /219/
         c_fueltax 'tax on fossil fuels' /258.5/
         c_CHPfuel 'Pr MJ'               /144/
         c_CHP2fuel 'Pr MJ'              /72/
         co2                             /57/
         cinf  'Infeasibility cost'      /1000/
         cbio 'Biomass supplment'        /150/
         rtax                            /1.2/
         sloss                           /1.05/
 ;

parameter Rxs_chp(d,tt), Rxs_chp2(d,tt), Rxs_hp(d,tt), Rxs_eb(d,tt),  Rxd_eb(d,
    tt), Rxd_chp(d,tt),
          Rxd_chp2(d,tt), Rxd_hp(d,tt);

Parameter Rp_chp2(d,tt), Rp_chp(d,tt), Rb_sup(d,tt)
Parameter Rst(d,tt), Rst1(d,tt), Rxd_st(d,tt);

Parameter R1xs_chp2(d,tt), R1xs_hp(d,tt), R1xs_eb(d,tt), R1xd_eb(d,tt),
    R1xd_chp2(d,tt),R1xd_hp(d,tt), R1st(d,tt)
         ,R1st1(d,tt), R1xd_st(d,tt), Rxd_st1(d,tt), R1xs_chp(d,tt),R1xd_chp(d,
             tt), Rin(d,tt), R1p_chp(d,tt) ;
```

```
Parameter R1xs_chp2_s(d,tt,s), R1xs_hp_s(d,tt,s), R1xs_eb_s(d,tt,s),  R1xd_eb_s(d,tt,s), R1xd_chp2_s(d,tt,s),R1xd_hp_s(d,tt,s), R1st_s(d,tt,s)
         ,R1st1_s(d,tt,s), R1xd_st_s(d,tt,s), R1xd_st1_s(d,tt,s), R1xs_chp_s(d,tt,s),R1xd_chp_s(d,tt,s), R1in_s(d,tt,s), R1p_chp_s(d,tt,s), R1p_chp2_s(d,tt,s) ;
Parameter R1b_chp(d,tt), R1b_chp_s(d,tt,s), R1b_hp(d,tt), R1b_hp_s(d,tt,s), R1b_chp2(d,tt), R1b_chp2_s(d,tt,s), R1b_sup_s(d,tt,s);
Parameter p_spot_dd(tt), R1p_fuel, R1p_s_fuel;
          R1b_chp(d,tt)$(ord(d)=d_start 1)=0;
          R1b_chp_s(d,tt,s)$(ord(d)=d_start 1)=0;
          R1b_hp(d,tt)$(ord(d)=d_start 1)=0;
          R1b_hp_s(d,tt,s)$(ord(d)=d_start 1)=0;
          R1b_chp2(d,tt)$(ord(d)=d_start 1)=0;
          R1b_chp2_s(d,tt,s)$(ord(d)=d_start 1)=0;

          R1xs_hp(d,tt)$(ord(d)=d_start 1)=0   ;
          R1xs_eb(d,tt)$(ord(d)=d_start 1)=0 ;
          R1xd_hp(d,tt)$(ord(d)=d_start 1)=0   ;
          R1st(d,tt)$(ord(d)=d_start 1)=200      ;
          R1st1(d,tt)$(ord(d)=d_start 1)=0     ;
          R1xd_st(d,tt)$(ord(d)=d_start 1)=0   ;
          Rxd_st1(d,tt)$(ord(d)=d_start 1)=0   ;

          R1xs_chp2_s(d,tt,s)$(ord(d)=d_start 1)=0;
          R1xs_hp_s(d,tt,s)$(ord(d)=d_start 1)=0   ;
          R1xs_eb_s(d,tt,s)$(ord(d)=d_start 1)=0 ;
          R1xd_chp2_s(d,tt,s)$(ord(d)=d_start 1)=0;
          R1xd_hp_s(d,tt,s)$(ord(d)=d_start 1)=0   ;
          R1st_s(d,tt,s)$(ord(d)=d_start 1)=0      ;
          R1st1_s(d,tt,s)$(ord(d)=d_start 1)=0     ;
          R1xd_st_s(d,tt,s)$(ord(d)=d_start 1)=0   ;
          R1xs_chp_s(d,tt,s)$(ord(d)=d_start 1)=0   ;
          R1xd_chp_s(d,tt,s)$(ord(d)=d_start 1)=0    ;
          R1xd_st1_s(d,tt,s)$(ord(d)=d_start 1)=0    ;

          R1xs_chp(d,tt)$(ord(d)=d_start 1)=0   ;
          R1xs_chp2(d,tt)$(ord(d)=d_start 1)=0   ;
          R1xd_chp(d,tt)$(ord(d)=d_start 1)=0   ;
          R1xd_chp2(d,tt)$(ord(d)=d_start 1)=0   ;
          R1p_fuel(d,tt)$(ord(d)=d_start 1)=0;
          R1p_chp(d,tt)$(ord(d)=d_start 1)=0;
          R1p_chp_s(d,tt,s)$(ord(d)=d_start 1)=0;
          R1p_s_fuel(d,tt,s)$(ord(d)=d_start 1)=0;

Parameter Rz(d);

********************************************************************************
*         VARIABLES                                                            *
********************************************************************************
Variables
         heat_chp
         heat_chp_s(s)
         xs_chp(tt)       'CHP heat for storage'
         xd_chp(tt)       'CHP heat to cover demand'
         xd_hp(tt)        'heat from HP to fulfill demand'
         xs_eb(tt)        'heat produced by EB to storage'
         xd_eb(tt)        'heat produced by EB to demand'
         xd_st(tt)        'heat taen from storage to cover demand'
         xs_hp(tt)        'From hp to storage'
```

```
         xd_st1(tt)      'From small storage to demand'
         st(tt)          'amount in storage'
         e(tt)           'Total amount of electricity bought at time t'
         y_chp(tt)       'amount of fuel used at time t'
         st1(tt)         'Storage in distribution net'
         p_fuel(tt)      'Power corresponding to fuel use'
         xs_chp2(tt)
         xd_chp2(tt)
         y_chp2(tt)      'fuel consumption from chp2'
         p_chp2(tt)      'Power prod from chp2'
         z               'profit'
         b_chp2(tt)      'Binary variable to be one if CHP2 1 is turned on'
         c_chp2(tt)      'Cost of start up '
         b_chp(tt)       'Binary variable to be one if CHP2 1 is turned on'
         c_chp(tt)       'cost of startup'

         b_hp(tt)        'Binary variable to be one if CHP2 1 is turned on'
         c_hp(tt)        'cost of start up '
         Q
         bs_chp2(tt)     'Binary variable to be one if CHP2 1 is turned on'
         bs_chp(tt)      'Binary variable to be one if CHP2 1 is turned on'
         bs1_chp         'Binary variable to be one if CHP2 1 is turned on'

         bs_hp(tt)       'Binary variable to be one if CHP2 1 is turned on'
         ec(tt)          'consumed el'
         in_s(tt,s)
         xs_s_chp(tt,s)
         xd_s_chp(tt,s)  'CHP heat to cover demand'
         xd_s_hp(tt,s)   'heat from HP to fulfill demand'
         xs_s_eb(tt,s)   'heat produced by EB to storage'
         xd_s_eb(tt,s)   'heat produced by EB to storage'
         xd_s_st(tt,s)   'heat taen from storage to cover demand'
         xs_s_hp(tt,s)   'From hp to storage'
         xd_s_st1(tt,s)  'From small storage to demand'
         st_s(tt,s)      'amount in storage'
         e_s(tt,s)       'Total amount of electricity bought at time t'
         y_s_chp(tt,s)   ' amount of fuel used at time t'
         st1_s(tt,s)     'Storage in distribution net'
         p_s_fuel(tt,s)  'Power corresponding to fuel use'
         xs_s_chp2(tt,s)
         xd_s_chp2(tt,s)
         y_s_chp2(tt,s)  'fuel consumption from chp2'
         p_s_chp2(tt,s)  'Power prod from chp2'
         b_s_chp2(tt,s)  'Binary variable to be one if CHP2 1 is turned on'
         b_s_chp(tt,s)   'Binary variable to be one if CHP2 1 is turned on'

         b_s_hp(tt,s)    'Binary variable to be one if CHP2 1 is turned on'
         bs_s_chp2(tt,s) 'Binary variable to be one if CHP2 1 is turned on'
         bs_s_chp(tt,s)  'Binary variable to be one if CHP2 1 is turned on'
         bs1_s_chp(s)    'Binary variable to be one if CHP2 1 is turned on'

         bs_s_hp(tt,s)   'Binary variable to be one if CHP2 1 is turned on'
         ec_s(tt,s)      'consumed el'
         d_CHP_su(tt,s)
         d_CHP2_su(tt,s)
         d_CHP2_sd(tt,s)
         d_CHP_sd(tt,s)
         d_HP_su(tt,s)
         p_chp(tt)
```

```
         p_s_chp(tt,s)
         b_sup_s(tt,s)   'amount to be substracted for bio tilskud'
         b_sup(tt)
         bsd_chp(tt)
         bsd_s_chp(tt,s)
         bsd_chp2(tt)
         bsd_s_chp2(tt,s)
;

Variable z;
positive variable heat_chp, heat_chp_s, in(tt), in_s(tt,s),
                  xs_chp, xd_chp, xd_hp, xs_hp, xs_eb, xd_eb, xd_st, y_chp, st,
                  st1, xd_st1, p_fuel, xd_chp2, xs_chp2, y_chp2, p_chp2,
                  ec, e, p_chp, b_sup;

binary variable   b_chp2, bs_chp2, b_chp, bs_chp, b_hp, bs_hp, bsd_chp,
                  bsd_chp2, bsd_s_chp, bsd_s_chp2;

binary variable   b_s_chp2,  b_s_chp, b_s_hp, bs_s_chp, bs_s_chp2, bs_s_hp;

// FIX THE FIRST STAGE SOLUTION
xd_s_chp.fx(tt,'s1')=0;
p_s_chp.fx(tt,'s1') =0;
p_s_chp2.fx(tt,'s1')=0;
xs_s_chp2.fx(tt,'s1')=0;
xd_s_chp2.fx(tt,'s1')=0;
xs_s_eb.fx(tt,'s1')=0;
xd_s_eb.fx(tt,'s1')=0;
xs_s_hp.fx(tt,'s1') =0;
xd_s_hp.fx(tt,'s1') =0;
xd_s_st.fx(tt,'s1') =0;
xd_s_st1.fx(tt,'s1') =0;
st1_s.fx(tt,'s1') =0;
st_s.fx(tt,'s1') =0;
d_CHP2_su.fx(tt,'s1')=0;
d_CHP_su.fx(tt,'s1')=0;
d_CHP_sd.fx(tt,'s1')=0;
d_hp_su.fx(tt,'s1')=0;
d_CHP2_sd.fx(tt,'s1')=0;


*******************************************************************************
*        EQUATION                                                             *
*******************************************************************************

Equations
        cost        'define objective function'
        stoch
        demand1(d,tt)   'Ensure demand fulfilled'
        demand_s(d,tt,s)
        eb_el(tt)
        eb_el_s(tt,s)
        elprod(tt) 'Define the required amount of el to be bought/sold'
        elprod_s(tt,s)
        elcon(tt) , elcon_s(tt,s)

        powerEq_s(tt,s)
        Fuel_tax
        Fuel_tax_s(s)
```

```
        EBcap(tt)   'capacity for EB'
        EBcap_s(tt,s)

        Stcap(tt)   'capacity for storage'
        Stcap_s(tt,s)
        chp2cap(tt)
        chp2cap_s(tt,s)

        HPcap(tt)   'capacity for HP' ,   HPcap_s(tt,s)
        St1cap(tt)    , St1cap_s(tt,s)

        chp_sdown1(tt), chp_sdown1_s(tt,s)
        chp_sdown(tt), chp_sdown_s(tt,s)
        chp2_sdown1(tt), chp2_sdown1_s(tt,s)
        chp2_sdown(tt), chp2_sdown_s(tt,s)

        chp_on(tt),   chp_on_s(tt,s)
        chp_cost(tt), chp_cost_s(tt,s)

        Storageeq(tt) ' storage equilibrium '
        Storageeq_s(tt,s)
        Storageeq1(d,tt), Storageeq1_s(d,tt,s)

        fuel(tt)     'Relationsip between fuel and heat production'
        fuel_s(tt,s)

        rh_up(tt),  rh_up_s(tt,s)     'up ramp contraint for heat'
        rh_up1(d,tt), rh_up1_s(d,tt,s)
        rh_do(tt),   rh_do_s(tt,s)   'down ramp contraint for heat'
        rh_do1(d,tt) , rh_do1_s(d,tt,s)    'Constraints for t=1'
        rh_up_chp2(tt),  rh_up_chp2_s(tt,s)      'up ramp contraint for heat'
        rh_up1_chp2(d,tt),  rh_up1_chp2_s(d,tt,s)
        rh_do_chp2(tt), rh_do_chp2_s(tt,s)      'down ramp contraint for heat'
        rh_do1_chp2(d,tt), rh_do1_chp2_s(d,tt,s)

        st1eq(tt)     , st1eq_s(tt,s)
        st1eq1(d,tt)      , st1eq1_s(d,tt,s)

        st_flowOut(tt) 'Maximum to be delivered from storage'
        st_flowOut_s(tt,s)
        st_flowIn(tt) 'Maximum to be delivered in storage'
        st_flowIn_s(tt,s)

        chp2_upper(tt),  chp2_upper_s(tt,s)
        chp2_lower(tt) , chp2_lower_s(tt,s)
        chp2_prod(tt)  , chp2_prod_s(tt,s)
        chp2_fuel(tt)  , chp2_fuel_s(tt,s)
        chp2_on(tt)    , chp2_on_s(tt,s)
        chp2_cost(tt)  , chp2_cost_s(tt,s)
        chp2_cost1(d,tt)  , chp2_cost1_s(d,tt,s)

        chp_cost1(d,tt), chp_cost1_s(d,tt,s)
        chp2_minl(tt)  , chp2_minl_s(tt,s)
        chp_minl(tt)   , chp_minl_s(tt,s)

        hp_on(tt)      , hp_on_s(tt,s)
        hp_cost(tt)    , hp_cost_s(tt,s)
        hp_cost1(d,tt)    , hp_cost1_s(d,tt,s)
        hp_minl(tt)    , hp_minl_s(tt,s)
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```
        HPs_con(tt,s)
        HPs_con1(tt,s)
        EBs_con(tt,s)
        EBs_con1(tt,s)
        CHPs_con(tt,s)
        CHP2s_con(tt,s)
        CHPs_con1(tt,s)
        CHP2s_con1(tt,s)
        Ps_con(tt,s)
        Ss_con(tt,s)
        S1s_con(tt,s)
        XDst1_con(tt,s)
        XDst_con(tt,s)

        delta_CHP_su(tt,s)
        delta_CHP2_su(tt,s)
        delta_HP_su(tt,s)
        delta_chp2_sd(tt,s)

        chpcap(tt,s)
        chp2pcap(tt,s)
        chp_power(tt)
        chp_power_s(tt,s)
        bio_substract_s(tt,s)
        bio_substract(tt)
        bio_con(tt,s)
         ;

cost .. z =e=    sum((tt), sum(dd,p_spot_s(dd,tt,'s1'))*( e(tt)+ec(tt)) +
                 (1/HP_cop)*(xd_hp(tt)+xs_hp(tt))*(tax+tax_net)
                 (p_chp(tt) b_sup(tt))*c_sub+ (xd_chp(tt)+xs_chp(tt))*nox
                 +c_CHPfuel*y_CHP(tt)+c_CHP2fuel*y_chp2(tt)+in(tt)*c_inf )+
                 c_start*(sum(tt, bs_chp(tt) +bs_chp2(tt) +bsd_chp(tt)+bsd_chp2(tt) ))
                 +sum(tt,bs_hp(tt))*hp_start +heat_chp + Q   ;

stoch .. Q =e=  sum(s,prob(s)*( (sum((tt), (sum(dd,p_spot_s(dd,tt,s)))*( e_s(tt,s)+ec_s(tt,s))+
                 (1/HP_cop)*(xd_s_hp(tt,s)+xs_s_hp(tt,s))*(tax+tax_net)
                  (p_s_chp(tt,s) b_sup_s(tt,s))*c_sub+(xd_s_chp(tt,s)+xs_s_chp(tt,s))*nox
                 +c_CHPfuel*y_s_CHP(tt,s)+c_CHP2fuel*y_s_chp2(tt,s) +in_s(tt,s)*c_inf+
                  hp_start*d_HP_su(tt,s)
                  + c_start*(  d_CHP_su(tt,s)+d_CHP_sd(tt,s)
                  d_CHP2_su(tt,s)+d_CHP2_sd(tt,s) ) )   +heat_chp_s(s) )   )  );

//Total production and demand
delta_CHP_su(tt,s)..  d_CHP_su(tt,s)=e= bs_s_chp(tt,s) bs_chp(tt)   ;
delta_chp_sd(tt,s) .. d_chp_sd(tt,s) =e= bsd_s_chp(tt,s) bsd_chp(tt);
delta_CHP2_su(tt,s)..  d_CHP2_su(tt,s)=e= bs_s_chp2(tt,s) bs_chp2(tt)   ;
delta_chp2_sd(tt,s) .. d_chp2_sd(tt,s) =e= bsd_s_chp2(tt,s) bsd_chp2(tt);
Delta_HP_su(tt,s)..  d_HP_su(tt,s)=e= bs_s_hp(tt,s) bs_hp(tt)   ;

Fuel_tax ..  heat_chp =g= (co2+nox+c_fueltax)*sum(tt,xd_chp2(tt)+xs_chp2(tt) ) *(1/rtax) ;
Fuel_tax_s(s) ..  heat_chp_s(s) =g= (co2+nox+c_fueltax)*sum(tt, xd_s_chp2(tt,s)+xs_s_chp2(tt,s) ) *(1/rtax) ;
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demand1(dd,tt) .. xd_chp(tt)+xd_hp(tt)+xd_st(tt)+xd_st1(tt)+xd_chp2(tt)+xd_eb(tt
    ) =e= dm_s(dd,tt,'s1') in(tt);
demand_s(dd,tt,s) .. xd_chp(tt)+xd_hp(tt)+xd_st(tt)+xd_st1(tt)+xd_chp2(tt)+xd_eb
    (tt)+
                 xd_s_chp(tt,s)+xd_s_hp(tt,s)+xd_s_st(tt,s)+xd_s_st1(tt,s)+
                     xd_s_chp2(tt,s)+xd_s_eb(tt,s) =e= dm_s(dd,tt,s) in(tt)
                     in_s(tt,s);


eb_el(tt) .. xs_eb(tt)+xd_eb(tt)=l=p_chp(tt)+ p_chp2(tt);
eb_el_s(tt,s) .. xs_eb(tt)+xs_s_eb(tt,s)+xd_eb(tt)+xd_s_eb(tt,s)=l=p_chp(tt)+
    p_s_chp(tt,s)+ p_chp2(tt)+p_s_chp2(tt,s);

elprod(tt) .. e(tt) =e=   p_chp(tt)   + p_chp2(tt)   ;
elprod_s(tt,s) .. e_s(tt,s) =e=   p_s_chp(tt,s)   + p_s_chp2(tt,s)   ;

elcon(tt)   .. ec(tt) =e= xs_eb(tt)+xd_eb(tt) +(1/HP_cop)*(xd_hp(tt)+xs_hp(tt))
     ;
elcon_s(tt,s)   .. ec_s(tt,s) =e= xs_s_eb(tt,s)+xd_s_eb(tt,s) +(1/HP_cop)*(
    xd_s_hp(tt,s)+xs_s_hp(tt,s))      ;


powerEq_s(tt,s) .. e_s(tt,s) ec_s(tt,s)=e= 0 ;

//Production and fuel  equations
chp_power(tt) .. p_chp(tt)   =e= (xs_chp(tt)+xd_chp(tt))*k    ;
chp_power_s(tt,s) .. p_s_chp(tt,s)   =e= (xs_s_chp(tt,s)+xd_s_chp(tt,s))*k ;

fuel(tt) .. y_CHP(tt) =e= (p_chp(tt)+xs_chp(tt)+xd_chp(tt))/eta_chp;
fuel_s(tt,s) .. y_s_CHP(tt,s) =e= (p_s_chp(tt,s)+xs_s_chp(tt,s)+xd_s_chp(tt,s))/
    eta_chp;

chp2_upper(tt) .. p_chp2(tt) =l= k_ex1*(xd_chp2(tt)+xs_chp2(tt))+pmax ;
chp2_upper_s(tt,s) .. p_s_chp2(tt,s) +p_chp2(tt) =l= k_ex1*(xd_chp2(tt)+xs_chp2(
    tt)+xd_s_chp2(tt,s)+xs_s_chp2(tt,s))+pmax ;

chp2_lower(tt) .. p_chp2(tt) =g= k_ex2*(xd_chp2(tt)+xs_chp2(tt))+pmin  ;
chp2_lower_s(tt,s) .. p_s_chp2(tt,s) +p_chp2(tt)=g= k_ex2*(xd_chp2(tt)+xs_chp2(
    tt)+xd_s_chp2(tt,s)+xs_s_chp2(tt,s))+pmin  ;

chp2_prod(tt) .. p_chp2(tt) =e= k_ex1*(xd_chp2(tt)+xs_chp2(tt))+p_fuel(tt) ;
chp2_prod_s(tt,s) .. p_chp2(tt)+p_s_chp2(tt,s) =e= k_ex1*(xd_chp2(tt)+xs_chp2(tt
    )+xd_s_chp2(tt,s)+xs_s_chp2(tt,s))+(p_fuel(tt)+p_s_fuel(tt,s)) ;

chp2_fuel(tt) .. y_chp2(tt) =e= (1/k_fuel)*p_fuel(tt);
chp2_fuel_s(tt,s) .. y_s_chp2(tt,s) =e= (1/k_fuel)*p_s_fuel(tt,s);

//Ramping

rh_up(tt)$(ord(tt)>1).. (p_chp(tt)) (p_chp(tt 1)) =l=   Ramp_chp;
rh_up_s(tt,s)$(ord(tt)>1).. (p_chp(tt)+p_s_chp(tt,s)) (p_chp(tt 1)+p_s_chp(tt 1,
    s)) =l=   Ramp_chp;
rh_up1(ddd,tt)$(ord(tt)=1).. (p_chp(tt)) (R1p_chp(ddd,'tt24')) =l=   Ramp_chp;
rh_up1_s(ddd,tt,s)$(ord(tt)=1).. (p_chp(tt)+p_s_chp(tt,s)) (R1p_chp(ddd,'tt24')+
    R1p_chp_s(ddd,'tt24',s)) =l=   Ramp_chp;

rh_do(tt)$(ord(tt)>1).. (p_chp(tt)) (p_chp(tt 1)) =g=    Ramp_chp;
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rh_do_s(tt,s)$(ord(tt)>1).. (p_chp(tt)+p_s_chp(tt,s)) (p_chp(tt 1)+p_s_chp(tt 1,s)) =g=   Ramp_chp;
rh_do1(ddd,tt)$(ord(tt)=1).. (p_chp(tt)) (R1p_chp(ddd,'tt24')) =g=   Ramp_chp;
rh_do1_s(ddd,tt,s)$(ord(tt)=1).. (p_chp(tt)+p_s_chp(tt,s)) (R1p_chp(ddd,'tt24')+R1p_chp_s(ddd,'tt24',s)) =g=   Ramp_chp;

rh_up_chp2(tt)$(ord(tt)>1).. (p_fuel(tt)) (p_fuel(tt 1)) =l=   Ramp_chp2;
rh_up_chp2_s(tt,s)$(ord(tt)>1).. (p_fuel(tt)+p_s_fuel(tt,s)) (p_fuel(tt 1)+p_s_fuel(tt 1,s)) =l=   Ramp_chp2;
rh_up1_chp2(ddd,tt)$(ord(tt)=1).. (p_fuel(tt)) (R1p_fuel(ddd,'tt24')) =l=   Ramp_chp2;
rh_up1_chp2_s(ddd,tt,s)$(ord(tt)=1).. (p_fuel(tt)+p_s_fuel(tt,s)) (R1p_fuel(ddd,'tt24')+R1p_s_fuel(ddd,'tt24',s)) =l=   Ramp_chp2;

rh_do_chp2(tt)$(ord(tt)>1).. (p_fuel(tt)) (p_fuel(tt 1)) =g=   Ramp_chp2;
rh_do_chp2_s(tt,s)$(ord(tt)>1).. (p_fuel(tt)+p_s_fuel(tt,s)) (p_fuel(tt 1)+p_s_fuel(tt 1,s)) =g=   Ramp_chp2;
rh_do1_chp2(ddd,tt)$(ord(tt)=1).. (p_fuel(tt)) (R1p_fuel(ddd,'tt24')) =g=   Ramp_chp2;
rh_do1_chp2_s(ddd,tt,s)$(ord(tt)=1).. (p_fuel(tt)+p_s_fuel(tt,s)) (R1p_fuel(ddd,'tt24')+R1p_s_fuel(ddd,'tt24',s)) =g=   Ramp_chp2;

//Startup
chp_on(tt) .. xs_chp(tt)+xd_chp(tt) =l= b_chp(tt)*chp_cap;
chp_on_s(tt,s) .. xs_chp(tt)+xd_chp(tt)+xs_s_chp(tt,s)+xd_s_chp(tt,s) =l= b_s_chp(tt,s)*chp_cap;

chp_cost(tt) $(ord(tt)>1).. bs_chp(tt)=g= b_chp(tt) b_chp(tt 1);
chp_cost_s(tt,s)$(ord(tt)>1) .. bs_s_chp(tt,s)=g= b_s_chp(tt,s) b_s_chp(tt 1,s);
chp_cost1(dd,tt)$(ord(tt)=1).. bs_chp(tt) =g= b_chp(tt) R1b_chp(dd,'tt24');
chp_cost1_s(dd,tt,s)$(ord(tt)=1) .. bs_s_chp(tt,s)=g= b_s_chp(tt,s) R1b_chp_s(dd,'tt24',s);

chp2_on(tt) .. p_chp2(tt)+xs_chp2(tt)+xd_chp2(tt) =l= b_chp2(tt) *(CHP2_cap+CHP2_ex);
chp2_on_s(tt,s) .. p_chp2(tt)+xs_chp2(tt)+xd_chp2(tt)+p_s_chp2(tt,s)+xs_s_chp2(tt,s)+xd_s_chp2(tt,s) =l= b_s_chp2(tt,s) *(CHP2_cap+CHP2_ex);

chp2_cost(tt)$(ord(tt)>1) .. bs_chp2(tt)=g= (b_chp2(tt) b_chp2(tt 1))   ;
chp2_cost_s(tt,s)$(ord(tt)>1) .. bs_s_chp2(tt,s)=g= (b_s_chp2(tt,s) b_s_chp2(tt 1,s))   ;
chp2_cost1(dd,tt)$(ord(tt)=1) .. bs_chp2(tt)=g=(b_chp2(tt) R1b_chp2(dd,'tt24'))   ;
chp2_cost1_s(dd,tt,s)$(ord(tt)=1) .. bs_s_chp2(tt,s)=g= (b_s_chp2(tt,s) R1b_chp2_s(dd,'tt24',s))   ;

hp_on(tt) .. xs_hp(tt)+xd_hp(tt) =l= b_hp(tt)*Hp_cap;
hp_on_s(tt,s) .. xs_hp(tt)+xd_hp(tt)+xs_s_hp(tt,s)+xd_s_hp(tt,s) =l= b_s_hp(tt,s)*Hp_cap;

hp_cost(tt)$(ord(tt)>1) .. bs_hp(tt)=g= (b_hp(tt) b_hp(tt 1) ) ;
hp_cost_s(tt,s)$(ord(tt)>1) .. bs_s_hp(tt,s)=g= (b_s_hp(tt,s) b_s_hp(tt 1,s) ) ;
hp_cost1(dd,tt)$(ord(tt)=1) .. bs_hp(tt)=g= (b_hp(tt) R1b_hp(dd,'tt24') ) ;
hp_cost1_s(dd,tt,s)$(ord(tt)=1) .. bs_s_hp(tt,s)=g= (b_s_hp(tt,s) R1b_hp_s(dd,'tt24',s) ) ;

// shudown costs
chp_sdown(tt)$(ord(tt)>1).. bsd_chp(tt)=g= b_chp(tt 1) b_chp(tt)   ;
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chp_sdown_s(tt,s)$(ord(tt)>1).. bsd_s_chp(tt,s)=g= b_s_chp(tt 1 , s) b_s_chp(tt,s) ;
chp_sdown1(tt)$(ord(tt)=1).. bsd_chp(tt)=g= b_chp('tt24') b_chp(tt) ;
chp_sdown1_s(tt,s)$(ord(tt)=1).. bsd_s_chp(tt,s)=g= b_s_chp('tt24',s) b_s_chp(tt,s) ;

chp2_sdown(tt)$(ord(tt)>1).. bsd_chp2(tt)=g= b_chp2(tt 1 ) b_chp2(tt) ;
chp2_sdown_s(tt,s)$(ord(tt)>1).. bsd_s_chp2(tt,s)=g= b_s_chp2(tt 1 , s) b_s_chp2(tt,s) ;
chp2_sdown1(tt)$(ord(tt)=1).. bsd_chp2(tt)=g= b_chp2('tt24') b_chp2(tt) ;
chp2_sdown1_s(tt,s)$(ord(tt)=1).. bsd_s_chp2(tt,s)=g= b_s_chp2('tt24',s) b_s_chp2(tt,s) ;

//Minimum load

chp2_minl(tt)..  p_fuel(tt)=g=b_chp2(tt)*CHP2_min;
chp2_minl_s(tt,s)..  p_fuel(tt)+p_s_fuel(tt,s)=g=b_s_chp2(tt,s)*CHP2_min;

chp_minl(tt)..  p_chp(tt)=g=b_chp(tt)*CHP_min;      ;
chp_minl_s(tt,s)..  p_chp(tt)+p_s_chp(tt,s)=g=b_s_chp(tt,s)*CHP_min ;

hp_minl(tt) ..  xs_hp(tt)+xd_hp(tt)=g=b_hp(tt)*Hp_min;
hp_minl_s(tt,s) ..  xs_hp(tt)+xd_hp(tt)+xs_s_hp(tt,s)+xd_s_hp(tt,s)=g=b_s_hp(tt,s)*Hp_min;

//Storage constraints
Storageeq(tt)$(ord(tt)>1) .. st(tt)=e= st(tt 1 )+xs_chp(tt)+xs_chp2(tt)+xs_eb(tt) sloss*xd_st(tt)  ;
Storageeq_s(tt,s)$(ord(tt)>1) .. st_s(tt,s)=e= st_s(tt 1 , s)+xs_s_chp(tt,s)+xs_s_chp2(tt,s)+xs_s_eb(tt,s) sloss*xd_s_st(tt,s)  ;  //change in storage
Storageeq1(dd,tt)$(ord(tt)=1) .. st(tt)=e= R1st(dd,'tt24')+xs_chp(tt)+xs_chp2(tt)+xs_eb(tt) sloss*xd_st(tt)  ;
Storageeq1_s(dd,tt,s)$(ord(tt)=1) .. st_s(tt,s)=e= R1st_s(dd,'tt24',s)+xs_s_chp(tt,s)+xs_s_chp2(tt,s)+xs_s_eb(tt,s) sloss*xd_s_st(tt,s)  ;  //change in storage

st1eq(tt)$(ord(tt)>1) .. st1(tt)=e=st1(tt 1 ) +xs_hp(tt) sloss*xd_st1(tt);
st1eq_s(tt,s)$(ord(tt)>1) .. st1_s(tt,s)=e=st1_s(tt 1 , s) +xs_s_hp(tt,s) sloss*xd_s_st1(tt,s);
st1eq1(dd,tt)$(ord(tt)=1) .. st1(tt)=e=R1st1(dd,'tt24') +xs_hp(tt) sloss*xd_st1(tt);
st1eq1_s(dd,tt,s)$(ord(tt)=1) .. st1_s(tt,s)=e=R1st1_s(dd,'tt24',s) +xs_s_hp(tt,s) sloss*xd_s_st1(tt,s);

st_flowOut(tt) .. xd_st(tt)=l=flow_max;
st_flowOut_s(tt,s) .. xd_st(tt)+xd_s_st(tt,s)=l=flow_max;

st_flowIn(tt) .. xs_chp(tt)+xs_eb(tt)+xs_chp2(tt)=l=flow_max;
st_flowIn_s(tt,s) .. xs_chp(tt)+xs_eb(tt)+xs_chp2(tt)+xs_s_chp(tt,s)+xs_s_eb(tt,s)+xs_s_chp2(tt,s)=l=flow_max;

//b Capacity constraints
EBcap(tt) ..  xs_eb(tt)+xd_eb(tt)=l=eb_cap;
EBcap_s(tt,s) ..  xs_s_eb(tt,s)+xs_eb(tt)+xd_s_eb(tt,s)+xd_eb(tt)=l=eb_cap;

HPcap(tt) .. xd_hp(tt)+xs_hp(tt) =l= hp_cap;
HPcap_s(tt,s) .. xd_s_hp(tt,s)+xs_s_hp(tt,s) +xd_hp(tt)+xs_hp(tt) =l= hp_cap;
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chp2cap(tt).. xd_chp2(tt)+xs_chp2(tt)=l=CHP2_cap;
chp2cap_s(tt,s).. xd_s_chp2(tt,s)+xs_s_chp2(tt,s)+xd_chp2(tt)+xs_chp2(tt)=l=
    CHP2_cap;

Stcap(tt) .. st(tt)=l= st_cap ;
Stcap_s(tt,s) .. st(tt)+st_s(tt,s)=l= st_cap ;

St1cap(tt) .. st1(tt)=l= st1_cap;
St1cap_s(tt,s) .. st1(tt)+st1_s(tt,s)=l= st1_cap;

chpcap(tt,s) .. p_chp(tt)+p_s_chp(tt,s) =l= 60;
chp2pcap(tt,s) .. p_chp2(tt)+p_s_chp2(tt,s) =l= 250;

//Constraining the stochastic variables
HPs_con(tt,s)..  xs_s_hp(tt,s)=l=xs_hp(tt)    ;
HPs_con1(tt,s)..  xd_s_hp(tt,s)=l=xd_hp(tt)    ;

XDst_con(tt,s)..  (xd_s_st(tt,s))=l=xd_st(tt)    ;
XDst1_con(tt,s)..  (xd_s_st1(tt,s))=l=xd_st1(tt)    ;

EBs_con(tt,s)..  (xs_s_eb(tt,s))=l=xs_eb(tt)    ;
EBs_con1(tt,s)..  (xd_s_eb(tt,s))=l=xd_eb(tt)    ;

Ss_con(tt,s)..  (st_s(tt,s))=l=st(tt)    ;
S1s_con(tt,s)..  (st1_s(tt,s))=l=st1(tt)    ;
CHPs_con(tt,s)..  xs_s_chp(tt,s)=l=xs_chp(tt)    ;
CHPs_con1(tt,s)..  xd_s_chp(tt,s)=l=xd_chp(tt)    ;

CHP2s_con(tt,s)..  xs_s_chp2(tt,s)=l=xs_chp2(tt)    ;
CHP2s_con1(tt,s)..  xd_s_chp2(tt,s)=l=xd_chp2(tt)    ;

Ps_con(tt,s)..  (p_s_chp2(tt,s))=l=p_chp2(tt)    ;

bio_substract(tt).. b_sup(tt) =g= ec(tt) p_chp2(tt);
bio_substract_s(tt,s).. b_sup_s(tt,s) =g= ec_s(tt,s) p_s_chp2(tt,s);
bio_con(tt,s)..  (b_sup_s(tt,s))=l=b_sup(tt);

Model transport /all/ ;

loop(d$(ord(d)>d_start and ord(d)<d_slut)     ,
dd(d) = yes;
ddd(d1)$(ord(d1)=ord(d) 1) = yes;

display dd;
display ddd;

display dm_s;
Solve transport using mip minimizing z ;

R1xs_chp(dd,tt) = xs_chp.L(tt);
R1xd_chp(dd,tt) = xd_chp.L(tt);
R1xs_chp2(dd,tt) = xs_chp2.L(tt);
R1xd_chp2(dd,tt) = xd_chp2.L(tt);
Rin(dd,tt)=in.L(tt);
Rp_chp(dd,tt) = p_chp.L(tt);
Rp_chp2(dd,tt) = p_chp2.L(tt);

Rb_sup(dd,tt)= b_sup.L(tt);
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```
Rxs_chp(dd,tt) = xs_chp.L(tt);
Rxd_chp(dd,tt) = xd_chp.L(tt);
Rxs_chp2(dd,tt) = xs_chp2.L(tt);
Rxd_chp2(dd,tt) = xd_chp2.L(tt);
Rxs_hp(dd,tt) = xs_hp.L(tt);
Rxd_hp(dd,tt) = xd_hp.L(tt);
Rxs_eb(dd,tt) = xs_eb.L(tt);
Rxd_eb(dd,tt) = xd_eb.L(tt);
Rst(dd,tt) = st.L(tt);
Rst1(dd,tt) = st1.L(tt);
Rxd_st(dd,tt) = xd_st.L(tt);
Rxd_st1(dd,tt) = xd_st1.L(tt);

R1b_chp(dd,tt) = b_chp.L(tt);
R1b_hp(dd,tt) = b_hp.L(tt);
R1b_chp2(dd,tt) = b_chp2.L(tt);

R1p_chp_s(dd,tt,s)=p_s_chp.L(tt,s);
R1p_chp(dd,tt)=p_chp.L(tt);
R1p_chp2_s(dd,tt,s)=p_s_chp2.L(tt,s);
R1p_fuel(dd,tt)=p_fuel.L(tt);
R1p_s_fuel(dd,tt,s)=p_s_fuel.L(tt,s);

R1xs_chp_s(dd,tt,s) = xs_s_chp.L(tt,s);
R1xd_chp_s(dd,tt,s) = xd_s_chp.L(tt,s);
R1xs_chp2_s(dd,tt,s) = xs_s_chp2.L(tt,s);
R1xd_chp2_s(dd,tt,s) = xd_s_chp2.L(tt,s);
R1xs_hp_s(dd,tt,s) = xs_s_hp.L(tt,s);
R1xd_hp_s(dd,tt,s) = xd_s_hp.L(tt,s);
R1xs_eb_s(dd,tt,s) = xs_s_eb.L(tt,s);
R1xd_eb_s(dd,tt,s) = xd_s_eb.L(tt,s);
R1in_s(dd,tt,s)=in_s.L(tt,s);
R1st_s(dd,tt,s) = st_s.L(tt,s);
R1st1_s(dd,tt,s) = st1_s.L(tt,s);
R1xd_st_s(dd,tt,s) = xd_s_st.L(tt,s);
R1xd_st1_s(dd,tt,s) = xd_s_st1.L(tt,s);
R1b_chp_s(dd,tt,s) = b_s_chp.L(tt,s);
R1b_chp2_s(dd,tt,s) = b_s_chp2.L(tt,s);
R1b_hp_s(dd,tt,s) = b_s_hp.L(tt,s);
R1b_sup_s(dd,tt,s)= b_sup_s.L(tt,s);
Rz(dd)=z.L                          ;

execute_unload 'C:\Users\Maria\Dropbox\Speciale2014\Master Thesis\
    StochSolutionVar_w3_C5.gdx', Rxd_st, Rxd_st1, Rxd_hp, Rxs_hp, Rxs_eb, Rxd_eb
    , Rz,  Rin,
                 Rxd_chp, Rxd_chp2, Rxs_chp, Rxs_chp2, Rst, Rst1, R1b_chp,
                     R1b_hp, R1b_chp2, R1xs_hp_s, Rp_chp2, Rp_chp;

execute_unload 'C:\Users\Maria\Dropbox\Speciale2014\Master Thesis\
    StochSolutionVar3d_w3_C5.gdx', R1xs_hp_s, R1xd_hp_s ,R1xs_chp_s, R1xs_chp2_s
    , R1xd_chp_s, R1xd_chp2_s,
                 R1xs_eb_s, R1xd_eb_s, R1xd_st_s, R1xd_st1_s, R1st_s, R1st1_s,
                     R1b_hp_s, p_spot_s, dm_s, R1in_s, R1p_chp_s, R1p_chp2_s ;
```