Determination of Optimal Electricity Reserve Requirements

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Abstract

The continuous and safe delivery of electricity is a matter of concern amongst almost everyone, from single customers at their homes to industry owners and transmission system operators. Due to the nature of electricity, the amount produced and consumed must be equal. If, for some unexpected reason, those two quantities are not equal then the system is imbalanced and load demand has to be shed. This occurrence is costly and undesired. The main tool that transmission system operators have to avoid it is to allocate electricity reserves and use them to balance up the system if required

An alternative probabilistic formulation to the traditional deterministic "n-1" criteria is given in this thesis by using a stochastic programming framework. The solution of the optimization model indicates the total amount of reserves that must be allocated at the lowest possible cost. Moreover, two ways of accounting for the risk are discussed, namely the Loss of Load Probability (LOLP) and the Conditional Value at Risk (CVar) formulation. The scenarios are computed from a function of reserve needs which takes into account the power load demand forecast error, the wind production forecast error and the failures of the power plants. Finally, the usefulness of the methods is tested with data from West Denmark electricity area. The results show that the models are able to account for the market principles and provide reasonable levels of optimal reserves, with some room for improvement if the methodology is intended to be implemented in a real system.

Preface

This thesis was prepared at the department of Informatics and Mathematical Modeling at the Technical University of Denmark in collaboration with Energinet.dk, in fulfillment of the requirements for acquiring an M.Sc. in Mathematical Modeling and computation.

The thesis deals with the optimization of electricity reserve requirements.

The project was carried out in the period from 14h February 2012 to 1st October 2012.

Javier Sáez Gallego

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Secondly, my gratitude goes to the team at ENFOR S/A for providing the necessary scenarios and also to the Nord Pool Spot's Power Data Services for their valuable information. Moreover, I would like to thank all the participants at the OSR Nordic project for their positive comments and for providing interesting different points of view.

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Contents

Chapter 1

Introduction

Electricity is a commodity that must be supplied continuously at all times at certain frequency. When this requirement is not fulfilled and there is shortage of electricity, consumers can face the very costly consequences of outages: their production being stopped or their systems collapsed. In a lesser extent, service interruptions also affect electricity producers as they are not able to sell the output of their plants. Therefore it is of high importance that the demand is always covered. The main tool that market operators have in order to avoid electricity interruptions is the allocation of reserves also called *ancillary services*. In practice, scheduling reserves means that the system is operating at less than full capacity and the extra capacity will only be used in case of disturbances, also named *contingencies*.

In a system with thousands of components [1] the failure of a single component is not a rare event. The reasons why components fail can go from temperature changes in the weather or in the operating temperature to simple human mistakes. Nevertheless, other kind of disturbances than failures also have an effect on the amount of load that must be shed, namely changes in the wind and demand predictions. Unexpectedly big increase in the demand makes the system imbalanced and corrective actions must be taken, either activating the reserves or on the worse case, shedding load. Similarly happens with wind power production. If the wind turns out to blow at less speed than forecast, the energy input into the system will be lower than expected and thus imbalanced.

If the security of the system wants to be maximized then the allocated ancillary services should be as great as possible. This means that all power plants must have their generators ready to be connected at any time if they are not producing energy already. Besides the technical issues this constrain arises, the overall probability of not meeting the demand will be decrease. From the economical point of view this constrain would be highly expensive since producers are paid to have their plants ready to produce at all times. On the contrary, if few reserve is allocated, the cost of allocating reserve would decrease. This would lead to an increase of the probability of facing an unprotected contingency and in case one would happen, a high societal cost. The power system operator faces a trade-off between the cost of allocating reserve and the societal cost of not allocating enough.

The **motivation** of this thesis becomes obvious when studying the specific case of West Denmark, also called *DK1* area. Nowadays, a simple deterministic rule is used to compute the amount of reserves that should be allocated. In the case of manual reserves (ie. one of the three types or reserves) it is set equal to the largest production unit online. This method is known as the *n-1* criterion does not take into account the probability of occurrence of contingencies such as failures in the system or sudden changes in the demand and wind power production.

The **objective** of this thesis is to develop an alternative method capable of determining the optimal amount of reserve. The final aim is to minimize the cost that the society has to pay in order to obtain a continuous and safe electricity delivery, taking into account both the cost of purchasing reserves as well as the cost derived from shedding load. The methodology proposed uses a stochastic programming framework to deal with the stochastic nature of the power plant failures, the wind power production and the electricity demand. Furthermore, the performance of the models developed is tested in a case study. The data related to DK1 area and includes information of wind power production, net electricity demand and failures in the system from the 1st January 2009 at 00:00 CET to the 30th June 2012 at 23:00 CET.

1.1 Thesis overview

Chapter 2 starts by giving an overview of the electricity market structure in Denmark, followed by a explanation of the reserve market and a definition of the three types of reserves that can be allocated. Then the data used along the thesis is briefly presented and graphically studied.

Chapter 3 deals with the core of the thesis which is the three different models for optimizing the total reserve level, first as a general formulation and then specific to the study case.

Chapter 4 elaborates on the estimation of the price for allocating reserves and also on the generation of scenarios characterized by a reserve need.

Chapter 5 shows the results of applying the models to the data.

Chapter 2

Electricity market, reserve market and data presentation

Firstly in this chapter an overview of Danish electricity market and the way it works is presented. Secondly, it is defined what reserve is and how is it managed in West Denmark and finally the data used for the study case is shown.

2.1 Electricity as a commodity

Electricity is one of the most necessary elements in contemporary society. It is used by millions of people in their houses and offices to empower appliances, cool down or heat the air, amongst many other applications; industry owners need it to create products or services. The energy produced by power plants is sold as a commodity in markets. However, the market structure is quite different to other products. The main differences with other markets are caused by the nature of the electricity itself and the reasons can be summarized as[2]:

1. The physical system works much faster than any other market, electricity

can be transported long distances in much faster way than other commodities. However, it requires special and expensive infrastructure called a transmission system which have limitations on how much energy can be transported simultaneously.

2. The energy from the generators is often pooled on the way to the consumer, making the consumer unable to determine from which power plant electricity comes from. The system operators play an important role in this pooling, controlling it.

3. The electricity must be produced at the same time when it is consumed as it cannot be stored at a reasonable price.

4. The demand is very inelastic and consumers do not modify their consumption depending on the price. One reason is that electricity is hard to substitute and another reason is that small consumers are not affected by prices changes instantly. This fact could change if smart grids were installed [3].

2.2 Market structure in Western Denmark

The electricity market in West Denmark is nowadays a competitive market, where any qualified competitor can participate. It is liberalized as opposed to centralized market so the competence amongst companies increases and therefore the efficiency is increased, hence the quality of the services with a minimum cost [4]. West Denmark has been integrated into the Nordic Power exchange area since 1999, trading energy through the Nord Pool Spot market[5]. The Nord Pool Spot AS market is an organization that offers both day-ahead and intraday markets to its customers, 370 companies from 20 countries. It is is owned by the transmission system operators of Norway, Statnett SF, the Sweedish, Svenska Kraftnat, the Finnish ,Fingrid Oyj, and the Danish Energinet.dk.

The grid corresponding to the West Denmark area or DK1 covers the Jutland peninsula, Funen island and the rest of the islands west of the Great Belt. A map illustrating the division is shown in Figure 2.1

Depending on the time when the electricity is traded one could distinguish between two markets: The Financial Market for futures and forward contracts, and the Nord Pool market and its three submarkets for short term transactions. A summary time line is presented in Figure 2.2

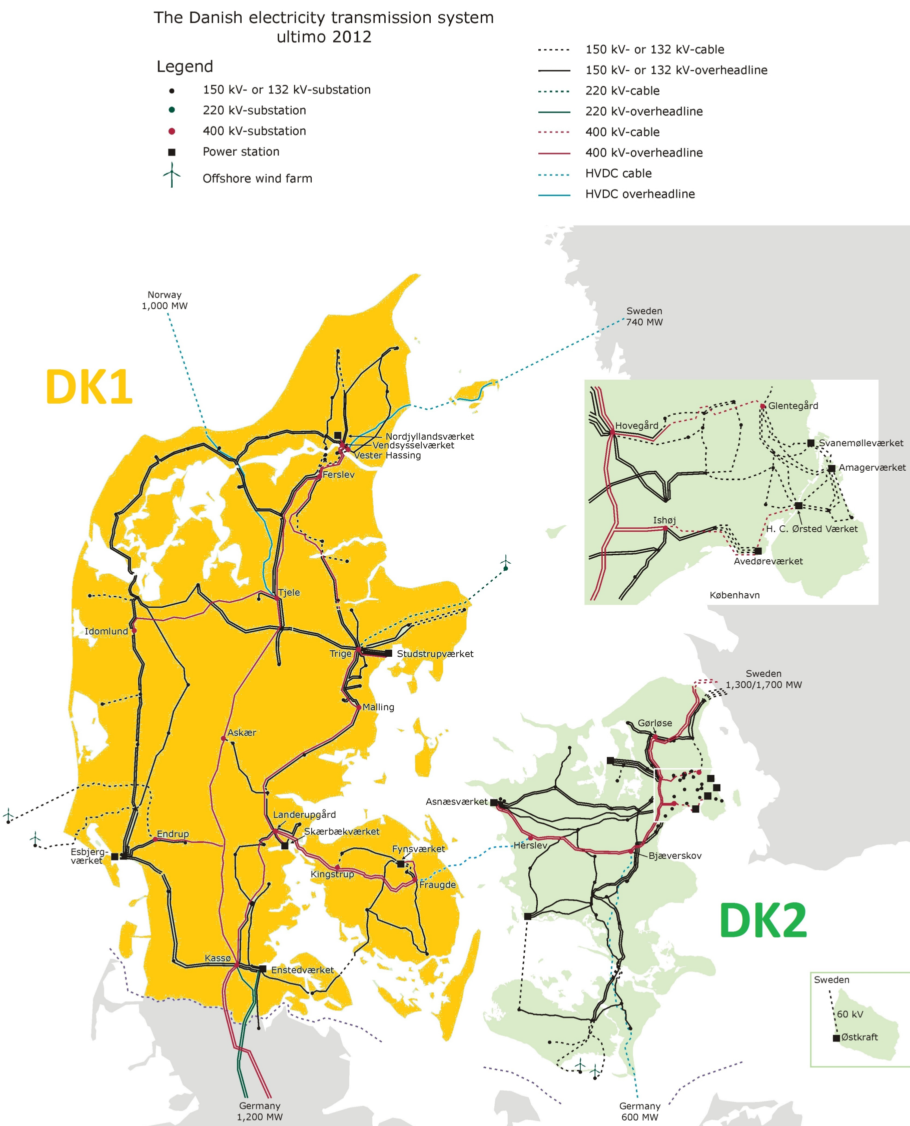

Figure 2.1: Map of Denmark. The West Denmark DK1 gird region is highlighted in orange while East Denmark or DK2 is colored in green. The main transmission lines and power plants are included as well [3]

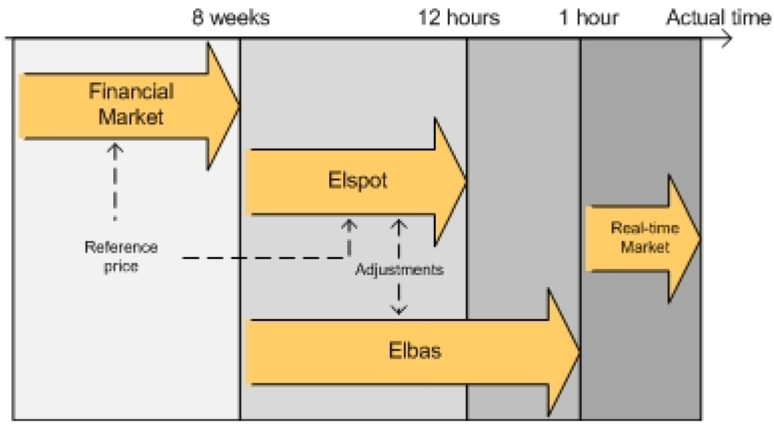

Figure 2.2: Time line showing the organization of the markets in Denmark along time

2.2.1 Financial Market

The financial market is used to settle contracts of energy delivered from 6 weeks to three years into the future. The derivates are base and peak load futures and forwards, options, and Contracts for Difference. The market itself is run by the NASDAQ OMX Commodities group and it has been designed to satisfy the needs of various participants[6]:

1. Producers, retailers and end-users who use the products as risk management tools. The price is volatile and can vary a lot from day to day, affected by many factors like weather, failures, or politics. The risk that producers and consumers are facing is big. In order to minimize it, the energy delivered in the future is sold at the reference System Price of the total Nordic power market

2. Traders who profit from volatility in the power market, and contribute to high liquidity and trade activity.

There is no physical delivery of financial market power contracts. Cash settlement is made throughout the trading- and/or the delivery period, starting at the due date of each contact (depending on whether the product is a future or forward). Financial contracts are entered into without regard to technical conditions, such as grid congestion, access to capacity, and other technical restrictions.[7]

2.2.2 Nord Pool market

The Nord pool area consists of three sub-markets, all with different functions at different trading times: Elspot, Elbas and Regulating Market.

2.2.2.1 Elspot

The Elspot Market, also known as the day-ahead market, allows Nordic market participants trade power contracts for next-day physical delivery. At noon each day, bids for either purchase or sale are collected. Figure 2.3 shows a timeline with the process of bidding, accepting and production planning.

Three bidding types are available, namely hourly bids, block bids, and flexible hourly bids. As soon as the noon deadline for participants to submit bids has passed, all buy and sell orders are gathered into two curves for each power-delivery hour: an aggregate demand curve and an aggregate supply curve. The spot price for each hour is determined by the intersection of the aggregate supply and demand curves. This spot price is also called the System Price.[8]

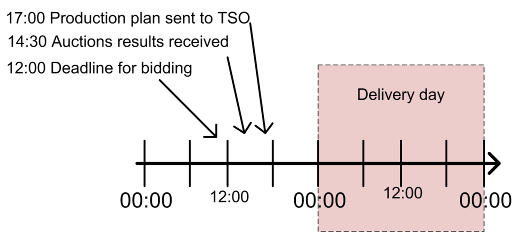

Figure 2.3: Time line showing the process of bidding, accepting and planning at Elspot [2]

2.2.2.2 Elbas

Elbas is a continuous market where power trading takes place until one hour before the power is delivered. Members submit bids stating how much power they want to sell and buy and at what price. Trading is then set based on a first-come, first-served basis between a seller and a buyer. West Denmark joined this market in 2008. Since energy already traded on the Elspot market is higher prioritized than energy traded on the Elbas market, transactions between areas where transmission capacities are already fully utilized are not allowed.

This market allows members to adjust their power production or consumption plans close to delivery in case the production or consumption schedules deviate from the original plan.

2.2.2.3 Real-time Market

In DK1 energinet.dk is in charge of the real-time market, also called regulating market.

Cleared up to 45 minutes prior to the upcoming delivery hour, the regulating market allocates load following bands among production units with capability to provide this service and interest in providing it. A load following power plant is a power plant that adjusts its power output as demand for electricity fluctuates throughout the day.

The bids must be submitted to Energinet.dk and may cover an entire day of operation. The entered prices and volumes can be adjusted by the player up to 45 minutes prior to the upcoming delivery hour. Players must be able to fully activate a given bid in maximum 15 minutes from receipt of the activation order.

2.2.3 Ancillary services in West Denmark

The ancillary services guarantee that enough back-up generation is available in case of equipment failure, drastic fluctuations of production from intermittent sources and sudden demand changes [4]. Security os achieved trough electricity reserves that both consumers and producers can offer. In DK1 the TSO Energinet.dk is responsible for purchasing security on behalf of the users of the system. Energinet.dk pays the providers of the ancillary services and recover the cost from the users trough taxes. Note producers are paid for the availability of the energy, even though in the future it might not have to be consumed.

Depending on certain technical conditions, Energinet.dk buys several kinds of reserves: primary reserves, secondary reserves, manual reserves and short-circuit power, reactive reserves and voltage control reserves, being the three first types the most relevant for this study. A generating unit can participate offering the three first types of reserve as shown in Figure 2.4 even though in practice it might provide none or one or two [9]

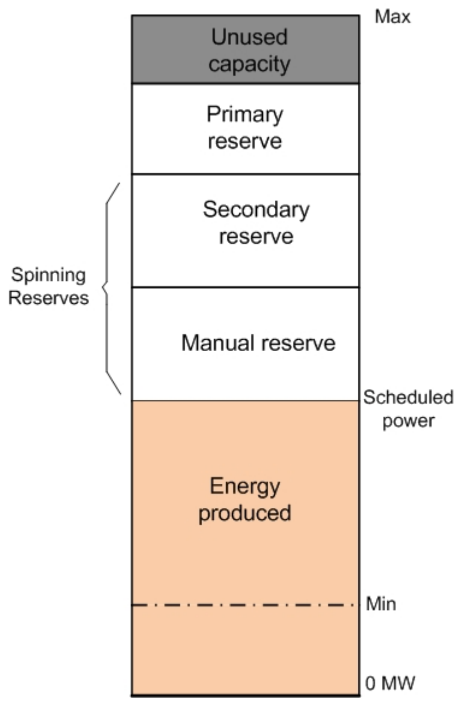

Figure 2.4: Allocation of the capacity of a generating unit that participates offering the three kinds of reserve plus a scheduled power

2.2.3.1 Primary reserves

The primary reserve regulation ensures that the balance between production and consumption is restored after a deviation from the 50Hz of frequency. The rotor of a generator spins at a different speed when the demand changes ie if the demand increases the rotor spins slower. The first half of the reserve must be activated within 15 seconds, while the second half must be fully supplying within 30 seconds. The reserve must be supplied maximum for 15 minutes. Energinet.dk buys two types of primary reserve, upwards regulation power and downward regulation power, in case of under frequency or over frequency respectively. An auction is held once a day for the coming day of operation. The bids are sent before 15:00, stating an hour-by-hour volume and price having the 24-hour period divided into six equally sized blocks. In 2011 the quantity of the primary reserves sums up to +/- 27MW, having the option of buying +/-90MW from other European transmission system operator as well as from East Denmark area, the Nordic countries and Germany.

2.2.3.2 Secondary reserves

The secondary reserve serves two proposes. One is to release the primary reserve which has been activated and the other is to restore any imbalances on the interconnections to follow the agreed plan. The requested energy must be supplied within 15 minutes and it can be supplied by a combination of unit in operation and fast-start units. It consists of upward and downward regulation that can be provided by several of production or consumption units. Energinet.dk currently buys approx. +/- 90MW on a monthly basis, based on a recommendation from the ENTSO-E RG Continental Europe organization.

2.2.3.3 Manual reserves

Also called tertiary reserves, relieves the secondary reserve in the event of minor imbalances and ensures that the demand is fulfilled, in the event of outages, restrictions affecting productions plants and international connections. It is used to release the secondary reserve as it is usually less costly. It must be supplied in full within 15 minutes of activation, so usually it is players with fast start units as gas turbines who usually bid into this market. Players must send their hourly volume and bids before 9:30 on the day before the day of operation and each bid must be of a minimum of 10MW and a maximum of 50MW. The bids are sorted according to the price per MW and the requirements are covered by selecting cheaper bids first. Bids are always accepted in their entirely or not at all, meaning that in situations where acceptance of a bid of more than 25MW will lead to excess of fulfillment of the requirement for reserves during the hour in question, such bids can be disregarded.[10]

Energinet.dk activates the reserve by manually ordering upward and downward regulation to the suppliers. The method used to determine the requirements for reserve is known as the *n-1 rule*: setting the minimum amount of manual reserves to be the capacity of the largest online generator. In West Denmark, the amount of manual reserve bought during the considered period varies considerably as shown in Figure 2.5.

2.3 Spinning Reserve definition

An alternative way of differentiating between types of reserves is to split them into spinning reserves and non-spinning reserves. From [9] “The spinning reserve is the unused capacity which can be activated on decision of the system operator

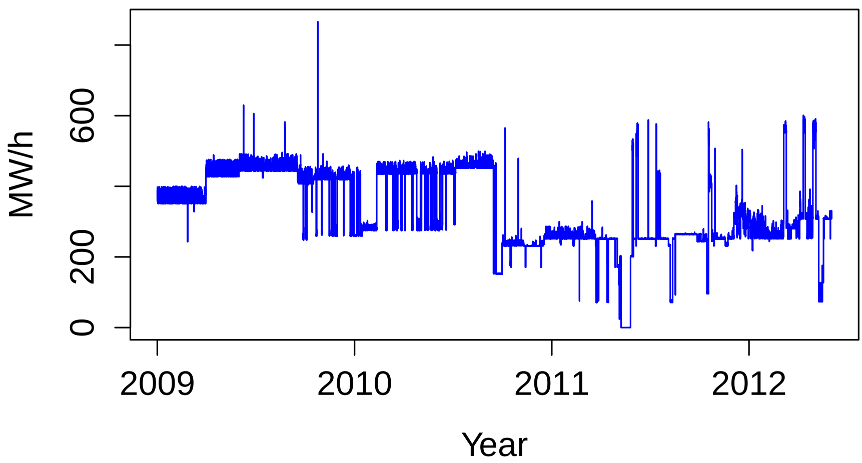

Figure 2.5: Total purchase of upward manual reserve in MW/h in DK1

and which is provided devices which are synchronized to the network and able to affect the active power". Similarly, non-spinning reserves are the reserves that are not activated on the decision of the system operator. According to this definition, spinning reserves are equal to the manual reserves. Secondary and primary reserves are not included in the definition since they are controlled automatically.

2.4 Presentation of data

All the data used for the study case in this project refers to the period from the 1st January 2009 at 00:00 CET to the 30 June 2012 at 23:00 CET. The training period goes from the 1st January 2009 at 00:00 CET to the 30st June 2011 at 23:00 CET, while the test period is set to be from the 1st July 2011 at 00:00 CET to the 30th June 2012 at 23:00. The resolution is hourly with a total of 30646 observations, 21863 belong to the training set and 8783 to the test set. All the data refers to the are of West Denmark, also called DK1 area .

The optimization model will use the following data information to draw empirical results:

1. Scenarios of wind power production and power load demand forecast errors
2. Scenarios of the quantity of MW as a consequence of power plant outages
3. Power net demand forecasts

The data is briefly presented in the following subsections.

2.4.1 Scenarios of wind power production and power load demand

Scenarios for wind power and power load have been provided by ENFOR A/S. Scenarios are created in pairs so the correlation between both variables is already taken into account. There are 5000 pairs of scenarios each hour covering the whole test data period. They are generated with a lead time of 24 hours at 9:00 CET in the morning.

The input of the models developed in this project is the forecast error of both the power load and the wind power production. It is assumed that wind power producers bid into the electricity market their expected production. If a scenario has a greater value of wind power production than what was expected then there will be extra power to sell; if, on the other hand, the realized wind is less than the expected value and some reserves will be needed. Similarly with the power load, it is assumed that the amount of power purchased in Nordpool is equal to the expected power load demand.

Scenarios are assumed to be all the possible realizations of reality. The more scenarios, the more accurate representation of reality, but on the other hand more computational complexity having here a trade-off between accuracy and complexity.

2.4.2 Mega Watts failed

In order to have the most realistic picture possible of how many MW failed at DK1, it is needed to gather information about failures of all central power stations, combined heat and power (CHP) plants, wind and solar farms and transmission lines. Due to the fact that the information is many times confidential, only known by the owners of the generating units, or it simply does not exist, not all the data required for a perfect representation of reality has been acquired. Only the most relevant electricity sources were considered, a complete list of the considered power plants is displayed in Table 2.1. The gathering of the missing data and its modeling has been left for future work. The reason why information about certain power plants is known is because they are big enough to be required to send Urgent Market Messages (UMM) to the Nordpool application. All members of the market are obliged to publish a UMM online when planned outages and unplanned outages happen, as well as failures in the power lines. In this project, only the unplanned outages or failures are considered, being published if the outage fulfills the following conditions [11]:

1. More than 100MW for one transmission facility, including changes of such plans in the next 6-week period. This means that small failures of less than 100MW are not registered in the system and therefore not taken into account in this project.

2. More than 400MW for one transmission facility for the current calendar year and three calendar years forward, including changes of such plans.

Plant name	Capacity biggest unit in MW
Asnæsværket	640
Enstedværket	262
Esbjergværket	378
Fynsværket	362
Horns Rev	160
Nordjyllandsværket	411
Skærbækværket	392
Studstrupværket	350

Table 2.1: Central power plants in Denmark which failures affect DK1

A UMM shall be published immediately and no later than 60 minutes after the information occurred and includes information about the amount of MW scheduled and failed, the power plant name, the company running it, the area affected and the cause for the event. If an outage lasts for less than 60 minutes, it is not mandatory to send any UMM.

One should note that the coal-fire power plant Asnæsværket is located at Sjælland and belongs to the DK2 region; however, according to the urgent Market Messages the plant has sent, its failures have an effect on DK1 and therefore have been considered.

A complete list of all UMM can be found at [5]. Thanks to the help of Power Data Service at Nord Pool Spot, it was possible to access their FTP servers and use their database to select only the messages tagged as "failures" and such that the area affected is DK1. A script in R was used to read all the messages and can be found in Appendix. During the considered period there was a total of 278 outages. Putting the same information into a time series, for every hour it is known how many MW of electricity have failed and also from which power plant. The data is shown in Figure 2.6.

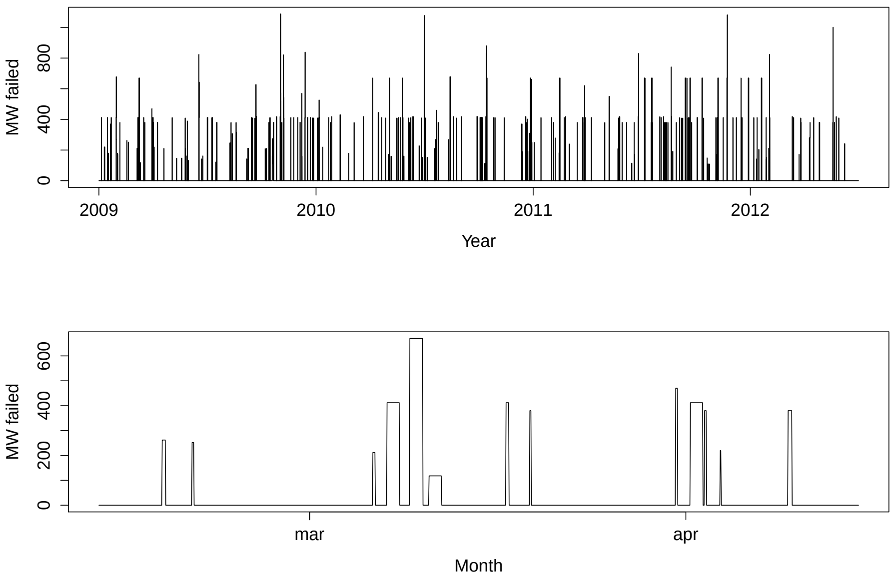

Figure 2.6: On the upper plot, the total amount of MW of electricity that failed from power plants that affected DK1, as recorded in the Urgent Market Messages application of Nord Pool Spot. The botton plot is a zoomed version of the upper plot on the year 2009.

The outage data is characterized by being very sparse, containing many zeros, since most of the times there are no failures and plants work as planned. There is a total of 29952 observations of which 27662 are zeros, or approximately 92.22%. At a first glance it is beleived that seasonality has to be taken into account. As one can see in on the left side of Figure 2.7, the mean of the MW failed at each hour of the day tends to be higher at certain hours of the day; similar issue happens for the week days and the mean of every month. This all indicates that seasonality should be studied. The seasonality of the MW that failed could be induced by the total production of the power plants, or similarly by the power load, since the higher the power load is, usually the more power plants will be ON and therefore the MW failed will increase. This effect can be reduced by dividing the total MW failed by the power load. The right side of Figure 2.7 reveals that there is still seasonality to be studied.

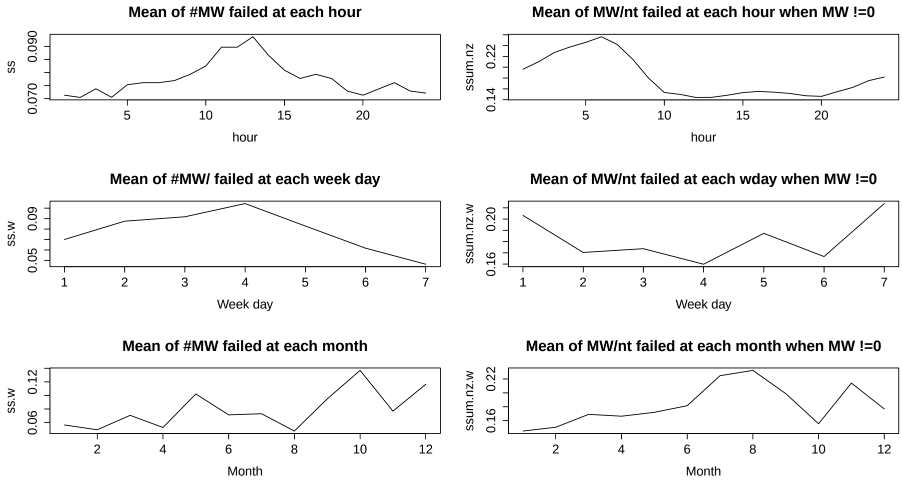

Figure 2.7: On the left side: The mean of the amount of MW failed per hour, week day and month. On the right side: The mean of the amount of MW failed divided by the net load ear hour, week day and month

Due to the nature of the data, the modeling presents several challenges. At a fist glance, one could say that autoregressive methods will not perform very well since they are not able to capture such jumps. Methods of Generalized Linear Models and Hidden Markov Models have been proved to be adequate and are deeply explained in Section 4.2.2.

2.4.3 Net demand in DK1

The Net Demand is defined as the sum of the West Danish consumption excluding transmission loss. The data was download from [3] and it is shown in Figure 2.8.

The net demand was chosen to be as an indicator of how much the system is being used. It seems reasonable to state that the more energy is demanded, the more power plants are activated and more generators are subject to fail.

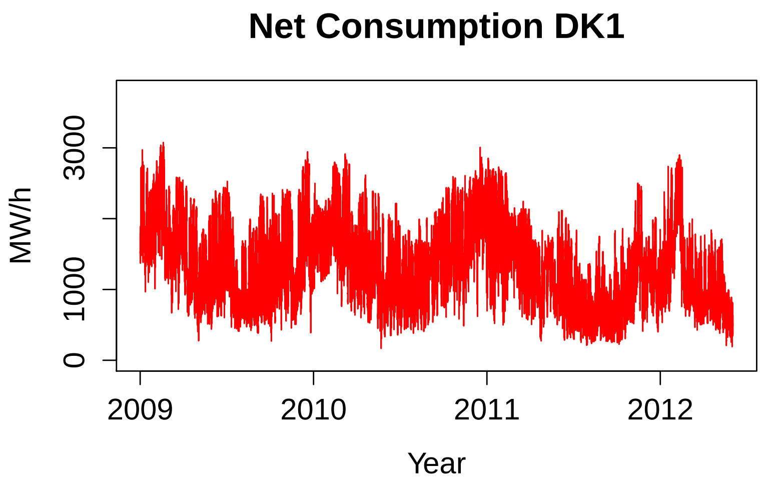

Figure 2.8: Net demand in DK1

Another reason for choosing this variable is the fact that generated scenarios characterized by the power load are available for this thesis given by the team at ENFOR S/A. Other variables could have been included in the model as the total production at DK1; however, in that case scenarios characterized by the new variables should be generated since they are not available and thus this task is left for future work.

Chapter 3

Models for optimizing Spinning reserve

3.1 Previous work

Traditionally market operators use a deterministic criterion to calculate the amount of reserves that should be scheduled. In Denmark [10], the amount of tertiary reserve is equal to the capacity of the largest generator. This criteria is commonly named as the "n-1" criteria. In other systems like in Spain [12] the amount of tertiary reserves is equal to the capacity of the largest generator plus 2% of the forecasted load. The UCTE reccomends a minimum requirement of secondary reserves calculated for each control area with the following formula [13]:

$$R = \sqrt{a \times L_{max} + b^2} - b$$

Where the parameters a and b are calculated empirically, currently set at $a = 10$ MW and b at 150 MW. The L_{max} is defined as the hourly maximum of the load of the day. Similarly, the The Western Interconnection of North America organization requires an amount of contingency reserves equal to the greater of (1) the most severe single contingency, and (2) the amount equal to five percent of the total load served by hydro generation, and seven percent of the total load served by thermal generation in the balancing authority or reserve sharing group [13].

Deterministic methods have been used in systems with very low penetration of renewable energy and fairly predictable load, since the biggest potential reserve needs arises from outages of large generation units. With increasing share of renewables (and decentralized production in general) in the production portfolio, renewable will naturally have a larger influence on the system's imbalance - both because of their own increasing imbalances and the consequent decommissioning of conventional power plants. Hence the potential outages or contract deviations of these plants has to be accounted for when reserve power is allocated once their share in the production portfolio becomes significant.

The second group of methods can be tagged as probabilistic. In [14] the author explains the computation of two reliability metrics, the Expected Energy Not Served and the Loss Of Load Probability and imposes a bound on them when calculating the traditional Unit Commitment (UC) problem. Reference [15] proposes a method to determine the spinning reserves requirements for each period of optimization horizon in an auxiliary computation prior to the UC commitment. This auxiliary computation consists of solving minimizing a cost function consisting of the sum of the running costs plus the cost of not serving energy. In [16] it is studied how wind power generation and load forecast errors as well as the possible contingencies affect the optimization of SR. Reference [17] uses outage probability information and assumes wind power and load forecast errors to be Gaussian distributed. The author does not tackle the UC problem. In [18] proposes a two-stage stochastic programming model to account for the stochastic nature of the wind power generation when clearing the market.

3.2 Problem identification

As explained in previous section, the manual reserves in Denmark are determined before 9.30 am when Energynet.dk collects the bids from producers who are willing to offer some quantity of reserve at some price. Energinet.dk sorts the bids for upward and downward regulation capacity according to price per MW and covers its requirements by selecting bids according to increasing price. Bids are always accepted in their entirely or not at all[10]. In situations where acceptance of a bid for more than 25 MW will lead to excess fulfillment of the requirement for reserves during the hour in question, Energinet.dk can disregard such bids.

Recall that the the Elspot market is cleared at 12:00 for the day ahead deployment. This means that at 9.30 am, when the manual reserves are settled, the production bids are not known yet hence the unit commitment problem cannot be addressed. In the following sections the production schedule of electricity is neglected. As a consequence, the production ramp up and ramp down limits and start-up costs are not relevant for this thesis either.

It should be noted that the models presented in this chapter will deal with the total reserve needs, meaning that no difference is made between primary, secondary or manual reserve. It is assumed that the TSO would take care of distributing the total needs amongst the three types of reserves.

All in all, there are five **assumptions** needed to be done:

1. Wind power producers bid into the Elspot market their expected production.
2. Energinet.dk buys the expected power load demand at the day ahead market.
3. Only producers who offer their reserves at the reserve market can take part into the real-time market.
4. Providing down-regulation is easier than up-regulation, hence it will be neglected.
5. The regulating power coming form the neighbor countries, namely Germany, Sweden, Norway and Denmark East, is neglected.

3.3 General formulation

This section presents three different model formulation for the optimal computation of the total reserve needs in a general form.

The total reserves that should be purchased are assumed to be affected by three main factors or uncertainty sources:

1. Wind power production
2. Electricity demand
3. Forced outages of power plants, namely failures in the plants that make the production stop.

Other factors could be considered as the reserves provided by the interconnections with neighbor countries, failures of transmission lines and CHP plants or solar energy production , but for simplicity only those three factors will be taken into account in this project.

As explained in 2.4.1, it is assumed that wind power producers bid into the electricity market their expected production. If the actual wind power production is greater than what was expected then there will be extra power to sell; if, on the other hand, the realized wind is lower than the expected value some reserves will be needed. One could say that if the forecasts were perfect and the errors equal to zero, no reserve would be needed. Likewise, if the errors are huge, big reserves are necessary to account for the possible variations. Similarly with the power load, it is assumed that the amount of power purchased in Nordpool is equal to the expected power load demand. Finally the predicted outages of power plants will lead directly to reserve needs.

The three reserve factors can be combined into one by convolving their distributions. If ϵ_w the forecast error of the wind power production with a distribution f_{ϵ_w}, ϵ_d is the forecast error of the electricity demand with distribution f_{ϵ_d} and f_{out} the forecast distribution of the forced outages, then the distribution of the reserve needs $f(z)$ will be given by the convolution of the three distributions:

$$f_z = f_{\epsilon_w} * f_{\epsilon_d} * f_{out}$$

This procedure is considered as a general formulation and will not be used in the study case. It could be useful in a very simplified approach where the three distributions are known. In a more realistic example, the distributions are not given in a closed form which would lead to more complex numerical issues. An alternative way of solving this is by scenario generation techniques, characterizing the possible scenarios by different realizations of the stochastic variables.

3.3.1 Expected Power Not Served (EPNS) model

The aim of this model is to minimize the function representing the total cost of allocating reserve plus the cost of the Expected Power Not Served (EPNS). The EPNS is incurred when the allocated reserves are smaller than actually needed. 1 MW of not served power costs to the society an amount represented by the Value of Loss Load V^{lol}. The optimal solution is one such that the total cost is minimized, as represented in Figure 3.1.

In mathematical terms, the objective function is to minimize the cost which

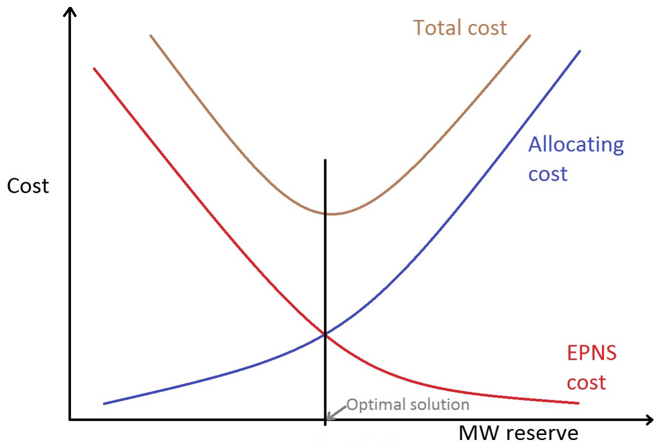

Figure 3.1: Representation of the total cost function of the EPNS model. The total cost is the sum of the allocating costs plus the cost of not allocating enough power.

depends on the variables R_i:

$$\min_{R_i} \sum_i \lambda_i R_i + V^{LOL} \times EENS \tag{3.1}$$

With a number of constrains:

$$R^T = \sum R_i \tag{3.2}$$
$$R_i \leq R_i^{max} \tag{3.3}$$
$$EPNS = \int_{R^T}^{\infty} z f(z) dz \tag{3.4}$$
$$R_i \geq 0 \qquad \forall i \tag{3.5}$$

where the variables are

R_i — Reserve provided by generator i in MW

R^T — Total amount of reserve. This is the quantity we are more interested in and the data

λ_i	Price to which generator i sells its availability of providing reserve in Euro. The actual activation costs (if the reserves are deployed) is not known at the time when the reserve market closes.
V^{LOL}	Value of Loss Load, or the cost that society pays for shedding 1 MW of load demand in Euro.
$EPNS$	Expected Power Not Served in MW.
R_i^{max}	Maximum amount of power offered by generator i in MW.
$f(z)$	Density function of variable Z, computed as the convolution of the distribution of the wind power production forecast error, the electricity demand forecast error and the forecast outages of power plants.

Equation 3.2 defines the total reserve needs as the sum of the contribution of each individual producer. When the producers submit a bid it is stated what is the maximum amount of reserve they can provide, this fact is indicated in Equation 3.3. Other alternative types of bids can be formulated as well, like stating a minimum for their reserve provided or the possibility of accepting either the whole quantity bid or none.

The Value of Loss Load V^{LOL} represents cost that society pays for shedding 1 MW of load demand (in Euro). The estimation os this parameter is quite complex to estimate. Some authors like [19] suggest that it should be approximately 100 times higher than the average price of electricity. Another possible approximation could be the maximum bid allowed to enter into the electricity market. In any case, it is supposed to be a very high value to be determined by the system operator.

When solving the problem with a computer software like GAMS, unless $f(z)$ has a close and easy form, equation 3.4 must be modified: the integral must be discretized. A grid of W points is drawn from all the possible values of Z. Each z_w has its corresponding probability $f'(z_w)$. Since, in practice, one cannot discretize a function by infinite number of points, the probabilities must be scaled by $f(z_w) = \frac{f'(z_w)}{\sum_{w=1}^{W} f'(z_w)}$ so they satisfy that $\sum_{w=1}^{W} f(z_w) = 1$. Then equation 3.4 must be substituted by

$$EENS = \sum_{w=1}^{W} y_w z_w f(z_w) \qquad \forall w \tag{3.6}$$

$$R^T - z_w > M(1 - y_w) \qquad \forall w \tag{3.7}$$

$$-(R^T - z_w) \leq M y_w \qquad \forall w \tag{3.8}$$

$$y_w \in \{0, 1\} \qquad \forall w \tag{3.9}$$

Where M is a relatively big value at least greater than the maximum of z_w. The auxiliary binary variables y_w model the same idea as the integral $\int_{R^T}^{\infty}$. Only values of reserve greater than R^T are taken into account in the calculation of the EENS. Namely,

$$y_w = \begin{cases} 1, & \text{if } z_w > R^T \\ 0, & \text{if } z_w \leq R^T \end{cases}$$

The same formulation can be applied in the case that the reality is represented trough scenarios as in [4] instead of directly specifying $f(z)$. If so, the formulation corresponds to a two-stage stochastic programming, with:

1. First stage variables: R_i and R^T, representing the decision that has to be made at the current time.
2. Second stage variable: z_w, representing the reserve needs at the time of deployment on scenario w. Each scenario is characterized by a realization of the stochastic variable Z *reserve needs*. More on how this variables is modeled and scenario generation is found at Section 4. The probability of each scenario is $\pi_w = f(z_w)$.

3.3.2 Loss of Load Probability (LOLP) model

The Loss Of Load Probability (LOLP) is "the probability that the available generation, including spinning reserve, cannot meet the system load" [14]. Another way of defining the same idea more suited to this thesis would be as the probability that the reserves needed exceed the scheduled reserve.

The objective function is the minimization of the cost:

$$\min_{R_i} \sum_i \lambda_i R_i \tag{3.10}$$

Constrained by:

$$R^T = \sum R_i \tag{3.11}$$
$$R_i \leq R_i^{max} \tag{3.12}$$
$$LOLP = \int_{R^T}^{\infty} f(z)dz \tag{3.13}$$
$$LOLP \leq \beta \tag{3.14}$$
$$R_i \geq 0 \qquad \forall i \tag{3.15}$$

As $f(z)$ represents the reserve needs, the area located under the curve is the probability of having a certain amount of reserve needed. Therefore, the area situated under the curve from $z = R^T$ to $z = \infty$ is the probability of not having enough reserves, namely the Loss Of Load Probability. It is constrained by a parameter target β at Equation 3.14, which must be determined by the transmission system operator. The smaller β is, the more reserves will be needed, as the LOLP has to be small. On the other hand, if β would be equal to 1, no reserves are needed at all.

Similarly as in the previous section, Equation 3.13 cannot be easily modeled into a computer software like GAMS unless it has a close and easy form, so it has to be discretized. A grid of W points is drawn from all the possible values of Z, each z_w with its corresponding probability $f(z_w)$. Equation 3.13 must be substituted by

$$LOLP = \sum_{w=1}^{W} y_w f(z_w) \tag{3.16}$$
$$R^T - z_w > M(1 - y_w) \qquad \forall w \tag{3.17}$$
$$-(R^T - z_w) \leq M y_w \qquad \forall w \tag{3.18}$$
$$y_w \in \{0, 1\} \qquad \forall w \tag{3.19}$$

Where M is a relatively big value, in this case it has to be greater of equal than the maximum of z_w. The auxiliary binary variables y_w means that

$$y_w = \begin{cases} 1, & \text{if } z_w > R^T \\ 0, & \text{if } z_w \leq R^T \end{cases}$$

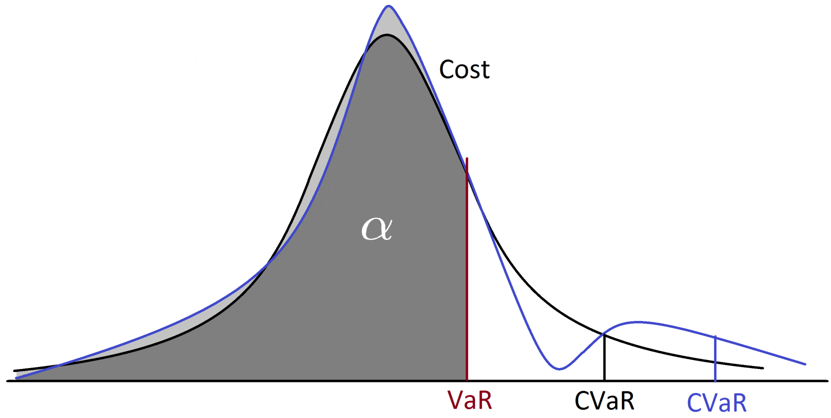

Figure 3.2: Representation of a probability density function and its corresponding VaR and CVaR values with a level of α

As in the previous subsection, the same formulation is valid if z_w is seen as a scenario characterized by a reserve need and $f(z_w) = \pi_w$ the probability of it to realize.

3.3.3 Conditional Value at Risk (CVaR) formulation

Before the model itself is presented it will be introduced the risk measure called Conditional Value at Risk (CVaR) also known as mean excess loss, mean shortfall, or tail Value at Risk. Defined as in [20], with respect to a specified probability level α, the α-VaR of a portfolio is the lowest amount ξ such that, with probability α, the loss will not exceed ξ, whereas the α-CVaR is the conditional expectation of losses above that amount ξ. An intuitive idea of how it works can be seen in Figure 3.2. The gray shaded area corresponds to an area of α situated on the left of the VaR. Both distributions, black and blue, have the same VaR value. However, the conditional expectation above the VaR, namely the CVaR, is greater on the blue distribution that on the black one. If the probability distributions refer to costs as they do in this project, minimizing the CVaR is a way of reducing the risk of having very high costs; in other words, minimizing the CVaR is similar to minimizing the worst cases scenarios.

The value of α should be decided by the TSO depending on how averse to risk they are. If $\alpha = 0$ then minimizing the CVaR is equivalent to minimizing the total cost, which is exactly what has been done in Section 3.3.1 in the EPNS model. As α increases risk is reduced. In this project, increasing α will make the solution R^T increase too. Having more reserves allocated reduces the risk.

The proposed formulation uses scenarios to characterize the reserve that will be needed in the future. How to generate those scenarios is addressed in Section 4. The formulation corresponds to a two-stage stochastic programming, with R_i and R^T as first stage variables, representing the decision that has to be made at the current time, and z_w as a second-stage variable, representing the reserve needs at the time of deployment on scenario w. Each scenario is characterized by a realization of the stochastic variable Z *reserve needs*. The probability of each scenario is $\pi_w = f(z_w)$.

The objective function is the minimization of the CVaR, computed in a linear way similarly as in [20]. The final objective is to minimize the cost.

$$\min_{R_i, R_t, L_w} CVaR_\alpha = \xi - \frac{1}{1-\alpha}\sum_{w=1}^{W} \pi_w \eta_w \tag{3.20}$$

Constrained by:

$$\eta_w \geq -ECost_w + \xi \qquad \forall w \tag{3.21}$$

$$Cost_w = \sum_i \lambda_i R_i + V^{LOL} L_w \qquad \forall w \tag{3.22}$$

$$R^T_w = \sum R_i \qquad \forall w \tag{3.23}$$

$$L_w = \begin{cases} 0 & \text{if } z_w < R^T \\ z_w - R^T & \text{if } z_w \geq R^T \end{cases} \qquad \forall w \tag{3.24}$$

$$R_i \leq R_i^{max} \qquad \forall i \tag{3.25}$$

$$R_i \geq 0 \qquad \forall i \tag{3.26}$$

$$\eta_w \geq 0 \qquad \forall w \tag{3.27}$$

Where ξ is the VaR, η_w an auxiliary variable indicating the difference between the VaR and the cost of scenario w and L_w represents the amount of lacking reserve. The objective function 3.20 and the first constrain 3.21 are used to linearly define the $CVaR_\alpha$. Once the optimal solution is obtained, one could calculate the Expected Power Not Served: $EPNS = \sum_{w=1}^{W} \pi_w L_w$.

Equation 3.24 has to be defined in a linear way that GAMS can compile. The two implications can be formulated as a linear set of constrains. An extra auxiliary continuous variable S_w and a binary variable y_w must be defined so 3.24 is substituted by

$$L_w = (z_w - R^T) - S_w \qquad \forall w \qquad (3.28)$$
$$-y_w \bar{R} \leq L_w \leq y_w \bar{R} \qquad \forall w \qquad (3.29)$$
$$-(1 - y_w)\bar{R} \leq S_w \leq (1 - y_w)\bar{R} \qquad \forall w \qquad (3.30)$$
$$-(1 - y_w)\bar{R} \leq (z_w - R^T) \leq y_w \bar{R} \qquad \forall w \qquad (3.31)$$
$$y_w \in \{0, 1\} \qquad \forall w \qquad (3.32)$$

With $\bar{R}$ as an upper bound of $z_w - R^T$. In such a way, when $z_w < R^T$, $z_w - R^T < 0$, therefore $y_w = 0$ and $L_w = 0$. Similarly if $z_w > R^T$ then $y_w = 1$.

3.4 Scenario formulation. Study case.

When the optimization models are implemented in a real set up, the transmission system operation, namely Energinet.dk, would run the optimization model at 9:30 am for the next 24 hours right after the reserve market is closed. At that time of the day, the values of the prices λ_i are known. Those prices, together with the corresponding R_{max} composes the bid of each producer.

A simplification of the real procedure must be done when dealing with the study case. The historical values of λ_i and R_{max} were not available for this project and an alternative similar formulation must be defined according to the data that we have. A function of reserve costs $g(z)$ is estimated, representing the cost of allocation of upward reserve in Eur per z MW. The product $\sum \lambda_i R_i$ will be replaced by the piecewise approximation of $\hat{g}(z)$, having the variables R_i removed. The estimation of $g(z)$ as a piece-wise constant approximation of a third degree polynomial is discussed in Section 4.1. The mid-point of each interval is named R_q^{mid} and the corresponding fitted value of the cost at the mid-point of interval q is represented by $\hat{\lambda}_q^{mid}$.

In order to express a piecewise constant function in GAMS it is necessary to define a new continuous positive variable $0 \leq R_q^{int} \leq \bar{R}^{int}$ which indicates how much of the interval q is being accounted for. $\bar{R}^{int}$ is the length of the interval, being set to 30 with a total of 62 intervals going from 0 to 1890. The equivalence $R^T = \sum_{q=1}^{63} R_q^{int}$ holds and defines the total scheduled reserves.

For the study case, the modified LOLP models remains as

$$\min_{R^T} \hat{g}(R^T) = \min_{R^T} \hat{\lambda}_q^{mid} R_q^{int} \tag{3.33}$$

Constrained by:

$$
\begin{aligned}
LOLP &= \sum_{w=1}^{5000} y_w \pi_w && && (3.34)\\
LOLP &\leq \beta && && (3.35)\\
R^T &= \sum_{q=1}^{62} R_q^{int} && && (3.36)\\
R^T - z_w &> M(1 - y_w) && \forall w && (3.37)\\
-(R^T - z_w) &\leq M y_w && \forall w && (3.38)\\
R^T &\geq 0 && && (3.39)\\
0 \leq R_q^{int} &\leq 30 && \forall q && (3.40)\\
y_w &\in \{0, 1\} && \forall w && (3.41)
\end{aligned}
$$

The optimal solution of this problem can be computed analytically. Recall that $\pi_w = \frac{1}{W} = \frac{1}{5000}$, $\forall w$. At the optimal solution it will be satisfied that the $LOLP = \beta$. By constrain 3.34 we know that in the optimal solution there will be $W \times \beta$ scenarios for which $z_w > R^T$. Therefore, the optimal R^{T*} is equal to the $(1 - \beta)$-quantile of the set of scenarios $z_w : w = 1...W$

The CVaR model (equivalent to the EPNS model when $\alpha = 0$) is

$$\min_{R_i, R_t^{int}, L_w} CVaR_\alpha = \xi - \frac{1}{1 - \alpha} \sum_{w=1}^{5000} \pi_w \eta_w \tag{3.42}$$

Constrained by:

$$\eta_w \geq -Cost_w + \xi \qquad \forall w \tag{3.43}$$

$$Cost_w = \hat{\lambda}_q^{mid} R_q^{int} + V^{LOL} L_w \qquad \forall w \tag{3.44}$$

$$R^T = \sum_{q=1}^{62} R_q^{int} \tag{3.45}$$

$$L_w = (z_w - R^T) - S_w \qquad \forall w \tag{3.46}$$

$$-y_w \bar{R} \leq L_w \leq y_w \bar{R} \qquad \forall w \tag{3.47}$$

$$-(1 - y_w)\bar{R} \leq S_w \leq (1 - y_w)\bar{R} \qquad \forall w \tag{3.48}$$

$$-(1 - y_w)\bar{R} \leq (z_w - R^T) \leq y_w \bar{R} \qquad \forall w \tag{3.49}$$

$$y_w \in \{0, 1\} \qquad \forall w \tag{3.50}$$

$$0 \leq R_q^{int} \leq 30 \qquad \forall q \tag{3.51}$$

$$R^T \geq 0 \qquad \forall i \tag{3.52}$$

$$\eta_w \geq 0 \qquad \forall w \tag{3.53}$$

The implementation of the model in GAMS is included in Appendix B.1

The optimization models described in this chapter do not include any time dependencies, meaning that the models can be run independently from one hour to another. In reality, ramp up and down constrains and start-up cost are relevant facts to take into account, so future studies should consider implementing this fact. Furthermore, if time dependencies are allowed it would be possible to include block contracts into the model.

Chapter 4

Estimation of functions. Scenario generation

When applying the optimization models described above to the real life problem it is necessary to estimate two functions: the price of allocating reserves $\hat{g}(z)$ and the function of reserve need from which scenarios will be drawn, both discussed in this section.

4.1 Estimation of the function price of reserve

The estimation of the reserve cost function $\hat{g}(R)$ is elaborated in this section. The estimation of a function that represents the cost of providing reserve was introduced in Section 3.4. The bids that producers submit to the reserve market are available for the transmission system operator before the market closes, but unfortunately that data is not available for this project; in order to adjust the optimization models to the the available data and test the efficiency of such, the bids of producers λ_i and R_{max} are substituted by a cost function being $g(z)$ the cost of allocation of z upward reserve in Eur.

In practice the function of reserve costs has to fulfill two properties:

1. Monotonically increasing. Allocating up reserves implies first that the power plant cannot bid into the daily ahead market that capacity and therefore the availability of that capacity has to be paid for. Secondly, the cost of fuel, startup and equipment failure increases when up reserve increases and that has to be paid for too.

2. Non-negative. By the basic principles of a market, if allocating reserve would have a negative cost then producers would not bid into the market at all.

If $\lambda(z)$ is the price for allocating 1 MW of reserve, it seems reasonable to assume that $\lambda(z)$ will increase exponentially: the price per MW of increasing z from 500 MW to 501 MW has to be greater than increasing z from 0 MW to 1 MW. A second order polynomial with no intercept is assumed to be adequate; if so, the price of allocating a total of z MW of reserve will be fitted by a third order polynomial.

Given the total amount of purchases manual reserve at time t, PMR_t, and the market price of up reserve allocating of PMR_t MW at time t in Euro, λ_t^M, the estimated function of reserve costs is

$$\hat{\lambda}_t^M = 5.3158\ PMR_t - 0.0299\ PMR_t^2 + 0.000054 PMR_t^3 \tag{4.1}$$

The scatter plot of the data and the estimated function can be seen in Figure 4.1. The data appears to be quite heteroscedastic and hence other ways of estimating the function should be further studied in the future as weighted least squares methods. Also it could be interesting to study how the cost depends on the hour of the day, the week day or the month, or even on the wind power production and power load. It seems logical to think that in systems with high penetration of wind power, at night time a big share of the demand will be supplied by wind power producers and therefore other cheap sources might offer will offer their regulating power. Nevertheless for the sake of this project the third degree polynomial reflects the real relations well enough.

When the optimal solution is computed with a computer software like GAMS, there are two ways of specifying the cost function:

1. Complete specification of the polynomial. The functions becomes non linear, facing the drawback that the solving time increases.

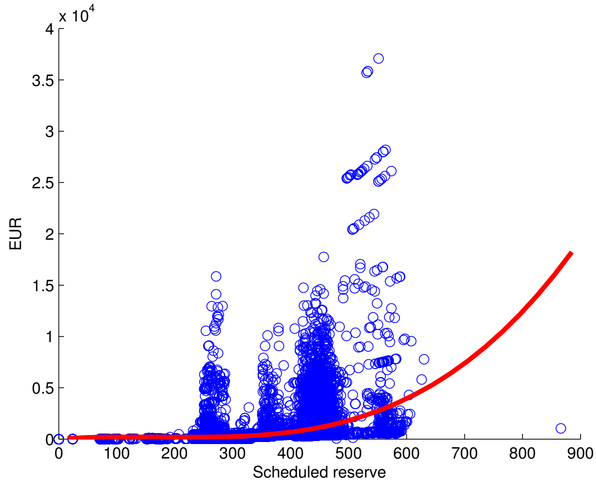

Figure 4.1: Plot of the upward manual reserves against the market price during the test period and a linear regression used to predict the price given the total amount of reserve to schedule.

2. Piece-wise constant approximation. The objective remains linear and therefore easy and fast to solve, with the drawback that it is only an approximation. Even so, it can be more suited to the reality because in practice when the bids are collected they form a "stairs" function. Another advantage is that more complex functions than three degree polynomials can be discretized using the same procedure.

The second option was chosen for being more simple and general too. There are a total of 62 intervals spanning from $R^I_1 = 0$ to $R^I_{63} = 1890$ with a length of $R^I_q - R^I_{q-1} = 30$ each. The reference point chosen is the mid-point of the interval, named R^{mid}_q and the corresponding fitted value of the cost at interval q is represented by $\hat{\lambda}^{mid}_q$. The function itself is defined as

$$\hat{g}_z = \hat{\lambda}^{mid}_q \text{ if } R^I_q < z \leq R^I_{q+1} \tag{4.2}$$

The way it is implemented in a linear programming shape was presented in Section 3.4.

4.2 Scenario generation

The adapted optimization models for the study case that are shown in Section 3.4 need as a input scenarios. Each scenario is characterized by a stochastic variable that represents the reserve needs. The reserve needs variable is composed by the sum of three variables: the forecast error of the wind power production + the forecast error of the power load + MW failed due to outages. Scenarios are generated of each individual variable and afterwards summed. The correlation between the first two forecast errors is taken into account and the MW failed is assumed to be independent from the other two variables. The remaining of the section explains how the scenarios for each of the three variables are computed.

4.2.1 Wind power production & power load

As explained in Section 2.4.1 the scenarios of wind power production and power load have been generated by ENFOR, having 5000 pairs of scenarios per hour for the whole test set period. The correlation between both variables is taken into account at the generation process and hence they have to be treated in pairs. Every given pair of scenarios is characterized by two values: A wind power production and a power load.

Instead of using the forecast values it is more interesting to compute the forecast error of both variables. The forecast error leads to a need of reserves: if the forecast error is greater more reserves will be needed to cover for the variation. If the forecast error is positive and big, then upward reserves are needed; if on the other hand is negative, downward reserve can be allocated. This later case is not taken into account since allocating down reserve is much easier than upwards.

4.2.2 Amount of MW failed

In order to generate scenarios characterized by the amount of MW that failed it is first necessary to build a model that characterizes the data and its dependencies. In this section, several statistical methods are explained and the reliability of their forecasts compared. Afterwards scenarios are drawn from the most suitable model, which in turns out to be a combination between a Bernoulli and Gamma Generalized Linear Model (GLM).

Define X_t as the amount of MW that have failed at time t due to outages and unforeseen events and Y_t as a binary variable which value is 1 if there is an outage at time t and 0 if there is no outage. The power load demand is denoted as n_t for every time t.

The first approach consists of a two-stage modeling. In the first stage, a model for the presence/absence of a failure y_t is done trough a Bernoulli General Linear Model (GLM). In the second stage it is modeled the amount of energy failed conditioned to knowing there is a failure $x_t|y_t = 1$, as a Poisson GLM and also as a Gamma GLM. The second approach consists of two Hidden Markov models: a binomial state distribution, with n_t as total number of trial and a Poisson state-dependent distribution with non homogeneous transition probabilities. All the modeling steps are explained in the subsections below.

4.2.2.1 Model y_t as a GLM Bernoulli

A simple first method to approach the modeling of this type of data is to consider only the presence or absence of a failure as a response variable, disregarding the amount of MW that occurred during the outage. The variable Y_t is defined as

$$Y_t = \begin{cases} 1 & \text{if failure occurs at time t} \\ 0 & \text{otherwise} \end{cases} \tag{4.3}$$

It is natural to assume that Y_t follows a binary distribution, $Y_t \sim bern(p_t)$ and model the response variable as a Generalized Linear Model. The link function chosen is the *logit* function. The explanatory variables are the hour of the day, the day of the week and the month, all represented through sinusoidal curves. Many sinusoidal terms were considered of the form $k^{(1)}cos(2\pi hour_t)$, $k^{(2)}cos(2\pi day_t)$ and $k^{(3)}cos(2\pi month_t)$ with $k^{(1)} = 1...24$. $k^{(2)} = 1...7$ and $k^{(3)} = 1...12$, also using the *sin* functions in a similar way. Only the most relevant were kept using an approximate χ^2-distribution test as in [21]. The final model is

$$\eta_t = log\left(\frac{p_t}{1-p_t}\right) = \mu + \alpha_1 cos\left(\frac{2\pi day_t}{7}\right) + \tag{4.4}$$

$$\alpha_2 sin\left(\frac{2\pi day_t}{7}\right) + \alpha_3 cos\left(2\frac{\pi month_t}{12}\right) + \tag{4.5}$$

$$\alpha_4 cos\left(5\frac{2\pi month_t}{12}\right) + \alpha_5 sin\left(\frac{2\pi month_t}{12}\right) + \tag{4.6}$$

$$\alpha_6 sin\left(2\frac{2\pi month_t}{12}\right) + \alpha_7 sin\left(3\frac{2\pi month_t}{12}\right) + \tag{4.7}$$

$$\alpha_8 sin\left(4\frac{2\pi month_t}{12}\right) + \alpha_9 sin\left(5\frac{2\pi month_t}{12}\right) \tag{4.8}$$

The final model shows that the hour of the day is not significant when predicting the probability of having an outage, being the day of the week and the month the only significant factors. The parameters of the model are relative to the train set data. In practice, the parameters can be updated every day at 9:30 am CET right before the reserve market is cleared including data from the previous 24 hours.

Since the aim of this model is to forecast future probabilities of failures, one way of illustrating how well it works is by constructing a reliability plot, shown in Figure 4.2. As explained in [22], it consists of a plot of the observed probabilities on the Y axis against the forecast probabilities on the X axis. Three lines are shown in the graph: the reliability of the training set in green; the reliability of the test set in red, with no parameter updates; and the reliability of the test set updating the model parameters every day at 9:00 am in blue. Ideally, all lines should be close to the diagonal. However, for probabilities greater than 0.1 the blue and red lines are quite far form it. All in all, this plot indicates that this model is relatively good when forecasting but still with a lot of room for improvement. As future work, other methods should be considered, for example including transition probabilities between the two "states" or by including information about previous observations as covariates.

Other ways to compare the quality of the forecast is by computing the ranked probability skill score, quantifying the extent to which a forecast strategy improved the predictions with respect to a reference forecast, as in [23]. This method was left for future work.

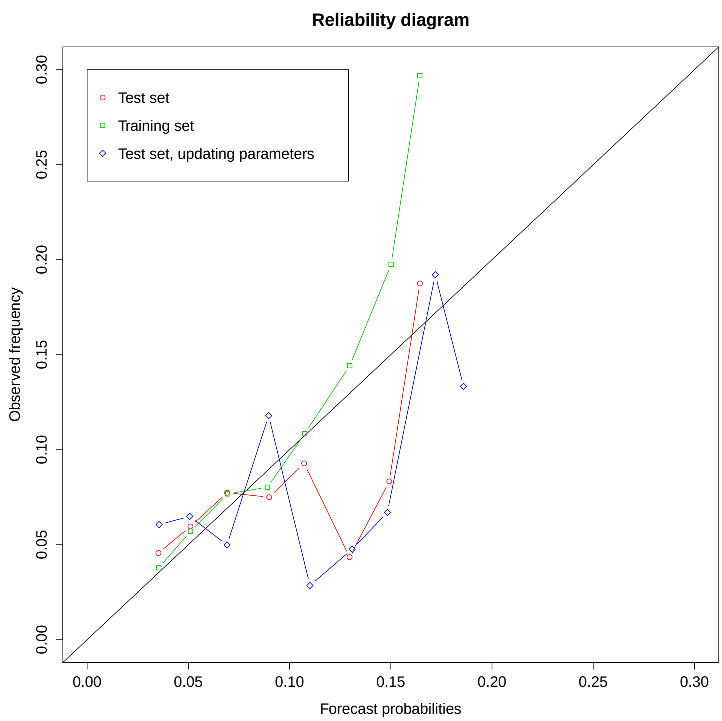

Figure 4.2: Reliability plot of the forecast of the presence of outages

4.2.2.2 Model $x_t|y_t = 1$ as a GLM Poisson

In this section it is presented a method to model $X_t|Y_t = 1$, namely the amount of MW that failed conditioned to knowing that there is an outage.

The time series $\{x_t|y_t = 1, t = 1...T\}$ is not properly modeled by an autoregressive model. The times between observations are either one hour because there as been an outage on the previous hour, or many hours, since most of the times there are no outages. Instead, a GLM model with sinusoidals of the time as regression variables was chosen.

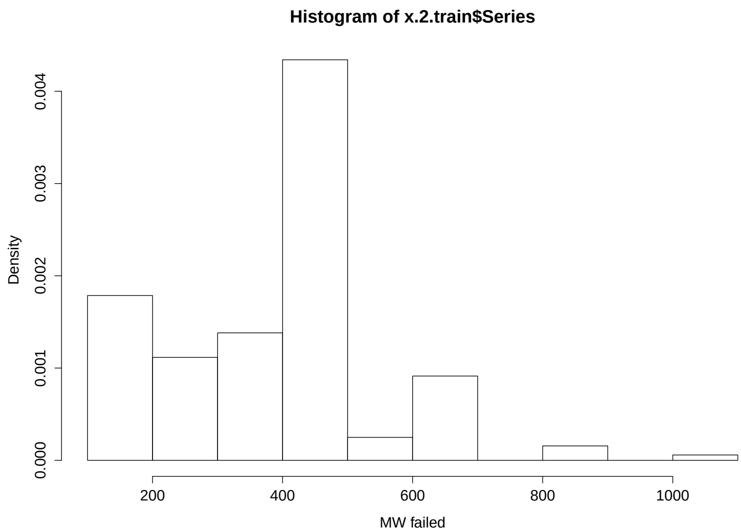

Figure 4.3: Histogram of $x_t|y_t = 1$

A histogram of the response variable $x_t|y_t = 1$ is shown in Figure 4.3. A positive distribution is needed. A GLM with Poisson response variable is chosen as a first approach. This means that it is assumed $X_t|Y_t = 1 \sim Pois(\lambda_t)$ with a logarithmic link. The explanatory variables are the hour of the day, the day of the week and the month, all represented through sinusoidal curves. Many sinusoidal terms were considered of the form $k^{(1)}cos(2\pi hour_t)$, $k^{(2)}cos(2\pi wday_t)$ and $k^{(3)}cos(2\pi month_t)$ with $k^{(1)} = 1...24$. $k^{(2)} = 1...7$ and $k^{(3)} = 1...12$, also using the *sin* functions in a similar way. Only the most relevant were kept using an approximate χ^2-distribution test as in [21]. The final model is

$$\eta_t = log(\lambda_t) = \mu + \alpha_1 cos\left(\frac{2\pi day_t}{7}\right) + \tag{4.9}$$

$$\alpha_2 sin\left(\frac{2\pi day_t}{7}\right) + \alpha_3 cos\left(3\frac{2\pi month_t}{12}\right) + \tag{4.10}$$

$$\alpha_4 cos\left(6\frac{2\pi month_t}{12}\right) + \alpha_5 sin\left(\frac{2\pi month_t}{12}\right) + \tag{4.11}$$

$$\alpha_6 sin\left(2\frac{2\pi month_t}{12}\right) + \alpha_7 sin\left(3\frac{2\pi month_t}{12}\right) + \tag{4.12}$$

$$\alpha_8 sin\left(4\frac{2\pi month_t}{12}\right) \tag{4.13}$$

As it turns out, the hour of the day is not significant when fitting the data.

In order to check up on how well the model performs at forecasting a reliability plot is built, see Figure 4.4. The green line corresponds to the reliability of the training set, the red line to the reliability of the model when predicting the test set, and the blue line to the reliability of the model that is updated every day at 9:00 am when predicting the test set. All three lines reveal that some of the model assumptions do not hold, since the "slope" of the line is clearly different than 1. In order to solve this issue, in the next section it is introduced a model with different assumptions and also more information through the ratio between X_t and the power load demand.

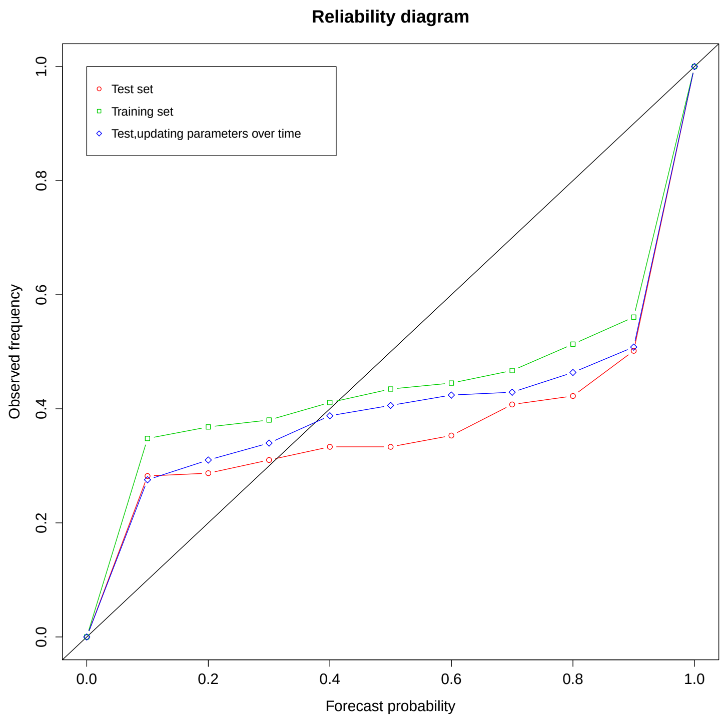

Figure 4.4: Reliability plot of the total amount of MW failed, X_t, when there is an outage

4.2.2.3 Model $\frac{x_t}{n_t}|y_t = 1$ as a GLM Gamma

As seen in previous section, modeling $x_t|y_t = 1$ as a Poisson GLM is not really adequate hence other distribution for the response variables will be studied in this section. There is one source of information that has not been used yet: the power load demand n_t. It seems reasonable to state that the more energy is demanded, the more power plants are activated and more generators are subject to fail. This section explores how n_t could affect our predictions of X_t. There are two ways of achieving this: by including n_t as an explanatory variable or alternatively to model directly $\frac{X_t}{n_t}|Y_t = 1$, both alternatives equally good at a first sight. The histogram of $\frac{X_t}{n_t}|Y_t = 1$ is depicted in Figure 4.5. It clearly resembles to the Gamma distribution density with the right parameters, therefore the second option was chosen.

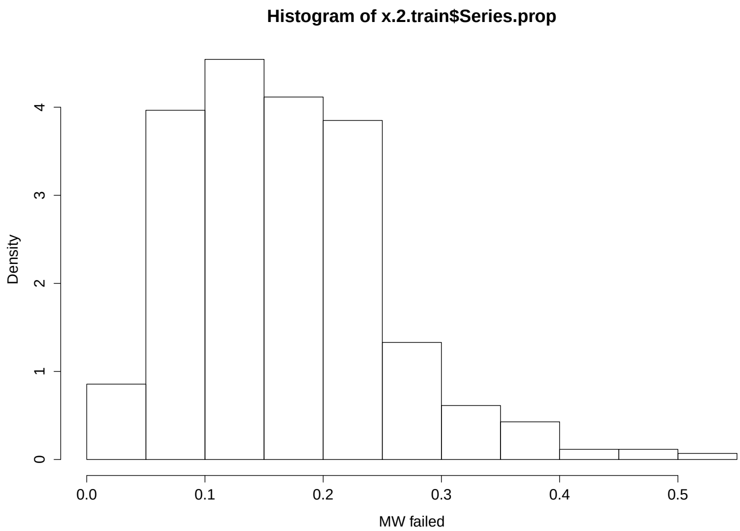

Figure 4.5: Histogram of $\frac{x_t}{n_t}|y_t = 1$

It is assumed that $\frac{X_t}{n_t}|Y_t \sim Gamma(s_t, k)$, where k is the shape parameter, common for all observations, and s_t the scale parameter at time t. The probability density function is defined as

$$f(x) = \frac{1}{\Gamma(k)s_t^k} x^{k-1} e^{-\frac{x}{s_t}} \tag{4.14}$$

With a mean of $\mu_t = ks_t$ and variance $\sigma^2 = ks_t^2$. The canonical link for the gamma distribution is the inverse link $\eta = 1/\mu$. As in previous sections 4.2.2 and 4.2.2.2 the explanatory variables are several sinusoidals. After keeping only the significant terms the mode in terms of the canonical link is

$$\eta_t = \frac{1}{\mu_t} = \mu + \alpha_1 cos\left(\frac{2\pi hour_t}{7}\right) + \tag{4.15}$$

$$\alpha_2 cos\left(2\frac{2\pi hour_t}{7}\right) + \alpha_3 sin\left(\frac{\pi hour_t}{12}\right) + \tag{4.16}$$

$$\alpha_4 sin\left(2\frac{2\pi hour_t}{12}\right) + \alpha_5 sin\left(\frac{3\pi hour_t}{12}\right) + \tag{4.17}$$

$$\alpha_6 cos\left(\frac{2\pi day_t}{12}\right) + \alpha_7 cos\left(2\frac{2\pi day_t}{12}\right) + \tag{4.18}$$

$$\alpha_8 sin\left(\frac{2\pi day_t}{12}\right) + \alpha_9 sin\left(3\frac{2\pi day_t}{12}\right) + \tag{4.19}$$

$$\alpha_{10} cos\left(\frac{2\pi month_t}{12}\right) + \alpha_{11} sin\left(2\frac{2\pi month_t}{12}\right) + \tag{4.20}$$

$$\alpha_{12} sin\left(3\frac{2\pi month_t}{12}\right) \tag{4.21}$$

It turns out that when predicting the ratio X_t/n_t the hour, the week day and the month are significant terms.

The reliability plot is shown in Figure 4.6. The reliability of the test set (blue line) when updating the parameters every day at 9:00 is slightly better than when they are not updated; The line is relatively close to the diagonal so the forecast can be considered satisfactory. In fact, this is the model chosen for generating scenarios; next subsections will explore the performance of other types of model that finally turn out being inadequate.

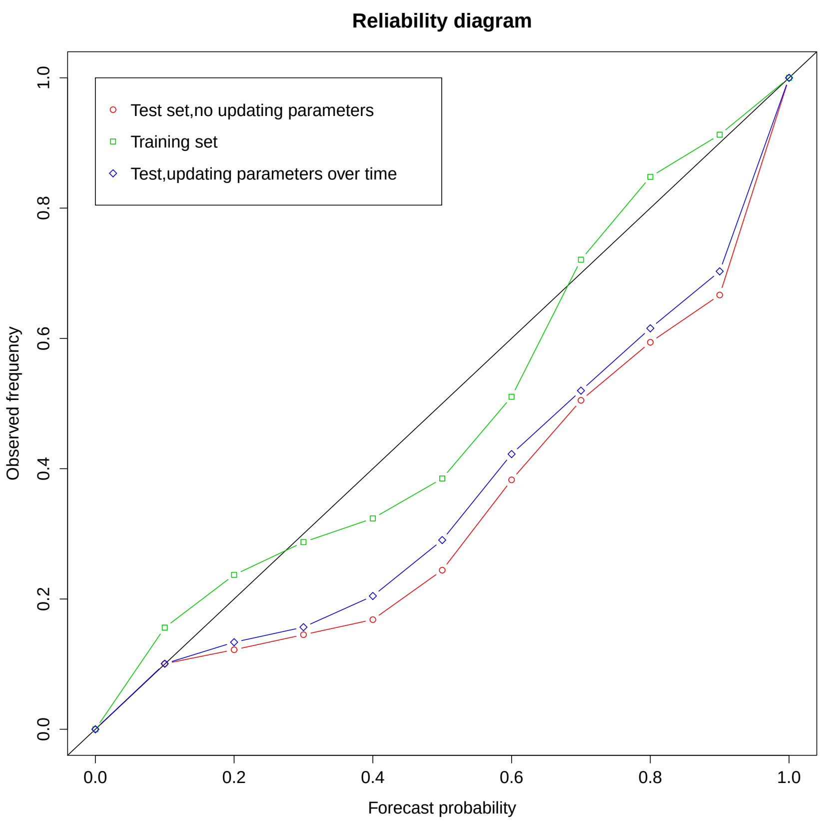

Figure 4.6: Reliability plot of the total amount of MW failed, $\frac{x_t}{n_t}$, when there is an outage

4.2.2.4 Model $\frac{x_t}{n_t}$ as a HMM with binomial state dependent distribution

In this section a brief introduction to a Hidden Markov Model (HMM) with binomial state-dependent distribution is explained and the results of applying the technique to the outages data.

Let X_t be a variable representing the amount of MW that failed from power plants at time t and n_t the power load demand at time t. Two assumptions have to be made: first, both X_t and n_t are considered integer non negative values. Second, it is assumed that $X_t \leq n_t$. In theory, if the exported quantity of MWh is positive, more MW can fail than than consumed in the producers' region. In practice, the study reveals that this assumption holds at least for the period considered in West Denmark. If n_t is not integer for any $t = 1, 2, ..., T$ then it is rounded to the closest integer.

The model consists of two parts. An unobserved "parameter process" called state of the Markov chain, represented as $\{C_t : t = 1, 2..T\}$ satisfying the Markov property, and secondly a "state-dependent process" $\{X_t : t = 1, 2 \ldots T\}$ such that the distribution of X_t is known when the state C_t is known [24]. This distribution is called state-dependent distribution and it is set to be binomial. For $i = 1 \ldots m$, being m the total number of states:

$$p_i(x) = P(X_t = x_t | C_t = i) = \left(\frac{n_t!}{x_t(n_t - x_t)} \right) \pi_i^{x_t} (1 - \pi_i)^{n_t - x_t} \tag{4.22}$$

Where π_i is the probability of having a success if the chain is at state i, and therefore the amount of MW failed X_t can be seen as the number of successes at time t. Note that it is assumed $0 \leq x_t \leq 1$ so the binomial distribution seems appropriate; however, different state-dependent distributions can be used like Gamma. This is left for future work.

The transition probability matrix indicates the probabilities of the chain moving from one state to the next one. Let $\gamma_{ij} = P(C_t = i | C_{t-1} = j)$ and

$$\mathbf{\Gamma} = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1m} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{m1} & \gamma_{m2} & \cdots & \gamma_{mm} \end{pmatrix}$$

In this case, since $\mathbf{\Gamma}$ does not depend on time it is called a homogeneous Markov chain. In Figure 4.7 it is shown a directed graph, where each arrow indicates which element depend on which, so X_2 depends on C_2 and C_2 depends only on C_1 and so on. Other types of structures as second order Markov chains could have also been implemented, where the transition probabilities depend not only on the last state but on two states back.

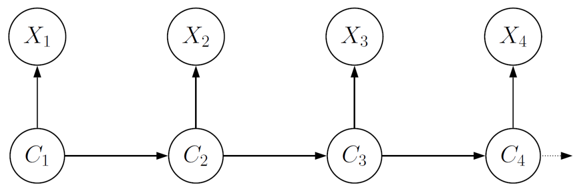

Figure 4.7: Directed graph of a basic HMM where arrows represent dependencies between variables.

The likelihood L_T of the model is given by

$$L_T = \boldsymbol{\delta}^{init}\mathbf{P}(x_1)\boldsymbol{\Gamma}\mathbf{P}(x_2)\boldsymbol{\Gamma}\mathbf{P}(x_2)\ldots\boldsymbol{\Gamma}\mathbf{P}(x_T)\mathbf{1}' \tag{4.23}$$

Where T is the total number of observations and $\boldsymbol{\delta}^{init}$ is the initial distribution of the Markov chain. The parameters of the model are estimated by maximizing L_T. Due to the great amount of data, direct optimization is too complex; instead, a Baunm-Welch algorithm also known as Expectation Maximization (EM) algorithm was implemented in R. It treats the states as missing data and exploits the fact that the complete-data log-likelihood (CDLL) may be straightforward to maximize even when the likelihood of the observed data is not. The sequence of states is represented by $c_1, c_2, ...c_T$ and the zero-one random variables defined as $u_j(t) = 1$ if and only if $c_t = j$,$(t = 1, 2, ..., T)$. Also define $v_{jk}(t) = 1$ if and only if $c_{t-1} = j$ and $c_t = k$ $(t = 2, 3, ..., T)$.

With this notation, the complete-data log-likelihood of an HMM is the probability of observing the data $x_1, x_2, ...x_T$ plus the missing values $c_1, c_2, ...c_T$:

$$log\left(P(\mathbf{x}^T, \mathbf{c}^T\right) = log\left(\delta_{c_1}^{init}\prod_{t=1}^{T}\gamma_{c_{t-1}c_t}\prod_{t=1}^{T}p_{c_t}(x)\right) = \tag{4.24}$$

$$log\ \delta_{c_1} + \sum_{t=1}^{T} log\ \gamma_{c_{t-1}c_t} + \sum_{t=2}^{T} log\ p_{c_t}(x) \tag{4.25}$$

By using the previously defined variables, the CDLL remains

$$log\left(P\left(\mathbf{x}^T, \mathbf{c}^T\right)\right) \tag{4.26}$$

$$= \sum_{j=1}^{m} u_j(t) log\ \delta_j^{init} + \sum_{j=1}^{m}\sum_{k=1}^{m}\left(\sum_{t=2}^{T} v_{jk}(t)\right) log\ \gamma_{jk}$$

$$+ \sum_{j=1}^{m}\sum_{t=2}^{T} u_j(t)\ log\ p_j(x_t) \tag{4.27}$$

$$= term1 + term2 + term3 \tag{4.28}$$

The algorithm follows two steps:

1. **E step**. Compute the conditional expectations of the missing data given the observations and the current estimate of the parameters, namely replace $u_j(t)$ and $v_{jk}(t)$ by their conditional expectations given the observations.

2. **M step**. Maximize, with respect to $\boldsymbol{\delta}^{init}$, $\boldsymbol{\Gamma}$ and $\boldsymbol{\pi}$, the complete-data log-likelihood with the functions of the missing data replaced in it by their conditional expectations. The CDLL splits into three terms, each of them depending only on one parameters set, which means they can be maximized independently from each other. The maximal value of term 1 and 2 are independent from the state-dependent distribution and can be found in [24]. Term 3 depends on the nature of the state-dependent distribution. In this case, the maximal solution is given by $\hat{\pi}_i = \frac{\sum_{t=1}^{T} \hat{u}_i(t) x_t}{\sum_{t=1}^{T} \hat{u}_i(t) n_t}$. See Appendix A for a justification.

The two steps are repeated until some convergence criterion is satisfied. The algorithm was implemented in R taking as a base the code in [24].

The next question that arises is: How many states should the chain have? One way of selecting the number of states is by computing the Bayesian Information Criteria (BIC) and the Akaike Information Criteria (AIC) of the model of the training set. One can see in Figure 4.8 that the more states the better both criteria become. However, for five or more states the improvement tends to slow down. Due to computational issues it was decided that five states is the most adequate.

In Table 4.1 one can see the estimated of the parameters after the likelihood is maximized. As expected, state number 1 accounts for all the observations when $x_t = 0$ and therefore $p_1 = 0$. The transition probability matrix $\boldsymbol{\Gamma}$ and the

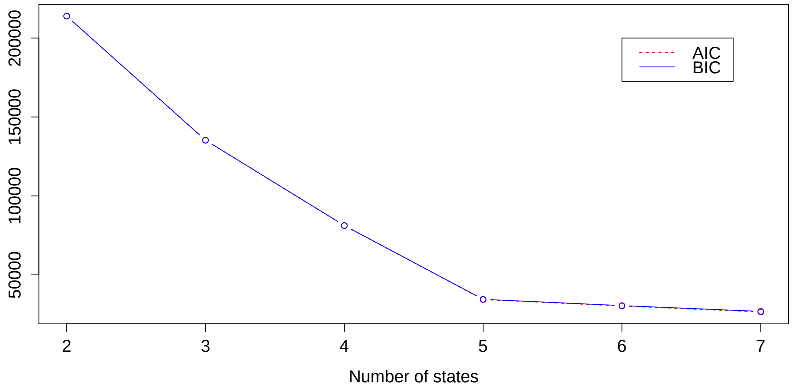

Figure 4.8: AIC and BIC of the binomial state-distribution HMM. Note that both lines seem overlapped.

stationary distribution $\boldsymbol{\delta}$reveal that state 1 is also more likely than the others. To give an example, when the chain is in state 1 at time $t-1$, there is a 0.99145 probability that at time t the chain is still at state 1.

p_i	δ_i	$\boldsymbol{\Gamma}$				
0.0000	0.9208	0.99145	0.00195	0.00373	0.00250	0.00034
0.0659	0.0189	0.10729	0.87404	0.01385	0.00479	0
0.1408	0.0273	0.11105	0.01657	0.80640	0.06112	0.00484
0.2197	0.0269	0.09179	0.00508	0.053757	0.82462	0.02474
0.3595	0.0058	0.05470	0	0.0233	0.11330	0.80867

Table 4.1: Estimation of the parameters $\mathbf{p}$ and $\boldsymbol{\Gamma}$ and the resulting stationary distribution $\boldsymbol{\delta}$ of a 5 state HMM with binomial as state-dependent distribution

In order to check the performance of the method the reliability of the model on the test set is computed. This is done in two steps: first, only considering how good the predicted probability of presence/absence of a failure is, and second by considering the amount of MW that failed given that there is a failure. In both cases it is considered the case where the model parameters are updated everyday at 9:00 am.

Figure 4.9 shows that the reliability of the model when the parameters are updated (green line) is relatively satisfactory. When the parameters are not updated, the forecast probability tends to the stationary distribution, so the probability of having a failure tends to $1 - 0.9208 = 0.0792$. Updating the parameters seems more adequate.

How well the model predicts the amount of MW that failed is shown in Figure 4.10. In this case, updating the parameters daily dot not make a big difference. Both lines tend to underestimate the real frequency as if the model is not able to capture some of the pattern of the data. By introducing another state-dependent distribution and time dependencies in the transition probabilities in the next section this issue has been tried to be solved.

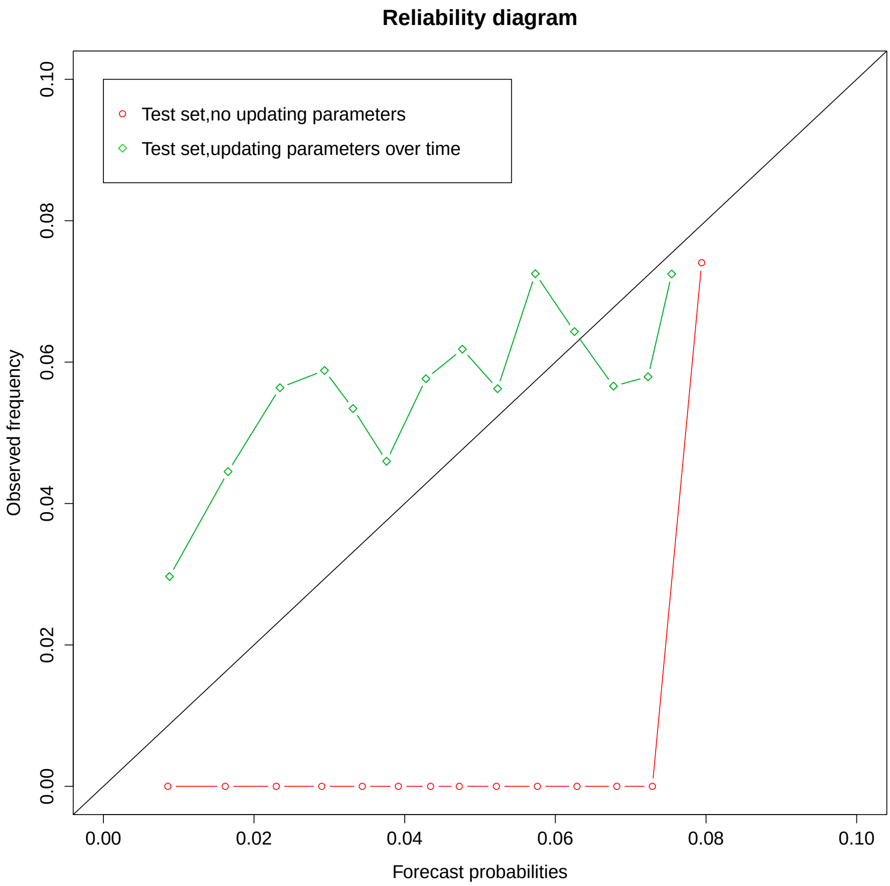

Figure 4.9: Reliability plot of the 5 state HMM with binomial state-distribution, considering only the presence/absence of a failure, y_t. Red line corresponds to the model where parameters are estimated one and green line to the model where parameters are updated daily

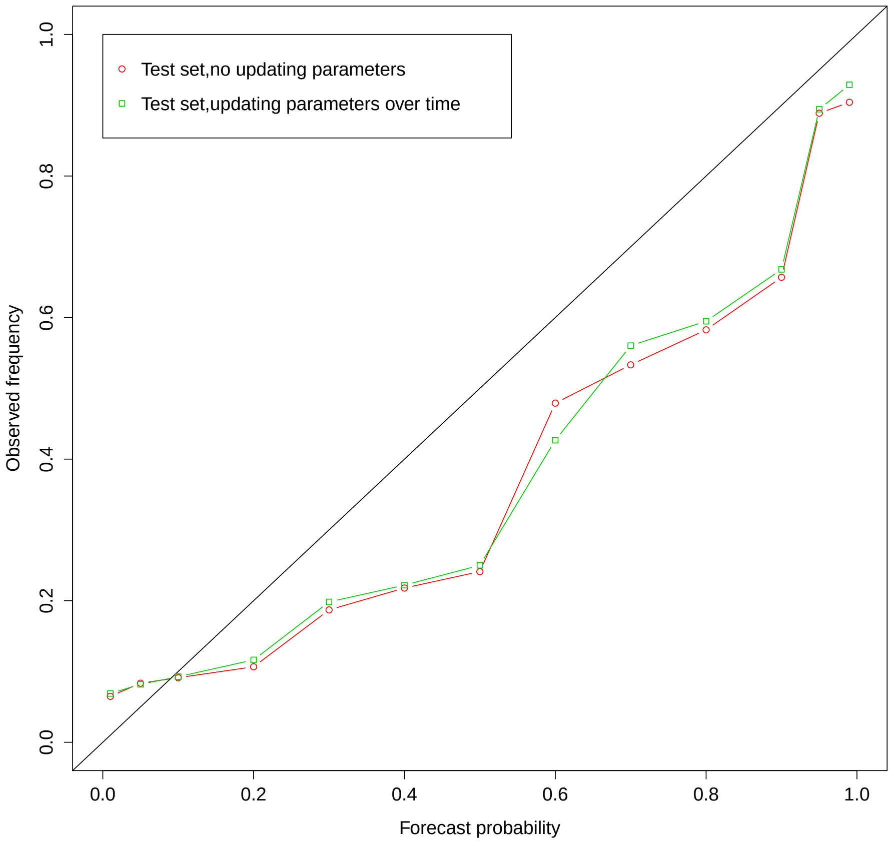

Figure 4.10: Reliability plot of the 5 state HMM with binomial state-distribution, considering only the amount of MW that failed conditioned to the fact that a failure occurred, namely $x_t|y_t = 1$. Red line corresponds to the model where parameters are estimated one and green line to the model where parameters are updated daily

4.2.2.5 Model x_t as a non-homogeneous HMM Poisson state dependent distribution, covariates on the transition probabilities

In this section is is presented a Hidden Markov Model for the total amount of MW that failed, having state-dependent distributions as Poisson and non-homogeneous transition probabilities, namely that the transition probabilities depend on time and also on the power load demand (n_t).

There are two possible ways of including covariates in a HMM: in the state-dependent probabilities or in the transition probabilities. Covariates (for example, time) should be incorporated in the state-dependent probabilities when time trend and seasonality could be present. However, by looking at the raw data plot 2.8 trend and seasonality do not seem very clear. It is logical to think that if the power plants installed at DK1 do not vary along the time period, the mean of the outages should not vary either. On the other hand, if covariates are on the transition probabilities, the probability of failure can vary with time as well as with other terms like n_t which in this case seems more reasonable.

The state-dependent distribution is set to be Poisson distribution; X_t is non-negative and integer so it is appropriate. The density is given by

$$p_i(x) = P(X_t = x_{(t)}|C_t = i) = e^{\lambda_i}\frac{\lambda_i}{x_t!} \tag{4.29}$$

With λ_i as the Poisson parameter of state i.

The main difficulty of this model lays on the transition probability matrix which will depend on the hour of the day and on n_t. Consider a HMM with m states. For $i \neq j$, the transition probabilities ${}_t\gamma_{ij}$ are modeled as

$$logit\left({}_t\gamma_{ij}\right) = \beta_{ij}(1) + \beta_{ij}(2)cos\left(\frac{2\pi t}{24}\right) + \beta_{ij}(3)sin\left(\frac{2\pi t}{24}\right) + \beta_{ij}(4)n_t \tag{4.30}$$

The case when $i = j$ can be deducted from the fact that $\sum_j^m \gamma_{i,j} = 1$ as in [25]. Note that $logit(p) = \frac{p}{1-p}$. After inverting the $logit$ function, the transition probability matrix remains

$$
{}_t\mathbf{\Gamma} = \begin{pmatrix}
\frac{1}{1+\sum_{j=1}^{m} exp(\beta_{11}y_t')} & \frac{exp(\beta_{12}y_t')}{1+\sum_{j=1}^{m} exp(\beta_{1j}y_t')} & \cdots & \frac{exp(\beta_{1m}y_t')}{1+\sum_{j=1}^{m} exp(\beta_{1j}y_t')} \\
\frac{exp(\beta_{21}y_t')}{1+\sum_{j=1}^{m} exp(\beta_{2j}y_t')} & \frac{1}{1+\sum_{j=1}^{m} exp(\beta_{2j}y_t')} & \cdots & \frac{exp(\beta_{2m}y_t')}{1+\sum_{j=1}^{m} exp(\beta_{2j}y_t')} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{exp(\beta_{m1}y_t')}{1+\sum_{j=1}^{m} exp(\beta_{mj}y_t')} & \frac{exp(\beta_{m2}y_t')}{1+\sum_{j=1}^{m} exp(\beta_{mj}y_t')} & \cdots & \frac{1}{1+\sum_{j=1}^{m} exp(\beta_{mj}y_t')}
\end{pmatrix} \tag{4.31}
$$

The total number of parameters is m due to $p_i(x)$ plus $4 \times m \times (m-1)$ due to $\mathbf{\Gamma}$. The likelihood of the model is:

$$
L_t = \boldsymbol{\delta}\mathbf{P}(x_1)\ {}_2\mathbf{\Gamma}\mathbf{P}(x_2)\ {}_3\mathbf{\Gamma}\mathbf{P}(x_2)\ldots\ {}_T\mathbf{\Gamma}\mathbf{P}(x_T)\mathbf{1}' \tag{4.32}
$$

The algorithm used to estimate the parameters is again the Expectation Maximization algorithm, see previous section 4.2.2.4 for more details about it. The implementation of the EM algorithm together with other useful functions in `R` code can be found in Appendix B.2. The biggest difficulty of the implementation comes when optimizing the second term of the complete-data log-likelihood. No direct optimization formula was found and therefore it has to be maximized numerically by the `R` function `optim`. Term 3 of the CDLL is maximized for

$$
\hat{\lambda}_i = \frac{\sum_{t=1}^{T} \hat{u}_j(t)x_t}{\sum_{t=1}^{T} \hat{u}_j(t)}
$$

where $\hat{u}_j(t)$ is the conditional expectation calculated in the expectation step.

Table 4.2 shows the AIC and BIC criteria of the model with two, three and four states. The more states are included, the better both criteria are so far. Ideally more states should be included until the BIC reaches a flat area or the AIC starts to increase. However, due to the rapidly growth of parameters it was not possible to achieve more complex model than with four states. Note that the 4-state model took around 52 hours to be fitted at the IMM-DTU servers; a model with more than 4 states is too complex given the big amount of data. For computational reasons, the parameters of this model are not updated and they are only computed once.

The estimates of the λ parameters are $\hat{\lambda} = (0, 166.72, 416.97, 714.68)$

The performance of the model can be seen in the reliability diagram on Figure 4.10. Red line corresponds to the model predicting only the presence/absence

States	AIC	BIC
2	79608.51	79682.77
3	55967.79	56168.29
4	29639.28	30053.27

Table 4.2: Table showing the AIC and BIC values for 2,3 and 4 states.

of a failure. The reason why the line does not go further than 0.4 is that the probability of having a failure ($P(y_t = 1)$) is low. The performance is satisfactory for small probabilities but not so good when greater than 0.2. Green line represents the performance of the model when predicting $X_t|Y_t = 1$. It it fairly good since it is close to the diagonal; however, it is similar to 4.6, being the GLM-gamma model much simpler. The conclusion is that this model is not more useful than simpler models and hence simple models are preferred.

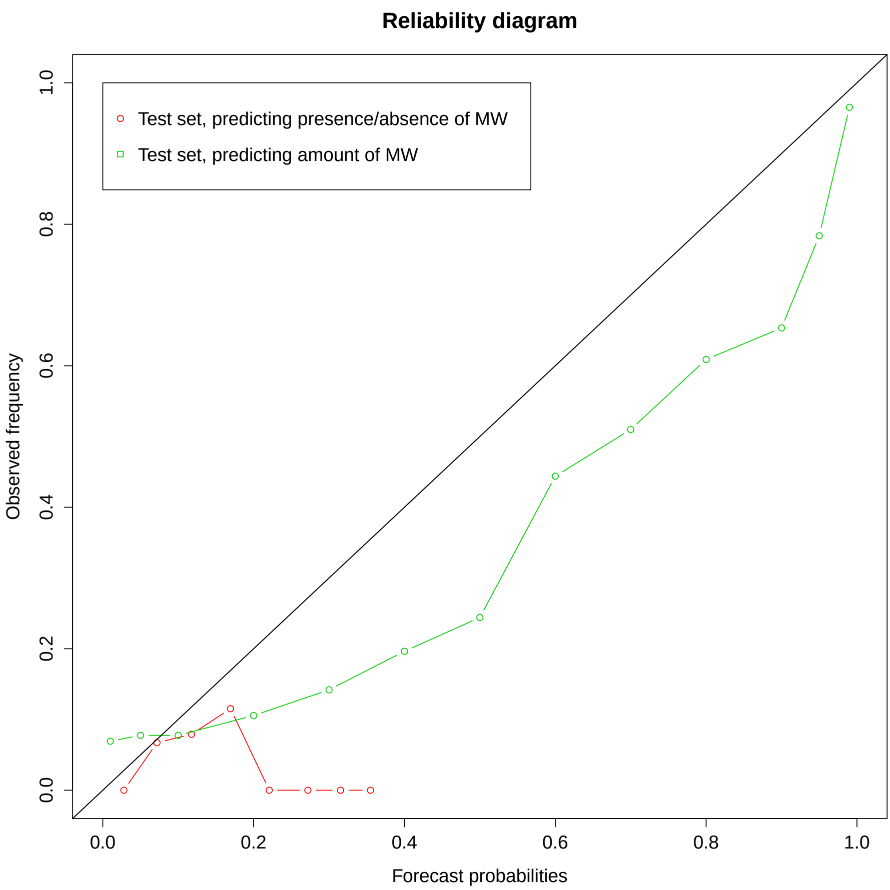

Figure 4.11: Reliability plot of the 4 state HMM with Poisson state-distribution and non-homogeneous transition probabilities. Red line is calculated only with $\hat{Y}_t$ and green line $\hat{X}_t|Y_t = 1$. No update of parameters is done.

4.2.3 Remarks on the chosen model

When looking at the reliability plots presented in the previous section it is clear that none of the model is perfect, none of then have a clearly outstanding performance. Nevertheless one of the models should be chosen.

The two stage approach was chosen for the scenario generation, with a Bernoulli GLM of the presence/absence of outage followed by a Gamma GLM of the amount of MW failed when an outage happens. The reliability of the Gamma GLM 4.4 is very similar to 4.6 and to 4.11 with the advantage that it is much simpler. The reliability of the forecasts of the binary HMM when predicting the presence/absence in 4.9 is slightly better than the Bernoulli GLM in 4.2 but the simpler model is preferred since the difference is not too big.

Some final remarks:

- The probability of having a failure in the system does depend on time, but not on the previous observation. By common sense one could say that it a unit failed on the previous hour, it is likely that next hour will be disconnected too. An attempt to account for this information was done by developing the HMM models, turning out not to be adequate.
- Since the outage probabilities do not depend on previous observations, the reality is not correctly modeled having here some source of future work. However, none of the optimization models include time dependencies. One could run the optimization of each hour and the result would not affect any other hour at all. So the fact that one scenario at time t depends on the value of the scenario at time $t-1$ will not affect the final result, or at least, not too much. Instead, what is more relevant is the marginal distribution for every hour.
- Updating the model parameters every day gives good results. The "ideal" model to be developed in the future should be fast to update.

Chapter 5

Results from study case

The optimization models described in the preceding chapter are tested on actual data. The performance of the LOLP model and CVaR model is examined and the validity of the results presented below.

5.1 Computational & data issues

The test set period goes from the 1st July 2011 at 00:00 CET to the 30th June 2012 at 23:00, summing up to a total of 8772 hours. The whole test period was used when comparing methods to generate scenarios and also when they were generated. If time and computational power was greater, the whole test year would be considered for the optimization model as well; however, due to limited resources and time, only four weeks randomly chosen along the period are used, one week per season and per position in the month:

1. Second week of September 2011
2. Third week of January 2012
3. Fourth week of April 2012

4. First week June 2012

With a total of 672 hours

The interpretation of the solution given by the optimization models should be taken cautiously. It represents the total reserve needs incurred by the deviation of the wind power production and the power load demand from its forecasts plus the effect of outages in power plants. Recall that an assumption states that those three factors are the only affecting the need of reserves, which in reality might be more like interaction with neighbors and failures of transmission lines. Secondly it should be noted that in the models no difference is made between primary, secondary or manual reserve. The solution representing the total reserve needs make no difference between the kinds of reserve; it is assumed that the TSO would take care of distributing the total needs amongst the three types of reserves.

Ideally the solution should be compared with the actual need of reserves. This information is not available and instead the performance of the optimization model is compared to the scenarios of reserve need, which are assumed to be all the possible realization of the reserve needs anyways. If the scenarios characterized by the reserve needs represent the actual process well, the comparison can be extrapolated to reality.

5.2 Energinet.dk policy

In this section, the actual strategy followed by Energinet.dk is compared to the scenarios generated. The total reserve scheduled by Energinet.dk is calculated by summing the primary, secondary and manual reserve at every hour, excluding the short-circuit power, reactive reserves, voltage control reserves and reserves provided by the neighbor areas.

The upper plot in Figure 5.1 shows in dark blue the sum of the primary, secondary and tertiary reserves at the four selected weeks. On the lower plot it is shown the proportion of the scenarios that have greater total reserve needs than the actual reserve scheduled by Energinet.dk: this ratio is equivalent to the LOLP. This plot tells us that some of the assumptions done do not necessary hold. In reality, during those four weeks no deficit of power has been recorded. However, the LOLP is quite high at certain times, suggesting that some load shedding could very possibly be incurred. This contradictions reveals that the generated scenarios of reserve needs do not necessarily adjust to the reality and further efforts should be done at studying how feasible the assumptions are.

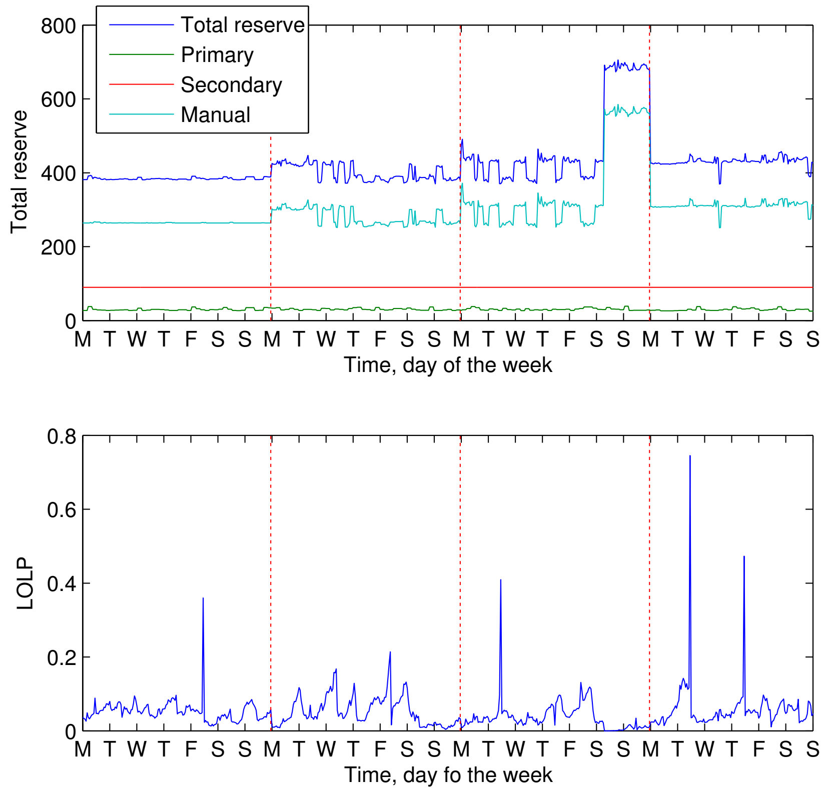

Figure 5.1: The upper plot shows the actual primary reserve, the secondary reserve, the manual reserve and the sum of the three at the four selected weeks. The bottom plot shows the percentage of the scenarios that are greater than the total reserves scheduled.

It should be noted in Figure 5.1 the behavior of the LOLP at the four main spikes, suggesting that the LOLP is much more high at those hours that at the rest and consequently the reserve needs too. One possible cause of this spikes is that much stronger wind than expected is realized being above the cut-out speed of the rotor thus causing production of the generators being reduced to zero. As a result, the reserve needs incurred by this event is very high. Surprisingly, the four spikes happen at 12:00 CET, which suggests that another unlikely but possible reason could be a systematic error at the generation stage or simply a human mistake. The beauty of the optimization models presented is that is those cases of high reserve needs are predicted, then more reserves will be bought and the system will keep running safe.

In Figure 5.2 it can be seen a similar plot showing the relationship between the actual reserves scheduled by Energinet.dk and the resulting LOLP. As expected, when R^{TOT} increases the LOLP tends to decrease.

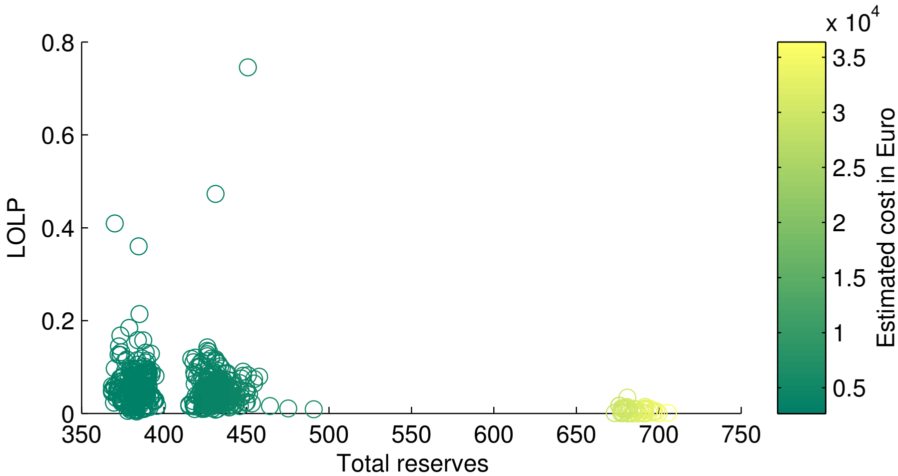

Figure 5.2: The LOLP is computed taking as total reserves the actual scheduled by Energinet.dk. When R^{TOT} increases the LOLP tend to decrease.

5.3 Results from the LOLP model

The implementation of this method was discussed at Section 3.4. The parameter that can vary in this model is β which is an upper bound to the LOLP. It turns out that in the optimal solution the relation $\beta = LOLP$ is satisfied. The value of β should be in practice given by the TSO; however, it is of interest to perform a sensitivity analysis and study how the modification of the parameter would affect the solution.

The solution along the four testing weeks can be seen in Figure 5.3. The level of R^{tot} varies depending on the time when the model is run and the predicted reserve needs at that time. As expected, when β decreases , the probability of not having enough reserves decreases and thus the total reserve is forced to increase.

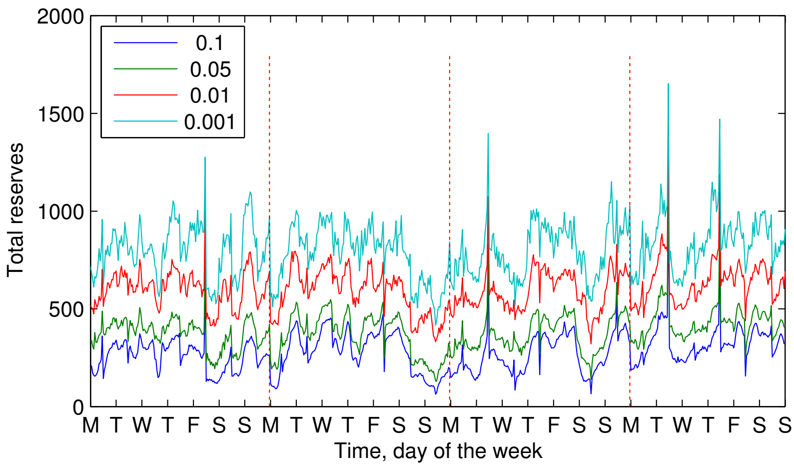

Figure 5.3: The solution of the LOLP-model along the four testing weeks. As β decreases, R^{tot} increases.

It is interesting to study how the total reserve varies along the day and if there is a specific time when more reserves are needed. This can be seen in Figure 5.4, where the median of R^{tot} is displayed along the 24 hours of the day. According to that plot, in general more reserves should be scheduled in the morning and specifically at 12:00, being 13:00 and several hours afterwards when least reserves are needed. At times when the load demand is higher like in the morning, more reserves are scheduled.

Finally it is analyzed how much improvement is achieved by using the LOLP

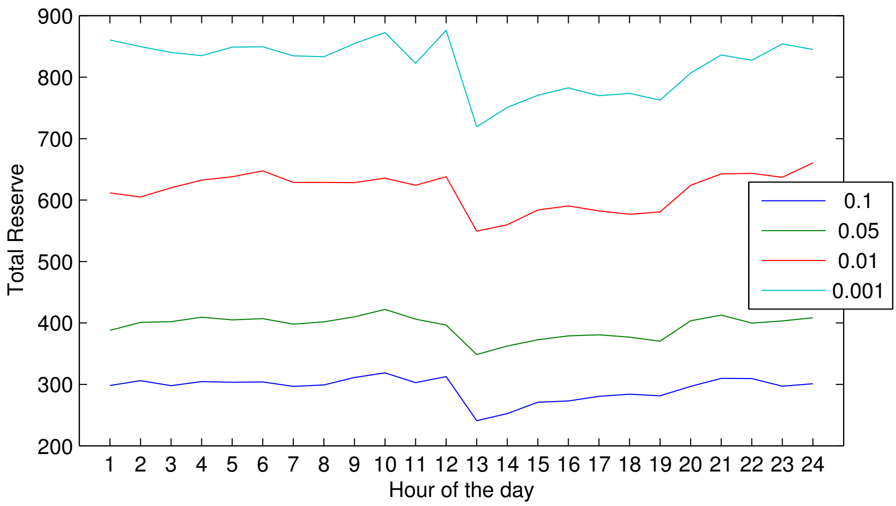

Figure 5.4: Median reserve level along the hours of the day, computer with the LOLP-model.

model compared to a very basic method which allocates constant reserves at all times. The blue line in Figure 5.5 corresponds to the solution of the LOLP model while red line corresponds to the basic constant model. The incurred LOLP by both solutions is displayed on the x-axis while the average solution of total reserves on the y-axis. To give an illustrative example, lets say the system operator wants to keep the probability of loosing load at 0.05. If the basic method is used then the total reserves to schedule should be constantly set to 417 MW; furthermore, if the LOLP method is used, on average 386 MW will be needed. The LOLP-model gives schedules less reserves for the same safety level.

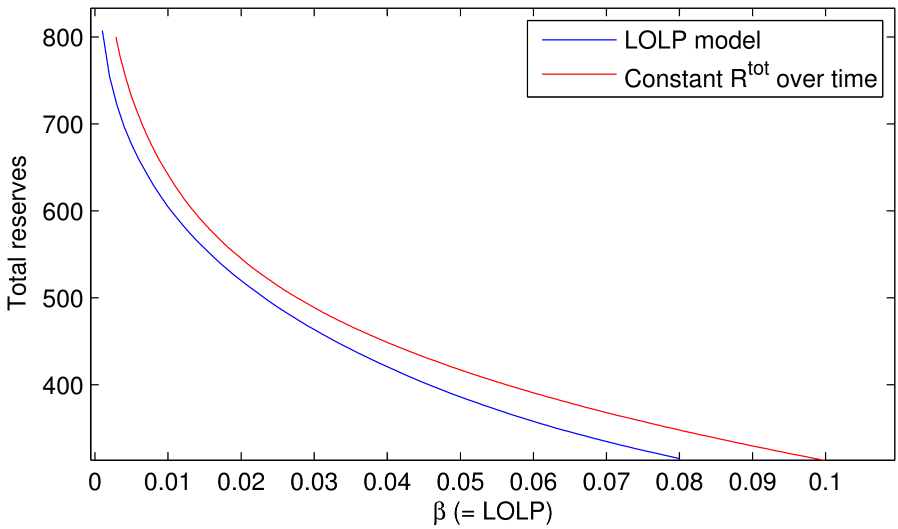

Figure 5.5: Efficient frontier plot, blue line corresponds to the solution of the LOLP model while red line corresponds to the basic constant model.

5.4 Results from the CVaR model

The implementation of this method was described at Section 3.4. Recall that there are two parameters to determine: α, which controls the CVaR risk measure, and the Value of Loss Load V^{LOL} which accounts for the cost to society when 1 MW of demand is shed. A sensitivity analysis is performed to find out how these two parameters affect the solution.

Figure 5.6 shows a efficiency frontier plot summarizing the behavior of the model. On the x axis it is displayed the average LOLP ie. the probability that the reserve needed is higher than the solution R^{tot^*}; on the y axis, the total expected cost, which accounts for providing reserve costs plus the expected power not served cost. The different colored lines correspond to solutions with same V^{LOL}, and finally the numbers drawn along the line correspond to the α parameter. As the LOLP decreases, the total cost increases because more reserves are scheduled. Also, increasing value of loss load implies increasing reserve: shedding 1 MW of load demand is more costly.

This is the kind of plot that the TSO would be using if this method is implemented in a system. According to the information they gather from the producers and consumers about the V^{LOL} and their policies regarding the LOLP and the total cost, α is chosen and the optimization model run for the following 24 hours.

The value of the solution $R_t^{tot^*}$ at time t along the four testing weeks is displayed in 5.7 on Page 68. On the upper plot the optimization was run with $V^{LOL} = 10000$ while on the bottom plot with $V^{LOL} = 75000$. For the same α and time t, the solution $R_t^{tot^*}$ is always greater with greater V^{LOL}. Similarly, greater α means the TSO is more risk averse and therefore $R_t^{tot^*}$ increases. If V^{LOL} would be ∞, then the solution $R_t^{tot^*}$ would be equal to the scenario in which the reserve needs is maximum, for any α. Likewise if $\alpha = 1$ for any V^{LOL}. Note the small "stairs" on the solution lines are caused by the discretization of the cost function.

In order to analyze the different reserve needs along the day, the median of the LOLP and the median of the Expected Power Not Served (EPNS) is shown in 5.8. The probability of loosing load changes along the day: in the afternoon, from 13:00 to 19:00 it is less probable not to cover all the reserve needs than at the rest of the hours. Similarly, at those hours the EPNS is lower than the rest. This fact could be caused by a lesser wind production and load demand and thus lower forecast errors.

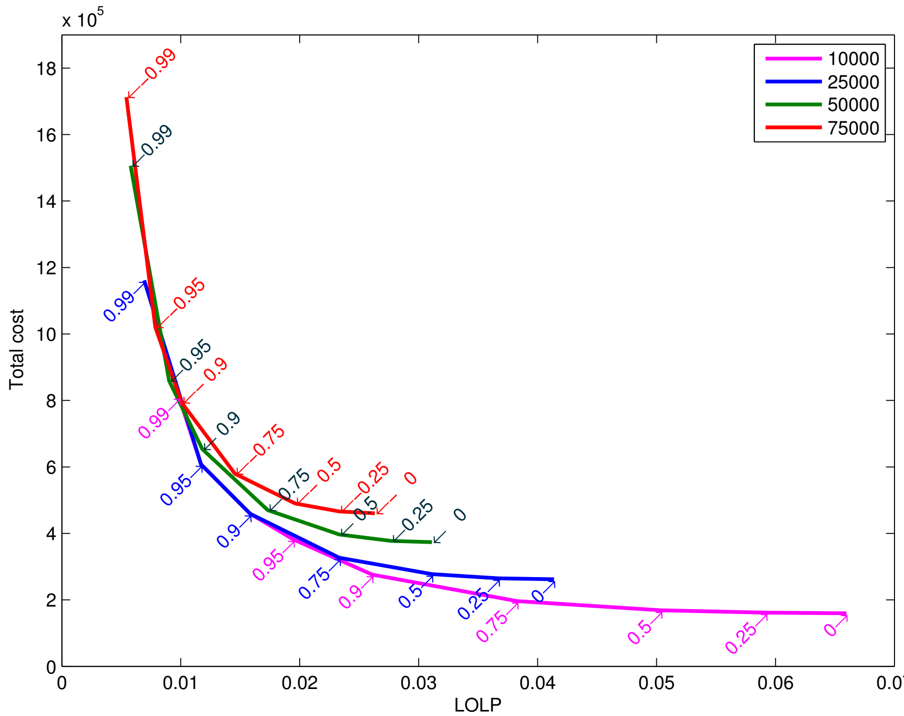

Figure 5.6: Each colored line corresponds to a different V^{LOl}, being the numbers along the line the parameter α. The values of the parameters will be chosen by the TSO depending on its preferences on the total cost and the LOLP.

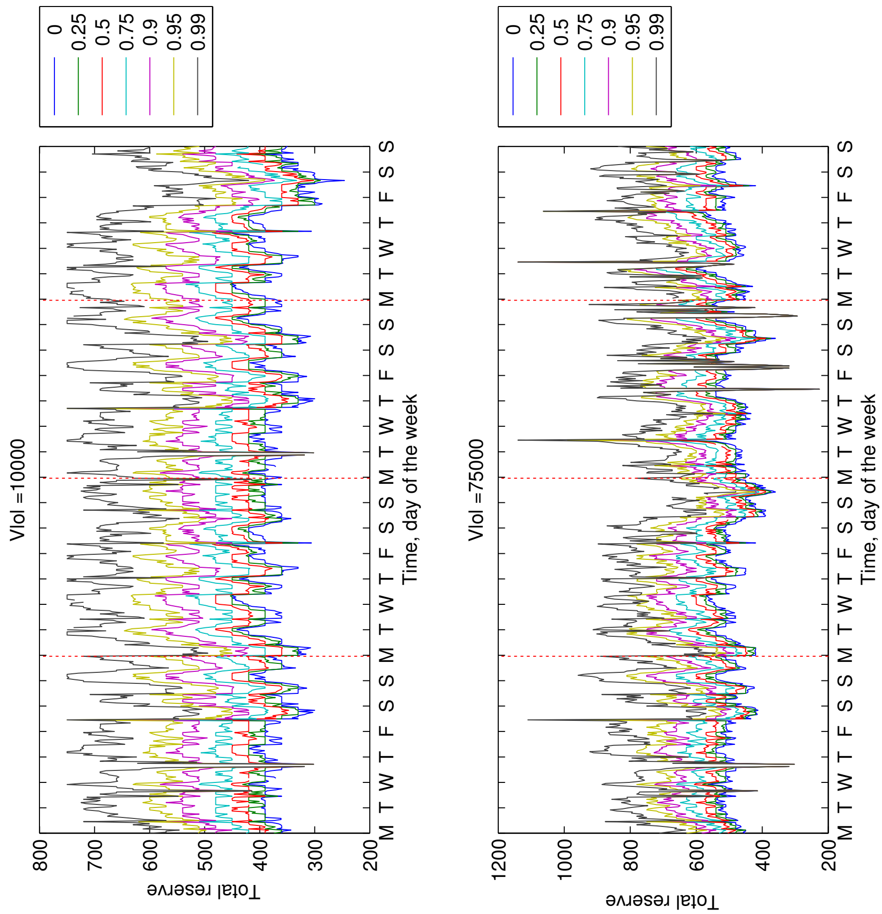

Figure 5.7: Optimal total reserve along the four testing weeks, computer with the CVaR model. On the upper plot the optimization was run with a V^{lol} of 10000, while on the bottom plot with V^{lol} equal to 75000. The colored lines correspond to different values of the parameter α. As V^{lol} and α increases, the solution R^{tot^*} increases too.

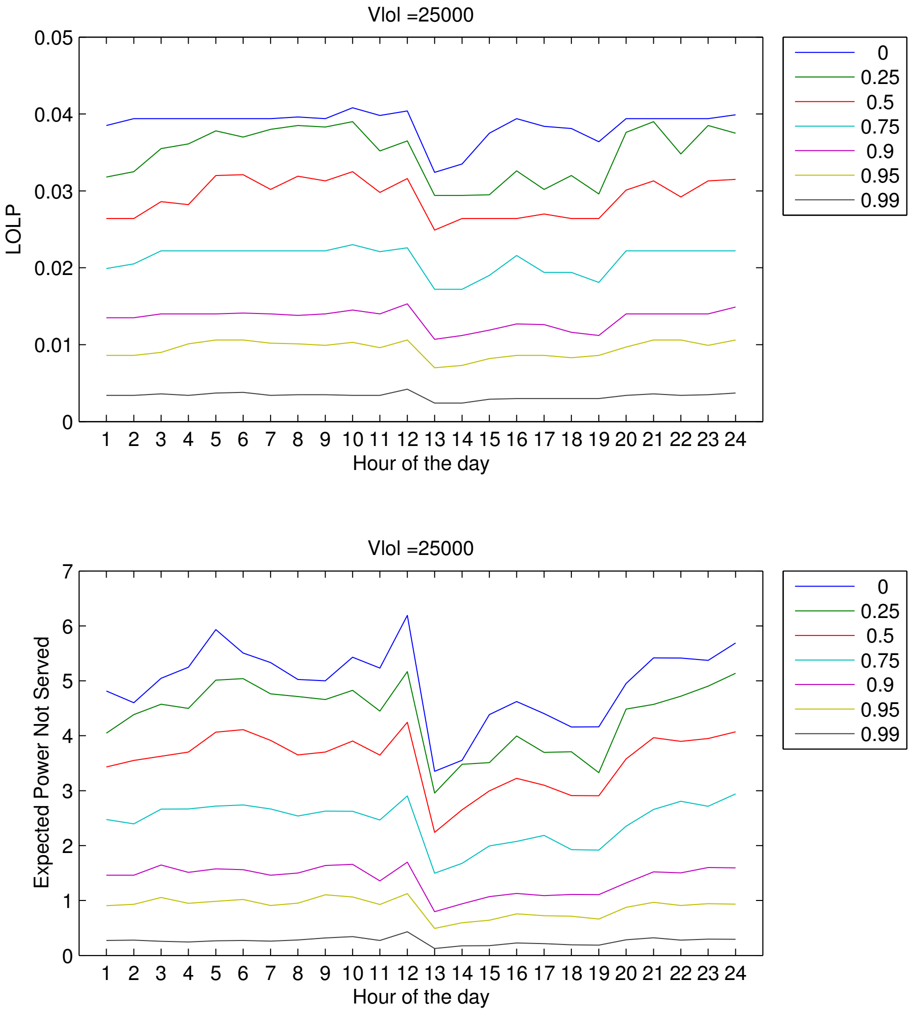

Figure 5.8: Median of the LOLP and R^{tot} along the hours of the day on the upper and lower plot, respectively, for different values of α represented trough lines with same color

5.5 General remarks about the results

The performance of the methods indicates that they could work in a real system providing reasonable levels of total reserve to schedule. However, the solutions had to be compared to the generated scenarios since the actual reserve needs was not known. This means that if the scenarios characterized by the reserve needs represent the actual process well, the comparison can be extrapolated to reality. The quality of the methods will directly depend on the quality of the scenarios.

There are several ways scenarios could represent better the real reserve needs. First, by simply generating more scenarios. This seems an fairly good approach specially in this case since one is more interested in the unlikely events of outages happening or big forecast errors occurring. The more scenarios, the more of this cases will be included. A second approach is to include more variables which have an effect on the reserve needs, for example transmission line failures or regulating power coming or going to neighbor areas. Last but not least, by improving the already existing statistical models of wind power production, power load demand and failure rate, specially this last one.

Which of the two methods is more useful should be addressed by the TSO depending on its preferences. If the main concern is the probability of not having enough reserves, or similarly the number of hours per year allowed to be load shedding, then the LOLP method is more appropriate. On the other hand, if the main concern are the cases when outages cause big societal costs, the CVaR method should be used. As a general observation, the CVaR method produces more safe solutions with greater reserve allocations than the LOLP method, basically because the cost of not serving power is accounted for in the objective function.

Chapter 6

Conclusion

Electricity must be supplied continuously and at certain level, balancing up the quantity that customers demand with the quantity that producers produce. If, for some unexpected reason, those two quantities are not equal then the system is imbalanced and load demand has to be shed. This occurrence is costly and undesired. The main tool that transmission system operators have to avoid it is to allocate electricity reserves and use them to balance up the system if required. During this thesis, the goal has been to define probabilistic methods to determine the optimal level of reserve required.

There is a main difference between the approach given in this thesis and the common approach given in the literature. The organization of the market structure is quite different from one country to another. Many of the Market Operators around the world organize theirs markets in such a way that at the day ahead market both unit commitment and reserves are cleared. This gives the MO information about which producers will be on and how much will they contribute with, information that can be used when determining the reserve levels. In some other markets, more specifically in West Denmark, the reserves are set more than two hours before the day ahead market closes, having no information about which units will be online and hence being the traditional probabilistic methods not useful for this case. The work of this thesis focuses on this last kind of market structure; the methods developed can however be used at both ways of arranging the markets.

Three methods described in Chapter 3 give an alternative to the traditional deterministic “n-1” criteria. All of them use as a main input a function representing the probability that certain reserves will be needed. The first half of the chapter explores methods which use this function in the case that it is known in a closed form. The second half approaches similar methods by a stochastic programming formulation using scenario generation techniques, which in fact proven to be useful at the study case, presented in Chapter 5.

The first of the two methods implemented imposes a target on the Loss of Load Probability (LOLP) while the function representing the cost is minimized. The second method uses the Conditional Value at Risk (CVaR) measure to account for the risk of having cases with big costs, either because the cost of providing reserve is big or because the cost of not allocating enough power is big. Both methods turn out to be useful, each having different capabilities. The LOLP method explores how the probability of not having enough reserves affects the solution and will be more useful if this is the concern of the Transmission System Operator. The CVaR approach gives the TSO more tools to account for the “worse case” scenarios.

The generation of scenarios is discussed in Chapter 4. It was assumed that three factors have an impact on the reserve needs, namely the forecast error of the wind power production, the forecast error of the power load demand and the forecast outages of power plants. The first two factors are modeled and given by ENFOR S/A. The third factor is subject to study in this thesis. The system is treated as a whole and when forecasting outages no difference is made between power plants. Efforts were made to model the failures as a hidden Markov model yet scenarios were finally generated from a combination of a Bernoulli Generalized Linear Model (GLM) and a Poisson GLM.

All in all, this thesis provides some useful methods to determine the optimal level of reserves that should be scheduled in a probabilistic way. Still more analyses should done, increasing the complexity while increasing the applicability in a real system as West Denmark. Nevertheless, a first step is done towards a method capable of using the available resources in the most efficient and economic way.

6.1 Future work

The West Denmark area, as well as the rest of the European electricity areas, are interconnected with their neighbor areas exchanging electricity and balancing power amongst them. With the increasing penetration of wind power production, the whole system is more subject to fluctuations being the safety of the system threatened. In the future it would be of interest to question to what extent safe individual areas implies a safe global system and try to develop a method capable of modeling the interconnections between the different areas.

The presented optimization methods seem to strongly rely on the scenarios generated. If such scenarios would be improved the overall practicability would increase too. In the first place a more adequate model of power plant failures should be worked on since the approach taken on this thesis did not perform as satisfactory as expected. For instance by adaptive models whose probability of observing a failure and its quantity would depend on the observation at the hour before. Secondly, including more factors into the scenario generation stage will represent the reality more precisely and practicability would definitely increase. Those factors could include outages from combined hear and power plants, transmission line failures or reserve needs coming from neighbor areas.

Further studies should concentrate their efforts on what would happen if the assumptions are dropped. Disregarding the assumption that only producers who bid into the reserve market can actually deploy it could be modeled as a three-stage stochastic model if the proper data is available. The models presented could account for this fact too if scenarios characterized by the amount of regulating power available that was not purchased in advanced were generated. Assuming that providing down-regulating power is allowed would be feasible to include in any of the cases.

Appendix A

Maximization of the third term from the CDLL

The Expectation-Maximization algorithm that is presented in 4.2.2.4 requires to maximize the third term of the complete data log-likelihood with respect to the parameters $\boldsymbol{\pi}$:

$$\max_{\boldsymbol{\pi}} term3 = \sum_{j=1}^{m} \sum_{t=1}^{T} \hat{u}_j(t) log \left({}_t p_j(x_t) \right) = \tag{A.1}$$

$$\sum_{j=1}^{m} \sum_{t=1}^{T} \hat{u}_j(t) log \left(\frac{n_t!}{x_t!(n_t - x_t)!} \pi_j^{x_t} (1 - \pi_j)^{n_t - x_t} \right) = \tag{A.2}$$

$$\sum_{j=1}^{m} \sum_{t=1}^{T} \hat{u}_j(t) \left(log \frac{n_t!}{x_t! n_t} + x_t log\ \pi_j + (n_t - x_t) log(1 - \pi_j) \right) \tag{A.3}$$

Deriving with respect to π_i:

$$\frac{\partial term3}{\partial \pi_i} = \sum_{t=1}^{T}\left(\frac{x_t}{\pi_i} - \frac{\hat{u}_i(t)(n_t - x_t)}{1-\pi_i}\right) \tag{A.4}$$

The last step is to make the previous Equation A.4 equal to zero and solve the equation with respect to π_i:

$$\sum_{t=1}^{T}\hat{u}_i(t)\left(\frac{x_t}{\pi_i} - \frac{(n_t - x_t)}{1-\pi_i}\right) = 0; \tag{A.5}$$

$$\frac{\sum_{t=1}^{T}\hat{u}_i(t)x_t}{\pi_i} = \frac{\sum_{t=1}^{T}\hat{u}_i(t)(n_t - x_t)}{1-\pi_i}; \tag{A.6}$$

$$(1-\pi_i)(\sum_{t=1}^{T}\hat{u}_i(t)x_t) = \pi_i(\sum_{t=1}^{T}\hat{u}_i(t)(n_t - x_t)); \tag{A.7}$$

$$\pi_i = \frac{\sum_{t=1}^{T}\hat{u}_i(t)x_t}{\sum_{t=1}^{T}\hat{u}_i(t)x_t + \sum_{t=1}^{T}\hat{u}_i(t)(n_t - x_t)}; \tag{A.8}$$

$$\pi_i = \frac{\sum_{t=1}^{T}\hat{u}_i(t)x_t}{\sum_{t=1}^{T}\hat{u}_i(t)n_t} \tag{A.9}$$

It has been proven that the maximum is achieved at $\hat{\pi}_i = frac\sum_{t=1}^{T}\hat{u}_i(t)x_t\sum_{t=1}^{T}\hat{u}_i(t)n_t$ for all $i = 1 \ldots m$

APPENDIX B

Selected code examples

B.1 GAMS code for the CVaR model formulation

Below it is presented the code used to solve the CVaR model introduced in Section 3.4. The script runs the optimization for several values of α given in line 21 to 27. Other input parameters as the Value of Loss Load (ie. `Vlol`), the scenarios of reserve needs (ie. `z`) and the steps piecewise approximation to the cost function `R_piece_cost(g)` are imported from a .gdx file called `input` created with MatLab. At the end of the script, in line 93, the solution is again exported to a .gdx file which is further analyzed with Matlab.

```
1  SETS
2         w Scenario /w1*w5000/
3         g Piecewise intervals /g1*g63/
4         a alpha  /a1*a7/;
5
6  SCALAR
7  alpha defines the CVaR
8  Vlol Value of Loss Load
9  Rbound Upper bound for R /1800/
10 status Status of the solver
11 pi Probability fo each scenario /0.0002/
12 ;
13
14 PARAMETER
15 z(w) capacity need at scenario w
```

```
Rtot_sol(a) Solution, total reserves to schedule
EPNS_sol(a) Solution, Expected Power Not Served
COST_sol(a) Solution, total cost
status_sol(a) Solution, solver status
alpha_param(a) alpha values
/ a1 0
a2 0.25
a3 0.5
a4 0.75
a5 0.9
a6 0.95
a7 0.99/
price_fit_rtot(g) Cost of Rtot estimated by a piecewise function;

*Import Data from gdx file, created with MatLab
$gdxin input
$load Vlol z  price_fit_rtot
$gdxin

VARIABLES
   Rtot total reserve
   CO(w) total cost
   CVar Conditional Value at Risk
   VaR Value at Risk
   EPNS Expected Power Not Served
   S(w) Auxiliary variable
   R_piece_cost(g) piecewise represnetation of the Rtot;
BINARY VARIABLES y(w);
POSITIVE VARIABLES Rtot,etta(w), L(w),R_piece_cost(g);

EQUATIONS
 FO  auxiliary constrain modeling the CVar
 Eq1(w) auxiliary constrain for modeling CVaR
 COST(w) cost of our decisions
 DefineL(w)
 bound1_eq1(w)
 bound1_eq2(w)
 bound2_eq1(w)
 bound2_eq2(w)
 bound3_eq1(w)
 bound3_eq2(w)
 EPNS_EQ define the EPNS
 R_piece_cost_eq Definition of the piecewise function
 max_r_piece_cost(g) ;


FO..      CVaR =e= VaR - (1/(1-alpha)) * sum(w,pi*etta(w));
Eq1(w).. etta(w) =g= CO(w) + VaR;

COST(w)..       CO(w) =e=  sum(g,price_fit_rtot(g)*R_piece_cost(g)) +  Vlol*L(w);

R_piece_cost_eq.. Rtot =e= sum(g,R_piece_cost(g));
max_r_piece_cost(g).. R_piece_cost(g) =l= 30;

DefineL(w)..  L(w) =e= (z(w) - Rtot) - s(w);

bound1_eq1(w).. -y(w)*Rbound =l= L(w);
bound1_eq2(w)..  L(w) =l= y(w)*Rbound;

bound2_eq1(w)..  -(1-y(w))*Rbound =l= S(w);
bound2_eq2(w)..  S(w) =l= (1-y(w))*Rbound;

bound3_eq1(w)..  -(1-y(w))*Rbound =l= z(w) - Rtot;
bound3_eq2(w)..  z(w) - Rtot =l= y(w)*Rbound;
```

```

EPNS_eq..       EPNS =e= sum(w, pi*L(w));


MODEL RESERVE /ALL/;

LOOP(a,alpha = alpha_param(a);
    SOLVE RESERVE USING mip maximizing CVaR;
    Rtot_sol(a) = Rtot.l;
    COST_sol(a) = sum(w,CO.l(w))/5000;
    EPNS_sol(a) = EPNS.l;
    status_sol(a) = RESERVE.modelstat;
)
execute_unload 'solution',Rtot_sol, EPNS_sol,COST_sol,status_sol;
```

B.2 R code. Non-homogeneous HMM with Poisson state-distribution

The following code is a compilation of functions used to fit a Hidden Markov Model with Poisson state-distribution and non-homogeneous transition probabilities. The code is based on the examples given in [24]. Function number 1 codes the same matrix as in Equation 4.31. The Expectation-Maximization algorithm is coded in function 3. Along the whole set of functions, `beta` represents the parameters of ${}_t\mathbf{\Gamma}$ arranged in a $K \times 4$ matrix with K as the number of parameters, in the way that `beta[1,p]` $= \beta_{12}(p)$, `beta[2,p]` $= \beta_{13}(p), \ldots,$ `beta[K,p]` $= \beta_{m-1,m}(p)$ for $p = 1 \ldots 4$. The number of states is represented as `m`, the Poisson parameter as a vector of length m is called `lambda` and the initial distribution `delta`. The series is given in `x` and the power load demand in `nMW`.

```
###############################################
### 1.Transition probabilities with covariates #
###############################################

gamma.nonH = function(beta,time,nMW,m){
gamma = matrix(NA,ncol=m,nrow=m)
    k = 1
    for(i in 1:m){
      for(j in 1:m){
        if(i == j){
          gamma[i,j] = 1
        }else{
          gamma[i,j] = exp( beta[k,1]  + beta[k,2]*cos(2*pi*time/24) + beta[k
               ,3]*sin(2*pi*time/24) + beta[k,4]*nMW )
          k = k+1
        }
      }
    }
    # Normalize
    gamma = gamma/rowSums(gamma)

  return(gamma)
}
```

```


##############################################
### 2. Forward and backward probabilities #
##############################################

pois.HMM.lalphabeta.nonH <-function(x,m,lambda ,delta,beta,nMW){
  if(is.null(delta)){
    print("Please provide delta")
    return(NA)
  }
  n <- length(x)
  lalpha <- lbeta <-matrix(NA ,m,n)
  allprobs <- outer(x,lambda ,dpois)
  foo <- delta*allprobs[1,]
  sumfoo <- sum(foo)
  lscale <- log(sumfoo)
  foo <- foo/sumfoo
  lalpha [,1] <- log(foo)+lscale
  for (i in 2:n){
    foo <- foo
    sumfoo <- sum(foo)
    lscale <- lscale+log(sumfoo)
    foo <- foo/sumfoo
    lalpha[,i] <- log(foo)+lscale
  }
  lbeta[,n] <- rep(0,m)
  foo <- rep (1/m,m)
  lscale <- log(m)
  for (i in (n-1) :1){
    foo <- gamma.nonH(beta,i,nMW[i],m)
    lbeta[,i] <- log(foo)+lscale
    sumfoo <- sum(foo)
    foo <- foo/sumfoo
    lscale <- lscale+log(sumfoo)
  }
  list(la=lalpha ,lb=lbeta)
}


########################################
### 3. EM estimation of a Poisson HMM  #
########################################

pois.HMM.EM.nonH <- function(x,m,lambda,beta ,delta , nMW, maxiter =1000 , reltol=0.00001, crittol=0.00001 ,track=TRUE,...){

  lambda.next <- lambda
  beta.next <- beta
  delta.next <- delta
  n = length(x)

  # Keep track of the params
  lambda.track = lambda
  beta.track = list()
  beta.track[[1]] = beta
  delta.track = delta

  for (iter in 1: maxiter){
    fb <- pois.HMM.lalphabeta.nonH(x,m,lambda ,delta=delta,beta,nMW) # return the log-lik
    la <- fb$la
    lb <- fb$lb

    # Scale the log-lik, Sec 3.2
```

```r
    c <- max(la[,n])
    llk <- c+log(sum(exp(la[,n]-c)))

    if(is.na(llk)) {
      print(paste("Loglikelihood is not a number. Stopped at iter ",iter))
      return(NA)
    }

    ## E-step
    # m number of states
    u = matrix(NA,ncol=n,nrow=m)
    v = list()
    for(t in 1:n){
      v[[t]] = matrix(NA,m,m)
      gamma.e.step = gamma.nonH(beta,t,nMW[t],m)
      for(j in 1:m){
        # Eq. 4.13
        u[j,t] = exp(la[j,t] + lb[j,t]- llk)

        for(k in 1:m){
          # Eq. 4.14
          if(t>1 && t<=n){
            v[[t-1]][j,k] =  exp( la[j,t-1] + log(gamma.e.step[j,k]) + log(
                 dpois(x[t],lambda[k])) + lb[k,t] - llk)
          }
        }
      }
    }

    ## M-step. Maximize the CDLL
    #Term 1
    delta.next = u[,1]/sum(u[,1])  # The sum should be equal to 1

    # Term 2. See function below.
    opt = optim(par = beta,term2.nonH,v.estim = v, nMW=nMW,m=m,
                control = list(maxit = 20000,reltol = reltol))
    beta.next = opt$par
    if(opt$convergence !=0) {
      print("Term 2 not sucessfully optimized")
      if(opt$convergence == 1) print("MaxIter reached.")
    }

    # Term 3.
    lambda.next =as.numeric( t( u %*% x / apply(u,1,sum)))

    ## Check for tolerance
    crit = sum(abs(lambda - lambda.next)) +
           sum(abs(beta -beta.next)) +
           sum(abs(delta -delta.next))
    if(crit <crittol){
      np <- 4*m*(m-1) + m
      AIC <- -2*(llk -np)
      BIC <- -2*llk+np*log(n)
      return(list(lambda=lambda ,beta=beta ,delta=delta ,
                  mllk=-llk ,AIC=AIC ,BIC=BIC,iterations = iter,
                  lambda.track = lambda.track, beta.track = beta.track,
             delta.track = delta.track))
    }
    lambda <- lambda.next
    beta <- beta.next
    delta <- delta.next

    print(paste("Iterarion number",iter))

    # Keep track of the parameters
```

```
    lambda.track = cbind(lambda.track,lambda)
    beta.track[[iter+1]] = beta
    delta.track = cbind(delta.track,delta)
  }
  print(paste ("No convergence after",maxiter ," iterations "))
  NA
}


###############
### 4. Term 2 #
###############

term2.nonH = function(beta,v.estim,nMW,m){
  beta = matrix(beta,ncol=4)
  ret = 0
  n = length(v.estim)

  for(t in 2:n){
    log.gamma = log(gamma.nonH(beta,t,nMW[t],m))
    ret = ret + sum(v.estim[[t-1]]*log.gamma)
  }

  return(-ret)
}


#############################
### 5.Forecast distribution #
#############################

pois.HMM.forecast <- function(x,m,lambda ,beta, nMW, delta=NULL ,xrange=NULL ,H,nMW.forecast ,...){
  if(is.null(delta)){
    print("Please provide delta")
    return(NA)
  }

  if(is.null(xrange))
    xrange <-qpois (0.001 , min(lambda)):
      qpois (0.999 , max(lambda))

  n <- length(x)
  allprobs <- outer(x,lambda ,dpois)
  allprobs <- ifelse (!is.na(allprobs),allprobs ,1)
  foo <- delta*allprobs [1,]
  sumfoo <- sum(foo)
  lscale <- log(sumfoo)
  foo <- foo/sumfoo
  for (i in 2:n){
    foo <- foo %*% gamma.nonH(beta,i,nMW[i],m)*allprobs[i,]
    sumfoo <- sum(foo)
    lscale <- lscale+log(sumfoo)
    foo <- foo/sumfoo
  }
  H = length(nMW.forecast)
  xi <- matrix(NA ,nrow=m,ncol=H)
  for (i in 1:H){
    foo <- foo %*% gamma.nonH(beta,n+i,nMW.forecast[i],m)
    xi[,i] <- foo
  }

  allprobs <- outer(xrange ,lambda ,dpois)
  fdists <- allprobs %*%xi[,1:H]
  list(xrange=xrange ,fdists=fdists)
```

```
}


####################################################################
### 6.Conditional distribution of one observation given the rest #
####################################################################

pois.HMM.conditionals <-function(x,m,lambda ,beta ,delta ,nMW,xrange=NULL ,...){
    if(is.null(delta)){
      print("Please provide delta")
      return(NA)
    }

    if(is.null(xrange))
      xrange <-qpois (0.001 , min(lambda)):
        qpois (0.999 , max(lambda))

    n <- length(x)
    fb <- pois.HMM.lalphabeta.nonH(x,m,lambda ,delta=delta,beta,nMW = nMW)
    la <- fb$la
    lb <- fb$lb
    la <- cbind(log(delta),la)
    lafact <- apply(la ,2,max)
    lbfact <- apply(lb ,2,max)
    w <- matrix(NA ,ncol=n,nrow=m)
    for (i in 1:n){
      foo <- (exp(la[,i]-lafact[i])%*% gamma.nonH(beta,i,nMW[i],m))*
        exp(lb[,i]-lbfact[i])
      w[,i] <- foo/sum(foo)
    }
    allprobs <- outer(xrange ,lambda ,dpois)
    cdists <- allprobs %*%w
    list(xrange=xrange ,cdists=cdists)
  }


##################################
### 7.Ordinary pseudo-residuals #
##################################

pois.HMM.pseudo_residuals <- function(x,m,lambda ,beta , delta ,nMW,...){
    n <- length(x)
    cdists <- pois.HMM.conditionals(x,m,lambda , beta, nMW = nMW, delta=delta ,xrange =0: max(x))$cdists
    cumdists <- rbind(rep(0,n),apply(cdists ,2,cumsum))
    ul <- uh <- rep(NA ,n)
    for (i in 1:n){
      ul[i] <- cumdists[x[i]+1,i]
      uh[i] <- cumdists[x[i]+2,i]
    }
    um <- 0.5*(ul+uh)
    npsr <- qnorm(rbind(ul ,um ,uh))
    npsr
  }
```

Bibliography

[1] D. S. Kirschen and G. Strbac, *Fundamentals of Power System Economics*. Wiley, 2004.

[2] T. Jonsson, *Forecasting of Electricity Prices Accounting for Wind Power Predictions*. PhD thesis, Informatics and Mathematical Modeling Department, Technical university of Denmark, 2008.

[3] Energinet.dk, August 2012. http://energinet.dk.

[4] A. Conejo, M. Carrión, and J. Morales, *Decision Making Under Uncertainty in Electricity Markets*. International Series in Operations Research and Management Science Series, Springer, 2010.

[5] "Nord pool spot," August 2012. http://www.nordpoolspot.com.

[6] "Nasdaq omx commodities," 2012. http://www.nasdaqomxcommodities.com.

[7] "Trading rules for financial electricity contracts and certificate contracts," 2010. Nord Pool ASA.

[8] "The nordic power market. electricity power exchange across national borders," April. Nord Pool ASA.

[9] D. K. Yann Rebours, "What is spinning reserve?," September 2005.

[10] "Ancillary services to be delivered in denmark. tender conditions," 2010. Energinet.dk.

[11] "Disclosure guidelines for urgent market messages," March 2012. Nord Pool Spot AS.

[12] "Boletín oficial del estado num. 173," 2006. Secretaría General de Energía. Gobierno de Espana.

[13] B. K. Erik Ela, Michael Milligan, "Operating reserves and variable generation," August 2011.

[14] F. Bouffard and F. D. Galiana, "An electricity market with a probabilistic spinning reserve criterion," vol. 19, no. 1, pp. 300–307, 2004.

[15] M. A. Ortega-Vazquez and D. S. Kirschen, "Optimizing the spinning reserve requirements using a cost/benefit analysis," 2007.

[16] M. A. Ortega-Vazquez and D. S. Kirschen, "Estimating the spinning reserve requirements in systems with significant wind power generation penetration," vol. 24, no. 1, pp. 114–124, 2009.

[17] R. Doherty and M. O'Malley, "A new approach to quantify reserve demand in systems with significant installed wind capacity," *Power Systems, IEEE Transactions on*, vol. 20, pp. 587 – 595, may 2005.

[18] J. M. Morales, A. J. Conejo, and J. Perez-Ruiz, "Economic valuation of reserves in power systems with high penetration of wind power," vol. 24, no. 2, pp. 900–910, 2009.

[19] M. Čepin, *Assessment of Power System Reliability: Methods and Applications.* Springer, 2011.

[20] R. T. Rockafellar and S. Uryasev, "Optimization of conditional value-at-risk," *Journal of Risk*, vol. 2, pp. 21–41, 2000.

[21] P. Thyregod and H. Madsen, *Introduction to General and Generalized Linear Models.* Chapman & Hall/CRC Texts in Statistical Science, Taylor & Francis, 2009.

[22] L. A. S. Jochen Brocker, "Increasing the reliability of reliability diagrams," *American Meteorological Society*, 2007.

[23] A. P. Weigel, M. A. Liniger, and C. Appenzeller, "Generalization of the Discrete Brier and Ranked Probability Skill Scores for Weighted Multimodel Ensemble Forecasts," *Monthly Weather Review*, vol. 135, no. 7, pp. 2778–2785, 2007.

[24] I. MacDonald and W. Zucchini, *Hidden Markov Models for Time Series: An Introduction Using R.* Monographs on Statistics And Applied Probability, Taylor & Francis, 2009.

[25] T. Hastie, R. Tibshirani, and J. H. Friedman, *The elements of statistical learning: data mining, inference, and prediction: with 200 full-color illustrations.* New York: Springer-Verlag, 2001.