ON THE DYNAMICS OF RAILWAY VEHICLES ON TRACKS WITH LATERAL IRREGULARITIES

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ABSTRACT

We examine theoretically the dynamics of the rolling motion of the loaded Cooperrider bogie model running on a straight and horizontal track with laterally sinusoidal irregularities. Both lateral and vertical degrees of freedom are included in the bogie model in order to allow for a coupling between the horizontal motion and roll, pitch and vertical motion. The car body however, can only move horizontally and roll. The wavelengths of the irregularities vary between 2.5 and 50 m with amplitudes up to 10 mm. The wavelengths are the same in both rails. We have investigated the two situations when the disturbances of the two rails are in phase and when they are half a wavelength out of phase. These cases correspond to a centre line and a gauge variation respectively. The results show that there is a high correlation between the lateral motion of the wheel sets and the centre line irregularities in most cases. They also demonstrate that only certain selected choices of forcing wavelengths and amplitudes of the gauge irregularities make the bogie oscillate for a fixed speed. In the other cases the bogie follows the centre line. The bogie oscillations are in the most cases symmetric, but we have also found many asymmetric motions, including phase locked synchronized oscillations with a period, which is an integer sub-multiple of the period of the forcing. In a few cases we find aperiodic motions that are presumably chaotic. Statistical methods are applied for the investigation. In the case of sinusoidal oscillations they provide information about the phase shift between the different variables, and they yield the amplitudes of the oscillations. In the case of aperiodic motion the statistical measures indicate some non-smooth transitions.

Keywords: non-linear railway vehicle dynamics, railway track standard analysis

1. INTRODUCTION

It is desirable to investigate the correlation between the disturbances of the railway track geometry and the response of a vehicle. Due to the nonlinear and non-smooth character of the dynamics a general answer to the problem probably does not exist. There are, however, apart from the scientific curiosity, economic interests connected with an investigation of this problem. As an example we mention the method of characterization of the track standard by dynamic measurements in regular trains instead of the measurements of the track geometry by special cars or trains. These last measurements are expensive as well in capital investments as in use. The test vehicles are operated by a highly professional crew and occupy time slots on the railway line, whereby the test runs interfere with the profitable use of the railway line.

Since the answer to the general problem is elusive, we suggest to begin with an investigation of the correlation between deterministic disturbances and the vehicle. Lieh and Hague [1] write that the behaviour of their wheel set is similar to a single degree-of-freedom system and that parametric resonance occurs when the frequency of excitation is twice or some multiple of the kinematical or Klingel frequency. However time varying systems with multiple degrees-of-freedom may experience resonance for some combinations of their natural frequencies. Therefore it is possible that the natural frequencies of the car body and the bogies will influence the parametrically excited behaviour of the vehicle dynamical system. The objective of this work is to investigate that possibility.

The aim of our investigation is to find out whether the response of a moving railway vehicle to well-defined *laterally sinusoidal track disturbances* yields a reliable characterization of the laterally sinusoidal disturbance.

This article is an abstract of Christiansen's thesis [2]. The complete version is available on the web.

2. THE DYNAMICAL SYSTEM

We investigate a theoretical model of a half passenger car on a Cooperrider bogie (see Fig. 1) running on a straight and horizontal railway line. Both lateral and vertical degrees of freedom are included in the bogie model in order to allow for a coupling between the horizontal motion and roll, pitch and vertical motion. The car body can, however, only move horizontally and roll. The rail profile is a standard UIC60 profile with an inclination of 1/40, and the gauge is standard 1435 mm. The wheel profile is the DSB97-1 profile, which is a S1002 standard profile that is modified for use on tracks with gauges, which are narrower than the standard 1435 mm.

All elements in the model are rigid with exception of the elements in the suspension, which have linear characteristics. The deformations in the wheel-rail contact surface are elastic and Hertz's theory applies. The wheel-rail contact geometry is calculated numerically using the routine RSGEO [3] and tabulated. The tangential force in the contact plane – the creep force – is calculated step by step using the Shen-Hedrick-Elkins model.

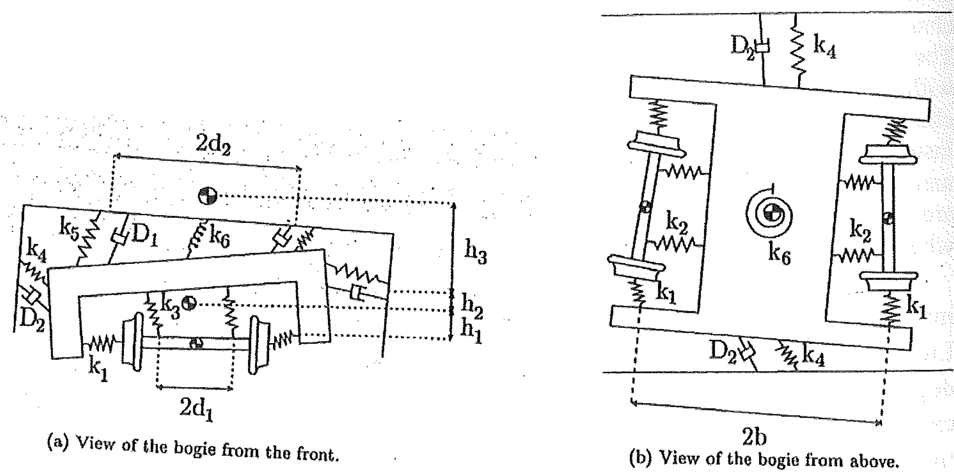


Figure 1 The Cooperrider bogie model

The imposed lateral track variations are all sinusoidal. The wavelengths vary between

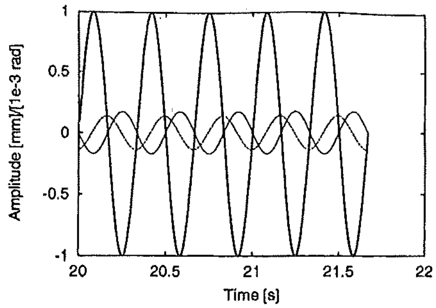
2.5 and 50 m with amplitudes up to 10 mm. The wavelengths are the same in both rails. We have investigated the two situations when the disturbances of the two rails are in phase and when they are half a wavelength out of phase. These cases correspond to an alignment and a gauge variation respectively. Newton's laws of motion are used for the mathematical formulation of the vehicle dynamical model. The model is formulated in a Cartesian coordinate system that moves along the undisturbed track centre line with the speed of the vehicle, V. The vehicle dynamical model consists of 14 second order ordinary differential equations plus 2 first order ordinary differential equations for the calculation of the differences between the actual speed of rotation of the wheels and the theoretical value $\Omega = V/r_0$, where r_0 is the nominal rolling radius of the wheel. The dynamical system can be found in [2].

The integrator is an explicit Runge Kutta method with variable step size. The method is of order five and uses a Cash-Karp-Butcher tableau. We checked the accuracy of the solver with different values for the tolerance. It turns out that a convergence problem appears, when the tolerance is set to be less than 10^{-9}. A comparison with the performance of a Runge Kutta solver of order four, which is slower, gave the same result, and the tolerance was therefore chosen to 10^{-9}.

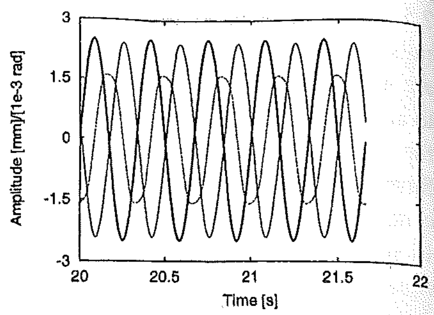
4. SIMULATION RESULTS

In the case of centre line disturbances we find a good correlation between the track disturbance and the displacement of the leading wheel set when the wavelengths are larger than 20 m and the amplitudes are larger than 4 mm. The amplification rate of the response depends, however, on as well the wavelength as the amplitude of the track disturbance. If the disturbance amplitude is smaller than 4 mm, then the amplification rate becomes large and a phase lag appears. It is therefore impossible to determine the amplitude of the track disturbance even at a fixed wavelength by a measurement of the displacement of the wheel set. When the speed is higher than the critical speed, then it is not possible to determine even the wavelength of the track disturbance, because the transients of the wheel set oscillations become very long and the period may differ from the period of the track disturbances. At a speed of 60 m/s we found cases where a transient of 100 s was needed. In 100 s the bogie travels 6 km(!)

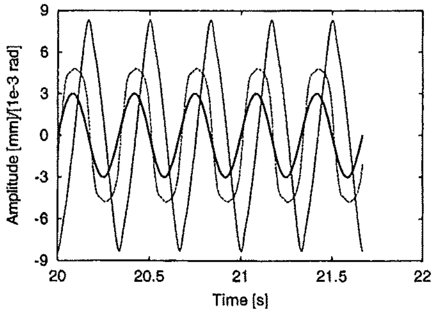
When the wavelengths of the centre line disturbances are shorter than 20 m, then the amplification rate still depends on as well the wavelength as the amplitude of the track disturbance. In addition phase lags of one half or one quarter of the wavelength are found. An example of the results are shown on Fig. 2. For sufficiently large amplitudes of the track disturbance a symmetry breaking bifurcation apparently takes place (see plot d), so the oscillations of the wheel sets are off-set towards one of the rails. For disturbance amplitudes larger than 4 mm the oscillation of the wheel sets is far from sinusoidal and may even become aperiodic (see plot e).



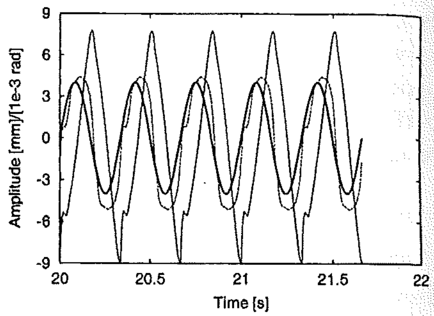
(a) Amplitude = 1 mm



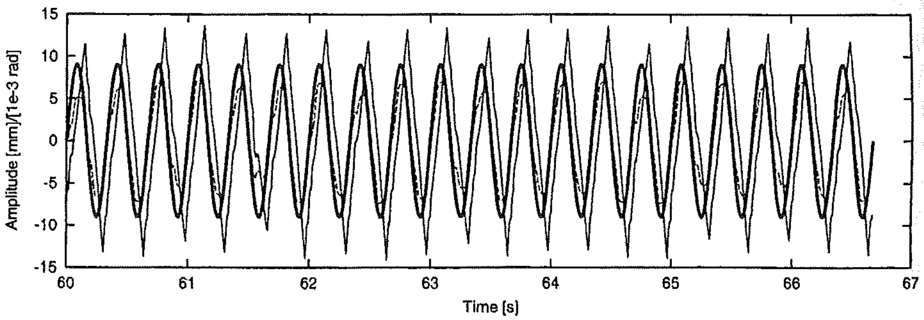
(b) Amplitude = 2.5 mm



(c) Amplitude = 3 mm



(d) Amplitude =4 mm



(e) Amplitude = 9 mm

Figure 2 Time series of the front wheel set and the track disturbance. The speed is 30 m/s and the wavelength is 10 m. The bold line is the track position, the unbroken line shows the lateral position and the dashed line the yaw angle of the leading wheel set. In (e) a longer transient was needed, and the shown time interval is larger than that of the other plots.

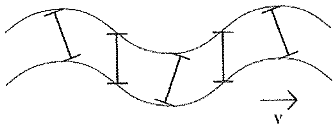
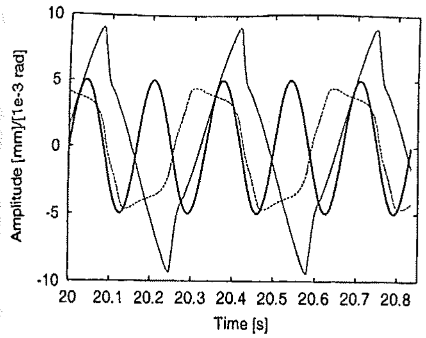
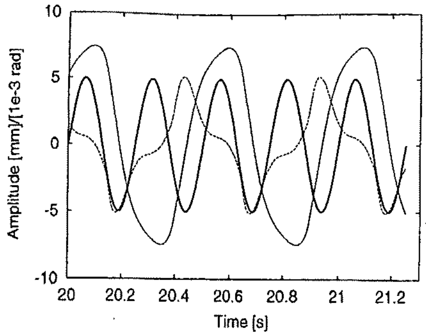


Figure 3 Illustration of the result of a positive correlation between the yaw angle and the track displacement

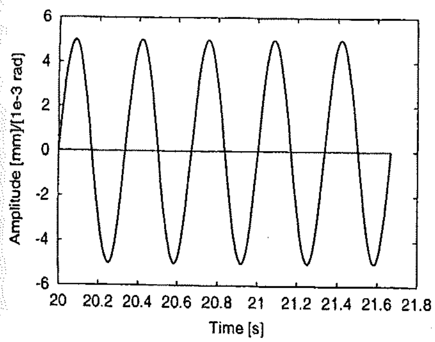
For disturbance wavelengths of the order of the wheel base or twice the wheel base of the bogie we find that the oscillations of the wheel sets are damped and with phase lag,



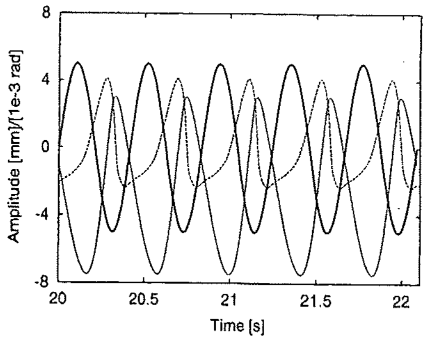
(a) Wavelength = 5 m.



(b) Wavelength = 7.5 m.



(c) Wavelength = 10 m.



(d) Wavelength = 12.5 m.

Figure 4 Time series of the motion of the leading wheel set and the left rail with gauge variations under it. The speed is 30 m/s and the forcing amplitude is 5 mm. The bold line is the position of the left rail, the full line is the lateral position and the dashed line the yaw angle of the leading wheel set. Notice the period doubling of the response on (a) and (b), the lack of response on (c) and asymmetric motion on (d) and the oscillation becomes non-sinusoidal for sufficiently large disturbance amplitudes.

In the case of sinusoidal gauge disturbances we only find a domain with a good correlation for short wavelengths less than 5 m and forcing amplitudes less than 6 mm. It is, however, a domain with a very small wheel set response, so it is not interesting for practical applications. For certain wavelengths of the disturbance the amplitude is even indistinguishable from zero(!) An example of the results is shown on Fig. 4.

When the speed of the vehicle is higher than the critical speed then the transients are very long in the case of centre line variations, and for wavelengths smaller than 10 m the response differs from the forcing. In the case of gauge variations the response remains periodic, but again it differs from the sinusoidal forcing. Some examples are shown on Fig. 5.

In the case of gauge variations frequency locking dominates the response in such a way that the wheel set oscillates with a period between 1.5 and 3 Hz with most frequencies a little less than 2 Hz. These frequencies are the primary frequencies.

The primary frequency, f, is found by dividing the speed by the wavelength and multiply the result by the inverse of the period ratio. As an example we find in the 7.5 m wavelength case, which is a period 2:1 solution, Fig. 4b:

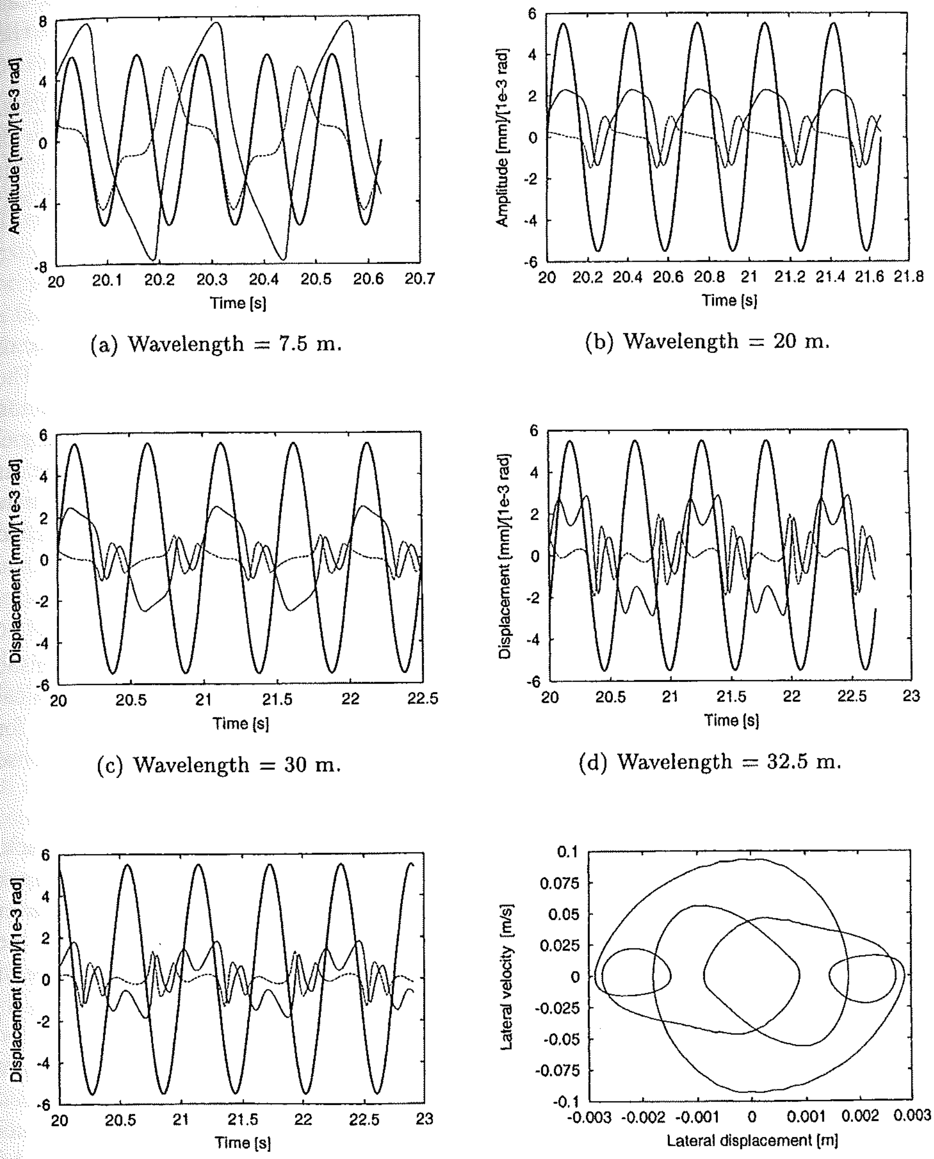
$$f = (30 \text{ m/s} / 7.5 \text{ m}) \cdot \tfrac{1}{2} = 2 \text{ Hz}.$$

For wavelengths larger than 27.5 m the wheel set does not respond at all. It follows the track centre line. Therefore the wheel set response cannot be used as a sensor for neither the wavelength nor the amplitude of the disturbance of the gauge.

The Klingel frequency of our model is 1.005 Hz for $V = 30$ m/s, so another frequency locking than one with the Klingel frequency must be active in our case. We must therefore look for possible sources for frequency locking.

We have therefore calculated the eigenvalues of the Jacobian for the fix point solution on the undisturbed track at V = 30 m/s. Four of them have frequencies between 1Hz and 10 Hz, and for these eigenvalues we calculated the corresponding eigenvectors. In this short article we only point out one remarkable example of the influence of the parametric resonance. In the case of gauge irregularities with a wavelength of 10 m and an amplitude of 5 mm at 30 m/s (see Fig. 4c) the bogie does not respond with a lateral motion, *but it rolls*!

Both eigenvalues f_{10} with 2.644 Hz and f_{11} with 1.728 Hz have eigenvectors with a roll motion. They are the only eigenvalues with frequencies in the interval (1.5 , 3.0) Hz. The eigenvector corresponding to f_{11} has three components with a positive real part. They represent bogie frame roll, leading wheel set roll and trailing wheel set roll respectively. The eigenvalue f_{12} with 1.206 Hz has an eigenvector with positive real



(a) Wavelength = 7.5 m.

(b) Wavelength = 20 m.

(c) Wavelength = 30 m.

(d) Wavelength = 32.5 m.

(e) Wavelength = 35 m.

(f) Phase portrait of (d).

Figure 5, (a)-(e): Some cases with gauge irregularities at a speed of 60 m/s, which is larger than the critical speed. The forcing amplitude is 5.5 mm. The bold line is the position of the left rail, the unbroken line shows the lateral position and the dashed line the yaw angle of the leading wheel set. (f) is a phase portrait of the periodic oscillation on (d).

parts representing the vertical motion of the leading and trailing wheel sets and the bogie frame. Due to the nonlinear couplings in the dynamical model they together can produce a parametric resonance at the value ½(1.728 + 1.206) Hz = 2.934 Hz ~ 3 Hz, which equals the forcing frequency 30/10 Hz.

The rolling response of the bogie due to resonance coupled with a vanishing lateral response, is remarkable. The response clearly demonstrates how the interaction with the natural frequencies of the bogie will influence the parametrically excited behaviour of the vehicle dynamical system and 'falsify' the response as suggested by Lieh and Haque [1].

5. CONCLUSIONS

In this work we have investigated the dynamical effects of a sinusoidal disturbance of the track centre line and the dynamical effects of a sinusoidal gauge variation. The results of the work show that it is impossible in general to extract accurate information about the track geometry from measurements of the motion of a wheel set or a bogie. Lie and Haque [4] predicted in their conclusion "that the natural frequences of the car body and the bogies will influence the parametrically excited behaviour of the vehicle dynamical system". It turned out to be true! The full thesis with many more results and more detailed explanations is available, see reference [2].

6. DISCUSSION

In this work we have demonstrated through numerical examples that certain deterministic variations of the track geometry cannot be detected correctly by the dynamical response of a wheel set. The result should not, however, be perceived as a basis for arguments against Hehenberger's proposition [4], [5] to apply the dynamical response of a railway vehicle to the diagnosis of the track.

It is important to define the target of a characterization of the standard of a track. Hehenberger [10] writes: "It is not the track geometry that is crucial for the track maintenance but rather the dynamic influence of the track on the vehicles that run over it" (authors' translation). The statement moves the target of a characterization of the standard of a track from the conventional measurements of the track geometry to measurements of the dynamical response of the vehicles that run on the track.

It is also important to be specific about the properties of the inputs. In this paper we only consider deterministic inputs to the dynamical systems. A *single isolated irregularity* is defined as an irregularity with a dynamical response that does not interfere dynamically neither with a preceding nor with a successive irregularity. An *isolated irregularity* is defined as an irregularity with a dynamical response, which is measurably larger than the dynamical response in a sufficiently small neighbourhood of the irregularity. A track irregularity, which is not an isolated irregularity, is a *non-isolated irregularity*. A non-isolated irregularity may be a couple of single irregularities, which are not isolated in the sense defined above, but it may also be a continuous function of the track centre line like in this paper. Our irregularities are laterally periodic, but in a real life situation the irregularities are more general.

Dynamical measurements of the standard of a track are applied worldwide today mainly on high-speed railway lines. They furnish information about the general state of the track and about the occurrence of isolated irregularities. The measurements are an important tool for the track maintenance departments in the decision-making process. The in-time repairs of isolated track irregularities stop the growing degradation of the track in between the periodic surveillance of the track quality. Thereby money is saved, the intervals between the regular track re-alignments extended, and the disturbance of the traffic on the line is thereby reduced. Three main problems with the dynamical measurements are: i) the determination of the location of a fault, ii) the determination of the type of a fault and iii) the influence of the state of the measuring vehicle on the results of the measurements.

The location can today be determined accurately by a GPS system. At a test on the Copenhagen s-train system the wheel revolutions and the passage between the track circuit zones were monitored and used for the localization of the fault. The uncertainty was ± 5 m. An inspector then had to find the accurate position and the type of fault. DSB then had conventional s-trains with München-Kassel bogies and Linke-Hoffmann-Busch train sets with Professor Friedrich's steered single-axle bogies. The measurements of the accelerations in three mutually orthogonal directions were performed in the car body above a bogie. The results were qualitatively independent of the type of the vehicle and the mileage covered since the last revision. Only vehicles that satisfied the safety and comfort requirements were used.

Hehenberger [4],[5] described a method for a fast and cheap evaluation of the standard of a track by measurements of the dynamics of a vehicle. The standards must then of course be expressed in limits of the horizontal and vertical accelerations at the points of measurement. Hehenberger [4] suggested *safety values* for the accelerations, but they are not acceptable for the passenger trains on a railway line. Instead *comfort values* must be used, such as was the case on the Copenhagen s-train system.

The problem with the determination of the type of fault is still unsolved. Either a visual inspection or some track measuring equipment is still needed for that purpose, but the inspection can be limited to the few fault positions and therefore be performed much faster than the conventional all-over track inspection.

The common practice today is to measure the accelerations of an axle box or a bogie frame. We do not believe that such measurements yield more information about the track standard than measurements in the car body do. The main reason is that it is *the dynamical reaction of the car body* to the track irregularities, which is wanted. The axle box measurements contain a large proportion of high frequency input, which is unimportant for the characterization of the track and therefore must be filtered out. It complicates the instrumentation and add to the costs. The results of this article demonstrate that measurements of the displacement of the axle boxes do not help to solve the problem with the determination of the type of the track irregularity. Another question is, if the dynamical measurements are helpful in a *vehicle diagnosis system*, but it is beyond the scope of this article.

The track irregularities are in general neither isolated nor purely periodic nor only lateral. This work is limited in scope and should only illustrate the problems that arise through the nonlinear interactions between the parametric track excitation on the vehicle and the dynamics of the vehicle. It may be considered as an investigation of the influence of the *periodic component* of a more general track irregularity on the vehicle, but this is a dangerous interpretation. The nonlinearity of the dynamical problem makes it impossible to separate the influences of the single modes of excitation, because the principle of superposition does not hold for nonlinear operators. It is very obvious from the results of this article. The problem of understanding the reaction of a railway vehicle to general disturbances acting in space and consisting of a combination of different periodic and aperiodic excitations is therefore still open.

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