

# Statistical analysis of Exchange Traded Funds for investment purposes

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# Summary (English)

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The aim of the thesis is twofold. First the thesis sets out to examine the replicative capacity of 20 widely diversified exchange traded index funds. The means is an analysis and subsequent modelling of the deviance of returns, over the life time of the funds. The life time of the considered funds span from 5 to 12 years. It is shown that the deviance processes, with one exception, can be modelled in an ARMA-GARCH framework with mean deviances in the range of  $[-4.57e-3, 3.81e-3]$  on weekly returns and  $[-1.13e-3, 8.54e-4]$  on daily returns. Considering correlation between returns, one out of 20 funds is only vaguely correlated in weekly returns, while this is true for four out of 20 funds considering daily returns. The remaining pairs of fund and index returns are highly correlated. It is thus concluded that on the bottom line the funds do in fact replicate the indices.

Given the replicative capacity of the funds, deduced from the close correlation and low mean deviance, a selection of the underlying indices are applied as proxies for the funds in a scenario generation intended to form the basis of a portfolio optimisation. The indices are considered due to lack of historic information on the funds. Four methods are applied and evaluated for four point predictions, namely bootstrapping, an ARMA-GARCH model, a Markov Switching Autoregressive model and lastly a dependent mixture model. While the first two models consider the index returns in their entirety, the latter two part data into regimes and estimate

separate models in each regime. The models are further distinguished as the dependent mixture model considers the market, as represented by the selected indices, in its whole, while the three models consider each asset individually.

It is found that bootstrapping returns in an attempt to predict new ones falls short in capturing the economic changes, relative to the remaining models. ARMA-GARCH and the Markov Switching autoregressive model perform roughly equal in generating out of sample predictions, although with a favour towards ARMA-GARCH which is generally more accurate. The dependent mixture model is the preferred model amongst the considered, for portfolio optimisation purposes, due to its superior ability to adequately reflect the financial situation.

# Summary (Danish)

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Formålet med denne afhandling er todelt. Først undersøges 20 bredt diversificerede børshandlede indeksfonde i relation til deres underliggende finansielle indeks. Som udgangspunkt udføres en analyse af afvigelsen mellem fondenes afkast og afkastet fra de underliggende indeks. Det påvises at tidsrækken af afvigelser, med en enkelt undtagelse, kan modelleres i en ARMA-GARCH model med gennemsnitlige afvigelser på  $[-4.57e-3, 3.81e-3]$  for ugentlige afkast og  $[-1.13e-3, 8.54e-4]$  for daglige afkast. Når man betragter korrelation mellem fond- og indeks afkast udviser én ud af 20 fonde svag korrelation på de ugentlige afkast, mens fire ud af 20 fonde udviser svag korrelation med daglige indeks afkast. De resterende par af fond og indeks afkast er højt korrelerede. Det konkluderes således at fondene på bundlinjen replikerer de underliggende indeks tilfredsstillende.

I betragtning af dette udvælges ni indeks til scenarie generering. Indeks bliver anvendt i stedet for fondene på grund af manglende historisk information omkring fondene. De genererede scenarier skal danne grundlag for en portefølje optimering over de udvalgte indeks. Fire metoder anvendes til genereringen, og vurderes på deres evne til at forudsige fire punkter frem i tiden sammenholdt med hvad der bliver observeret i samme periode. De fire anvendte metoder er bootstrapping, ARMA-GARCH, en Markov Switching autoregressive model og en dependent mixture model. Mens de første to modeller modellerer afkast-tidsrækken i sin helhed,

deler de to sidstnævnte data ind i regimer og estimerer separate modeller i hvert regime. Modellerne kan yderligere adskilles ved at dependent mixture modellen betragter markedet, som repræsenteret ved de valgte indeks, i sin helhed, mens de tre foregående modeller betragter hvert enkelt aktiv individuelt.

Det findes, at forsøg på at forudsige fremtidige afkast ved at bootstrappe tidligere afkast er betragteligt mindre pålideligt i forhold til de øvrige modeller. ARMA-GARCH og Markov Switching autoregressiv model er nogenlunde lige gode til at generere nøjagtige forudsigelser, omend ARMA-GARCH foretrækkes på grund af moderate bedre præcision i at replikere udviklingen i data. Dependent mixture modellen er den foretrukne model blandt de undersøgte, til formålet porteføljeoptimering, på grund af en fremtrædende evne til at opfange og replikere den finansielle situation.

# Preface

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This thesis was prepared at the department of Informatics and Mathematical Modelling at the Technical University of Denmark in partial fulfilment of the requirements for acquiring a M.Sc. in Mathematical Modelling and Computing.

The thesis deals with different aspects of a financial product known as exchange traded funds. The main focus is on data analysis and non-linear time series modelling. The funds are evaluated based on their performance relative to the underlying index and subsequently four models are applied towards scenario generation.

I would like to express my gratitude to my supervisors on this projects, Lasse Engbo Christiansen and Kourosch Marjani Rasmussen, for helpful supervision and instructive conversations. Gratefulness should also be directed at Jørgen Bjerreskov, Claus Nielsen and Kourosch Marjani Rasmussen for assistance in selecting the funds and providing data.

Lyngby, August 5th-2012

Mie Sustmann Helledie





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# Introduction

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*“Buy low, Sell high – And get paid taking risks!”* These are the basic guidelines for any investor. Yet, to predict the right time to buy respectively sell is an ever-challenging task that only few people master.

The financial markets are constantly changing and adapting to their environments, by continuously inventing new products and abandoning old ones. One of the more recent favourites on the world exchanges is exchange traded funds, which are the topic of this thesis. Much debate persists about whether exchange traded funds are the root of all evil (= volatility) or whether they provide a harmless and much needed opportunity for the average investor to join the investment game [1]. – Regardless of the debate, exchange traded funds have become progressively more popular among professional and individual investors alike, over the last two decades since the first exchange traded fund was introduced in 1993.

As of June 2012, 7,510 exchange traded funds were listed for trading, with a monthly trading volume of \$ 668,576.5 M [2]. According to Dansk Aktionærforening the Danish population have DKkr 135.000 per capita and the United States have DKkr 192.000 per capita invested in ETFs [3].

These numbers signify the importance of ETFs as investment products and validate the need for thorough analysis.

A complete analysis of exchange traded funds is not the objective of this thesis. Instead the primary focus will be on statistical finance, applied towards determining the ability of the funds to mirror the underlying index. As the funds claim to offer an opportunity to indirectly invest in the underlying index, the quality with which the fund replicates the index is of immense importance to the reputation of the product in general and to the involved investor in specific.

The second part of the thesis returns to the question of when to buy and when to sell. Is detailed information about the individual fund sufficient – or perhaps preferred, or is general knowledge of the surrounding financial market required? – Four model frameworks are established and tested in out of sample predictions to determine this.

## Outline

Chapter 2 provides the foundation for understanding exchange traded funds as investment vehicles. ETFs and some of the more popular competing products are described, as well a motivation for investing in ETFs. Chapter 3 describes the necessary measures applied to prepare data for analysis, and section 4 contains a description and introductory analysis of the funds. Chapter 5 provides an analysis of the tracking performance of the funds, as well as the theoretic foundation for the models. The aim of Figure 5 is to determine to which extent the funds deliver as promised, and if the indices can be applied as proxies for the funds in further analysis. Chapter 6 goes on to make predictions about the future return of the funds, using four different model frameworks. The chapter also contains an evaluation of the methods applied. Finally Figure 7 applies the predictions towards asset allocation under different investment strategies. In chapters 8 and 9 the content and implications of the thesis will be summarised and discussed.

## CHAPTER 2

# The concept of Exchange Traded Funds

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The subject of analysis in this thesis is a financial product known as exchange traded funds. An ETF, as the name suggests, is a predetermined basket of securities (fund) which trade on the exchanges in the same way as normal company stock. Typically the content of the basket is determined by a financial index, like the S&P500 or the Dow Jones Industrial Average, which the fund replicates, following some defined guidelines. This is called an index fund, and is the most common practise.

The ETFs are issued and managed by so-called investment companies. An investment company is essentially a business that specialise in pooling funds from individual investors and investing them based on some investing guidelines. Essentially the investor owns a fraction of all of the securities held by the fund, corresponding to the value of the investment. For example the DIA ETF aims at providing investors with a security whose initial market value is approximately one-hundredth (1/100th) the value of the Dow Jones Industrial Average index, meaning that one share of DIA roughly corresponds to one-hundredth of each of the 30 stocks in the Dow Jones Industrial Average index [4].

When it comes to investment companies methodology distinguishes between actively and passively managed funds. Where as the manager of an actively managed fund will seek to outperform the investment benchmark by actively trading, the passively managed funds seek only to replicate as closely as possible the investment benchmark. Typically the actively managed funds come with higher fees, due to the increased costs of the active management. Both types of investment company experience great success, and the most common types are listed here

- **Managed investment companies.** These include closed end funds and open end mutual funds. Trading is conducted directly with the fund. Due to this process orders may be received throughout the day, but will only be settled end-of-day at the closing price.
- **Un-managed investment companies.** This include most ETFs. The funds trade at the exchange and very close to the net asset value (NAV, ref (2.1)) because of the possibility of arbitrage between the ETF and the basket of securities it represents.

$$NAV = \frac{\text{Market value of assets} - \text{Liabilities}}{\text{Shares outstanding}} \quad (2.1)$$

- **Other investment organizations.** This includes REITS which invest in real estate or real estate related assets and hedge funds.

An index fund can adopt many different replication techniques. In the considered data the relevant forms are *physical replication* under which the fund manager replicates the index by acquisition of securities held in it. Thus the fund consists of all or a representative subset of the securities in the index. One of the selected funds employ *synthetic replication* under which the fund lends its assets to a counter party via a collateralised repurchase agreement, and then swap the yield on that loan for the total return of the underlying index. For the purpose of this analysis it suffices to know that there is a difference, and that the replication techniques carry different risks.

## 2.1 Diversification

The appeal of ETFs for investment purposes are several, but the most distinct is the increased possibility of portfolio diversification, it represents to the private investor.

Diversification in the context of finance refers to the action of distributing your means of risk and income over several, non or negatively-correlated areas. The financial markets present periodic up- and downturns in every sector. A perfectly diverse portfolio ensures that at any given time some fraction of investments will be the market winners. At the same time some investments will be temporary losers. – Because of the effects of diversification, an investor will never receive the highest return possible from a single asset. However, the investor will also never receive the lowest return. More importantly, even though an investor does give up the potential “home run” investment, the reduction in return is more than offset by the reduction in risk. In other words, you give up a little return for a lot less risk. Financial theory shows that the expected return on a portfolio of equally weighted securities is exactly equal to the unweighted average of the expected returns on the individual securities. But the standard deviation of such a portfolio is less than the unweighted average of the standard deviations of its individual securities. Thus diversification among securities reduces risk, but not return.

Below is shown the algebraic proof for the diversification benefit where risk is defined as variance of return. C.f. [5] portfolio variance is defined as

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + 2 \sum_i \sum_{j>i} w_i w_j \text{cov}_{i,j}, \quad \sum_i w_i = 1 \quad (2.2)$$

In this expression  $w_i$  refers to the portfolio weight of asset  $i$  and  $\sigma_i$  to the standard deviation of the return of asset  $i$ .

For large portfolios  $w_i^2 \simeq 0$  meaning that (2.2) is reduced to

$$\sigma_p^2 \simeq 2 \sum_i \sum_{j>i} w_i w_j \text{cov}_{i,j}$$

showing that the contribution of each assets variance is (almost) eliminated and the contribution of the covariance terms increase. Hence the

behaviour of each individual asset only matters in relation to the portfolio.

From (2.2) it is apparent that the more non or negatively correlated assets in a portfolio the lower the aggregate variance and the more secure the return. This insight is an important factor in understanding why ETFs have become so popular. Also notice that while perfectly negatively correlated assets will completely cancel out the effect of each other, leaving the portfolio at status quo, non-correlated assets may experience growth respectively decline on the same periods, or may just as likely move opposite each other. It is thus possible to benefit from an increase in one asset, as it will not be neutralized by a descent in another asset. Thus to mirror the long term growth of the economy a portfolio must be composed of non-correlated assets. Yet, to shield from the periodic ups and down of various markets, negatively correlated assets are advised.

## CHAPTER 3

# Preprocessing cause of action

---

Data analysed in this thesis consists of daily observations of the net asset value, (2.1), from 20 different funds and their underlying index. Data is extracted from four different sources, namely the iShares webpage, the SPDR webpage, Bloomberg and <http://www.bullionvault.com/gold-price-chart.do>. These institutions operate in different parts of the world, meaning they operate with different calendars and banking holidays. As data consists of price registrations, consequently there will be no observations on banking holidays where the exchanges will be closed. In order to make the funds comparable, similar date vectors must be defined so as not to compare April 2nd in one fund to April 4th in a different fund or risking that a vector of 1000 observations in one fund covers a longer period than a vector of 1000 observations in a different fund. This imposes some instances of missing values in the sets where some countries/institutions have celebrated a banking holiday while other countries/institutions have been working and registering prices. Apart from the banking holidays, additional few instances of unexplained missing observations occur in all of the datasets. Lastly, few instances of very extreme observations occur. A threshold is set for determining how extreme an observation

can be, relative to its adjacent observations, before it is categorised as an erroneous registration and treated as missing. This will be elaborated on shortly.

A number of operations are implemented to prepare data for analysis. These are described in the following.

**Merge data columns to contain all unique dates** This step builds one vector for each set, containing all business days between the earliest and the latest date. These are extracted and the function interpolates with daily intervals, omitting weekends. Afterwards the NAV observations are added, implying NA values where there is no price. NAs are overcome by interpolation, as described in the next step.

**Remove missing data** The data gaps are interpolated. When a missing value is detected, the cell is interpolated using the previous value and the first real value going forward and calculating the number of missing values in between:

```
a = min(which(!is.na(x[i:length(x)])))
if(i>1)
{
  if(i<5)
  {
    sd = sd(x[(i+2):(i+a+10),j], na.rm=T)
    e = rnorm(1,mean = 0, sd = sd)
    x[i,j] = x[i-1,j] + (x[i+-1,j]-x[i-1,j])*1/a + e
  }else
  {
    sd = sd(c(x[(i-5):(i-1),j], x[(i+a):(i+a+5),j]), na.rm=T)
    e = rnorm(1,mean = 0, sd = sd)
  }
  x[i,j] = x[i-1,j] + (x[i+a-1,j]-x[i-1,j])*1/a + e
}
```

A small amount of noise,  $e$ , is added to the interpolation.  $e$  is normally distributed with zero mean and the standard deviation is determined



---

based on the standard deviation of observations in a small interval on either side of the relevant point. It is possible that the real number of observations used to compute the standard deviation of the noise are fewer than intended going ahead of the relevant point, due to the possibility of small intervals of missing observations. Thus fewer points are used to determine the standard deviation. It should also be noted that points in the end of such an interval will be added noise with standard deviation computed using up to four artificial observations.

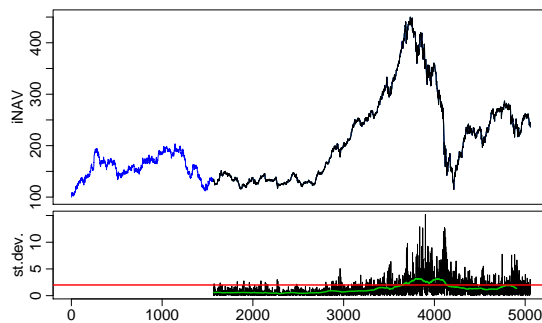
For few missing values a linear interpolation is our best guess and a reasonable estimate. Yet for larger gaps of missing data, it is no longer reasonable to assume that a linear interpolation is suitable. In the examined data the gaps span from 1 to at most 5 missing observations. Within this range the method is considered applicable without loss of quality in data. Table 3.1 shows how the amount of missing data effect each pair. The length of the interval is determined by the time period used for analysis of difference in returns, that is the time since the fund inception date. The fractions stated here are representative for the fractions of missing data over the lifetime of the index as well, with one exception.

TOPIX fund is the dataset most widely affected by missing data, with a fraction of 5.55 percent of the final dataset being computed.

When expanding the focus area to the lifetime of the index, RWX carry only monthly observations in the period January 1st, 1993 to December 31st, 1998. In this period the method of expanding data to contain all banking days leaves 21-23 missing values for every one observed value. In this case the above described method for determining the standard deviation of added noise is not applicable. Instead it is assumed that the standard deviation over the period is fixed, which is supported by figure 3.1. The standard deviation of the added noise is computed as the standard deviation of the gaps between the observations, adjusted for the number of values to be interpolated (on average 22 days). The applied standard deviation is 1.99. This is marked by the red line in the bottom panel of figure 3.1. It is seen that 1.99 roughly corresponds to the standard deviation in observations 4500-5000 of data. This period of the observed data is what closest resemble the interpolated period, thereby supporting the computed standard deviation of 1.99.

Date	Length	Index		Fund	
		Missing	Fraction	Missing	Fraction
DGT	2978	112	3.76	3	0.10
ELR	1641	58	3.53	28	1.71
EMBI	1139	50	4.39	50	4.39
FEZ	2442	111	4.55	65	2.66
FXC	1940	24	1.24	24	1.24
GLD	1920	74	3.85	59	3.07
IBCI	1659	23	1.39	23	1.39
IBGL	1384	22	1.59	22	1.59
IBGS	1518	22	1.45	22	1.45
IEEM	1659	18	1.08	18	1.08
IHYG	1194	60	5.03	0	0
IJPN	1954	28	1.43	28	1.43
IMEU	1234	16	1.30	16	1.30
INAA	1519	18	1.18	18	1.18
LQDE	2314	55	2.38	55	2.38
RWX	1357	48	3.54	0	0
STN	2735	49	1.79	2	0.07
STZ	2735	49	1.79	2	0.07
TOPIX	1621	42	2.59	90	5.55
XOP	1486	51	3.43	30	2.02

**Table 3.1:** A specification of the amount of missing data in pairs. TOPIX fund is the dataset with the highest fraction of missing data, with 5.55 percent of the points in the final dataset being computed.



**Figure 3.1:** The top panel shows the NAV process for index RWX. The blue line is the interpolated part where 22 out of every 23 values have been generated by interpolation. The bottom panel shows the one-point standard deviation for the observed data. The green line is a 200 point moving average and the red line is the applied standard deviation of 1.99.

**Level outliers** This step identifies the extreme outliers in data and levels them by interpolation. It is assumed that extreme outliers are a result of erroneous registration. Thus there is an issue in defining the limit for what is accepted likely effects of the market, and what is the threshold for which the deviation can no longer be ascribed to valid market dynamics. By default the threshold is set to a factor of 2.5 relative to the previous value.

A noteworthy part of the outliers come in pairs of two. Thus, interpolating with the immediate neighbour will result in “half a mis-registration”. The standard deviation is computed using the adjacent 10 point on either side of the relevant observation. Going forward the first adjacent point is skipped, due to the often seen pairing of outliers. This is also the reason to expand the interval of points used to compute the standard deviation. By expanding the interval, any extreme events in the interval will become linearly less prominent in computing the standard deviation. The script is shown below, where  $y$  defaults to 2.5 and  $x$  is the relevant data:

```
a = as.numeric(x[1:(length(x)-2)])
b = as.numeric(x[3:length(x)])
for(i in
first non-NA combination of a and b :
last non-NA combination of a and b
&& i > 5 && i < (b-4))
  if(abs(x[i-1]/x[i]) > y || abs(x[i-1]/x[i]) < 1/y)
  {
    sd = sd(c(x[(i-5):(i-1)],x[(i+1):(i+5)]))
    e = rnorm(1,mean = 0, sd = sd)
    x[i] = (x[i-1]+x[i+1])/2 + e
  }
```

This method presents some problems in either end of the vector, where it is not possible to compare the observations to observations before and after, respectively. In these cases outliers are detected by deviation from a  $y$  multiple of the mean of the proceeding respectively trailing 10 observations.

### 3.1 Data transformation

There is a tradition in econometrics for considering the log returns instead of the simple return. This transformation is implemented to improve the behaviour of data in relation to modelling purposes. Log transformation will decrease the magnitude of extreme events, making it easier to establish a model. Further, log transformation might have beneficial effect on factors such as stationariness in mean and volatility and normality. However, as data takes on numerically small and very close values, the transformation is irrelevant. In this dataset the transformation did not impart substantial improvement in data quality. Tests for normality as well as stationariness in mean and volatility were performed and produce equal results for either data series.

For this reason the simple return, as depicted in (5.1), is considered, in order not to impose any more complexity to the models than what is necessary.

The KPSS test for the null hypothesis that data is level or trend stationary [6] was performed and confirm stationarity in all processes with p-values exceeding 0.1. The Shapiro-Wilks test for normality [7] uniformly reject normality with p-values smaller than 5.7e-04.

# The funds and indices examined

---

The considered funds are listed in table 4.3 in which each fund is represented by its short-name and listed with the name of the underlying index as well as the morningstar<sup>®</sup> asset class classification. Data is collected from various sources but have some common features. All data as a minimum contain observations of the NAV assuming reinvested dividends along with a registration of the date of the observation. Thus, for the models to be correct the investor must reinvest all gains, when the fund does not reinvest automatically. SPDR supply data about the fund only. Information about the related indices is obtained from Bloomberg.

All data related to the iShares funds is supplied by the issuer and is available on the description page for each fund on the internet<sup>1</sup>. An illustrative subset of data on an iShares UK administered fund is shown in table 4.1. The 20 pairs of fund and index NAV are shown in appendix section A.2.

---

<sup>1</sup>e.g. <http://uk.ishares.com/en/rc/products/IEEM?utrack=true>,  
<https://www.spdrs.com/product/fund.seam?ticker=ELR>

Date	NAV	Total Return NAV	Total Net Assets (000)	Shares in issue	Benchmark level
04/05/12	187.58	187.58	525,235	2,800,000	N/A
03/05/12	187.32	187.32	524,524	2,800,000	190.2568
02/05/12	186.99	186.99	523,591	2,800,000	189.9174
01/05/12	186.93	186.93	523,430	2,800,000	189.8713
30/04/12	186.93	186.93	523,410	2,800,000	189.8713

**Table 4.1:** Illustrative subset of iShares UK data.

	DGT	ELR	EMBI	FEZ	FXC
Fund	25/09/2000	08/11/2005	17/12/2007	15/10/2002	05/10/2004
Index	11/09/2008	31/21/1999	30/06/2006	26/02/1998	16/03/2001
	GLD	IBCI	IBGL	IBGS	IEEM
Fund	12/11/2004	18/11/2005	08/12/2006	05/05/2006	18/11/2005
Index	-	31/12/1999	31/12/1998	31/12/1997	31/12/1987
	IHYG	IJPN	IMEU	INAA	LQDE
Fund	10/07/2007	01/10/2004	06/07/2007	02/06/2006	16/05/2003
Index	31/12/2005	31/12/1969	31/12/1987	31/12/1969	31/12/1998
	RWX	STN	STZ	TOPIX	XOP
Fund	15/12/2006	06/08/2001	06/08/2001	28/10/2005	19/06/2006
Index	31/12/1992	31/12/1969	31/12/1969	04/01/1989	04/01/1989

**Table 4.2:** Fund and index inception dates.

Table 4.2 shows for each pair the fund and the index inception dates.

## 4.1 Fund correlation

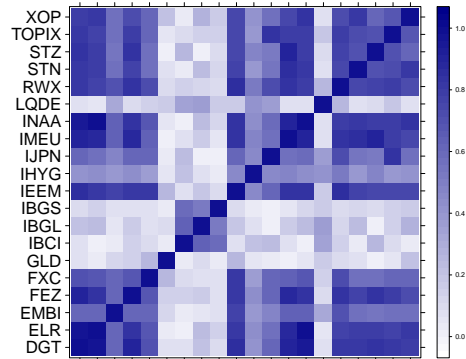
Table 4.3 shows the diversification amongst the selected funds which represent various different markets. As was stated in section 2.1, the primary concern when building a portfolio is the correlation between the portfolio and the individual security. For this reason the correlation matrix for the weekly returns of the 20 funds is shown in table 4.4. The correlation has been computed based on the 1078 most recent observations, from December 18th, 2007, marking the latest inception date, until February third, 2012. The progression in fund NAV is shown in figure 4.2 where the date December 18th, 2007 is marked with a vertical line. The funds have been indexed to 100 at that date, to facilitate comparison. It is

important to note that the following discussion is based on correlation coefficients obtained on returns during or after the recent crisis. Figure 4.2 gives reason to believe that the correlation between funds is different today than it was before the crisis.

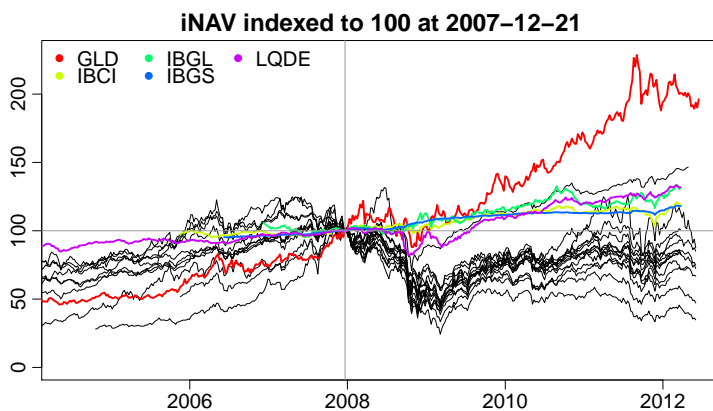
To clarify table 4.4 the correlation is illustrated in a heatmap in figure 4.1. GLD, IBCI, IBGL, IBGS and LQDE have low correlation with the rest of the funds, although some correlation, in the amount of fifty to sixty percent, is seen between IBCI, IBGL and IBGS. IBCI, IBGL and IBGS all hold European government bonds, which explains their mutual correlation. A complete analysis of the funds in relation to table 4.4 can be found in appendix Figure A.3.

In figure 4.2 the coloured lines show the five funds GLD, IBCI, IBGL, IBGS and LQDE. The funds in question are moving with the remaining funds before the beginning of the crisis in late 2007 - 2008, but are clearly affected differently afterwards, at what point all funds continue a steady growth as opposed to the equity funds which uniformly decline. The gold fund is seen to rapidly and steadily increase over the observed period.

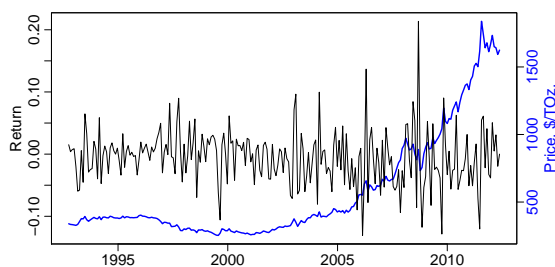
This is consistent with the development in the price of gold, as illustrated in figure 4.3 where monthly observations of the gold price going back 20 years is plotted in blue. Evidently the price of gold has been largely unaffected by the intervening crises. It can also be noticed that the monthly return (in black) is reasonably stable over the period, supporting the stability and uncorrelated nature of the gold market to the dynamics which drive the stock market.



**Figure 4.1:** Absolute correlation between the funds.



**Figure 4.2:** The indices move with the remaining indices before the crisis, but are clearly affected differently afterwards, at what point they continue a steady growth as opposed to the remaining indices which uniformly decline.



**Figure 4.3:** Price of gold over 20 years is shown in blue on the right vertical axis. The black line shows the return over the same period on the left vertical axis. The return is stable over the period, despite several crises and unstable markets.



Ticker	Index	Asset class
DGT	The Global Dow Index	International large cap <b>equity</b>
ELR	Dow Jones U.S. Large Cap Total Stock Market Index <sup>SM</sup>	U.S. large cap <b>equity</b>
EMBI	J.P. Morgan EMBI <sup>SM</sup> Global Core Index <sup>SM</sup>	Emerging markets <b>government bonds</b>
FEZ	EURO STOXX 50 <sup>®</sup>	European large cap <b>equity</b>
FXC	FTSE China 25 Index	Emerging markets large cap <b>equity</b>
GLD	Gold bullion	Gold bullion
IBCI	Barclays Capital Euro Government Inflation-Linked Bond Index	EUR inflation linked <b>government bonds</b>
IBGL	Barclays Capital Euro Government Bond 15-30 Year Term Index	EUR <b>government bonds</b>
IBGS	Barclays Capital Euro Government Bond 1-3 Year Term Index	EUR <b>government bonds</b> (short term)
IEEM	MSCI Emerging Markets Index	Global emerging markets <b>equity</b>
IHYG	Markit iBoxx Euro Liquid High Yield Index	EUR high yield <b>bonds</b>
IJPN	MSCI Japan Index	Developed Asia large cap <b>equity</b>
IMEU	MSCI Europe Index	Europe large cap <b>equity</b>
INAA	MSCI North America Index	U.S large cap <b>equity</b>
LQDE	Markit iBoxx \$ Liquid Investment Grade Top 30 Index	USD <b>corporate bonds</b>
RWX	Dow Jones Global ex-U.S. Select Real Estate Securities Index <sup>SM</sup>	Global (ex-U.S.) <b>real estate</b>
STN	MSCI Europe Energy	Sector <b>equity</b> Energy
STZ	MSCI Europe Financials	Sector <b>equity</b> Financials
TOPIX	Tokyo Stock Price Index	Developed Asia large cap <b>equity</b>
XOP	The oil and gas exploration and production sub-industry portion of the S&P Total Markets Index <sup>TM</sup>	Sector <b>equity</b> energy

**Table 4.3:** The selected funds listed along with the index each of them attempt to track and the morningstar<sup>®</sup> asset class classification.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	DGT	0.96																	
2	ELR	0.61	0.61																
3	EMBI	0.62	0.83	0.61															
4	FEZ	0.91	0.70	0.62	0.69														
5	FXC	0.70	0.67	0.62	0.15	0.27													
6	GUD	0.08	0.04	0.13	0.15	0.09	-0.04												
7	IBCI	0.08	0.00	0.02	0.15	0.09	-0.09	-0.04											
8	IBGL	-0.21	-0.23	-0.06	-0.17	-0.08	-0.09	0.63	0.61	0.51									
9	IBGS	-0.10	-0.13	-0.08	0.85	-0.04	-0.01	0.61	0.61	0.10	-0.10								
10	IBEM	0.86	0.82	0.76	0.85	0.87	0.27	0.06	-0.21	0.48	0.48								
11	IHYG	0.43	0.45	0.41	0.44	0.39	0.04	0.19	-0.07	0.48	0.47								
12	IIPN	0.61	0.55	0.48	0.61	0.62	0.03	0.24	0.01	0.63	0.48	0.58							
13	IMEU	0.91	0.89	0.63	0.88	0.67	0.02	0.06	-0.20	0.48	0.48	0.57	0.90						
14	INAA	0.96	1.00	0.62	0.84	0.68	0.06	0.01	-0.23	0.84	0.46	0.38	0.07	0.05					
15	LQDE	0.07	0.04	0.33	0.08	0.19	-0.17	0.35	0.17	0.15	0.44	0.84	0.84	0.80	0.26				
16	RWX	0.82	0.78	0.70	0.85	0.74	0.13	-0.12	-0.08	0.86	0.53	0.74	0.88	0.80	0.05	0.74			
17	STN	0.81	0.79	0.56	0.78	0.61	0.11	-0.04	-0.13	0.78	0.39	0.52	0.88	0.80	0.05	0.74	0.81	0.73	
18	STZ	0.85	0.81	0.57	0.87	0.61	0.01	-0.10	-0.04	0.76	0.48	0.55	0.92	0.82	0.06	0.81	0.72		
19	TOPIX	0.82	0.77	0.56	0.80	0.64	0.08	-0.16	-0.13	0.76	0.41	0.85	0.79	0.78	0.18	0.81	0.83	0.65	
20	XOP	0.82	0.82	0.57	0.76	0.63	0.22	-0.27	-0.15	0.79	0.41	0.54	0.78	0.84	0.05	0.74	0.83	0.65	0.71

Table 4.4: Correlation between weekly fund returns.

## CHAPTER 5

# Performance of funds relative to indices

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The previous chapters have focused on the concept of ETFs and specifics of the EFTs analysed in this thesis. This chapter will examine the funds in relation to the indices they aim to track. One of the problems inherent in ETF investing is the lack of history and experience. Referring to table 4.2 the fund are at most 12 years active, and some of them have as little as five years of history. This is not an atypical time frame to be facing when dealing with ETFs. As mentioned, ETFs are fairly new to the investment scene and have only recently become popular. This means that while ETFs possess a number of theoretically appealing features, there is a lack of historic performance available to support the investment decision.

This motivates the following chapter. A thorough analysis of the funds in relation to the indices is performed, exemplified by a modelling of the difference in returns between the two series, referred to as tracking error or deviance.

The return is considered, as opposed to the raw NAV for two reasons. Firstly the return possesses attractive statistical properties for analysis. The

return is a stationary process and further the returns provide a scale free assessment of the performance of the asset.

In general, tracking error is considered the single most important factor in the analysis of an index fund performance [8]. Tracking error can be defined in a variety of ways, for example by computing the difference in returns between the fund and the index, which is the approach taken in this thesis. Another approach is to compare the volatility of the fund with that of the benchmark, thus defining tracking error as the standard deviation of the difference between the return of the fund and that of the index,

$$TE = \sqrt{Var(r_i - r_f)}$$

where  $r_i$  denotes the return of the index and  $r_f$  is the return of the fund. Thirdly tracking accuracy can be measured by the correlation between the fund return and the index return. Both of the mentioned measures of tracking error are illustrated in table 5.1 for daily and weekly data. Measured solely on correlation especially IHYG greatly stands out, as does IBGS when considering the volatility of the deviance in returns. Because the goal of the present analysis is to extract information which can be applied towards giving information about the expected future performance of the funds, by examining past performance of the indices, we need a measure, not only of the volatility of the difference, but a way to model it. For this reason tracking error is defined as difference of returns.

For completeness the full dataset of daily observations as well as a reduced set of only weekly observations are analysed. The analysis carried in either example is the same. The reduced datasets consist only of observations made on Fridays, so the datasets have been reduced by four fifths. The argument for reducing the dataset is twofold. For once the weekly observations present with a more easily interpretable structure. Secondly, the aim of the analysis is to develop models for describing the deviance of the fund from the underlying indices. This is to be utilised for four week predictions, which further supports the notion to consider weekly data. Yet, for completeness, both datasets will be analysed, after which a decision about further progress will be made.

In figures 5.1, 5.2, 5.3 and 5.8, 5.9, 5.10 data is illustrated in low and high frequency.

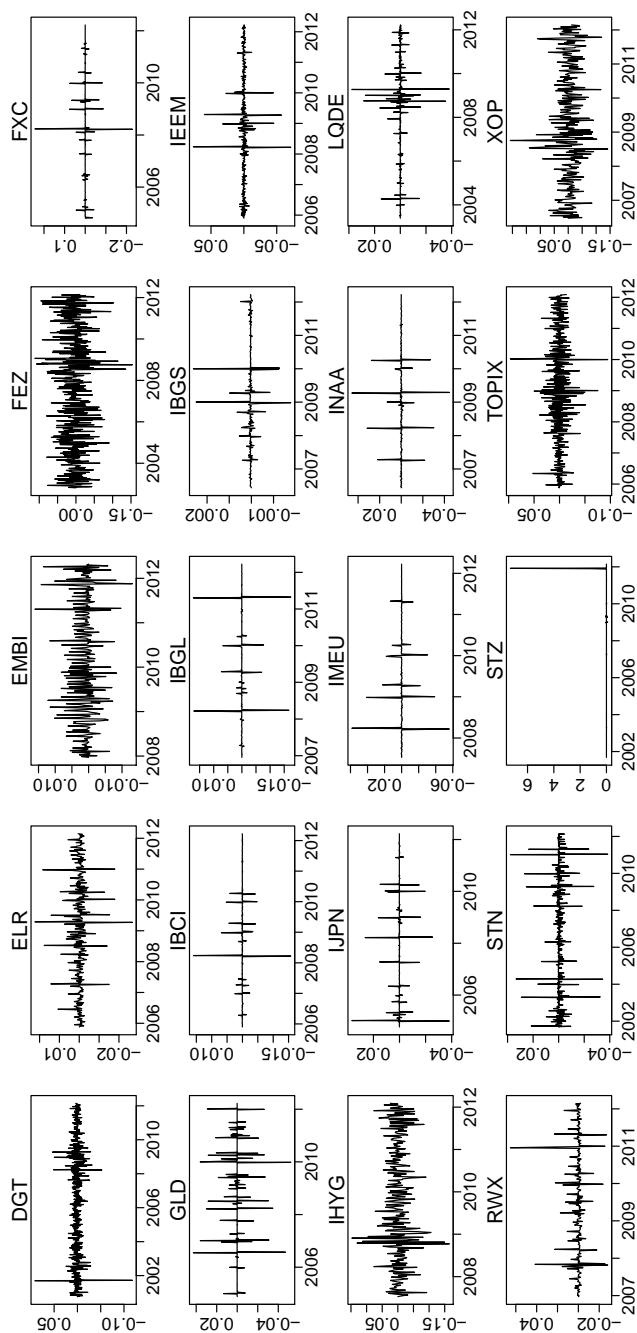
Fund	Correlation		$\sigma_{\text{Deviance}}$	
	Weekly	Daily	Weekly	Daily
DGT	0.9348	0.8132	0.0096	0.0085
ELR	0.9758	0.9131	0.0065	0.0064
EMBI	0.971	0.6168	0.0044	0.0055
FEZ	0.6735	0.431	0.0362	0.024
FXC	0.9499	0.8948	0.0138	0.0102
GLD	0.9619	0.7946	0.0087	0.0091
IBCI	0.9802	0.9355	0.0018	0.0013
IBGL	0.9922	0.957	0.0017	0.0017
IBGS	0.9876	0.9527	4e-04	3e-04
IEEM	0.9618	0.943	0.0112	0.0055
IHYG	0.432	0.1768	0.0398	0.0196
IJPN	0.9746	0.9694	0.006	0.0037
IMEU	0.9707	0.8999	0.0087	0.0073
INAA	0.9582	0.9684	0.0091	0.0039
LQDE	0.955	0.7471	0.0031	0.0031
RWX	0.9671	0.884	0.0105	0.008
STN	0.9904	0.9672	0.0048	0.0042
STZ	0.933	0.9284	0.0169	0.0075
TOPIX	0.8458	0.443	0.0159	0.0165
XOP	0.7238	0.5105	0.0389	0.0253

**Table 5.1:** Two alternative measures of tracking error. Correlation between index and funds returns is shown in columns two and three. Columns four and five show the standard deviation of the deviance processes.

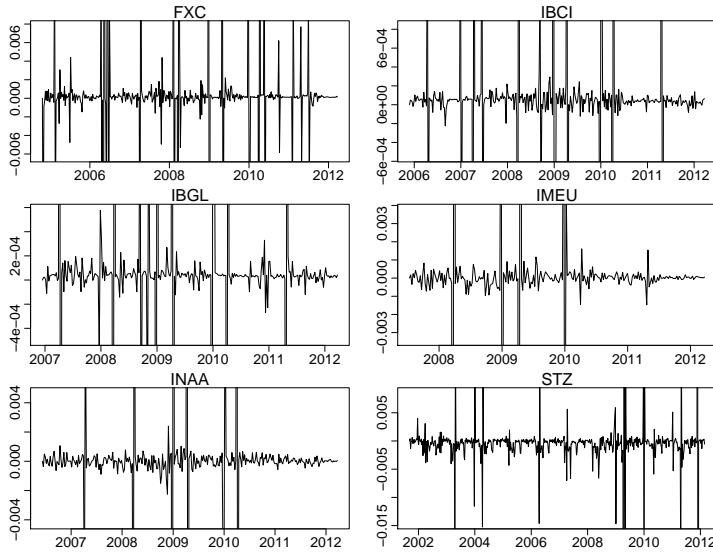
Figures 5.1 and 5.8 show the difference in returns between the index level and the fund net asset value over the life span of the fund. The return is computed as a simple return

$$\begin{aligned}
 \text{fund return}_i &= \frac{\text{NAV}_i - \text{NAV}_{i-1}}{\text{NAV}_{i-1}} \\
 \text{index return}_i &= \frac{\text{Index level}_i - \text{Index level}_{i-1}}{\text{Index level}_{i-1}} \\
 \text{DiffReturn} &= \text{index return} - \text{fund return} \tag{5.1}
 \end{aligned}$$

In figure 5.1 the second axis has been distorted in several of the plots by a few extreme observations. Figure 5.2 shows a subset of figure 5.1 where the second axis has been fixed. When manually fixing the second axis to only show a limited interval around the mean, the structure of data is made clear and it is evident that several of the series display periods of volatility clustering. As an increase in volatility is a sign of instability or insecurity, and instability tend to breed more instability, it is expected that the volatility clusters are more distinct with the high risk assets. However, this seem not to be unambiguously the case.



**Figure 5.1:** The difference in return between the fund NAV return and the index level return, as computed by (5.1), for each of the 20 series. The plots show clear examples of volatility clustering in several of the processes.



**Figure 5.2:** Zoom on six selected processes also displayed in figure 5.1. In this scale it is possible to see the volatility.

## 5.1 GARCH models

While the examined data is technically a derivative of two sets of financial data, it presents with certain characteristics, typical for financial data. A general high volatility is a common observation and the mentioned volatility clusters are a result of dependence on past volatility as well as dependence on past observations and has also been documented in surveys of financial return series [9] [10]. These characteristics of financial data are similar to the characteristics of prices in the electricity market, c.f. description of data in [11]. For electricity price forecast various models have been applied. [11] finds that the GARCH framework outperforms the general time series ARIMA model when volatility and price spikes are present, as is observed in the present data.

Based on the volatility clustering depicted in figures 5.1 and 5.2, as well as lessons of before mentioned studies, a generalised autoregressive conditional heteroskedastic model (henceforth abbreviated GARCH) is applied

to the series.

The GARCH model was originally proposed by Engle in [9] as an ARCH( $q$ ) model, and in 1986 generalised by Bollerslev in [10] to the generalised form applied in this analysis. The model is given by

$$\begin{aligned} y_t &= \gamma + \epsilon_t \\ \epsilon_t | \psi_{t-1} &\sim N(0, \sigma_t^2) \\ \sigma_t^2 &= \omega + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2 \end{aligned} \quad (5.2)$$

where

$$\begin{aligned} q &> 0, \quad p \geq 0 \\ \omega &> 0, \quad \alpha_i \geq 0, \quad i = 1, \dots, q \\ \beta_i &\geq 0, \quad i = 1, \dots, p \end{aligned} \quad (5.3)$$

and  $\gamma$  is some function determining the mean structure of  $y_t$  and  $\psi_{t-1} = y_1, \dots, y_{t-1}$  denotes the information set at time  $t$ . For  $p = 0$  the process reduces to an ARCH( $q$ ) process and for  $p = 0, q = 0$   $\epsilon_t$  is simply white noise with variance  $\omega$ . The error terms are considered conditionally normally distributed.

The non-negativity constraints on the GARCH parameters ensure a non-negative conditional variance. If non-negativity is ensured in all GARCH parameters then the unconditional variance is given by

$$Var(\epsilon_t) = E(\sigma_t^2) = \frac{\omega}{1 - \sum \alpha_i - \sum \beta_i} \quad (5.4)$$

Clearly this requires  $\sum \alpha + \sum \beta < 1$  in order to be meaningful, in which case the process is wide-sense stationary [10].

### 5.1.1 GARCH likelihood expression

The parameters in the GARCH model are estimated by maximum likelihood estimation. Given the observations  $\psi_n$ , we estimate  $\Theta$  as the values of the parameters for which the likelihood is maximized. C.f. [12] the



likelihood expression to be optimized is given by the conditional likelihood

$$l^b(\Theta) = -\frac{n-1}{2} \log(2\pi) - \frac{1}{2} \sum_{t=2}^n \log(\sigma_t^2) - \frac{1}{2} \sum_{t=q+1}^n \frac{\epsilon_t^2 + 1}{\sigma_t^2} \quad (5.5)$$

where  $\Theta = (\omega, \alpha, \beta)^T$  is the parameter vector. The error terms are determined by

$$\epsilon_t = y_t - E(\hat{y}_t | \psi_{t-1}, \Theta) \quad (5.6)$$

where  $y_t$  is the observation at time  $t$ ,  $\psi_{t-1}$  is the information set at time  $t$  and  $E(\hat{y}_t | \psi_{t-1}, \Theta)$  is the expected value of  $\hat{y}_t$  given the parameter values and all previous observations.  $E(\hat{y}_t | \psi_{t-1}, \Theta)$  is the mean process structure; in the GARCH model framework

$$E(\hat{y}_t | \psi_{t-1}, \Theta) = \gamma + E(\epsilon_t) = \gamma$$

### 5.1.2 Limitations of GARCH

The general limitations of the GARCH framework include first of all the assumption that only the magnitude and not the sign of the lagged error determines the conditional variance going forward. This has been proven wrong, as evidence is found that volatility tends to rise in response to bad news (negative error) and to fall in response to good news (positive error) [13].

A technical concern is the model parameters, on which a non-negativity constraint is imposed to enforce a positive conditional variance. If a GARCH model is estimated on a time series that contains parameter changes in the conditional variance process and these parameter changes are not accounted for, a distinct error in the estimation occurs: The sum of the estimated autoregressive parameters of the conditional variance converges to one. Simulations of the GARCH model show that the effect occurs for realistic parameter changes and sample sizes for financial volatility data [14].

Lastly, it is not possible to include time dependent parameter values. This is to some extent accounted for by the time varying conditional variance,

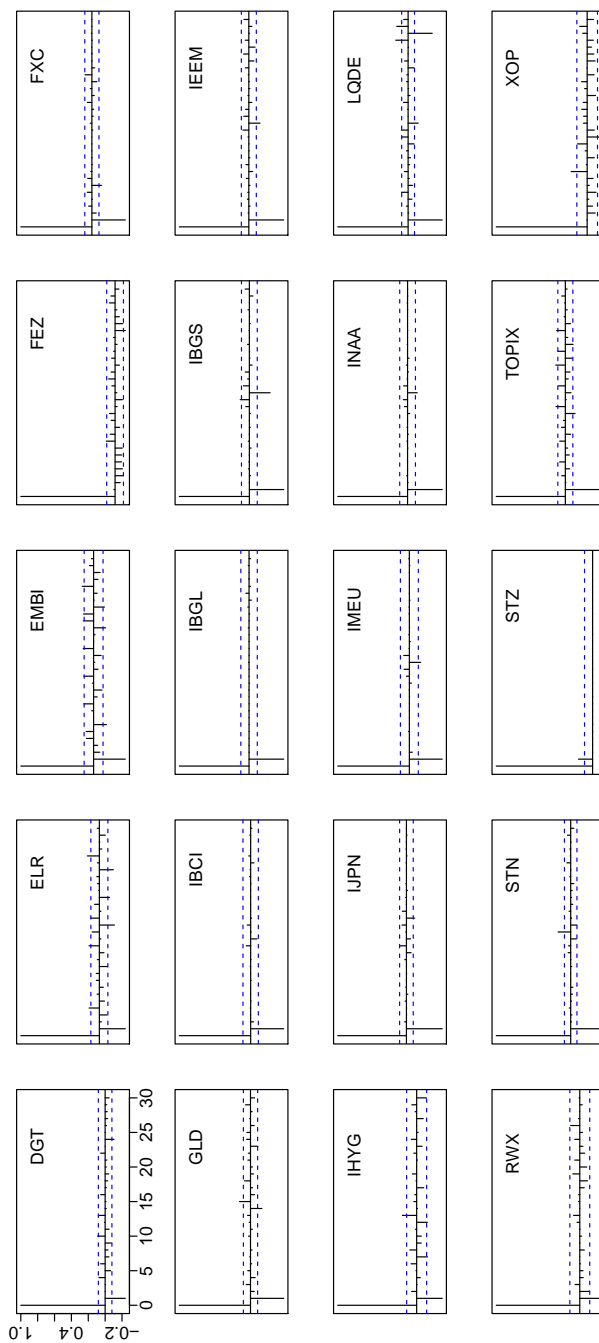
but more flexibility can be obtained by allowing more parameters to be time dependent.

## 5.2 Modelling the deviance series

The software used to model the processes is the `fGarch` package in R, specifically the `garchFit()` function. This can possibly operate with four different optimisation algorithms in the likelihood maximisation, namely `nlminb`, `lbfgsb`, `nlminb+nm` and `lbfgsb+nm`. Initially five randomly chosen funds were selected and nine models of different orders were tested on daily as well as weekly observation sets. The optimiser which succeeded in estimating a model most often out of the 90 attempts were selected as default algorithm for further progress. The success criteria is that a given model structure is applicable in all five datasets, given the specific optimiser. Reversion to the cause for estimation failure is in [Table 6.2.2](#).

On daily data L-BFGS-B outperformed the other algorithms in robustness by succeeding to estimate a model in five out of nine attempts. `nlminb` and `nlminb+nm` failed to estimate a single model in all five cases and `lbfgsb+nm` succeeded in two cases. Considering weekly data `nlminb` and `nlminb+nm` each succeeded in estimating one model, and `lbfgsb` succeeded twice while `lbfgsb+nm` successfully estimated three different model structures in all five sets. For the sake of consistency only one optimisation algorithm is chosen for all models, and the following models have been estimated using the L-BFGS-B optimisation algorithm [\[15\]](#).

Under the L-BFGS-B optimisation algorithm the central difference approximation for evaluating the hessian proved to perform superiorly to the alternative, the `optimHess()` function in R.



**Figure 5.3:** Auto correlation in the deviance processes. All except FEZ and XOP show significant auto correlation in lag one.

Figure 5.3 shows the auto correlation of the 20 series of weekly observations. Several of the series display significant auto correlation in lag 1, indicating serial dependence on past observations. Thus in (5.2)  $\gamma$  is computed as an ARMA(m,n) process, so consequently the deviance series are modelled as ARMA-GARCH. Where as the traditional ARMA framework assumes constant variance in the error terms, this expansion allows the conditional variance to be a function of past errors as well as past conditional variance. The framework used to model the difference in returns (5.1) is, c.f. (5.2)

$$\begin{aligned}
 y_t &= \mu + \sum_{i=1}^m \phi_i y_{t-i} + \sum_{i=1}^n \theta_i \epsilon_{t-i} + \epsilon_t \\
 \epsilon_t | \psi_{t-1} &\sim N(0, \sigma_t^2) \\
 \sigma_t^2 &= \omega + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2
 \end{aligned} \tag{5.7}$$

where

$$\begin{aligned}
 p &\geq 0, \quad q > 0, \quad \omega > 0 \\
 \alpha_i &\geq 0 \quad i = 1, \dots, q \\
 \beta_i &\geq 0, \quad i = 1, \dots, p
 \end{aligned}$$

### 5.2.1 ARMA-GARCH likelihood expression

The expression of the likelihood in the ARMA-GARCH framework is identical to that of the GARCH framework, with a single adjustment to the structure of the error terms and to the parameter vector  $\Theta$ , such that  $\Theta = (\mu, \phi, \theta, \omega, \alpha, \beta)$ . In the ARMA-GARCH framework the error terms are again determined by

$$\epsilon_t = y_t - E(\hat{y}_t | \psi_{t-1}, \Theta)$$

with the same notation as in (5.6). The adjustment to this revolves the expectation term under which the mean structure is altered to include the ARMA parts. In an ARMA-GARCH process the conditional mean is determined by

$$E(\hat{y}_t | \psi_{t-1}, \Theta) = \mu + \sum_{i=1}^m \phi_i y_{t-i} + \sum_{i=1}^n \theta_i \epsilon_{t-i}$$

### Limitations of `fGarch` in R

The `fGarch` package has some limitations. One of them is an inability to handle pure GARCH(0,p) processes. While this is in line with the model constraints, it proved to impose some restrictions on modelling the deviance processes. The function can handle GARCH(q,0) processes, as well as ARMA(0,0)-GARCH(q,0) processes. But it also cannot handle pure ARMA(m,n) processes. For this purpose `auto.arima` from the `forecast` library is applied.

Also, the package is unable to include seasonality in the models.

During the work with data it further became apparent that the `fGarch` package does not impose non-negativity constraints on the GARCH parameters. The user must thus be aware that negative conditional variance may occur.

## 5.3 Low frequency data, results

Estimating the models entails numerous attempts of different model orders. In selecting the preferred model emphasis is put first and foremost on parameter significance. A model with insignificant parameters is rejected, and a model nested by a larger model is rejected in favour of the larger model. In the case of two or more un-nested, relevant models, AIC is used to select the preferred model. This approach leads to the models listed in table 5.3, where the preferred model is highlighted in grey. ELR, IMEU and XOP did not exhibit significant ARMA parameters.

Columns five to eight state the estimated values along with corresponding uncertainty of the mean values of respectively the ARMA and the GARCH parts. Generally the mean deviance between the index return and the fund return is very low, in the magnitude of  $10^{-3}$  to  $10^{-6}$ . 13 of the 20 processes have zero mean deviance and two of the 20 processes have a negative mean value, meaning that they on average produce a higher return than the index they are tracking. This is the case in STN

	Est.	$\sigma$	t value	$\Pr(> t)$
$\mu$	-0.00	0.00	-2.33	0.02
$\phi_1$	0.86	0.06	14.52	0.00
$\theta_1$	-0.95	0.03	-29.49	0.00
$\omega$	0.00	0.00	13.93	0.00
$\alpha_1$	0.42	0.12	3.58	0.00

(a) STN modelled by arma(0,1) + garch(1,0) model.

	Est.	$\sigma$	t value	$\Pr(> t)$
$\mu$	-0.00	0.00	-2.27	0.02
$\omega$	0.00	0.00	1.59	0.11
$\alpha_1$	0.08	0.03	2.97	0.00
$\beta_1$	0.89	0.04	23.75	0.00

(b) XOP modelled by an arma(0,0) + garch(1,1) model.

and XOP. The fitted model coefficient for funds STN and XOP are given in tables 5.2a and 5.2b.

The mean conditional variance,  $\omega$  is in all instances non-negative, but in one instance, XOP, insignificant. This is a violation of the model constraints, as given by (5.3). The model coefficient matrix is shown in table 5.2b. In addition to the non-significant mean conditional variance, the sum of the remaining GARCH parameters exceed 1 in the 95 percent confidence intervals. This implies a non-stationary process. But subject to the overall dubious fit to this particular process, the parameter estimates are revised with caution.

The process IBGS proved impossible to model in the applied framework in both high and low frequency data. Inspecting the raw data suggests a certain pattern, rejecting the premise of complete randomness. It is outside the scope of this thesis to investigate alternative models, and the fund will no longer be a subject of analysis. Plots of the deviance process as well as ACF and PACF are given in appendix Figure A.4

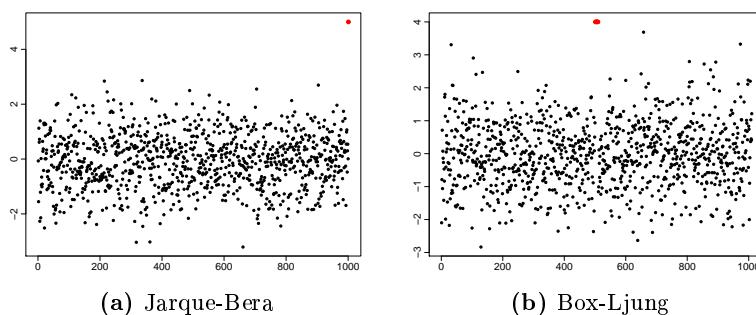
	ARMA (m,n)	GARCH (p,q)	AIC	$\mu$	$\sigma_\mu$	$\omega$	$\sigma_\omega$
DGT	0,1	1,1	-6,579126	1,24E-3	2,85E-4	1,35E-5	4,47E-6
DGT	1,0	1,1	-6,577468				
ELR	0,0	1,0	-7,654219	3,06E-4	2,78E-4	2,39E-5	2,02E-6
EMBI	2,0	1,1	-8,360577	1,91E-3	2,46E-4	9,41E-6	1,42E-6
FEZ	1,1	1,0	-3,811546	3,81E-3	1,57E-3	1,09E-3	9,14E-5
FXC	2,0	1,1	-6,593415	6,66E-4	3,69E-4	1,59E-5	2,10E-6
GLD	2,0	1,1	-7,202282	5,13E-4	3,39E-4	1,84E-5	5,35E-6
GLD	0,1	1,1	-7,163977				
IBCI	1,1	1,0	-10,26134	6,43E-5	4,55E-5	1,85E-6	1,49E-7
IBGL	1,0	1,0	-10,40312	7,76E-6	7,70E-5	1,55E-6	1,36E-7
IBGL	0,1	1,0	-10,38829				
IBGS	-	-	-	-	-	-	-
IEEM	1,0	1,0	-6,713379				
IEEM	0,1	1,0	-6,719467	-3,10E-4	2,82E-4	6,37E-5	5,14E-6
IHYG	1,0	1,1	-3,825355				
IHYG	0,1	1,1	-3,832723	-1,43E-3	1,64E-3	1,53E-4	5,99E-5
IJPN	1,0	1,0	-7,954982	5,57E-4	2,17E-4	1,77E-5	1,37E-6
IMEU	0,0	1,0	-7,257321	-2,33E-4	3,93E-4	3,65E-5	3,36E-6
INAA	0,1	1,1	-8,127345	-1,83E-4	3,83E-4	8,04E-6	1,78E-6
INAA	1,0	1,1	-7,080148				
LQDE	3,0	1,1	-9,809857				
LQDE	3,1	1,0	-9,827306	-4,45E-7	1,93E-6	1,79E-6	1,59E-7
RWX	2,0	1,0	-6,690533	6,81E-4	5,05E-4	6,20E-5	5,74E-6
RWX	0,1	1,0	-6,669214				
STN	1,1	1,0	-8,238585	-9,34E-5	4,00E-5	1,24E-5	8,89E-7
STZ	1,0	1,0	-6,214845				
STZ	0,1	1,0	-6,215137	-3,85E-4	3,12E-4	1,09E-4	6,72E-6
TOPIX	1,0	1,0	-5,803026	6,16E-4	6,53E-4	1,37E-4	1,63E-5
XOP	0,0	1,1	-12,44596	-4,57E-3	2,01E-3	5,89E-5	3,70E-5

**Table 5.2:** Best fit models for each dataset in the low frequency data. Column two and three states the orders of the ARMA and GARCH parts respectively. The grey highlight points out the selected model for each process.

The model residuals are evaluated based on a Jarque-Bera test for normality [16], and a Ljung-Box test for independence in the residuals [17], as well as visual inspection of various plots of the residuals.

The p-values for the Jarque-Bera test are zero or extremely close to zero in all of the models, indicating that the hypothesis must be rejected and that the residuals are not normally distributed. The p-values of the Ljung-Box test for accepting independence in the residuals, using 95 percent confidence intervals, span from 0.98 to 0 in the selected models. However, performing a small test proves that the Jarque-Bera test is extremely sensitive to outliers. When generating 1000 random numbers from a nor-

mal distribution the hypothesis of normality is accepted, with a p-value of 0.24. However, when adding one single outlier to the same dataset, the null hypothesis is rejected with a p-value of 0.0091. Once again, a simple test proves that also the Ljung-Bow test is sensitive to outliers, although not as sensitive as the Jarque-Bera test. 10 data points are added to a 1000 point dataset to distort the independence. To emphasise the sensitivity of these tests, data is plotted in figures 5.4a and 5.4b, and the outliers are emphasised in red.



**Figure 5.4:** Sensitivity of residual tests

By observing the data series depicted in figure 5.1 it is apparent that every dataset contains a number of extreme events. In some series the outliers occur on a regular basis and are caused by quarterly distributions from the fund, while others occur irregularly and are caused by less obvious market effects. The method in question is not able to detect single outliers and thus these will be mirrored in the residuals. For this reason, not much attention is paid to the two test statistics, and the models are instead selected based on parameter significance, AIC and visual inspection of the residuals.

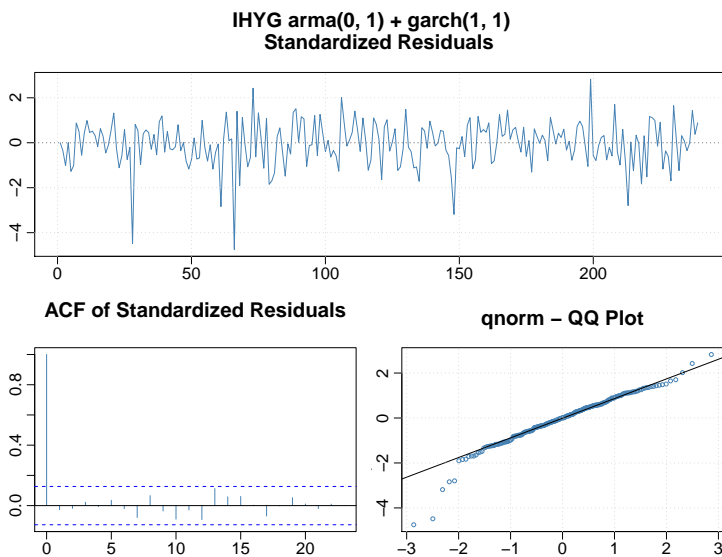
AIC provides a measure of the trade-off between size and goodness of fit in a statistical model. It is computed

$$AIC = 2k - 2\log(L)$$

where  $k$  is the number of parameters in the model and  $L$  is the maximised value of the likelihood function. The selection criteria is to pick the model with the smallest AIC.

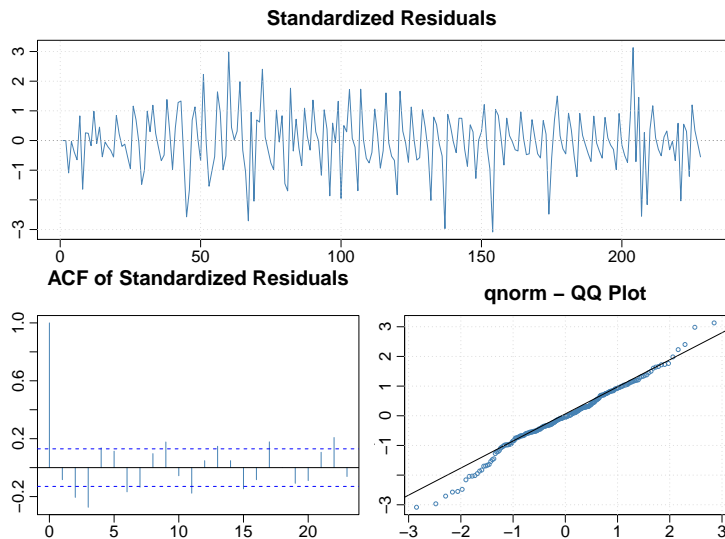


When inspecting the residuals attention is focused on a simple visualisation of the standardised residuals (top panel), the autocorrelation of the standardised residuals (bottom left panel) and a QQ-Plot of the standardised residuals (bottom right panel). The plots are inspected for patterns in the residuals which could indicate an inadequate model which fails to capture the structures of data. Firstly, figure 5.5 shows an example of a well fit model. No autocorrelation is left in the residuals and only few observations deviate from the QQ line.



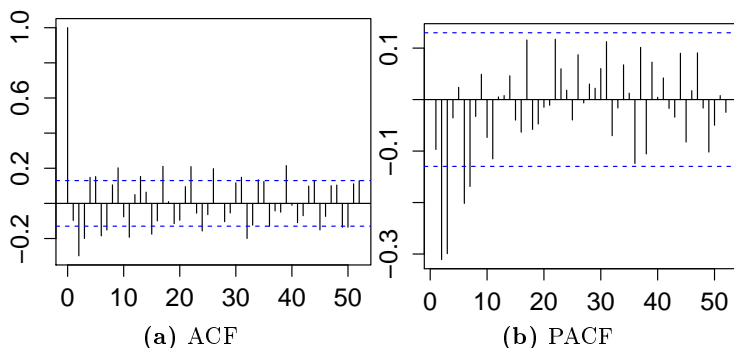
**Figure 5.5:** This series is well described by the model. The standardised residuals are uncorrelated and the residuals are largely unaffected by extreme observations.

Figure 5.6 shows the residuals of an inadequate model. Judged by the autocorrelation of the standardised residuals a seasonal trend seems to be present. But the applied R package cannot include seasonal trends. The depicted residuals come from an ARMA(2,0)-GARC(1,1) model of EMBI. Extending to model to an ARMA(3,3)-GARCH(1,0) – which is the highest obtainable order of the AR respectively MA parts – reduces the autocorrelation of the residuals, but does not correct for the seasonal trend.



**Figure 5.6:** This series is inadequately described by the model. The standardised residuals are correlated and show signs of a seasonal trend.

Extending the autocorrelation plot to include more lags reveals, in figure 5.7, that the seasonality persists, well beyond lag 3, explaining why the extended model did not suffice either.



**Figure 5.7:** ACF and PACF of model with seasonal trend, extended to lag 52 (one year). The plots show that the seasonality persists, thus disabling non-seasonal models from adequately describing data.

The remaining models all present residual plot similar to those of figure 5.5, although some models are more affected by outliers than others. Also some models remain with a small amount of autocorrelation in the residuals at lags 13-15, suggesting regular quarterly events. This is explained by quarterly distributions from the funds. As only a small amount of autocorrelation remains in these models, and an unambiguous explanation exists, no further action is taken to find a model structure which capture the quarterly trends. It thus remains up to the investor to take this into account.

In the fitted models FXC and LQDE have GARCH parameters summing to exceed one, and DGT, IHYG and XOP have GARCH parameters which sum exceed one if including the uncertainty. This suggest models which are not stationary, and contradicts tests confirming stationarity as well as plots showing no signs that the processes be non-stationary

Residual plots for all models along with coefficient matrices similar to those of tables 5.2a and 5.2b are given in appendix B.

## 5.4 High frequency data, results

The same analysis as described above applies to these datasets.

Data is depicted in figures 5.8, 5.9 and 5.10. The obtained model orders are depicted in table 5.3. STN and XOP show a negative mean deviance between the fund and the index returns, as was seen in the low frequency data.

12 models present with insignificant mean deviance, while no models are fitted with insignificant mean conditional variance and also no models are fitted with negative GARCH parameters. In the fitted models ELR, EMBI, IBGL, IEEM, IJPN and IMEU have GARCH parameters summing to exceed one, and DGT, FEZ, GLD, IHYG, STN, TOPIX and XOP have GARCH parameters which sum exceed one if including the uncertainty. This suggest models which are not stationary, and contradicts tests confirming stationarity as well as plots showing no signs that

the processes be non-stationary

Residual plots for all models along with coefficient matrices similar to those of tables 5.2a and 5.2b are given in appendix B.

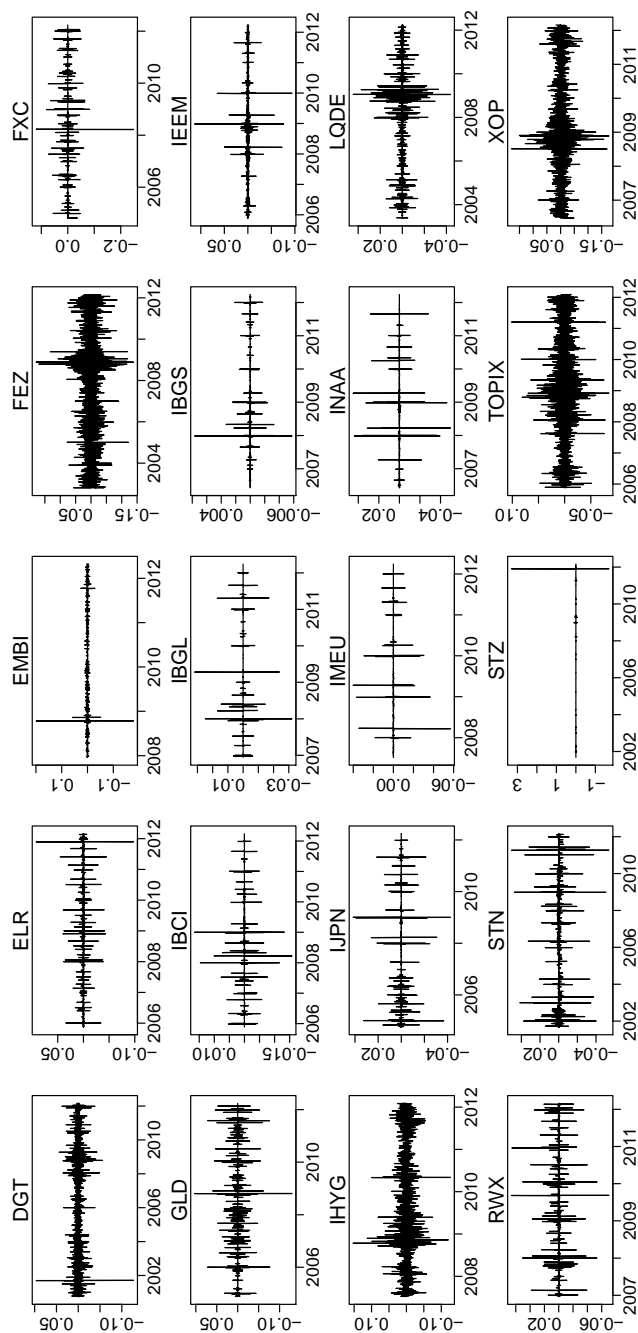
## 5.5 Discussion of deviance in returns

The increase in models fitted with GARCH parameter estimates exceeding 1 between the weekly and daily deviance in return processes is quite remarkable. This suggests non-stationary processes, although both datasets are confirmed to be stationary by KPSS test. The suspected cause of this behaviour is an increase in noise and extreme observations. This will be discussed further in the succeeding chapters.

In spite of the estimation challenges, there clearly is a structure to the way the funds deviate from their underlying indices in returns over time. This can widely be modelled by ARMA in the conditional mean and GARCH in the conditional variance. The models present with largely uncorrelated residuals but many outliers. QQ plots are of varying quality – in the worst case as few as one third of data points lie on the QQ-line (GLD weekly). On an overall view, approximately half or two thirds of the data points lie on the QQ-line. Further, all models are fitted with a mean deviance in the range of  $[-4.57e-3, 3.81e-3]$  on weekly returns and  $[-1.13e-3, 8.54e-4]$  on daily returns. This, however, is subject to reservations due to the mentioned challenges in fitting the models.

A large fraction of the weekly residual series present with a small amount of autocorrelation around lag 13-15. This periodic trend is due to quarterly distributions from the funds. The models presented in tables 5.2 and 5.3, as well as the model coefficient tables and residual plots listed in appendix Figure B, are the largest possible models, following the model selection guidelines described. As the example of low frequency EMBI showed, the way to capture the quarterly trend will be to include a seasonality in the models. This is not a feature of the applied R package.

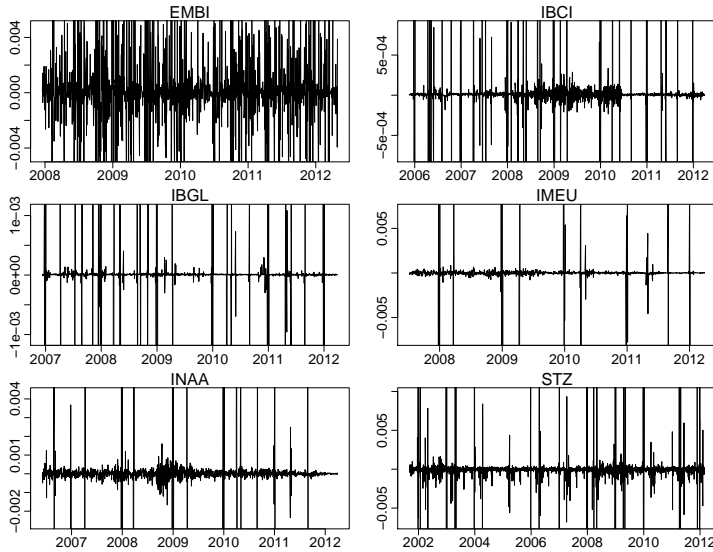
It is interesting to note that most of the 13-week autocorrelation is posi-



**Figure 5.8:** The difference in return between the fund NAV return and the index level return, as computed by (5.1), for each of the 20 series. The plots show clear examples of volatility clustering in several of the processes.

	ARMA (m,n)	GARCH (p,q)	AIC	$\mu$	$\sigma_\mu$	$\omega$	$\sigma_\omega$
DGT	1,1	1,0	-7,132836				
DGT	2,0	1,1	-7,204607	1,51E-4	1,07E-4	7,40E-6	1,13E-6
ELR	0,1	1,0	-7,871961	6,77E-5	2,37E-5	1,58E-5	2,08E-6
ELR	2,0	1,0	-7,834532				
EMBI	2,0	1,1	-8,333665				
EMBI	3,3	1,0	-8,589423	4,27E-4	3,14E-5	5,69E-6	4,27E-7
FEZ	3,0	1,2	-4,951145	8,54E-4	3,81E-4	1,25E-5	2,91E-6
FEZ	0,1	5,0	-4,900276				
FXC	0,0	3,0	-6,993581				
FXC	0,2	1,2	-7,055985	5,84E-5	3,98E-5	2,81E-5	2,36E-6
GLD	0,2	1,1	-7,19076	7,57E-6	4,78E-6	8,41E-7	3,52E-7
GLD	0,3	1,0	-7,189512				
IBCI	0,1	1,1	-11,4171				
IBCI	3,0	1,0	-11,44323	6,79E-6	1,86E-5	5,23E-7	1,92E-8
IBGL	2,0	1,0	-10,88397	4,80E-5	2,42E-5	8,35E-7	3,56E-8
IBGL	1,2	1,1	-10,88311				
IBGS	-	-	-	-	-	-	-
IEEM	4,0	3,0	-8,859649	2,52E-6	5,02E-5	1,17E-6	2,45E-7
IEEM	0,2	3,0	-8,764289				
IHYG	3,0	1,1	-5,330347	-1,22E-4	4,16E-4	4,13E-6	1,64E-6
IHYG	1,1	3,0	-5,172578				
IJPN	4,0	2,0	-9,162141	1,17E-4	5,18E-5	4,93E-6	1,67E-7
IJPN	0,2	1,0	-9,143911				
IMEU	0,4	2,0	-8,596118	1,73E-5	7,39E-5	8,01E-6	3,45E-7
IMEU	1,0	2,1	-8,578448				
INAA	0,1	1,2	-9,270186				
INAA	2,0	1,1	-9,272207	1,08E-5	5,82E-5	3,66E-6	4,07E-7
LQDE	3,2	2,1	-9,46963				
LQDE	0,2	1,1	-9,792716	1,56E-6	4,53E-6	3,00E-8	4,63E-9
RWX	1,1	1,0	-7,296514	-7,24E-6	4,28E-5	3,51E-5	1,49E-6
RWX	2,0	1,0	-7,286354				
STN	3,0	1,2	-8,968861				
STN	0,1	1,2	-8,96934	-1,12E-4	3,23E-5	3,84E-6	2,18E-7
STZ	1,0	1,1	-7,54554	3,00E-6	9,05E-5	2,34E-5	1,94E-6
STZ	0,1	1,0	-7,510444				
TOPIX	3,0	1,1	-6,082997				
TOPIX	0,2	1,1	-6,212725	9,02E-5	1,78E-5	5,42E-6	1,10E-6
XOP	3,0	2,0	-4,806823				
XOP	0,1	1,1	-4,942836	-1,13E-3	3,63E-4	4,84E-6	1,67E-6

**Table 5.3:** The table shows the best fit models for each dataset when the high frequency data. Column two and three states the orders of the ARMA and GARCH parts respectively. The grey highlight points out the selected model for each process.

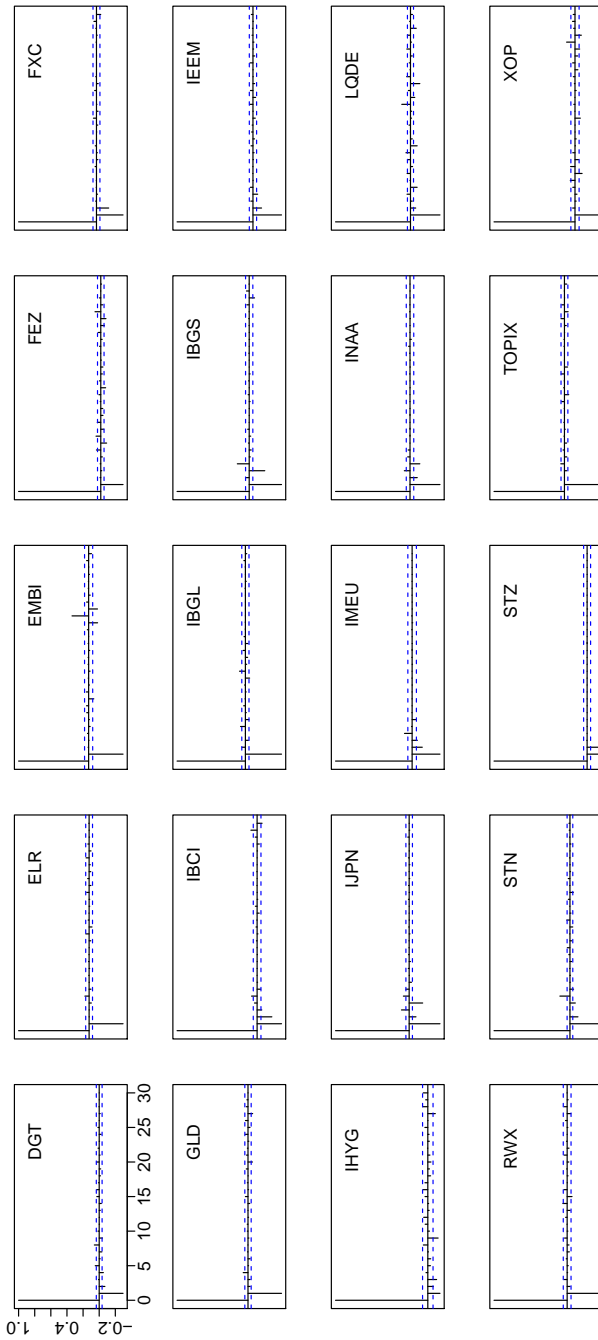


**Figure 5.9:** Zoom on six selected processes also displayed in figure 5.1. In this scale it is possible to see the volatility.

tive. In conjunction with the general negative or zero mean deviance, this shows that the return on the fund increase around the time the distributions are paid. As the funds assume distributions are reinvested, this is expected as the fund increases its value.

The residuals are of equal quality in both high and low frequency models. But the parameter estimation is generally more reliable in the weekly returns. In addition, for the purpose of long term investing as well as four week predictions it is more reasonable to consider weekly data. For this reason the remainder of the thesis deal only with weekly data.

The numerically small mean deviances show that the well fitted models can be used as proxies for funds, in estimations of future performance. Caution should be demonstrated regarding the models which were fitted with dubious GARCH parameters.



**Figure 5.10:** Auto correlation in the deviance processes. All series show significant, negative auto correlation in lag one. Some auto correlation is inherent in lag two in five of the processes.



## CHAPTER 6

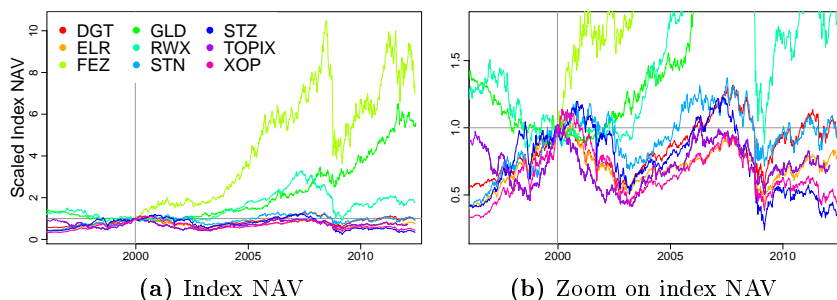
# Scenario generation

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The aim of this chapter is to test and apply a model for generating four week predictions of weekly fund returns. The scenarios are to be applied towards robust asset allocation using the funds.

It was concluded in chapter 5 that generally it is well-founded to apply the indices as proxies for the funds. Due to the lack of historic information on the funds, this proxy will be adapted, and the following analysis will be conducted on index returns. The indices for analysis are selected based on the amount of historical data it was possible to obtain. On this ground the nine indices DGT, ELR, FEZ, GLD, RWX, STN, STZ, TOPIX and XOP are selected and the considered period is December 24th, 1999 to February 3rd, 2012, that is 12 years of data and 633 observations of weekly return for each index. Amongst these DGT and XOP deviance processes were modelled with dubious GARCH parameters. Both were within limits in the estimate, but not when including the uncertainty. The indices are applied here after all, as the overall impression from [Figure 5](#) validate this.

The NAV progression of the indices is shown in figure [6.1](#), where it is also

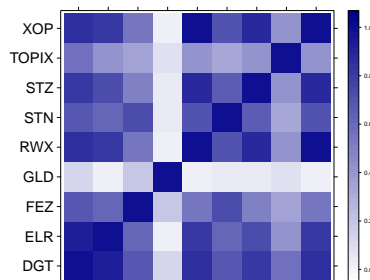


**Figure 6.1:** NAV of the nine selected indices. The left panel shows the full picture, while the right panel shown a zoom on the seven indices in the bottom of the lefts panel.

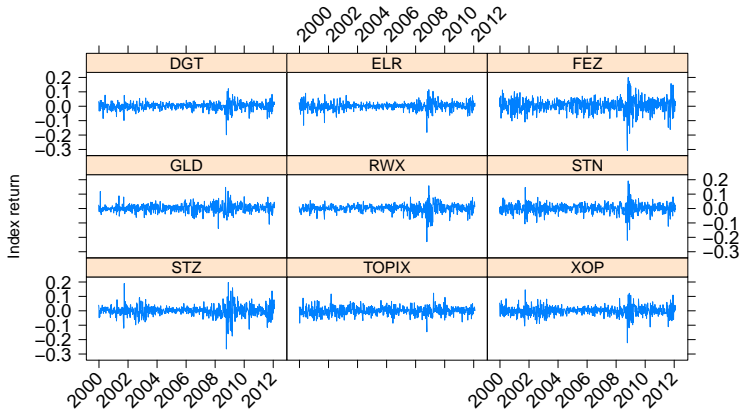
depicted that the considered period of time covers several up and down turns, thus providing a sound foundation for estimation. Return and ACF of returns is shown in figures 6.2 and 6.3. Indices DGT, RWX, STZ and XOP display signs of a seasonal trend of 13 weeks. RWX, STZ and XOP employ quarterly rebalancing of the indices. But this is also true for ELR and STN, without this being depicted in the autocorrelation of the index return, so the rebalancing is not the only reason for the observed seasonality. Measures as described in section 3.1 are taken and confirm mean stationariness in the returns with p-values  $\geq 0.1$ .

A correlation matrix of the index returns is shown in table 6.1 and further in figure 6.4. Figure 6.4 is highly similar to what was seen in figure 4.1, which is of cause expected, given the high correlation between the funds and the indices (c.f. table 5.1).

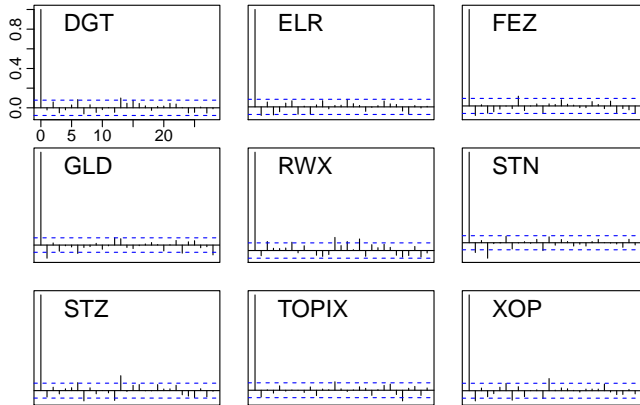
In the following, four frameworks will be applied towards scenario generation in the indices. First scenarios will be bootstrapped. This method was applied by [18] and [19] and will serve as base reference for the succeeding methods. Secondly an ARMA-GARCH framework similar to what was implemented in Figure 5 will be applied towards



**Figure 6.4:** Absolute correlation between the indices.



**Figure 6.2:** Return of the nine selected indices. The processes are seen to be mean stationary with volatility clusters primarily around the period of 2008–2009.



**Figure 6.3:** Autocorrelation of return series on the nine selected funds. DGT, RWX, STZ and XOP display some autocorrelation in lag 13, indicating a seasonal trend.

		1	2	3	4	5	6	7	8	9
1	DGT	1.00								
2	ELR	0.93	1.00							
3	FEZ	0.69	0.62	1.00						
4	GLD	0.12	0.02	0.20	1.00					
5	RWX	0.86	0.81	0.55	-0.01	1.00				
6	STN	0.68	0.61	0.72	0.04	0.71	1.00			
7	STZ	0.82	0.74	0.51	-0.05	0.89	0.67	1.00		
8	TOPIX	0.58	0.41	0.35	0.09	0.42	0.34	0.41	1.00	
9	XOP	0.86	0.81	0.55	-0.01	1.00	0.71	0.89	0.42	1.00

**Table 6.1:** Correlations of returns in the 9 selected indices over the observed period, from 1999-12-24 to 2012-02-03.

the index returns. Following this, a regime approach will be introduced, first in a Markov Switching Autoregressive model where the indices are considered individually and subsequently in a dependent mixture model where the covariance matrix of the indices is included.

## 6.1 Bootstrap

As a base reference scenarios are generated using simple bootstrapping. The scenarios are generated following the directions outlined below.

```

for(j in seq(0,floor(length(dates)/2),4))
{
  j.date = floor(length(dates)/2)+j
  index[,1] = sample(j.date,1,prob=dnorm(1:j.date,1,5), replace=T)
  for(t in 2:4)
  {
    index[,t] = sample(j.date,1,
      prob=dnorm(1:j.date,index[t-1],5), replace=T)
  }
}

```

The index of the existing return vector to replicate,  $\text{index}[,t]$  is sampled on the space  $[1, j.\text{date}]$  which is the length of the considered subset of data iteratively expanded by four observations. The sampling probability

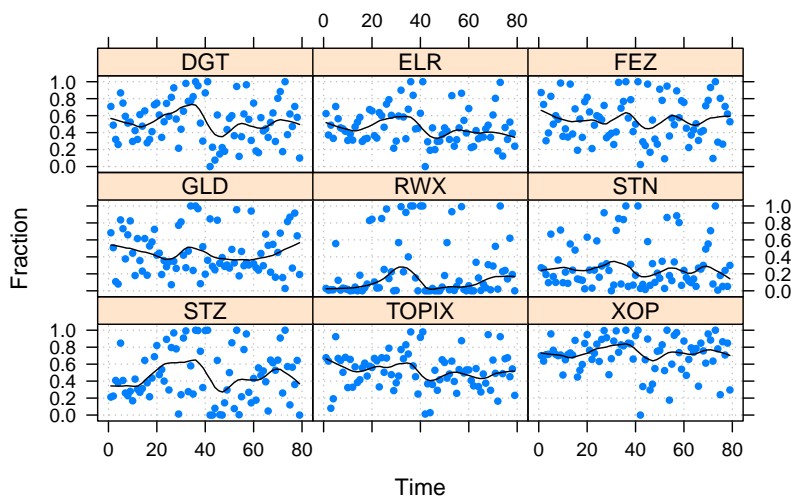
is specified to be normally distributed with mean equal to the previous index and standard deviation of five weeks. The optimal value of the mean and standard deviation in the bootstrapping procedure were examined, and determined to be as applied, by [18] and [19]. In each time period,  $j$ , 250 scenarios are generated.

### 6.1.1 Results

The scenarios are evaluated based on their distribution relative to the observed return in each period. From the four one-week predicted returns one four-week return is derived. This is compared to the observed four-week return over the same period. The aim of the asset allocation will not be on short term, thus the weekly returns are of little interest.

For each period and for each index 250 returns are generated. Referencing figure 6.5 the second axis shows which fraction of the 250 generated returns exceed the observed and along the first axis is the number of the relevant round of scenarios ( $1 = 2006-02-10$ ,  $2 = 2006-03-10$ , etc.). The plots are inspected for patterns indicating that the predictions are generally too high, too low or are not capturing the level changes in data. Assuming the generated scenarios and the observed returns follow the same distribution, that is the scenarios truly capture the dynamics of data, fractions should be uniformly distributed along the  $[0;1]$  space on the vertical axis. The black line is a linear smoothing of the points, and is shown to emphasise the patterns in data. The trendline should be flat, indicating that the scenarios at all times “pull” equally from the top and bottom.

For a more exact analysis of the scenarios, the fractions are examined in a variety of tests. Firstly the fractions are transformed from gaussian densities and the autocorrelation is examined to reveal patterns. Additionally the Kolmogorov-Smirnov test is performed to test for uniformity in the fractions and lastly the variance of the fractions is determined and compared to the theoretical variance of uniformly distributed numbers. The theoretical variance of uniform numbers is  $\frac{1}{12}(\max - \min)^2$  [20]. Since we are considering fractions,  $\max = 1$  and  $\min = 0$  implying a theoretic variance of  $\frac{1}{12} \simeq 0.0833$ . These measures are shown in figure 6.6 where the

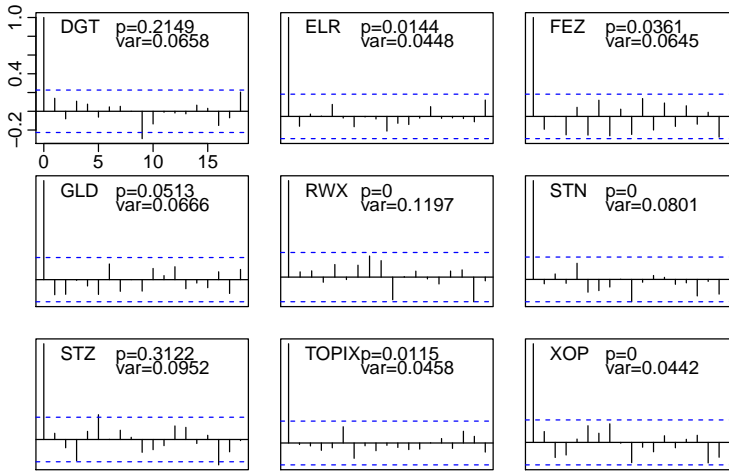


**Figure 6.5:** Fractions of scenarios which exceed the observed return (second axis) in each period (first axis).

ACF is plotted and the p-value for the Kolmogorov-Smirnov test along with the observed variance is written in the plots.

The results of bootstrapping the scenarios are clearly poor. As many as six of the nine examples are rejected by the Kolmogorov-Smirnov test for uniformity. GLD is accepted, but just barely and with a noticeably low variance. DGT and STZ are accepted as uniform, but the trendlines reveal a remarkable drop around period 40, and there are distinct white areas in the fraction plots. Period 40 is the four weeks trailing February 6th, 2009. In other words, these plots indicate that the bootstrap generated scenarios do not capture the economic crisis indicating a general inability to capture the changes in the economic environment.

A different way to illustrate and compare the scenarios distribution relative to the observed return is in a boxplot where the 79 periods of 250 scenarios are summarised in a boxplot with the usual marks. In figure 6.7 such a plot is depicted. It clearly shows that the distribution of the bootstrapped scenarios is largely identical over the 79 periods. The median and 25th and 75th percentile hardly change. It also shows that in the



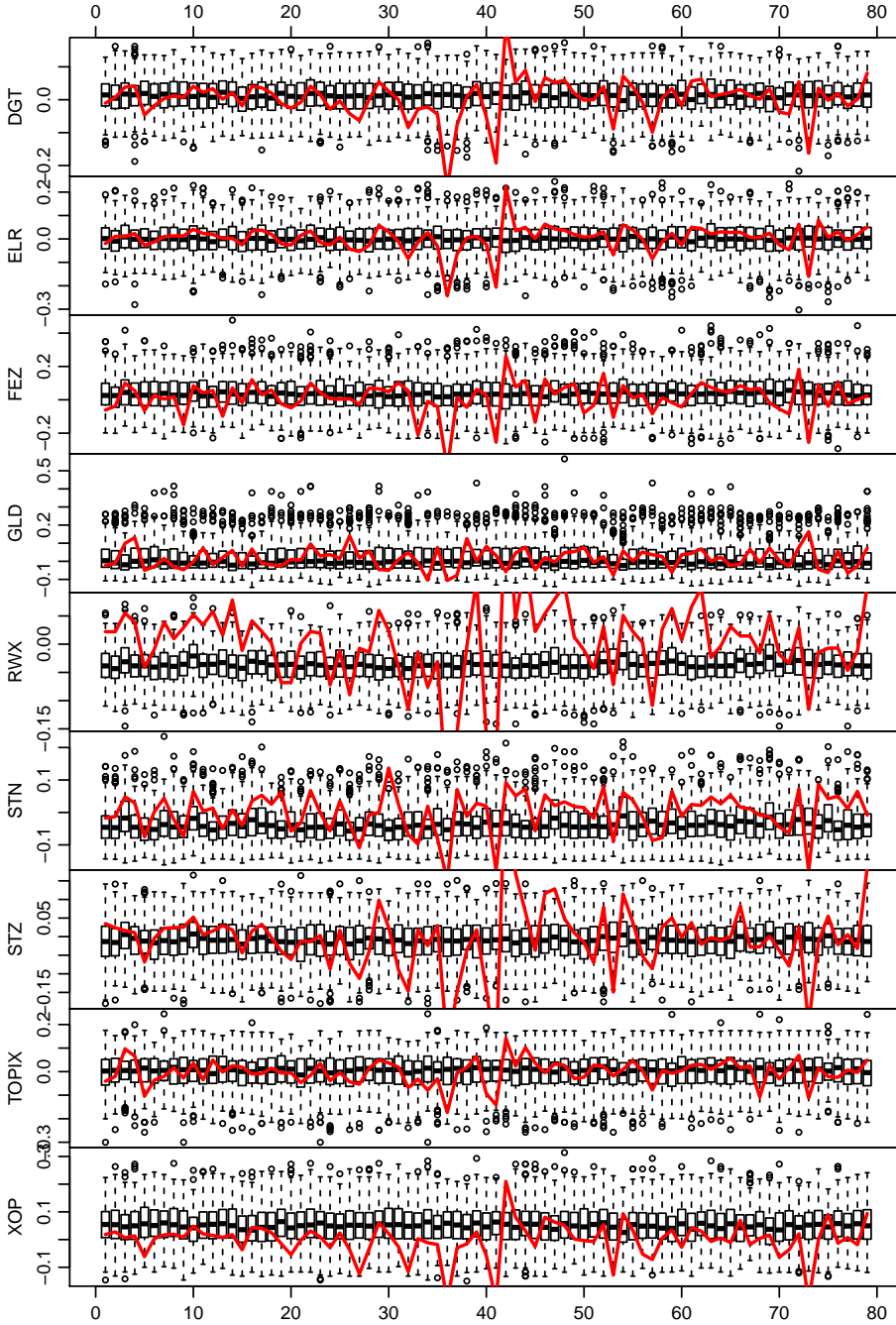
**Figure 6.6:** Autocorrelation of the 79 fractions along with p-value for test for uniformity as well as sample variance of the fractions.

worst case (RWX), the scenarios appear to have no coherence with the observed returns. In STZ the scenarios seem to have detected the mean return, but otherwise are in no way following the observed returns.

The conclusion is, that bootstrapping returns to obtain prediction of future returns will likely capture the mean, but only by random chance will it capture the periodic market fluctuations. Judged by the severity and duration of the crises in the considered period of time, implying return oscillation of considerable magnitude, held together with concurrent lack of adaptability in the scenarios, bootstrapping will be plain wrong in unstable times.

## 6.2 GARCH

In line with common practise within econometrics, and following the lessons from [Figure 5](#), the indices are also modelled in an ARMA-GARCH framework. Causes of action will be to determine the best fit model structure for each dataset, using the full length of data, and subsequently to generate a series of four week predictions using the chosen models.



**Figure 6.7:** Distribution of the 250 bootstrap scenarios in each period represented by boxplot. Shown along with observed return in red.



	ARMA (p,q)	GARCH (p,q)	Jarque Bera	Box-Ljung
DGT	0,0	1,1	0	0.9497
ELR	1,1	1,1	0	0.8725
FEZ	1,1	1,1	0	0.4342
GLD	2,1	1,1	0	0.3499
RWX	0,0	1,1	0	0.3561
STN	0,0	1,1	0	0.6499
STZ	0,0	1,1	0	0.6143
TOPIX	0,0	1,1	0	0.8659
XOP	0,0	1,1	0	0.7579

**Table 6.2:** ARMA-GARCH model structure of the nine series of index returns.

### 6.2.1 Models

Following the approach described in section 5.2 the resulting models are given in table 6.2. The models generally produce satisfactory residuals. However, RWX, STN and XOP leave a small but significant amount of autocorrelation in lag 13 in the residuals. This shows that the seasonal events in the indices are unaccounted for in the model, and thus transfers to the residuals. Referencing figure 6.3 these funds clearly have the most significant autocorrelation in lag 13.

### 6.2.2 Scenario generation

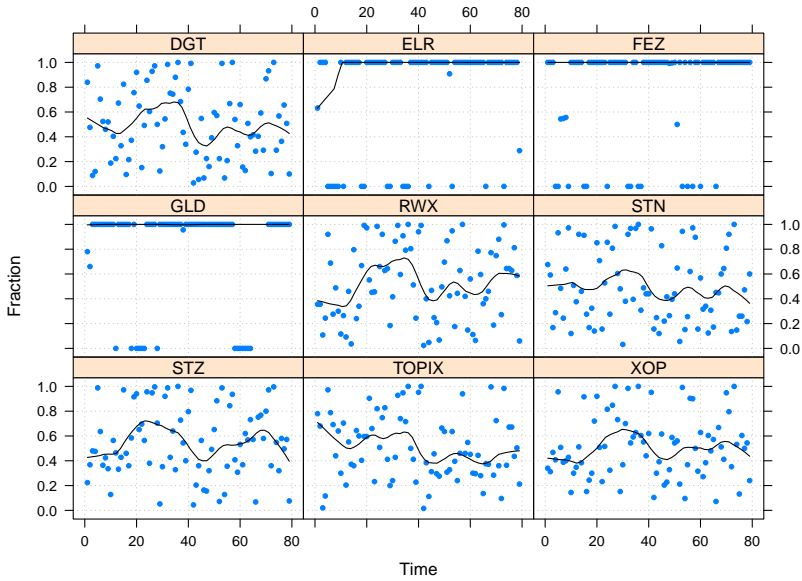
As mentioned previously this analysis aims at generating a series of four week predictions of the index returns. For each index the model structure outlined in table 6.2 is applied. The scenarios start half way through the observed dataset and are re-run for every four weeks going forward. That is, on the base date, January 13th, 2006, a number of four week predictions are made. The model parameters have at that point been re-estimated using only data available at the time, that is data from the period 1999-12-24 – 2006-01-13. The next round of predictions are started four weeks later, at February 10th, 2006 using data from the period 1999-12-24 – 2006-02-10. This repeats until the end of the observed dataset, resulting in 79 periods of 250 four-week predictions for each of the nine indices.

Because the dataset used to estimate the model parameters change along

the course, the best fitting model on the full set might not be the best fitting model along the course of the 79 subsets of data. The approach taken is to successively update the model parameters, but maintain the model order. This imposes some limitations on quality. Firstly the model order might not be correct for the current subset of data. The model may be too large, leaving insignificant parameters, or it may be too small, meaning a better fit and thus more accurate predictions could have been obtained. [21] also employs this approach to successive model estimation in a number of model structures, but uses AIC to select the model order in each consecutive estimation. He finds that successively updating the model order is generally not changing the model orders from the ones determined on the full dataset. It is the experience here that estimation error and subsequently execution failure do occur. This happens when the model order is wrong, causing the residuals to break the underlying assumptions of normality or leaving the Hessian non-invertable.

In the case of insignificant parameters or the possibility of an insufficient model, the obtained model is applied. The argument is that the model structure has been determined as optimal for the particular index. Further, parameters which are determined insignificantly different from zero will carry very small coefficients, and thus will not affect the predictions greatly. There are two cases of modelling errors. One result in a warning that the Hessian is singular and cannot be inverted to obtain the parameter uncertainty. This usually means that the obtained optimum is a saddle point, and thus not a true optimum. When this happens the estimation is re-attempted using a Nelder Mead [22] optimisation algorithm (lbfgsb+nm in R). The second type of error is complete failure to model data. In that case the parameter values obtained in the previous round are re-used. The argument for this is that the difference between two rounds is only four observations. Thus the estimate of one round will be a reasonable proxy for the estimate of the second round. In the special case that the model fails in the very first round, so that no prior model is available, a random walk with mean and standard deviation obtained from data is applied instead, thereby eliminating any ARMA or GARCH effects. This is reasoned by the original low model-orders, implying that if these models do not apply, a random walk will be a reasonable estimate.

In total 66 attempted model fits out of 711 fail and the Nelder-Mead



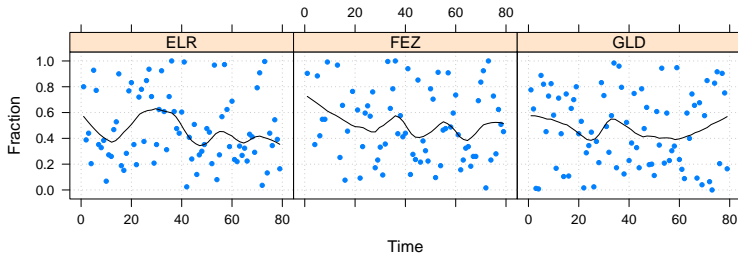
**Figure 6.8:** Fractions of scenarios which exceed the observed return (second axis) in each period (first axis). ELR, FEZ and GLD fail.

algorithm is applied. Of these 66, 34 attempts also fail under the Nelder Mead algorithm, and the model is substituted by its predecessor.

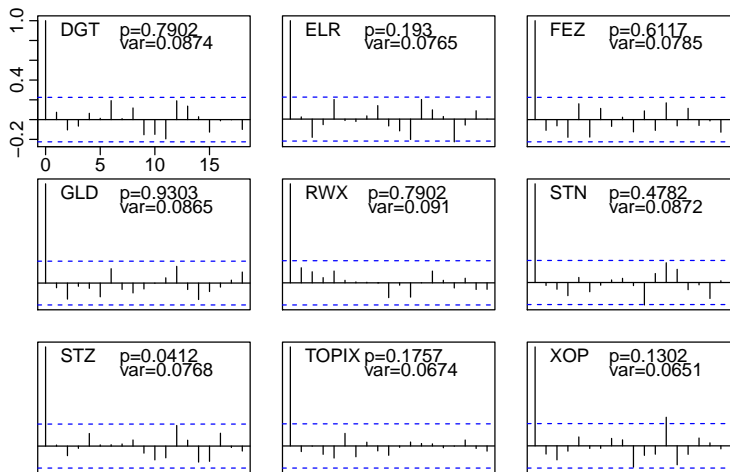
### 6.2.3 Results

Considering figure 6.8 clearly three of the models did not generate well fitting scenarios. Referring to table 6.2 the three models producing very poor scenarios are the ones estimated by the bigger ARMA-GARCH models. A study of the relevant models over the 79 periods reveal that only very few of the estimated model turn out to be a good fit for the relevant subsets of data. In the vast majority of periods only  $\beta_1$  – that is the GARCH parameter, is significant. This means that the model estimate is a random walk with zero mean and conditional variance relative to the previous ditto.

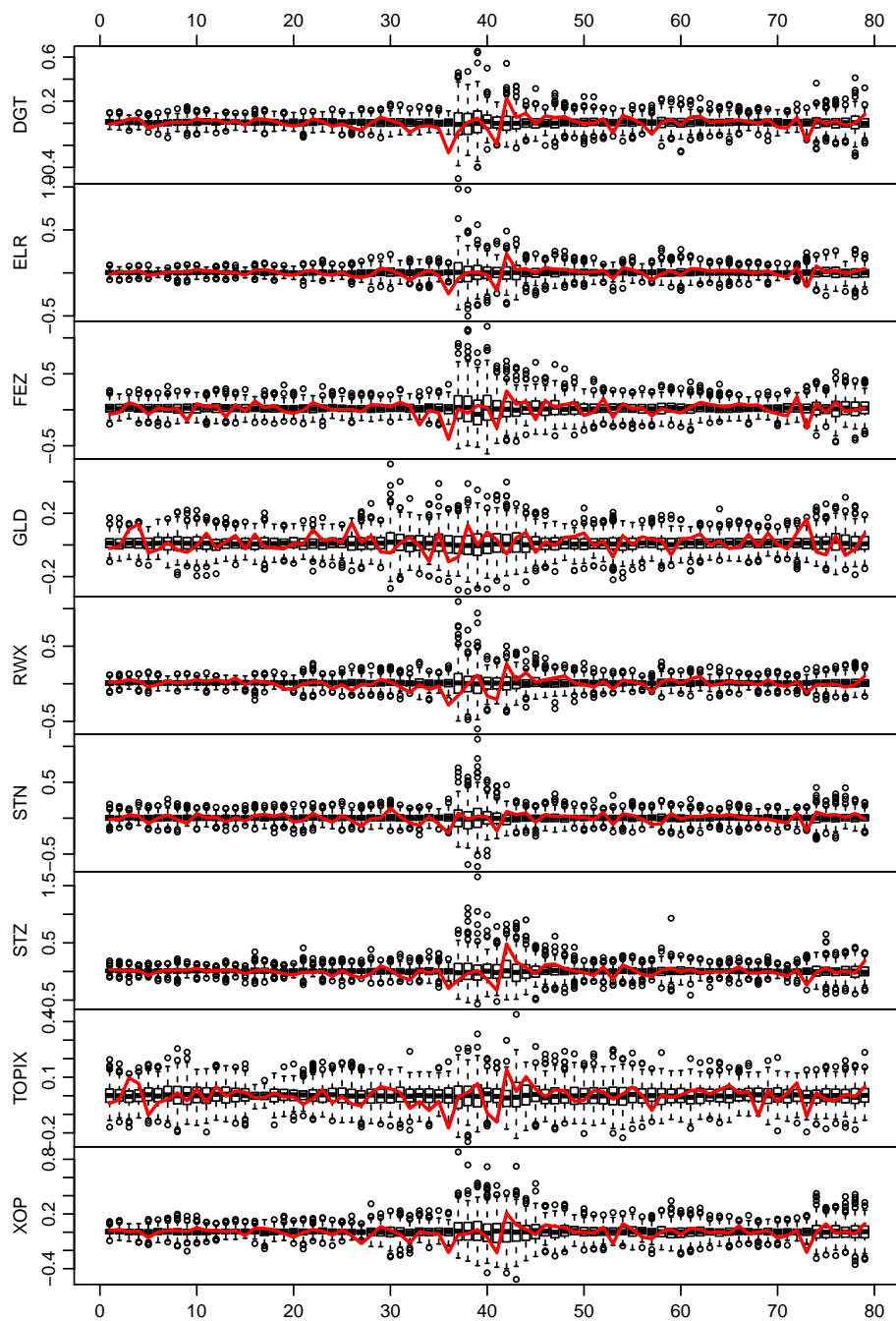
The results in figure 6.8 indicate that although the model structure out-



**Figure 6.9:** Fractions of scenarios which exceed the observed return (second axis) in each period (first axis). ELR, FEZ and GLD under reduced model.



**Figure 6.10:** Autocorrelation of the 79 fractions along with p-value for test for uniformity as well as sample variance of the fractions.



**Figure 6.11:** Distribution of the 250 GARCH scenarios in each period represented by boxplot. Shown along with observed return in red.

lined in table 6.2 proved to be the better fit on the full dataset, this is not the case for the subsets. For that reason the three indices are instead estimated in the smaller model ARMA(0,0)-GARCH(1,1), effectively the simplest version of a pure GARCH(p,q) process. This produce the fraction plots illustrated in figure 6.9 where the generated scenarios are seen to be nicely distributed over the whole interval.

Referencing figures 6.8 and 6.9 the smoothed curves lie in the area of 0.5, although all of the nine indices show a more or less radical drop around period 40. Further DGT, STN, STZ and XOP show a similar drop towards the end of the periods, corresponding to the increased volatility displayed in figure 6.2 in that same period.

According to the Kolmogorov-Smirnov test illustrated in figure 6.10 all except STZ produce uniformly distributed fractions. There is hardly any autocorrelation and the observed variance is in line with the theoretical, although low in TOPIX and XOP. ELR and TOPIX show a high density of fractions around 0.4 which is also reflected in the lowered variance. XOP is also clustering around the middle of the plot, and further shows signs of a trumpet shape in the scenario fractions. DGT and RWX seems to be nicely distributed, but the trendline shows a clear pattern i the fraction, that the scenarios do not respond to the financial downturn.

The boxplots of the scenarios are shown in figure 6.11. The result of the GARCH effect is clearly depicted. The range of the scenarios clearly increase in unstable times. The situation in the financial markets over the past 12 years is accurately depicted in the scenarios, where very low volatility is observed prior to 2008 after which the same level of stability is never completely restored. In addition most of the indices show beginning increase in volatility towards the end of the considered period, accurately depicting what is seen in figure 6.2.

There seem to be a tendency towards a small delay in the volatility of the scenarios to increase, relative to when the observed return begin to show signs of financial distress. In seven of the indices there is a steep drop in the observed returns at time 36. This is captured at time 37 by the model fits, where the drop triggers an increase in the conditional variance. In the affected models the subsequent graduate decrease of

scenario-volatility follows the observed returns.

#### 6.2.4 Discussion

4.7 percent of the model fits failed and were replaced by previous models. The most common error is negative  $\omega$  and  $\alpha$  parameters, causing the conditional variance to be negative. This impose inaccuracy in the scenario generation. Still the scenarios are following the observations nicely, judged from the boxplot, but perhaps too tight judged by the fraction plots. These show that still a noticeable amount of structure is undetected by the scenarios. The conclusion is that while volatility does follow data the delay has implications. Also, although the volatility increase, the median of the scenarios is more or less constant over the periods, because the mean of the generating model is constant. This means that the oscillations in return can only be captured by a small fraction of the scenarios.

An thorough analysis of selected failed models is enclosed in appendix [Figure A.6](#).

### 6.3 Considering data as classified into regimes

The volatility clusters detected in figure 6.2 inspired the application of GARCH methodology to capture the non-constant variance. In the following a different approach will be applied. Instead of capturing the volatility clusters with the adaptive learning approach of GARCH, the perspective is altered and instead data is looked upon as regime dependent, where the regimes have different characteristics. As such separate models are fitted in each regime. Several studies [18] [19] [21] [23] [24] [25] have employed this approach towards similar data<sup>1</sup>. In choosing the

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<sup>1</sup>[23] describes a dataset with characteristics similar to what is witnessed in the present data. [24] models the US GNP and [21] consider the unemployment rate and argues it is an important business cycle indicator. As such both are considered structurally similar to the business cycles modelled in this thesis. [25] consider real estate prices which are assumed similar to RWX.

means of how to switch between regimes the most common practise in the literature is to compare a Self-Excited Threshold Autoregressive (SETAR) model with that of a Markov Switching Autoregressive (MSAR) approach. The most fundamental difference between the models is that the state variable is not observed in the case of the Markov model, but must be specified in advance in the SETAR model. A priori specification of a switching variable could be viewed as either an advantage or a disadvantage of the SETAR formulation. Either there may not be a single variable which determines the state of the series, or this state variable may not be readily observable. Additionally, in the case of a univariate time series the regime shifts are necessarily determined by past realisations of the process. It should be noted, though, that if we can specify an appropriate state variable, we will know with certainty which regime the system is in at any given time, rather than merely having some probabilistic knowledge of it, as in the Markov switching framework.

[24] and [25] find that the methods perform equally well in describing and predicting data, while [23] find that MSAR is superior to SETAR. To avoid the problem of defining the threshold for regime shifts this study will proceed only with the MSAR framework.

## 6.4 MSAR

The MSAR model provides a framework for modelling data as independent AR processes in the regimes and assumes that an unobserved underlying process governs the switching. For an  $R$  regime model with data in each regime characterised by an  $AR(m_R)$  process, the  $MSAR(R; m_1, \dots, m_R)$  model is given by

$$y_t = \mu_{s_t} + \sum_{i=1}^{m_{s_t}} \phi_{i,s_t} y_{t-i} + \sigma_{s_t} \epsilon_t \quad (6.1)$$

where  $\epsilon_t \sim N(0, 1)$  and  $\sigma_k^2$  is the variance of the noise process in the  $k$ th regime and  $s_t$  is the sequence of regimes. Thus the regime at time  $t$  is determined only based on the regime at time  $t - 1$ , meaning that

$$P(s_t = j | s_{t-1} = i, s_{t-2}, \dots, s_0) = P(s_t = j | s_{t-1} = i), \quad \forall i, j, t$$



For the two-regime case this corresponds to the transition matrix

$$\mathbf{P} = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}$$

where  $p$  denotes the probability of staying in regime 1 once there,  $1-p$  subsequently corresponds to leaving regime 1. In a similar fashion  $q$  is the probability of staying in regime two once there, and  $1-q$  is the probability of transitioning to regime one from regime two.

The parameters are estimated using the EM algorithm, which is a two-step algorithm where likelihood maximisation is obtained through alternating between determining the expectation of the regime sequence, given the parameters, and adjusting the parameters to maximise the likelihood, given the expected regime sequence. If  $\Theta = (\theta_1, \theta_2, \theta_3)$  is the general parameter vector consisting of three sub-vectors with parameters for the prior model, transition model, and response models respectively the joint log-likelihood is given by

$$\begin{aligned} \log P(\psi_n, S_n | \Theta) &= \log P(s_1 | \theta_1) + \sum_{t=2}^n \log P(s_t | s_{t-1}, \theta_2) \\ &\quad + \sum_{t=1}^n \log P(y_t | s_t, \theta_3) \end{aligned} \quad (6.2)$$

where  $S_n$  is the vector of regimes assigned to each time  $t \in 1, \dots, n$  and  $\psi_n = y_1, \dots, y_n$  and  $\theta_{1,j} = \pi_j$ ,  $\theta_{2,j} = \mathbf{P}$  and  $\theta_{3,j} = (\mu_j, \phi_j, \sigma_j^2)$  where  $\pi$  is the initial state probability,  $\mathbf{P}$  contain the transition probability matrix and  $(\mu_j, \phi_j, \sigma_j^2)$  contain the AR model parameters.

The marginal log-likelihood of the observations is expressed by

$$l_T = \sum_{t=1}^n \log (P(y_t | \psi_{t-1})) = \sum_{t=1}^n \log \Phi_t \quad (6.3)$$

where  $\Phi_t = \sum_{i=1}^R \varphi_t(i)$  and

$$\begin{aligned} \varphi_1(j) &= P(y_1, s_1 = j) = \pi_j \mathbf{b}_j(y_1) \\ \varphi_t(j) &= P(y_t, s_t = j | \psi_{t-1}) \\ &= \sum_{i=1}^R (\varphi_{t-1}(i) \mathbf{P}_{ij} \mathbf{b}_j(y_t)) \times (\Phi_{t-1})^{-1} \end{aligned} \quad (6.4)$$

and

$$\begin{aligned}\pi_j &= P(s_1 = j) \\ \mathbf{b}_j &= -\frac{1}{2} \log(\sigma_j^2) - \frac{1}{2\sigma_j^2} \epsilon_t^2(\boldsymbol{\theta}_{3,j})\end{aligned}\quad (6.5)$$

$$\begin{aligned}\epsilon_t(\boldsymbol{\theta}_{3,j}) &= y_t - E(\hat{y}|\psi_{t-1}, \boldsymbol{\theta}_{3,j}) \\ \mathbf{P}_{ij} &= P(s_{t+1} = j | s_t = i)\end{aligned}\quad (6.6)$$

In these expressions  $\mathbf{b}_j(y_t)$  provides the conditional densities of observations,  $y_t$  given regime  $j$  and is supplied by [26],

$$E(\hat{y}|\psi_{t-1}, \boldsymbol{\theta}_{3,j}) = \mu_{s_t} + \sum_{i=1}^{m_{s_t}} \phi_{i,s_t} y_{t-i}$$

In (6.3)  $P(y_t|\psi_{t-1})$  reduces to  $P(y_t|y_{t-1})$  because of the Markov property [27] and  $n$  denotes the number of observations. In (6.4)  $j$  is the regime and runs from 1 to  $R$ . In this model we consider two regimes, so  $R = 2$ .  $s_t$  is the regime at time  $t$ .

$\mathbf{b}_j$  in expression (6.5) is not directly maximised in the applied algorithm, but it can be shown that the applied algorithm asymptotically maximises that likelihood. For a detailed walk-through of the parameter estimation process please refer to [23] or [28].

### 6.4.1 Models

To model data in the MSAR framework a library of functions has been supplied by Pierre-Julien Trombe, PhD student at Department of Informatics and Mathematical Modelling at the Technical University of Denmark, to which minor modifications have been made. The functions applied do not yield the model residuals nor the fitted model. It was beyond the scope of this thesis to alter the existing functions in such a way as to supply this. For that reason the criteria for model selection had to rely on other parameters than residuals. In selecting the models emphasis is put first and foremost on model BIC. BIC provides a measure of the trade-off between size and goodness of fit in a statistical model. It is computed

$$BIC = \log(n)k - 2 \log(L)$$

Index	AR(m,n)	Intercept	BIC	Exp. duration	Par. significance
DGT	0,0	x	-5236,286	12.50857 63.87707	int 2
ELR	0,0	x	-5155,773	58.94034 78.02196	int 2
FEZ	0,0	x	-2238,147	28.39696 230.35163	int 2
GLD	0,0	x	-5525,087	36.58532 19.04899	int 2
RWX	0,0	x	-4468,917	17.43270 31.74602	int 2
STN	0,0	x	-3756,508	9.412034 116.849997	int 2
STZ	0,0	x	-3763,724	60.44992 30.81178	int 1
TOPIX	1,1		-4967,782	22.68193 12.62767	AR(1)1
XOP	0,0	x	-5499,034	40.10461 70.62847	int 2

**Table 6.3:** Selected models in each index. Column three indicate whether the model was fitted with different intercepts in the two regimes.

where  $k$  is the number of parameters in the model and  $L$  is the maximised value of the likelihood function of the relevant model and  $n$  is the number of observations. BIC differs from AIC in the penalty put on model size. For samples larger than 75 data points  $\log(n) > 2$  and thus BIC penalises model size harder than AIC. BIC is selected in evaluation of the MSAR models based on the learnings from [Figure 6.2](#) which point towards a general tendency that smaller models perform better through out the subsets of data. As with AIC the selection criteria is to pick the model with the smallest BIC. [Table 6.3](#) show the selected models. The process of model estimation is illustrated in [table A.2](#) in appendix [Table A.7](#) represented by a large subset of rejected models.

Apart from BIC also parameter significance and the transition matrix is considered. From the transition matrix it is possible to derive the expected duration of stays in each regime. This is computed by

$$\left(1 - \begin{pmatrix} p \\ q \end{pmatrix}\right)^{-1} = \begin{pmatrix} \text{length of expected stay in regime 1} \\ \text{length of expected stay in regime 2} \end{pmatrix}$$

where the result is interpreted in weeks. The idea is to consider the economic sense in the expected duration of stays. As it turns out, within each set of data this is very similar.

The last consideration in this model selection is parameter significance. Parameter significance is computed based on the Hessian of the optimisation function at the estimated maximum. Parameter standard deviation is derived from the diagonal of the Hessian and the confidence intervals

are determined the usual way by subtracting respectively adding a multiple of 2 standard deviations to the estimate. The estimates which are significantly different from zero in the confidence intervals are given in the rightmost column of table 6.3. Generally the selected models are ARMA(0,0) models with one intercept significantly different from zero. The exception is TOPIX which is best modelled by an AR(1) process is in both regimes, each with zero mean. For TOPIX the preferred model is

$$R_1 : y_t = -0.002433 y_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, 0.04124)$$

$$R_2 : y_t = 0.003017 y_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, 0.01797)$$

$$\mathbf{P} = \begin{pmatrix} 0.9559 & 0.04409 \\ 0.07919 & 0.9208 \end{pmatrix}$$

The difference between the regimes lie both in the AR parameter and the variance of the error term. The high volatility regime has negative AR parameter meaning that the return shift sign every period. This corresponds to a high volatility, and also makes sense in relation to unstable financial markets. The transition matrix reveals that this index is expected to remain in the calm regime for approximately twice as long as the regime representing economic instability.

The remaining eight indices are modelled with intercept and without AR dependence. Below is shown an example of such a model, here exemplified by ELR

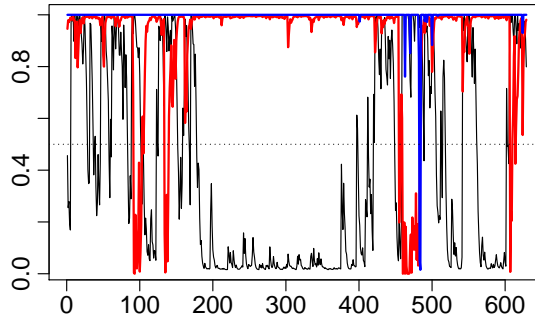
$$R_1 : y_t = \epsilon_t, \quad \epsilon_t \sim N(0, 0.03777)$$

$$R_2 : y_t = 0.002341 + \epsilon_t, \quad \epsilon_t \sim N(0, 0.01560)$$

$$\mathbf{P} = \begin{pmatrix} 0.9830 & 0.01697 \\ 0.01282 & 0.9872 \end{pmatrix}$$

The high volatility regime has zero mean, while the other regime is less than half as volatile and has positive mean return. The transition matrix tells that the duration of a stay in each regime is approximately equally long, with a favour towards the calm regime. The sign of the mean return in each regime differs between models.

Column five in table 6.3, shows that the indices respond differently to the financial environment. FEZ and STZ hardly use the second regime while ELR uses both regimes with almost equal emphasis. Figure 6.12 illustrates three different examples. The lines show the estimated probability



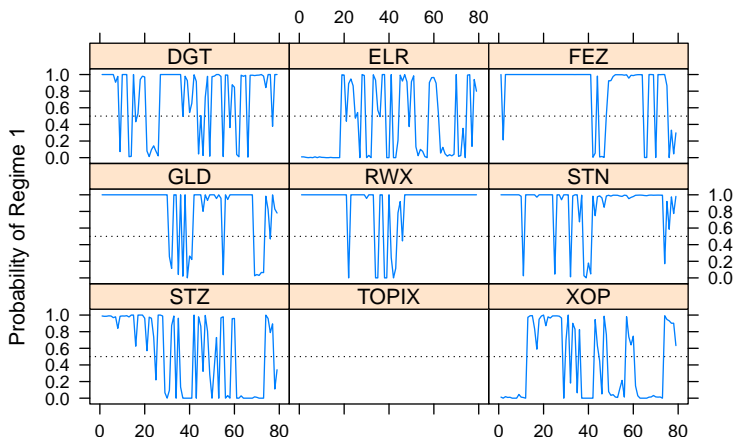
**Figure 6.12:** The lines show the estimated probability of being in regime 1 at time  $t$ . Red is STN, blue is STZ and black is ELR.

of the model being in one regime at time  $t$ . Thus the black line shows a model which is sensitive towards the market dynamics and thus jump between regimes in unstable periods (ELR). The red line on the other hands illustrate a semi-stable index which is largely unaffected by market instability (STZ). The blue line shows an index which is almost completely unaffected by the market dynamics of the depicted period (STN). STN is the European financial sector. The blue line indicates that the financial industry generally is unaffected by the market dynamics, or at least it is unaffected to an extend where the mean return is not significantly altered unless in very extreme events.

### 6.4.2 Scenario generation

As in the previous frameworks, a model is fitted to data each four weeks, every time using all past observations. Following the model fits the model parameters are extracted for scenario generation.

From the ending values of the fitted model, predictions are made by generating values in (6.1). The first regime is sampled with probabilities  $(p, 1 - p)$  of being regime 1 respectively 2 where  $p$  is equal to the probability of the fitted model ending in regime 1 as extracted from the model. The initial regime is used to determine the parameters used to generate the first prediction. Subsequently the following regime is sampled as a function of the current regime, now using the estimated transition prob-



**Figure 6.13:** Ending regimes of the 9 indices over the course of the 79 periods.

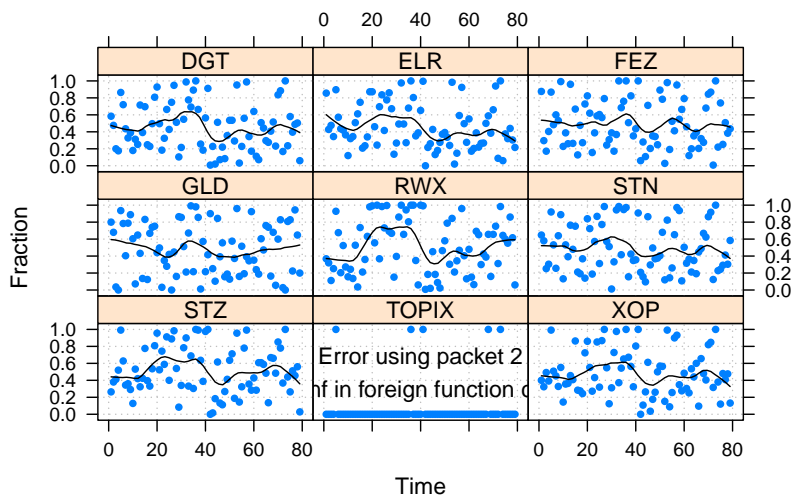
abilities, also extracted from the fitted model. The new regime is used to make the next prediction, etc.

The inferred ending regime of the 79 models fitted to each index is shown in figure 6.13. The regime sequence jumps back and forth. Some indices are more widely affected than others, but overall the models are fairly certain of the current regimes to pass to the scenario generation, that is, only few models end in a stage where the current regime is determined with less than 80 percent certainty. This implies that the scenarios are very likely generated from the depicted regimes through out the periods.

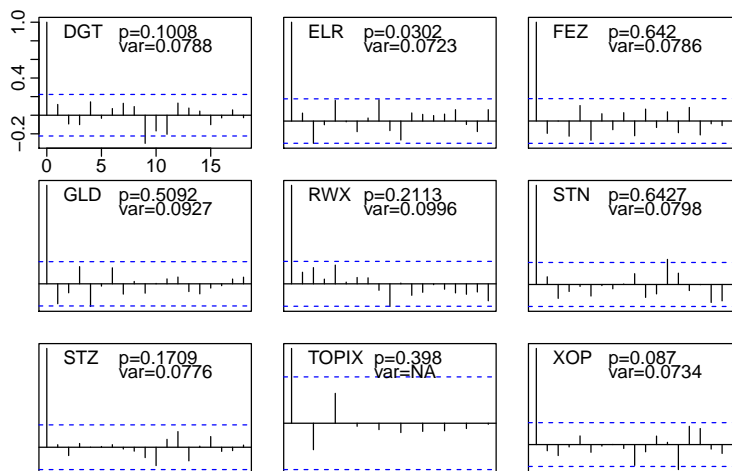
### 6.4.3 Results

The fraction of scenarios that exceeds the observed return is plotted in figure 6.14. There was an error in the scenario generation on the TOPIX index, making that plot invalid. TOPIX follows a different model structure than the other eight indices. Under the time restrictions of the project the cause of error was not detected.

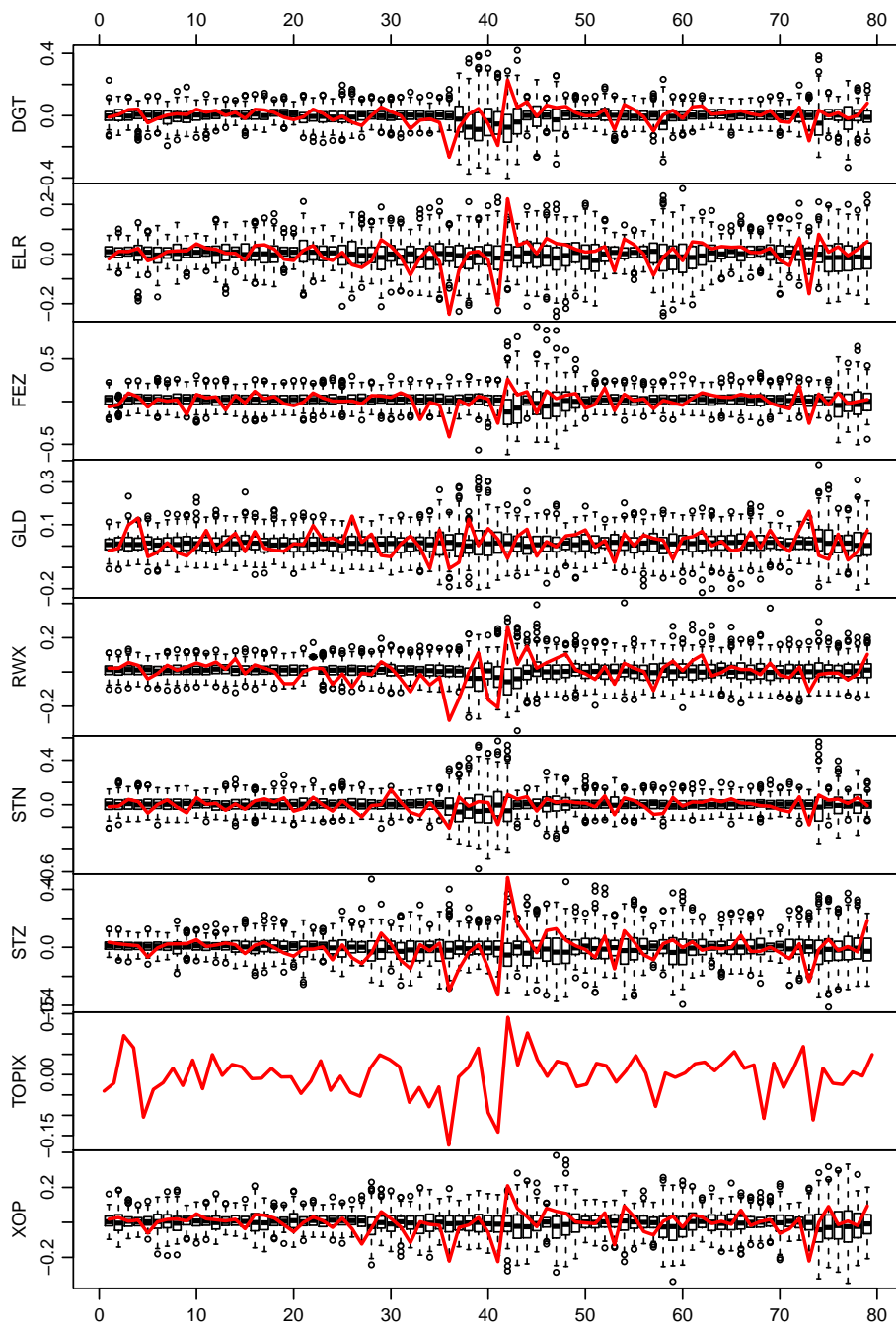
Considering figure 6.14 DGT looks evenly distributed, but with a steep drop in the trendline around period 40. ELR and XOP both favour the bottom half of the  $[0,1]$  interval and both show a tendency towards



**Figure 6.14:** Fractions of scenarios which exceed the observed return (second axis) in each period (first axis). TOPIX fail.



**Figure 6.15:** Autocorrelation of the 79 fractions along with p-value for test for uniformity as well as sample variance of the fractions.



**Figure 6.16:** Distribution of the 250 MSAR scenarios in each period represented by boxplot. Shown along with observed return in red.



trumpet shape, where the fractions cluster around 0.5 in the left side of the plots. This behaviour cause the Kolmogorov-Smirnov test to reject uniformity in ELR and only barely accept it in XOP. The scenarios of RWX are accepted as uniform, although they seem heavily biased towards one regime, causing a concurrent overrating of the returns during the housing crisis, periods 15–40. RWX also show a variance 20 percent greater than the theoretic variance.

The boxplots in figure 6.16 show that the scenarios generally encapsulate the observed returns prior to 2008. The range of the scenarios is overall increased after 2008, but in the individual assets the volatility increase does not perfectly reflect the times of change in the observed returns. The 2008 crisis is well marked in most indices, but other regime shifts are less distinct. The median of the scenarios is non-constant and adjust to reflect the decreasing mean return during times of high volatility. But a general delay in the scenarios means that the models in which the scenario median decreases, the effect is diminished anyway. In the case of RWX the boxplot shows that the model fits take too long to identify the regime shift, causing the scenarios to be generated from the wrong regime. Judging by the range of the scenarios the model fits have ended in regime 2 only in periods 38–44, while the observed returns indicate a highly volatile period starting around period 28 and not ending until approximately period 50. This is also reflected in the fraction plots. The boxplot also reveals that the regime shift in FEZ is delayed and the volatility increase overstated, in STN it is overstated and in STZ it is insufficient. Holding together figures 6.13 and 6.16 and considering STN, the prior figure shows that the ending regimes are uncertain towards the end. The boxplot shows that in the last 7–8 periods the scenarios have been generated from alternating regimes, with clearly different variance.

#### 6.4.4 Discussion

Referring to figure 6.12 ELR is seen to be “indecisive” regarding one regime, which is also reflected in the boxplot. Although figure 6.12 only shows the regime shifts estimated in the last period, the pattern applies though out the subsets. The result of this is clearly illustrated in figure 6.13 where in particular ELR is seen to change regime basically every period. A part

of the explanation for that is likely found in changing parameter values, so simply the labels of the regime change. This claim is supported by the boxplot in which a certain change in volatility is present in the models, but yet not as bad as figure 6.13 might suggest. In further some of the indices, data seem to be well fitted, GLD and DGT in particular. Yet in others the increase in volatility is delayed, or the transition probabilities appear to be erroneously estimated, causing the model fits to switch regime too often. A third sign of inadequacy is that the variance terms in the regimes are overstated (STN) or understated (STZ).

When the effect of figure 6.13 is not clear in the other figures it is assumed that the parameter values adjust correspondingly, such that effectively only the label of the regimes change, but the applied parameters – mean and variance – remain largely unaffected. Additional plots which support this claim are attached in appendix section A.8.1.

Rothman1998 examined regime shifting models in [21]. Under successive model estimation, similar to what is carried here, he found that while most of the examined methods were unaffected in the optimal model orders, one systematic change did occur for the SETAR model. As the sample size available for estimation increased, there was variation in the value of the threshold identified. Rothman finds that the threshold for changing regimes change. In MSAR there is no threshold to determine the regime shifts, but some parallels can possibly be drawn to the continuously adjusting regime parameter values of these models.

## 6.5 Dependent mixture model

The MSAR framework is often applied towards modelling a single time series [23] [24] [21] [25] [29] [30], yet as the purpose of this analysis is to generate scenarios for asset allocation, the correlation between the indices is of great importance and cannot be ignored.

The dependent mixture model, as applied in this section, models the indices simultaneously. As such it is able to fit the transition model while allowing for time dependent covariance matrices. Since it was concluded

in the previous section that when applying regimes, the AR effect disappears (in all but one instance), now both regimes are modelled as a multivariate normal distribution. In the multivariate normal distribution the mean and variance structure in each regime is determined by

$$\begin{aligned}\boldsymbol{\mu}_R &= E(\mathbf{Y}_R) \\ \boldsymbol{\Sigma}_R &= Cov(\mathbf{Y}_{Ri}, \mathbf{Y}_{Rj}), \quad i = 1, \dots, 9, \quad j = 1, \dots, 9\end{aligned}$$

where  $\mathbf{Y}_R$  is a vector of observations belonging to regime  $R$ .  $\boldsymbol{\mu}_R$  is a vector of nine elements and  $\boldsymbol{\Sigma}_R$  is a  $9 \times 9$  covariance matrix.

The fundamental assumption of the dependent mixture model is that at any time point, the observations are distributed as a mixture with  $R$  regimes, and that time dependencies between the observations are due to time-dependencies between the mixture components, that is transition probabilities between the components. These latter dependencies are assumed to follow a first-order Markov process on the finite space  $(1, \dots, R)$  [27].

The parameters in the dependent mixture model are estimated by the EM algorithm also mentioned in section 6.4. Expression (6.2) gives the joint log-likelihood of the observed data and the estimated regimes, given the parameters. This is the same method as described above, with a modification to the parameter vector as well as the the observation densities,  $\mathbf{b}_j(\mathbf{y}_t)$  which are now given by the expression

$$\mathbf{b}_j(\mathbf{y}_t) = -\frac{1}{2} \log |\boldsymbol{\Sigma}_j| - \frac{1}{2} \boldsymbol{\epsilon}_t^T \boldsymbol{\Sigma}_j^{-1} \boldsymbol{\epsilon}_t$$

which is the likelihood function for the multivariate normal distribution, where in the usual notation

$$\boldsymbol{\epsilon}_t = \mathbf{y}_t - E(\hat{\mathbf{y}}_t | \boldsymbol{\psi}_{t-1}, \boldsymbol{\Theta}) = \mathbf{y}_t - \boldsymbol{\mu}_j$$

Referring to (6.3), (6.4) and (6.5) the observations,  $\mathbf{Y}$ , is now a matrix with nine rows and  $\mathbf{y}_t$  is a vector of nine observations of returns at time  $t$ . In the parameter vector  $\boldsymbol{\Theta}$ ,  $\boldsymbol{\theta}_{1,j}$  and  $\boldsymbol{\theta}_{2,j}$  remain unchanged relative to the previous specification, but  $\boldsymbol{\theta}_{3,j}$  is updated in the dependent mixture model to  $(\boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)$ , that is the model parameters of the relevant model.

### 6.5.1 Scenario generation

As in the previous frameworks, a model is fitted to data each four weeks, every time using all past observations. Following the model fits the estimated parameters are extracted for scenario generation. As mentioned the obtained model in each regime is very simple, as all dependence on past observations has been removed. Thus predictions are made by generating one random multivariate normally distributed return, with mean  $\boldsymbol{\mu}_R$  and covariance structure  $\boldsymbol{\Sigma}_R$  in each regime. Additionally the prediction start-off regime as well as the transition probabilities between regimes are needed.

$\boldsymbol{\mu}_R$ ,  $\boldsymbol{\Sigma}_R$  and  $\mathbf{P}$  are extracted from the fitted models. The start-off regime is the regime which the model fit has deemed the most likely regime of the last observation. This probability, along with corresponding probabilities for all previous observations, is also extracted from the model.

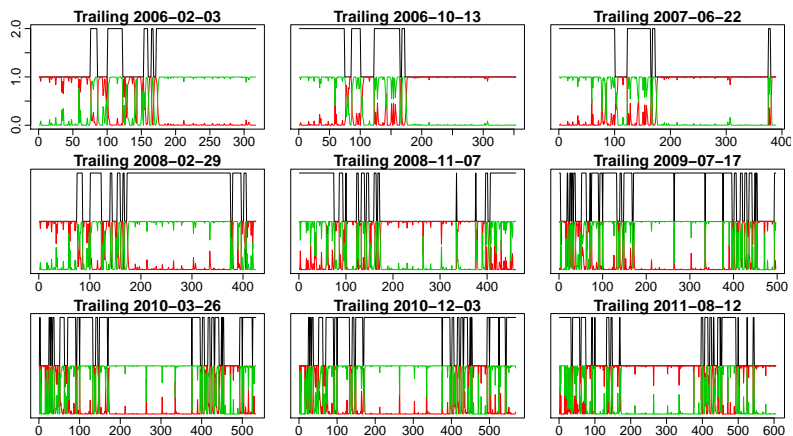
From the extracted parameter values, predictions are made. The first regime is sampled with probabilities  $(p, 1 - p)$  of being regime 1 respectively 2 where  $p$  is equal to the probability of the fitted model ending in regime 1, as extracted from the model. The initial regime is used to determine the parameters used to generate the first prediction. Subsequently the following regime is sampled as a function of the current regime, now using the estimated transition probabilities, also extracted from the fitted model. The new regime is used to make the next prediction, etc.

In figure 6.17 the estimated regime sequence for 9 iso-distant models are shown. The figures depict the evolution in the regime sequence as more observations are added to the model fitting. Each plot is 32 observations apart, corresponding to approximately eight months. The black line shows the estimated regime sequence in each time step, and the red and green line shows the estimated probability of being in either regime at each time step. Starting in the upper right corner, going from graph 1 to graph 2 the certainty of current regime between observations 50 and 150 is increased, and thus the number of shifts is reduced. Adding additional 32 points from graph 2 to graph 3 does not make a difference to the estimated regime shifts.

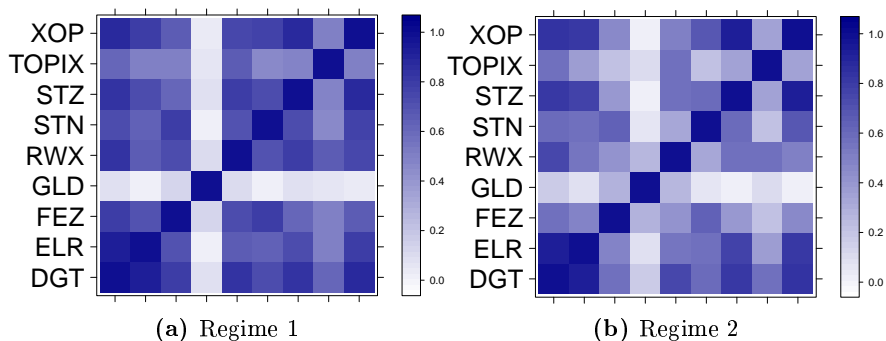
From graph 6 and onwards it is evident that the regime corresponding to calm waters is easily determined while the other regime, corresponding to financial instability, is a lot less obvious, resulting in periods with numerous regime shifts in very short intervals. The explanation for this is found, at least in part, by referring to figure 6.12, where it was seen that the indices change regime at very different points in time. Exemplified by ELR one regime is well determined, while the other regime is somewhat undetermined and the state probabilities jump back and forth. The plot illustrates the process of estimating the regimes as more observations are included. Note in the last plot that the regimes have been shifted, so the green regime is now e.g. high volatility where it was low volatility in the previous models. This has no influence on the estimation.

When considering the correlation matrices in the regimes in figure 6.18, it is apparent that the correlation is generally higher in one regime, thus implying the indices “agree” about where they are, whereas the other regime show less correlation. While there seem to be a general trend similar to that of figure 6.4, the correlation is lower in figure 6.18b. The correlation coefficient change slightly along the course of the 79 periods, but the general trend as described remains. In the dependent mixture model, as mentioned, the model must determine one common point in time to change regime. Yet considering the very different behaviour of data, this apparently causes the model to move the regime shifts back and forth when new data is added to the estimation, causing the model to appear confused.

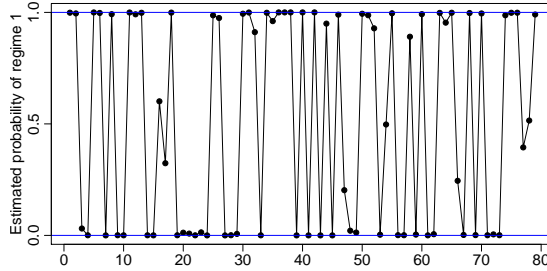
The result is that the models along the course of the 79 periods end in different regimes every other time. In figure 6.19 is shown the estimated probabilities of the model ending in regime 1, through the course of the 79 periods. This is also the probability used to sample a starting regime for the scenario generation. Due to the high probabilities in almost all periods, this is with fair certainty also the start regimes for the scenario generation. To be fair there is a short period of consistency around period 20 and again between 30 and 40. These are the only periods of time where the nine indices have to a sufficient degree agreed on an appropriate regime. – this despite their highly correlated returns.



**Figure 6.17:** The red and green lines show the probability of being in regime 2 respectively 1. The black line show the chosen regime at time  $t$ . The plots illustrate the progression of the models attempts to determine the time of regime shifts as a compromise between the indices as more information is added.



**Figure 6.18:** Full dataset correlation matrices of the nine indices in respectively regime 1 and 2.



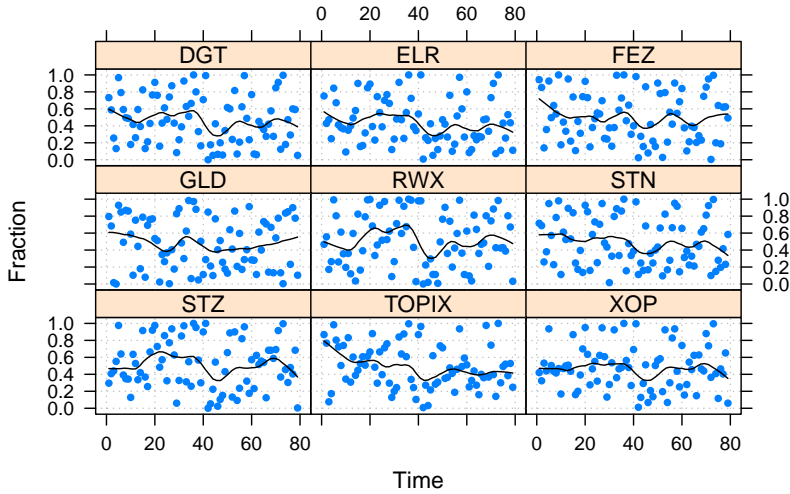
**Figure 6.19:** Sequence of scenario generation start regimes over the course of the 79 considered periods.

### 6.5.2 Results

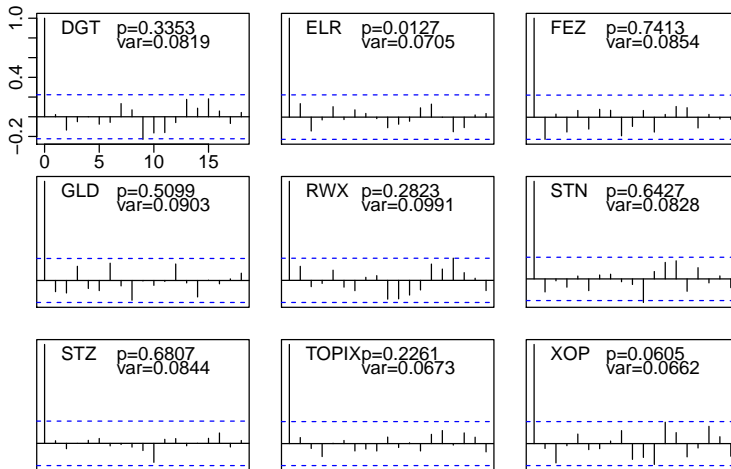
Evaluation of the method is carried following the same method as previously. Figure 6.20 shows that all of the trends apparent in the previous similar plots are also present in this case, however at a less pronounced level. The fractions are generally evenly distributed over the interval without apparent white spots in the plots. XOP is trumpet-shaped, and is only barely accepted as uniformly distributed. The variance is noticeably lower than the theoretic ideal of 0.0833. ELR is rejected as uniformly distributed. There is a tendency that the fractions cluster towards the bottom of the  $[0,1]$  interval. Whereas TOPIX also seem to have the fraction points cluster, the fraction span most of the  $[0;1]$  interval, distinguishing it from ELR.

Figure 6.19 shows that the regimes jump back and forth, but this is not reflected in the variance of the scenarios, as depicted in figure 6.22. The reason is that while the regimes jump back and forth, the model parameters do the same. The result is a realised mean return and variance of returns as illustrated in figure 6.23. The plots show the model parameters related to each period assuming the scenarios are generated from the exiting regime of the model fit. Supporting plots are attached in appendix Figure A.8.2.

Looking over figure 6.22 it is seen that there is a brief but consistent increase in scenario variance in periods 15–17. There appear also to be a permanent increase in scenario variance starting in period 22, although

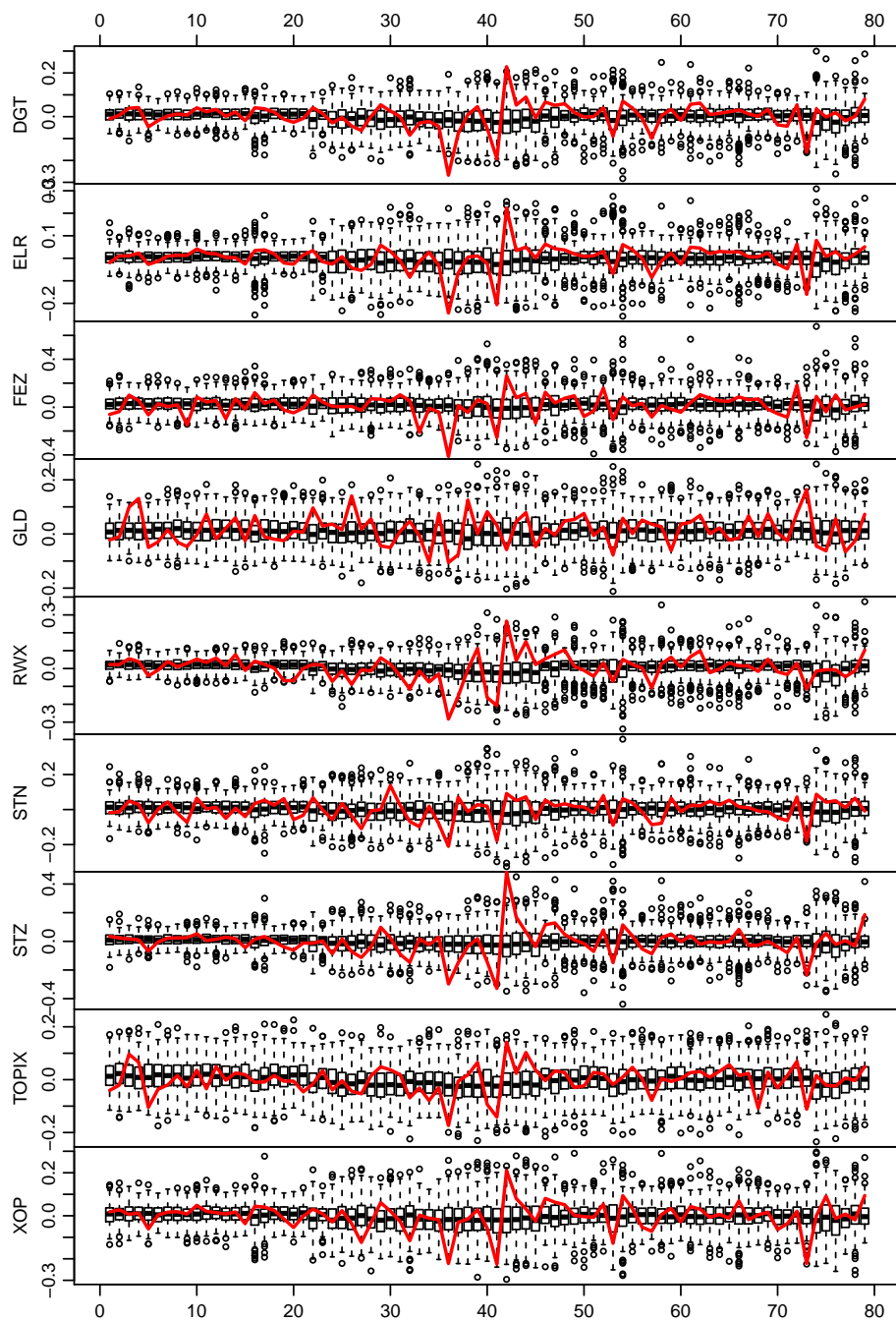


**Figure 6.20:** Fractions of scenarios which exceed the observed return (second axis) in each period (first axis).

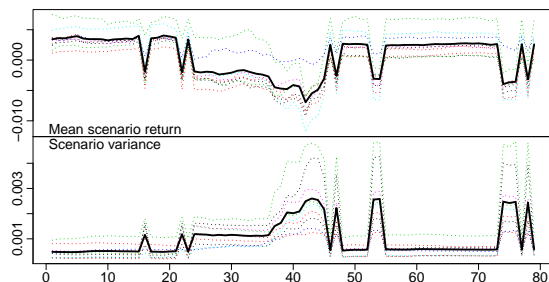


**Figure 6.21:** Autocorrelation of the 79 fractions along with p-value for test for uniformity as well as sample variance of the fractions.





**Figure 6.22:** Distribution of the 250 dependent mixture model scenarios in each period represented by boxplot. Shown along with observed return in red.

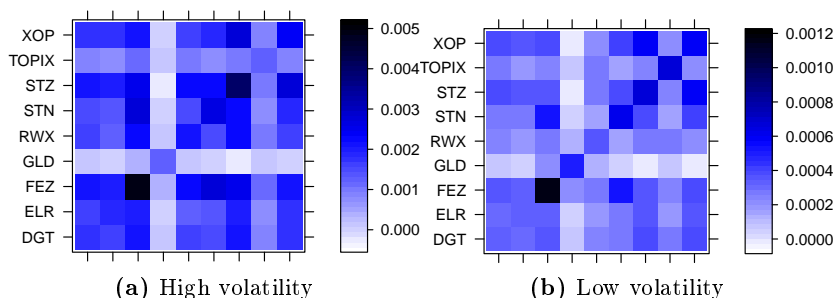


**Figure 6.23:** Top panel shows the realised mean scenario return, where each index is shown with a coloured dotted line and the mean is shown in fat black. The bottom panel shows the realised variance, using the same notation.

this is less distinct. This is coherent with figure 6.23, although it is not with figure 6.19, again supporting the idea of changing model parameters during the course of the regime shifts.

The dotted lines in figure 6.23 show that the level for each index is different, but the evolution is similar. Because of the nature of the model, all indices experience the regime shift simultaneously. In figure 6.22 it is seen to be a good fit for particularly STZ, and to some extent XOP and RWX, while the remaining funds do not change behaviour until 10–15 periods later. Thus for the majority of the indices, the compromise causes a long period of undue volatility in the scenarios.

In the multivariate normal distribution the scenarios are generated from a mean value and a covariance matrix. Thus the parameter determining the range of the scenarios is the covariance matrix,  $\Sigma_R$ .  $\Sigma_R$  is illustrated in a heatmap in figure 6.24. These heatmaps only show the covariance matrices for the last period. The covariance is subject to change throughout the subsets of data, but the plots illustrate the difference between the indices, in relation to the behaviour witnessed in figure 6.22. The difference in level of variance between the regimes is almost five times. It is also interesting to note that in the calm regime emphasis is on the variance terms. In the unstable regime all covariance terms increase, and in particular FEZ, STN and XOP display high levels of covariance with the other indices, meaning that the behaviour of each index to a greater extent affect the predictions in the other indices.



**Figure 6.24:** Full dataset covariance matrices of the nine indices in respectively regime 1 and 2.

### 6.5.3 Discussion

This method accounts for correlation in the market, which was missed in the previous methods. But it does not consider the assets individually. As the models are fitted to all indices simultaneously, the predictions on each index will be generated by a model which is not fitted specifically to that index. Regime shifts are determined based on the portfolio, as a compromise between the considered assets, and the result is a loss of accuracy with regards to the individual assets. This, however, is not reflected in figures 6.20 which show very good results. It is reflected in the boxplot, from which it appears that the model philosophy is “better safe than sorry” causing it to conservatively increase variance and decrease expected return at the first sign that one index is unstable.

Further, while the ability to model and predict data is undeniable, plots show that the characteristics of each regime almost consistently change with the addition of only four points. For the estimation the label of the regimes is clearly not important, but if the model is intended for extracting information about the portfolio, it is a problem. – The regime label will tell nothing about the current state, as this could today mean the opposite of what it meant yesterday. Likewise is the current volatility of the portfolio useless in trying to make a guess at the regime. Thus in order for this model to be useful, you need all the information. But if you have all the information, you do not need the model.

## 6.6 Discussion of scenario generation

The visualisations of the scenarios, in figures 6.5, 6.8, 6.9, 6.14 and 6.20 as well as figures 6.7, 6.11, 6.16 and 6.22 show that while bootstrapping does not catch any changes in data, the other methods capture some dynamics in an increased variance. Bootstrapping is in all regards inferior to the remaining methods and will be left out of further comparison.

Neither of the applied methods manage to fully transfer the dynamics of data to predictions. This is proved by the trendlines in figures 6.8, 6.9, 6.14 and 6.20 which for each index are very similar. The same trends and tendencies appear three times, although with decreasing magnitude. The dependent mixture model performed better than the two previous models on this measure.

When turning to the boxplots the conclusion is another. In these plots the GARCH generated scenarios to a significantly wider extent reflect the volatility of the individual assets. This is of course due to the GARCH model, under which changes in volatility are triggered by data itself. Under the regime models changes in volatility within the scenarios result from probability. In only four predictions a change in regime is unlikely. The conclusion is that any difference in quality of predictions is due to the quality of the model fit used to generate the predictions.

The GARCH scenarios fall short in following the observed return when this decrease in high volatility periods. This is nicely captured by MSAR, although with delay. The dependent mixture model is encapsulating the observed returns almost completely, and generate data which replicate the characteristics of the financial markets very well. Yet, it is also very conservative in determining the applied variance, causing the predictions in most periods to be generated with excess volatility. Due to the model specification the changes in mean return are considerably less accurate than in MSAR, but instead follow a common trend. GARCH and MSAR produce tighter scenarios, with no excess volatility. The 25 to 75 percentiles span around half the range of the observed returns in the close proximity. Subsequently, the range of the 25 to 75 percentiles increase when the range of the returns increase. Again, a very tight fit is obtained. But this also calls for periodic mis-estimations, which these models are

seen to be more prone to than the wide-ranging dependent mixture model scenarios.

For that reason the dependent mixture model with two multivariate gaussian processes will be the recommendation for the risk aware investor. But the investor is also advised to remember that the model predictions reflect the general state of the economy rather than the expectation of the specific assets. This recommendation is based on the fact that MSAR is delayed in decreasing the mean returns anyway, and GARCH model the volatility with higher accuracy. To obtain predictions on one specific asset GARCH is recommended. To obtain a complete impression both models should be applied. This will reveal not only the general state of the economy, but also provide a basis for understanding the individual assets.

While praising the dependent mixture model, it should be noted that six of the considered indices are highly correlated, making it plausible that one common trend can adequately describe all indices.

It was outside the scope of this thesis to examine the implications of modelling with more than two regimes. The extensive use of the highly volatile regime in both MSAR and the dependent mixture models suggest that a third regime, to describe the period after the 2008 crisis, would be in place.

From the persistent drops of the trendlines in the fraction plots it is clear that the exact behaviour of data cannot be predicted, in any of the considered models. But the boxplots show that all except bootstrapping are able to accurately detect changes in volatility, and subject to the desired application purpose, well describing models have been detected.



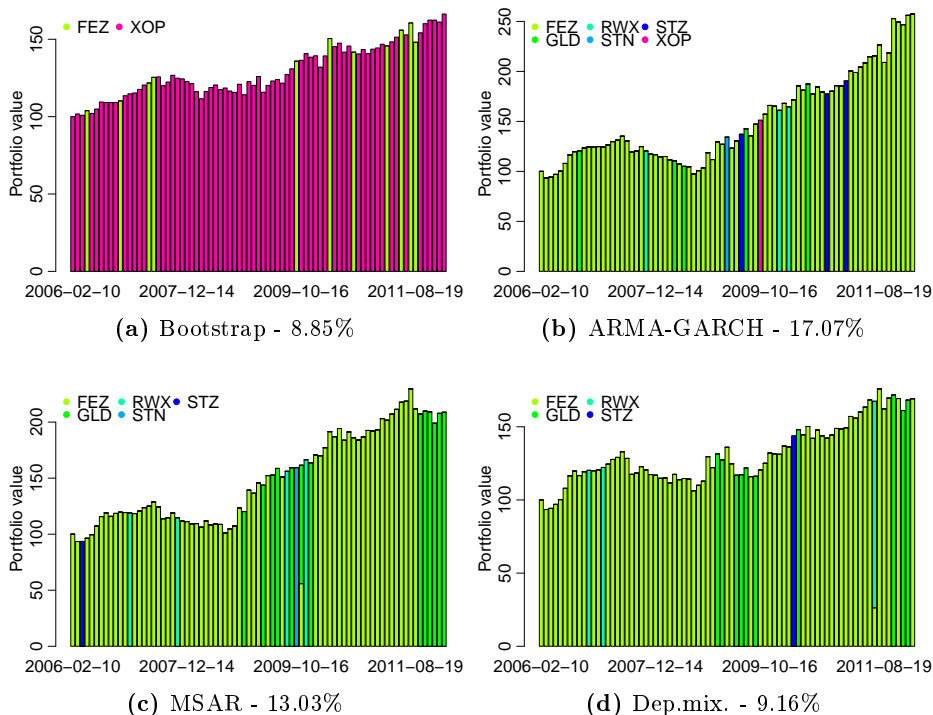
# Asset allocation

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Studies have shown that asset allocation is the most significant factor in determining the performance of a portfolio. When deciding on the asset allocation the investor must have an investment strategy, including a risk profile. The risk profile is determined by factors such as tolerance for loss and risk averseness in regards to gains and losses of the investor.

In constructing the portfolio the investor must keep in mind the investment strategy and select asset classes and weights accordingly. For example, equity is associated with a higher degree of risk than fixed income, but also comes with a significantly higher expected return. Precious metals are basically uncorrelated with any other asset (refer to table 4.4), making it a much used tool for diversification [31] [32] [33]. According to the Capital Asset Pricing Model [34], two different risk measures are relevant to a portfolio manager – the portfolios total risk and its systematic risk. Total risk is measured by the variance of the portfolio. Systematic risk is measured by the portfolio's beta. The total risk of a portfolio is a function of the correlation coefficients and variances of the different investments that form the portfolio. The key to reducing the total risk of a portfolio is diversification, c.f. (2.2).

When investing in ETFs the investor improves the possibility of diversification and frees himself of the decision about which particular asset to invest in, thereby reducing the decision to determining weights in each asset subcategory and choosing between the ETFs represented in the relevant fields.



**Figure 7.1:** Dynamic Max Average (risk lover) under the four scenario generation methods

## 7.1 Asset allocation under different strategies

The scenarios generated in [Table 6](#) are applied in portfolio optimisation, under three different investment strategies. As optimisation is not a topic of this thesis, the task of constructing the portfolios was performed by Kourosh Marjani Rasmussen, from the Management Engineering depart-

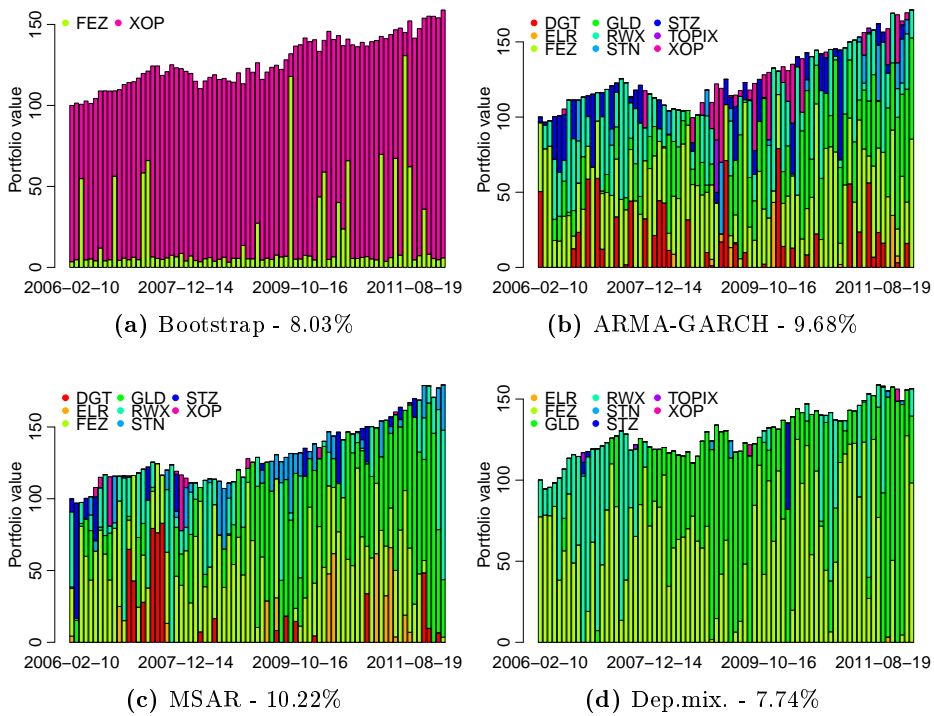


ment at the Technical University of Denmark. The strategies are

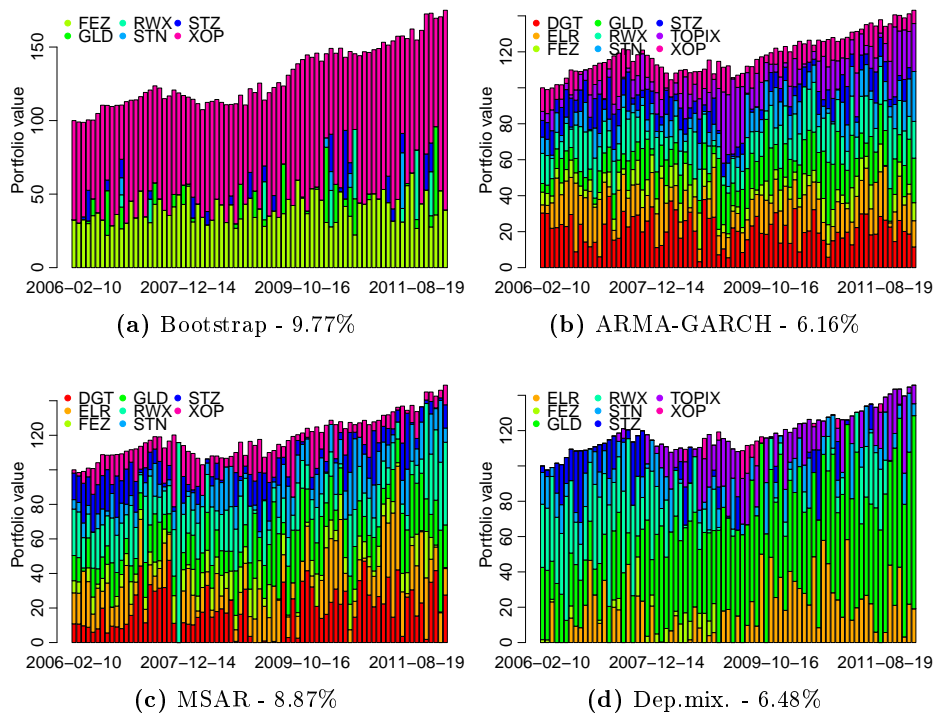
- **DynamicMaxAvg** Risk loving investor. The return of the portfolio is maximised each period, regardless of the associated risks. No consideration for diversification or market risk in the assets. This strategy carry high risk, which should result in a greater return.
- **MaxRiskAdjustedReturn** Risk neutral investor. Assemble the portfolio by selecting assets which increase the portfolio return while maintaining or reducing portfolio risk (relative to the risk free asset.)
- **DynamicMinCVaR** Risk averse investor. The selection strategy is to minimise the five percent greatest losses each period. Formally the conditional variance at risk (CVaR) at 95 percent confidence level is minimised. See [35] [36] for elaboration on CVaR.

In figures 7.1, 7.2 and 7.3 asset allocation under the three strategies is illustrated. The asset allocation is performed based on the generated scenarios, and in fulfilment of the above stated guidelines. In the captions are given the average annual return on each portfolio. Each panel represent one method of scenario generation.

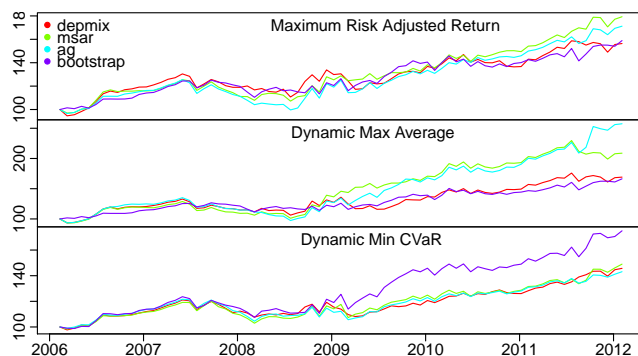
The return on portfolios in Figure 7 is a weighted average of the returns of the nine assets. Thus as TOPIX is excluded from MSAR these portfolios cannot be included in the asset allocation analysis. Yet, considering the risk loving strategy, neither of the three other methods facilitate the use of TOPIX in the portfolio, so with reservations for the unknown, it is plausible that the portfolios are comparable under this strategy. Likewise arguments apply to the risk neutral strategy, under which the other methods only to a very limited extend use TOPIX. Under the risk averse strategy TOPX plays a significant role in ARMA-GARCH as well as the dependent mixture model, so these portfolios cannot be compared to MSAR. It is interesting to note, how the optimisation have replaced TOPIX under this strategy. Around mid-2009 there is a slightly increased emphasis on STZ but otherwise the weight assigned to TOPIX in ARMA-GARCH and Dep. mix. is equally distributed amongst the other assets in MSAR.



**Figure 7.2:** Maximum Risk Adjusted Return (risk neutral) under the four scenario generation methods



**Figure 7.3:** Dynamic Min CVaR (risk averse) under the four scenario generation methods



**Figure 7.4:** Development in portfolio value, indexed to 100 at the starting point.

What is apparently most significant about the portfolios is the consistent emphasis on XOP in all of the bootstrapped scenarios (7.1a, 7.2a, 7.3a), regardless of risk profile. Additionally, the bootstrapped portfolios generally use fewer different assets, relative to the three other considered methods. Given that the bootstrap scenarios were rejected as representative for the observed returns, the portfolios based on them are now exhausted.

Secondly, the risk lover takes on a very narrow portfolio consisting primarily of FEZ. This is the case in all methods (fig. 7.1). While the ARMA-GARCH generated scenarios produce an average annual return of 17.07 percent the numbers for MSAR respectively the dependent mixture model are 13.05 and 9.16 percent. As the portfolios are roughly identical the difference must lie in the mean prediction under each method.

Finally, the methods are distinguished by their use of DGT, ELR and FEZ. In the risk neutral strategy FEZ is approximately taking the place of DGT and ELR. This suggests that when taking the correlation between indices into consideration, the highly correlated indices become obsolete. Referring to table 6.1, ELR and DGT are highly correlated in both regimes. Also FEZ is correlated with this group with coefficients of respectively 0.69 and 0.62. Given the relatively higher return of FEZ, more emphasis is put on FEZ.

GLD is overrepresented in the dependent mixture model generated scenario portfolios. The exception is the risk lover strategy, where emphasis is solely on return. In the two risk-considering portfolios, the exposure to gold bullion is enhanced, underlining the risk-decreasing feature of this asset [31] [32] [33].

Figure 7.4 shows how the value of each portfolio progress. Disregarding the bootstrap portfolios the dependent mixture model is generally producing lower returns. This is a result of the extensive use of high volatility and corresponding low returns. MSAR and AG are very similar, indicating that the apparent excessive use of regime 2 under MSAR is in fact averagely corresponding to the effect of the constant mean in the AG models.

## Discussion

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In [Figure 5](#) it was concluded that the deviance processes can in fact be modelled. Weekly and daily returns were considered, and due to similarity in quality of the residuals and consistently better model fits, weekly returns were preferred.

The deviance processes were visually determined to exhibit volatility clusters. For that reason a GARCH framework was applied. Autocorrelation of the processes revealed a serial dependence on past observations, inspiring an ARMA specification of the conditional mean. The `fGarch` package in R was applied. This turned out to contain important shortcomings which may have restricted the modelling. The unimplemented model assumptions, intended to ensure non-negative conditional variance was not an issue in [Figure 5](#), but proved to be of importance in [Table 6](#).

The fitted models in [Figure 5](#) are fitted with parameter values inducing  $\sum \alpha_i + \sum \beta_i > 1$ , implying deviance processes with non-finite unconditional variance. As this is not observed in the visualisation of the processes, nor tests for stationarity, it is suspected that the erroneous parameter estimation results from the outliers. While the index returns are

fitted nicely in small models and are producing satisfactory results, the deviance processes are fitted in models with as many as nine parameters and generally produce acceptable, but not impressive, residuals.

Neither the problem with GARCH parameter estimation occurred in the nine models fitted to the index returns, although these carry many of the same characteristics as the deviance processes, with the exception that the index returns do not have the extreme observations seen in the deviance processes. This supports that the main problem with fitting the deviance processes lie in the extreme observations.

Preceding [Figure 5](#) the optimisation algorithms in `garchFit` were tested on data for robustness and L-BFGS-B was selected. Yet, in [Figure 6.2](#) it was found that it is disposed towards settling on points which are not true minima (e.g. saddle points), thus causing the parameter estimation to fail. The Nelder Mead optimisation algorithm was introduced to estimate the models which failed under L-BFGS-B. This of course raises the question of whether the algorithm should have been introduced sooner, and how it would perform on the deviation processes. However, as stated, the L-BFGS-B algorithm proved more robust on data, and consistency was prioritised.

It is important to emphasise that outperforming the underlying index is not a goal of the fund, and should not be considered a label of quality. In fact, outperforming the indices, which are widely undiversified, should be an easy task, as the funds are free to employ any investment policy they desire. Thus, within the small deviances observed, perhaps a better measure of replication-ability is found in the correlation between fund and index returns.

The funds are generally highly correlated with the indices. The correlation coefficients are in all funds higher on weekly returns relative to daily returns, meaning that the funds may observe ultra-short term fluctuations, but overall replicated the returns of the index very accurately. On the bottom line only the result counts in finance, but as index fund investing is in itself rather obscure, the investor is helping himself by selecting funds which comply with their index.

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Four methods were tested towards scenario generation. Bootstrapping was applied as a base reference and proved unable to capture the dynamics of data. Under ARMA-GARCH all ARMA parts were insignificant and consequently the index returns were modelled solely in GARCH framework. The addition of time dependent conditional variance in ARMA-GARCH proved to very accurately model data, and showed significant improvement to the base reference, as expected.

Thirdly a new approach was applied under which the observed changes in volatility were captured by regimes. This approach was applied in two models, namely the Markov Switching Autoregressive model and a dependent mixture model. In MSAR the AR effect disappears when regimes are introduced. This means that dependence on previous observations, error terms or conditional variance, is unfounded in data, but solely serves to capture the important fluctuations in financial markets. Thus all models support the weak form market efficiency hypothesis that information of past returns are useless in predicting future returns. The last step was to apply a dependent mixture model under which each regime is modelled as a multivariate random walk with time dependent covariances.

The scenarios definitely show improvement over the four considered methods, yet the strongest improvement is found from bootstrap to ARMA-GARCH, in which case time dependent volatility is introduced. It can be concluded that regardless of the specific methods, time dependent model parameters are essential. The fraction plots show subtle improvement going forward from bootstrap. But the boxplots expose that the improvement in the dependent mixture model is primarily based on a considerably higher variance in the scenarios. The bootstrapped scenarios are generated with constant variance, which is expressed in the plots by homogeneous scenarios. The remaining methods allow the conditional variance to change, which is reflected in the range of the scenarios and the presence of outliers.

Separating the observations into regimes imposed a lack of accuracy, relative to the results of ARMA-GARCH. Both MSAR and the dependent mixture model accurately refrain from returning to regime 1 after the 2008 crisis, but with the cost that the predictions are generated with excess variance. This inspires the application of additional regimes, to

characterise post-crisis markets, where naivety and blissful ignorance following a decade of prosperity, have been replaced by financial markets under constant pressure and highly alert players.

In all three methods the parameter most influential to the volatility of the predictions is the estimate of current volatility / regime at the end of the observed period. Followed by only four predictions the impact of the predictions to radically alter that is limited.

The asset allocation in [Figure 7](#) does not consider the deviances between the indices and the funds. The asset allocation is performed solely on the scenarios, generated on index returns. Thus, as the investor cannot invest directly in the indices, he must collate these results with the results of [Figure 5](#).

The returns of the portfolios in the asset allocation in an indication of the level of the mean prediction under the different methods. Considering the high risk strategy, the portfolios are roughly identically assembled, but the ARMA-GARCH portfolio produce higher returns. As ARMA-GARCH does not adjust the mean return during high volatility periods, this is expected.

## 8.1 Sources of inaccuracy

The limitations of the `fGarch` package, which may have resulted in better fitting models not being identified due to e.g. convergence error. Additionally, the ARMA-GARCH generated scenarios suffered under negative GARCH parameters. In the scenario generation the model structure for each index is determined using the full dataset, while the scenarios are generated using only subsets of this. As was seen in the ARMA-GARCH case the initially optimal models may not be the best along the course of the 79 periods. This practise is a source of inaccuracy, which affect the scenario generation to an undocumented extend. Finally the asset allocation is imprecise and lacks comparability between scenario generation methods due to the error in MSAR.



## Conclusion

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The thesis set out to examine the replicative capacity of 20 widely diversified index funds. Both daily and weekly returns were examined, and it was found that the deviances in returns can generally be described by a GARCH process with ARMA mean structure. The mean deviance is in most cases statistically zero, and in any case numerically very small. Thus it is concluded that the funds do in fact replicate their underlying indices.

This paved the way for substituting the underlying indices for the funds, in a scenario generation intended towards portfolio optimisation. Four methods for scenario generation were applied, of which one was intended as a point of reference. Expectedly, the three methods imposing time dependent variance strongly outperformed bootstrapping in capturing the dynamics of data. The results are inconclusive regarding the ranking of the linear ARMA-GARCH and the non-linear dependent mixture model. However, for the risk averse investor the latter will be the cautious choice, while the risk loving investor may prefer the accurate modelling of each asset, produced by ARMA-GARCH.

## 9.1 Future work

During the work with the present thesis many questions and ideas arose. Due to the time restriction of the project, many were left untouched. This section collects some of these ideas.

Include negatively correlated assets in the scenario generation. This is interesting in the dependent mixture model under which the covariance of the assets is included in the estimation. Neither of the bond indices from [section 4](#) are represented in the scenario generation. Due to the low correlation between the bond funds and the other funds, combined with the higher correlation amongst the bond funds, this would have been an interesting addition to the scenario generation. However, sufficient historical data was not acquired.

Based on [figure 6.19](#) and the corresponding inconsistency in adjacent models, it is of interest to examine the behaviour if including a third regime. This is intended to describe the post crisis situation, where the markets in fact are more volatile than seen prior to 2007, yet no-where near as flighty as they were during 2007–2009.

Considering the ARMA-GARCH models in [Table 6](#) the indices showing correlation in the residuals are all purely GARCH processes. This could perhaps be overcome by introducing arma parameters. However, these were not ruled significant / errors were produced by the software. This inspires a study of the different optimisation algorithms available in the `fGarch` package. Further, the models fitted in [Figure 5](#) leaves room for improvement, by e.g. including a seasonal trend. Further, high model orders were obtained, often due to failure in simpler models. Perhaps this could be improved by considering a different optimisation algorithm in the relevant cases.

Consider external regressors as explanatory for future stock performance. For the purpose of scenario generation and portfolio optimisation, considering the stock price as a function of previous values as well as under influence of external regressors may definitely be an interesting study. [\[25\]](#) also suggest to employ the use of external regressors in determining the current regime. This idea is supported in this dataset by the supe-

rior performance of the dependent mixture model, which suggests regime shifts should be determined based on market dynamics rather than single asset behaviour.



## APPENDIX A

# Additional

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### A.1 ETFs as investment products

As an investment product, ETFs in general allow investors to gain from intraday moves of an index since they trade as individual company stocks do, with continuously updated bid/ask quotes. Thus, if investors believe that large, stable companies are going to have a good (bad) day on average, but are unable to pinpoint exactly which of the relevant companies might experience the most significant price increases (declines), buying into an ETF which invest in said large, stable companies provide a vehicle with which they can try to profit from their expectation. This is not possible with a traditional index mutual fund, since these funds calculate their net asset value once a day, after the markets have closed, so buyers and sellers of shares do so at the same price. Additionally, unlike mutual funds, ETFs can be sold short to gain if the index is expected to decline. This is usually not recommended for private investors, which is why it is only listed as a benefit for professional investors.

The appeal of ETFs for investment purposes are several, but the emphasis

is slightly different between different types of investors. Common for all investors is a significant reduction in fees, compared to competing products. The expenses on regular mutual funds may take a lot of the return. This means that the ETFs are significantly more return-efficient than many of the open-ended funds. Another real return optimising feature is favourable taxation compared to the discussed competing products. In Denmark, ETFs purchased by means of available funds are taxed like capital income, so the rate is subject to the total income of the investor. For comparison share income is taxed at 27 percent for annual income less than Dkk. 48,300 and 42 percent for annual income in surplus of Dkk. 48,300 [37]. ETFs purchased with retirement funds are taxed at a substantially lower rate [3]. In The United States, which account for the majority of the ETF market, taxation on ETFs is in general advantageous to the taxation of e.g. mutual funds [38]. Yet for the private investor the primary benefit lies in the combination of ease of access, low costs and ease of diversification and entrance to high barrier markets. The professional investor also benefits from the low costs and particularly from the speculation enabled by the exchange trading.

However, the diversification benefit is distinct with private investors who invest with limited funds. Achieving a diversified portfolio can be a demanding task which requires many hours spend on research and a substantial amount of money available to invest. For the private investor funds are usually strictly limited, and in most cases so is the time available for research. The vast majority of private investors do not have the necessary funds to build a well-diversified portfolio. Instead they own shares in a few companies that they for some reason feel familiar with. Now imagine a private investor who wants to enter the real estate market as a strategic step towards greater diversification in his portfolio, and perhaps believes that real estate prices in Spain are undervalued. Making just a single investment in the Spanish real estate market requires a heavy investment and this might be too much for the private investor. Additionally, making a huge investment in one area could create too much exposure on this particular area, and consequently reduce the overall diversification.

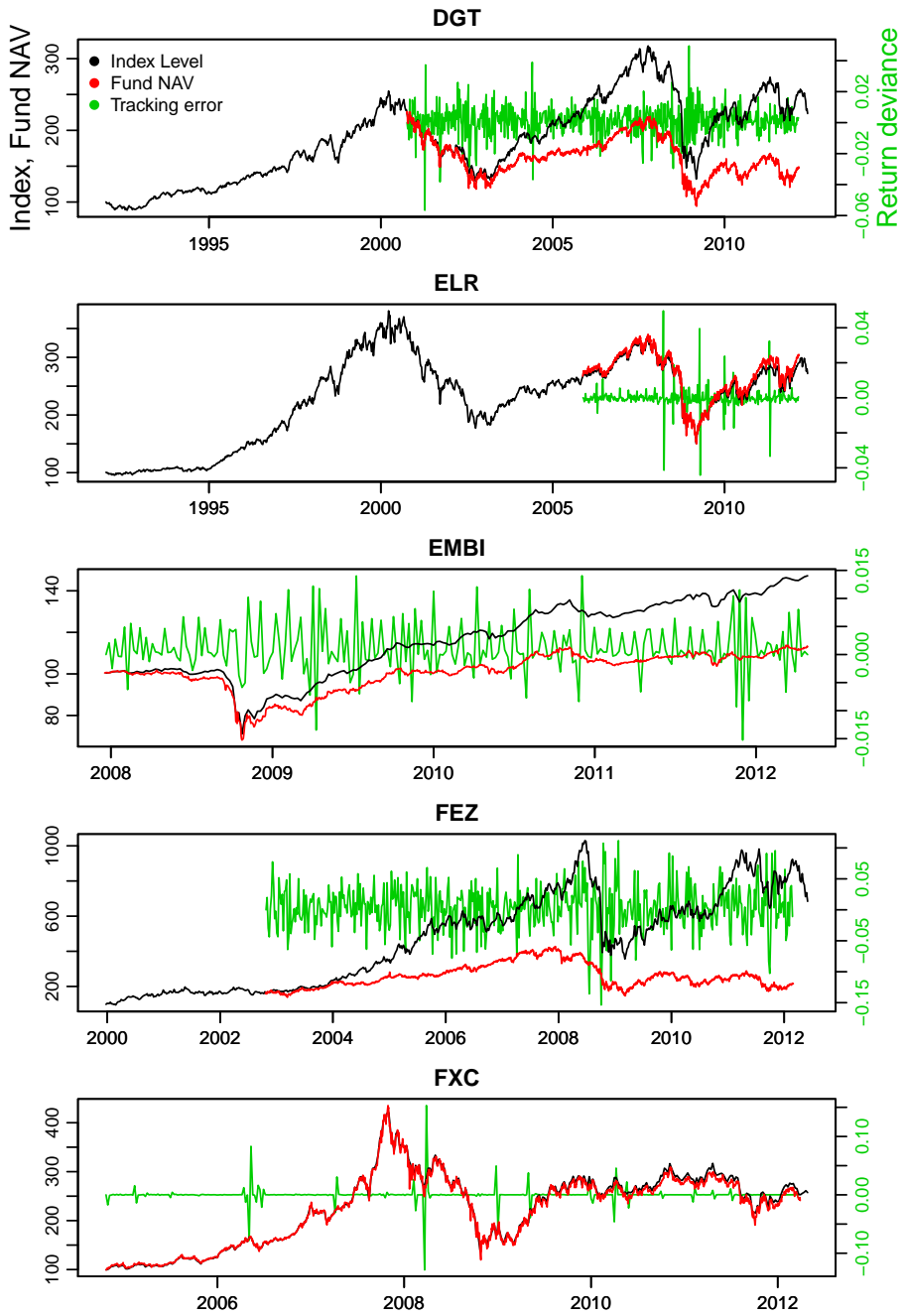
The investor also faces the question of which particular investment to make, in the desired asset class. To overcome these issues he can buy shares in an ETF which invests in European real estate. These will typ-

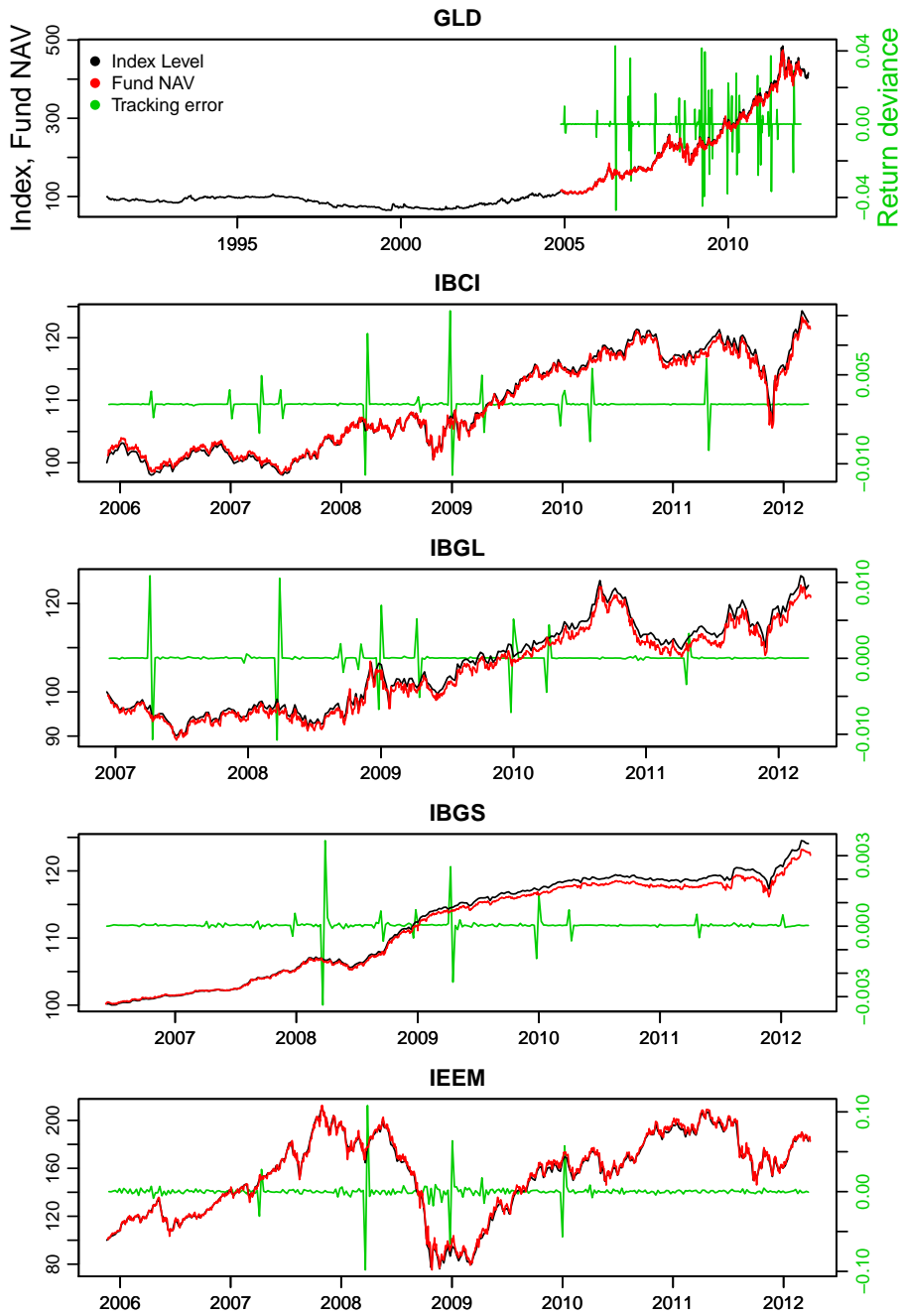
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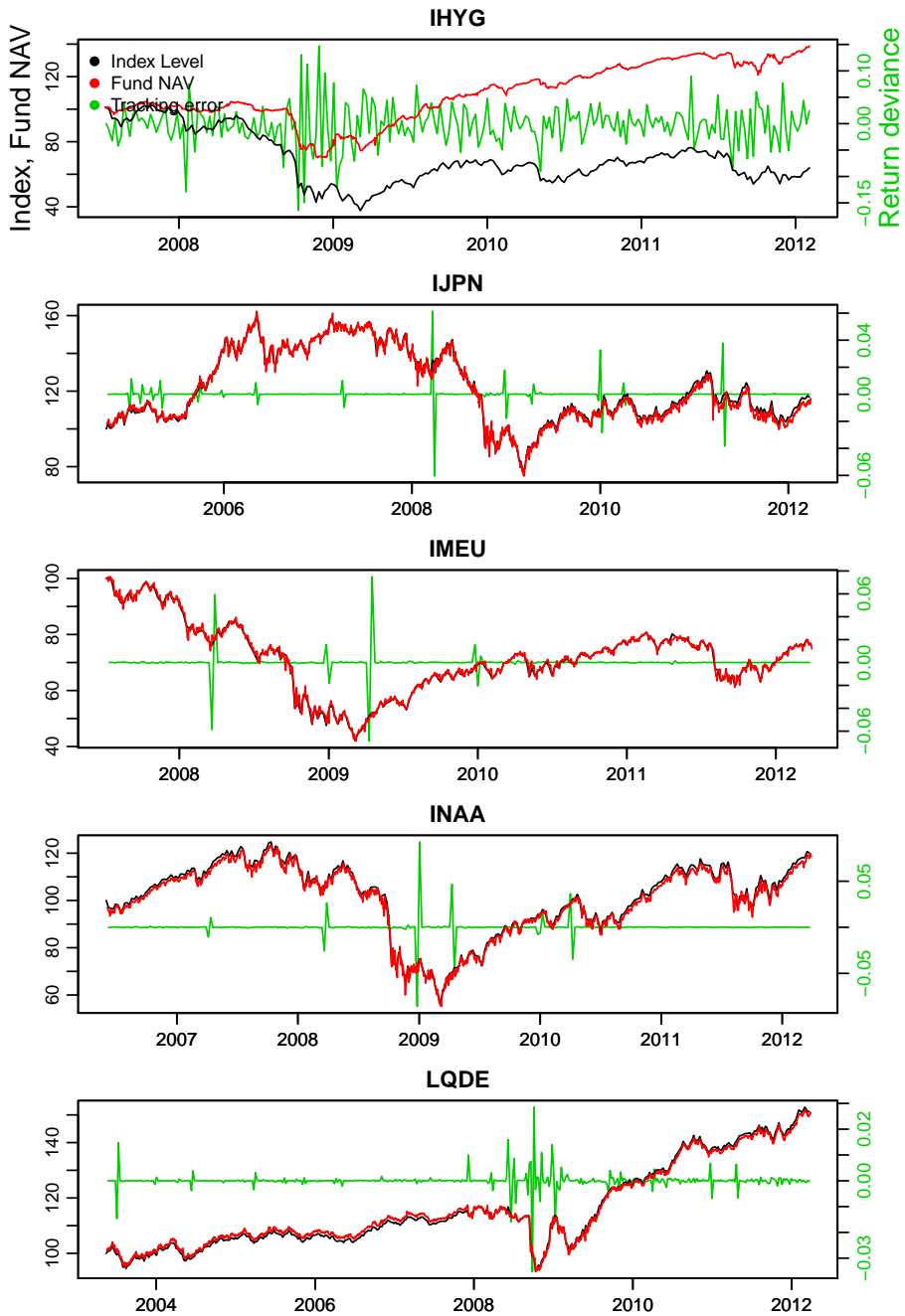
ically sell for as low as \$100/share. Thereby the investor achieves the desired diversification of his portfolio. Many of the ETFs sell for less than \$100 per share, making it possible for everyone to benefit from this. Now, while it is reasonable to think that European real estate is a rather broad investment to engage in, when only Spanish summerhouses was the original intention, the diversification of this fund provides insurance against fluctuating prices in the markets represented. This means that while Spanish summerhouses may be a blooming market, Greek summerhouses may be simultaneously crashing. By taking on both investments as part of the security basket that is the fund, the full potential gain of the Spanish market will never be realized. However, neither will the full potential loss of the Greek market, and on average the total outcome is a net gain. This effects is quantified in the term diversification which is a key factor in the theory known as Modern Portfolio Theory.

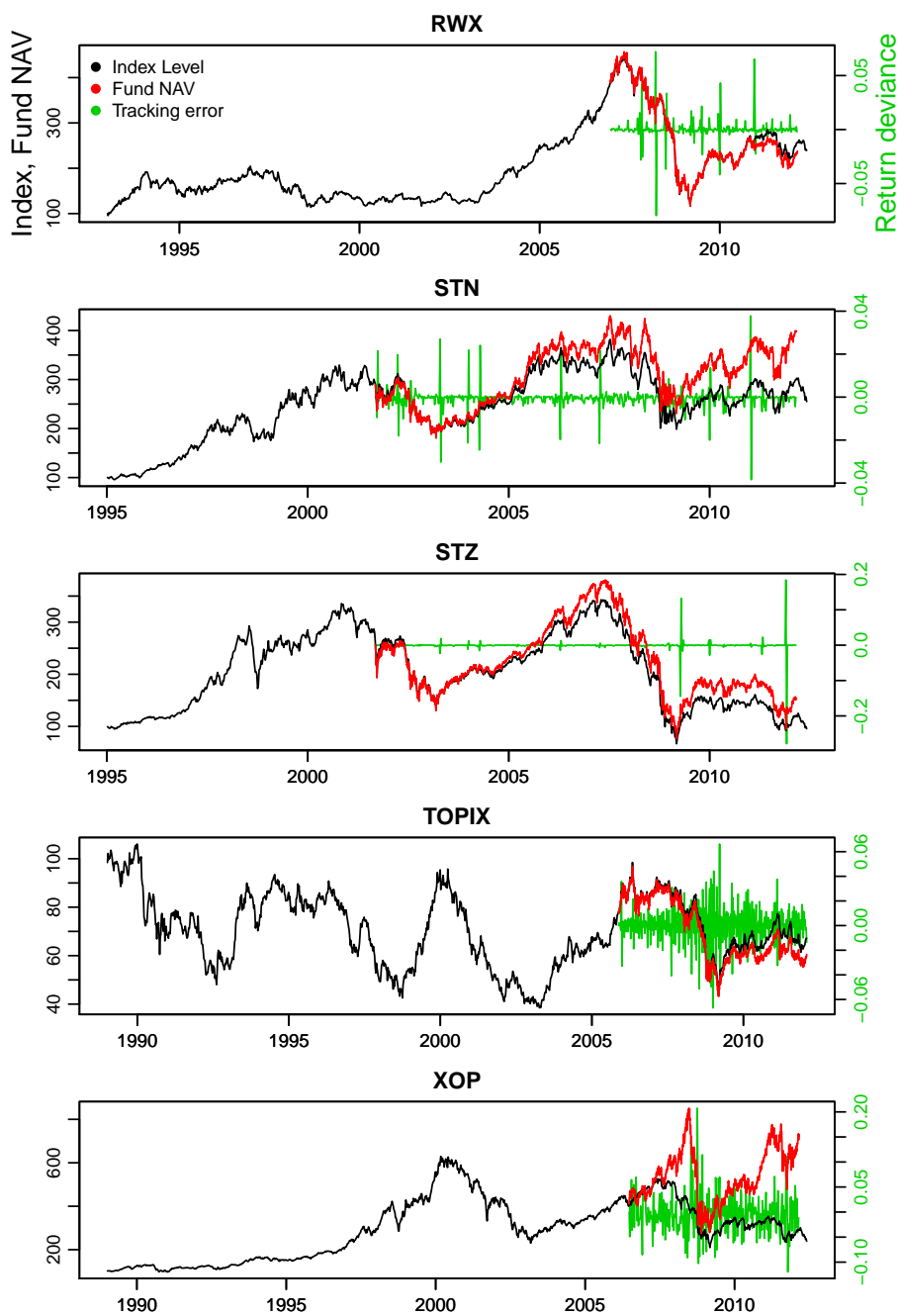
## A.2 Fund and index NAV and deviance processes











## A.3 Fund correlation analysis

GLD, IBCI, IBGL, IBGS and LQDE have low correlation with the rest of the funds, although some correlation, in the amount of fifty to sixty percent, is seen between IBCI, IBGL and IBGS. IBCI, IBGL and IBGS all hold European government bonds, which explains their mutual correlation. This also serves to explain the negative correlation with the remaining funds of which the majority hold equity. It is a well known fact that bonds and equity tend to move opposite each other, as they are considered alternative investments [39] [40]. However, in times of very low interest rates and low inflation, the return on fixed income is reduced, and the asset classes become more correlated. This fact is supported in the examined data where it is seen that IBGL and IBGS are only slightly negatively correlated with the remaining funds. The exceptions are IBCI which is also a European bond fund, LQDE which hold the 30 largest, most liquid investment grade US corporate bonds and IJPN with which IBGL and IBGS are practically uncorrelated. IJPN is an equity fund, and as such, negative correlation is generally expected.

LQDE is generally little but positively correlated with the other funds. The only exception is GLD which track the price of the gold bullion. Historically gold exhibit low correlation with all other assets [33] - less than fifty percent for all except silver, and generally less than 40 percent to all major asset classes [31]. The present data confirms this, also in today's market. Here GLD exhibit correlations of -17 percent up to 27 percent, which is low enough to consider gold an independent asset which will enhance diversification in any portfolio. This is documented in [33] and [32]. Table 4.4 also show that the highest correlation with GLD is found in XOP. XOP is holding oil and gas related stock, making it an applicable proxy for oil as a commodity. As such the value of the basket is closely related to the crude oil price, with which gold is relatively high correlated, c.f. [31].

GLD, IBCI, IBGL, IBGS and LQDE are moving with the remaining funds before the beginning of the crisis in late 2007 - 2008, but are clearly affected differently afterwards, at what point all funds continue a steady growth as opposed to the equity funds which uniformly decline. The gold fund is seen to rapidly and steadily increase over the observed period.

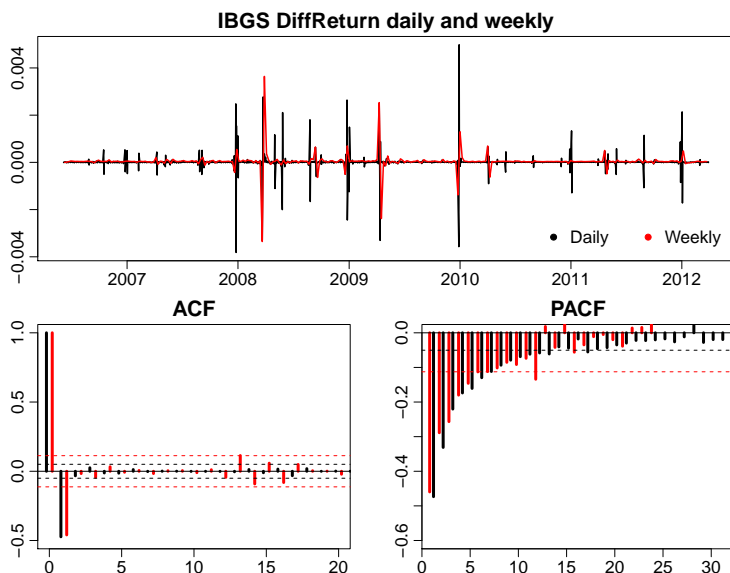
This is consistent with the development in the price of gold, as illustrated in figure 4.3 where monthly observations of the gold price going back 20 years is plotted in blue. Evidently the price of gold has been largely unaffected by the intervening crises. It can also be noticed that the monthly return (in black) is reasonably stable over the period, supporting the stability and uncorrelated nature of the gold market to the dynamics which drive the stock market.

Apart from the funds mentioned in the analysis in Table 4.1 data consists of additional two fixed income funds, namely EMBI and IHYG. Along with IJPN these are medium correlated (up to 65 percent) with the remaining funds. EMBI holds 80 percent government bonds issued by emerging market countries and 20 percent corporate bonds issued by companies registered in emerging markets. Around 50 percent of its holdings are in BBB rated bonds while another 45 percent are split roughly evenly between ratings BB, B and *Not rated*, leaving a mere 6 percent for A rated bonds. IHYG holds EUR denominated high yield corporate bonds, where *High yield* is a label used to mark a bond with a certain low credit rating of BBB- or below from Standard and Poors or Ba or below from Moody's. The two rating agencies characterize these ratings as "Considered lowest investment grade by market participants" [41] respectively "Obligations rated Ba are judged to have speculative elements and are subject to substantial credit risk." [42]. 90 percent of the holdings mature within 7 years, with approximately 30 percent of the holdings in each category 1-3 years, 4-5 years, 6-7 years and the remainder in the category 8-10 years to maturity. Naturally time to maturity is considerably important when investing in high yield bonds.

The remaining funds (DGT, ELR, FEZ, FXC, IEEM, IMEU, INAA, RWX, STN, STZ, TOPIX, XOP) are highly correlated with coefficient between 0.60 and 0.98. Very high correlations (>90 percent) are seen between DGT:ELR, DGT:FEZ, DGT:IMEU, DGT:INAA, ELR:INAA, IMEU:STZ. Again referring back to table 4.3 it is seen that these all represent large cap equity indices. Further DGT is an international index covering both developed and emerging countries and holding as many as 150 equities. Thus this index is a wide representation of the equity market in general. The ELR fund holds as many as 750 different company shares, selected to represent the top 750 US companies ranked by capi-

talization. Intuitively this is a very good proxy of the DGT, which is also mirrored by a correlation coefficient of 0.96. Similar reasoning apply to FEZ, IMEU and INAA which all represent the large cap equity category of the worlds leading developed regions. INAA and ELR are both holding USD large cap equity, explaining the perfect correlation between the two. From the correlation between IMEU (EUR large cap equity) and STZ (EUR financials sector) it is deduced that STZ primarily hold large cap stocks. This is confirmed by consulting the relevant morningstar<sup>®</sup> webpage, listing that 53 percent of the funds holding are giant cap, 34 percent are large cap and merely 13 percent are medium cap.

Due to the high correlation between these indices they can to a wide extend be used interchangeably in a portfolio.



**Figure A.1:** The raw data of IBGS DiffReturn in high and low frequency data. The top panel shows the processes, and the bottom panels show the autocorrelation and partial autocorrelation of the daily and weekly observations. The plots clearly reject any resemblance to independent random data.

## A.4 Modelling failure of IBGS

The process IBGS proved impossible to model in the applied framework in both high and low frequency data. Inspecting the raw data suggests a certain pattern, rejecting the premise of complete randomness. This is illustrated in figure A.1. It is outside the scope of this thesis to investigate alternative models, and the fund will no longer be a subject of analysis.



## A.5 Sensitivity of Jarque-Bera and Ljung-Box tests

If the residuals are normally distributed, the Jarque-Bera statistic is asymptotically  $\chi^2$  distributed with two degrees of freedom. It is computed

$$JB = \frac{n}{6} \left( S^2 + \frac{1}{4} (K - 3)^2 \right)$$

where

$$S = \frac{\frac{1}{n} \sum_{t=1}^n (r_t - \hat{\mu}_r)^3}{\left( \frac{1}{n} \sum_{t=1}^n (r_t - \hat{\mu}_r)^2 \right)^{3/2}}$$

$$K = \frac{\frac{1}{n} \sum_{t=1}^n (r_t - \hat{\mu}_r)^4}{\left( \frac{1}{n} \sum_{t=1}^n (r_t - \hat{\mu}_r)^2 \right)^2}$$

that is,  $S$  is the skewness of the distribution and  $K$  is the excess kurtosis.  $r_t$  and  $\hat{\mu}_r$  denotes the return at time  $t$  and the sample mean return, respectively.

The Box-Pierce test is computed

$$BL = n(n+2) \sum_{k=1}^h \frac{\hat{\rho}_k^2}{n-k}$$

where  $\hat{\rho}_k$  is the sample autocorrelation at lag  $k$  and  $h$  is the number of lags being tested. The hypothesis of independence in the residuals is rejected for  $BL > \chi_{1-\alpha, h}^2$  where  $\alpha$  is the significance level and  $h$  is the degrees of freedom.

The p-values for the Jarque-Bera test are zero or extremely close to zero in all of the models, indicating that the hypothesis must be rejected and that the residuals are not normally distributed. However, performing a small test proves that the Jarque-Bera test is extremely sensitive to outliers. When generating 1000 random numbers from a normal distribution the null hypothesis is accepted. However, when adding one single outlier to the same dataset, the null hypothesis is rejected.

```
> x = rnorm(1000); jarque.bera.test(x)

      Jarque Bera Test

data:  x
X-squared = 2.8493, df = 2, p-value = 0.2406

> y = c(x, 5); jarque.bera.test(y)

      Jarque Bera Test

data:  y
X-squared = 9.403, df = 2, p-value = 0.009082
```

Additionally the residuals are inspected for independence qua the Ljung-Box test. The p-values for this test, using 95 percent confidence intervals, span from 0.98 to 0 in the selected models. Once again, a simple test proves that also the Ljung-Box test is sensitive to outliers, although not as sensitive as the Jarque-Bera test. 10 data points are added to the 1000 point dataset to distort the independence.

```
> x = rnorm (1000); Box.test(x, type="Ljung")

      Box-Ljung test

data:  x
X-squared = 0.0035, df = 1, p-value = 0.9527

> y = c(x[1:500],rep(4,10), x[501:1000]); Box.test(y, type="Ljung")

      Box-Ljung test

data:  y
X-squared = 15.864, df = 1, p-value = 6.806e-05
```

## A.6 Failed models under ARMA-GARCH scenario generation

The ARMA-GARCH models experienced a number of problems during the recursive model estimation in the scenario generation. In the following the problems with FEZ, which was most widely affected, are described in details. The described models are the ones which failed to be properly fitted, and were thus replaced by their predecessor in generating the scenarios. In such cases the model parameters are estimated, but the hessian is non-invertible causing the uncertainty on the parameters to remain unknown. Additionally the conditional variance takes negative values. Out of the initial 66 failed attempts under the L-BFGS-B algorithm the Nelder Mead algorithm managed to successfully estimate a model in 32 cases, while also failing the remaining 34. This suggests that the Nelder Mead optimisation algorithm is more robust on the selected data.

Table A.1 shows the range of the FEZ scenarios over the 79 periods, prior to replacing the erroneous models. Pointing out that the scenarios represent four week returns on a stock, the range in period two, three, 10 and 11 is clearly wrong. The estimation of these models caused a warning that some parameters could not be properly estimated, as described in Table 6.2.2.

In periods two and three, the conditional variance explodes, ending at  $7.27e+72$ , thereby causing the range of the generated returns to explode. In periods 10 and 11  $\alpha$  and  $\beta$  are both estimated to be 1. This causes the conditional variance to continually increase with a slope of  $\omega$ , in this case 0.0055, thereby causing the error terms to be increasingly bigger. The implications for the unconditional variance is a denominator of -1 and thus an unconditional variance of  $-\omega$  which is meaningless. In period 18 and 20 the scenario generation fail. In these cases the model parameters are very similar to those in period 2 and 3, yet slightly shifted, causing the conditional variance to be negative, ending at respectively  $-3.58e+65$  and  $-5.45e+68$ . This happens because the GARCH parameters obtain negative values in the model estimation. This is a violation of the model assumptions, c.f. (5.3). When the conditional variance remain positive

in the first two cases, it has to do with the level of the three relevant parameters.

Period	$\omega$	$\alpha$	$\beta$
2 + 3	-0.000309	-0.151	1.72
10 + 11	0.00549	0.999	0.999
18	-0.000358	-0.325	1.51
20	-0.000377	-0.331	1.52

Periods 2, 3, 18 and 20 violate the non-negativity constraint. In 2 and 3 the relationship between  $\alpha$  and  $\beta$  cause the conditional variance to remain positive, while in 18 and 20 the conditional variance is negative after 4 observations. Due to the very high, positive coefficient to  $\beta$  in either of the cases, the remaining parameters are not sufficient to turn the effect of  $\beta$ , thereby causing the conditional variance to explode, towards  $\infty$  in 2 and 3 and towards  $-\infty$  in 18 and 20. 10 and 11 fulfil the non-negativity in coefficient condition, but  $\alpha + \beta > 1$ .

Periods 2 and 3 respectively 10 and 11 are identical due to the procedure described in [Table 6.2.2](#) for handling erroneous models. At the time of writing the author is not able to explain why some models are detected as erroneous, causing the procedures to be followed as described, while other erroneous models remain undetected by the algorithm.

	1	2	3	4	5	6	7	8	9	10
min	-0.20	-2.7e147	-2.4e147	-0.19	-0.19	-0.22	-0.20	-0.19	-0.18	-1.1e3
max	0.27	1.7e147	2.5e147	0.28	0.24	0.24	0.33	0.26	0.26	877.09
	11	12	13	14	15	16	17	18	19	20
min	-860.82	-0.10	-0.18	-0.19	-0.17	-0.18	-0.17	NA	-0.12	NA
max	1.3e4	0.16	0.29	0.24	0.28	0.18	0.26	NA	0.20	NA
	21	22	23	24	25	26	27	28	29	30
min	-0.23	-0.14	-0.14	-0.13	-0.16	-0.17	-0.14	-0.17	-0.18	-0.16
max	0.22	0.21	0.19	0.23	0.19	0.17	0.22	0.22	0.21	0.27
	31	32	33	34	35	36	37	38	39	40
min	-0.15	-0.16	-0.14	-0.15	-0.16	-0.18	-0.64	-0.44	-0.62	-0.55
max	0.29	0.24	0.25	0.32	0.30	0.23	0.73	0.83	1.40	1.67
	41	42	43	44	45	46	47	48	49	50
min	-0.42	-0.38	-0.36	-0.29	-0.32	-0.42	-0.36	-0.25	-0.27	-0.31
max	0.53	1.16	0.61	0.42	0.73	0.41	0.57	0.44	0.37	0.49
	51	52	53	54	55	56	57	58	59	60
min	-0.26	-0.23	-0.24	-0.25	-0.21	-0.19	-0.14	-0.28	-0.21	-0.37
max	0.46	0.27	0.32	0.24	0.28	0.31	0.37	0.33	0.35	0.38
	61	62	63	64	65	66	67	68	69	70
min	-0.23	-0.20	-0.18	-0.16	-0.24	-0.15	-0.10	-0.16	-0.21	-0.19
max	0.36	0.37	0.31	0.25	0.27	0.26	0.21	0.24	0.21	0.24
	71	72	73	74	75	76	77	78	79	
min	-0.17	-0.15	-0.23	-0.34	-0.24	-0.37	-0.31	-0.45	-0.33	
max	0.27	0.37	0.25	0.58	0.36	0.47	0.46	0.76	0.70	

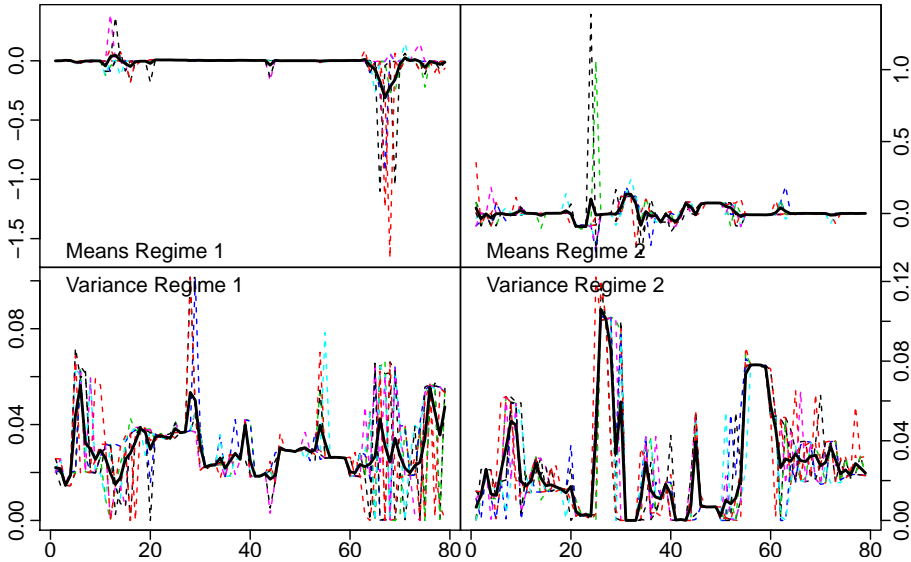
Table A.1: Range of FEZ predictions in 79 periods

## A.7 Rejected MSAR models

This table shows a large subset of rejected models for the nine indices. The index return series were attempt fitted with MSAR models of varying model orders and the preferred models were selected based on BIC and parameter significance. The selected models are highlighted in grey.

Data	AR(m,n)	Intercept	BIC	Exp. duration	Par. significance
DGT	0,0	x	-5236,286	12.50857 63.87707	int 2
DGT	1,0	x	-5229,38	12.30991 63.85709	int 2
DGT	1,1	x	-5223,393	12.54744 63.86098	int 2
DGT	1,1	x	-5220,875	77.57261 13.23333	-
DGT	1,2		-5212,853		-
DGT	2,2	x	-5206,185		-
ELR	2,2		-5125,857	57.30306 83.98200	-
ELR	1,1	x	-5153,799	60.53816 81.21398	int2, AR(1)2
ELR	2,1	x	-5142,2	61.83453 82.16936	int2, AR(1)2
ELR	1,1		-5142,901		-
ELR	1,0	x	-5152,991	57.04495 78.84465	int 2
ELR	0,0	x	-5155,773	58.94034 78.02196	int 2
FEZ	1,0	x	-2232,454	27.43766 230.28821	int 2
FEZ	1,1		-2229,735	246.97633 28.19155	-
FEZ	1,1	x	-2229,645	236.51295 29.18852	int 2
FEZ	2,2	x	-2214,764		-
FEZ	2,2		-2214,754		-
FEZ	0,0	x	-2238,147	28.39696 230.35163	int 2
GLD	0,0	x	-5525,087	36.58532 19.04899	int 2
GLD	1,0	x	-5520,308	18.93800 36.35756	int 1, int 2
GLD	1,1	x	-5513,447	36.72257 19.17539	int 2
GLD	1,1		-5521,103	36.30355 18.63641	-
RWX	1,1		-4459,042	37.34789 17.48983	-
RWX	1,1	x	-4457,353	32.34645 17.27798	int 1, int 2
RWX	2,1	x	-4450,383		-
RWX	2,1	x	-4450,383		-
RWX	1,0	x	-4463,665	17.34338 32.14350	int 2
RWX	0,0	x	-4468,917	17.43270 31.74602	int 2
STN	1,1		-3755,718	10.06519 96.83689	AR(1)1
STN	1,1	x	-3748,526	9.401092 113.925817	int 2
STN	2,1	x	-3736,797		-
STN	2,2		-3737,111		-
STN	1,0	x	-3754,182	9.229262 104.100518	int 2
STN	0,0	x	-3756,508	9.412034 116.849997	int 2
STZ	1,1		-3750,986	74.27124 29.02466	-
STZ	1,1	x	-3753,655	30.64846 64.25441	int 2
STZ	2,1	x	-3741,793		-
STZ	2,1		-3744,173		-
STZ	2,2	x	-3737,398		-
STZ	1,0	x	-3757,203	30.06586 60.80649	int 2
STZ	0,0		-3763,724	60.44992 30.81178	int 1
TOPIX	1,1		-4967,782	22.68193 12.62767	AR(1)1
TOPIX	1,1	x	-4913,229		-
TOPIX	2,1	x	-4897,65		-
TOPIX	2,1		-4960,304	12.97564 22.68945	AR(1)2
TOPIX	2,2		-4953,364	13.16962 22.57209	AR(1)2
TOPIX	1,0	x	-4956,674	12.79537 19.23521	int 2
TOPIX	0,0		-4959,791	21.11069 11.81463	-
XOP	1,1		-5480,799	51.16639 86.88108	-
XOP	1,1	x	-5487,764	70.81641 38.26960	int 2
XOP	2,1	x	-5475,016		-
XOP	2,1		-5469,474		-
XOP	2,2		-5469,558		-
XOP	1,0	x	-5493,07	39.13828 70.61969	int 2
XOP	0,0	x	-5499,034	40.10461 70.62847	int 2

**Table A.2:** The process of selecting the best models in the MSAR framework. The selected models are highlighted in grey. Column three indicate whether the model was fitted with different intercepts in the two regimes.



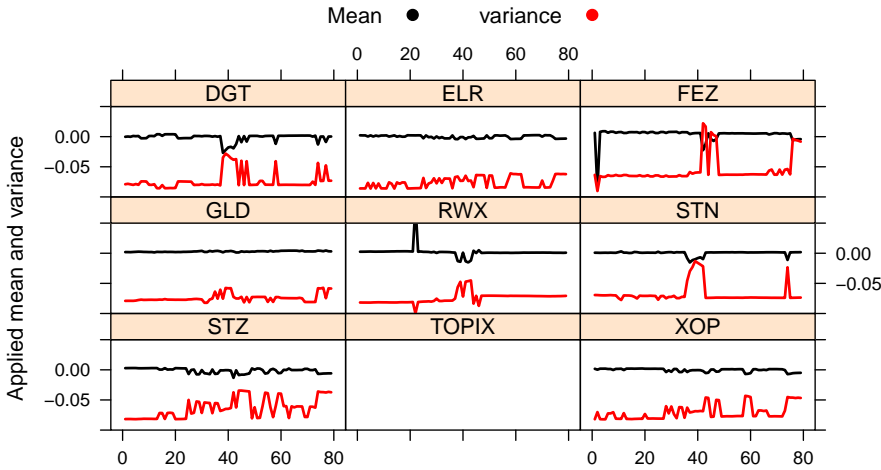
**Figure A.2:** Evolution in the mean scenario and variance in each index of the scenarios over the course of the 79 periods. The black line show the mean of the nine processes which are each represented by a coloured dotted line.

## A.8 Regime estimation

### A.8.1 MSAR

Figure A.2 shows in the top two panels the estimated mean return for the two regimes, over the course of the 79 periods. The values are extracted from the fitted models, used to generate scenarios from. The bottom two panels show the estimated variance in each regime. All of the indices are to a wide extent in agreement about the mean return, but the variance is different. Regime one shows a wide range of boundaries for the variance to change level, especially towards the end there is a wide disagreement. This is also present in regime two, but to a lesser extent, and generally the indices go together most of the way.

Figure A.3 shows the applied mean and variance, considering the selected regimes in each period. This plots clearly illustrate that the level of volatility and mean return is not very correlated.



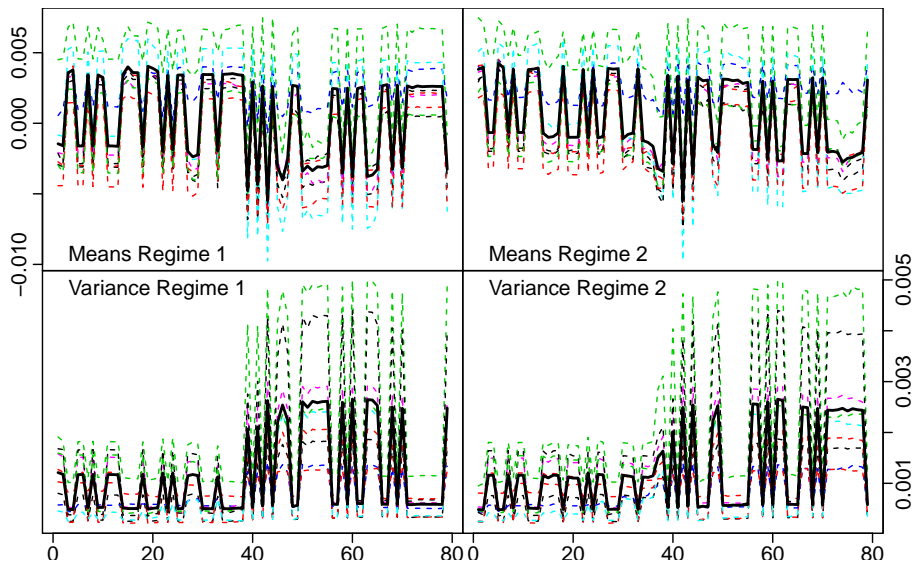
**Figure A.3:** Applied mean and variance over MSAR scenarios over the course of the 79 periods. The black line show the applied mean and the red line show a parallel shifting of the variance. The variance has been shifted by -0.1 for improved visibility.

### A.8.2 The dependent mixture model

Figure A.4 shows the evolution in the mean scenario and variance of the scenarios over the course of the 79 periods. The black line show the mean of the nine processes which are each represented by a coloured dotted line. Similar to the regime sequence the mean and variance associated with each regime fluctuate and are highly inconsistent.

The dotted lines show that the level for each index is different, but the evolution is similar – due to the nature of the model.





**Figure A.4:** Evolution in the mean scenario and variance of the scenarios over the course of the 79 periods. The black line show the mean of the nine processes which are each represented by a coloured dotted line.

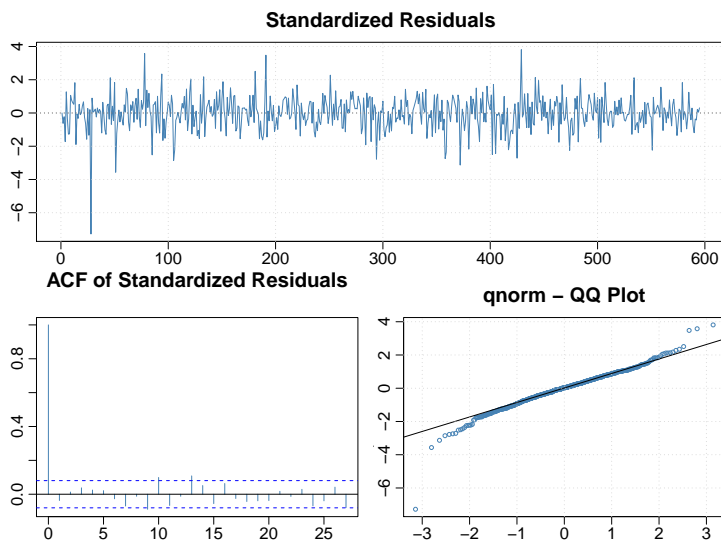


## APPENDIX B

# ARMA-GARCH models

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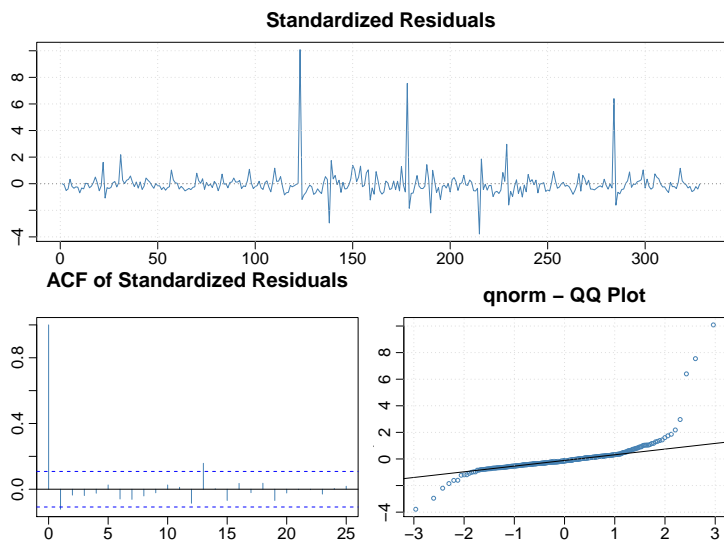
### B.1 Weekly data



## DGT

	Estimate	Std. Error	t value	Pr(> t)
mu	0.00	0.00	4.35	0.00
mal	-0.15	0.05	-2.88	0.00
omega	0.00	0.00	3.02	0.00
alpha1	0.27	0.06	4.23	0.00
beta1	0.61	0.07	8.25	0.00

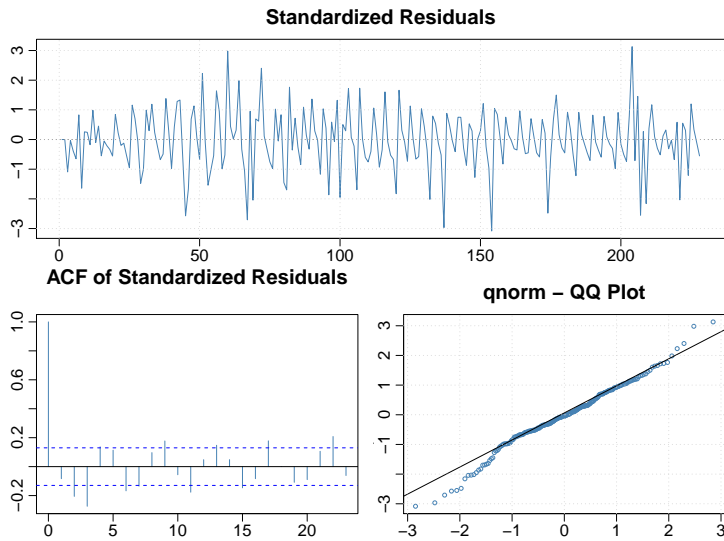
**Table B.1:** Estimated model coefficient and related uncertainty for DGT modelled by an arma(0, 1) + garch(1, 1) model.



## ELR

	Estimate	Std. Error	t value	$\Pr(> t)$
mu	0.00	0.00	1.10	0.27
omega	0.00	0.00	11.83	0.00
alpha1	0.32	0.10	3.19	0.00

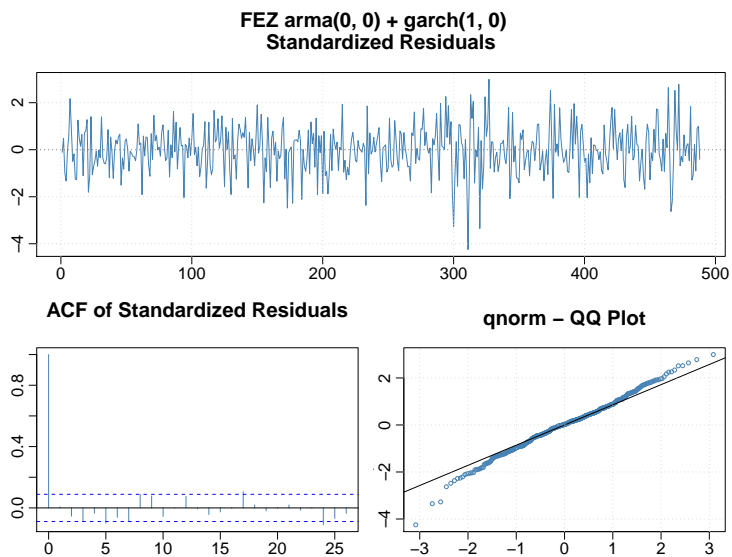
**Table B.2:** Estimated model coefficient and related uncertainty for ELR modelled by an arma(0, 0) + garch(1, 0) model.



## EMBI

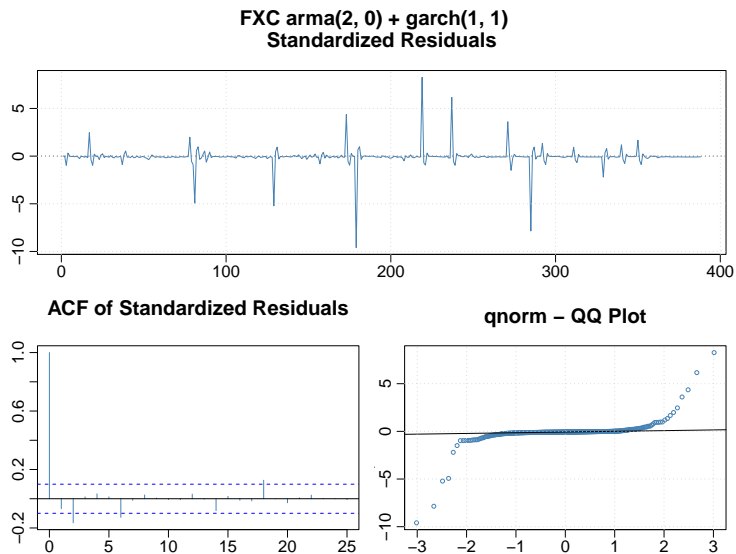
	Estimate	Std. Error	t value	Pr(> t)
mu	0.00	0.00	7.77	0.00
ar1	-0.51	0.07	-6.97	0.00
ar2	-0.15	0.06	-2.68	0.01
omega	0.00	0.00	6.63	0.00
alpha1	0.37	0.15	2.48	0.01

**Table B.3:** Estimated model coefficient and related uncertainty for EMBI modelled by an arma(2, 0) + garch(1, 0) model.

**FEZ**

	Estimate	Std. Error	t value	Pr(> t)
mu	0.00	0.00	2.43	0.02
omega	0.00	0.00	11.95	0.00
alpha1	0.16	0.06	2.62	0.01

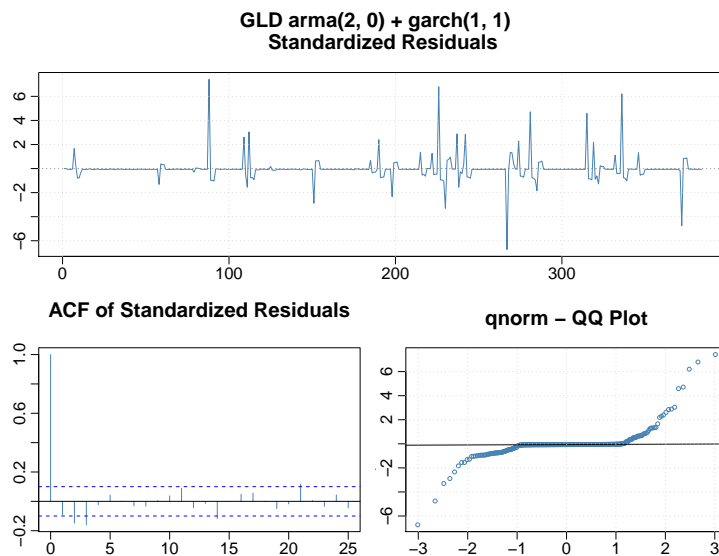
**Table B.4:** Estimated model coefficient and related uncertainty for FEZ modelled by an arma(0, 0) + garch(1, 0) model.

**FXC**

	Estimate	Std. Error	t value	Pr(> t)
mu	0.00	0.00	1.81	0.07
ar1	-0.51	0.15	-3.41	0.00
ar2	0.33	0.16	2.12	0.03
omega	0.00	0.00	7.56	0.00
alpha1	1.00	0.22	4.50	0.00
beta1	0.53	0.04	12.07	0.00

**Table B.5:** Estimated model coefficient and related uncertainty for FXC modelled by an arma(2, 0) + garch(1, 1) model.

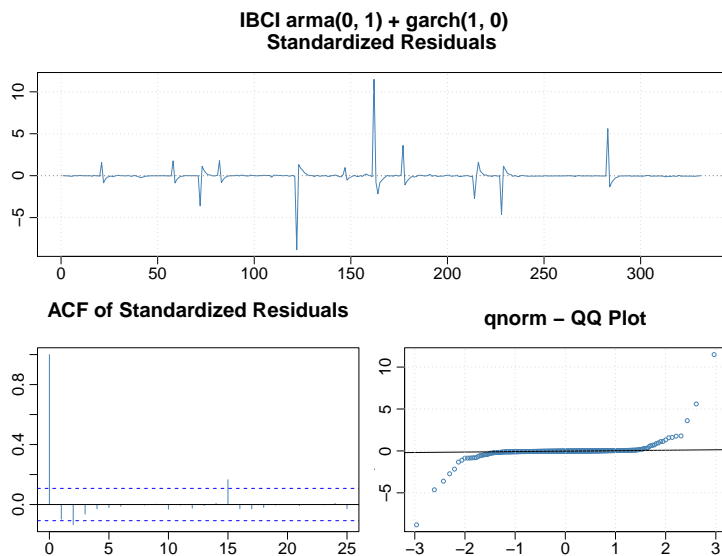




## GLD

	Estimate	Std. Error	t value	Pr(> t)
mu	0.00	0.00	1.52	0.13
ar1	-0.63	0.08	-8.30	0.00
ar2	-0.31	0.07	-4.77	0.00
omega	0.00	0.00	3.45	0.00
alpha1	0.22	0.09	2.32	0.02
beta1	0.43	0.15	2.83	0.00

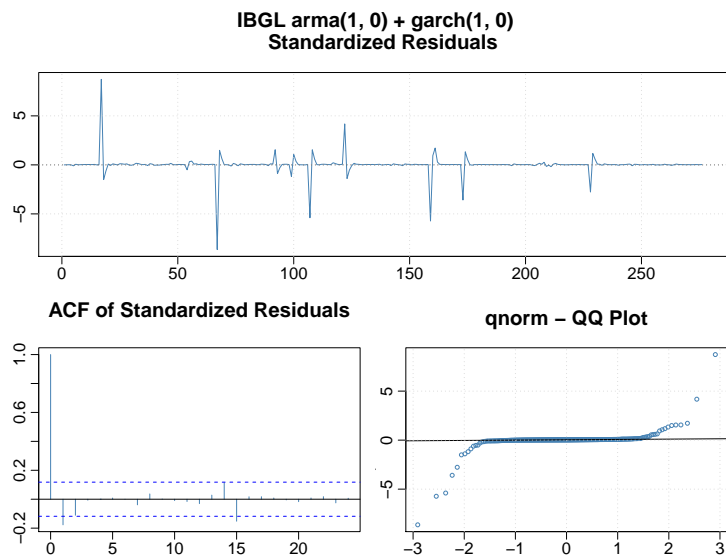
**Table B.6:** Estimated model coefficient and related uncertainty for GLD modelled by an arma(2, 0) + garch(1, 1) model.



## IBCI

	Estimate	Std. Error	t value	Pr(> t)
mu	0.00	0.00	1.41	0.16
mal	-0.42	0.12	-3.53	0.00
omega	0.00	0.00	12.46	0.00
alpha1	0.17	0.07	2.34	0.02

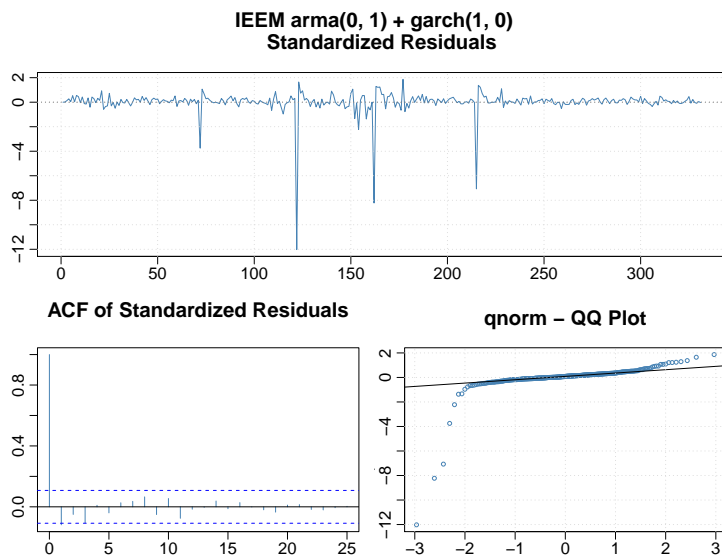
**Table B.7:** Estimated model coefficient and related uncertainty for IBCI modelled by an arma(0, 1) + garch(1, 0) model.



## IBGL

	Estimate	Std. Error	t value	Pr(> t)
mu	0.00	0.00	0.10	0.92
ar1	-0.24	0.08	-2.89	0.00
omega	0.00	0.00	11.37	0.00
alpha1	0.23	0.09	2.55	0.01

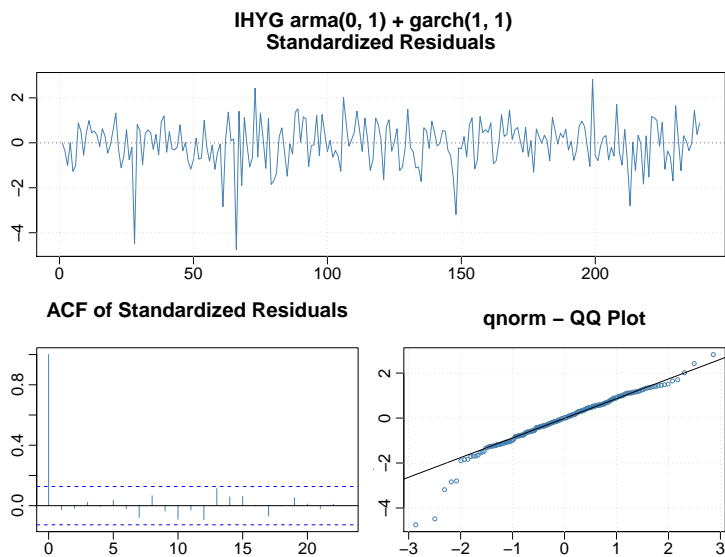
**Table B.8:** Estimated model coefficient and related uncertainty for IBGL modelled by an arma(1, 0) + garch(1, 0) model.



## IEEM

	Estimate	Std. Error	t value	Pr(> t)
mu	-0.00	0.00	-1.10	0.27
mal	-0.41	0.10	-4.15	0.00
omega	0.00	0.00	12.38	0.00
alpha1	0.18	0.08	2.39	0.02

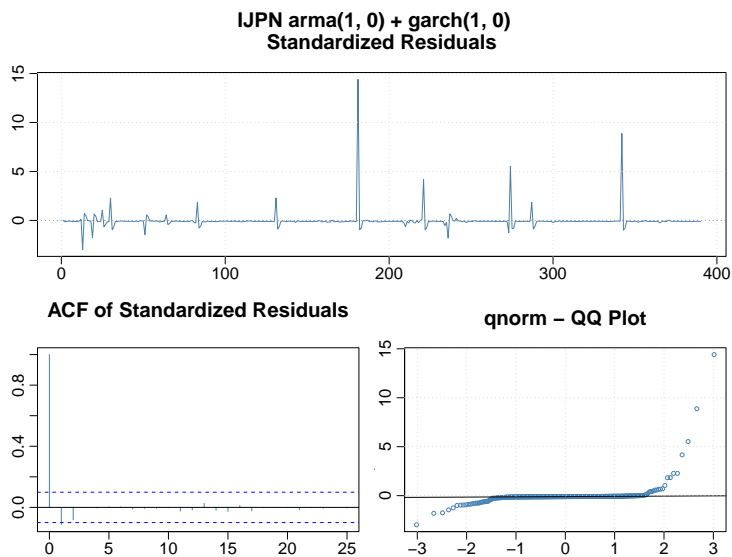
**Table B.9:** Estimated model coefficient and related uncertainty for IEEM modelled by an arma(0, 1) + garch(1, 0) model.



## IHYG

	Estimate	Std. Error	t value	Pr(> t)
mu	-0.00	0.00	-0.87	0.38
ma1	-0.26	0.07	-3.59	0.00
omega	0.00	0.00	2.55	0.01
alpha1	0.12	0.05	2.42	0.02
beta1	0.77	0.07	10.63	0.00

**Table B.10:** Estimated model coefficient and related uncertainty for IHYG modelled by an arma(0, 1) + garch(1, 1) model.



## IJPN

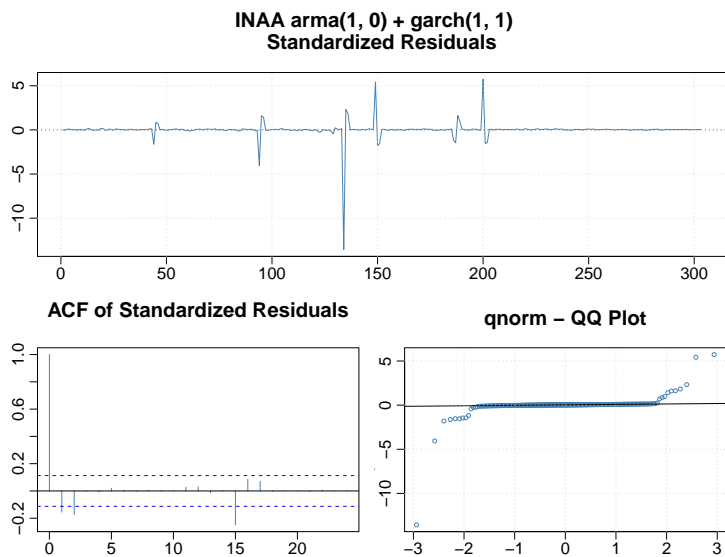
	Estimate	Std. Error	t value	Pr(> t)
mu	0.00	0.00	2.57	0.01
ar1	-0.34	0.09	-3.76	0.00
omega	0.00	0.00	12.94	0.00
alpha1	0.46	0.20	2.30	0.02

**Table B.11:** Estimated model coefficient and related uncertainty for IJPN modelled by an arma(1, 0) + garch(1, 0) model.

## IMEU

	Estimate	Std. Error	t value	Pr(> t)
mu	-0.00	0.00	-0.59	0.55
omega	0.00	0.00	10.88	0.00
alpha1	0.47	0.24	2.00	0.05

**Table B.12:** Estimated model coefficient and related uncertainty for IMEU modelled by an arma(0, 0) + garch(1, 0) model.

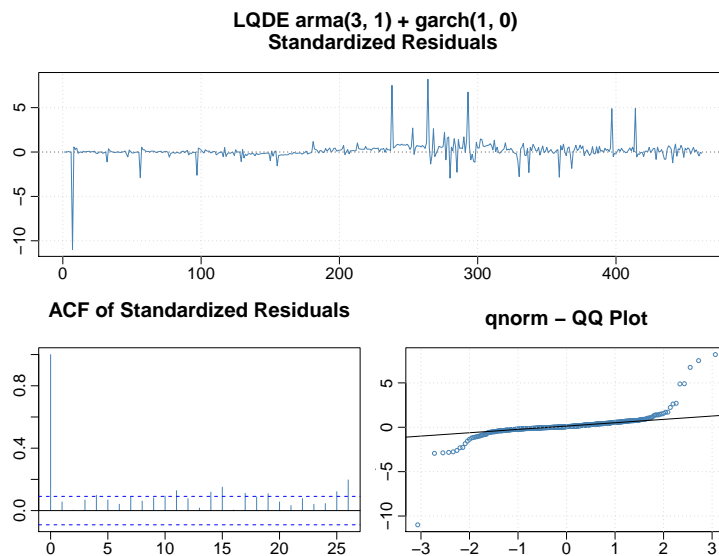


## INAA

	Estimate	Std. Error	t value	Pr(> t)
mu	-0.00	0.00	-0.48	0.63
ar1	-0.48	0.08	-5.95	0.00
omega	0.00	0.00	4.52	0.00
alpha1	0.06	0.02	2.71	0.01
beta1	0.80	0.04	20.22	0.00

**Table B.13:** Estimated model coefficient and related uncertainty for INAA modelled by an arma(1, 0) + garch(1, 1) model.

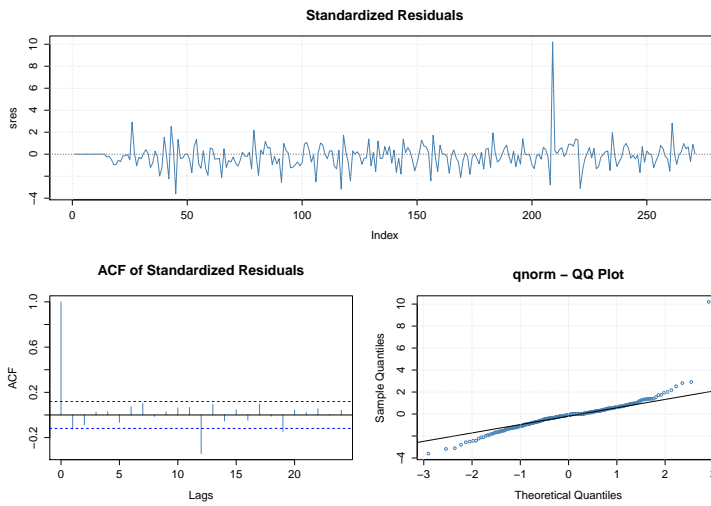




## LQDE

	Estimate	Std. Error	t value	Pr(> t)
mu	-0.00	0.00	-0.23	0.82
ar1	0.55	0.07	8.17	0.00
ar2	0.15	0.03	5.40	0.00
ar3	0.08	0.03	2.67	0.01
ma1	-0.97	0.01	-175.36	0.00
omega	0.00	0.00	11.21	0.00
alpha1	1.00	0.14	7.03	0.00

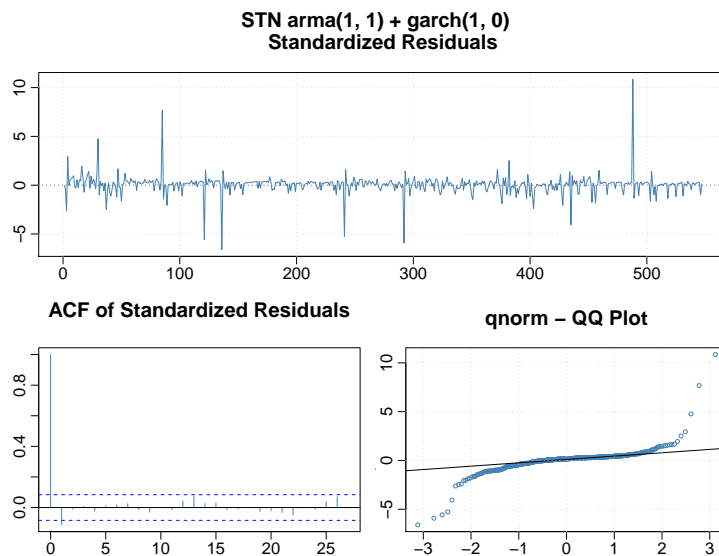
**Table B.14:** Estimated model coefficient and related uncertainty for LQDE modelled by an arma(3, 1) + garch(1, 0) model.



## RWX

	Estimate	Std. Error	t value	Pr(> t)
mu	0.00	0.00	1.35	0.18
ar1	-0.44	0.11	-4.10	0.00
ar2	-0.19	0.07	-2.70	0.01
omega	0.00	0.00	10.80	0.00
alpha1	0.23	0.10	2.39	0.02

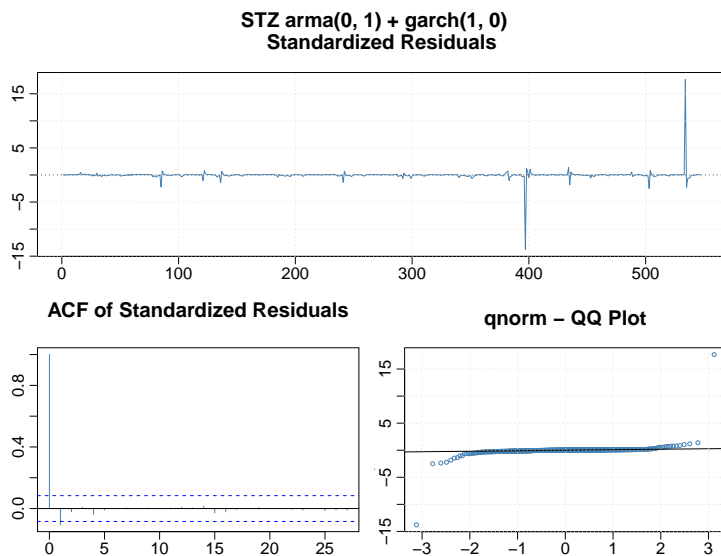
**Table B.15:** Estimated model coefficient and related uncertainty for RWX modelled by an arma(2, 0) + garch(1, 0) model.



STN

	Estimate	Std. Error	t value	Pr(> t)
mu	-0.00	0.00	-2.33	0.02
ar1	0.86	0.06	14.52	0.00
ma1	-0.95	0.03	-29.49	0.00
omega	0.00	0.00	13.93	0.00
alpha1	0.42	0.12	3.58	0.00

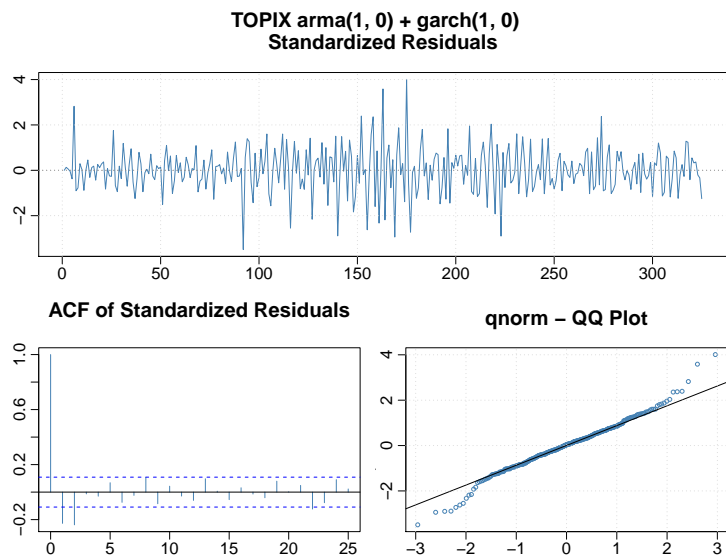
**Table B.16:** Estimated model coefficient and related uncertainty for STN modelled by an arma(1, 1) + garch(1, 0) model.



## STZ

	Estimate	Std. Error	t value	Pr(> t)
mu	-0.00	0.00	-1.23	0.22
mal	-0.32	0.10	-3.19	0.00
omega	0.00	0.00	16.28	0.00
alpha1	0.25	0.11	2.25	0.02

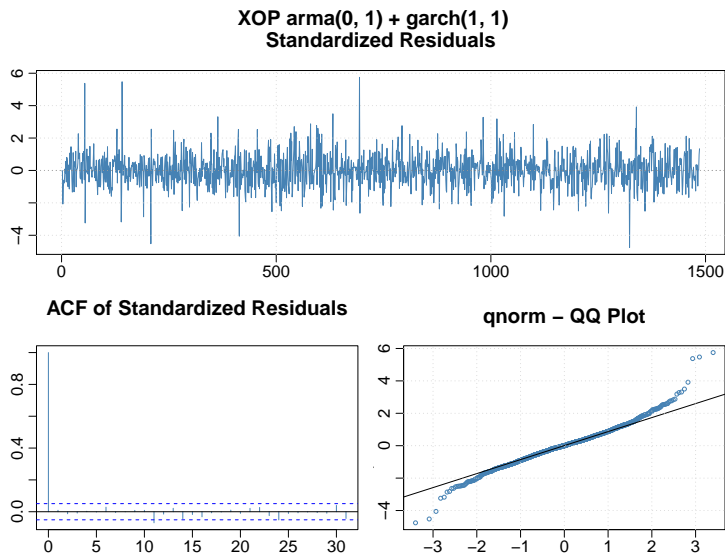
**Table B.17:** Estimated model coefficient and related uncertainty for STZ modelled by an arma(0, 1) + garch(1, 0) model.



## TOPIX

	Estimate	Std. Error	t value	Pr(> t)
mu	0.00	0.00	0.94	0.35
ar1	-0.41	0.06	-6.80	0.00
omega	0.00	0.00	8.36	0.00
alpha1	0.26	0.11	2.37	0.02

**Table B.18:** Estimated model coefficient and related uncertainty for TOPIX modelled by an arma(1, 0) + garch(1, 0) model.

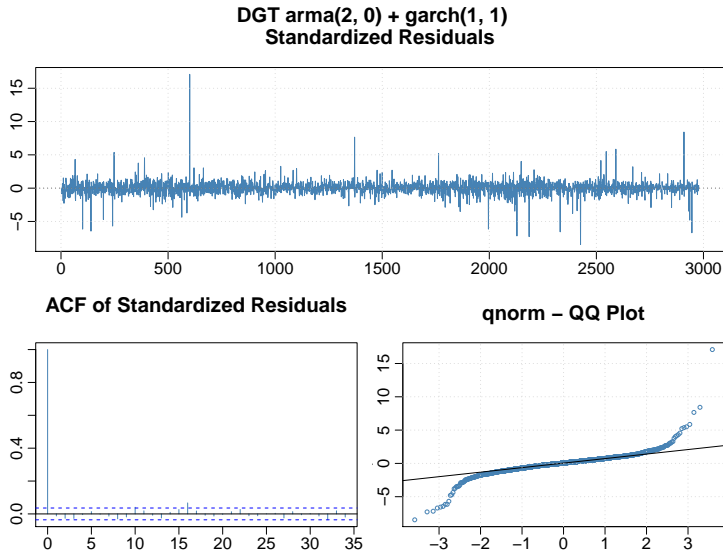


## XOP

	Estimate	Std. Error	t value	Pr(> t)
mu	-0.00	0.00	-2.27	0.02
omega	0.00	0.00	1.59	0.11
alpha1	0.08	0.03	2.97	0.00
beta1	0.89	0.04	23.75	0.00

**Table B.19:** Estimated model coefficient and related uncertainty for XOP modelled by an arma(0, 0) + garch(1, 1) model.

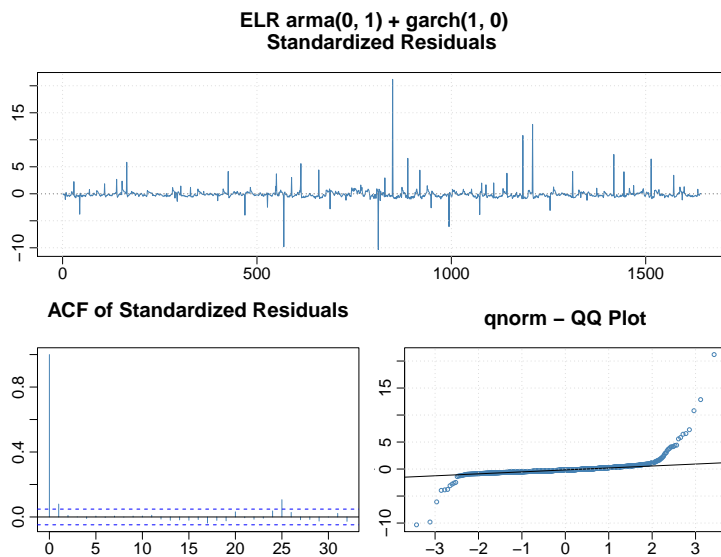
## B.2 Daily data



## DGT

	Estimate	Std. Error	t value	Pr(> t)
mu	0.00	0.00	1.41	0.16
ar1	-0.40	0.02	-16.79	0.00
ar2	-0.14	0.02	-5.92	0.00
omega	0.00	0.00	6.54	0.00
alpha1	0.30	0.04	7.68	0.00
beta1	0.63	0.04	14.96	0.00

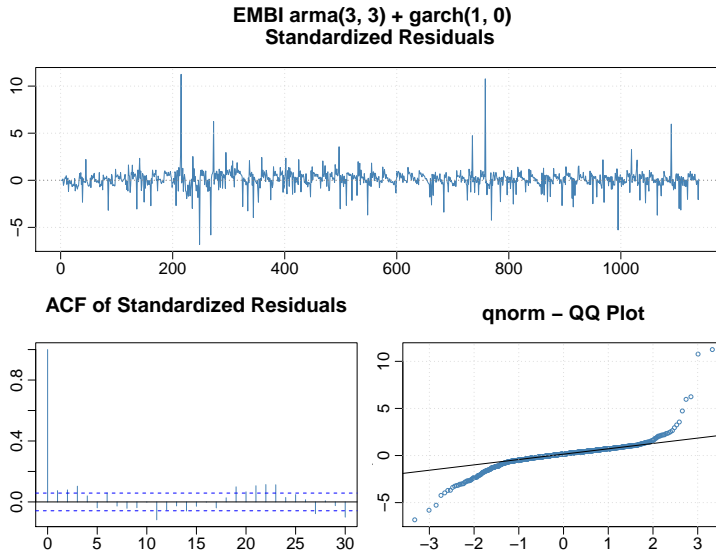
**Table B.20:** Estimated model coefficient and related uncertainty for DGT modelled by an arma(2, 0) + garch(1, 1) model.

**ELR**

	Estimate	Std. Error	t value	Pr(> t)
mu	0.00	0.00	2.85	0.00
mal	-0.82	0.02	-33.43	0.00
omega	0.00	0.00	7.58	0.00
alpha1	1.00	0.32	3.10	0.00

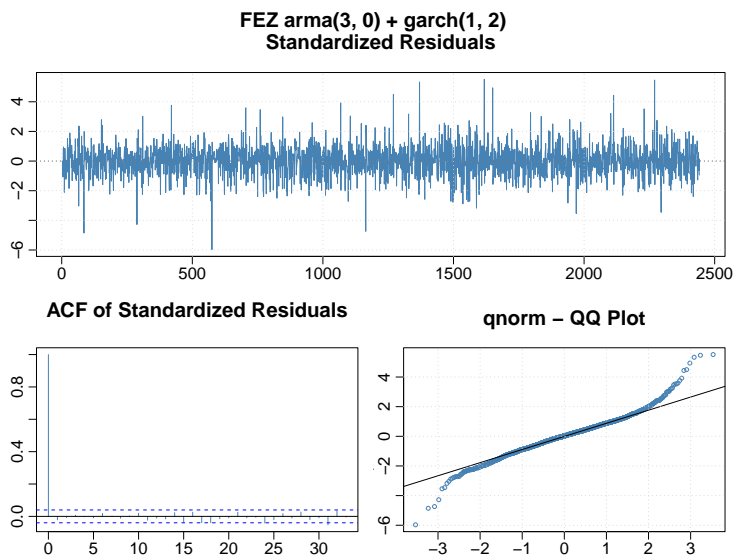
**Table B.21:** Estimated model coefficient and related uncertainty for ELR modelled by an arma(0, 1) + garch(1, 0) model.



**EMBI**

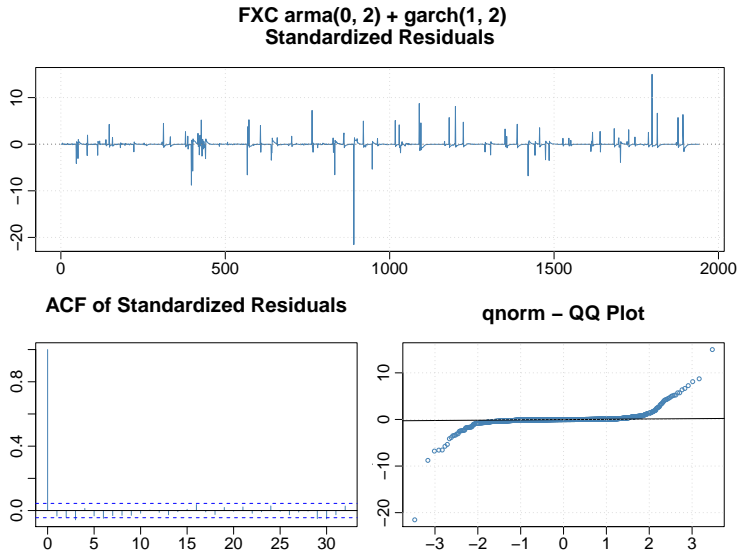
	Estimate	Std. Error	t value	Pr(> t)
mu	0.00	0.00	13.57	0.00
ar1	-0.54	0.06	-9.61	0.00
ar2	-0.71	0.05	-14.56	0.00
ar3	0.18	0.05	3.83	0.00
ma1	-0.16	0.02	-10.21	0.00
ma2	0.16	0.02	7.47	0.00
ma3	-0.88	0.03	-34.88	0.00
omega	0.00	0.00	13.33	0.00
alpha1	1.00	0.09	10.55	0.00

**Table B.22:** Estimated model coefficient and related uncertainty for EMBI modelled by an arma(3, 3) + garch(1, 0) model.

**FEZ**

	Estimate	Std. Error	t value	Pr(> t)
mu	0.00	0.00	2.24	0.02
ar1	-0.20	0.02	-9.04	0.00
ar2	-0.07	0.02	-3.17	0.00
ar3	-0.04	0.02	-2.10	0.04
omega	0.00	0.00	4.28	0.00
alpha1	0.10	0.01	7.49	0.00
beta1	0.13	0.05	2.45	0.01
beta2	0.75	0.05	13.87	0.00

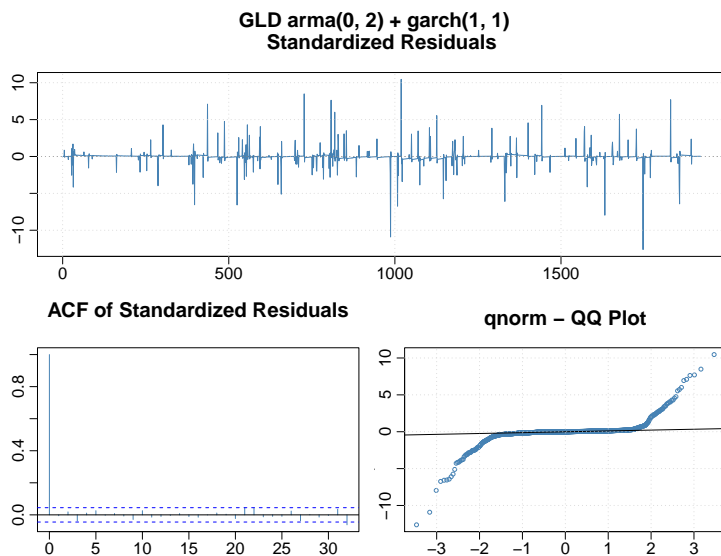
**Table B.23:** Estimated model coefficient and related uncertainty for FEZ modelled by an arma(3, 0) + garch(1, 2) model.



## FXC

	Estimate	Std. Error	t value	Pr(>t)
mu	0.00	0.00	1.47	0.14
ma1	-0.64	0.05	-12.51	0.00
ma2	-0.11	0.05	-2.29	0.02
omega	0.00	0.00	11.91	0.00
alpha1	0.25	0.05	5.44	0.00
beta1	0.21	0.07	2.98	0.00
beta2	0.12	0.04	2.76	0.01

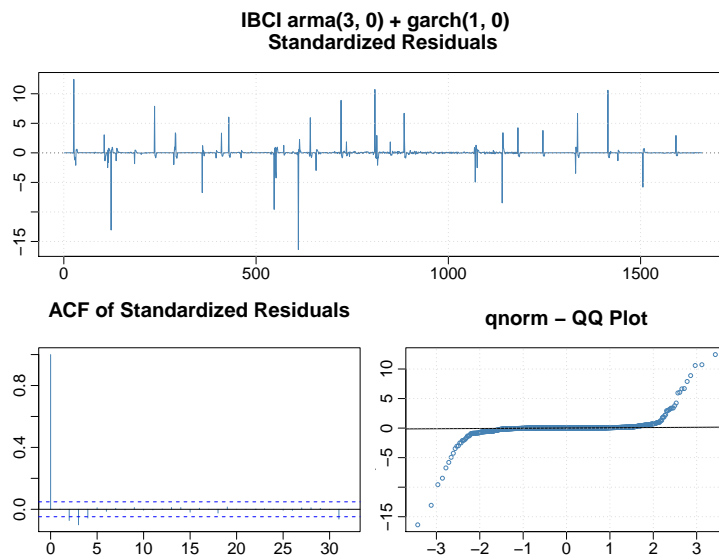
**Table B.24:** Estimated model coefficient and related uncertainty for FXC modelled by an arma(0, 2) + garch(1, 2) model.



## GLD

	Estimate	Std. Error	t value	Pr(>t)
mu	0.00	0.00	1.58	0.11
ma1	-0.88	0.03	-33.66	0.00
ma2	-0.09	0.03	-3.41	0.00
omega	0.00	0.00	2.39	0.02
alpha1	0.01	0.00	3.90	0.00
beta1	0.97	0.01	98.23	0.00

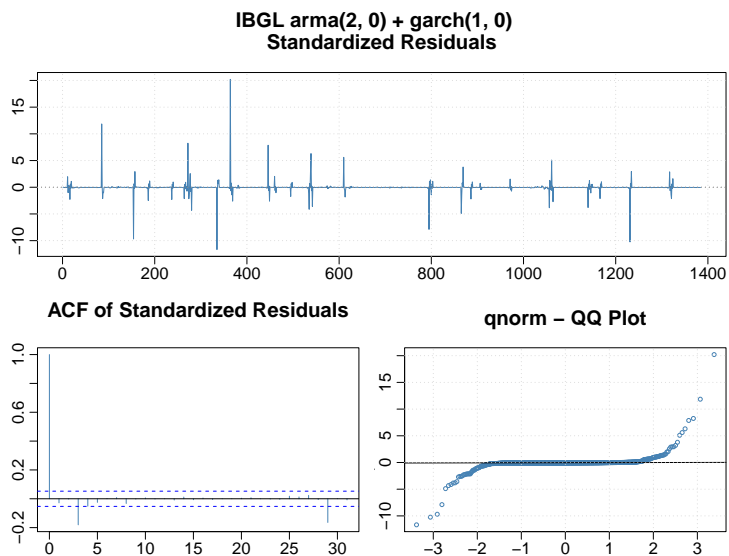
**Table B.25:** Estimated model coefficient and related uncertainty for GLD modelled by an arma(0, 2) + garch(1, 1) model.



## IBCI

	Estimate	Std. Error	t value	Pr(>t)
mu	0.00	0.00	0.36	0.72
ar1	-0.55	0.04	-13.53	0.00
ar2	-0.32	0.03	-9.70	0.00
ar3	-0.13	0.03	-3.66	0.00
omega	0.00	0.00	27.21	0.00
alpha1	0.74	0.12	6.35	0.00

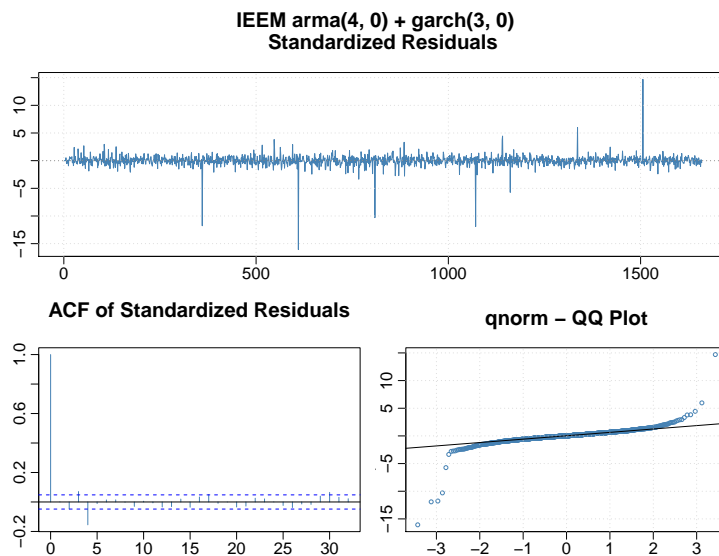
**Table B.26:** Estimated model coefficient and related uncertainty for IBCI modelled by an arma(3, 0) + garch(1, 0) model.



## IBGL

	Estimate	Std. Error	t value	Pr(>t)
mu	0.00	0.00	1.99	0.05
ar1	-0.35	0.02	-15.64	0.00
ar2	-0.49	0.03	-16.59	0.00
omega	0.00	0.00	23.46	0.00
alpha1	1.00	0.11	9.07	0.00

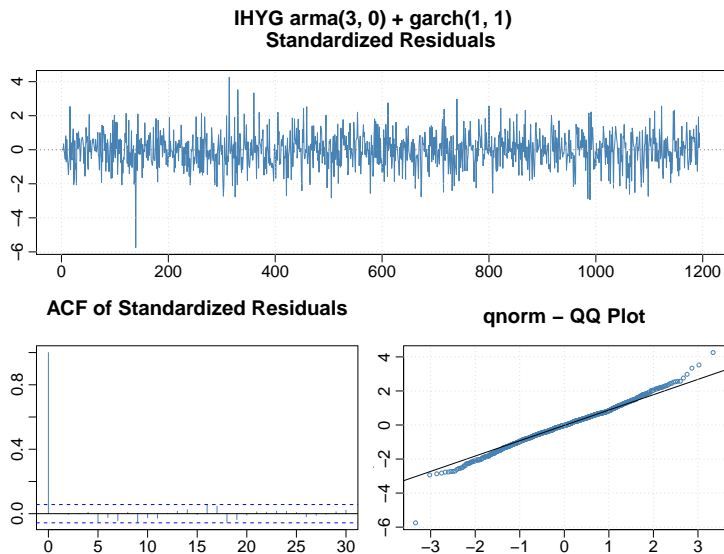
**Table B.27:** Estimated model coefficient and related uncertainty for IBGL modelled by an arma(2, 0) + garch(1, 0) model.



## IEEM

	Estimate	Std. Error	t value	Pr(>t)
mu	0.00	0.00	0.05	0.96
ar1	-0.49	0.03	-14.11	0.00
ar2	-0.18	0.03	-5.94	0.00
ar3	-0.45	0.07	-6.68	0.00
ar4	0.26	0.04	7.01	0.00
omega	0.00	0.00	4.76	0.00
alpha1	0.56	0.09	6.15	0.00
alpha2	0.82	0.19	4.31	0.00
alpha3	1.00	0.16	6.15	0.00

**Table B.28:** Estimated model coefficient and related uncertainty for IEEM modelled by an arma(4, 0) + garch(3, 0) model.

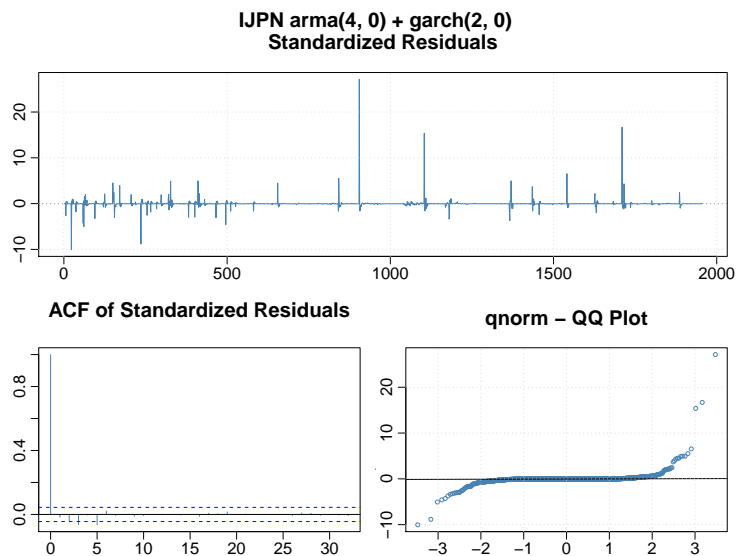


## IHYG

	Estimate	Std. Error	t value	Pr(>t)
mu	-0.00	0.00	-0.29	0.77
ar1	-0.10	0.03	-3.25	0.00
ar2	-0.08	0.03	-2.64	0.01
ar3	-0.10	0.03	-3.40	0.00
omega	0.00	0.00	2.51	0.01
alpha1	0.10	0.02	6.17	0.00
beta1	0.89	0.02	57.65	0.00

**Table B.29:** Estimated model coefficient and related uncertainty for IHYG modelled by an arma(3, 0) + garch(1, 1) model.

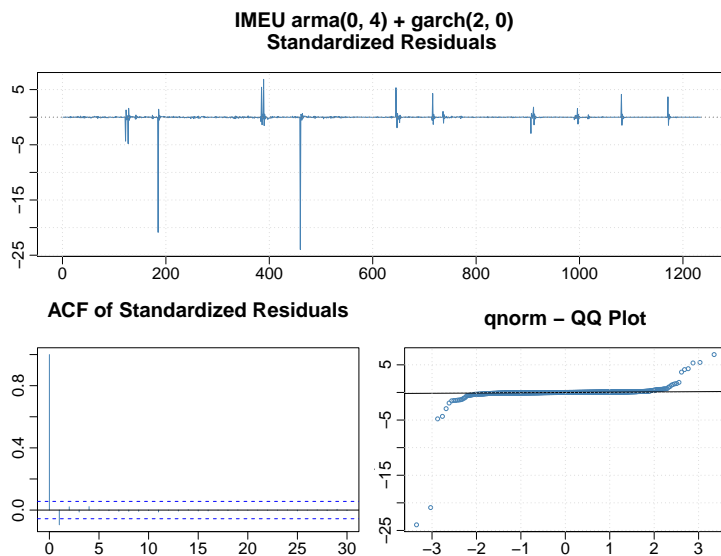




## IJPN

	Estimate	Std. Error	t value	Pr(>t)
mu	0.00	0.00	2.25	0.02
ar1	-0.49	0.05	-9.62	0.00
ar2	-0.32	0.03	-9.52	0.00
ar3	-0.08	0.03	-2.38	0.02
ar4	-0.11	0.02	-6.39	0.00
omega	0.00	0.00	29.52	0.00
alpha1	1.00	0.13	7.50	0.00
alpha2	0.07	0.03	2.24	0.03

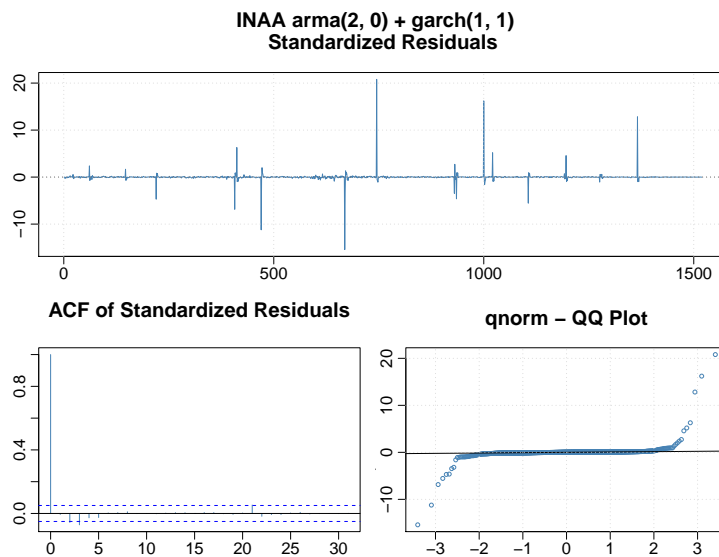
**Table B.30:** Estimated model coefficient and related uncertainty for IJPN modelled by an arma(4, 0) + garch(2, 0) model.



## IMEU

	Estimate	Std. Error	t value	Pr(>t)
mu	0.00	0.00	0.23	0.81
ma1	0.49	0.07	6.62	0.00
ma2	-0.28	0.08	-3.50	0.00
ma3	-0.30	0.06	-5.11	0.00
ma4	-0.08	0.03	-2.80	0.01
omega	0.00	0.00	23.23	0.00
alpha1	1.00	0.24	4.22	0.00
alpha2	1.00	0.31	3.20	0.00

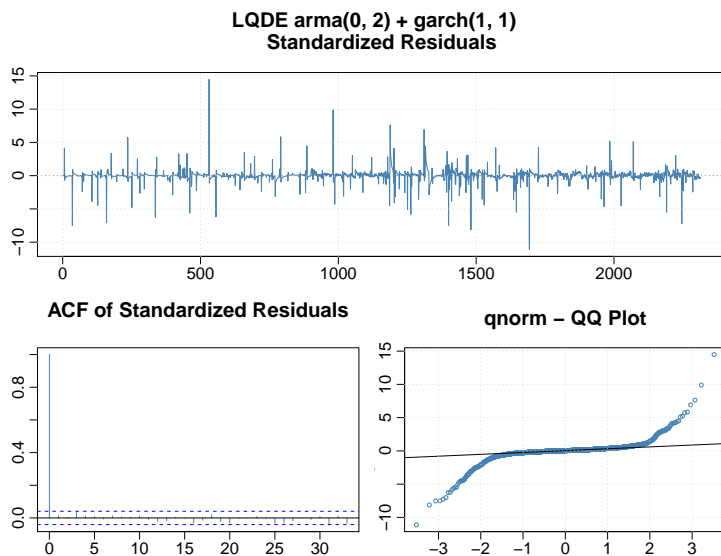
**Table B.31:** Estimated model coefficient and related uncertainty for IMEU modelled by an arma(0, 4) + garch(2, 0) model.



## INAA

	Estimate	Std. Error	t value	Pr(>t)
mu	0.00	0.00	0.19	0.85
ar1	-0.40	0.10	-4.03	0.00
ar2	-0.26	0.06	-4.53	0.00
omega	0.00	0.00	8.99	0.00
alpha1	0.37	0.09	3.96	0.00
beta1	0.25	0.08	3.31	0.00

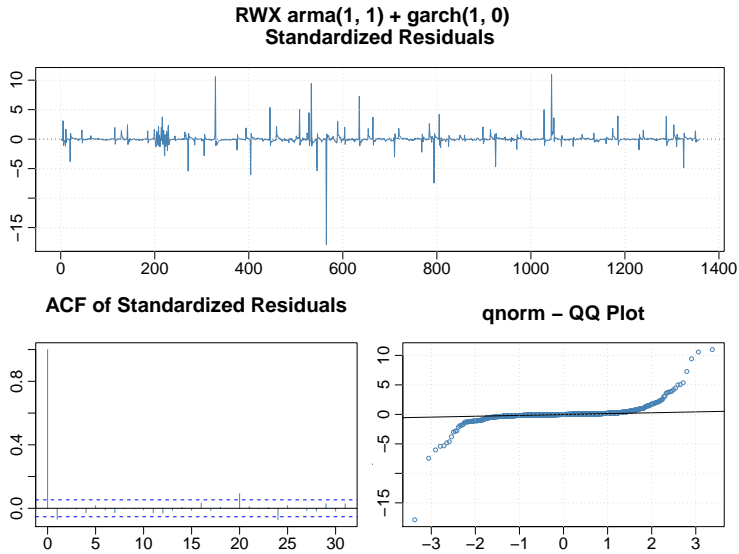
**Table B.32:** Estimated model coefficient and related uncertainty for INAA modelled by an arma(2, 0) + garch(1, 1) model.



## LQDE

	Estimate	Std. Error	t value	Pr(>t)
mu	0.00	0.00	0.34	0.73
ma1	-0.80	0.03	-31.70	0.00
ma2	-0.06	0.03	-2.18	0.03
omega	0.00	0.00	6.47	0.00
alpha1	0.02	0.00	8.82	0.00
beta1	0.97	0.00	331.16	0.00

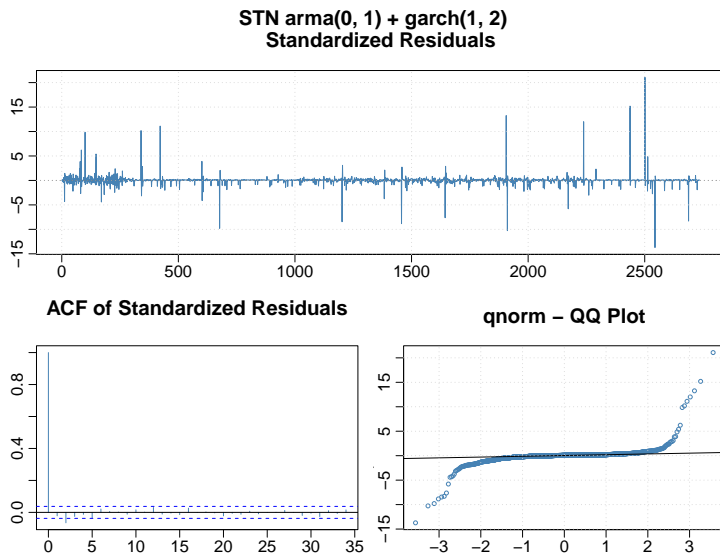
**Table B.33:** Estimated model coefficient and related uncertainty for LQDE modelled by an arma(0, 2) + garch(1, 1) model.



## RWX

	Estimate	Std. Error	t value	Pr(>t)
mu	-0.00	0.00	-0.17	0.87
ar1	0.34	0.09	3.97	0.00
ma1	-0.74	0.05	-13.85	0.00
omega	0.00	0.00	23.49	0.00
alpha1	0.31	0.08	3.82	0.00

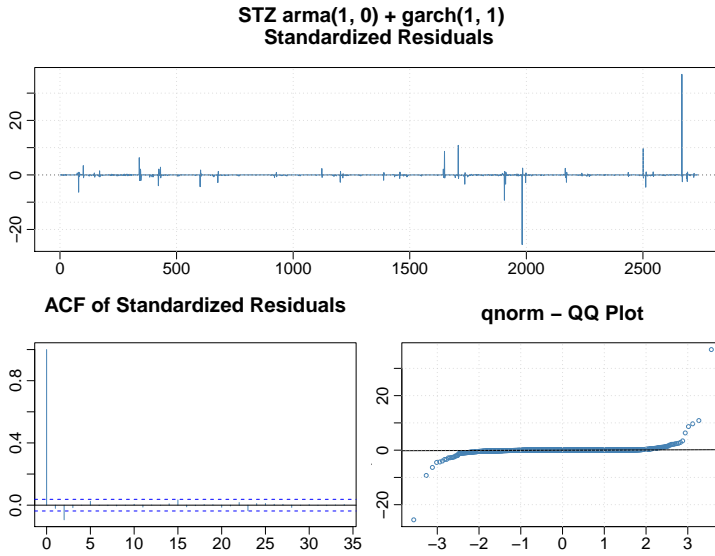
**Table B.34:** Estimated model coefficient and related uncertainty for RWX modelled by an arma(1, 1) + garch(1, 0) model.



## STN

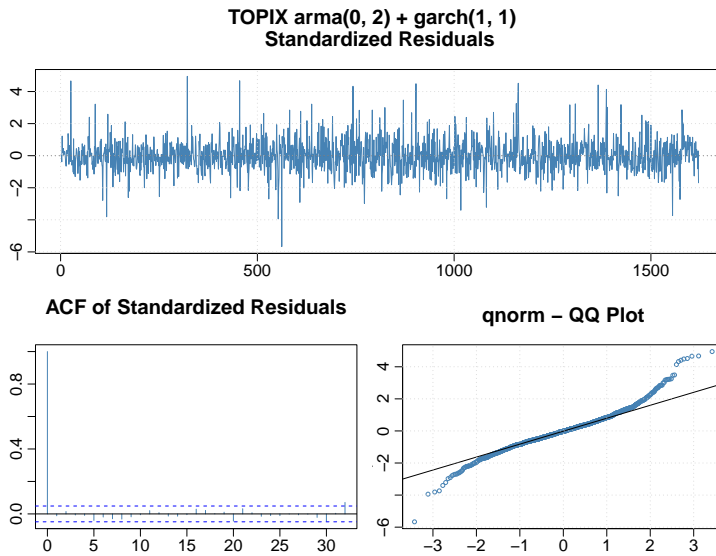
	Estimate	Std. Error	t value	Pr(>t)
mu	-0.00	0.00	-3.47	0.00
ma1	-0.35	0.05	-7.56	0.00
omega	0.00	0.00	17.59	0.00
alpha1	0.45	0.06	7.82	0.00
beta1	0.08	0.03	3.03	0.00
beta2	0.25	0.03	8.49	0.00

**Table B.35:** Estimated model coefficient and related uncertainty for STN modelled by an arma(0, 1) + garch(1, 2) model.

**STZ**

	Estimate	Std. Error	t value	Pr(>t)
mu	0.00	0.00	0.03	0.97
ar1	-0.33	0.06	-5.62	0.00
omega	0.00	0.00	12.07	0.00
alpha1	0.37	0.08	4.71	0.00
beta1	0.18	0.06	3.04	0.00

**Table B.36:** Estimated model coefficient and related uncertainty for STZ modelled by an arma(1, 0) + garch(1, 1) model.

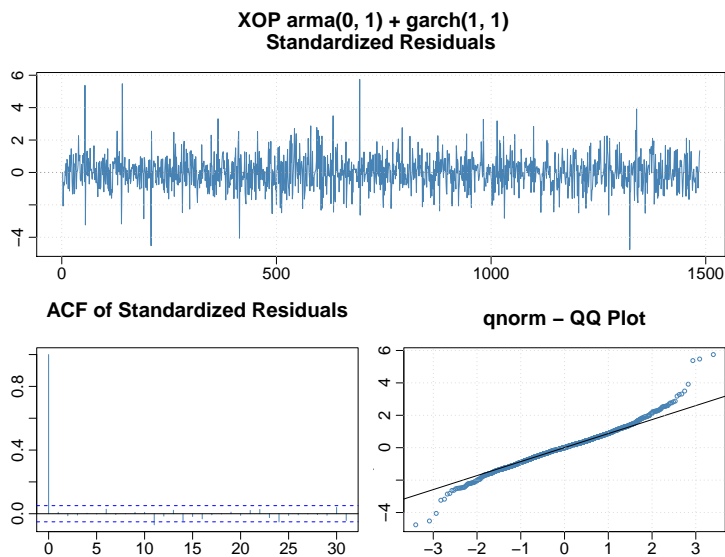


## TOPIX

	Estimate	Std. Error	t value	Pr(>t)
mu	0.00	0.00	5.07	0.00
ma1	-0.86	0.03	-30.69	0.00
ma2	-0.07	0.03	-2.49	0.01
omega	0.00	0.00	4.93	0.00
alpha1	0.14	0.02	7.09	0.00
beta1	0.83	0.02	39.40	0.00

**Table B.37:** Estimated model coefficient and related uncertainty for TOPIX modelled by an arma(0, 2) + garch(1, 1) model.





## XOP

	Estimate	Std. Error	t value	Pr(>t)
mu	-0.00	0.00	-3.11	0.00
ma1	-0.22	0.03	-8.21	0.00
omega	0.00	0.00	2.91	0.00
alpha1	0.05	0.01	5.79	0.00
beta1	0.94	0.01	86.73	0.00

**Table B.38:** Estimated model coefficient and related uncertainty for XOP modelled by an arma(0, 1) + garch(1, 1) model.



## APPENDIX C

# R code

---

### C.1 Functions

```
rm(list=ls())
options(width=150)
library(lattice)
library(xtable)
library(forecast)
library(fGarch)
library(chron)
library(mvtnorm)
library(depmixS4)
setwd("C://Users//Mie//Dropbox//Speciale//CSVdata")
load(".RData")

# Function to collect and isolate all unique dates in two date
# vectors.
All.Dates = function(x,y)
{
  start.date = min(c(x$Date, y$Date), na.rm=T); slut.date = max(c(x$
    Date, y$Date), na.rm=T)
  all.dates = seq(from = chron(as.numeric(start.date)), to = chron(
    as.numeric(slut.date)))
  all.dates = all.dates[!is.weekend(all.dates)]
}

# Fucton to merge two datasets. Used to merge fund and index data.
```

```

Add.Data = function(x,y){merge(x, y, all.x=T)}

# Interpolate NA values in x and adds normally distributed noise.
Remove.NA = function(x)
{
  if(is.null(dim(x)))
  {
    for(i in which(is.na(x[-length(x)])))
    {
      firstnonNA.after.i = min(which(!is.na(x[i:length(x)])))
      if(i>1)
      {
        e = rnorm(1,mean = 0, sd = sd(c(x[(i-5):(i-1)],j], x[(i+
          firstnonNA.after.i):(i+firstnonNA.after.i+5)],j), na.rm
          =T))
        x[i] = x[i-1] + (x[i+firstnonNA.after.i-1]-x[i-1])*1/
          firstnonNA.after.i + e
      }
    }
  } else
  for(j in 2:dim(x)[2])
  {
    for(i in which(is.na(x[-dim(x)[1],j])))
    {
      if(length(which(!is.na(x[i:dim(x)[1],j]))) < 1)
      {break} else
      {
        firstnonNA.after.i = min(which(!is.na(x[i:dim(x)[1],j])))
        if(i>1)
        {
          if(i<5)
          {
            e = rnorm(1,mean = 0, sd = sd(x[(i+2):(i+firstnonNA.
              after.i+10)],j], na.rm=T))
            x[i,j] = x[i-1,j] + (x[i+firstnonNA.after.i-1,j]-x[i-1,
              j])*1/firstnonNA.after.i + e
          }else
          {
            e = rnorm(1,mean = 0, sd = sd(c(x[(i-5):(i-1)],j], x[(i+
              firstnonNA.after.i):(i+firstnonNA.after.i+5)],j)],
              na.rm=T))
          }
          x[i,j] = x[i-1,j] + (x[i+firstnonNA.after.i-1,j]-x[i-1,j]
            )*1/firstnonNA.after.i + e
        }
      }
    }
  }
  return(as.data.frame(x))
}

# Removes outliers by interpolation of the adjacent cells.
# The sensitivity defaults to 2.5, but can be adjusted as an input
  to the function.
Remove.Outliers = function(x,y=2.5)
{
  if(is.null(dim(x)))

```

```

{
  for(i in (First.combi.nonNA(x,x)+2):(Last.combi.nonNA(x,x)-2))
  {
    if(abs(x[i-1]/x[i]) > y || abs(x[i-1]/x[i]) < 1/y)
    {
      e = rnorm(1,mean = 0, sd = sd(c(x[(i-5):(i-1)],j], x[(i+1):(i
+5)],j)), na.rm=T))
      x[i] = (x[i-1]+x[i+2])/2 + e
    }
  }
} else
for(j in which(names(x) %in% c("fNAV", "iNAV")))
{
  a = as.numeric(x[1:(length(x[,j])-2),j])
  b = as.numeric(x[3:length(x[,j]),j])
  for(i in First.combi.nonNA(a,b):Last.combi.nonNA(a,b))
  {
    if(!is.numeric(x[i,j]))
    {
      x[i,j] = 1
    }
    if(i < 3)
    {
      if(abs(x[i,j]) < 1/y*mean(x[(i+1):(i+10)],j) || abs(x[i,j]) > y
*mean(x[(i+1):(i+10)],j))
      {
        e = rnorm(1,mean = 0, sd = sd(x[(i+1):(i+10)],j), na.rm=T))
        x[i,j] = mean(x[(i+1):(i+10)],j) + e
      }
    }
    else if(i > Last.combi.nonNA(a,b))
    {
      if(abs(x[i-2,j]/x[i,j]) > y || abs(x[i-2,j]/x[i,j]) < 1/y)
      {
        e = rnorm(1,mean = 0, sd = sd(x[(i-10):(i-1)],j), na.rm=T))
        x[i,j] = (x[i-2,j]+x[i-3,j])/2 + e
      }
    }
    else
    if(abs(x[i-1,j]/x[i,j]) > y || abs(x[i-1,j]/x[i,j]) < 1/y)
    {
      e = rnorm(1,mean = 0, sd = sd(c(x[(i-5):(i-1)],j], x[(i+1):(i
+5)],j)), na.rm=T))
      x[i,j] = (x[i-1,j]+x[i+2,j])/2 + e
    }
  }
}
return(as.data.frame(x))
}

Return = function(x){c(0,(x[-1]-x[-length(x)])/x[-length(x)])}
LogReturn = function(x){c(0,log(x$iNAV[-1]/x$iNAV[-length(x$iNAV)]))
- log(x$fNAV[-1]/x$fNAV[-length(x$fNAV)]))}
# Finds the first non-NA cell in a vector
First.combi.nonNA = function(x,y){min(which(!is.na(x+y)))}

```

```

Last.combi.nonNA = function(x,y){max(which(!is.na(x+y)))}
Find.NA = function(x){which(is.na(x))}
# Scale x relative to y. y defaults to 100.
Skalering = function(x,y = 100)
{
  if(length(y)==1){faktor = x[1]; z = x/faktor*y; return(z)}else
    faktor = y[First.combi.nonNA(x,y)]/x[First.combi.nonNA(x,y)]; z
    = x*faktor; return(z)
}

# Transform data from daily to daily, weekly or monthly and add
  return columns.
# a defaults to Friday.
# b default to the day of the month which contains more
  observations.
# y accepts the values "daily", "weekly" and "monthly"
# x is a data.frame to be reduced.
Periodic = function(x,y,a = "fredag",b = min(which(table(as.POSIXlt
  (x$date)$mday)==max(table(as.POSIXlt(x$date)$mday))))))
{
  if(y=="daily"){x=x}
  else if(y=="weekly"){x=x[which(weekdays(x$date)== a),]}
  else if(y=="monthly"){x=x[which(as.POSIXlt(x$date)$mday == b),]}

  x$iReturn = Return(x$iNAV)
  x$fReturn = Return(x$fNAV)
  x$DiffReturn = x$iReturn - x$fReturn

  return(as.data.frame(x))
}

```

C:/Users/Mie/Dropbox/Specialer/R/Specialer/Funktioner-print.R

## C.2 Dataload

```

lec = c("DGT", "ELR", "EMBI", "FEZ", "FXC", "GLD", "IBCI", "IBGL",
  "IBGS", "IEEM", "IHYG",
  "IJPB", "IMEU", "INAA", "LQDE", "RWX", "STN", "STZ", "TOPIX", "
  XOP")

## Load and merge fund and index data.
## Data.frame reduced in both ends to span the lifetime of the fund
.
#####
filmatrix = matrix(NA, ncol=2, nrow=length(lec))

for(i in 1:length(lec))
{
  filer = paste(lec[i], c("f.csv","i.csv"), sep="")
  filmatrix[i,] = filer
  rm(filer)
}

```

```

for(i in 1:length(lec))
{
  for(j in 1:ncol(filmatrix))
  {
    fil = read.table(filmatrix[i,j], sep=";", header = TRUE, dec = "
      .", na.strings=c("", "N/A", "NA", "Na", "null", 0))
    fil$Date=as.Date(substr(fil$Date,1,10), "%d-%m-%Y")
    fil$NAV = as.numeric(fil$NAV)
    fil = fil[sort.list(fil$Date),]

    names(fil)[names(fil)=="NAV"]=paste(substr(filmatrix[i,j],(nchar(
      filmatrix[i,j])-4),nchar(filmatrix[i,j])-4),"NAV", sep="")

    filIJ = paste("fil", j, sep="")
    assign(filIJ, fil)
  }

  all.dates = data.frame(Date=All.Dates(fil1, fil2))
  dat = Add.Data(all.dates, fil2)
  dat = Add.Data(dat, fil1)
  dat = Remove.NA(dat)
  dat = dat[(First.combi.nonNA(dat$iNAV, dat$fNAV)+1):Last.combi.
    nonNA(dat$iNAV, dat$fNAV),]
  data = Remove.Outliers(dat)
  data$iNAV.raw = data$iNAV
  data$iNAV = Skalering(data$iNAV, data$fNAV)
  data$Date = as.Date(data$Date)

  d1 = fil1$Date[min(which(fil1$Date >= min(data$Date))):max(which(
    fil1$Date <= max(data$Date)))]
  d2 = fil2$Date[min(which(fil1$Date >= min(data$Date))):max(which(
    fil1$Date <= max(data$Date)))]
  dd = all.dates$Date
  a = which(dd %in% d1)[-1]-which(dd %in% d1)[-length(which(dd %in%
    d1))]-1
  b = which(dd %in% d2)[-1]-which(dd %in% d2)[-length(which(dd %in%
    d2))]-1

  assign(lec[i], data)
  assign(paste("a", lec[i], sep="."), a)
  assign(paste("b", lec[i], sep="."), b)
}
df = mget(lec, envir=.GlobalEnv)
all.a = mget(paste("a", lec, sep="."), envir = .GlobalEnv)
all.b = mget(paste("b", lec, sep="."), envir = .GlobalEnv)

## Load index data. Data.frame spans the lifetime of the index.
#####

ind = c("DGT", "ELR", "FEZ", "GLD", "RWX", "STN", "STZ", "TOPIX", "
  XOP")
filer = paste(lec[!(lec %in% ind)], "i.csv", sep="")
filer = c(filer, paste(ind, "i2.csv", sep=""))
filer = filer[sort.list(filer)]

```

```

for(i in which(filer!="RWXi2.csv"))
{
  fil = read.table(filer[i], sep=";", header = TRUE, dec = ".", na
    .strings=c("", "N/A", "NA", "Na", "null", 0))
  fil$Date=as.Date(substr(fil$Date,1,10), "%d-%m-%Y")
  fil$NAV = as.numeric(fil$NAV)
  fil = fil[sort.list(fil$Date),]
  names(fil)[names(fil)=="NAV"] = "iNAV"

  fil$iNAV = Skalering(fil$iNAV)
  all.dates = data.frame(Date=All.Dates(fil, fil))
  dat = Add.Data(all.dates,fil)
  dat = Remove.NA(dat)
  data = Remove.Outliers(dat)
  data$Date = as.Date(data$Date, "%d-%m-%Y")
  data = Periodic(data, "weekly")
  data = data[,which(names(data) %in% c("iNAV", "Date", "iReturn"))
  ]

  d1 = fil$Date [min(which(fil$Date >= min(data$Date))):max(which(
    fil$Date <= max(data$Date)))]
  dd = all.dates$Date
  a = which(dd %in% d1)[-1]-which(dd %in% d1)[-length(which(dd %in%
    d1))]-1

  assign(paste("a", lec[i], sep="."), a)
  assign(paste("index",lec[i], sep="."), data)
}

# CORRECTING RWX TO ADD NOISE TO THE INTERPOLATION OF THE FIRST
# COUPLE OF YEARS.
#####
i = which(lec == "RWX")
fil = read.table(filer[i], sep=";", header = TRUE, dec = ".", na.
  strings=c("", "N/A", "NA", "Na", "null", 0))
fil$Date=as.Date(substr(fil$Date,1,10), "%d-%m-%Y")
fil$NAV = as.numeric(fil$NAV)
fil = fil[sort.list(fil$Date),]
names(fil)[names(fil)=="NAV"] = "iNAV"
fil$iNAV = Skalering(fil$iNAV)
all.dates = data.frame(Date=All.Dates(fil, fil))
dat = Add.Data(all.dates,fil)
x = dat$iNAV
for(ii in which(is.na(x[-length(x)])))
{
  firstnonNA.after.i = min(which(!is.na(x[ii:length(x)])))
  last.observed = max(which(!is.na(dat$iNAV[1:ii])))
  if(ii>1)
  {
    sd = sd(diff(dat$iNAV[which(!is.na(dat$iNAV[1:1561]))]))
    e = rnorm(1,mean = 0, sd = sd/sqrt(22))
    #er[j] = e
    x[ii] = x[ii-1] + (x[ii+firstnonNA.after.i-1]-x[ii-1])*1/
      firstnonNA.after.i + e
  }
}

```



```

    }
}
## Plot of the added noise.
sd5 = c(); sd10=c()
for(i in 1:(length(dat$iNAV)-10))
{
  sd5[i] = sd(dat$iNAV[i:(i+1)], na.rm=T)
  sd10[i] = sd(dat$iNAV[i:(i+22)], na.rm=T)
}
graphics.off()
mf <- layout(matrix(c(2,1),2,1,byrow=TRUE), heights=c(5,3), TRUE)
par(mgp=c(2,0.7,0), mar=c(3, 3, 0, 1))
plot(sd5, type="l", ylab = "st.dev.", main="", xlab="")
lines(c(rep(NA,1568),rollmean(sd5[which(!is.na(sd5))], 150)), col
      =3, lwd=2)
abline(h=2, col=2, lwd=2)
par(mar=c(0.5,3,1,1))
plot(dat$Date, dat$iNAV, type='l', xlab='', ylab='iNAV', col=3, xaxt="n"
      , main="")
lines(dat$Date, x ,xaxt='n', xlab='time step', ylab='iNAV', col=4)
lines(dat$Date, dat$iNAV)
path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//data//"
dev.print(device=pdf, height = 5, width = 8, file = paste(path,"
  RWXdatagen.pdf", sep=""))
###
dat$iNAV = x
data = Remove.Outliers(dat)
data$Date = as.Date(data$Date, "%d-%m-%Y")
data = Periodic(data, "weekly")
data = data[,which(names(data) %in% c("iNAV", "Date", "iReturn"))]

d1 = fil$Date[min(which(fil$Date >= min(data$Date))):max(which(fil$
  Date <= max(data$Date)))]
dd = all.dates$Date
a = which(dd %in% d1)[-1]-which(dd %in% d1)[-length(which(dd %in%
  d1))]-1

assign(paste("a", lec[i], sep="."), a)
assign(paste("index",lec[i], sep="."), data)
rm(list=objects(pattern = "a.")[1:20])

i.df = mget(ls(pattern = "index."), envir = .GlobalEnv)
names(i.df)=lec
rm(list=objects(pattern = "index.")[-21])

for(i in 1:length(lec))
{
  print(range(i.df[[i]]$Date))
}

# select dates for common span of the indices.
# MIN DATA RANGE
min.date = max(min(i.df$DGT$Date),
  min(i.df$ELR$Date),

```

```

        min(i.df$FEZ$Date),
        min(i.df$GLD$Date),
        min(i.df$RWX$Date),
        min(i.df$STN$Date),
        min(i.df$STZ$Date),
        min(i.df$TOPIX$Date),
        min(i.df$XOP$Date))

# MAX DATA RANGE
max.date = min(max(i.df$DGT$Date),
               max(i.df$ELR$Date),
               max(i.df$FEZ$Date),
               max(i.df$GLD$Date),
               max(i.df$RWX$Date),
               max(i.df$STN$Date),
               max(i.df$STZ$Date),
               max(i.df$TOPIX$Date),
               max(i.df$XOP$Date))

dates = unique(c(i.df$DGT$Date,
                 i.df$ELR$Date,
                 i.df$FEZ$Date,
                 i.df$GLD$Date,
                 i.df$RWX$Date,
                 i.df$STN$Date,
                 i.df$STZ$Date,
                 i.df$TOPIX$Date,
                 i.df$XOP$Date))
dates = dates[sort.list(dates)]

dates = dates[which(dates==min.date):which(dates==max.date)]
data = data.frame(Date = dates)
for(i in which(names(i.df) %in% ind))
{
  names(i.df[[i]])[which(names(i.df[[i]])=="iReturn")] = lec[i]
  data = Add.Data(data, i.df[[i]][, -which(names(i.df[[i]])=="iNAV")
    ])
  names(i.df[[i]])[which(names(i.df[[i]])==lec[i])] = "iReturn"
}
index.matrix = data
rm(data)

## historic price of gold bullion.
#####
GOLD20Y = read.table("GOLD20years.csv", sep=";", header=TRUE, dec="
.")
GOLD20Y$Date = as.Date(substr(GOLD20Y$Date,1,10), "%d-%m-%Y")
GOLD20Y = GOLD20Y[, -c(5,6,7)]

rm(all.dates, dat, fil, filmatrix, e, filer, firstnonNA, after.i, i, ii, ind,
   last.observed, lec, nf, obs, path, sd, sd10, sd5, x)

```

C:/Users/Mie/Dropbox/Speciale/R/Speciale/Dataload-print.R

## C.3 Analysis of difference in returns

```

# INDEX ANF FUND NAV WITH DEVIANCE
graphics.off()
path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//data/"
for(j in seq(1,20,5))
{
  pdf(paste(path, "test", j, ".pdf", sep=""), width=5.5, height=8)
  par(mar=c(2,3.5,2,3.5), mgp=c(2,0.7,0), mfrow=c(5,1)) #ADJUST THE
    MARGINS
  temp = Periodic(df[[j]], "weekly")
  plot(i.df[[j]]$Date, i.df[[j]]$iNAV, type="n", ylab="", xlab="",
    ylim=c(min(temp$DiffReturn),max(temp$DiffReturn)), yaxt="n",
    main = names(df)[j])
  lines(temp$Date, temp$DiffReturn, type="l", col=3, xaxt="n", yaxt
    ="n", xlab="", ylab="", main = names(df)[j])
  axis(4, col.axis=3)
  mtext("Return deviance", side=4, line=2, col=3)
  par(new=TRUE)
  st.date = which(i.df[[j]]$Date == min(Periodic(df[[j]], "weekly")
    $Date))
  plot(i.df[[j]]$Date, i.df[[j]]$iNAV, col=1, type="l", ylab="",
    xlab="",
    ylim = c(min(min(i.df[[j]]$iNAV),min(df[[j]]$fNAV/df[[j]]$
    fNAV[1]*i.df[[j]]$iNAV[st.date])), max(max(i.df[[j]]$
    iNAV),max(df[[j]]$fNAV/df[[j]]$fNAV[1]*i.df[[j]]$iNAV[st
    .date]))))
  lines(df[[j]]$Date, df[[j]]$fNAV/df[[j]]$fNAV[1]*i.df[[j]]$iNAV[
    st.date], col=2)
  mtext("Index, Fund NAV", side=2, line=2, col=1)
  legend("topleft", col=c(1,2,3),pch=19,legend=c("Index Level","Fund
    NAV", "Tracking error"), bty="n")
  for(i in (j+1):(j+4))
  {
    temp = Periodic(df[[i]], "weekly")
    plot(i.df[[i]]$Date, i.df[[i]]$iNAV, type="n", ylab="", xlab=""
      ,
      ylim=c(min(temp$DiffReturn),max(temp$DiffReturn)), yaxt="n
        ", main = names(df)[i])
    lines(temp$Date, temp$DiffReturn, type="l", col=3, xaxt="n",
      yaxt="n", xlab="", ylab="", main = names(df)[i])
    axis(4, col.axis=3)
    par(new=TRUE)
    st.date = which(i.df[[i]]$Date == min(Periodic(df[[i]], "weekly
      ")$Date))
    plot(i.df[[i]]$Date, i.df[[i]]$iNAV, col=1, type="l", ylab="",
      xlab="",
      ylim = c(min(min(i.df[[i]]$iNAV),min(df[[i]]$fNAV/df[[i]]$
        fNAV[1]*i.df[[i]]$iNAV[st.date])), max(max(i.df[[i]]$
          iNAV),max(df[[i]]$fNAV/df[[i]]$fNAV[1]*i.df[[i]]$iNAV[
            st.date]))))
    lines(df[[i]]$Date, df[[i]]$fNAV/df[[i]]$fNAV[1]*i.df[[i]]$iNAV
      [st.date], col=2)
  }
}

```

```

    dev.off()
}

## various measures of tracking error
for(i in 1:20)
{
  x = Periodic(df[[i]], "daily")
  y = Periodic(df[[i]], "weekly")
  a=First.combi.nonNA(x$fReturn, x$iReturn)
  b=Last.combi.nonNA(x$fReturn, x$iReturn)
  c=First.combi.nonNA(y$fReturn, y$iReturn)
  d=Last.combi.nonNA(y$fReturn, y$iReturn)
  print(c(lec[i], round(cor(y$fReturn[c:d], y$iReturn[c:d]),4),
        round(cor(x$fReturn[a:b], x$iReturn[a:b]),4),
        round(sd(y$DiffReturn[c:d]),4), round(sd(x$DiffReturn[a:b]
        ),4)))
}
rm(x,y,a,b,c,d)

# Difference in volatility between diff return and diff log return
for(i in 1:20)
{
  a = Periodic(df[[i]], "daily")
  a = a$DiffReturn
  y = Periodic(df[[i]], "daily")
  x = log(y$iNAV[-1]/y$iNAV[-length(y$iNAV)]) - log(y$fNAV[-1]/y$fNAV[-length(y$fNAV)])
  print(paste(lec[i], round(sd(a), digits=5), round(sd(x), digits
    =5), round((sd(a)-sd(x))/sd(a)*100, digits=2), sep="&"))
}
rm(a,x,y)

# Compare ACF of diff return and diff log return
par(mfrow=c(4,5))
for(i in 1:20)
{
  a = Periodic(df[[i]], "daily")
  c = a$DiffReturn
  acf(c)
}
par(mfrow=c(4,5))
for(i in 1:20)
{
  a = Periodic(df[[i]], "daily")
  b = log(a$iNAV[-1]/a$iNAV[-length(a$iNAV)]) - log(a$fNAV[-1]/a$fNAV[-length(a$fNAV)])
  acf(b)
}
rm(a,b,c)

# plot of density distributions of diff return and diff log return,
  relative to normal distribution.
for(i in 1:length(lec))

```

```

{
  a = Periodic(df[[i]], "daily")
  a = a$DiffReturn
  assign(names(df)[i], a)
}
#DiffReturn = mget(lec, envir=.GlobalEnv)
#rm(list=lec)
graphics.off()
par(mfrow=c(1,3))
for(i in c(3,4,19))
{
  y = Periodic(df[[i]], "daily")
  x = LogReturn(y)
  z = y$DiffReturn

  plot(density(x, na.rm=T), main = lec[i], ylab = "", xlab = "")
  lines(density(z, na.rm=T), col = 3)
  lines(seq(min(x, na.rm=T),max(x, na.rm=T),(max(x, na.rm=T)-min(x,
    na.rm=T))/500),
    dnorm(seq(min(x, na.rm=T),max(x, na.rm=T),(max(x, na.rm=T)-
    min(x, na.rm=T))/500),
    mean = mean(x, na.rm=T),
    sd = sd(x, na.rm=T)),
    col = "red")
}
cpgram(z, main = "Periodogram")
rm(a,x,y,z)

# Shapiro Wilks test for normality in diff return and diff log
return.
for(i in 1:20)
{
  x = Periodic(df[[i]], "weekly")
  y = LogReturn(x)
  z1 = shapiro.test(x$DiffReturn)
  z2 = shapiro.test(y)
  print(c(z1$p.value, z2$p.value))
}

# test for stationaritet.
library(tseries)
for(i in 1:20)
{
  x = Periodic(df[[i]], "daily")
  y = LogReturn(x)
  z1 = kpss.test(x$DiffReturn, null="Level")
  z2 = kpss.test(y, null="Trend")
  print(c(z1$p.value, z2$p.value))
}
rm(x,y,z1,z2)

## PLOTS OF DIFF RETURNS AND ACF
#####
path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//data//"
y = "daily"

```

```

if(y=="daily") c = c(3,7,8,13,14,18) else c=c(5,7,8,13,14,18)

graphics.off()
par(mfrow=c(4,5), mgp=c(2,0.7,0), mar=c(2,3,1.5,1))
for(i in 1:20)
{
  a = Periodic(df[[i]], y)
  plot(a$Date, a$DiffReturn, type="l",
       xlab = "", ylab = "",
       main = substitute(paste(RSS), list(RSS = lec[i])))
}
dev.print(device=pdf, height = 5.5, width = 11.7, file = paste(path
, "DiffReturn4x5", y, ".pdf", sep=""))

graphics.off()
par(mfrow=c(3,2), mgp=c(2,0.7,0), mar=c(2, 3,1.5,1))
for(i in c)
{
  a = Periodic(df[[i]], y)
  plot(a$Date, a$DiffReturn, type="l",
       xlab = "", ylab = "",
       main = substitute(paste(RSS), list(RSS = lec[i])),
       ylim = c(mean(a$DiffReturn)+min(a$DiffReturn)/20, mean(a$
DiffReturn)+max(a$DiffReturn)/20)
  )
}
dev.print(device=pdf, height = 6, width = 8, file = paste(path,"
DiffReturn2x3", y, ".pdf", sep=""))

graphics.off()
par(mfrow=c(4,5), mgp=c(2,0.7,0), mar=c(2,2,1,1))
a = Periodic(df[[1]], "weekly")
acf(a$DiffReturn, lag.max = 30, ylab = "", xlab = "")
legend("topright", lec[1], bty="n")
for(i in 2:length(lec))
{
  a = Periodic(df[[i]], y)
  acf(a$DiffReturn, lag.max = 30, ylab = "", xlab = "", xaxt = "n",
      yaxt="n")
  legend("topright", lec[i], bty="n")
}
dev.print(device=pdf, height = 5.5, width = 11.7, file = paste(path
, "DiffReturnACF4x5", y, ".pdf", sep=""))

rm(y,a)

## JARQUE-BERA AND BOX-LJUNG SENSITIVITY TESTS
graphics.off()
par(mgp=c(2,0.7,0), mar=c(2.5, 2.5, 1, 1))
path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//ARMA-GARCH/
/"

# PROOF OF SENSITIVITY IN JARQUE-BERA TEST.
# FAILS WHEN ONE SLIGHT OUTLIER IS INTRODUCED.
x = rnorm(1000); jarque.bera.test(x)

```

```

y = c(x, 5); jarque.bera.test(y)
plot(y[1:1001], pch=20, ylab="", xlab="")
points(1001,y[1001], pch=19, cex = 1.1, col=2)
dev.print(device=pdf, height = 6, width = 8, file = paste(path,"
jarque-bera.pdf", sep=""))

# PROOF OF SENSITIVITY IN BOX-LJUNG TEST.
# FAILS WHEN FEW OUTLIERS ARE INTRODUCED.
x = rnorm(1000); Box.test(x, type="Ljung")
y = c(x[1:500],rep(4,10), x[501:1000]); Box.test(y, type="Ljung")
plot(y, pch=20, ylab="", xlab="")
points(c(501:510), rep(4,10), pch = 19, col=2)
dev.print(device=pdf, height = 6, width = 8, file = paste(path,"box
-ljung.pdf", sep=""))

rm(x,y)

## RANDOMNESS OF IBGS
graphics.off()
layout(matrix(c(1,1,2,3), 2, 2, byrow = TRUE))

x = Periodic(df$IBGS, "daily")
y = Periodic(df$IBGS, "weekly")

ddd = acf(x$DiffReturn, plot=FALSE, max.lag=20)
www = acf(y$DiffReturn, plot=FALSE, max.lag=20)
ddd1 = pacf(x$DiffReturn, plot=FALSE, max.lag=20)
www1 = pacf(y$DiffReturn, plot=FALSE, max.lag=20)

par(mgp=c(2,0.7,0), mar=c(2, 3.5, 2, 1))
plot(x$Date,x$DiffReturn, type="l", lwd=1.5, xlab="", ylab="", main
="IBGS DiffReturn daily and weekly")
lines(y$Date,y$DiffReturn, col=2, lwd=1.5)
legend("bottomright", c("Daily","Weekly"), col=c(1,2), pch=19, cex
=1, ncol=2, bty="n")

plot(ddd$lag-0.2, ddd$acf, type="h", lwd=3, col=1, main="ACF", xlab
="", ylab="", ylim=c(-0.5,1), xlim=c(0,20))
lines(www$lag+0.2, www$acf, type="h", lwd=3, col=2)
abline(h=0)
abline(h=qnorm(0.975)/sqrt(ddd$n.used), lty="dashed", col=1)
abline(h=-qnorm(0.975)/sqrt(ddd$n.used), lty="dashed", col=1)
abline(h=qnorm(0.975)/sqrt(www$n.used), lty="dashed", col=2)
abline(h=-qnorm(0.975)/sqrt(www$n.used), lty="dashed", col=2)

plot(ddd1$lag+0.2, ddd1$acf, type="h", lwd=3, col=1, main="PACF",
xlab="", ylab="", ylim=c(-0.6,0))
lines(www1$lag-0.2, www1$acf, type="h", lwd=3, col=2)
abline(h=0)
abline(h=-qnorm(0.975)/sqrt(ddd1$n.used), lty="dashed", col=1)
abline(h=-qnorm(0.975)/sqrt(www1$n.used), lty="dashed", col=2)

path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//data//"
dev.print(device=pdf, height = 6, width = 8, file = paste(path,"

```

```

    IBG$randomness.pdf", sep=""))
rm(x,y,ddd,www,ddd1,www1)

C:/Users/Mie/Dropbox/Speciale/R/Speciale/Diffreturn-analysis.R

```

### C.3.1 ARMA-GARCH models

```

i = 3
y = Periodic(df[[i]], "weekly")
x = y$DiffReturn

model = garchFit(formula = ~arma(1,0)+garch(1,0), data = x, trace =
  F, algorithm = "lbfgsb" , hessian = "rcd")

summary(model)
assign(paste("AGweekly",names(df)[i], sep="."), model)
xtable(model@fit$matcoef,
  caption = paste("Estimated model coefficient and related
    uncertainty for ", names(df)[i], " modelled by an ",
    model@formula, " model.", sep="")[3],
  label = paste("tab:",names(df)[i], "AGweekly", sep=""))

graphics.off()
layout(matrix(c(1,1,2,3), 2,2, byrow=T))
par(mgp=c(2,0.7,0), mar=c(2, 2, 4, 1))
plot(model, which = 9)
title(paste(names(df)[i], model@formula, sep=" ")[3], outer=T, line
  =-1)
plot(model, which = 10)
plot(model, which = 13)
path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//ARMA-GARCH/
/"
dev.print(device=pdf, height = 6, width = 8, file = paste(path,"w",
  names(df)[i], ".pdf", sep=""))

mod = auto.arima(x)
summary(mod)

## PLOTS OF RWX 52 week ACF and PACF
#####
std.res = model$residuals/model@sigma.t #AGmodels.weekly$
  AGweekly.RWX@residuals/AGmodels.weekly$AGweekly.RWX@sigma.t
acf(std.res, lag.max = 52, main = "ACF of Standardized Residuals")
pacf(std.res, lag.max = 52, main = "PACF of Standardized Residuals"
)
#####

C:/Users/Mie/Dropbox/Speciale/R/Speciale/GARCHmodeller-print.R

```



## C.4 Scenario generation

```
#####
## BOOTSTRAP
#####
returns = matrix(NA,633,10)
for(i in 1:9)
{
  returns[,i+1] = as.numeric(ind.matrix[,i+1])
}

set.seed(1)
scen.list.bootstrap<-list()
scen.collect.bootstrap<-numeric(0)
obs.afkast.bootstrap = c()
a.bootstrap = list(); models.bootstrap = list()
l = 250
for(j in seq(0,floor(length(ind.matrix$Date)/2),4)[-80])
{
  j.date = floor(length(ind.matrix$Date)/2)+j
  w=matrix(NA,l,4)
  w[,1] = sample(j.date,l,prob=dnorm(1:j.date,1,5), replace=T)
  for(t in 2:4)
  {
    w[,t] = sample(j.date,l,prob=dnorm(1:j.date,w[t-1],5), replace=
      T)
  }

  ytpovery0.bootstrap = (returns[(w[,1]),-1]+1)*(returns[(w[,2]),
    -1]+1)*(returns[(w[,3]),-1]+1)*(returns[(w[,4]),-1]+1)
  scen.bootstrap = ytpovery0.bootstrap -1

  rownames(scen.bootstrap) = paste("scen",1:l, sep="")
  colnames(scen.bootstrap) = names(ind.matrix)[-1]
  date0.bootstrap = as.character(ind.matrix[which(ind.matrix$Date==
    "2006-01-13")+j+4,1])
  scen.collect.bootstrap <- c(scen.collect.bootstrap, paste(date0.
    bootstrap,".",rep(names(ind.matrix)[-1],each=1),".scen",1:l,"
    ",scen.bootstrap,sep=""))
  date0.ind = which(ind.matrix$Date == date0.bootstrap)
  a.bootstrap[[date0.bootstrap]] = ((ind.matrix[date0.ind-3,-1]+1)*
    (ind.matrix[date0.ind-2,-1]+1)*
    (ind.matrix[date0.ind-1,-1]+1)*(ind.matrix[date0.ind,-1]+1))-1
  obs.afkast.bootstrap = c(obs.afkast.bootstrap, paste(date0.
    bootstrap,".",names(ind.matrix)[-1],".obs", " ",ind.matrix[
    date0.ind,-1],sep=""))
  scen.list.bootstrap[[date0.bootstrap]]<-scen.bootstrap
}
obs.afkast = a.bootstrap
rm(returns,scen.list.bootstrap,scen.collect.bootstrap,obs.afkast.
  bootstrap, a.bootstrap)
rm(l,j.date,w,ytpovery0.bootstrap,scen.bootstrap,date0.bootstrap,j,
  i,y,obs,scen,dat2)
```

```
#####
## ARMA - GARCH
#####
data=ind.matrix
set.seed(1)
scen.list.AG<-list()
scen.collect.AG<-numeric(0)
models.AG = list()
l = 250
taller = 0
last.warning <-NULL

for(j in seq(0,floor(length(data$Date)/2),4)[-80])
{
  scen.AG = matrix(NA, l, ncol(data)-1)
  for(i in 2:10)
  {
    taller = taller + 1
    y<-data[1:(which(data$Date=="2006-01-13")+j),i]

    form<-switch(names(data)[i],
      GLD = ~arma(0,0)+garch(1,1), #~ arma(2,1)+garch
        (1,1),
      ELR = ,
      FEZ = ~arma(0,0)+garch(1,1), #~ arma(1,1)+garch
        (1,1),
      ~arma(0,0)+garch(1,1))

    mod = try(garchFit(formula = form, data = y, trace = F,
      algorithm = "lbfgsb" , hessian = "rcd"))
    if(inherits(mod, "try-error")) mod.na = 0 else mod.na = sum(
      which(is.na(mod@fit$matcoef)))
    if(mod.na > 0 | !is.null(warnings()) | inherits(mod, "try-error
      "))
    {
      last.warning <-NULL
      mod = try(garchFit(formula = form, data = y, trace = F,
        algorithm = "lbfgsb+nm" , hessian = "rcd"))
      print(c(taller, "Nelder-Mead"))
    }

    if(inherits(mod, "try-error")) mod.na = 0 else mod.na = sum(
      which(is.na(mod@fit$matcoef)))
    if(mod.na > 0 | !is.null(warnings()) | inherits(mod, "try-error
      "))
    {
      if(j==0)
      {
        mod<-models.AG[[taller-1]]
        mod@fit$matcoef<-cbind(estimate=c(mu=mean(y),omega=var(y)),
          NA,NA,NA)
        mod@fit$series$x<-y
        mod@fit$series$h<-rep(mod@fit$matcoef[2,1],length(y))
        mod@fit$series$z<-y-mod@fit$matcoef[1,1]
        print(c(taller,"Manuelt"))
      }
    }
  }
}

```

```

    }else
    {
      mod = models.AG[[taller-9]]
      print(c(taller,"prev-model1"))
    }
  }
  if(taller > 9 && sum(is.na(mod@fit$matcoef)) > 0 | min(mod@fit$
    series$h) < 0)
  {
    mod = models.AG[[taller-9]]
    print(c(taller,"prev model2"))
  }

  models.AG[[taller]] = mod

  fitParm = numeric(7)
  names(fitParm) = c("mu", "ar1", "ar2", "ma1", "omega", "alpha1",
    "beta1")
  fitParm[match(rownames(mod@fit$matcoef),names(fitParm))] =
    mod@fit$matcoef[,1]

  lz = length(mod@fit$series$z)
  z = matrix(rep(c(mod@fit$series$z[(lz-1):lz],rep(NA,4)),1),
    nrow=1, byrow=T)
  h = matrix(rep(c(mod@fit$series$h[(lz-1):lz],rep(NA,4)),1),
    nrow=1, byrow=T)
  x = matrix(rep(c(mod@fit$series$x[(lz-1):lz],rep(NA,4)),1),
    nrow=1, byrow=T)
  for(t in 3:6)
  {
    h[,t] = fitParm[5] + fitParm[7]*h[,t-1] + fitParm[6]*z[,t
      -1]^2
    for(k in 1:l)
    {
      z[k,t] = rnorm(1,mean=0,sd=sqrt(h[k,t]))
    }
    x[,t] = z[,t] + fitParm[1] + sum(fitParm[2:3]*x[, (t-1):(t-2)
      ]) + fitParm[4]*z[,t-1]
  }
  scen.AG[,i-1] = ((x[,3]+1)*(x[,4]+1)*(x[,5]+1)*(x[,6]+1))-1
}
rownames(scen.AG) = paste("scen",1:l, sep="")
colnames(scen.AG) = names(data)[-1]
date0.AG <- as.character(data$Date[which(data$Date=="2006-01-13")
  +j+4])
scen.collect.AG <- c(scen.collect.AG, paste(date0.AG,".",rep(
  names(data)[-1],each=1),".scen",1:l," ",scen.AG ,sep=""))
scen.list.AG[[date0.AG]]<-scen.AG
}

rm(scen.collect.AG, scen.list.AG, models.AG, a.AG, obs.afkast.AG,
  date0.AG, rets.AG,scen.AG)
rm(h,x,z,fitParm,form,i,j,k,l,lz,mod,t,taller,y, cb,wh, scen.list,
  obs.afkast,scen)

```

```
#####
## MSAR
#####
lgt = 250
scen.2.collect = numeric(0)
a = list()
obs.afkast.2 = numeric(0)
scen.list.2 = list()
models.MSAR = list()
taller = 0

for(j in seq(0, floor(length(data$Date)/2), 4)[-80])
{
  scen.2 = matrix(NA, lgt, (length(names(data))-1))
  for(i in c(1:9))
  {
    taller = taller+1
    print(c(i,j))
    y = data[1:(which(data$Date=="2006-01-13")+j), (i+1)]
    Regimes <- 2
    if(names(data)[i+1]=="TOPIX"){intercept0 = 0; orderAR0 = c(1,1)
      ; ARpar = c(-0.11, 0.01); int=c()} else
    {intercept0 <- 1; orderAR0 = c(0,0); ARpar = c(); int = c(0.1,
      -0.1)}
    #if(intercept0==1){int = c(0.1, -0.1)} else{int=c()}
    pAR0 <- matrix(0, nrow=Regimes, ncol=max(orderAR0)+intercept0,
      byrow=TRUE)
    pAR0[1,]<-c(int[1], ARpar[1])
    pAR0[2,]<-c(int[2], ARpar[2])
    P0 <- matrix(0.2, nrow=Regimes, ncol=Regimes)
    diag(P0) <- 0.8
    sig0 <- c(2,2)
    indexMissing0<-flagNA(y, nhor=1, orders=1:max(orderAR0), exogen=
      FALSE)
    orderX0 <- c(0,0)
    x <- rep(0, length(y))

    models.MSAR[[taller]] <- fitMSARX(R=Regimes, y=y, orderAR=
      orderAR0, x=x, orderX=orderX0, pARX=pAR0, P=P0, sig=sig0,
      intercept=intercept0,
      indexMissing=
      indexMissing0)

    dat = models.MSAR[[taller]]
    delta = dat$more$deltaT[length(dat$more$deltaT)]
    sd = dat$est_sig
    rt = i.df[[which(names(i.df) == names(data)[i+1])]]$iReturn[dat
      $more$Tnew]
    if(names(data)[i+1]=="TOPIX")
    {
      int = rep(0, length(delta))
      AR = dat$est_pARX
    } else{int = dat$est_pARX[,1]; AR = dat$est_pARX[, -1]}
    r = matrix(NA, ncol=5, nrow=lgt); r[,1] = rt
    R = matrix(NA, ncol=5, nrow=lgt)
  }
}

```

```

R[,1] = sample(2, lgt, prob = c(1-delta, delta), replace=T)
for(t in 1:lgt)
{
  index = R[,t]==1
  sid = sum(index)
  r[,t+1] = int[R[,t]] + ifelse(is.na(AR[R[,t]]*r[,t]), 0, AR[R
    [,t]]*r[,t]) + rnorm(lgt,0,sd[R[t]])
  R[which(R[,t]==1),t+1] = sample(2, length(which(R[,t]==1)),
    prob = dat$est_P[1,], replace=T)
  R[which(R[,t]==2),t+1] = sample(2, length(which(R[,t]==2)),
    prob = dat$est_P[2,], replace=T)
}

ytpovery0.2 = ((r[,2]+1)*(r[,3]+1)*(r[,4]+1)*(r[,5]+1))
scen.2[,i] = ytpovery0.2 - 1
}
rownames(scen.2) = paste("scen", 1:lgt, sep="")
colnames(scen.2) = names(data)[-1]
date0.2 = as.character(data$Date[which(data$Date=="2006-01-13")+j
  +4])
scen.2.collect = c(scen.2.collect, paste(date0.2, ".", rep(names(
  data)[-1], each=lgt), ".scen.", 1:lgt, " ", scen.2, sep=""))
a[[date0.2]] = ((data[which(data$Date == date0.2) -3, -1]+1)*(data[
  which(data$Date == date0.2) -2, -1]+1)*(data[which(data$Date ==
  date0.2) -1, -1]+1)*(data[which(data$Date == date0.2), -1]+1))
  -1
obs.afkast.2 = c(obs.afkast.2, paste(date0.2, ".", names(data)[-1],
  " obs", " ", data[which(data$Date == date0.2), -1], sep=""))
scen.list.2[[date0.2]]<-scen.2
}

scen.list.MSAR = scen.list.2
models.MSAR = mod
rm(scen.list.2, mod)

rm(lgt, scen.2.collect, a, obs.afkast.2, scen.list.2, mod, j, scen.2, i, y,
  Regimes, intercept0, orderAR0, ARpar, int, pAR0, P0, sig0,
  indexMissing, orderX0, x)
rm(dat, delta, sd, rt, AR, r, R, index, t, sid, ytpovery0.2, date0.2)

#####
## DEPMIX
#####

set.seed(1)
scen.list.depmix<-list()
scen.collect.depmix<-numeric(0)
l = 250
models.depmix = list()
taller=0
for(j in seq(0, floor(length(data$Date)/2), 4)[-80])
{
  taller=taller+1
  y<-with(data[1:(which(data$Date=="2006-01-13")+j)], cbind(DGT,
    ELR, FEZ, GLD, RWX, STN, STZ, TOPIX, XOP))
}

```

```

rModels <- list()
rModels[[1]] <- list(MVNresponse(y~1))
rModels[[2]] <- list(MVNresponse(y~1))

trstart=c(0.9,0.10,0.05,0.95)

transition <- list()
transition[[1]] <- transInit(~1,nstates=2,data=data.frame(1),
  pstart=c(trstart[1:2]))
transition[[2]] <- transInit(~1,nstates=2,data=data.frame(1),
  pstart=c(trstart[3:4]))

instart=runif(2)
inMod <- transInit(~1,ns=2,ps=instart,data=data.frame(1))

mod <- makeDepmix(response=rModels,transition=transition,prior=
  inMod)
fm2 <- fit(mod)
models.depmix[[taller]] = fm2

delta = fm2@posterior[nrow(fm2@posterior),2:3]
sigma = list(sigma1 = show(fm2@response[[1]][[1]]), sigma2 = show
  (fm2@response[[2]][[1]]))
rt = data[which(data$Date=="2006-01-13")+j,-1]
int = list(int1 = fm2@response[[1]][[1]]@parameters$coefficients,
  int2 = fm2@response[[2]][[1]]@parameters$coefficients)
prob = matrix(c(fm2@trDens),2,2, byrow=T)

rets = list()
R = matrix(NA,1,5); R[,1] = sample(2, 1, prob = delta, replace=T)

for(t in 1:4)
{
  index1 <- R[,t]==1
  sid1 <- sum(index1)
  rets[[t]]<-matrix(NA,1,9)
  if(sid1>0){
    rets[[t]][index1,] = rmvnorm(sid1, mean = int[[1]], sigma =
      sigma[[1]])
    R[index1,t+1] = sample(2, sid1, prob = prob[1,], replace=T)
  }
  if ((1-sid1)>0){
    rets[[t]][!index1,] = rmvnorm(1-sid1, mean = int[[2]], sigma
      = sigma[[2]])
    R[!index1,t+1] = sample(2, 1-sid1, prob = prob[2,], replace=T)
  }
}
}
ytpovery0.depmix = (rets[[1]]+1)*(rets[[2]]+1)*(rets[[3]]+1)*(
  rets[[4]]+1)
scen.depmix<-ytpovery0.depmix-1

rownames(scen.depmix) = paste("scen",1:1, sep="")
colnames(scen.depmix) = names(data)[-1]

```

```

date0.depmix <- as.character(data$Date[which(data$Date=="
  2006-01-13")+j+4])
scen.collect.depmix <- c(scen.collect.depmix, paste(date0.depmix,
  ".", rep(names(data)[-1], each=1), ".scen", 1:1, " ", scen.depmix,
  sep=""))
scen.list.depmix[[date0.depmix]]<-scen.depmix
}

rm(scen.list.depmix, scen.collect.depmix, l, models.depmix, taller, j, y,
  rModels, trstart, transition, instart, inMod, mod)
rm(fm2, delta, sigma, rt, int, prob, rets, R, index1, sid1, ytpobery0.depmix,
  scen.depmix, date0.depmix, obs, scen, dat2)

C:/Users/Mie/Dropbox/Speciale/R/Speciale/scengen-print.R

```

### C.4.1 MSAR library

```

#####
#####

#function interfacing R and the .C function
loopMSARX_wrap<- function(Tnew,R,dnst,Pvec,delta,indexMissing,nll2
,deltaT) {
  res.2loops<- .C("loopMSARX", Tnew=as.integer(Tnew), R=as.integer(R),
  dnst=as.double(dnst), Pvec=as.double(Pvec), delta=as.double(
  delta), indexMissing=as.integer(indexMissing), nll=as.double(
  nll2), deltaT=as.double(deltaT))
}
#####
#####

flagNA<-function(y,nhor,orders,exogen){
  bool<-rep(1,length(y))
  orderMAX<-max(orders)
  for (ii in (orderMAX+nhor):length(y)) {
    dataVect<-y[(ii-nhor+1-orders)]
    if (exogen==FALSE) {
      dataVect<-c(y[ii],dataVect)
    }
    if (any(is.na(dataVect))) {
      bool[ii]<-0
    }
  }
  bool[1:(orderMAX+nhor-1)]<-0
  return(bool)
}
#####
#####

fromNATtoWORK <- function(R,pARX,P,sig,intercept,orderAR,orderX) {
  #This function transforms natural parameters into working
  parameters (to avoid constrained optimization)

```

```

tP<-c()
for (ii in 1:R) {
  tP<-c(tP,log(P[ii,-ii]/P[ii,ii]))
}
if(intercept)
  #tmpVect<-pARX[,1]
  tmpVect<-pARX[,1]
else
  tmpVect<-numeric(0)
for (i in 1:R){
  if(orderAR[i]>0)
    tmpVect<-c(tmpVect,pARX[i,intercept+(1:orderAR[i])])
}
if(any(orderX>0)){
  for (i in 1:R){
    if(orderX[i]>0)
      tmpVect<-c(tmpVect,pARX[i,intercept+max(orderAR)+(1:orderX[
        i])])
  }
}
#wkVect<-c(as.vector(t(pARX)),tP,log(sig))
wkVect<-c(tmpVect,tP,log(sig))

indexNA<-which(is.na(wkVect))
if (length(indexNA)>0) {
  wkVect<-wkVect[-indexNA]
}

return(wkVect)
}
#####
#####

fromWORKtoNAT <- function(R,orderAR,orderX,wkVect,intercept) {

  #This function transforms working parameters into working back
  into natural parameters

  pARX<-matrix(0,nrow=R,ncol=(intercept+max(orderAR)+max(orderX)))
  index<-0
  for (ii in 1:R) {
    pARX[ii,1:(intercept+orderAR[ii]+orderX[ii])]<-wkVect[(index+1)
      :(index+intercept+orderAR[ii]+orderX[ii])]
    index<-index+intercept+orderAR[ii]+orderX[ii]
  }

  P <- matrix(0,nrow=R,ncol=R)
  for (ii in 1:R) {
    P[ii,-ii] <- exp(wkVect[(index+1):(index+(R-1))])/(1+sum(exp(
      wkVect[(index+1):(index+(R-1))])))
    P[ii,ii] <- 1/(1+sum(exp(wkVect[(index+1):(index+(R-1))])))
    index <- index+(R-1)
  }
  delta <- try(solve(t(-diag(R)+P+1),rep(1,R)))
  if (inherits(delta,'try-error')) {

```



```

    print("Continuing")
    print(P)
    delta<-rep(1/R,R)
  }

  sig <- exp(wkVect[(index+1):(index+R)])

  return(list(pARX=pARX,P=P,sig=sig,delta=delta))
}
#####
#####

nllMSARX <- function(parVect,R,Z,y,orderAR,orderX,intercept,
  indexMissing,listOut=FALSE) {
  parNat <- fromWORKtoNAT(R=R,orderAR=orderAR,orderX=orderX,wkVect=
    parVect,intercept=intercept)
  pARX <- parNat$pARX
  P <- parNat$P
  sig <- parNat$sig
  delta <- parNat$delta
  delta_t<- matrix(parNat$delta,nrow=1)

  nll <- 0

  #print(pARX)

  T <- length(y)
  orderMAX <- max(c(orderAR,orderX))

  yHat<- Z % * % t(pARX)

  dens_t<-matrix(0,nrow=T-orderMAX,ncol=R)
  for (ii in 1:R) {
    dens_t[,ii]<-dnorm(as.vector(y[(orderMAX+1):T]),mean=as.vector(
      yHat[,ii]),sd=rep(sig[ii],T-orderMAX))
  }

  indexNA<-which(is.na(apply(dens_t,1,sum)))
  dens_t[indexNA,]<-1

  ### C variable
  Pvec<-as.vector(P)
  Tnew<-T-orderMAX
  denst<-as.vector(dens_t)
  nll<-0
  deltaT<-denst*0 #rep(0,R)
  ###

  msarxC<-loopMSARX_wrap(Tnew,R,denst,Pvec,delta,indexMissing[(
    orderMAX+1):T],nll,deltaT)
  nll <- unlist(msarxC[7])

  if (is.na(nll) | is.infinite(nll)) {
    nll<- .Machine$double.xmax
  }
}

```

```

}
else {
  nll<--nll
}
#print(nll)

if (listOut){
  msarxC
}
else
  nll
}

#####
#####

prepDataMSARX <- function(R,y,orderAR,x,orderX,intercept) {

  T<-length(y)
  orderMAX <- max(c(orderAR, orderX))
  if (intercept==1) {
    Z<-rep(1,T-orderMAX)
  }
  else {
    Z<-c()
  }
  if(orderMAX>0){
    for (ii in 1:max(orderAR)) {
      Z<-cbind(Z,y[(orderMAX+1-ii):(T-ii)])
    }
  }
  if (max(orderX)>0) {
    for (ii in 1:max(orderX)) {
      Z<-cbind(Z,x[(orderMAX+1-ii):(T-ii)])
    }
  }
  return(Z)
}

#####
#####

fitMSARX <- function(R,y,orderAR,x,orderX,pARX,P,sig,intercept,
  indexMissing, stepmax=10) {

  wkVect0 <- fromNATtoWORK(R=R,pARX=pARX,P=P,sig=sig,intercept=
    intercept,orderAR=orderAR,orderX=orderX)
  Z<-prepDataMSARX(R,y,orderAR,x,orderX,intercept)
  estMSARX <- nlm(nllMSARX,p=wkVect0,R=R,Z=Z,y=y,orderAR=orderAR,
    orderX=orderX,intercept=intercept,indexMissing=indexMissing,
    iterlim=2000, hessian=TRUE, stepmax=stepmax)
  moreOut <- nllMSARX(p=estMSARX$estimate,R=R,Z=Z,y=y,orderAR=
    orderAR,orderX=orderX,intercept=intercept,indexMissing=

```

```

      indexMissing, listOut=TRUE)
estVect <- fromWORKtoNAT(R=R, orderAR=orderAR, orderX=orderX, wkVect=
  estMSARX$estimate, intercept=intercept)
nll <- estMSARX$minimum
nllNorm<-nll/(sum(indexMissing))
nbParam <- R*(R-1) + R*intercept + sum(orderAR)+sum(orderX) + R #
  R*(orderZt+1)
AIC <- 2*(nll+nbParam)
BIC <- 2*nll+nbParam*log(sum(indexMissing))
list(est_pARX=estVect$pARX, est_P=estVect$P, est_sig=estVect$sig,
  nll=nll, nllNorm=nllNorm, BIC=BIC, AIC=AIC, code=estMSARX$code,
  more=moreOut, hessian=estMSARX$hessian)
}

```

C:/Users/Mie/Dropbox/Speciale/R/Speciale/MSARlibrary.R

## C.4.2 Plots for Table 6

```

# NINE SELECTED INDICES INAV RAINBOW PLOT
#####
min.da = c(); max.da = c()
for(i in which(names(i.df) %in% ind))
{
  min.da = c(min.da, min(i.df[[i]]$Date))
  max.da = c(max.da, max(i.df[[i]]$Date))
}
graphics.off()
par(mgp=c(2,0.7,0), mar=c(2, 3.5, 1, 1))
plot(i.df[[19]]$Date[400:1205], i.df[[19]]$iNAV[400:1205]/i.df
  [[19]]$iNAV[which(i.df[[19]]$Date == max(min.da))],
  type="l", ylab="", xlab="", cex.lab=1.5, main="", ylim=c
  (0.2,1.8), cex=2)
abline(h=1, v=max(min.da), col="grey60")
j = 0
for(i in which(names(i.df) %in% ind))
{
  j = j+1
  a = i.df[[i]]$iNAV/i.df[[i]]$iNAV[which(i.df[[i]]$Date == max(min
    .da))]
  lines(i.df[[i]]$Date, a, col = rainbow(9)[j])
}
legend("topleft", legend = ind, col = rainbow(9), pch = 19, ncol =
  3, cex = 1.5, box.col="white", inset=0.005) #bty="n")
path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//Scengen//"
dev.print(device=pdf, height = 5, width = 8, file = paste(path, "
  RainbowIndices2.pdf", sep=""))

rm(min.da, max.da, j, a)
#####

# NINE SELECTED INDICES IRETURN PROCESSES
#####
min.da = c(); max.da = c()

```

```

a=c()
for(i in 1:9)
{
  a = cbind(a, data[,i+1])
}

dat2 = data.frame(Return = as.numeric(a), time = rep(data[,1], 9),
  index = rep(names(data)[-1], each=length(data$Date)))
xyplot(Return~time|index,data=dat2, type="l", xlab="", ylab="Index
return",
  scales=list(x=list(rot=45, cex=1.2), y=list(cex=1.2)),
  layout=c(3,3,1),
  index.cond= list(c(7:9, 4:6, 1:3)))
dev.print(device=pdf, height = 5, width = 8, file = paste(path,"3
x3iReturn.pdf", sep=""))
#####

## ACF OF NINE SELECTED INDEX RETURNS
#####
par(mgp=c(1,0.7,0), mar=c(1.5, 1.9, 0.5, 0.1), mfrow=c(3,3))
acf(ind.matrix[,2], xlab="", ylab="")
legend("topleft", ind[1], cex=1.5,bty="n")
for(i in 3:10)
{
  acf(ind.matrix[,i], xlab="", ylab="", xaxt="n", yaxt="n")
  legend("topleft", ind[i-1], cex=1.5, bty="n")
}
dev.print(device=pdf, height = 4, width = 6, file = paste(path,"3
x3iACF.pdf", sep=""))
#####

## FRACTION OF PREDICTIONS TO EXCEED OBSERVED RETURN PLOT
#####
path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//Scengen//"
y = matrix(NA,79,9)
for(i in 1:9)
{
  for(j in 1:79)
  {
    obs = obs.afkast[[j]][,i]
    scen = scen.list.AG[[j]][,i]
    y[j,i] = sum(scen > obs)/length(scen)
  }
}
dat2 = data.frame(Fraction = as.numeric(y), Time = rep(1:79, 9),
  index = rep(ind, each=79))
par(mgp=c(2,0.7,0), mar=c(3.5, 3.5, 1, 1))
xyplot(Fraction ~ Time | index , data = dat2,
  panel = function(x, y){panel.grid(h=-1,v=-1, col="grey", lty
=3);
  panel.xyplot(x,y, pch=20);
  panel.loess(x,y, col=1, evaluation
=50, span = 1/4)},
  layout=c(3, 3, 1),
  index.cond= list(c(7:9, 4:6, 1:3)))

```

```

dev.print(device=pdf, height = 6, width = 8, file = paste(path,"
  AGFraction.pdf", sep=""))
#####

## TEST FOR UNIFORM DISTRIBUTION OF FRACTIONS
#####
par(mgp=c(1,0.7,0), mar=c(1.5, 1.9, 0.5, 0.1), mfrow=c(3,3))
acf(qnorm(y[,1])[is.finite(qnorm(y[,1]))], xlab="", ylab="")
legend("top", paste(c("p","var"), c(round(ks.test(y[,1], "punif")$p
  ,4), round(var(y[,1]),4))), sep="=", bty="n")
legend("topleft", ind[1], bty="n")
for(i in 2:9)
{
  acf(qnorm(y[,i])[is.finite(qnorm(y[,i]))], xlab="", ylab="", xaxt
    ="n", yaxt="n")
  legend("top", paste(c("p","var"), c(round(ks.test(y[,i], "punif")
    $p,4), round(var(y[,i]),4))), sep="=", bty="n")
  legend("topleft", ind[i], bty="n")
}
dev.print(device=pdf, height = 4, width = 6, file = paste(path,"
  AGACF.pdf", sep=""))
#####

## BOXPLOT SCENARIO FRACTION PLOT
#####
load("ObsAfkast.RData")
load("bootstrapsценарии.RData")

scen.list = scen.list.2
boxdata=c()
for(i in 1:79)
  boxdata = c(boxdata, scen.list[[i]])

dat2 = data.frame(returns = boxdata, time=rep(1:79, each=9*250),
  index = rep(ind, 79, each=250))

graphics.off()
pdf("C://Users//Mie//Dropbox//Speciale//Latex//figs//Scengen//
  MSARtest.pdf", width=5.5, height=8)
nf<- layout(matrix(c(1:11),11,1,byrow=TRUE), heights=c(0.8,rep(3,9)
  ,0.8), TRUE)
par(mgp = c(2,0.7,0), mar=c(0,3,0,0.5))

plot(rep(1,79), xaxt="n", yaxt="n", bty="n", type="n", ylab="",
  xlab="")

boxplot(returns ~ time, data=dat2[which(dat2$index==ind[1]),], ylab
  =ind[1], xlab="", xaxt="n")
axis(side=3, at=seq(0,80,10))
lines(sapply(sapply(obs.afkast, function(x) x)[1,], function(x) x),
  col=2, lwd=2)

for(j in 2:8)
{
  boxplot(returns ~ time, data=dat2[which(dat2$index==ind[j]),],

```

```

        ylab=ind[j], xlab="", xaxt="n")
    lines(sapply(sapply(obs.afkast, function(x) x)[j,], function(x) x
    ), col=2, lwd=2)
}

boxplot(returns ~ time, data=dat2[which(dat2$index==ind[9]),], ylab
    =ind[9], xlab="", xaxt="n")
axis(side=1, at=seq(0,80,10))
lines(sapply(sapply(obs.afkast, function(x) x)[9,], function(x) x),
    col=2, lwd=2)
dev.off()

rm(boxdata, dat2)
rm(list=objects(pattern = "scen.list"))
#####

## MSAR MODELS POSTERIOR PLOTS
#####
load("MSARmodeller.RData")

par(mgp=c(2,0.7,0), mar=c(3.5, 3.5, 1, 1), mfrow=c(4,4))
for(i in seq(1,711,9)[seq(1,79,5)])
{
    matplot(matrix(models.MSAR[[i]]$more$deltaT, ncol=2), type="l",
        lty=1, ylab="")
}

lmat = length(matrix(models.MSAR[[1]]$more$deltaT, ncol=2)[,1])

MSARregime=matrix(NA, ncol=9, nrow=lmat)
MSARregime[,1] = rep(2, length(matrix(models.MSAR[[1]]$more$deltaT,
    ncol=2)[,1]))
MSARregime[which(matrix(models.MSAR[[1]]$more$deltaT, ncol=2)[,1] <
    0.5),1] = 1
plot(MSARregime[,1], type="l", col=rainbow(9)[1], ylim = c(0.9,
    2.1))
for(i in 2:9)
{
    MSARregime[,i] = rep(2, length(matrix(models.MSAR[[i]]$more$deltaT
        , ncol=2)[,1]))
    MSARregime[which(matrix(models.MSAR[[i]]$more$deltaT, ncol=2)[,1]
        < 0.5),i] = 1
    lines(MSARregime[,i], col=rainbow(9)[i])
}

dat2 = data.frame(Regime=as.numeric(MSARregime), time = rep(1:lmat,
    9), index = rep(ind, each=lmat))
xyplot(Regime ~ time | index, data = dat2, type="l")

graphics.off()
plot(matrix(models.MSAR[[i]]$more$deltaT, ncol=2)[,1], type="l",
    col=rainbow(9)[i])
for(i in 1:9)
{
    lines(matrix(models.MSAR[[i]]$more$deltaT, ncol=2)[,1], col=

```

```

    rainbow(9)[i])
}

rm(lmat,MSARregime,dat2)
#####

## EXAMPLE OF 2-DIM MULTIVARIATE NORMAL DISTRIBUTION
#####
sigma = matrix(c(4,2,2,3), ncol=2)
x <- rmvnorm(n=2500, mean=c(3,2), sigma=sigma, method="chol")
plot(x, pch=20, col=4, xlab = "Index 1 (mu = 3, sd = 4)", ylab = "
    Index 2 (mu = 2, sd = 3)")
points(colMeans(x)[1], colMeans(x)[2], col=2, pch=19)
path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//Scengen//
dev.print(device=pdf, height = 3, width = 4, file = paste(path,"
    MVNeksempel.pdf", sep=""))

rm(sigma,x)
#####

## HEATMAP OF COVARIANCE AND CORRELATION MATRIX DEPMIX
#####
covmat1 = show(models.depmix[[55]]@response[[1]][[1]])
covmat2 = show(models.depmix[[55]]@response[[2]][[1]])
colnames(covmat1) = ind; rownames(covmat1)= ind
colnames(covmat2) = ind; rownames(covmat2)= ind
pal <- colorRampPalette(c("white","blue", "black"), space = "rgb")

par(mar=c(3,3.5,0.5,0.5), mgp=c(2,0.7,0))
levelplot(covmat1, xlab="", ylab="", col.regions=pal(100), cuts=45)
dev.print(device=pdf, height = 4, width = 4, file = paste(path,"
    depmixCovmat1.pdf", sep=""))

rm(covmat1,covmat2,pal)
#####

## MSAR COMPARISON OF REGIMESHIFTS BETWEEN SENSITIVE AND STABLE
MODEL
#####
par(mfrow=c(1,2), mgp=c(2,0.7,0), mar=c(2,2,0.5,0.5))
model = models.MSAR[[708]]
matplot(matrix(model$more$deltaT, ncol=2), type="l", lty=1, ylab="
)
model = models.MSAR[[701]]
matplot(matrix(model$more$deltaT, ncol=2), type="l", lty=1, ylab="
, yaxt="n")
dev.print(device=pdf, height = 3, width = 5, file = paste(path,"
    test.pdf", sep=""))

graphics.off()
par(mgp=c(2,0.7,0), mar=c(2,2,0.5,0.5))
model1 = models.MSAR[[708]]
model2 = models.MSAR[[704]]
plot(matrix(model2$more$deltaT, ncol=2)[,1], col=1, type="l", lty
    =1, ylab="", lwd=1)

```

```

lines(matrix(model1$more$deltaT, ncol=2)[,1], col=2, type="l", lty
      =1, ylab="", yaxt="n", lwd=2)
abline(h=0.5, lty=3)
dev.print(device=pdf, height = 3, width = 5, file = paste(path,"
      test.pdf", sep=""))

rm(model,model1,model2)
#####

## DEPMIX MODELS POSTERIOR PLOTS
#####
par(mfrow=c(3,3), mgp=c(2,0.7,0), mar=c(2, 1.8, 1.5, 0.5))
matplot(models.depmix[[1]]@posterior, type="l", lty=1, xlab = "",
      ylab = "",
      main = paste("Trailing", ind.matrix[316+1*4,1],sep=" "))
for(i in seq(10,79,by=9))
{
  matplot(models.depmix[[i]]@posterior, type="l", lty=1, xlab = "",
      ylab = "", yaxt="n",
      main = paste("Trailing", ind.matrix[316+i*4,1],sep=" "))
}
dev.print(device=pdf, height = 5, width = 9, file = paste(path,"
      regimesequence3x3.pdf", sep=""))

par(mgp=c(2,0.7,0), mar=c(3.5, 3.5, 1, 1))
rgb.palette <- colorRampPalette(c("white", "darkblue"), space = "
  rgb")
lab1 = show(models.depmix[[34]]@response[[1]][[1]])
corrmat1 = cov2cor(lab1)
colnames(corrmat1) = ind
rownames(corrmat1) = ind
levelplot(abs(corrmat1), col.regions=rgb.palette(200), cuts=50,
      xlab = "", ylab = "", main = "Heatmap of numeric
      correlation matrix",
      scales = list(tch = 0, cex = 2))
dev.print(device=pdf, height = 6, width = 6, file = paste(path,"
      depmixCorrmat12.pdf", sep=""))

lab2 = show(models.depmix[[4]]@response[[2]][[1]])
corrmat2 = cov2cor(lab2)
colnames(corrmat2) = ind
rownames(corrmat2) = ind
levelplot(abs(corrmat2), col.regions=rgb.palette(200), cuts=50,
      xlab = "", ylab = "", main = "Heatmap of numeric
      correlation matrix",
      scales = list(tch = 0, cex = 2))
dev.print(device=pdf, height = 6, width = 6, file = paste(path,"
      depmixCorrmat22.pdf", sep=""))

rm(lab1,lab2,corrmat1,corrmat2)
#####

## DEPMIX MODEL FINAL REGIME
#####
load("C:/Users/Mie/Dropbox/Speciale/CSVdata/DepmixModeller.RData")

```



```

Regime = c()
for(i in 1:79)
{
  lpos = length(models.depmix[[i]]@posterior$state)
  Regime = c(Regime, models.depmix[[i]]@posterior$state[lpos])
}

ProbRegime=c()
for(i in 1:79)
{
  lpos = length(models.depmix[[i]]@posterior$state)
  ProbRegime = c(ProbRegime, models.depmix[[i]]@posterior$X2[lpos])
}
par(mgp=c(2,0.7,0), mar=c(2,3,0.5,0.5))
plot(ProbRegime, type="o", pch=19, at=seq(0,80,10), yaxt="n", ylab=
"Estimated probability of regime 1")
abline(h=0, col=4)
abline(h=1, col=4)
axis(side=2,at=c(0,0.5,1))
path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//Scengen//"
dev.print(device=pdf, height = 4, width = 8, file = paste(path,"
depmixRegimes.pdf", sep=""))

rm(Regime, lpos)
#####

## DEPMIX MODELS DEVELOPMENT IN MEAN AND VARIANCE
#####
#load("DepmixModeller.RData")

var1 = c(); var2=c()
int1=c(); int2=c()
for(i in 1:79)
{
  fm2 = models.depmix[[i]]
  sigma = list(sigma1 = show(fm2@response[[1]][[1]]), sigma2 = show
(fm2@response[[2]][[1]]))
  int = list(int1 = fm2@response[[1]][[1]]@parameters$coefficients,
int2 = fm2@response[[2]][[1]]@parameters$coefficients)
  var1 = c(var1, diag(sigma[[1]]))
  var2 = c(var2, diag(sigma[[2]]))

  int1 = c(int1, int[[1]])
  int2 = c(int2, int[[2]])
}

temp.int.1 = matrix(int1, nrow=79, byrow=T)
temp.int.2 = matrix(int2, nrow=79, byrow=T)
temp.int = list(temp.int.1, temp.int.2)

temp.var.1 = matrix(var1, nrow=79, byrow=T)
temp.var.2 = matrix(var2, nrow=79, byrow=T)
temp.var = list(temp.var.1, temp.var.2)

```

```

graphics.off()
nf = layout(matrix(c(1,2,3,1,1,4,5,1,1,1,1,1), 3,4, byrow=T),
  heights=c(2.5,2.5,0.25), widths=c(0.3,4,4, 0.3), TRUE)
par(mgp=c(2,0.7,0), mar=c(0,0,0,0))

plot(rep(1,79), xaxt="n", yaxt="n", bty="n", type="n", ylab="",
  xlab="")

matplot(temp.int.1, type="l", lty=2, xaxt="n")
lines(rowMeans(temp.int.1), lwd=2)
legend("bottomleft", "Means Regime 1", bty="n")

matplot(temp.int.2, type="l", lty=2, xaxt="n", yaxt="n")
lines(rowMeans(temp.int.2), lwd=2)
legend("bottomleft", "Means Regime 2", bty="n")

matplot(temp.var.1, type="l", lty=2, yaxt="n")
lines(rowMeans(temp.var.1), lwd=2)
legend("topleft", "Variance Regime 1", bty="n")

matplot(temp.var.2, type="l", lty=2, yaxt="n")
axis(side=4, at=seq(0,0.005,0.001))
lines(rowMeans(temp.var.2), lwd=2)
legend("topleft", "Variance Regime 2", bty="n")

path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//
  appendixplots//"
dev.print(device=pdf, height = 4, width = 8, file = paste(path,"
  depmixmeanvar.pdf", sep=""))

rm(var1,var2,int1,int2,fm2,sigma,int,temp.int.1,temp.int.2,temp.int
  ,temp.var.1,temp.var.2,temp.var,nf)

Regime = c()
for(i in 1:79)
{
  lpos = length(models.depmix[[i]]@posterior$state)
  Regime = c(Regime, models.depmix[[i]]@posterior$state[lpos])
}

int.total = matrix(NA, ncol=9, nrow=79); var.total = matrix(NA,
  ncol=9, nrow=79)
for(i in 1:79)
{
  j = Regime[i]
  int.total[i,] = temp.int[[j]][i,]
  var.total[i,] = temp.var[[j]][i,]
}

graphics.off()
nf = layout(matrix(c(1:3), 3,1, byrow=T), heights = c(3,3,0.5),
  TRUE)
par(mgp=c(2,0.7,0), mar=c(0,2,0,0.5))
matplot(int.total, type="l", lty=3, xaxt="n")
lines(rowMeans(int.total), lwd=2)

```

```

legend("bottomleft", "Mean scenario return", bty="n")
matplot(var.total, type="l", lty=3, yaxt="n", xaxt="n")
axis(side=2, at=seq(0,0.004,0.001))
axis(side=1, at=seq(0,80,10))
lines(rowMeans(var.total), lwd=2)
legend("topleft", "Scenario variance", bty="n")
path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//Scengen//"
dev.print(device=pdf, height = 3, width = 6, file = paste(path,
  depmixmeanvar.pdf", sep=""))

rm(lpos,Regime,int.total,j,var.total,nf)
rm(models.depmix)
#####

## MSAR MODEL FINAL REGIME
#####
load("C:/Users/Mie/Dropbox/Speciale/CSVdata/MSARModeller.RData")

graphics.off()
par(mgp=c(2,0.7,0), mar=c(2,3,0.5,0.5), mfrow=c(3,3))
RegProb = matrix(NA,ncol=9, nrow=79)
taller=0
for(i in c(1:7,9))
{
  RegimeProb = c()
  taller=0
  for(j in 1:79)
  {
    taller=j*9-(9-i)
    len = models.MSAR[[taller]]$more$Tnew
    RegimeProb = c(RegimeProb, matrix(models.MSAR[[taller]]$more$
      deltaT,ncol=2)[len,1])
  }
  RegProb[,i] = RegimeProb
  plot(RegimeProb, type="l")
}

dat2 = data.frame(RegimeProb = as.numeric(RegProb), time=rep(1:79,
  9), index = rep(ind, each=79))
xyplot(RegimeProb ~ time|index, data=dat2,
  ylab="Probability of Regime 1", xlab="",
  panel = function(x, y){panel.abline(h=0.5, lty=3);
    panel.xyplot(x,y, type="l")},
  layout=c(3, 3, 1),
  index.cond= list(c(7:9, 4:6, 1:3)))

path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//Scengen//"
dev.print(device=pdf, height = 4, width = 6, file = paste(path,
  MSARRegimes.pdf", sep=""))

rm(RegProb,taller,RegimeProb,len,dat2)
rm(models.MSAR)
#####

## MSAR MODELS DEVELOPMENT IN MEAN AND VARIANCE

```

```
#####

var1 = c(); var2=c()
int1=c(); int2=c()
for(i in c(1:7,9))
{
  for(j in 1:79)
  {
    fm2 = models.MSAR[[j*9-(9-i)]]
    sigma = list(sigma1 = fm2$est_sig[1], sigma2 = fm2$est_sig[2])
    int = list(int1 = fm2$est_pARX[1], int2 = fm2$est_pARX[2])
    var1 = c(var1, sigma[[1]])
    var2 = c(var2, sigma[[2]])

    int1 = c(int1, int[[1]])
    int2 = c(int2, int[[2]])
  }
}

temp.int.1 = matrix(int1, nrow=79, byrow=F)
temp.int.2 = matrix(int2, nrow=79, byrow=F)
temp.int = list(temp.int.1, temp.int.2)

temp.var.1 = matrix(var1, nrow=79, byrow=F)
temp.var.2 = matrix(var2, nrow=79, byrow=F)
temp.var = list(temp.var.1, temp.var.2)

graphics.off()
nf = layout(matrix(c(1,2,3,1,1,4,5,1,1,1,1,1), 3,4, byrow=T),
  heights=c(2.5,2.5,0.25), widths=c(0.3,5,5, 0.3), TRUE)
par(mgp=c(2,0.7,0), mar=c(0,0,0,0))

plot(rep(1,79), xaxt="n", yaxt="n", bty="n", type="n", ylab="",
  xlab="")

matplot(temp.int.1, type="l", lty=2, xaxt="n", ylim=c(-0.05,0.05))
lines(rowMeans(temp.int.1), lwd=2)
legend("bottomleft", "Means Regime 1", bty="n")

matplot(temp.int.2, type="l", lty=2, xaxt="n", yaxt="n", ylim=c
  (-0.1,0.1))
axis(side=4, at=pretty(temp.int.2))
lines(rowMeans(temp.int.2), lwd=2)
legend("bottomleft", "Means Regime 2", bty="n")

matplot(temp.var.1, type="l", lty=2, ylim=c(0,0.07))
lines(rowMeans(temp.var.1), lwd=2)
legend("topleft", "Variance Regime 1", bty="n")

matplot(temp.var.2, type="l", lty=2, yaxt="n", ylim=c(0,0.07))
axis(side=4, at=pretty(temp.var.2))
lines(rowMeans(temp.var.2), lwd=2)
legend("topleft", "Variance Regime 2", bty="n")

path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//
```

```

Presentation//"  

dev.print(device=pdf, height = 4, width = 8, file = paste(path,"  

  MSARmeanvar-appendix.pdf", sep=""))  

rm(var1,var2,int1,int2,fm2,sigma,int,temp.int.1,temp.int.2,temp.int  

  ,temp.var.1,temp.var.2,temp.var,nf)  

Reg.mat=matrix(NA,ncol=9,nrow=79)  

for(i in c(1:7,9))  

{  

  Regime=c()  

  for(j in 1:79)  

  {  

    fm2 = models.MSAR[[j*9-(9-i)]]  

    len = fm2$more$Tnew  

    Regime = c(Regime, which(matrix(fm2$more$deltaT,ncol=2)[len,] >  

      0.5))  

  }  

  Reg.mat[,i] = Regime  

}  

int.total = matrix(NA, ncol=9, nrow=79)  

var.total = matrix(NA, ncol=9, nrow=79)  

for(i in 1:79)  

{  

  for(j in c(1:7,9))  

  {  

    m = Reg.mat[i,j]  

    j = ifelse(j==9,8,j)  

    int.total[i,j] = temp.int[[m]][i,j]  

    var.total[i,j] = temp.var[[m]][i,j]  

  }  

}  

graphics.off()  

nf = layout(matrix(c(1:3), 3,1, byrow=T), heights = c(3,3,0.5),  

  TRUE)  

par(mgp=c(2,0.7,0), mar=c(0,2,0,2))  

matplot(int.total, type="l", lty=3, xaxt="n", ylim=c(-0.02, 0.02))  

lines(rowMeans(int.total, na.rm=T), lwd=2)  

legend("bottomleft", "Mean scenario return", bty="n")  

matplot(var.total, type="l", lty=3, yaxt="n", xaxt="n")  

axis(side=4, at=pretty(var.total))  

axis(side=1, at=seq(0,80,10))  

lines(rowMeans(var.total, na.rm=T), lwd=2)  

legend("topleft", "Scenario variance", bty="n")  

path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//  

  Presentation//"  

dev.print(device=pdf, height = 4, width = 8, file = paste(path,"  

  MSARmeanvar-appendix2.pdf", sep=""))  

graphics.off()  

plot(int.total[,i], type="l")  

lines(var.total[,i], col=2)

```

```

dat2=data.frame(Mean = c(as.numeric(int.total[,1:7]), rep(NA,79),
  as.numeric(int.total[,8])),
  variance = c(as.numeric(var.total[,1:7]), rep(NA
    ,79), as.numeric(var.total[,8])),
  time=rep(1:79, 9), index=rep(ind,each=79))
xyplot(Mean + (variance-0.1) ~ time|index, data = dat2, type="l",
  lwd=2, col=c(1,2),
  ylim = c(-0.1,0.05),
  layout=c(3,3,1),
  index.cond= list(c(7:9, 4:6, 1:3)),
  ylab="Applied mean and variance", xlab="",
  key=list(text = list(names(dat2)[1:2]),
    points=list(pch=19, col=c(1,2)),
    columns = 2))
path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//Scengen//"
dev.print(device=pdf, height = 4, width = 7, file = paste(path,"
  MSARmeanvar2.pdf", sep=""))

rm(lpos,Regime,int.total,j,var.total,nf, dat2, Reg.mat,m,len)
rm(models.MSAR)
#####

## FOUR-WEEK RETURNS OF GLD HAS NEGATIVE MEAN
#####
ld = length(data$Date)
fwr = (data[-(1:4),5]-data[-((ld-3):ld),5])/data[-((ld-3):ld),5]

par(mgp=c(2,0.7,0), mar=c(3.5, 3.5, 1, 1))
plot(fwr, ylim=c(-10,10))
abline(h=mean(fwr[which(is.finite(fwr))]), col=2)
path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//Scengen//"
dev.print(device=pdf, height = 4, width = 8, file = paste(path,"
  GLD4weekReturn.pdf", sep=""))

rm(ld,fwr)
#####

## EXPLORATION OF FEZ IN ARMA-GARCH SCENARIOS
#####
plot(models.AG[[156]]@fit$series$h, type="l")
models.AG[[156]]@fit$matcoef

(sapply(scen.list.AG, function(x){range(x[,3])}))
(lapply(models.AG, function(x){which(x@fit$matcoef[-1,] < 0)})
  [1:100])
(lapply(models.AG, function(x){which(x@fit$series$h < 0)})) [1:200]
(lapply(models.AG, function(x){sum(is.na(x@fit$matcoef))>0}))
  [1:100])

models.AG[[84]]@fit$matcoef
plot(models.AG[[105]]@fit$series$h, type="l")

xtable(sapply(scen.list.AG, function(x){range(x[,1])}))

```

```

plot(sapply(scen.list.AG, function(x){range(x[,i])})[2,], type="l",
      ylim=c(-max(sapply(scen.list.AG, function(x){range(x[,i])})
                  [2,], na.rm=T), max(sapply(scen.list.AG, function(x){range
                  (x[,i])})[2,], na.rm=T)))
lines(sapply(scen.list.AG, function(x){range(x[,i])})[1,], col=4)

for(i in c(12,21,84,93,156,174))
{
  mat = models.AG[[i]]@fit$matcoef[-1,1]
  print(mat)
  print(mat[1]/(1-mat[2]-mat[3]))
}
rm(mat)
#####
C:/Users/Mie/Dropbox/Special/R/Special/scengen-plots.R

```

## C.5 Asset allocation

```

files = paste(c("MRAR", "DMA", "DMCV"), rep(c("depmix", "msar", "ag
", "bootstrap"), each=3), sep="")
files=files[1:3]

for(i in 1:length(files))
{
  fil = read.table(paste(files[i], ".csv", sep=""), sep=";",
                  header = TRUE, dec = ",", na.strings=c("", "N/A", "NA", "Na",
                  "null", 0))
  fil$Date=as.Date(substr(fil$Date,1,10), "%Y-%m-%d")
  for(j in 3:ncol(fil))
  {
    fil[which(is.na(fil[,j])),j] = 0
  }
  assign(files[i], fil)
}
allokeringer = mget(files, envir=.GlobalEnv)
rm(list=files)
rm(fil,i,files,)

## STACKED BAR PLOTS OF THE THREE STRAGETIES AND THE FOUR SCENGEN
## METHODS
#####
par(mgp=c(2,0.7,0), mar=c(2,3,1,0.5))
path = "C://Users//Mie//Dropbox//Speciale//Latex//figs//AssetAlloc/
/"
for(i in 1:length(allokeringer))
{
  matr = as.matrix(allokeringer[[i]][,-c(1,2)])
  rownames(matr) = as.character(allokeringer[[i]]$Date)
  barplot(t(matr), col=rainbow(9)[which(ind%in%colnames(matr))], ylab
          ="Portfolio value")
  legend("topleft",colnames(matr), col=rainbow(9)[which(ind%in%
          colnames(matr))], pch=19, ncol=3, bty="n")
}

```

```

dev.print(device=pdf, height = 4, width = 6, file = paste(path,
  names(allokeringer)[i], ".pdf", sep=""))
}
rm(matr,path)
#####

temp = sapply(allokeringer, function(x){rowSums(x[-c(1,2)])})
graphics.off()
par(mgp = c(2,0.7,0), mar=c(0,2,0,0.5), mfrow=c(4,1))
temp1 = temp[,which(substr(colnames(temp),1,3)=="MRA")]
ylim = range(temp[,which(substr(colnames(temp),1,3)=="MRA")])
plot(allokeringer[[1]]$Date, temp1[,1], type="l", col=rainbow(4)
  [1], ylim=ylim, ylab="", xlab="", xaxt="n")
for(i in 2:4)
{
  lines(allokeringer[[1]]$Date, temp1[,i], col=rainbow(4)[i])
}
legend("topleft", substr(colnames(temp1),5,nchar(colnames(temp1))),
  col=rainbow(4), pch=19,bty="n")
legend("top", "Maximum Risk Adjusted Return", bty="n")

temp1 = temp[,which(substr(colnames(temp),1,3)=="DMA")]
ylim = range(temp[,which(substr(colnames(temp),1,3)=="DMA")])
plot(allokeringer[[1]]$Date, temp1[,1], type="l", col=rainbow(4)
  [1], ylim=ylim, ylab="", xlab="", xaxt="n")
for(i in 2:4)
{
  lines(allokeringer[[1]]$Date, temp1[,i], col=rainbow(4)[i])
}
legend("top", "Dynamic Max Average", bty="n")

temp1 = temp[,which(substr(colnames(temp),1,3)=="DMC")]
ylim = range(temp[,which(substr(colnames(temp),1,3)=="DMC")])
plot(allokeringer[[1]]$Date, temp1[,1], type="l", col=rainbow(4)
  [1], ylim=ylim, ylab="", xlab="")
for(i in 2:4)
{
  lines(allokeringer[[1]]$Date, temp1[,i], col=rainbow(4)[i])
}
legend("top", "Dynamic Min CVaR", bty="n")
dev.print(device=pdf, height = 4, width = 6, file = paste(path,"
  test.pdf", sep=""))

rm(temp,temp1,ylim)

## AVERAGE ANNUAL RETURN
#####
rets = lapply(allokeringer, function(x){rowSums(x[-c(1,2)])})

(prod(1+(rets[[1]]-rets[[1]][1]))^(1/79)-1)*100

for(j in 1:12)
{
  taller=0
  a=c()

```



```

for(i in seq(1,79,13)[-7])
{
  taller = taller+1
  a[taller] =100*((1+(rets[[j]][i+13]-rets[[j]][i])/rets[[1]][i])
              -1)
}
print(a)
print(c(j,mean(a)))
}

```

```

dat2 = data.frame(return = as.numeric(temp),
                  Date = rep(allokeringer[[1]]$Date, 12),
                  strategy = rep(c("MRAR", "DMA", "DMCV"), each=79,
                                 4),
                  method = rep(c("depmix", "msar", "arma-garch", "
                                 bootstrap"), each=79*3))

```

```

for(j in c("MRA", "DMA", "DMC"))
{
  x11()
  par(mgp = c(2,0.7,0), mar=c(2,2,3,0.5))

  taller=0
  temp = sapply(allokeringer, function(x){range(x$DGT)})
  max.y = max(temp[,which(substr(names(allokeringer),1,3) == j)][is
    .finite(temp[,which(substr(names(allokeringer),1,3) == j)])])
  min.y = min(temp[,which(substr(names(allokeringer),1,3) == j)][is
    .finite(temp[,which(substr(names(allokeringer),1,3) == j)])])
  plot(allokeringer[[k]]$Date, rep(1,length(allokeringer[[k]]$Date)
    ), type="n",
       ylim = c(min.y,max.y+1),ylab="", xlab="",
       main = main)
  for(k in names(allokeringer)[which(substr(names(allokeringer)
    ,1,3) == j)])
  {
    taller = taller+1
    lines(allokeringer[[k]]$Date, allokeringer[[k]]$DGT, type="l",
          col=taller)
  }
  legend("topleft", bty="n", legend=paste("DGT", c("depmix", "MSAR"
    , "AG", "bootstrap"), sep="-"),
        col=1:4, pch=19)
}

rm(taller,temp,min.y,max.y)

```

C:/Users/Mie/Dropbox/Speciale/R/Speciale/AssetAllocation-plots.R



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