# Skeleton-based and Interactive 3D Modeling 

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## Summary

The costs of developing blockbuster video games are growing fast with a large part spent on artists creating 3D models. New procedural modeling techniques, such as the Skeleton to Quad-dominant Mesh (SQM) method by Bærentzen et al. [15], could potentially reduce the costs of creating 3D models. The SQM method is skeleton based, and allows the artist to create 3D models from a tree with nodes.

The thesis explores the limits of SQM and the Skeleton Modeler by setting modeling goals and trying to model them. The SQM method and associated Skeleton Modeler is shown to be unfit for modeling non-organic shapes and concavities.

The SQM method and Skeleton modeler is improved by extending the skeleton with seven new nodes types. The new node types will not change the SQM mesh generation, but extend it through pre- and post-processing steps.

The results show that the new node types enabled new shapes, but did not improve the modeling goals. The results also show that concavities are not easily representable in a skeleton based system.

The results for non-organic shapes can be improved by continuing with the selected approach, but it is recommended to instead change the mesh generation to simplify the algorithms. This will in turn require simplifications of SQM. Concavities need a different approach, and integrating sculpting tools is a potential solution that could to be explored.

## Preface

This master thesis was written as a part of the requirement for obtaining a degree in Master of Science in Digital Media Engineering at the Technical University of Denmark (DTU). The work was supervised by J. Andreas Bærentzen from The Institute of Informatics and Mathematical Modelling at DTU.

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I would like to thank my thesis supervisor J. Andreas Bærentzen. The thesis is an attempt to expand the Skeleton to Quad-dominant Method (SQM) by him and others [15]. The thesis topic and some of the proposed methods were suggested by him.

I would finally like to thank my family and my Fiancé for supporting me and cheering me on.

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## Chapter 1

## Introduction

The goal of the thesis is to improve the Skeleton to Quad-dominant Mesh (SQM) algorithm presented in Bærentzen et al. [15]. The SQM Skeleton Modeler tool uses lines to represent the bones in the skeleton and each bone is connected by a sphere. The SQM algorithm is good for modeling objects that have a skeleton structure, e.g. trees and base meshes for game characters. It, however, seems to be limited to tapered branching objects. The work will be to analyze these limitations and find solutions for them.

I will first present the motivation for why it is interesting to extend SQM, and state the goals and scope of the thesis. Some of the background theory necessary for understanding SQM will then be presented, and it can be skipped and referred to later if needed. The SQM algorithm and the Skeleton will then be described more in-depth before going into the meat of the thesis. Four modeling goals are presented and modeled with SQM. These models reveal some of the limitations of SQM, and ways around them are discussed in the subsequent chapter. The discussion on the limitations in turn leads to the development of seven new node types that are documented in-depth in their own chapters. The new node types are then finally used to model the modeling goals again to see if they have improved SQM.

### 1.1 Motivation

Over the last many years the cost of game development "skyrocketed", according to Folmer [18], with console games costing on average between 3 and 10 million US dollars to develop. A large part of the cost is caused by the development time and team sizes, which have "nearly doubled over the last decade". Folmer [18] focuses on how using commercial of the shelf (COTS) engines and tools can reduce the programming time and costs.

One way that costs have already been brought down is by licensing game engines such as the Unreal Engine [8]. That way a game developer does not need to develop a whole engine, and the cost is shared among all the game developers licensing the engine. Licensing game engines is not the only way that costs have been brought down. There is also tools for generating content such as SpeedTree[7] for generating trees and foliage procedurally.

Folmer [18] chose to focus on game engines and tools, but says that "the majority of the game development costs are spent on art and animation". Another way to curb the cost is therefor to make the artists more efficient, which could be done through better animation and modeling tools.

There are many modeling tools for helping the artist be more effective and produce higher quality work. Some of these are dedicated sculpting tools such as Mudbox[4] and ZBrush[9], while others are more general tools such as Softimage[6] and Blender[1]. Most of these tools work directly with surfaces. SQM is different because the artists specifies the surface implicitly by creating a skeleton connected by spheres and the SQM algorithm generates the surface from it. The artist can, therefore, quickly model the basic structure without worrying about the surface.

One interesting property of using a skeleton structure is that it is suitable for integration with procedural generation such as L-systems [22]. Bærentzen et al. [15] showed that a tree could be procedurally generated and skinned with the SQM algorithm. Using procedural generation would relieve the artist from manually creating all the game content. He could instead generate a number of models by specifying a few parameters, and then choose the best result.

Ji et al. [19] also generates meshes from a skeleton in their B-mesh method, and showed good results when using it to generate base meshes for game characters. The B-mesh tends to generate "three to four times more irregular vertices" [15, p. 8] than SQM, and SQM is therefore, a more interesting starting point for improving a skeleton based system. Leblanc et al. [20] uses blocks instead of spheres and is suitable for modeling non-organic shapes. It might be possible
to combine the ideas and extend SQM to make it possible to create non-organic shapes.

### 1.2 Goals and Scope

There are two goals for the thesis. The first is to document some of the limitations of SQM. The second goal is to find simple methods to extend SQM with new shapes.

The article by Bærentzen et al. [15] represents a large body of work, and it is therefore out of the scope of the thesis to fundamentally change how SQM works. The vertex and mesh generation is considered to be the fundamentals of SQM, and the thesis will therefore focus on how it can be improved through pre- and post-processing steps.

The scope of the implementation is to demonstrate the new methods for creating shapes. The implementation is meant as a prototype and proof-of-concept, and the focus will not be on code quality. It will need further work if it is to be used in a production environment.

It is too early, and out of the scope of this thesis, to research if SQM is more efficient than other modeling methods. There are no standardize methods for comparing the efficiency of modeling techniques, and that is also out of a scope of the thesis. I believe it is still important to develop new methods, as it is the first step to become more efficient.

## Chapter 2

## Background Theory and Concepts

This chapter contains an introduction to the background theory and concepts necessary to understand the SQM algorithm. It can be skipped and referred to later if needed.

### 2.1 Skeleton and Rooted Tree

SQM is a skeleton based method and uses a rooted tree to represent the skeleton. The two terms will be used interchangeably throughout the thesis.

An informal introduction to rooted trees will now be given based on the definition in Cormen et al. [17, p. 1087] and Bærentzen et al. [15]. A rooted tree consists of nodes that are connected by edges. The rooted tree starts in the root node. The root node can be connected to other nodes via edges. The nodes that are directly connected to the root node are called child nodes, and the root is their parent node. Similarly the child nodes can have child nodes of their own, to which they are their parent.

A node that does not have any children is called a leaf node. A node that has
exactly one child is called a connection node, and a node with more than one child is called a branch node.

One important difference between a graph and a tree, is that a tree does not have any loops(also called cycles). That means it must not be possible to follow a path of edges in one direction and visit a node more than once.

### 2.2 Meshes

A mesh is a graph that consists of vertices connected by edges. The vertices and edges form polygons. These polygons are typically triangles or quadrilaterals. A mesh consisting of triangles is called a triangle mesh, and a mesh the consisting of quadrilaterals is called a quad mesh. A quad-dominant mesh is a mesh that primarily consists of quadrilaterals and has few extraordinary vertices. SQM produces meshes that are quad-dominant.

### 2.3 Valency and Extraordinary Vertices

The valency is the term for describing the number of neighbors a vertex has. In a triangle mesh a regular vertex has the valency of six [23]. Vertices in a triangle mesh that do not have a valency of six are called extraordinary vertices. The valency of a regular vertex in a quad mesh is four, and the extraordinary vertices in a quad mesh are those who do not have a valency of four.

## Chapter 3

## SQM and the Skeleton Modeler

The Skeleton to Quad-dominant Mesh (SQM) method by Bærentzen et al. [15] is an algorithm for generating a skin from a skeleton. The SQM article has not been published at the time of the writing, so the description here will build on the draft and source code from July 12th 2011, which can be found on the accompanying CD-ROM or Zip-file. The SQM method has been improved in the meantime, and has been accepted for the Shape Modeling International 2012 conference, so there will be some discrepancy between the final SQM description and the one given here.

The skeleton in SQM is represented by a rooted tree. Each node in the tree has a position and a radius, and a mesh is generated from the tree as seen on Figure 3.1. The input skeleton for the SQM algorithm can be generated using an interactive modeling tool, like the Skeleton Modeler written for SQM, or by a program using a procedural approach like L-systems [22]. The Skeleton Modeler represents the skeleton as a number of spheres connected by lines. The spheres represent the nodes and their size and position can be manipulated by the user.

A quick overview of how the mesh is generated will now be given. The SQM algorithm creates a branch node polyhedron (BNP) at each branch node and connects them with tubes consisting of quads. If the node is a leaf node, then the end of the tube is collapsed into a triangle fan. The radius of the node


Figure 3.1: The Skeleton Modeler in action. The skeleton is seen on the left and the generated mesh on the right.
controls the size of the BNPs.

A more thorough description of the mesh generation will now be given. There are are six steps in the SQM algorithm :

1. BNP generation
2. BNP refinement
3. Bridging
4. Final vertex placement

The BNP generation works by first finding the vertices by intersecting the edges coming into the branch node with its associated sphere. Each BNP is then created by performing a spherical Delaunay triangulation on each of these sets of vertices. The SQM algorithm greedily refines the BNPs until they can be connected by a tube of quadrilaterals. The BNPs are then bridged to form a single mesh. The final vertex placement improves the look of the mesh by running multiple iterations of Laplacian smoothing and finding an optimal position on a tube using a quadric error measure.

The SQM is interesting because of the quality of the mesh it generates. The SQM algorithm produces a quad-dominant mesh with extraordinary vertices in the joints (BNPs) and at the tips. SQM generates fewer extraordinary vertices compared to the B-Mesh method by Ji et al. [19]. The quads tend to be aligned in the direction of the curvature lines, which is similar to what an artist would do [14]. The tips are where the tubes were collapsed. The tips form polar regions, which means that they are triangle fans surrounded by quads. This makes the mesh suitable for Bi-3 $C^{2}$ polar subdivision ( $C^{2} \mathrm{PS}$ ) by Myles and Peters [21]. If the mesh is subdivided with $C^{2} \mathrm{PS}$, then it will not develop unwanted creases in the tips, as would otherwise be the case if the more popular Catmull-Clark subdivision [16] is used. The Skeleton Modeler has a subdivision implementation that can handle polar regions, but it is not a full implementation of $C^{2} \mathrm{PS}$.

## Chapter 4

## Goals and Limitations

A good way to analyze the limitations of the SQM algorithm and the Skeleton Modeler is to set a goal for modeling various objects, and try to see if the goal is possible to model. In [15] several objects were modeled to understand the performance and limitations of the SQM method. Some of these objects can be seen on Figure 4.1 (a-c) on page 12. The goat creature on Figure 4.1a and the tree on Figure 4.1b are both organic objects, and they look fairly convincing. The car on Figure 4.1c, which is a non-organic object, however, looks too organic to be convincing. This suggests that SQM or the Skeleton Modeler might be less than ideal for modeling non-organic objects.

It is not clear which part of the SQM that makes it difficult to produce nonorganic looking objects. It could very well be that the SQM is suitable for modeling non-organic objects, and it is just the associated Skeleton Modeler tool that makes it difficult to produce the correct input for the SQM algorithm.

In this chapter the limitations of SQM and the Skeleton Modeler will be analyzed through modeling goals. The first goals are a ladder and a gear. They were chosen because they are more simple than a car but still complicated enough to reveal some of the limitations. A lollipop will also be modeled to see if it is possible to create spherical shapes. This will also serve as a starting point for modeling a head. The head was chosen because it contains concavities.


Figure 4.1: Examples of some of the objects modeled by Bærentzen et al. [15] with SQM.

### 4.1 Modeling a Ladder

A simple ladder, as seen on Figure 4.2, was chosen as a modeling goal. It was modeled using Blender[1], which is a traditional 3D content creation suite. The ladder was chosen because it is symmetric, it consists of rectangular and round shapes, and it has loops. It consists of two elongated boxes connected by three tubes that are slightly offset in relation to the boxes edges. The ladder is clearly symmetric around a vertical center line. It can also been seen that the ladder could be cut into three identical parts using two horizontal planes.


Figure 4.2: A simple ladder created with Blender.

The Skeleton Modeler starts with a red root node in the middle of the screen in symmetric mode. Figure 4.3 shows the Skeleton Modeler started up and rotated slightly around the red root node to reveal the default vertical green symmetry plane. Any nodes that are added to the root node will be mirrored around the symmetry plane. It, therefore, seems obvious to crate one of the round tubes by going through the root node as seen on Figure 4.4a on page 14. This simple shape reveals the first limitations: The Skeleton Modeler cannot skin the bone structure and outputs the error message "Root must be a branch node!".


Figure 4.3: The root node and the symmetry plane.

It is possible to work around the limitation that the root node must be a branch node by placing a node in the exact same position as the root node, as seen on Figure 4.4b. This is, however, difficult because there is no automatic snapping


Figure 4.4: Creating a straight tube with Skeleton Modeler.
and the node has to be placed in the exact same position as the root node. It is also difficult to move the node afterwards without selecting the root node.

After the straight piece has been created it can be skinned by pressing the space bar. The result can be seen on Figure 4.4c. The resulting model does not look like a tube because it is pointy instead of flat in both ends. The Skeleton Modeler has a feature called "consolidate leaves" that can make the leaves less pointy by placing an extra node at a small distance from all the nodes in their respective directions. The distance is dependent on radius of the spheres in the leaf nodes. It can be activated by pressing shift-c and the result can be seen on Figure 4.4d. Please note that the new node on top of the node on the root node has been deleted with the x key. Skinning the skeleton with consolidated leaves produces the result seen on Figure 4.4e. The ends are still not flat but they can be made more flat by manually moving the new nodes closer to the original leaf nodes. It is, however, again like in the case of the root node, difficult to move the nodes directly on top of leaf nodes. Creating flat ends is, therefore, clearly another limitation that needs to be addressed.

The tube with pointy ends on Figure 4.4e looks similar to two boxes next to each other, rotated $90^{\circ}$, and with pyramids on the ends. When it is subdivided it looks close similar to a tube, except for the round ends, as seen on Figure 4.4f. This result should be good enough for creating the tube sections in the ladder, but there is no way to explicitly create boxes, which is another limitation that needs to be solved.

With the current limitations in mind we will now try to create a ladder. The skeleton structure is first created as seen on Figure 4.5a. It is then skinned which results in the mesh seen on Figure 4.5 b . The mirror symmetric leaf nodes close to each other can be merged after the skinning process by pressing the $m$ key. The Skeleton Modeler does this by modifying mesh. It removes the vertices corresponding to close mirror leaf nodes and bridges the one rings. The result of merging the close leaf nodes can be seen on Figure 4.5c. There is no way to specify how close the leaf nodes need to be, so it could potentially merged some


Figure 4.5: A ladder created with the Skeleton Modeler.


Figure 4.6: The ladder with sharper joints.
wrong leaf nodes. This is another limitation. It would be helpful to be able to either model loops, or specify which nodes need to be merged.

The ladder was then subdivided as seen on Figure 4.5d. The parts that were supposed to be boxes are rounded as seen before when creating the straight piece. This is the same limitations as noted before. It is not possible to specify hard edges nor control the rotation. It can also be seen that joint where the tube parts meet the boxes is rounded instead of creating sharp $90^{\circ}$ angled joints. The joints can be made sharper by using a workaround similar to the consolidate leaves one. If an extra node is placed in the joint, then the joint is forced to be sharper as can be seen on Figure 4.6. They are, however, still not completely sharp, so this is another limitation.

Another detail that was left out from the modeling the ladder on Figure 4.6


Figure 4.7: The ladder with offset and smaller tubular steps.


Figure 4.8: A comparison of the goal created with Blender and the result created with the Skeleton Modeler.
was that the tubular steps connecting the boxes were slightly offset from the edge, and that they are smaller than the boxes. There is no way to specify this explicitly, but it can be done by scaling down the size of the spheres inserted for creating the sharper joints. The result of this can be seen on Figure 4.7.

The final result is compared to the goal on Figure 4.8. The ladder created with the Skeleton Modeler looks somewhat similar to the goal on, if we ignore the exact dimensions and the thickness of the parts. The biggest difference is that the boxes on the side have been rounded, the tops ends are round, the joints are not $90^{\circ}$ angles, and there is a glitch in the middle of the bottom step. There is, therefore, an opportunity to improve SQM to better handle theses cases.

### 4.2 Modeling a Gear

In this section the goal is to create a simple gear with the Skeleton Modeler. The goal seen on Figure 4.9 was created procedurally with Blender's "Extra Objects" plug-in. The gear can be composed into simpler objects like in the case of the ladder. The main part is a tube with thick walls, and it has eight
teeth around its side each separated by 45 degrees angles.


Figure 4.9: A simple gear created with Blender.
There is no way to directly create a tube with thick walls, so one approach would be to create a ring structure as seen on Figure 4.10a with eight spokes. Placing the spokes and creating 45 degree angles had to be done manually, but the skeleton modeler could be extended to help with the placement. Aside for the non-perfect node placement, the bottom part of the ring looks odd when skinned as seen on Figure 4.10b. This is because the nodes at the bottom have not been merged. The SQM algorithm does not support cycles as mentioned earlier in the previous section, but the modeler can merge the mesh if two leaf nodes are close to the symmetry plane. The two bottom nodes in the ring are, however, not leaf nodes, so they are not merged. The bottom spoke on the other hand is a leaf node, which is why the merged mesh on Figure 4.10c looks odd. It could in principal be worked around by placing a pair of leaf nodes on top of the nodes in the bottom of the ring, but it would be difficult to place the nodes precisely. It can, therefore, be seen as earlier, that it is a limitation that cycles cannot be expressed explicitly.

On Figure 4.10 it can also be seen that the hole in the middle of the ring is much larger than the reference on Figure 4.9. The spheres in the ring can be made larger, which makes the walls of the tube thicker as seen on Figure 4.11, but it also makes the tube thicker in the other direction. It can, therefore, be seen that it is not possible to independently control the width and height of nodes.

The final result can be seen next to the goal on Figure 4.12. It can be seen that the gear becomes rounded when subdivided, whereas a flat tube was the goal. Another limitation is, therefore, that it is not possible to make big flat sides on non-leaf nodes. It can also be seen that the teeth are not box shaped and the corners have been rounded. This is the same limitation with sharp angles and box shapes that was seen when modeling the ladder.

(a) Gear ring

(b) Odd bottom

(c) Odd merge

Figure 4.10: An attempt to create a gear with the skeleton modeler.

(a) Thicker ring
(b) Skinned

(c) Skinned side view

(d) Original side view

Figure 4.11: Making the spheres in the ring larger makes the wall of the tube thicker in both directions.


Figure 4.12: The final result compared with the goal.

### 4.3 Modeling a Lollipop

In this section the goal is to create a lollipop in order to find out if it is possible to create spherical shapes. The goal seen on Figure 4.13 was created with Blender. It consists of a sphere on top of a cylinder with closed ends.


Figure 4.13: A lollipop created with Blender.

Creating a sphere with the Skeleton Modeler requires more than just placing a node. For example, the root node must be a branch node as seen earlier. The SQM algorithm tries to create branch like structures, which means that in order for the joint to become round, there must be branches in all directions. This means that six nodes are needed as seen on Figure 4.14. The joint between the sphere and the cylinder was made sharper by inserting an extra node right under the root node, and finally the bottom of the cylinder was made more flat by inserting an extra node at the bottom.

The final result was textured and rendered in Blender and is compared with the goal on Figure 4.15. The result and goal look similar, and the biggest difference is that the sphere in the result is not completely round but slightly elongated at the bottom.


Figure 4.14: A lollipop modeled with the Skeleton Modeler.


Figure 4.15: The result compared with the goal.

### 4.4 Modeling a Head

In this section the goal is to create a simple head model in order to uncover potential limitations of SQM and the Skeleton Modeler. The goal seen on Figure 4.16 was modeled using Blender. A subdivided cube was used as the base mesh, and the features were added by moving the vertices using the grab sculpting tool. The main features are the eye sockets, nose and mouth. The eye sockets are almost ellipsoid concavities, whereas the mouth curves down at the sides.


Figure 4.16: A simple head modeled with Blender.
Figure 4.17 shows the result of creating a simple head with the Skeleton Modeler and SQM. The round shape was accomplished in the same way as for the lollipop in Section 4.3, by placing nodes on all sides of the root node, and then scaling the root node up in size until it encompasses the nodes. The concavities for the eye sockets and the mouth were created by placing nodes under the surface of the root node, which forces the mesh back in.


Figure 4.17: The result of creating a simple head with the Skeleton Modeler.

The biggest limitation that was discovered was that it is difficult to create concavities. The nodes used for creating the concavities have to be placed inside other nodes. That makes it difficult to see what the result of the input tree will be. It also makes it difficult to manipulate the concavities, as the node that encompasses it will have to be scaled down or moved before the node used for the concavity can be moved.

It is also not possible to control the shape of the concavity. The mouth for example was supposed to be an elongated concavity, but it can be seen that there is a tube sticking out of the mouth. This is because three nodes were used to create the concavity and the SQM tries to build a mesh that follows that tree structure. There is also no way to express concavities in relation to the generated mesh.

### 4.5 Summary of Limitations

The following limitations were found in this chapter:

- The root node must be a branch node.
- Difficult to produce flat end pieces.
- Not possible to create boxes.
- Not possible to mark sharp edges for the subdivision.
- Not possible to create sharp joints.
- Difficult to place nodes according to a pattern, e.g. 45 degree angles.
- Cycles cannot be expressed explicitly.
- Not possible to control node width and height independently.
- Not possible to make flat parts in non-leaf nodes.
- Difficult to create spheres.

The next chapter discusses how to improve SQM and the Skeleton Modeler to remove the limitations.

## Chapter 5

## Improving SQM and The Skeleton Modeler

In this chapter we will look into how SQM could be extended to handle more shapes. The chapter is meant as an overview and the details of the implementation and results will be described in later chapters.

### 5.1 New Leaf Node Types

In the previous chapter it was seen that it was difficult to produce flat end pieces. The SQM algorithm connects the two nodes with a cylinder but collapses leaf nodes to a point which result in a cone as seen on the simplified 2D illustration on Figure 5.1. The figure shows a section of the SQM tree with a connection node and a leaf node. The solid line represents the bone and the stippled line represents the generated mesh.

One way to solve the problem with the flat end pieces could be to add a special leaf node that is flat, e.g. a hemisphere leaf node. There is, however, no reason to stop at flat end pieces. One could also think other types of end pieces that could expressed with new leaf node types. Four proposed leaf node types can be seen on Figure 5.2 on page 27 drawn in a light gray. The four proposed leaf
node types are:

- Hemisphere leaf nodes
- Sphere leaf nodes
- Cone leaf nodes
- Cube leaf nodes

Common for the proposed leaf node types is that they do not change the basic mesh properties. The leaf nodes are still polar regions. We will now go through the four new node types.


Figure 5.1: The current behavior of a leaf node.
The hemisphere node would allow the creation closed cylinders. The hemisphere leaf node is represented by a closed hemisphere. It is rotated so that the normal of the flat side points in the direction of the bone and away from the center of the parent. The two nodes should be connected by a cylinder with a flat closed end or a truncated cone.

The new leaf sphere node changes the behavior so that the end is round instead of pointy. This makes the generated mesh correspond more closely to the input skeleton. The representation of the new leaf sphere node looks just like the old sphere node, but the nodes will be connected by a cylinder or truncated cone with a hemisphere on the top.

The cone node provides a way to explicitly make ends pointy. The cone node should behave in the same way as the old leaf sphere node on Figure 5.1. The representation should be a cone which changes its size and direction automatically when the user moves the tip of the cone. The bottom of the cone should


Figure 5.2: The four proposed node types.
go through the center of the parent node, and the normal of the bottom side should be aligned with the bone and point towards the center of the parent. The bone ends in the tip of the cone.

The cube leaf node would enable box shapes. It is represented by a cube that can be scaled in all directions. It can also be rotated around the bone axis, which means the bottom normal should point towards the center of the parent node. The SQM algorithm should be modified so that the bottom of the cube is connected with the sphere using a tubular shape consisting of quads. One end of the tube should approximates a circle end other end should approximate a rectangle shape. The cube is subdivided until it is possible to connect the two kinds of shapes. Finally the top should be triangle fan so that the part of the mesh resulting from the leaf node remains a polar region.

All of these nodes should be possible to implement using a tree transform, which is explained in the next section.

### 5.2 The Extended SQM Tree

A number of new node types were suggested in the previous section in this chapter. One way to implement these new node types is to modify the SQM algorithm to directly generate the new shapes that the new node types represent. Another way would be to create an Extended SQM tree (ESQM tree). The ESQM tree would contain the new nodes types and would be converted into the old SQM tree before the SQM algorithm is run. That way the SQM algorithm itself would not need to be changed.

In the section where a ladder was modeled, it was seen that the leaf nodes could be more flat by using the "consolidate leaves" operation on the leaf nodes. This was done by inserting extra nodes close to the leaf nodes as seen on Figure 4.4d on page 14. One could imagine that the end would become completely flat if new extra node is placed directly on top of its parent instead of close to it. It is, however, difficult to place them on top of each other with a mouse. Furthermore, it is cumbersome to move the nodes together manually, if the position of the original node needs to be edited.

The leaf hemisphere node could be implemented using the ESQM tree approach. It would then be implemented by converting it to a node with a an extra node on top of it similar to how "consolidate leaves" operation worked. The user would only see and move the new hemisphere leaf node type, thus the problem with the exact placement and editing would be solved.


Figure 5.3: Transforming the new leaf sphere node in the ESQM tree into multiple nodes in the SQM tree.

The new leaf sphere node type could also be implemented using the ESQM approach. Instead of just adding an extra node as in the case of the leaf hemisphere node, multiple nodes could be used to approximate a sphere as seen on

Figure 5.3. The subdivision depends on the number of nodes, and the level of subdivision could be made user controllable.

### 5.3 Cube Node

New leaf node types were defined in the previous sections. The idea could be expanded to also include new non-leaf node types. A new cube node could for example make it possible to create cube shapes, e.g. for creating a ladder. Like the leaf cube node, the non-leaf cube could be scaled in any direction, and rotated along the bone axis.

One way to implement cube nodes could be to modify the final vertex placement step. The vertices could be moved out into corners of the cube and the remaining vertices could be distributed on the edges. This would not require changes to SQM but could be applied as a post-process.

### 5.4 Root Branch Node Requirement

The SQM algorithm could be modified so that it does not require the root node to be a branch node. The BNP creation would have to change because the root node must have at least three children in order to create a BNP. Remember that this requirement comes from when the intersection between the root node's sphere and the paths are performed. This results in two opposite triangles when it is has three children. It would produce two line segments if there are two children, and two points if there is one child. This would then cause a problem in the subsequent subdivision because there would not be any triangles to subdivide. One way to get around the subsequent subdivision problem is to make sure that triangles are created even with fewer than three connected nodes. If there are two children, then the orthogonal direction to the directions of the paths could be used to create a pseudo path. The intersection could then be performed between the node's sphere and the pseudo path to create the missing points. Two pseudo paths could similarly be created if the root node only has a single child.

Finally if the root has no children, then a sphere mesh could be created during the BNP creation and the remaining SQM steps could be skipped. This would not really be helpful in the number of new shapes it enables(only one), but it would be helpful in giving the artist some more logical feedback than an error message.

The suggested method for removing the root branch node requirement will require changes to the BNP creation, which means that it is a fundamental change to SQM. It is, therefore, considered out of the scope to implement this change.

### 5.5 Concavities

There is no way to explicitly specify a concavity in the Skeleton Modeler. It seems to be a general problem with any bone editing system that there is no obvious way to create concavities. For example, neither [19] nor [20] specify any way to create concavities. Both add details in a post process using traditional sculpting methods. The problem seems to be that in the real world there are no objects that take away space. Putting flesh and skin on top of a bone structure, however, corresponds well to how the real world works. Similarly sculpting and carving is how an artist would remove material from an object. Sculpting tools, therefore, seem like a natural way of creating concavities.

It is possible to create concavities with the SQM algorithm by moving a leaf node back into its parent. It is, however, difficult to control. The first problem is that the new leaf node is inside the node and therefore not possible to select and manipulate with the mouse. The second, and probably most important problem, is that there is no clear relationship between the position and size of the leaf node, and the size and shape of the concavity. We will outline three possible ways to improve the creation of concavities.

One way could be to add negative nodes leaf nodes. They would point out of the parent, but would be moved back into the parent just before the SQM algorithm is run. They would be easier to manipulate because they stick out of the parent node. A negative node that is further away from its parent would create a deeper concavity. Similarly a bigger negative node would create a wider concavity. Negative leaf nodes could be implemented using an ESQM to SQM tree transformation, which moves the negative node into the parent.

A second way could be to add a deformable node type. The deformable node would have a mesh that could be deformed with traditional sculpting tools. The SQM algorithm would have to be changed so that the final vertex placement follows the deformation of the node. It is, however, not obvious how it would affect the rest of the SQM algorithm, and how many changes would be needed.

Finally a third way to create concavities could be to display the generated mesh and let the user manipulate it using sculpting tools. The sculpting operations could then somehow be associated with nearby nodes and reapplied after the
mesh is regenerated. That would make it possible to move and create nodes in one part of the tree, while retaining details created in another part of the tree. For example, a list of type, magnitude, relative position, and direction of the sculpting operations could be stored in the closest node.

The first way would be easiest to implement as it would not require any changes to the core of the SQM algorithm. It could be implemented as a simple transformation of the input tree. The two last ways would require more work to implement. The second way would require changes to the core of the SQM algorithm, and it is not clear how much will need to be changed. The third way could be done without changing the SQM algorithm, as it would be a post process applied to the mesh after generating the mesh. The sculpting operations would be coupled to the tree, so the mesh could be regenerated without discarding all the sculpting operations. The negative node will be implemented as it requires the fewest changes.

### 5.6 Cycles

The SQM algorithm cannot handle cyclic graphs. It is possible to add cycles using a built-in post-process. After the mesh generation the symmetric twin leaf nodes that are close to each other are found. If they are closer than some threshold then the parts of the meshes representing the nodes are bridged. This means that it is not possible to explicitly control which nodes are merged.

A special cycle node could be added. The Skeleton Modeler would need to have a way to select two nodes and mark them for merging. The artist would then see a special node type in the Skeleton Editor. The merged node would be broken into two separate nodes before the SQM algorithm is run and marked for merging later. The SQM algorithm would be run to create the mesh, and then finally the nodes marked for merging with each other would be merged.

Cycles have been chosen not to be implemented to limit the scope and will, therefore, not be discussed further.

### 5.7 Remaining Limitations

Section 4.5 listed a number of limitations. All of them have not been addressed in this chapter to limit the scope. They are, therefore, good choices for further work.

## Chapter 6

## Hemisphere Node

### 6.1 Analysis

During the modeling of a ladder in Chapter 4, it was seen that it was difficult to produce flat end pieces. In Chapter 5 it was suggested that adding a hemisphere node could make it easier to create rounded shapes such as cylinders with flat tops and truncated cones. The hemisphere node illustration is repeated here on Figure 6.1. The stippled line represents the generated mesh. The mesh between the hemisphere node and the sphere node is a truncated cone or cylinder, where the top radius matches the radius of the hemisphere node, and the bottom radius matches the radius of the sphere node. The hemisphere node is represented by a closed hemisphere. It should always be rotated so that the normal of the flat side points in the direction of the bone and away from the center of the parent. That way the flat end will be represented by the flat end of the filled hemisphere.

One straightforward way to implement the hemisphere node without changing the SQM algorithm would be to use a tree transform as described in Section 5.2. The hemisphere node in the ESQM tree would be replaced with two sphere nodes in the SQM tree as seen on Figure 6.2. The SQM algorithm creates a single vertex for the leaf node, so if the leaf node is in the same position as its parent's position, then a flat top with a polar region should be generated.


Figure 6.1: A 2D illustration of the hemisphere node.


Figure 6.2: The hemisphere node in the ESQM tree is replaced by two sphere nodes in the SQM tree.

Another approach to implementing the hemisphere node could be again to use a tree transformation, but changing the positions of the new nodes. The hemisphere node is again replaced by two sphere nodes as seen on Figure 6.3, but the position of the leaf node is not directly on top of its parent. The position of the leaf node is moved away in the direction of the bone. This is somewhat similar to how the existing "consolidate leaves" feature works. The single vertex resulting from the leaf node is then moved to the parent node's position during the final vertex placement.

The hemisphere node should also work with the subdivision scheme used in Skeleton Modeler. The edges around the vertex created from the leaf node would have to be marked in a special way. The subdivision scheme should be


Figure 6.3: An alternative way to implement the hemisphere. The hemisphere node in the ESQM tree is again replaced by two sphere nodes in the SQM tree, but the leaf node is not in the same position as its parent. The vertex in the tip is moved down during the final vertex placement.
able to split the edges, but not move the original vertices. Furthermore, the new vertices should remain in the same plane as the original vertices. This is illustrated in 2D on Figure 6.4 where the mesh is subdivided once to better approximate a disk. The mesh is illustrated with thick black lines, the stippled circle is the limit surface, and the black dots are the vertices. It was decided that modifying the subdivision implementation is out of the scope of the thesis, so it will not be implemented.


Figure 6.4: An example of how the subdivision should progress to approximate a disk.

### 6.2 Implementation

The tree transform as describe in the previous section was implemented. It did however not work. The original SQM implementation by [15] cannot handle nodes in the same position. In the method Graph::compute_paths_and_-
distances a 2D grid is initialized with the distances between the nodes. A non-calculated distance is represented by zero, so it is not possible to differentiate between the distance not being calculated and a distance of zero. The code was changed so that the grid was initialized with -1 instead of zero. That way a distance of zero could be stored in the grid. That was unfortunately not enough to make it work as there are also multiple places in the source code where the distance between a parent and a child node is used in a division (e.g. in the final vertex placement). This problem would require extensive changes to the original implementation, so it was chosen to instead try the second approach described in the previous section. This meant that the SQM tree had to be changed so that nodes resulting from the transformation had to be marked as hemisphere nodes. That way the vertex created by the leaf node could be identified and moved to the same position as its parent node during the final vertex placement.

The visualization of the hemisphere node was implemented by modifying the source code of the freeglut project[10]. The code for visualizing a sphere was modified so that all the vertices belonging to the top half of the sphere would all be in the plane separating the top from the bottom half. This was done by setting the $y$ coordinate to zero for all the vertices that had a positive $y$ coordinate ( $y$ was the same direction as the up vector).

The top of the hemisphere node should always be rotated, so that the normal points in the same direction as the bone. The rotation around the bone is not important as long as the mesh is not twisted with that rotation. The GEL library [2] contains the function make_rot that can "Construct a Quaternion rotating from the direction given by the first argument to the direction given by the second.". The direction of the bone and the default up vector is used as the input to construct the quaternion. The rotation axis and angle is then be found by using the get_rot function in GEL. The rotation axis and angle is then supplied to glRotated in OpenGL [5] before drawing the hemisphere node.

### 6.3 Results

A comparison of the straight piece created with the "consolidate leaves" feature and the hemisphere node can be seen on Figure 6.5. The end piece created with the hemisphere node is flat, whereas the one created with the "consolidate leaves" feature is pointy. That is a clear improvement in the quality of the shape.

If the flat end needs to be moved, then the two nodes crated with the "consolidate leaves" feature will have to be moved together, which can be cumbersome.

The hemisphere is a single node, so it is no more difficult to move than a normal node.


Figure 6.5: Comparison of a straight piece created with "consolidate leaves" feature and the hemisphere node.

The results on Figure 6.5 shows a simple straight piece. The part of the mesh resulting from each hemisphere node consists of only five vertices, four in the circumference and one in the middle. That means that the flat end does not approximate a disc very well. The number of vertices is dependent on the BNP subdivision, and this can be forced higher by inserting more nodes. The result on Figure 6.6 shows that the mesh in the top of the hemisphere node does indeed approximate a disc when the BNP is more subdivided. Please not that the BNP subdivision or refinement should not be confused with the subdivision scheme that is included with the Skeleton Modeler and can be applied as a post-process.


Figure 6.6: The top of the hemisphere node approximates a disc.
Figure 6.7 shows the straight piece created with hemisphere nodes and subdivided using the subdivision scheme in the Skeleton Modeler. The straight piece is not flat in the end. This is because hard edges were not implemented, nor was the subdivision implementation improved to handle hard edges.

Figure 6.7: Straight piece created with hemisphere nodes and subdivided using the subdivision in the Skeleton Modeler.

### 6.4 Sub-Conclusion

It was not possible to create flat ends with SQM. The Skeleton Modeler came with a feature called "consolidate leaves" that could insert extra nodes in order to create a more flat end, but it was still pointy. Another disadvantage of this feature was that two nodes would have to be moved together, if the artist wanted to move the flat end. It was assumed that a completely flat end could be created if the nodes were moved close enough together. This was discovered not to be true during the implementation phase. Here it was discovered that the current implementation assumed that the distance was non-zero, and that it became numerically unstable when the nodes were close together. The second approach therefore, had to be used where the nodes were placed at a distance, and the leaf node vertex moved during the final vertex placement.

The hemisphere node solved the problem of having to use more than one node to create a flat end. A flat end is represented by a single node that can easily be moved.

It was briefly outlined how the mesh generated from the hemisphere node could be subdivided. The subdivision implementation was, however, not changed because it was deemed out of the scope of the thesis. That meant that marking hard edges was also not implemented. This is something that could be looked into into as further work.

## Chapter 7

## Cone Node

### 7.1 Analysis

It was seen in Chapter 4 that using a leaf sphere node results in a single vertex being created. This means that the resulting mesh, before being subdivided, is shaped like a cone. It seems odd that the shape of the resulting mesh and the shape of the node do not correspond to each other. A cone node could provide a better connection between the input tree and the resulting shape.

The cone leaf node should behave in the same way as the old sphere leaf node on Figure 5.1 and the SQM algorithm does, therefore, not need to be changed for this node type. Figure 7.1 shows a simplified 2D illustration of how the cone node should work. The representation should be a cone which changes its size and direction automatically when the user moves the tip of the cone. The bottom of the cone should go through the center of the parent node, and the normal of the bottom side should be aligned with the bone and point towards the center of the parent. The bone ends in the tip of the cone. Finally the radius of the cone should be the same as the radius of the parent node.

The cone leaf node should also work with the subdivision scheme in the Skeleton Modeler. None of the original vertices should be moved during the subdivision and the new vertices should be placed in the same plane as the bottom plane


Figure 7.1: The cone node illustration.
of the cone. This is the same way as the subdivision scheme described for the hemisphere node in Chapter 6 .

### 7.2 Implementation

The implementation was straightforward. The ESQM cone node is translated to a single sphere node in the SQM tree.

The visualization of the cone node was implemented using the glutSolidCone function in GLUT [3]. The distance between the cone node and the parent node is used as the height, and the radius of the parent node is used as the radius of the cone.

### 7.3 Results

The result of the implemented cone node can be seen on Figure 7.2. The generated mesh looks more like the input tree. The two results should have been identical because the cone node is translated into an SQM sphere node. The small difference is likely cause by the position of the extra node on top of the root node. The extra node on top of the root node was added to work around the requirement that the root must be a branch node.

One thing that was noticed, but cannot be shown on a figure, is that the cone
node can become difficult to select when it is small or thin. This is because the picking test is only done against the tip of the cone node out to the radius of the resulting sphere node. The picking should be changed so that the whole cone is used instead of just the tip.

(a) Old Input

(c) New input

(b) Old result

(d) New result

Figure 7.2: Comparison between the result of the old node sphere type and the new cone node type.

### 7.4 Sub-Conclusion

The introduction of the cone node was motivated by the observation that there was a poor correspondence between the shape of the sphere node and the resulting mesh. The resulting mesh was conical, so it was assumed that a cone node would offer a better correspondence between the input tree and the resulting mesh. The results confirmed this, but the usability has not been tested on any artists.

The cone node in the current implementation can be difficult to manipulate because only the tip of the cone node is used in the picking test. The picking should be changed so that the whole cone is used instead of just the tip. Furthermore, the proposed changes to the subdivision implementation was not implemented. These two extensions could be good candidates for further work.

## Chapter 8

## Leaf Sphere Node

### 8.1 Analysis

The leaf sphere node in the original Skeleton Modeler resulted in a conical shape. It seemed odd that a round node would result in a conical shape. The cone node was therefore introduced in the previous chapter. It gave a direct mapping between the input tree and the output mesh. In a similar way it could be interesting to change the semantics of the sphere node, so that it results in a spherical mesh instead of a cone.


Figure 8.1: The new leaf sphere node.

Figure 8.1 shows a 2D illustration of how the new sphere node should work. Multiple vertices are inserted so that the mesh better approximates a hemisphere. One way to implement this would be to use a tree transformation. Figure 8.2 shows how the new sphere node type in the ESQM tree could be translated into multiple nodes in the SQM tree. Each SQM sphere node will result in a edge ring being generated. The SQM sphere nodes radii decreases so that the resulting mesh approximates a hemisphere.


Figure 8.2: The new leaf sphere node in the ESQM tree is translated into several SQM sphere nodes.

Figure 8.3 shows how the radii of the generated sphere nodes could decay in order to approximate a hemisphere. The left shows the 90 degrees arc that we want to approximate, and the vertical lines represents the radii of the generated sphere nodes in the SQM tree. The right shows a line segment connecting the radii of the generated sphere node, and it can be seen that the line segments


Figure 8.3: The position and radii of the sphere nodes that could be used for the new sphere node. The left shows a 90 degrees arc with each radius as a vertical line. The right shows the radii connected by line segments.
approximate a 90 degrees arc. The equation for the circle is:

$$
\begin{equation*}
x^{2}+y^{2}=r^{2} \tag{8.1}
\end{equation*}
$$

Where $x$ and $y$ are the coordinates in $R^{2}, r$ is the radius of the circle. If $x$ is already known then it is just a matter of isolating the $y$ coordinate. Furthermore, only a 90 degrees arc is needed, which means that only the positive solution is needed:

$$
\begin{equation*}
y=\sqrt{r^{2}-x^{2}} \tag{8.2}
\end{equation*}
$$

The distance between each sphere node was chosen to equidistant. It is found by dividing the radius of the new leaf sphere node by the number of generated nodes:

$$
\begin{equation*}
d=r / N \tag{8.3}
\end{equation*}
$$

Where $r$ is the radius of the new leaf sphere node, and $N$ is the number of SQM sphere nodes used to approximate the hemisphere. This method was chosen because it was the simplest. There might be a better way to place the generated sphere nodes in order to reduce the approximation error, but this was not investigated. Also, the subdivision could be controlled through the number of number of generated sphere nodes.

The $x$ coordinate of the $i$ 'th sphere can now be found:

$$
\begin{equation*}
x_{i}=i \cdot d \tag{8.4}
\end{equation*}
$$

The radius of $i$ 'th sphere node in the SQM tree is therefore:

$$
\begin{equation*}
r_{i}=\sqrt{r^{2}-x_{i}^{2}}=\sqrt{r^{2}\left(1-(i / N)^{2}\right)} \tag{8.5}
\end{equation*}
$$

Which will be used to approximate a hemisphere.

### 8.2 Implementation

The implementation was straightforward as the position and the radius calculations described in the previous section did not require much code. There were, however, problems modifying the final vertex placement, so that it would not interfere with the vertex placement, which can be seen in the next section.

### 8.3 Results

The results of the new sphere leaf node type is shown on Figure 8.4. The resulting mesh does not approximate a hemisphere, and the result is not satisfactory.

(a) Input for old sphere node.

(c) Input for new leaf sphere node.
(b) Old sphere node result.

(d) New leaf sphere node result.

Figure 8.4: Comparison of the result between the old leaf sphere node and the new leaf sphere node.


Figure 8.5: The new leaf sphere node approximates a hemisphere if the final vertex placement is disabled.

The final vertex placement was disabled and the mesh re-generated. The re-
sult can be seen on Figure 8.5, where it is clear that the new sphere node now approximates a hemisphere. More experimentation showed that it is the Laplacian smoothing applied during the final vertex placement that is the problem. There is currently no way to selectively apply the smoothing. This shows that although it is possible to create more advanced shapes by using multiple SQM sphere node, it is likely that the shape will be changed too much by the final vertex placement to be useful. It might be possible to modify the final vertex placement so that it does not apply smoothing to the whole mesh, but this is out of the scope of the thesis.

The interaction between the final vertex placement and the leaf sphere node is not as severe when there are more nodes in the skeleton. Figure 8.6 shows a skeleton with multiple nodes and the resulting mesh. The mesh in the nodes does still not follow a hemisphere as closely as without the final vertex placement, but the result is much better than what was seen on Figure 8.4d.


Figure 8.6: The leaf sphere node works better with the final vertex placement when there are many nodes in the skeleton.

### 8.4 Sub-Conclusion

The motivation for adding a leaf sphere node was, like in the case of the cone node, to achieve a better correspondence between the input tree and the resulting mesh. Round ends would be expected when the nodes in the tree are round.

The results showed that it is possible to create round ends using a tree transform with multiple SQM sphere nodes. The result is, however, not stable because the Laplacian smoothing, applied during the final vertex placement, can change the vertex position too drastically. If the smoothing could be removed from the final
vertex placement or disabled for parts of the mesh, then the leaf sphere node would perform well. It would also allow for the addition of other interesting shapes in the leaves.

## Chapter 9

## Negative Node

### 9.1 Analysis

During the modeling of a head it was seen that it was difficult to create concavities. Different ways of implementing concavities were analyzed in section 5.5. One of them was to create a negative node. The negative node would point out of the tree, but would be moved back into the parent node just before the SQM algorithm is run. It would be easier to manipulate because it sticks out of the tree. A negative node that is further away from its parent would create a deeper concavity. Similarly a wider negative node would create a wider concavity.

The negative node could be implemented using a tree transform. Two different ways were explored as seen on Figure 9.1, either using no extra child or using an extra child. The stippled line represents the generated mesh, the thick solid circles the nodes, and the solid lines represent the bones. The dark gray line illustrates how far the negative node will be moved into the parent node. The method using a no extra child, as seen on Figure 9.1a, will result in a concavity where the size of the negative node does not influence the size concavity in the parent. This is because there is nothing that forces the surface to follow the parent node. The mesh can be made to follow the parent by introducing an extra node during the tree transform as seen on Figure 9.1b. The size of the concavity can then be controlled by setting the size of the extra node to be the
same as the size of the negative node.

(a) No extra child.

(b) Extra child.

Figure 9.1: The two ways of the negative could work using a tree transform.
The shape of the concavity could be controlled by the shape of the negative node. The Skeleton Editor would have to have some means of manipulating the negative node to make it a spheroid. The vertices of the extra child in the SQM tree could then be projected onto the spheroid during the final vertex placement.

### 9.2 Implementation

The result of the first attempt to implement the new negative node using a tree transform can be seen on Figure 9.2. The result is not as expected. The resulting mesh contains degenerate triangles, and some of the triangles are showing the back faces. The problem is likely in the the initial mesh generation, but the exact location of the problem could not be established. The reason why it is suspected
to be in the initial mesh generation, is that each step of the SQM algorithm was visualized and the problem occurs before the final vertex placement.


Figure 9.2: The early results of implementing a negative node that inserts an extra node in the surface using a tree transformation. There is a clear problem with degenerate triangles.

The implementation was changed so that the position of the negative node is not modified. The negative node is a leaf node so it will result in a single vertex. This vertex is then moved back into the parent during the final vertex placement in the same way as the tree transform would have done. Changing the shape of the negative node was not implemented, as it was deemed to require extensive changes to the Skeleton Modeler.

### 9.3 Results

The result of implementing the negative node by moving the vertices during the final vertex placement, instead of relying on the tree transform, can be seen on Figure 9.3. The mesh does now no longer contain degenerated triangles. The depth of the concavity can be controlled by the distance of the negative node. The size of the hole can be controlled with the size of the negative node. The shape of the concavity can only be round, which is a clear limitation.

(a) Input

(b) Result

Figure 9.3: The result of the negative leaf node using the final vertex placement to move the negative node inwards.

### 9.4 Sub-Conclusion

The negative node was implemented and it is now possible to create concavities explicitly. The depth of the concavity is controlled by distance between the negative node and its parent, and the diameter of the concavity is controlled by the radius of the negative node. Manipulating the shape was not implemented because it was deemed to require extensive changes to the Skeleton Modeler. This is something that could be explored in further work.

The negative node still has some unsolved problems. What the artist wants is to specify the concavity in relation to the mesh and not a tree structure. It is difficult to visualize exactly how the negative node is going to affect the resulting mesh. Furthermore, the change in the tree structure can have unintended sideeffects. It would be better if the artist could manipulate the mesh directly when creating concavities, and that information would be stored in the tree. It would, therefore, be interesting to explore the sculpting tools described in Section 5.5 in future work.

## Chapter 10

## Leaf Cube Node

### 10.1 Analysis

A ladder was modeled during the exploration of the limits of SQM. This showed that it was not possible to create cuboid shapes. A leaf cube node would enable the ends of the ladder to be cuboid.

The behavior of the leaf cube node is illustrated in 2D on Figure 10.1, where the stippled line represents the mesh. The cube leaf node is represented by a cube that can be scaled in all directions. It can also be rotated around the bone axis, which means the bottom normal should point towards the center of the parent node. On Figure 10.1 the leaf cube node is connected to a sphere node. The tube connecting the two nodes should in this case be rectangular in one end and round in the other end.

The cube node could be implemented using a tree transform. The leaf cube node could also be implemented by modifying the SQM algorithm, so that its vertices are created and correctly placed during the generation of the mesh. This is, however, outside the scope of the thesis and will not be explored further.

We need an edge loop around the bottom and top of the cube in order to create a cuboid. Furthermore, the top needs to be flat. The edge loops could be


Figure 10.1: A 2D illustration of the behavior of the leaf cube node.
introduced in the same way as how it was done for the leaf sphere node in Chapter 8. The leaf cube node in the ESQM tree would be transformed into three sphere nodes in the SQM tree as illustrated on Figure 10.2. The new bottom sphere node should result in the bottom edge loop, and the new top sphere node in the top edge loop. The small new sphere inside the top sphere should result in a flat top.

The tree transform would not alone be able to achieve the cuboid shape. The final vertex placements moves the vertices so it would result in a closed cylinder instead. This is because the final vertex placement approximately places the vertices in the edge loop on a circle as illustrated on Figure 10.3. The final vertex placement, therefore, has to be modified so that the vertices form a cuboid. Figure 10.4 illustrates how the final vertex placement could be modified for the leaf cube node, so that the vertices are placed on a rectangle. A vertex would need to be placed in each corner and the remaining vertices could be placed on the sides in a way so it preserves the existing edge flow. One way to do this is to find each vertex that is closets to a corner and move it to the corner. The remaining vertices are then projected onto the sides of the rectangle.

The remaining vertices are projected orthogonally onto the sides of the rectangle. This is done by splitting the rectangle into four line segments, one for each side, and then trying to project the vertices onto each of the four line segments. The projection of the vertices onto the side of the rectangle is done using the scalar


Figure 10.2: The cube leaf node in the ESQM tree is translated into three spheres in the SQM tree.


Figure 10.3: Final vertex placement approximately places the vertices on a circle.
projection [13, p. 607-608]:

$$
\begin{equation*}
s=\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|} \tag{10.1}
\end{equation*}
$$

Where $s$ is the length of the projection of vector $\mathbf{u}$ onto vector $\mathbf{v}$. The vector $\mathbf{v}$ is a side of the rectangle and vector $\mathbf{u}$ is the vector from the start of the side to the position of the vertex being projected. The correct projection can be found by choosing the one where the length of the projection $s$ is the shortest.

The described method could also work for cuboid shapes if the radius of the sphere node inserted during the tree transformation is so big that the cube nodes fits within it. Additionally, the Skeleton Editor would need some way to manipulate the shape of the cube node, e.g. handles on each side that could be pulled or pushed could be displayed when the cube node is selected.


Figure 10.4: Proposed final vertex placement for the leaf cube node. The vertices are illustrated with black dots. The vertices that are closets to the corners are moved to the corners. The remaining vertices are projected onto the sides.

Finally the subdivision would also have to be modified so that the edges can be subdivided, and the vertices should not be moved. That would allow the subdivision to make blend between a round and rectangular shape.

### 10.2 Implementation

The tree transform was implemented as described earlier. There was again the problem with creating a flat top as seen during the implementation of the hemisphere node. The original implementation does still not support nodes that are on top of each other, so the leaf node was moved out in the same way as earlier. The vertex generated from the leaf node is then instead moved during the final vertex placement.

Cuboid shapes were not implemented as it was deemed that implementing push and pull handles in the skeleton editor was out of the scope of the thesis.

The rotation around the bone was also not implemented. The skeleton editor already uses the plus and minus key, and there is no easy way to add numerical input with the used GUI framework (GLUT). Changing the GUI framework to something more advanced like Qt [12] or GTK+ [11] would make it trivial to add a spinner input box where the rotation could be entered. It is however out of the scope of the thesis to change the GUI framework. It should not be difficult to add this feature, if the GUI framework is changed, as it is simply a matter of rotating the rectangle that is used for the projection around the axis of the bone.

Another method for finding out which edge of the rectangle a vertex belongs to, via the vertex numbering (VertexID), was also tried. For example, if two
corners are vertex 1 and 3 , then one would assume that vertex 2 should sit somewhere on the side between them. That would make it trivial to determine the edge a vertex belongs to. The numbering, however, turned out not to be consecutively, so this method will not work.

### 10.3 Results

The result of the projection method can be seen on Figure 10.5. The vertices that need to be placed on the edges of the cube are correctly identified and moved.


Figure 10.5: The cube leaf node implemented using the projection method.

### 10.4 Sub-Conclusion

It is now possible to create cube shapes for leaf nodes. The implementation does not support cuboid shapes, but this should be trivial to add if a way to manipulate the shape of the cube node is implemented. Also the implementation does not support rotation around the bone. This should also be trivial to implement if some way to enter the rotation value is implemented. This will, however, most likely require the GUI framework to be changed.

The chosen method is cumbersome, as it calculates many projections that are discarded. It is also potentially unstable as it assumes that the final vertex placement places the vertices approximately on a circle. The implementation could be made more stable and simplified if the vertex placement is calculated
during the mesh generation, instead of as a post-process. This was, however, out of the scope.

## Chapter 11

## Connection Cube Node

### 11.1 Analysis

With SQM the connection nodes are all connected by cylinders. This could be extended, so they could be connected by cuboids instead. This would be useful for mechanical structures, e.g. air ducts and molding. This could be achieved by introducing a connection cube node.

Figure 11.1 gives two examples of how the connection cube node could behave if it was in 2D. One way could be to share angle between the incoming and outgoing rectangular pieces equally. That means the angle at the interface should be half the angle between the incoming and outgoing bone. The connection cubes should automatically be angled so that the rectangular cut through the middle, indicated with a thick stippled line, is place in the interface, and so that it is orthogonal to the half-vector between the incoming and outgoing bone. The half-vector will not work if twisting occurs, but it will suffice for some shapes. One could also imagine the artist choosing how the angle should be shared.

The projection method, used for the leaf cube nodes in Chapter 10, can be used to move the connection cube node vertices into the rectangular cut in the middle. This will, however, not work well for obtuse angles, as the cuboid will be pinched in the interface. A different method will have to be used in order to
implement obtuse angles.

(a) 90 degrees.

(b) 45 degrees.

Figure 11.1: An example of how the connection cube node should work in relation to its parent and child.

### 11.2 Implementation

The code for implementing the leaf cube node was re-used and modified. The tree transformation is simple, as the connection cube node is converted to a sphere node in the SQM tree. The information about the node type has to be retained for the final vertex placement though.

### 11.3 Results

The result of implementing the connection cube node can be seen on Figure 11.2. The result looks decent if the rotation of the leaf node is such that the quads on the sides are flat, and if the connection node is scaled to a fitting size. It is however, difficult to manually adjust the size connection node, so it would be good to have a feature to automatically fit the size. It is also difficult in the Skeleton Modeler to control the angles between the nodes, so some feature to control this would also be helpful. The position is also difficult to control precisely with the Skeleton Modeler, so a feature to enter it manually would also be useful.

The exact source of the slight narrowing at the base on Figure 11.2b has not been identified, but it is likely caused by the difficult of manipulating the nodes in the Skeleton Modeler. The Skeleton Modeler could be ruled out as a source by creating the tree using a procedural approach instead of using the Skeleton Modeler. This was, however, not investigated further.


Figure 11.2: The results of the connection cube node.

The connection node implementation cannot handle obtuse angles as seen on Figure 11.3.


Figure 11.3: Obtuse angles do not work.

### 11.4 Sub-Conclusion

The connection cube node was implemented and it can handle "simple" angles between nodes. It does, however, not work for obtuse angles and when there is twisting. The exact behavior of how this should be handled was not analyzed, and this could be further explored.

## сhapter 12

## Branch Cube Node

### 12.1 Analysis

A ladder was modeled during the exploration of the limits of SQM in Section 4.1. This showed that it was not possible to create a branching structure that is cuboid.

My thesis supervisor suggested that it might be easy to create cuboid shapes in branch nodes by placing valency four vertices on a line. This method will be called the valency four method. Figure 12.1 shows a sphere branch node surrounded by six leaf cube nodes. The branch node's valency four vertices are almost on straight lines between the higher valency vertices. The branch node could possibly be made look cuboid if the non valency four vertices are moved to the corners of a cube, and the valency four vertices are moved to be on a line between the non valency four vertices. This could be done during the final vertex placement.

The non valency four vertices arise where the cylinders or tubes of more than two children meet. This means that there can be more than four non valency four vertices as seen on Figure 12.2. The valency four method therefore has to determine which non valency four vertices that need to be moved to the corners of a cube, e.g. the closets vertices. The remaining non valency four vertices can


Figure 12.1: The valency four vertices belonging to the branch node could be moved to create a cuboid shape.
then be projected onto the cube, e.g. using a ray-box intersection during the final vertex placement.


Figure 12.2: There can be more than four non valency four vertices (pointed to by arrow).

The branch cube node might not always need to be aligned to the axes. It would be useful to be able to control the rotation of the branch cube node.

### 12.2 Implementation

The final vertex placement was modified to first move the non valency four vertices belonging to a the branch cube nodes. The vertices closets to the corners are found and moved to the corners. The ray-box intersection in GEL was modified so it is possible to get the intersection point. The remaining
non valency four vertices are then moved to the intersection point. Finally the valency four vertices are then moved to the average position of the non valency four vertices belonging to the same node.

The code for visualizing the leaf cube node was re-used, which used the glutSolidCube function in GLUT [3].

### 12.3 Results

The results are good in the ideal case when there are six branches each going out of each side of the cube branch node as seen on Figure 12.3. The method, however, breaks down when this is not the case as can be seen on Figure 12.4. This is because there are no longer non valency four vertices placed in all four corners of the branch cube node. With four nodes there are only two non valency four vertices left, and the vertices are all placed on a line between them, which results in the branch node loosing its volume as seen on Figure 12.4c.


Figure 12.3: The valency four method works well for ideal cases.
The problem with the collapsing branch cube nodes could be worked around by creating extra nodes in the ESQM tree, so the total number of children nodes becomes six as seen on the mock-up on Figure 12.5. For example, if there are three nodes going out of the branch cube node, then an extra three nodes could be added. The extra nodes would have to be placed so they are in the middle of the sides that do not have a bone coming out of it. This method was unfortunately not implemented due to time constraints.


Figure 12.4: Less than six children nodes makes the branch cube node collapse when using the valency four method.


Figure 12.5: Mock-up of how extra nodes could be inserted to create a cuboid shape in a branch nodes with less than six children.

The problem with more than six children can be seen on Figure 12.6, where the projection is not enabled. The vertices are not on the surface of the branch cube node, which looks odd.

The result of moving the vertices down onto the the branch cube node by using a projection method can be seen on Figure 12.7. The vertices are now on the branch cube node but not necessarily on the edges, which would look better. This create a new problem in that the rectangle connecting the children to the parent are squeezed at the bottom, which also looks odd. Other ways of handling multiple children coming out one side should be investigated.


Figure 12.6: The branch cube node does not produce good results when multiple children are going out from one side.


Figure 12.7: The result of projecting the remaining vertices onto the branch cube node.

### 12.4 Sub-Conclusion

The branch cube node works well in the ideal case where there are six children, and they each come out of one side of the branch cube node. If there are fewer than six children, then the valency four method breaks down. It could be worked around by using a tree transform to insert the missing children. This was, however, not investigated due to time constraints.

More than six children did also not give a good result. The underlying problem with handling multiple children coming out of one side is that there is no "one correct way" to do this. It depends on what is being modeled, which could be anything. The block modeling method by Leblanc et al. [20] solves this by not allowing multiple children to be connected to one side of a block. It might be possible to change the SQM method to also have this restriction, but that would
change the core of the SQM method, which is out of the scope of this thesis.
The branch cube could also, like the other cube node types, be improved if the mesh generation is modified. That way it could make sure that there are always enough vertices to form a box. That was, however, out of the scope of the thesis, but could be done as future work.

## CHAPTER 13

## Goals and Limitations Revisited

In this chapter of the goals and limitations from Chapter 4 will be revisited to see if they have been addressed. This is done by modeling the goals again using the new node types. The lollipop model will not be revisited, as its primary function was to explore how to create spherical shapes that would serve as a starting point for the head model. Finally a model will be created that uses all the new node types. This is to make sure that all the new node types can work together.

### 13.1 The Ladder Revisited

The ladder in Section 4.1 was modeled again. This time with the branch and leaf cube nodes for the rectangular parts of the ladder. The round steps were modeled using the hemisphere node. The skeleton and the result can be seen on Figure 13.1. The input skeleton looks more intuitive than the old input skeleton. There are fewer nodes, and the rectangular parts are expressed with nodes that have a similar shape. The end result is, however, not very good.

The result of the old and new ladder are compared to the goal on Figure 13.2.


Figure 13.1: Modeling the ladder again using cube and hemisphere nodes.

The top and bottom pieces of the new ladder are flat, which is an improvement over the old result. The new result as a whole, however, looks worse than that of the old. The branch cube nodes are collapsed because there are not enough children to pull out the vertices. A method to solve this could be to automatically add extra children, and this was discussed Chapter 12.


Figure 13.2: Comparison of the old and new ladder model with the goal.

### 13.2 The Gear Revisited

The ladder in Section 4.2 was modeled again using leaf cube nodes. The input skeleton and the result can be seen on Figure 13.3. The branch cube node was not used for the ring part to avoid it collapsing as seen earlier.


Figure 13.3: Modeling the gear again using leaf cube and sphere nodes.

The result of the old and new gear is compared to the goal on Figure 13.4. The teeth of the new gear look closer to the goal. This is because the leaf cube nodes were used. Some of the teeth are however rotated, which is not what was wanted. This is because rotation around the bone was not implemented. Finally the width of inner ring was not addressed. It could possibly be solved if the branch cube node was improved, so it does not lose its volume, and so that they can be wider.


Figure 13.4: Comparison of the old and new gear model with the goal.

### 13.3 The Head Revisited

The simple head model in Section 4.4 was modeled again using the new node types. The skeleton and the result can be seen on Figure 13.5. The round shape was again achieved using nodes in six directions around the root node.

This time cone nodes were used instead of sphere nodes. The concavities were created with the negative nodes.


Figure 13.5: Modeling the head again with the new node types.


Figure 13.6: Comparison of the old and new head model with the goal.

A comparison of the old and the new result with the goal can be seen on Figure 13.6. The eye concavities are more well defined on new than the old. They are round and the impact on the surrounding mesh is smaller. The result, however, leaves a lot to be desired. It is not possible to control the shape of the eye concavities. The concavity for the mouth looks very different than the goal. The mesh sticks out of the parent where the part of the mesh generated from the negative nodes meet, which was not intended. It was instead intended to be flat.

The fundamental issue is that SQM is skeleton based and generates the mesh procedurally from the skeleton. Adding a node adds more vertices to the mesh and changes the shape of the surrounding branches. This means that the concavities are not localized. They cannot be combined, as it was seen with the mouth, the mesh will first go out into the surface of the parent node and then come back in. The mesh will therefore stick out where the mesh generated from
the negative nodes meet.
Overall the result is not promising. Using a negative node only works for some specific cases, and it is not good enough to be used for general modeling. The underlying problem is that SQM was designed for creating branching structures like trees. That also means that the artist is not able to specify the concavity in relation to a surface. One could display the generated mesh sooner, which would let the artist see the changes immediately, but it would not change the underlying problem that SQM only uses the skeleton as input. There is, therefor, no straightforward way to directly specify concavities in relation to the generated surface.

Two other ways to create concavities were discussed in Section 5.5. They could either be created using a deformable node, or by using mesh sculpting tools, where the operations are stored in the nearest node. The latter approach seems to be the most interesting because it would allow the operations be specified in relation to the mesh and be re-applied after the mesh is regenerated. The same sculpting operation would be applied even if the structure of the underlying mesh changes.

### 13.4 Interaction Between Nodes

The new nodes would not be very useful if they suddenly changed the whole shape of the mesh. One would expect that the effects of each new node type to be localized. One way to check this is to create a model that uses all the different node types together. This have been done on Figure 13.7, which shows the interaction between all the new node type, except for the negative node. The negative node was left out because it requires the mesh to have a significant volume. The remaining new nodes interact well together in the example. There might be problematic corner cases, but that is unlikely as the new nodes are designed only to move their own vertices.

The interaction between the nodes does not always result in a good looking mesh. The blend between the leaf cube node and the connection sphere on the right side of the tree does, for example, not look good because one of the quads narrows towards the top. It might be possible to improve the look by using a different method to place the leaf cube node vertices. The result will, however, always look somewhat odd, because a circle and a rectangle cannot be connected by flat quads. The quads will always be bent in some way. Triangles would be a better fit to connect the two shapes. One of the strengths of SQM was that it connected the nodes with flat quads oriented along the curvature lines. That


Figure 13.7: An example of how the new nodes interact with each other.
means mixing cube nodes and sphere nodes makes it lose that advantage.

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## Conclusion

Four modeling goals were proposed and modeled with SQM and the skeleton modeler. These goals revealed some of limitations of SQM. This led to a proposal to add seven new node types. The results obtained with the new node types on their own were good. Using them to model the goals did, however, not yield good results. A method for improving the result of the ladder model was given, and it should be trivial to implement with more time.

Tree transformations were shown to be a way to implement the new node types. It was, however, discovered that it was not sufficient for implementing the shapes by itself. This was because the interaction with the rest of the SQM algorithm caused the mesh to be changed too much. Post-processing of the mesh, therefore, also had to be employed. Some of the post-processing could possibly be avoided by changing the SQM algorithm. The post-processing could be discarded completely, and the implementation simplified, if the new node types were implemented as a part of the SQM mesh generation instead. Changing SQM to support the new node types directly is, therefore, an obvious candidate for further work, but it is expected to require a large amount work, that would be beyond the scope of a master's thesis.

Concavities were shown to possible to create with SQM, but that it was difficult to manipulate the size and position of the concavity because the nodes had to be placed inside another node. The implemented negative node made it possible
to place the node on the outside, thereby making it easier to manipulate the size and position of the concavity. The result showed that it worked well for simple round concavities, but is not good enough for use in complex situations, such as modeling a head. It is not possible to control the shape of the concavities, nor combine multiple concavities next to each other. The fundamental issue is that SQM is skeleton based and generates the mesh procedurally from the skeleton. Adding a node adds more vertices to the mesh and changes the shape of the surrounding branches. Other methods for adding a concavities could be explored for further work, such as sculpting operations that are associated with the nodes, and applied to the mesh as a post-process.

Skeleton based systems offer a new and interesting approach to modeling. The fundamental limits are not fully understood yet, and some of the perceived limits can be overcome, as was shown in the thesis. The utility of using a skeleton based system for general modeling was, however, not addressed in the thesis. This is something that I personally believe is the largest obstacle for the use of SQM and skeleton based systems in general. They seem like an obvious choice for procedural generation of trees, and for generating base meshes for game characters, but there is an impedance mismatch for modeling anything else. Other tools are more powerful by offering more direct control. They also use interaction concepts that are intuitive for adding details and creating nonorganic shapes. I, therefore, believe that the best direction to take SQM for further work is not to improve it as a general modeling tool, but to use its strengths and improve its capabilities for generating trees and game character base meshes.

## Appendix $A$

## Source Code

The source code can be found on the accompanying CD-ROM or Zip-file uploaded to CampusNet. The src folder contains the source code folders for SQM, GEL, RectangleLineIntersection. The RectangleLineIntersection folder contains the code used for projecting the vertices onto rectangles used by the leaf and connection cube node as well as test code. All the source code folders also contain project files for Visual Studio 2010.

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## User Guide

The accompanying CD-ROM or Zip-file uploaded to CampusNet. The bin folder contains the sqm. exe Windows executable file which can be used to start the extended Skeleton Modeler. The executable has been tested and verified to work on Windows 7 with an NVIDIA GeForce 250 GTS graphics card.

Most of the features already existed in the Skeleton Modeler and only few of those will be described here. The Skeleton Modeler starts with showing the red root node on a white background. A node can be selected by holding down the shift key and left clicking on it with the mouse. More nodes can be added by shift left-clicking on the white background. Nodes can be moved by holding down shift, and left-click dragging the node to a new location. Finally pressing space generates the mesh from the skeleton.

The main new feature that was added is the ability to change the node type. The node type can be changed by first selecting a node and then pressing the tab key. The types of nodes that can be used depends on whether the node is a branch, connection or leaf node. The tab key cycles between the node types, so if for example a leaf node is selected, then one press will change the type to a hemisphere node. Another press will change it to a cone node and so on.

Another feature that was added, but not described in the thesis, is the ability to change the projection mode to orthographic. This is useful when modeling
non-organic shapes. The orthographic projection mode can be toggled on and off with the 'o'-key (letter not number).

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