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Analysis of the Greenland ice sheet loss from remote sensing data*

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Introduction

During recent years, satellite measurements have indicated a trend of increasing melt-off of the Greenland ice sheet. We will examine this apparent trend and the statistical significance of our findings.
We are using gravity data from the twin GRACE satellites. By continuously, and very accurately, measuring the distance between the two satellites, one can infer the gravity for each location on Earth.
What we are considering are the equivalent water heights (EWH), i.e., the dynamic mass changes from the reference geoid, expressed in the equivalent amount of water. Thus, assuming the changing masses to be water or ice, we can estimate trends and seasonal variability in the ice sheet (and any other global mass changes).
The data used are the CNES/GRGS 10-day solutions with XMASCEN correction [2], available from the GRGS website [1].
We will create a regressive model to account for the various trends and seasonal variations in the data. To identify the relevant oscillations, a spectral analysis using the discrete Fourier transform (DFT) is used.
Any remaining phenomena in the residuals will be investigated using principal component analysis (PCA) and a modified approach, sparse PCA.

Spectral analysis

The usual formulation of the DFT assumes that the input signal is periodical and has equidistant samples. Neither criterion is true for the GRACE data; the implied periodicity is remedied using a tapering function (in this case the Hann function). Though the data are mostly contiguous 10-day averages, there are a number of missing solutions early in the dataset. To perform the spectral analysis anyway, we are using the DFT matrix formulation with columns corresponding to the missing data removed.

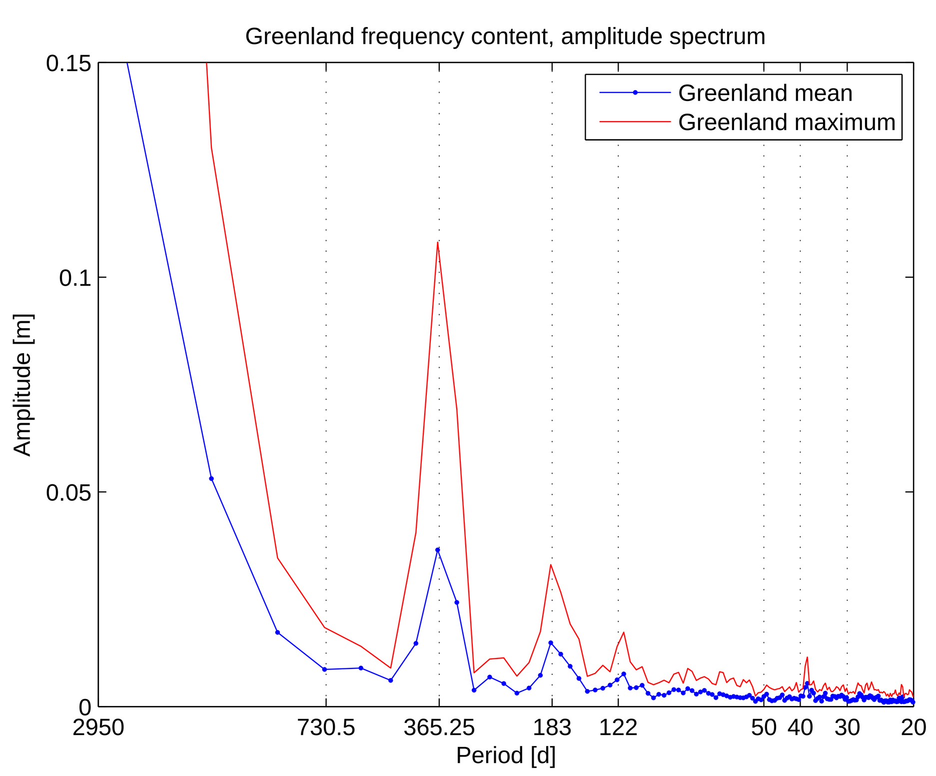

Figure 1: EWH amplitude spectrum for Greenland. Note the strong low-frequency parts, which appear to be a result of long-term trends in the time series. The second and third harmonics of the 1-year oscillation are likely a result of a sawtooth-shaped waveform, rather than subannual phenomena as such.

Regression model

Based on the results from the spectral analysis, the regressive model has been decided to consist of second-degree polynomial terms and whole-, half- and one-third-year oscillations, as well as a step function H to catch the effects of the 2004 Indian Ocean earthquake (a significant feature in the dataset). Thus, we get the following predictor for the EWH values (t in years):

$$\mathbf{X} = \begin{bmatrix} 1 & t_1 & \frac{1}{2}t_1^2 & H(t_1) & \sin(2\pi t_1) & \cos(2\pi t_1) & \sin(4\pi t_1) & \cos(4\pi t_1) & \sin(6\pi t_1) & \cos(6\pi t_1) \\ 1 & t_2 & \frac{1}{2}t_2^2 & H(t_2) & \sin(2\pi t_2) & \cos(2\pi t_2) & \sin(4\pi t_2) & \cos(4\pi t_2) & \sin(6\pi t_2) & \cos(6\pi t_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & t_N & \frac{1}{2}t_N^2 & H(t_N) & \sin(2\pi t_N) & \cos(2\pi t_N) & \sin(4\pi t_N) & \cos(4\pi t_N) & \sin(6\pi t_N) & \cos(6\pi t_N) \end{bmatrix}$$

Using the solution coefficients of the model, we then obtain the apparent melt-off rates and corresponding acceleration, as well as amplitude and phase of any annual variation. Also, we can determine a p-value for a test statistic for each parameter, expressing significant deviance (or not) from zero.

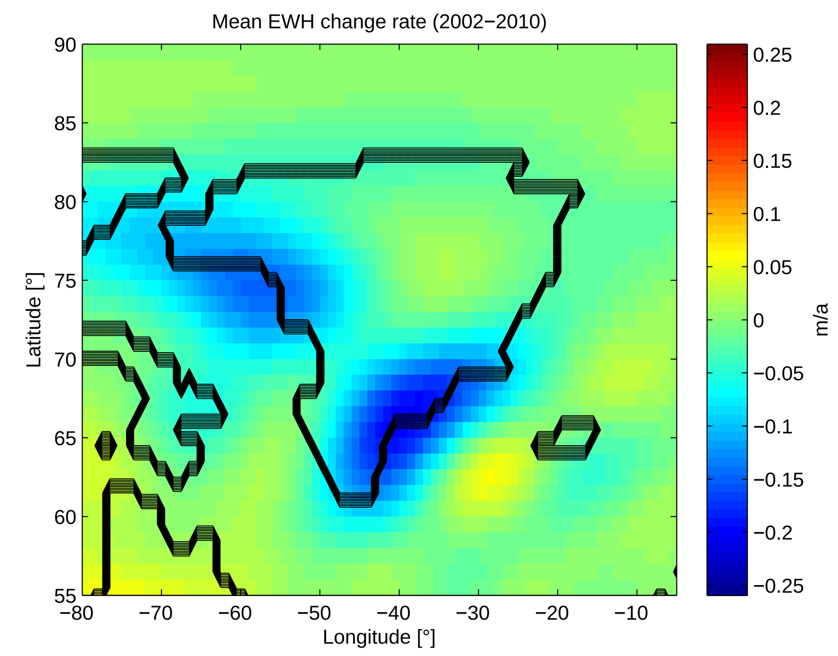

Figure 2: Mean EWH change rate over the time span observed.

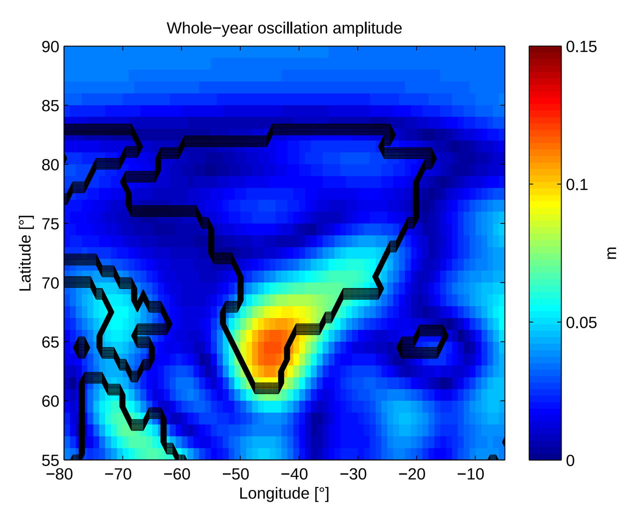

Figure 3: Amplitude of the one-year oscillation. The amplitude generally decreases with higher latitude.

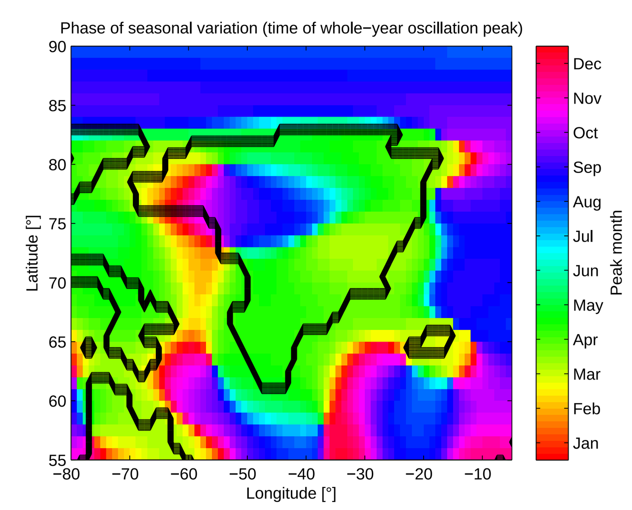

Figure 4: Phase (expressed as peak month) of the one-year oscillation. Note that the actual peak generally occurs later, as the annual melt-off is more rapid than the refreezing.

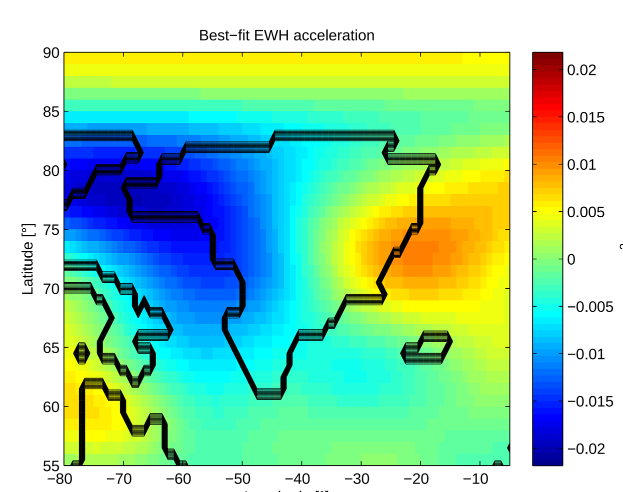

Figure 5: Acceleration term of the regressive model.

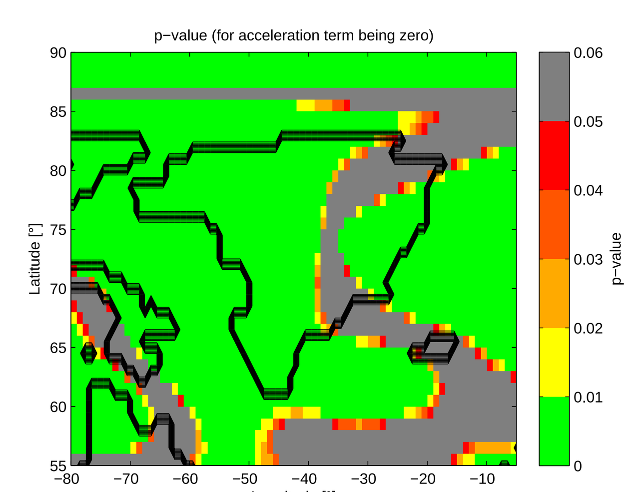

Figure 6: Corresponding p-values (null hypothesis being zero acceleration).

As we see, there is a clear downward trend (melting ice) in the EWH in Southeast and Northwest Greenland, with significant negative acceleration (i.e, an increasing melt-off rate) in Northwest Greenland, and even positive acceleration (slowing melt-off) in East Greenland. The results are perhaps surprisingly significant; as an alternative, a long-term oscillation might be able to yield the same quality of fit.

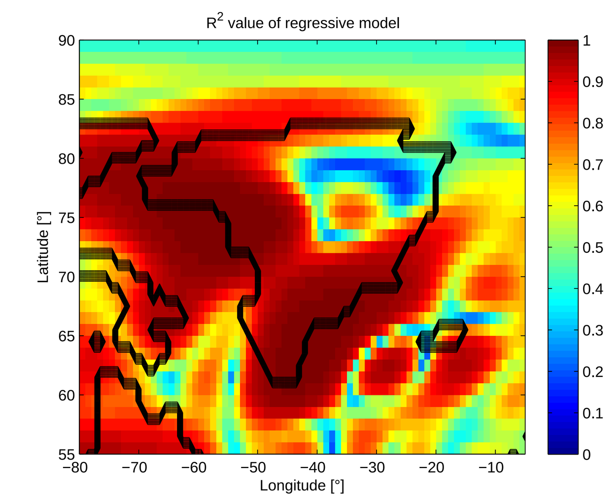

Figure 7: R^2 values for the regressions. The fit seems to be good in areas of clear trends and seasonal variation, which we are interested in modelling.

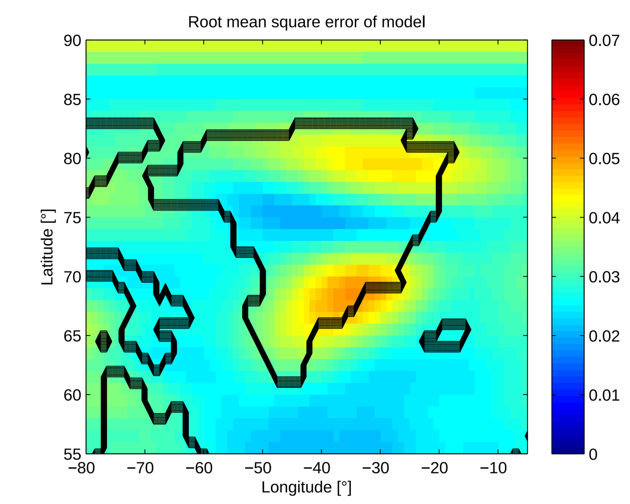

Figure 8: Residuals of the model, specifically root mean squared error. Note the large errors in Northeast Greenland, which also has low R^2.

PCA/sparse PCA and further possibilities

Principal component analysis (PCA), first described by Hotelling in 1933 [3], is a very popular way of orthogonalizing and compressing multi-temporal data. Because the principal components (PCs) are weighted, linear combinations of all the original variables they are sometimes difficult to interpret. Sparse PCA carries out the transformation in a way such that some or even many weights (also known as loadings) are forced to zero. This potentially facilitates easier interpretation of the resulting sparse PCs.
As an initial attempt to introduce sparsity into traditional empirical orthogonal function (EOF) analysis or principal component analysis, we have made a few tests with sparse PCA [5, 4]. Applied to the GRACE data over Greenland only, these show tendencies of a drastic transition in inland ice melt-off taking place in Greenland from mid-2004 to late 2006.
Further analysis of the GRACE data might include wavelet analysis to identify any time-limited phenomena. Various models for the background noise of the data and significance levels in the power spectra are being investigated. Also, the post-glacial rebound needs to be taken into account when examining the trends, as the land uplift affects the EWH data.
For validation of the results, other satellite data should be involved later on. Useful examples may be NOAA sea surface temperature data as well as altimetry from the now defunct ICESat satellite.

Conclusions

The spectral analysis has clearly identified annual variation in the data, as well as a number of harmonics to be included in the model. A regression model has been established to test for the presence of significant acceleration in the trends, with strongly significant results. Sparse PCA, while useful for identifying outliers and similar short-term phenomena, has not yielded particularly useful results when applied to the residuals of the regression model, as the residuals contain more or less continuous trends and periodical oscillations.

References

[1] Le Groupe de Recherche de Géodésie Spatiale. GRACE solutions release 02. `http://grgs.obs-mip.fr/index.php/fre/Donnees-scientifiques/Champ-de-gravite/grace/release02`, 2010.

[2] Martin Horwath, Jean-Michel Lemoine, Richard Biancale, and Stéphane Bourgogne. Improved GRACE science results after adjustment of geometric biases in the Level-1B K-band ranging data. *Journal of Geodesy*, 85:23–38, 2011. 10.1007/s00190-010-0414-2.

[3] Harold Hotelling. Analysis of a complex of statistical variables into principal components. *Journal of Educational Psychology*, 24:417–441, 498–520, 1933.

[4] Karl Sjöstrand. *Regularized statistical analysis of anatomy*. PhD thesis, Technical University of Denmark, 2007.

[5] Hui Zou, Trevor Hastie, and Robert Tibshirani. Sparse principal component analysis. *Journal of Computational and Graphical Statistics*, 15(2):265–286, 2006.

*) Abstract title: *Spatio-temporal analysis of multi-sensor observations of the Greenland ice sheet mass loss.* The project is a work in progress.