THE DYNAMICS OF A RAILWAY WAGON WHEEL SET WITH DRY FRICTION DAMPING

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ABSTRACT

Dry friction dampers have been used in car constructions for several hundred years, and are still extensively used by the railways today. The main reason is that they are much cheaper than hydraulic dampers and more rugged. Their disadvantages are that their function is variable and depends on weather conditions and their state of contamination (dirt, oil) and last but not least the state of wear. The designers have used old empirical rules for the application of dry friction dampers in railway vehicles. This contribution will help to unveil the dynamics of a suspension system containing a coupling between a dry friction damper and the basic nonlinearity in railway dynamics - the rail-wheel contact forces, which act as a nonlinear softening restoring force.

Keywords: non-linear dynamics, dry-friction damping, bifurcation, stability

1. INTRODUCTION

In this work we consider a simple model of a wheel-set that supports one end of a railway freight wagon by springs with linear characteristics and dry friction dampers. A simpler model, in which the gravitational stiffness term was neglected, has been investigated earlier by True and Trzepacz [1] and True and Asmund [2].

The stick-slip of the dry friction action introduces a non-smoothness in the dynamical system, and it is expected that this non-smoothness will introduce a chaotic behaviour in the system. In the earlier works by True and Trzepacz [1] and True and Asmund [2] it was indeed found to be the case, but it was also found that the wagon would derail in large speed intervals. Both works used very simple models of the wheel-rail interaction and this simplicity might be a contributing factor to the derailments. In this article a more realistic and acknowledged wheel-rail interaction model is applied, and it is found that the wagon no longer will derail, but the dynamics is still chaotic in large speed intervals.

2. THE DYNAMICAL MODEL

The wagon runs on an ideal, straight and level track with constant speed. We want to investigate the lateral dynamics of the wheel-set in dependence on the speed, which is the bifurcation parameter in the problem. The equations of motion are formulated in a coordinate system that moves along centre line of a straight and horizontal track with the constant speed of the wagon. The wheels have the DSB97-1 profile. It is an S 1002 profile, which is modified to run on gauges that might be slightly narrower than the standard 1435 mm. The track is a standard 1435 mm gauge with UIC60 rails inclined by 1/40, towards the centre of the track. We have included stick-slip and hysteresis in our model of the dry friction and assume that Coulomb's friction law holds during the slip phase.

The model system has three degrees of freedom: Translation of the car body and

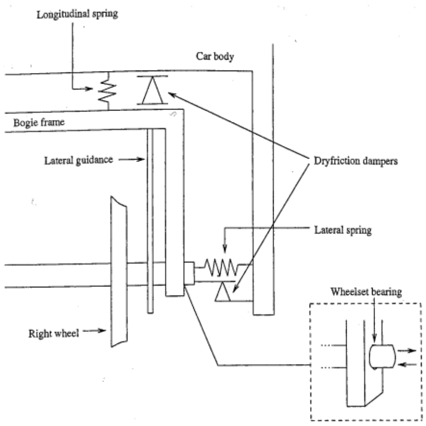

Figure 1. The wheel set model. The wheel set is restrained laterally in the car body by a linear spring and a dry friction damper in parallel, and the yaw motion of the bogie frame is also restrained by a linear spring in parallel with a dry friction damper.

translation of the single-axle bogie in the lateral (x-) direction and yaw of the bogie around the frictionless pivot in the bottom of the car body. A picture of the model is seen on figure 1.

The dry friction action is modeled by a smoothed step function (see [2]), and the restoring force of the linear spring in combination with the dry friction damper is plotted on figure 2. It is shown how the contact force between two parts reacts to an applied external force. The contact force is constant until the 'tear-loose force' between the surfaces is reached. When the applied force grows further, the linear spring comes into action after a very short smoothed step.

The wheel-rail contact parameters are calculated by Fujie Xia's program WRKIN,

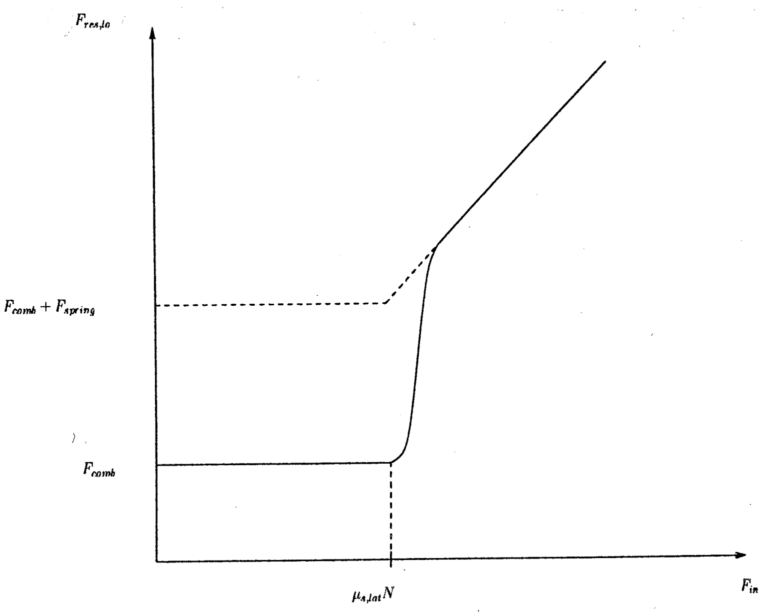

Figure 2. The action of the combined friction element and linear spring.

and the wheel-rail contact forces are calculated using the Shen-Hedrick-Elkins method.

3. EQUATIONS OF MOTION

The nonlinear dynamical system is formulated as a system of six first-order differential equations in combination with the equations for the wheel-rail contact parameters. It is solved numerically using the SDIRK method with appropriate initial conditions for increasing values of the speed. SDIRK is a Runge-Kutta type one-step solver with variable step-length and error control.

4. SIMULATION RESULTS

The dynamics of the wheel set is investigated in the speed range from 5 to 40 m/s. We have calculated time series of the components of the motion and the friction forces and two and three dimensional state space plots and in some cases the Lyapunov exponents. A few representative results will be shown next.

In the beginning the motion is periodic. Figure 3 illustrates the motion for V = 5 m/s. After V = 5 m/s a sequence of period doubling sequences seems to start that results in a mildly chaotic motion around V = 8.75 m/s. The chaos is verified by an investigation of the sensitivity of the motion to initial conditions.

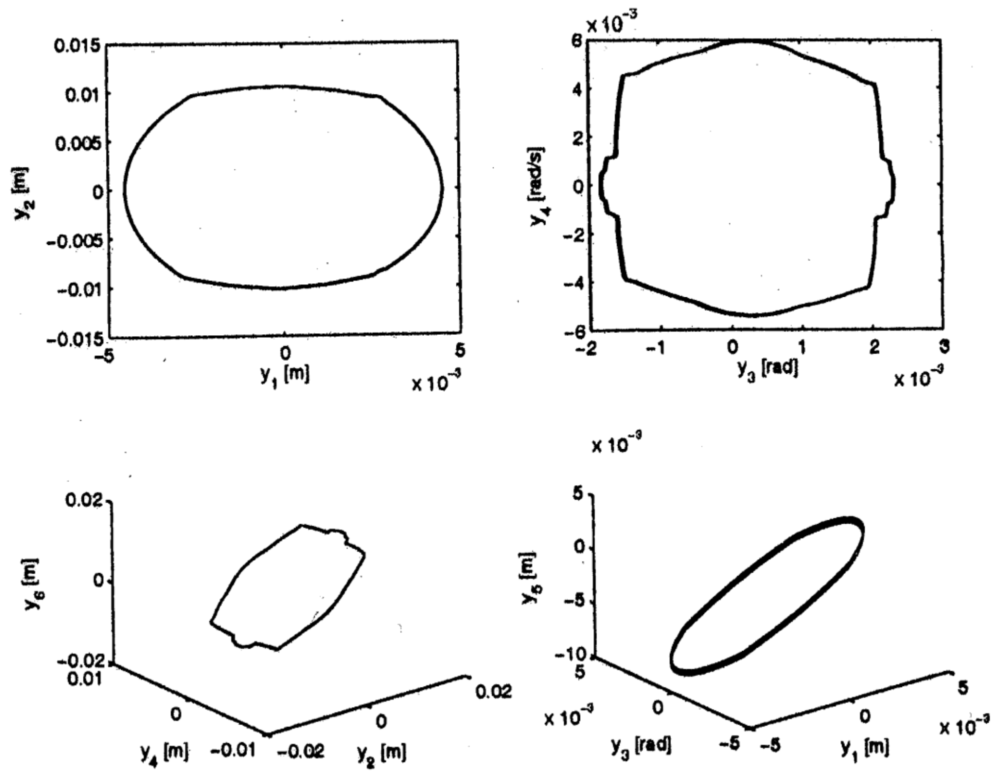

Figure 3. A few state space portraits at V = 5 m/s. The motion is periodic.

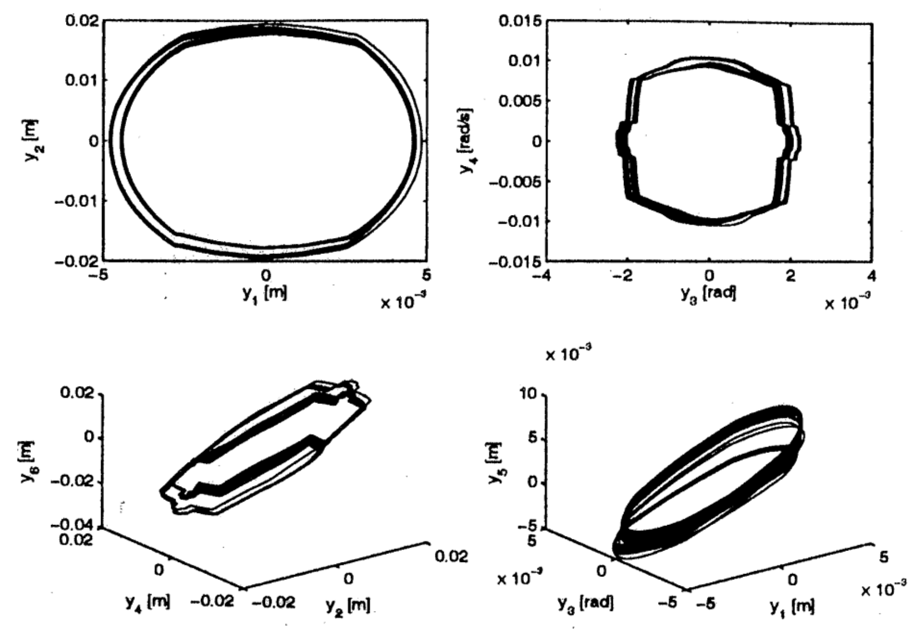

Figure 4. A few state space portraits at V = 8.75 m/s. The motion is 'mildly chaotic'.

Figure 4 illustrates the chaotic motion for V = 8.75 m/s.

The motion regains periodicity for V > 9.5 m/s, but at still higher speeds several transitions between periodic and chaotic motion takes place. The results of the investigation is summarized in table 1 and illustrated on the bifurcation diagram figure 5. We have found alternating small and large amplitude periodic motion, chaotic motion, mildly chaotic motion of two types, and what we call 'complete-stick motion'.

The chaotic attractors are characterized by erratic-looking time series. They are seen in figure 5 as dense clusters of extrema for approximately -4 mm < y < 4 mm. An example of chaotic motion is shown on the figures 6 and 7. We also found that the transients connected with the large amplitude periodic solutions are chaotic. It may indicate the existence of an attractor with smaller amplitude, which was not found. The 'mildly chaotic attractor' is characterized by erratic looking phase portraits and occur in two varieties. The perturbed initial condition clearly experiences an interval of significant growth before it levels off at a value, which is not of the same order of magnitude as the size of the attractor.

Time series of the 'complete-stick motion' are shown on figure 8. The 'complete-stick motion' occurs only at V = 8.75 m/s and around V = 37 m/s. With our initial conditions it is seen that the motion starts with erratic oscillations as the transients of the large-amplitude attractors. As soon as the amplitude reaches -9 mm the motion turns into complete stick and a stationary state starts.

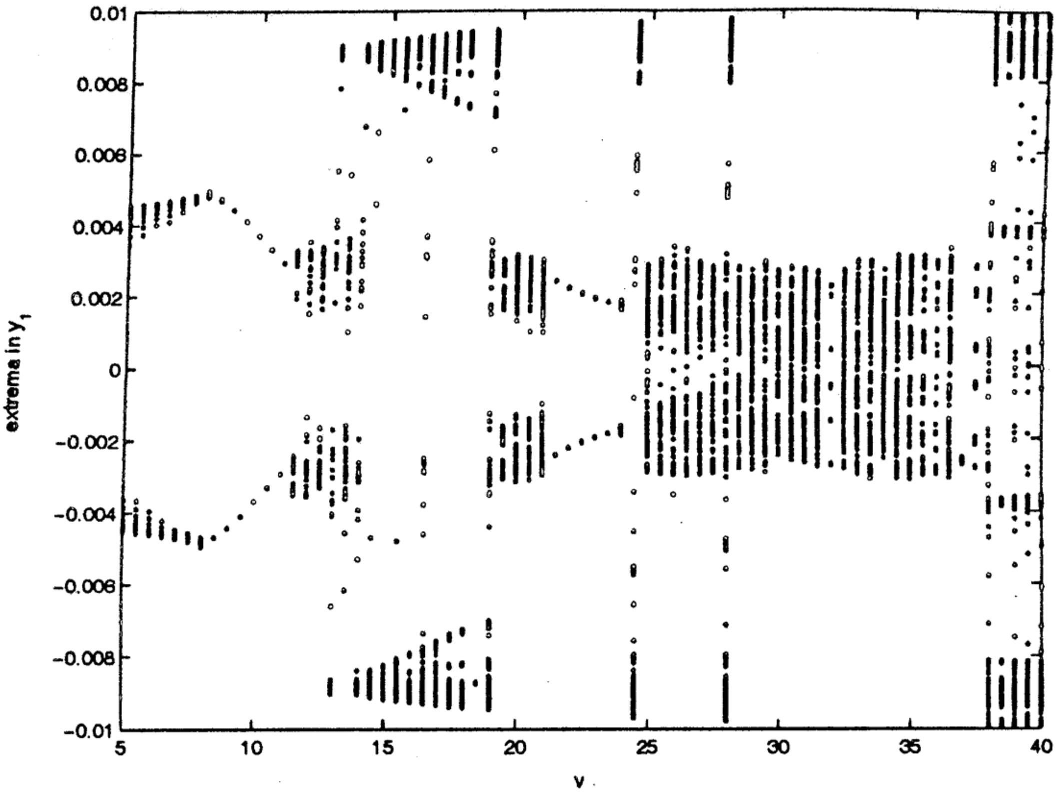

Figure 5. The complete bifurcation diagram showing max lateral displacement of the axle versus the speed.

Table 1

Speed intervals [-;-] m/s	**Attractor type**
[5; 8.71], [9.5; 11], [21.5; 24]	Periodic, small amplitude
[13; 19], [24.5; 28]	Periodic, large amplitude
[11.135; 12.765], [19.5; 21], [25; 36.5]	Chaotic
[8.72; 9.32]	Mildly chaotic, Type 1
Only for V = 32 and 37.5	Mildly chaotic, Type 2
Only for V = 18.5 and 37	Complete stick

The figures 6 and 7 illustrate a chaotic motion. Figure 6 shows time series and figure 7 shows projections of the motion in state space. y_1 is the lateral displacement [m] and y_2 the lateral velocity [m/s], y_3 is the yaw angle [rad] and y_4 the angular velocity [rad/s] – all of the wheel set. y_5 is the lateral displacement [m] and y_6 the lateral velocity [m/s] of the car body.

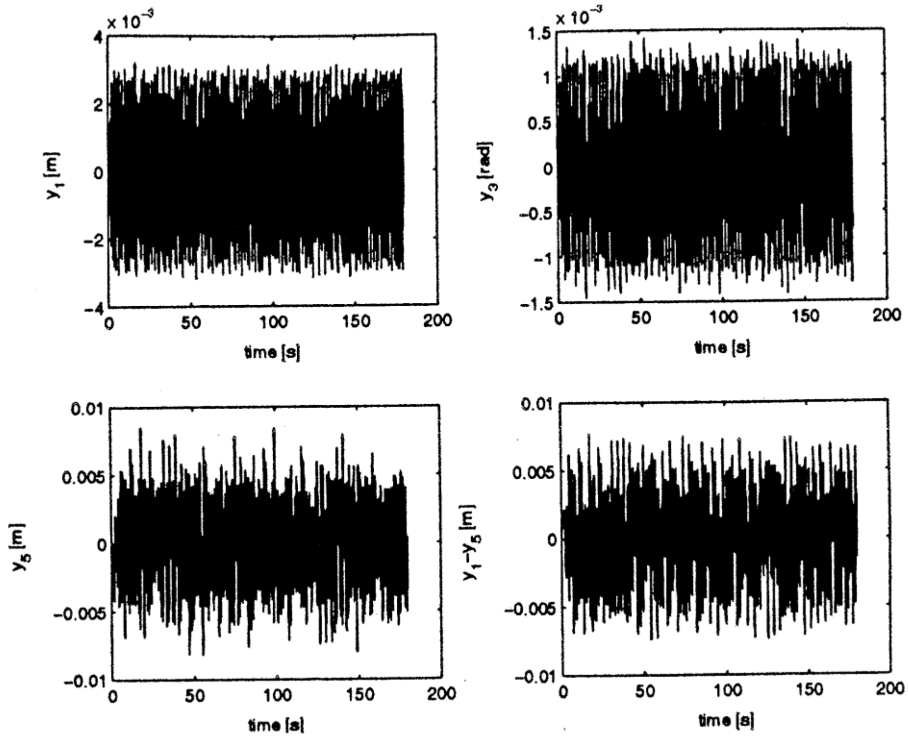

Figure 6. Time series at V = 20 m/s. Upper left: lateral displacement of the wheel set; lower left: lateral displacement of the car body; upper right: yaw of the wheel set; lower right: difference between the lateral displacements of the wheel set and the car body

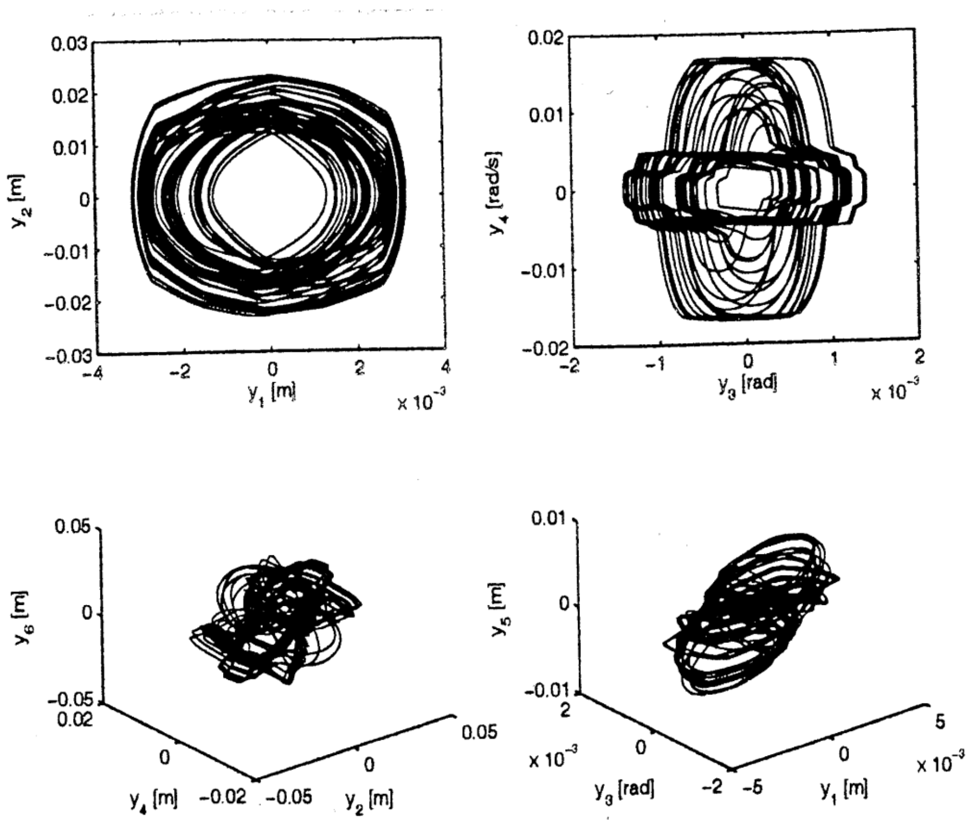

Figure 7. State space plots at V = 20 m/s. The motion is chaotic. For explanation see the text.

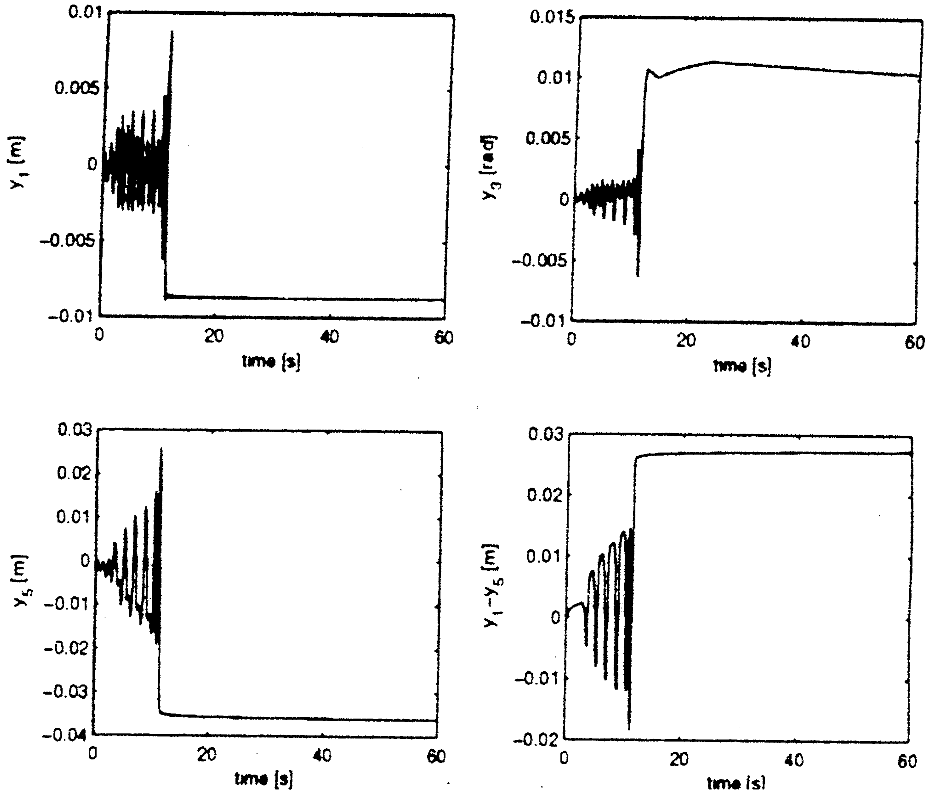

Figure 8. Time series that illustrate the complete-stick state

The existence of chaos in the problem is verified by calculations of the largest Lyapunov exponent. When chaos exists, the largest Lyapunov exponent is always larger than zero. The calculations are not presented in this article. They are the main topic of another article to be published later, because Gilles Brieuc developed a new method for the calculation of Lyapunov exponents in non-smooth dynamical problems, which was applied to the problem that is treated in this article.

5. CONCLUSIONS

The works by True and Trzepacz [1] and True and Asmund [2] demonstrated that chaos develops in simple models of a single rolling wheel set on a straight track when the damping is caused by dry friction. The typical damping was chosen in such a way that the attenuation is of the same order of magnitude as that of an equivalent hydraulic damper. The results were not unexpected since non-smoothnesses in dynamical systems very often produce chaos. True and Asmund [2] also varied the damping of as well the lateral as a yaw damper and found that the yaw damper for almost all values of the damping had a decremental effect on the dynamics of the wheel set. In large speed intervals the wheel set would derail. The derailments were a surprise and they were conjectured to be related to the too simplified model of the wheel-rail contact in the dynamical problems.

In this article we have used realistic wheel-rail contact models and demonstrated that the conjecture is true: *The wheel set no longer derails*, when realistic wheel-rail profiles are used. There are however still large speed intervals with chaotic dynamics and very narrow speed intervals, where the wheel set locks in a large amplitude off-set stationary state. It is caused by the stick in the friction damper. It demonstrates that it is necessary to include stick-slip in all realistic models of a plane dry friction damper.

This may not be the last word in these problems. Piotrowski [3] has shown in a very recent article that medium frequency dither may have a not only quantitative but even a qualitative effect on dry friction characteristics. The dither may stem from rail irregularities or wheel-rail contact noise. Piotrowski [3] gives strong arguments for the necessity of including dither in all realistic dynamical models of vehicles with dry friction contacts.

6. REFERENCES

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