Computational Methods in Contact Mechanics

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Abstract. *The present paper gives an introduction to a new approach to solve the problem of two elastic bodies in rolling contact. The technique is based on half space theory for quasi identical bodies, which covers most of the technical applications of interest. The advantage of this new method is that it is very flexible and thus covers a broad spectra of different contact situations and furthermore is much faster than discretisation methods.*

1 INTRODUCTION

Rolling contact plays an important role in the simulation of many physical systems e.g. roller bearing dynamics or the wheel/rail contact in railway dynamics. As such simulations often are made by time integration methods it is crucial to be able to calculate the contact forces fast and with a high degree of accuracy.

Contact problems are often divided into two groups:

1. **The Normal Problem:** How to calculate the normal stress distribution over the contact patch when two elastic bodies are pressed together

2. **The Tangential Problem:** How to establish a relation between the tangential stress distribution and the global relative velocity for two elastic bodies rolling over each other under a normal load

For very simple geometries it is possible to solve the two problems analytically, but for more realistic contact situations numerical methods must be applied. An obvious approach is to employ a Finite Element Method[1] which can handle very complicated geometries with a high accuracy. The drawback of the FEM is that the long computation time makes it impossible to use it in a dynamical simulation.

A more suitable way to solve the contact problem is to assume that the bodies can be defined as half spaces and then apply the constitutive equation of Cerruti-Boussinesq which states a relation between stress and displacements. To utilize the half space approximation it is necessary that the size of the contact patch is much smaller than the characteristic radii of the bodies and that the materials involved are homogeneous, which is the case of most steel-steel contacts.

The traditional way to solve a half space problem is to make a discretisation of the contact patch and linearise the functions within each element[2]. It is then possible by iterative algorithms and variational methods to find the stresses and displacements in every point. This method is more inaccurate than the FEM approach but much faster. However for several applications the discretisation method is to slow to be applied in a time integration routine.

So it is in the present paper sought to evolve a new method to solve the half space problem, which is much faster and which has at least the same degree of accuracy as the discretisation method. This is done by assuming that the stresses can be expressed as polynomial forms. This way the integral in the constitutive equation is transformed into a polynomial and the contact problem can then be solved straight forward. The advantage of this method is that it only is necessary to calculate the coefficients of the polynomials, which can be done directly.

1 CONTACT MECHANICS

In the present work only the case of elastic contact is investigated i.e. it is assumed that no plastic deformation of the bodies takes place and that there is no time-dependent behaviour in the materials relationship between stress and strain. An introduction to inelastic contact is given by Johnson[3] and a more thorough investigation of the two dimensional case of viscoelastic cylinders in rolling contact can be found in the work of Wang[4].

If furthermore the two bodies in contact are quasi identical i.e. have the same material properties, then the tangential stress does not influence on the normal stress distribution and so the tangential contact problem can be solved independently of the normal contact problem. As most contact problems of interest involve bodies made of identical materials, especially steel-steel contact, only the case of quasi identical bodies will be described in this paper. If the bodies in contact have different material properties an iterative method must be applied e.g. The Panagiotopoulos Process[5].

For the case of steel-steel contact the deformations of the bodies are so small, that the linear small strain theory[6] yields a very good approximation. As the size of the contact patch for these cases furthermore in general are much smaller than the characteristic sizes of the bodies in contact, the half space theory[3] can be applied. The methods described in the following are all based on the half space theory for two dimensional quasi identical bodies.

1.1. The Two Dimensional Normal Contact Problem

When two elastic bodies are pressed together under a normal load N, they deform around their mutual contact point and a contact patch is created. Now the normal contact problem consists in finding the size of the contact patch and the normal stress distribution.

Let Z_1 and Z_2 be the surfaces of the two bodies, then the gradient of the local normal displacements u_{z1} and u_{z2} of the bodies in the contact area reads

$$\frac{du_{z1}(x)}{dx} + \frac{du_{z2}(x)}{dx} = \frac{d}{dx}[-Z_1(x) + Z_2(x)] \quad , \quad x \in S \tag{1}$$

$$\frac{du_{z1}(x)}{dx} + \frac{du_{z2}(x)}{dx} > \frac{d}{dx}[-Z_1(x) + Z_2(x)] \quad , \quad x \notin S \tag{2}$$

where S denotes the contact patch. The relation between the normal stress $p(x)$ and the displacement gradient is given by the constitutive equation of Cerruti-Boussinesq

$$\frac{du_{z1}(x)}{dx} + \frac{du_{z2}(x)}{dx} = -\frac{4(1-\nu^2)}{\pi E}\int_S \frac{p(\zeta)}{x-\zeta}\,d\zeta \tag{3}$$

where ν is the Poisson ratio and E is the modulus of elasticity. The last equation necessary to solve the normal contact problem arises from the fact that the normal load equals the integral of the normal stress distribution:

$$N = \int_S p(x)\,dx \tag{4}$$

Thus the normal contact problem is defined by four equations including an inequality and two integrals. This rather complex system of equations implies that it only is possible to solve the normal contact problem analytically for very restricted cases, otherwise approximations or numerical methods must be applied.

1.2. The Two Dimensional Tangential Contact Problem

Now consider two bodies in elastic contact. If a torque is applied to one of the bodies a tangential force will be transmitted to the other body due to the friction in the contact patch and the bodies will thus roll over each other. The compression of material in the contact patch results in a global relative velocity - creep - between the two bodies and so the tangential contact problem treats the task of establishing a relation between the creep and the tangential stress distribution or in other cases just the tangential force.

Introducing the local relative velocity - slip - as s, then the kinematic constraint is defined as

$$s(x,t) = \xi(t) + \frac{\partial u_x(x,t)}{\partial x} - \frac{1}{V}\frac{\partial u_x(x,t)}{\partial t} \tag{5}$$

where V is the velocity of the bodies and ξ is creep. If the term $\partial u(x,t)/\partial t$ is negligible the tangential contact problem is said to be stationary.

The contact patch is divided into a stick zone and a slip zone

$$x \in \{\text{stick zone}\} \Leftrightarrow \begin{cases} s(x,t) &= 0 \\ |q(x,t)| &< \mu p(x,t) \end{cases} \tag{6}$$

$$x \in \{\text{slip zone}\} \Leftrightarrow \begin{cases} s(x,t) &\neq 0 \\ |q(x,t)| &= \mu p(x,t) \end{cases} \tag{7}$$

where $q(x,t)$ is the tangential stress distribution and μ is the friction coefficient according to the friction law of Coulomb.

The constitutive equation gives a relation between the tangential stress distribution and the tangential displacement of the material in the contact patch:

$$\frac{\partial u_{x1}(x,t)}{\partial x} + \frac{\partial u_{x2}(x,t)}{\partial x} = -\frac{4(1-\nu^2)}{\pi E} \int_S \frac{q(\zeta)}{x-\zeta}\, d\zeta \tag{8}$$

The solution to the tangential contact problem is not unique: for a given creep an infinity of stress distributions fulfill the equations (5)-(8). The physical explanation to this apparently non physical behaviour is, that only the solution that minimizes the tangential force is stable: all other solutions are unstable and will only occur in a transition phase.

2 SOLUTIONS TO THE TWO DIMENSIONAL CONTACT PROBLEM

2.1. The Hertzian Solution

The two dimensional normal contact problem was solved analytically by Hertz[7] in 1882 for the case of two infinite cylinders pressed together under a normal load N. Hertz made the assumption that the length of the contact patch is much smaller than the radii of the cylinders, and then applied a second order Taylor approximation of the surfaces in the vicinity of the contact point. With this inserted into equation (1)-(4) the normal stress distribution is found to be elliptic over a contact patch with the length $2a$:

$$p(x) = \frac{p_0}{a}\sqrt{a^2 - x^2} \ , \quad -a \leq x \leq a \tag{9}$$

$$p_0 = \sqrt{\frac{NE}{2(1-\nu^2)\pi R}} \tag{10}$$

$$a = \sqrt{\frac{8(1-\nu^2)RN}{\pi E}} \tag{11}$$

where the characteristic radius R is defined by the radii of the two cylinders as $1/R = 1/R_1 + 1/R_2$. The Hertzian solution is obviously also valid for the case of a cylinder rolling over a level surface, where the surface is defined as a cylinder with infinite large radius i.e. $R_1 \to \infty$.

2.2. The Carter Solution

The stationary tangential contact problem for a Hertzian normal stress distribution was solved by Carter[8] in 1926 and by Fromm[9] in 1927. They found that the tangential stress distribution can be calculated as the sum of two ellipses. A new coordinate system where the relation between the old and the new coordinate system is given as

$$x^* = x + a - a^* \tag{12}$$

is introduced and so one of the ellipses has its centre in $O(x)$ and the other in $O(x^*)$ as indicated in Figure 1.

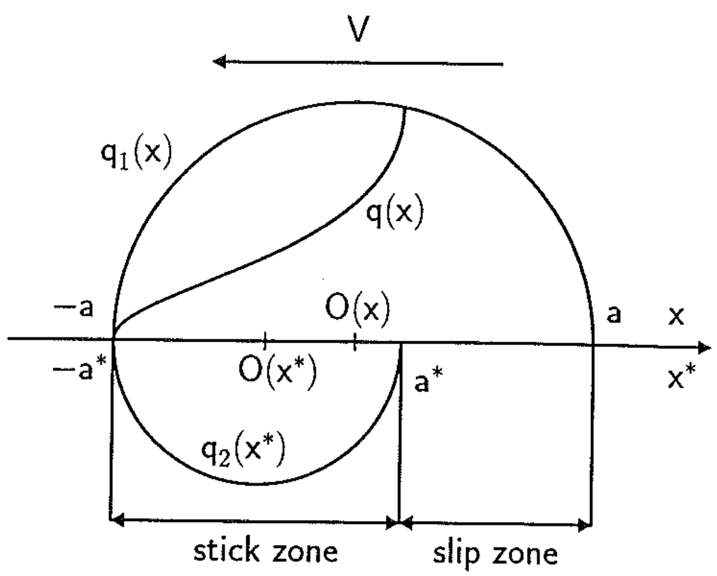

Figure 1: Tangential stress distribution for a cylinder rolling over a level surface

The tangential stress distribution $q(x)$ is then

$$q(x) = q_1(x) + q_2(x^*) \tag{13}$$

$$q_1(x) = \begin{cases} \frac{\mu p_0}{a}\sqrt{a^2 - x^2} & , \quad -a \leq x \leq a \\ 0 & , \quad \text{otherwise} \end{cases} \tag{14}$$

$$q_2(x^*) = \begin{cases} -\frac{\mu p_0}{a}\sqrt{a^{*2} - x^{*2}} & , \quad -a^* \leq x^* \leq a^* \\ 0 & , \quad \text{otherwise} \end{cases} \tag{15}$$

By inserting this tangential stress distribution into equation (5) the local relative velocity between the bodies is found to be

$$s(x^*) = \begin{cases} 0 & , \quad -a^* \leq x^* \leq a^* \\ \frac{\mu}{R}\sqrt{x^{*2} - a^{*2}} & , \quad a^* \leq x^* \leq 2a - a^* \end{cases} \tag{16}$$

This implies that the contact patch is divided into a stick zone with the length $2a^*$ at the leading edge and a slip zone at the trailing edge (Figure 1).

In the stick zone is $s(x) = 0$ while $s(x) \neq 0$ in the slip zone. The size of a^* depends on the size of the contact length and the size of the creep

$$a^* = a - \frac{R}{\mu}\xi \ , \quad 0 \leq \xi \leq \mu a/R \tag{17}$$

The classic way to evaluate a tangential contact problem is via a creep curve, where the tangential force is plotted as a function of the creep. The creep curve for the Carter solution is shown in Figure 2.

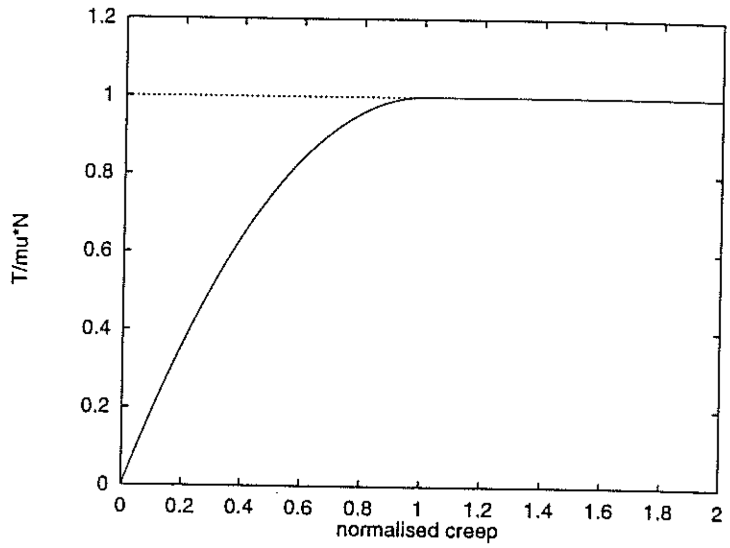

Figure 2: Creep curve for the Carter solution

It is seen that when the creep is small the tangential force is below the Coulomb value, whereas it for a certain size of the creep will reach the saturated regime where complete sliding occurs and the tangential force will be equal to the Coulomb value.

3 A POLYNOMIAL APPROXIMATION METHOD

As stated in the previous sections the Hertzian solution and the Carter solution are based on some fundamental assumptions. Some of these properties are however often violated in more realistic contact situations e.g.

1. the shape of the bodies cannot be described as $Z(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2$ in the vicinity of the contact point

2. the tangential problem does not fulfill the friction law of Coulomb

3. the tangential problem is instationary

So now the question arises: what to do if the solutions of Hertz and Carter cannot be applied? In those cases it is necessary to develop more refined methods to solve the contact problem. As the nature of the normal contact problem and the tangential contact problem are almost equivalent, the methods used for one of the problems can directly be applied the same way on the other problem. In this paper an approach based on polynomial approximations will be described.

3.1. The Basic Idea

The Hertzian solution was based on second order Taylor expansions of the bodies, but in fact it is possible to solve the problem for an arbitrary high order of the polynomial approximation.

Introduce the polynomial

$$\sum_{n=0}^{N} B_n x^n$$

where the boundary conditions

$$\sum_{n=0}^{N} B_n(-a)^n = \sum_{n=0}^{N} B_n(a)^n = 0 \tag{18}$$

are fulfilled. It can then be shown that the following unique relation exists:

$$\int_{-a}^{a} \frac{\sum_{n=0}^{N} B_n \zeta^n}{(x-\zeta)\sqrt{a^2-\zeta^2}} d\zeta = \sum_{n=0}^{N-1} \beta_n x^n + K\pi \sum_{n=0}^{N} \frac{B_n x^n}{\sqrt{x^2-a^2}} \tag{19}$$

$$K = \begin{cases} \text{sign}(x) & , \quad |x| > |a| \\ 0 & , \quad |x| \leq |a| \end{cases} \tag{20}$$

The connection between the B_n's and the β_n's is described by the matrix equation

$$\left\{\begin{array}{c}\beta_0\\ \beta_1\\ \beta_2\\ \beta_3\\ \beta_4\\ \dots\\ \beta_{N-1}\end{array}\right\} = \left[\begin{array}{ccccccc} A_1 & 0 & A_2 & 0 & A_3 & \dots & 0\\ 0 & A_1 & 0 & A_2 & 0 & \dots & A_{N/2}\\ 0 & 0 & A_1 & 0 & A_2 & \dots & 0\\ 0 & 0 & 0 & A_1 & 0 & \dots & A_{N/2-1}\\ 0 & 0 & 0 & 0 & A_1 & \dots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & 0 & 0 & \dots & A_1\end{array}\right] \left\{\begin{array}{c}B_1\\ B_2\\ B_3\\ B_4\\ B_5\\ \dots\\ B_N\end{array}\right\} \tag{21}$$

where the coefficients A_n are defined as

$$A_1 = -\pi \tag{22}$$

$$A_n = -\pi a^{2n-2}\prod_{i=2}^{n}\frac{2i-3}{2i-2}\ ,\quad n=2,3,\dots,N \tag{23}$$

Because the matrix always is nonsingular there exists a unique solution to the inverse problem i.e. that if the β_n's are given, then the B_n's can be determined from the equation

$$\left\{\begin{array}{c}B_1\\ B_2\\ B_3\\ B_4\\ B_5\\ \dots\\ B_N\end{array}\right\} = \left[\begin{array}{ccccccc} A_1^{-1} & 0 & A_2^{-1} & 0 & A_3^{-1} & \dots & 0\\ 0 & A_1^{-1} & 0 & A_2^{-1} & 0 & \dots & A_{N/2}^{-1}\\ 0 & 0 & A_1^{-1} & 0 & A_2^{-1} & \dots & 0\\ 0 & 0 & 0 & A_1^{-1} & 0 & \dots & A_{N/2-1}^{-1}\\ 0 & 0 & 0 & 0 & A_1^{-1} & \dots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & 0 & 0 & \dots & A_1^{-1}\end{array}\right] \left\{\begin{array}{c}\beta_0\\ \beta_1\\ \beta_2\\ \beta_3\\ \beta_4\\ \dots\\ \beta_{N-1}\end{array}\right\} \tag{24}$$

where the coefficients A_n^{-1} are defined as

$$A_1^{-1} = -\frac{1}{\pi} \tag{25}$$

$$A_n^{-1} = -\frac{1}{\pi}\frac{a^{2n-2}}{2n-3}\prod_{i=2}^{n}\frac{2i-3}{2i-2}\ ,\quad n=2,3,\dots,N \tag{26}$$

3.2. Application of the Theory

Now consider equation (3). With Z_1 and Z_2 defined as polynomials in x the normal stress distribution can be calculated straight forward as a function of a by applying equation (19) and (24). Inserting this normal stress distribution into equation (4) one equation with the unknown size a remains and this equation can then be solved numerically e.g. with the Hertzian value of a as an initial guess. This implies that the entire normal contact problem is reduced to solving one nonlinear equation with one unknown.

Compared with discretisation methods where an iterative process within each contact element must be applied the present approach is of course much faster. A further advantage is that the problem is solved without using variational methods which are very time consuming. The existence of many different values of a as a solution to the nonlinear equation is of course a problem, but provided the contact patch is coherent the iterative process will converge quickly towards the real a-value. The other solutions to the equation yields non physical stress distributions where the normal stress in some areas is negative.

According to the above described principle where the integral equation is transformed into an algebraic equation, a wide variety of contact problems can be solved as long as the properties included are evaluated as polynomials.

4 NUMERICAL EXAMPLES

In the following sections examples of non Hertzian contact situations will be solved with the introduced polynomial approximation method. For some of the cases the contact problem can be solved directly whereas in other cases an iterative process must be applied. Common for all three examples is it that the solution is found much faster and more precise than if a discretisation method was applied.

4.1. Non Smooth Surface

Consider a cylinder with the radius R rolling over a periodically varying surface Z

$$Z(x) = \sum_{m=1}^{M} \Phi_m \cos(k_m x) + \Psi_m \sin(k_m x) \tag{27}$$

If the characteristic wave length of the surface is small compared to the size of the contact patch, the second order Taylor expansion of the bodies from the Hertzian theory is much to primitive an approximation. Instead the polynomial method is applied. Making the complete Taylor expansion of the surface this yields the following expressions for the normal contact problem:

$$\begin{aligned} N &= \frac{\pi E}{4(1-\nu^2)} \left\{ \frac{a^2}{2R} + \sum_{m=1}^{M} a k_m \mathrm{J}_1(a k_m) \left[\Phi_m \cos(k_m(x+\Delta)) + \right.\right. \\ &\qquad\qquad \left.\left. \Psi_m \sin(k_m(x+\Delta)) \right] \right\} \end{aligned} \tag{28}$$

$$\Delta = \sum_{m=1}^{M} R k_m \mathrm{J}_0(a^* k_m) \left[-\Psi_m \cos(k_m(x+\Delta)) + \Phi_m \sin(k_m(x+\Delta)) \right] \tag{29}$$

where Δ is the distance from the centre of the contact patch to the vertical projection of the cylinder axis and J_0 and J_1 are Bessel functions of first kind. With two equations and the two unknowns a and Δ the normal contact problem is solved by applying a numerical method on the two nonlinear equations. A similar procedure is utilized for the tangential contact problem, and so the following two equations are derived

$$T = \mu N - \frac{\mu\pi E}{4(1-\nu^2)}\left\{\frac{a^{2*}}{2R} + \sum_{m=1}^{M} a^* k_m \mathrm{J}_1(a^* k_m)\left[\Phi_m \cos(k_m(x-a+a^*+\Delta)) + \Psi_m \sin(k_m(x-a+a^*+\Delta))\right]\right\} \quad (30)$$

$$\xi = \frac{\mu(a-a^*-\Delta)}{R} + \sum_{m=1}^{M} \mu k_m \mathrm{J}_0(a^* k_m)\left[-\Psi_m \cos(k_m(x-a+a^*+\Delta)) + \Phi_m \sin(k_m(x-a+a^*+\Delta))\right] \quad (31)$$

where T is the tangential force. So now there is two equations with the two unknowns a^* and T, which can be solved numerically without any difficulties.

A further advantage of the present model is that the semi analytical approach results in algebraic equations, which give a much better understanding of the different parameters influence on the problem, and this way the 'black box' nature of discretisation methods is avoided. Another possibility made feasible by the analytical expressions in equation(28)-(31) is that these equations can be linearised and then solved analytically. In Figure 3 is a typical stress distribution calculated with the polynomial approach compared with the classic solution of Hertz and Carter. It is clearly seen that the error introduced by the original solution is significant. Especially the varying size of the tangential stress in the slip zone is important in wear calculations[10].

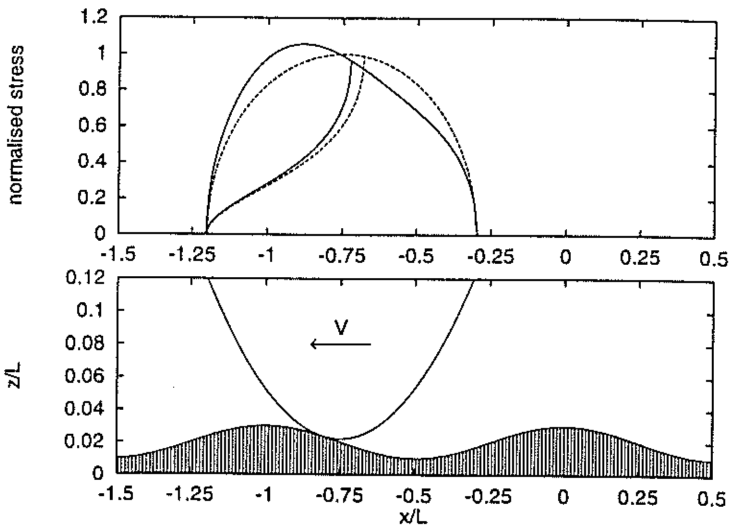

Figure 3: Top: tangential stress distribution for the Carter approximation (dashed line) and for the polynomial approximation (solid line). Bottom: the cylinders position on the surface.

4.2. Velocity Dependent Friction Coefficient

In the Carter solution it is assumed that the friction law of Coulomb is valid i.e. $T \leq \mu N$, where μ is constant. Experimental work[11] indicates however, that the friction coefficient is velocity dependent. It will thus be of interest to examine how a velocity dependent friction coefficient influences the tangential contact problem. One attempt to solve this problem was made by Ohyama[12] who investigated a friction coefficient with a static value and a kinematic value. With this assumption Ohyama showed that the creep curve will have a maximum and then decay as the creep increases. Unfortunately Ohyamas solution does not fulfill equation (5)-(8) and is thus wrong. Instead it can be shown that a friction coefficient defined as a step function does not influence the creep curve at all[13].

Now let the friction coefficient be a smooth function of the velocity. This implies that the tangential stress distribution in the slip zone is no longer elliptic because it is defined as $|q(x)| = \mu(s(x))p(x)$ where the slip $s(x)$ not is constant in the slip zone. The way to solve this problem is to apply an iterative method. Equivalent to the Carter solution it is assumed that the tangential stress distribution can be described as the sum of two polynomial forms, where the largest of them is defined as $q_1(x) = \mu(s(x))p(x)$.

With an initial guess of $s(x)$ it is possible to approximate $\mu(s(x))$ by a polynomial. This implies that q_1 and thus q_2 are polynomial forms and so the constitutive equation (8) is transformed into an algebraic equation and the tangential contact problem can be solved. This results in a new value for the slip which implies that $q_1(x)$ must be redefined and so the calculation is made once more. After just a few iterations the process converges and the tangential stress distribution is found[13].

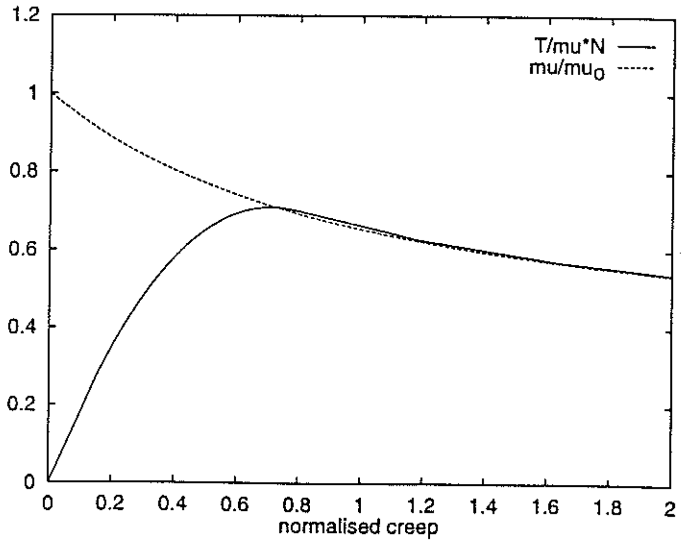

Figure 4: Velocity depending friction coefficient (dashed line) and corresponding creep curve (solid line).

In Figure 4 is the the creep curve shown for a given μ-function. When μ is a decaying function as in the present example, this will be reproduced in the shape of the creep curve which has a distinct maximum and then decays. Another interesting result is illustrated in Figure 5 where the tangential stress distribution for the μ-function shown in Figure 4 is compared with the Carter solution. When μ is a decaying function, a very sharp spike in the tangential stress distribution will appear in the limit between the stick zone and the slip zone.

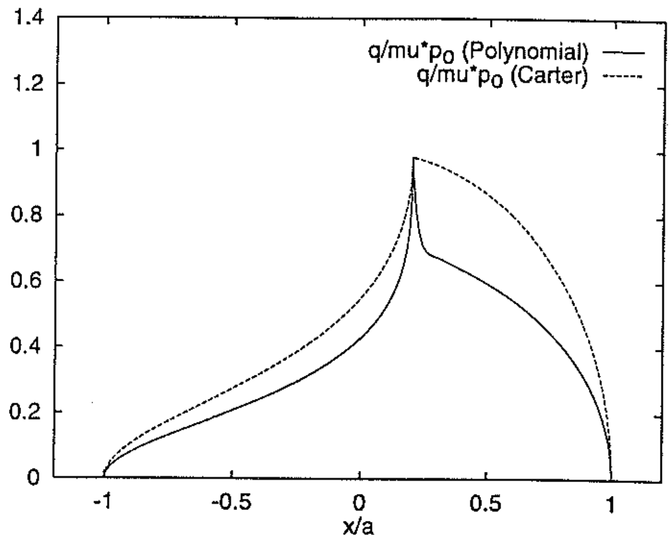

Figure 5: Normalised tangential stress distribution: Carter solution(dashed line), polynomial approximation for velocity depending friction coefficient (solid line)

4.3. Non Stationary Contact

In the solutions described in the previous sections it is always assumed that the contact is stationary. A rule of thumb states that if L is the characteristic wave length, then the problem can be considered stationary if $L/a > 20$. If this is not the case, the contact problem is instationary and the term with $\partial u(x,t)/\partial t$ must be included in the calculations. The main difficulty with the two dimensional non stationary contact problem is, that it is unsolvable! This is due to the fact that the size of $u(0,t)$ depends on the choice of datum of the displacements, and so the solution to the instationary two dimensional contact problem will always contain the unknown term $u(0,t)$[14]. It is however possible to calculate the tangential stress distribution for a given oscillating tangential force.

From equation (5) it is found that the displacements in the stick zone must fulfill

$$\frac{\partial u_x(x,t)}{\partial x} = \frac{1}{V}\frac{\partial u_x(x,t)}{\partial t} - \xi(t) \quad , \quad x \in \{stickzone\} \tag{32}$$

Since $u(x,t)$ is bounded this yields the solution

$$u(x,t) = \sum_{m=1}^{M} \Phi_m \sin(k_m x + \omega_m t) - \Psi_m \cos(k_m x + \omega_m t) + \int \xi(t) \qquad (33)$$

where

$$\frac{1}{V}\frac{\omega_m}{k_m} = 1 \ , \quad m = 1, 2, \ldots, M \qquad (34)$$

As $u(0,t)$ is unknown it is not possible to find ξ and so a relation between the creep and the tangential force can not be established.

With the expression for $u(x,t)$ from equation (33) the displacement gradient is found to be

$$\frac{\partial u(x,t)}{\partial t} = \sum_{m=1}^{M} \Phi_m \cos(k_m x + \omega_m t) + \Psi_m \sin(k_m x + \omega_m t) \qquad (35)$$

which is similarly to the displacement gradient from the case of a cylinder rolling over a periodically varying surface. Thus the tangential stress distribution for the instationary case can be found exactly the same way as for the case of a cylinder rolling over a periodically varying surface.

In Figure 6 is an example of a tangential stress distribution for the instationary contact problem compared with the Carter solution. It is clearly seen how the oscillating behaviour of the contact problem influences the tangential stress distribution.

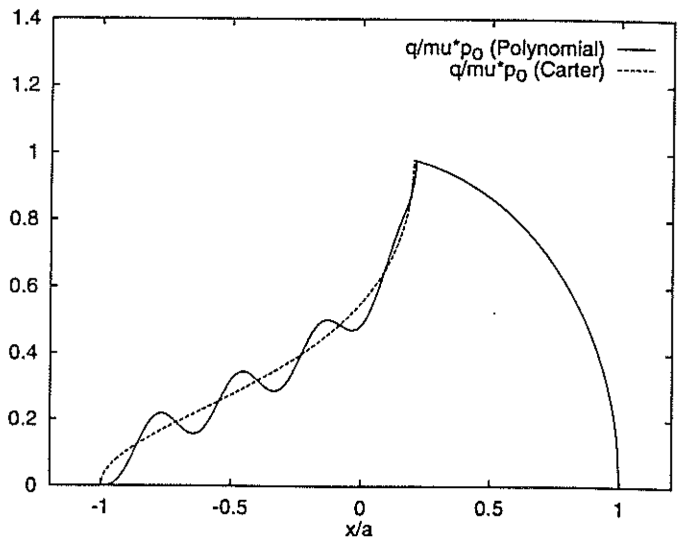

Figure 6: Tangential stress distribution for an instationary contact problem (solid line) compared with the equivalent Carter solution (dashed line)

5 CONCLUSION

In the present paper a new method to solve 2D contact problem is described. The method is very flexible and can thus cover a large variety of contact problems. One of the great advantages of the technique is that the contact problem is reduced to the problem of solving one or two nonlinear equations due to the similarities of the normal contact problem and the tangential contact problem.

The reason why variational calculations can be avoided is that some basic knowledge of the behaviour of rolling contact mechanics is included. The assumption that the contact patch is divided into a stick zone at the leading edge and a slip zone at the trailing edge provides that the found solution always is the one that minimizes the contact forces.

If the assumption of one coherent contact patch or one stick zone and one slip zone does not hold the model breaks down. This is the case for e.g. very rough surfaces or instationary contact with large amplitudes. The principle of polynomial approximation can still be applied by dividing the contact problem into several adjacent contact patches or several stick and slip zones. The advantages of the polynomial approximation method compared with discretisation techniques will however diminish.

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