

# Statistical framework for decision making in mine action

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## How do we construct a reliable detector?

- Empirical method: systematic **acquisition of knowledge** which is used to build a mathematical model to generate reliable results in real use cases
- Specifying the relevant scenarios and performance measures – end user involvement is crucial!!!
- Cross-disciplinary R&D involving very competences

# Knowledge acquisition

## Physical modeling

- Study physical properties and mechanism of the environment and sensors
- Describe the knowledge as a mathematical model

## Statistical modeling

Require real world related data  
Use data to learn e.g. the relation between the sensor reading and the presence/absence of explosives

# Why do we need statistical models and machine learning?



- Mine action is influenced by many uncertain factors
- The goals of mine action depends on difficult socio-economic and political considerations and constraints are to be built in

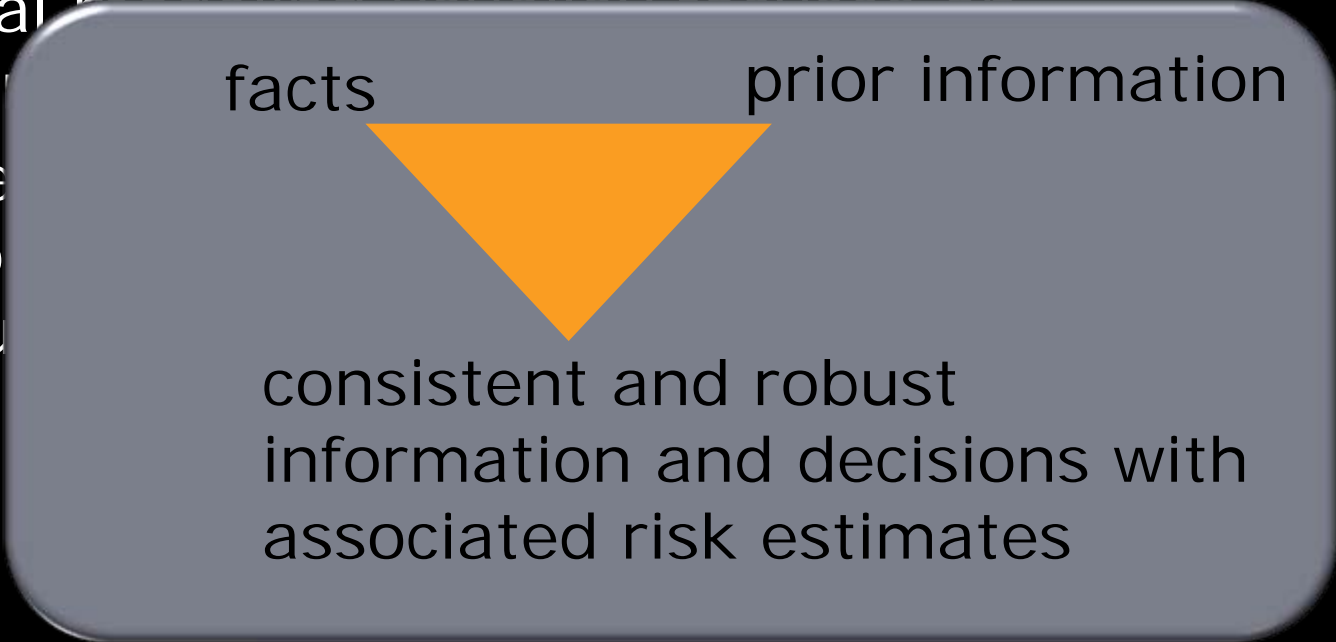
Scientist are born sceptical: they don't believe facts unless they see them often enough

# Why do we need statistical models and machine learning?

- statistical modeling is the **principled framework** to handle uncertainty and complexity
- statistical modeling handles features or identifying
- machine learning handles collection of new situations

facts

prior information



consistent and robust  
information and decisions with  
associated risk estimates

# There is no such thing as facts to spoil a good explanation!

- Pitfalls and misuse of statistical methods sometimes wrongly leads to the conclusion that they are of little practical use

After  
in the  
accid

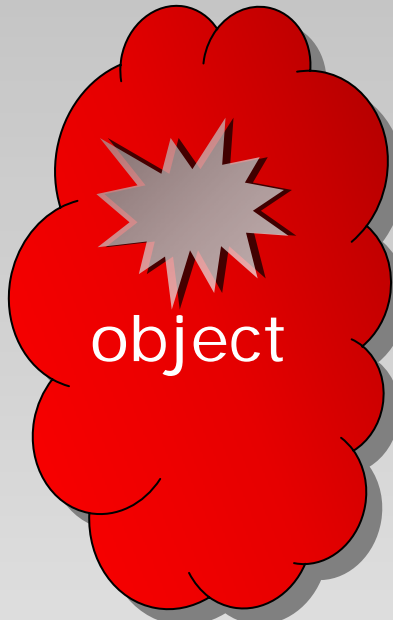
Some data are in the tail of the distribution: generalization from few examples is not possible

Smoking is not dangerous: mv

Most suspected areas have very few mines

# Information processing pipeline

Physical domain



environment

Technical/detection domain

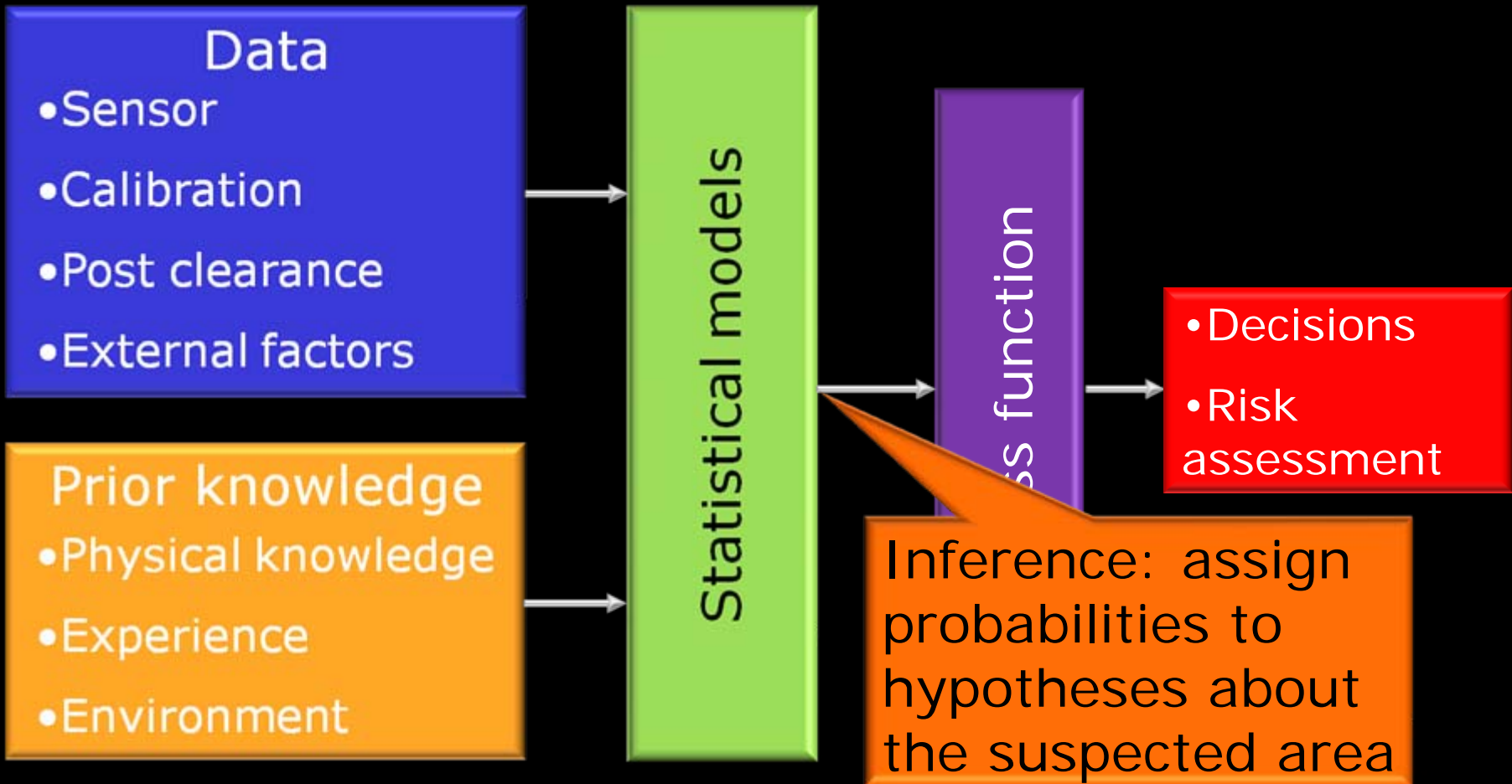


User /cognitive domain



HCI  
perception  
interpretation

# The elements of statistical decision theory





# Outline

- The design and evaluation of mine clearance equipment – the problem of reliability
  - Detection probability – tossing a coin
  - Requirements in mine action
  - Detection probability and confidence in MA
  - Using statistics in area reduction
- Improving performance by information fusion and combination of methods
  - Advantages
  - Methodology
  - DeFuse and Xsense projects

## Detecting a mine – tossing a coin

$$\textit{Frequency} = \frac{\text{no of heads}}{\text{no of tosses}}$$

*probability = frequency* when infinitely many tosses

## On 99,6% detection probability

$$\textit{Frequency} = \frac{9960}{10000} = 99,6\%$$

One more (one less) count will change the frequency a lot!

## Detection probability - tossing a coin

- $N$  independent tosses number of
- $y$  number of heads observed
- $\theta$  probability of heads

$$\hat{\theta} = \frac{y}{N}$$

$$P(y | \theta) = \text{Binom}(\theta | N) = \binom{N}{y} \theta^y \theta^{N-y}$$

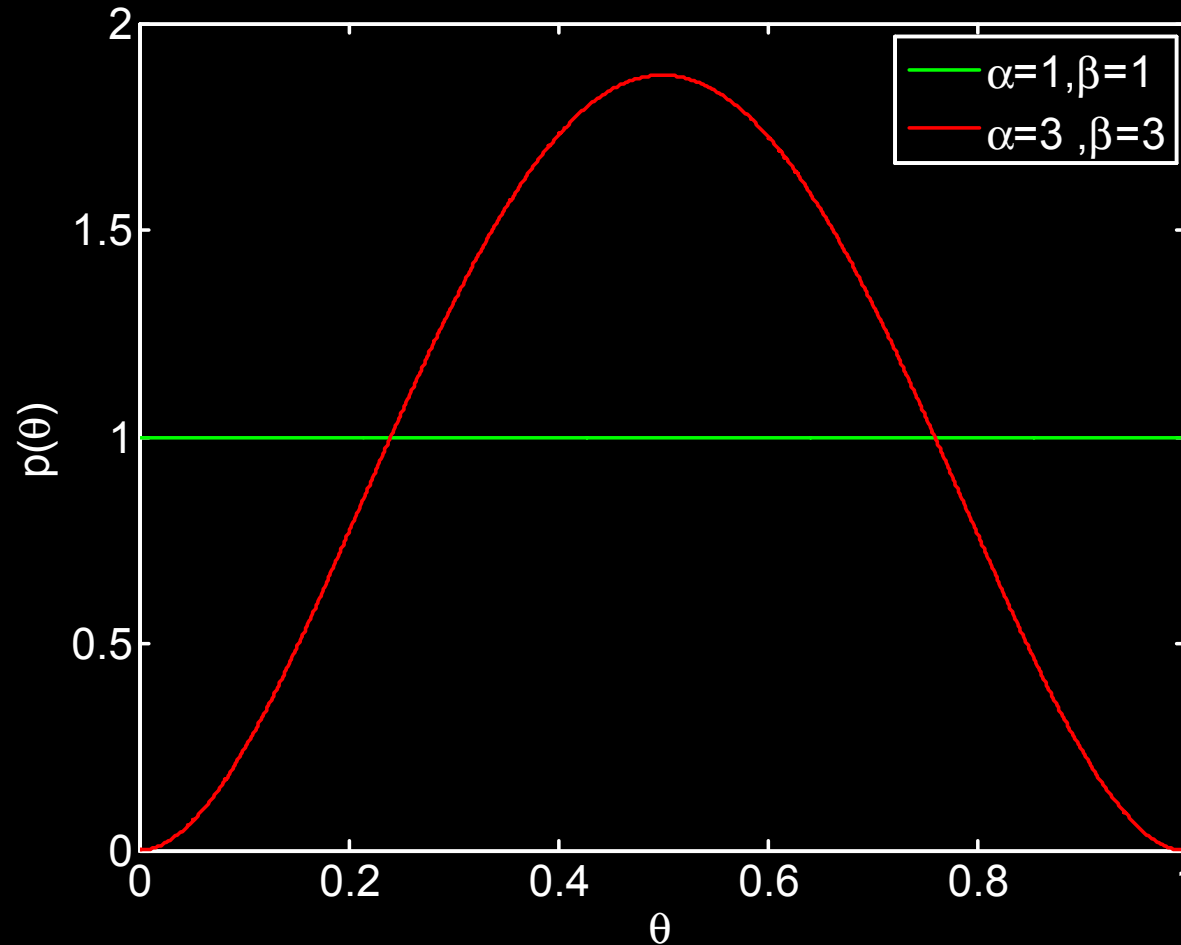
Data likelihood

## Prior beliefs and opinions

- Prior 1: the fair coin:  $\theta$  should be close to 0.5
- Prior 2: all values of  $\theta$  are equally plausible

$$p(\theta) = \text{Beta}(\theta \mid \alpha, \beta)$$

# Prior beliefs and opinions



# Bayes rule: combining data likelihood and prior

$$P(\theta | y) = \frac{P(y | \theta) p(\theta)}{P(y)}$$

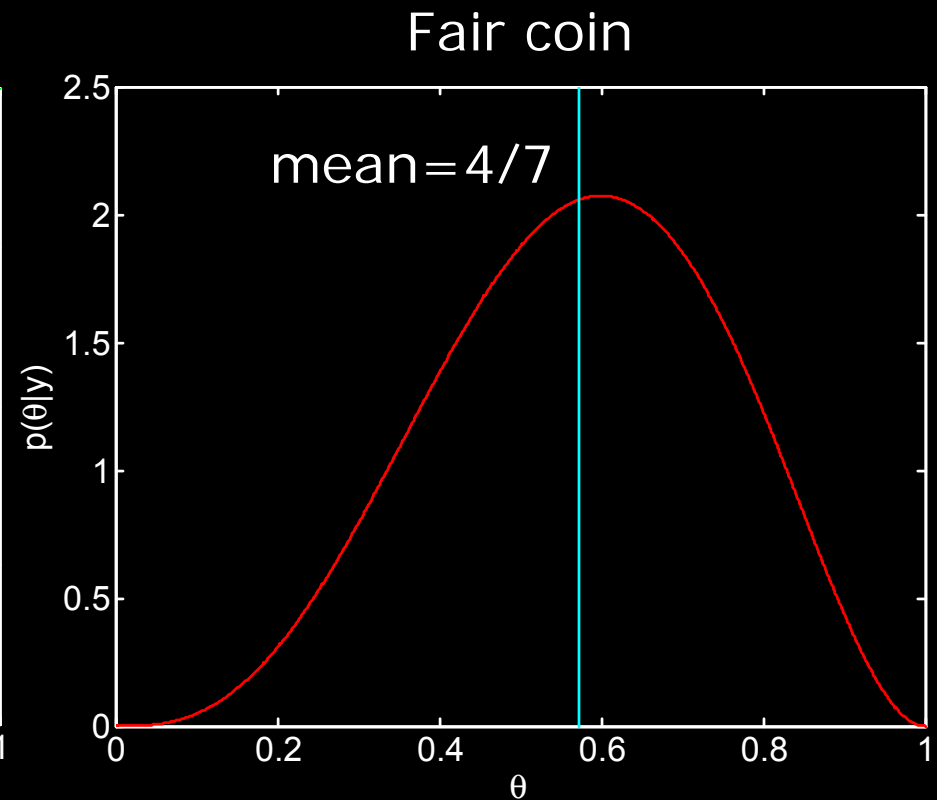
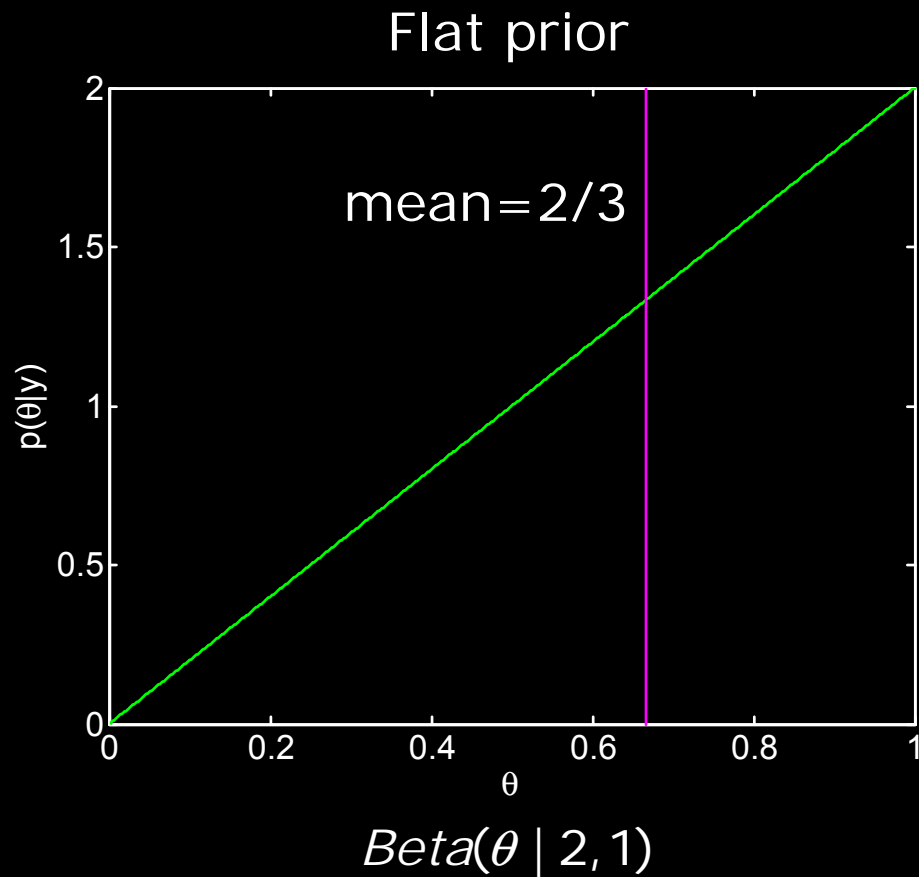
$$P(\theta | y) = \text{Beta}(\theta | y + \alpha, \beta + n - y) \sim \theta^{y+\alpha} \theta^{n-y+\beta}$$

# Posterior probability is also Beta

$$P(\theta | y) = \text{Beta}(\theta | y + \alpha, \beta + n - y) \sim \theta^{y+\alpha} \theta^{n-y+\beta}$$



# Posteriors after observing one head



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# What are the requirements for mine action risk

- Tolerable risk for individuals comparable to other natural risks

- As high cost efficiency as possible requires detailed risk

99.6% detection is not an unrealistic requirement

but... today's methods achieve at most 90% and are hard to evaluate!!!

management and control involving all partners (MAC, NGOs, commercial etc.)

GICHD and FFI are currently working on such methods  
[Håvard Bach, Ove Dullum NDRF SC2006]

# A simple inference model – assigning probabilities to data

- The detection system provides the probability of detection a mine in a specific area:  $\text{Prob}(\text{detect})$
- The land area usage behavior of an area provides the probability of mine encounter:  $\text{Prob}(\text{mine encounter})$

For discussion of assumptions and involved factors see

“Risk Assessment of Minefields in HMA – a Bayesian Approach”

PhD Thesis, IMM/DTU  
2005 by Jan Vistisen

$\text{Prob}(\text{casualty}) = (1 - \text{Prob}(\text{detect})) \times \text{Prob}(\text{mine encounter})$

# A simple loss/risk model

- Minimize the number of casualties
- Under mild assumptions this equivalent to minimizing the probability of casualty

# Maximum yearly footprint area in m<sup>2</sup>

P(detection)	$\rho$ : mine density (mines/km <sup>2</sup> )				
	0.1	1	10	100	1000
0.996	25000	2500	250	25	2.5
0.9	1000	100	10	1	0.1

Prob(causality) =  $10^{-5}$  per year

Reference: Bjarne Haugstad, FFI

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# Evaluation and testing in MA

- How do we assess the performance/detection probability?
- What is the confidence?

Changing environment

- mine types, placement
- soil and physical properties
- unmodeled confounds

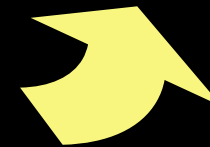


system design phase

operation phase

Overfitting

- insufficient coverage of data
- unmodeled confounding factors
- insufficient model fusion and selection



n phase



# Two types of error in detection of mines

## Sensing error

Example: metal detector

The system does not sense the mine object of the mine object

- Sensing error: the mine has low metal content
- Decision error: leakage from the mine was too low



probability

## Decision error

Example: mine detection dog

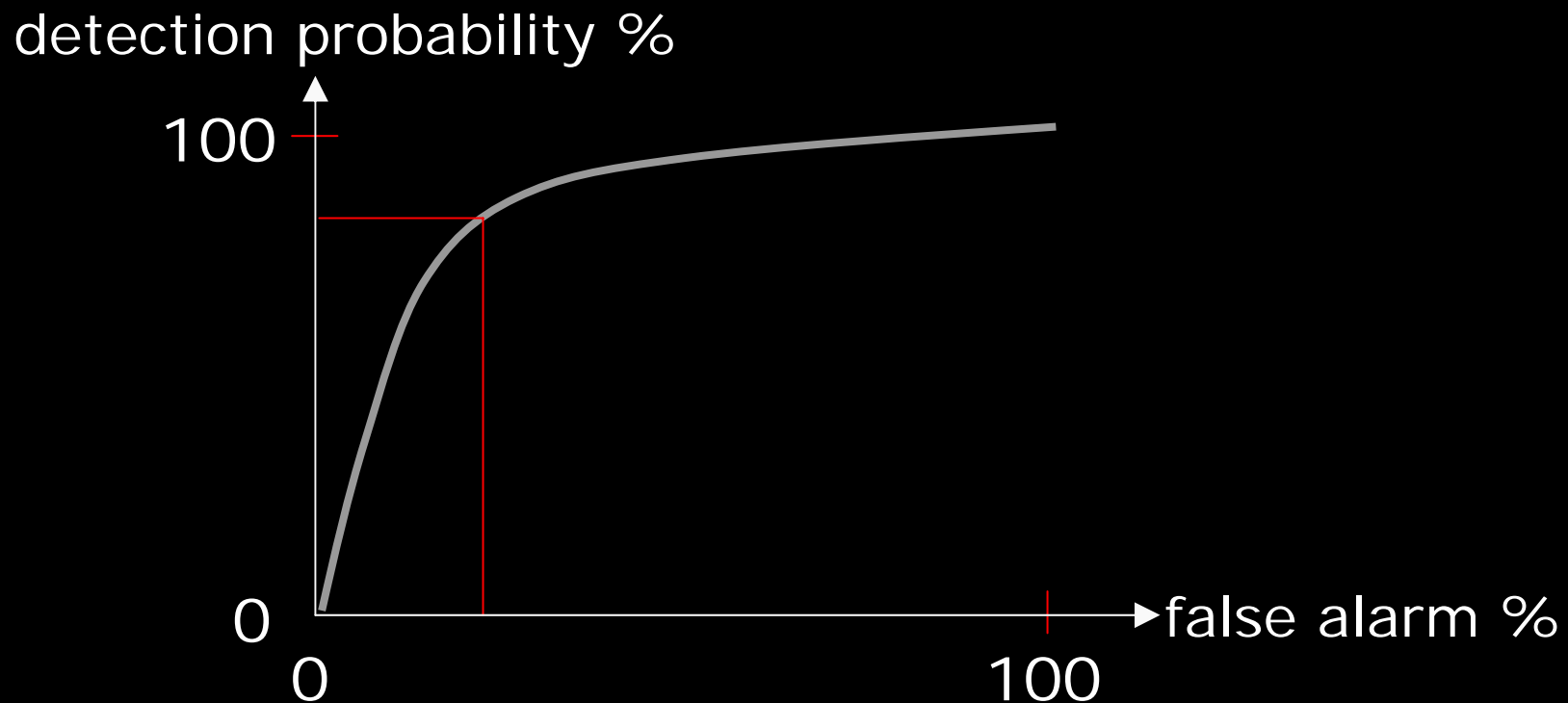
- Sensing error: the TNT leakage from the mine was too low
- Decision error: the dog handler misinterpreted the dogs indication

# Confusion matrix in system design and test phase which should lead to certification

		True	
		yes	no
Estimated	yes	a	b
	no	c	d

- Detection probability (sensitivity):  $a/(a+c)$
- False alarm:  $b/(a+b)$
- False positive (specificity):  $b/(b+d)$

# Receiver operation characteristic (ROC)

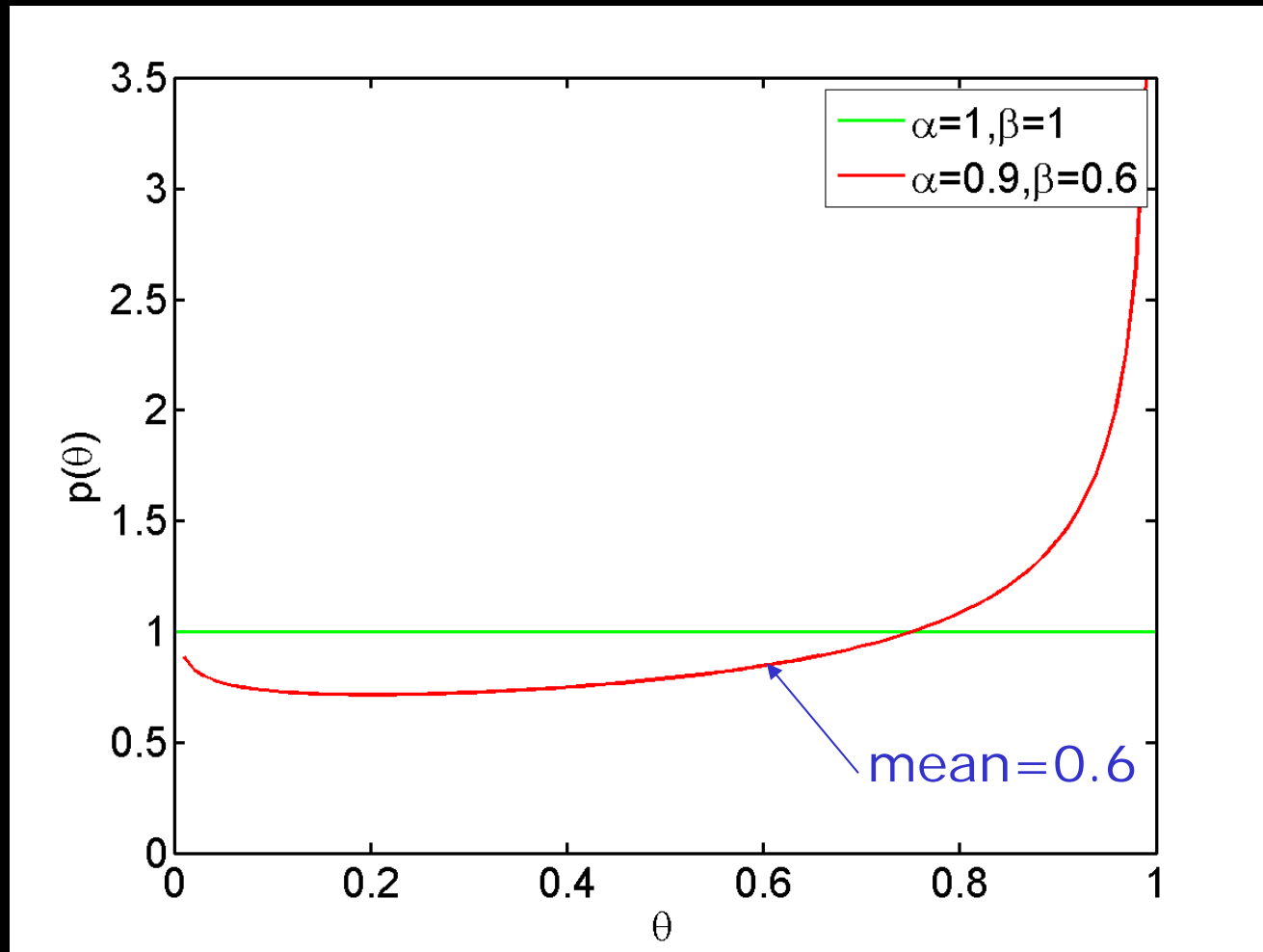


# Bayes rule: combining data likelihood and prior

The diagram illustrates the components of Bayes' rule. A red box labeled 'Likelihood' has a red arrow pointing down to the numerator term  $P(y | \theta)$  in the equation. A blue box labeled 'Prior' has a blue arrow pointing down to the numerator term  $p(\theta)$ . An orange box labeled 'Posterior' has an orange arrow pointing up to the entire equation  $P(\theta | y)$ .

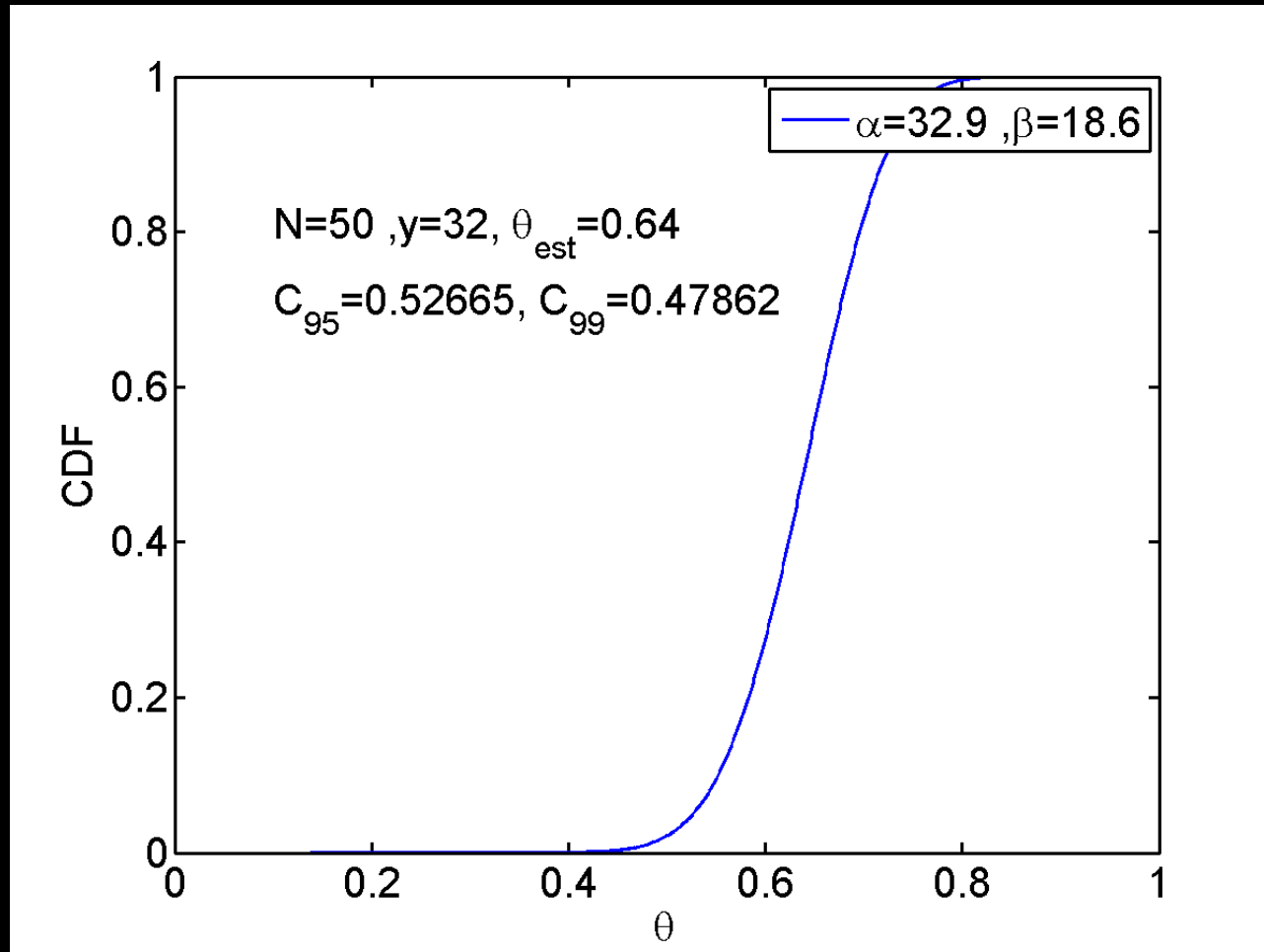
$$P(\theta | y) = \frac{P(y | \theta) p(\theta)}{P(y)}$$

# Prior distribution



# HPD credible sets – the Bayesian confidence interval

$$C_{1-\varepsilon} = \{\theta: P(\theta | y) \geq k(\varepsilon)\}, \text{CDF}(\theta | y) > 1 - \varepsilon$$



# The required number of samples $N$

- We need to be confident about the estimated detection probability

$$\text{Prob}(\theta > 99.6\%) = C_{1-\varepsilon}$$

	$C_{95\%}$	$C_{99\%}$		$C_{95\%}$	$C_{99\%}$
$\theta_{est} = 99.7\%$	9303	18994	$\theta_{est} = 99.7\%$	8317	18301
$\theta_{est} = 99.8\%$	2285	3995	$\theta_{est} = 99.8\%$	2147	3493

Uniform prior

Informative prior  
 $\alpha=0.9, \beta=0.6$

Prior info reduces the need for samples

# Credible sets when detecting 100%

Minimum number of samples  $N$

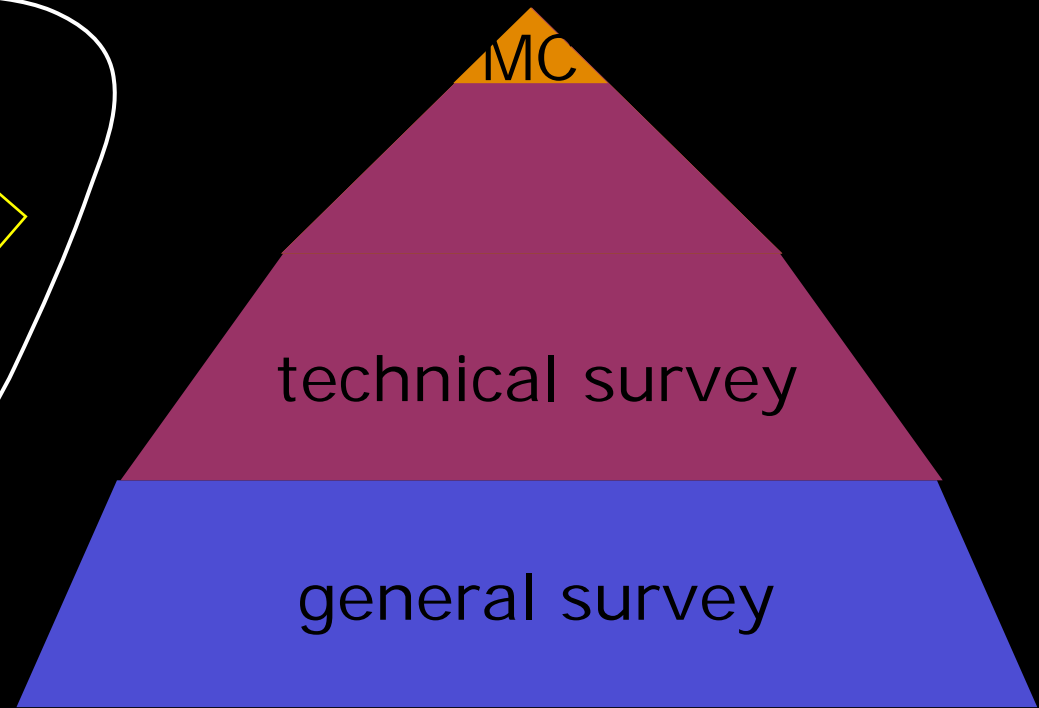
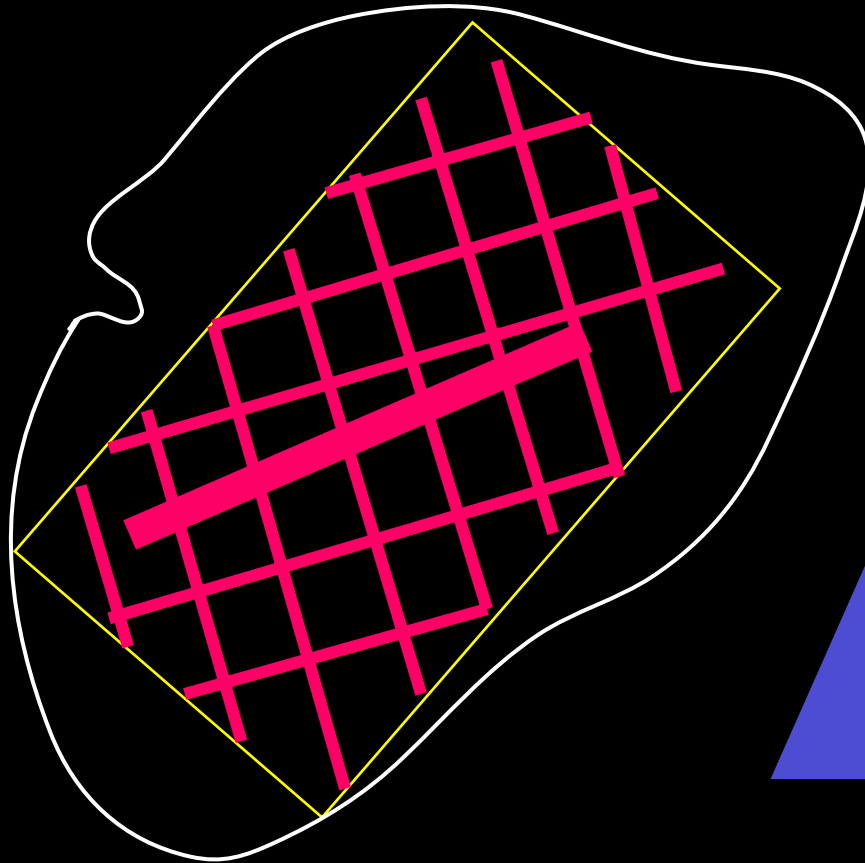
	Prob( $\theta > 80\%$ )	Prob( $\theta > 99.6\%$ )	Prob( $\theta > 99.9\%$ )
$C_{95\%}$	13	747	2994
$C_{99\%}$	20	1148	4602



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# Efficient MA by hierarchical approaches



Refs: Håvard Bach, Paul Mackintosh

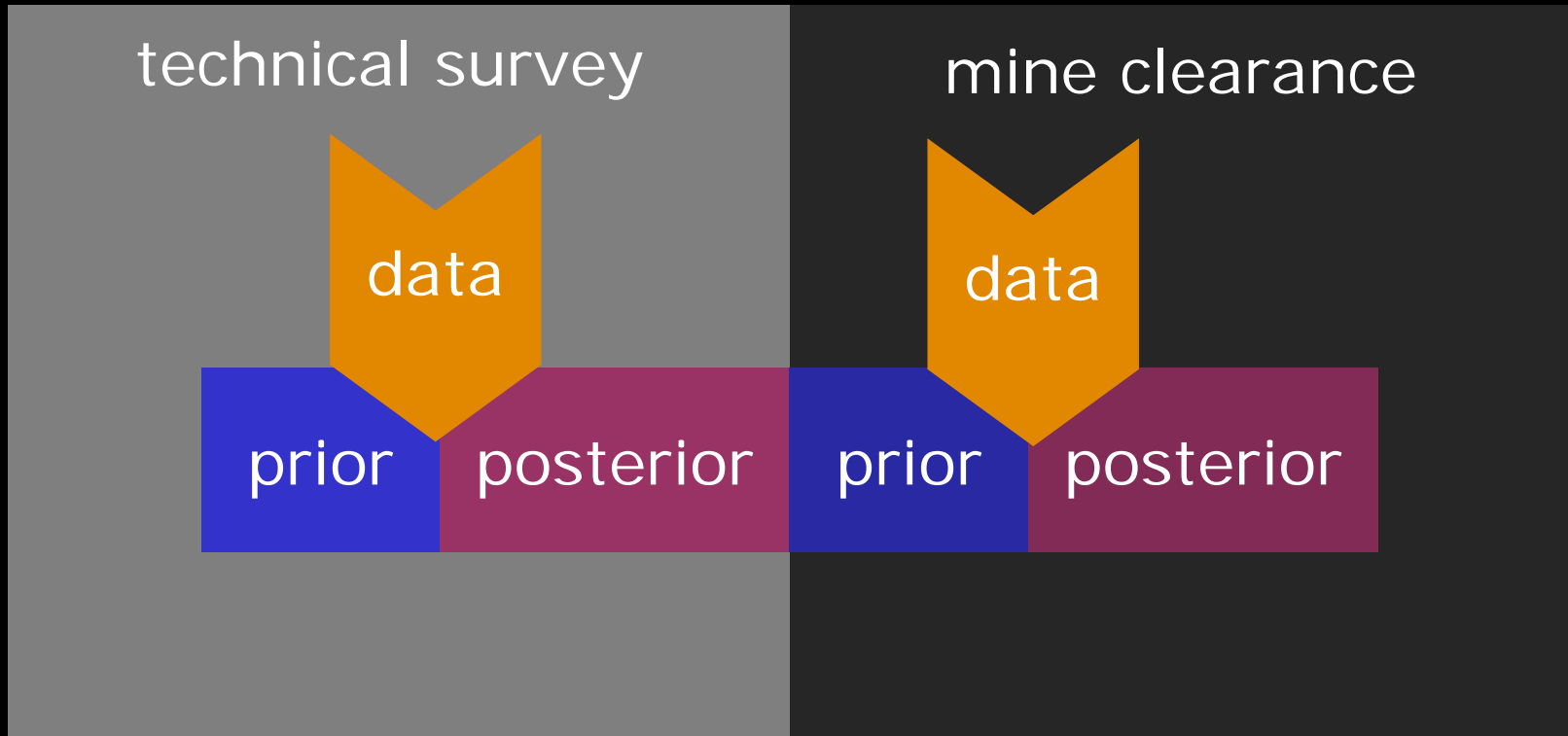
# Danger maps

- The outcome of a hierarchical surveys
- Information about mine types, deployment patterns etc. should also be used
- Could be formulated/interpreted as a prior probability of mines



SMART system described in GICHD: Guidebook on Detection Technologies and Systems for Humanitarian Demining, 2006

# Sequential information gathering



## Statistical information aggregation

- $e=1$  indicates encounter of a mine in a box at a specific location
- probability of encounter  $P(e = 1)$  from current danger map
- $d=1$  indicates detection by the detection system
- probability of detection  $P(d = 1)$  from current accreditation

$$P(e = 1 \wedge d = 0) = P(e = 1)(1 - P(d = 1))$$

$$P(\text{no mine}) = 1 - P(e = 1 \wedge d = 0)$$

## Statistical information aggregation

$$P(e = 1) = 0.2, P(d = 1) = 0.8$$

$$P(\text{no mine}) = 1 - P(e = 1 \wedge d = 0) = 1 - 0.2 * 0.2 = 0.96$$

$$P(e = 1) = 1, P(d = 1) = 0.96$$

$$P(\text{no mine}) = 1 - P(e = 1 \wedge d = 0) = 1 - 1 * 0.04 = 0.96$$

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## Where are we and how do we get further?

- No single existing method deliver sufficient detection performance
- No universal best method exists – every method has its pros and cons
- Fusion of sensors have been suggested in  
“Analysis and Fusion using Belief Function Theory of Multisensor Data for Close-range Humanitarian Mine Detection.

PhD Thesis RMA, 2001 by Nada Milisavljević

Does not immediately  
apply to fusion of  
heterogenous  
methods



# Advantages

- Combination leads to a possible exponential increase in detection performance
- Combination leads to better robustness against changes in environmental conditions

# Challenges

- Need for **certification procedure** of equipment under well-specified conditions (ala ISO)
- Need for new procedures which estimate **statistical dependences** between existing methods
- Need for new procedures for statistically **optimal combination**

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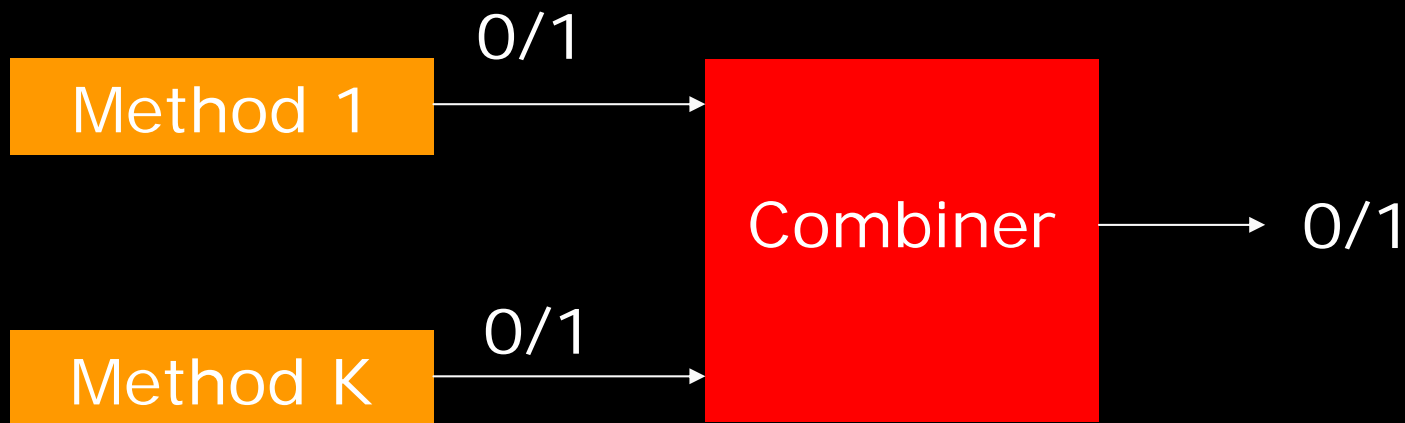
# Dependencies between methods

Contingency tables

		Method j	
		yes	no
Method i	yes	c11	c10
	no	c01	c00

Mine present

# Optimal combination



Optimal combination depends on contingency tables

# Optimal combiner

Method		Combiner						
1	2	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
0	1	0	0	0	1	1	1	1
1	0	0	1	1	0	0	1	1
1	1	1	0	1	0	1	0	1

OR rule is optimal for independent methods

$$2^{2^{K-1}} - 1 \text{ possible combiners}$$

## Example

$$p_{d1} = 0.8, p_{fa1} = 0.1 \quad p_{d2} = 0.7, p_{fa2} = 0.1$$

$$p_d = 1 - (1 - 0.8) \cdot (1 - 0.7) = 0.94$$

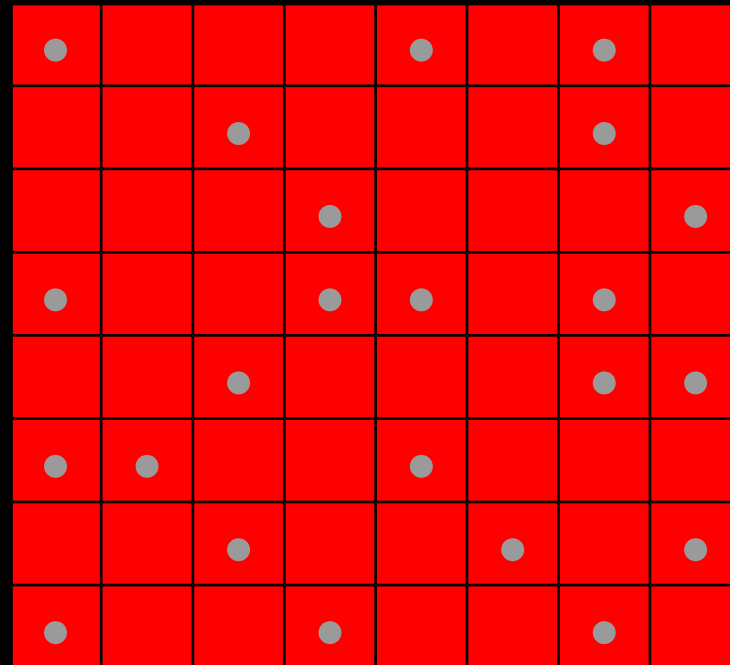
$$p_{fa} = 1 - (1 - 0.1) \cdot (1 - 0.1) = 0.19$$

Exponential increase in detection rate  
Linear increase in false alarm rate

Joint discussions with: Bjarne Haugstad

## Artificial example

- $N=23$  mines
- Method 1 (flail):  
 $P(\text{detection})=0.8$ ,  $P(\text{false alarm})=0.1$
- Method 2 (metal detector):  
 $P(\text{detection})=0.7$ ,  $P(\text{false alarm})=0.1$
- Resolution: 64 cells



How does detection and false alarm rate influence the possibility of gaining by combining methods?



# Confusion matrix for method 1

		True	
		yes	no
Estimated	yes	19	5
	no	4	36

## Confidence of estimated detection rate

- With  $N=23$  mines 95%-credible intervals for detection rates are extremely large!!!!

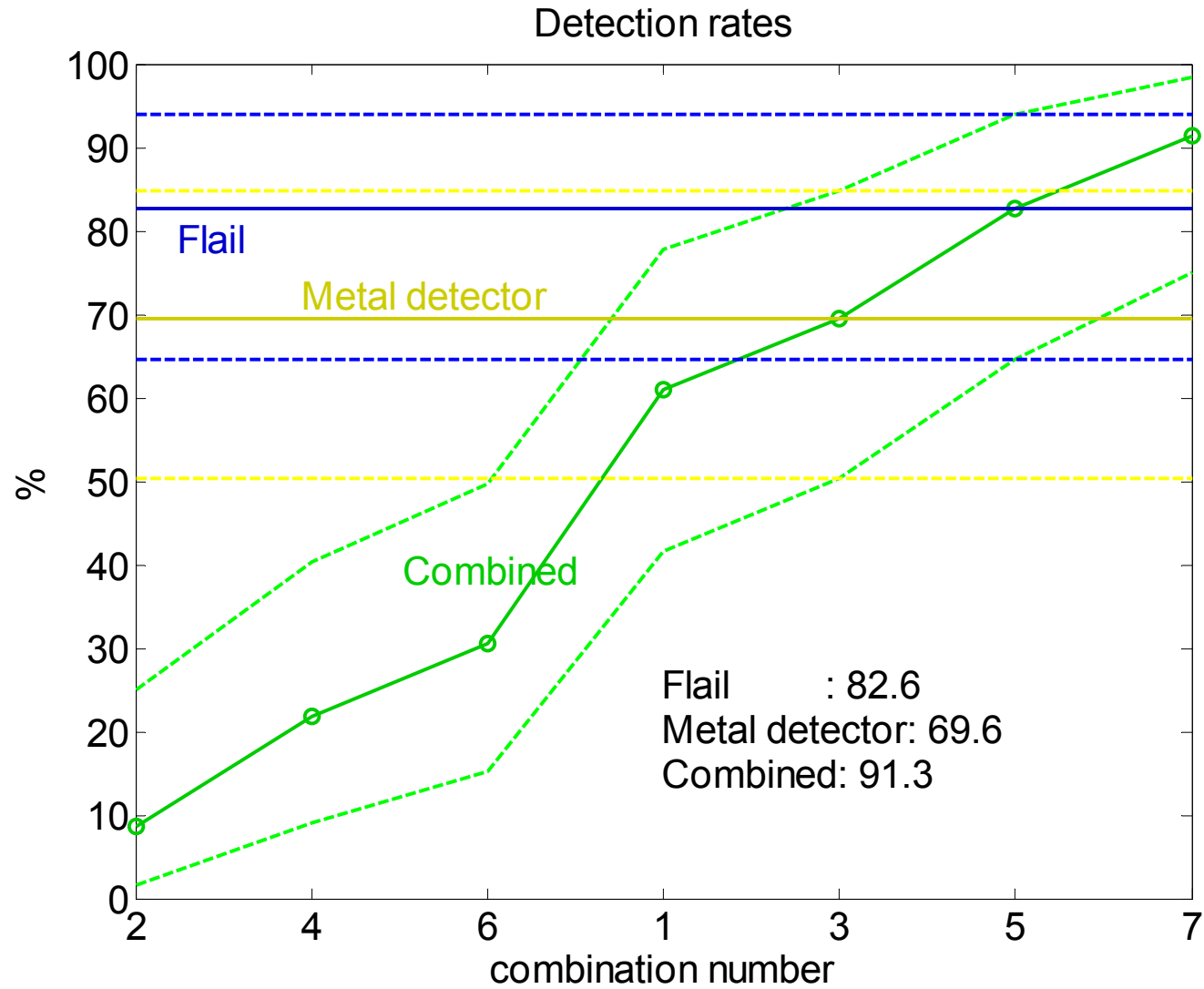
[64.5%    82.6%    93.8%]

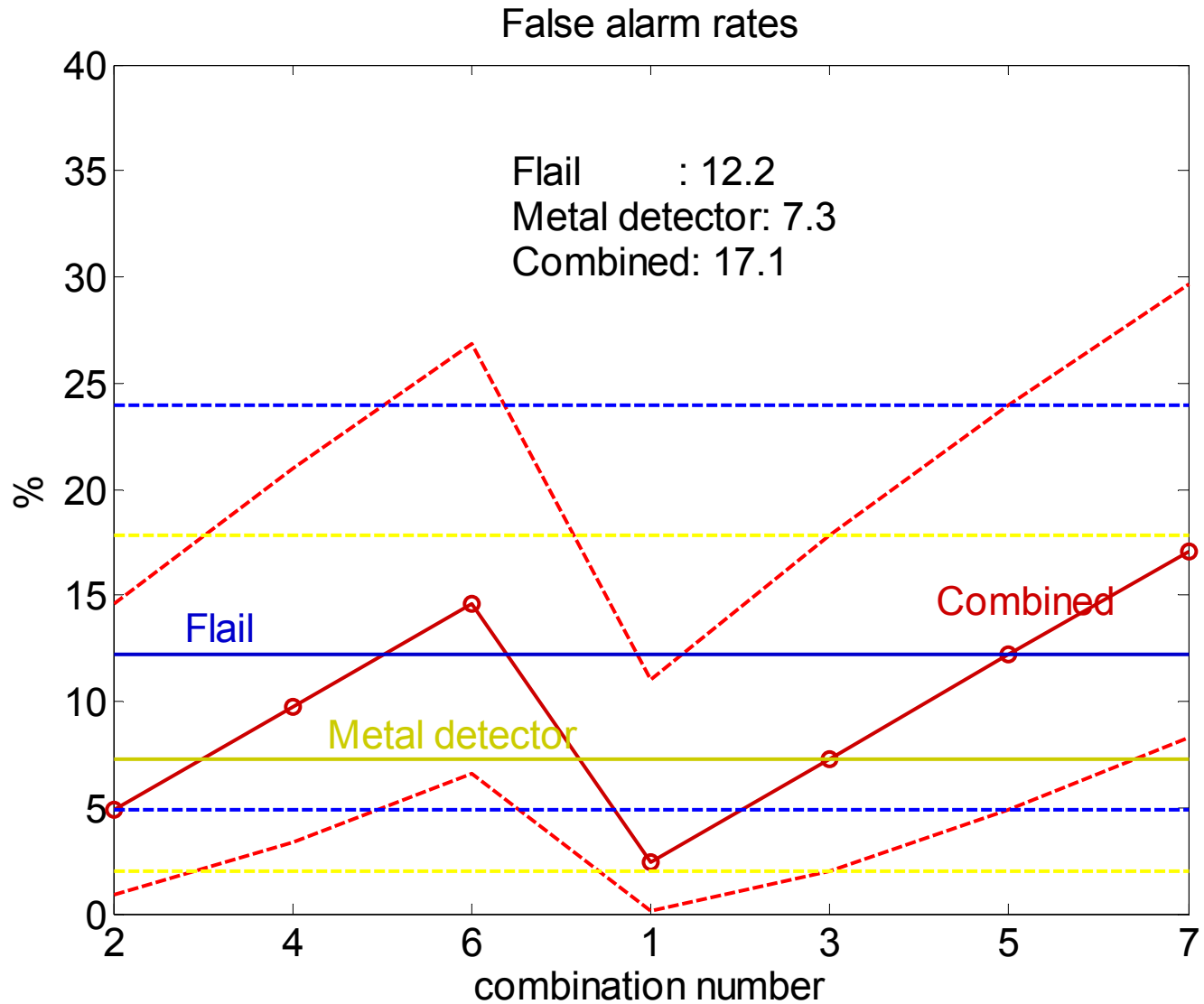
[50.4%    69.6%    84.8%]

## Confidence for false alarm rates

- Determined by deployed resolution
- Large resolution - many cells gives many possibilities to evaluate false alarm.
- In present case:  $64-23=41$  non-mine cells


[4.9%    12.2%    24.0%]






## Comparing methods

- Is the combined method better than any of the two original?
- Since methods are evaluated on same data a paired statistical McNemar with improved power is useful

Method1 (flail): 82.6% < 91.3% Combined 

Method2 (MD): 69.6% < 91.3% Combined 

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# DeFuse

## scientific objectives

- Obtain **general scientific knowledge** about the advantages of deploying a combined approach
- Eliminate confounding factors through **careful experimental design** and specific scientific hypotheses
- Test the general **scientific hypothesis is that there is little dependence** between missed detections in successive runs of the same or different methods
- To accept the hypothesis under **varying detection/clearance** probability levels
- To lay the foundation for new practices for mine action, but it is **not within scope** of the pilot project



# Xsense

- The scope of the Xsense program is to realize a reliable, sensitive, portable and low-cost explosive detector
- The detector will be miniaturized and will therefore be highly suitable for use in anti terror efforts, boarder control, environmental monitoring and demining
- The sensitivity will be optimized by a concentrated effort in data processing (reducing noise and pattern recognition) and emerging sensing principles
- The reliability of the detector will be ensured by combining several independent sensor technologies

# Conclusions

- A cross-disciplinary effort is required to obtain sufficient knowledge about physical, operational and processing possibilities and constraints as well as clear definition of a measurable goal – **the right tool for the right problem**
- Statistical decision theory and modeling is essential for optimal use of prior information and empirical evidence
- It is very hard to assess the necessary high performance which is required to have a tolerable risk of casualty
- The use of sequential information aggregation is promising for developing new hierarchical survey schemes (SOPs)
- Combination of methods is a promising avenue to overcome current

