The Aristotelian Rainbow: From Philosophy to Computer Graphics

Jeppe Revall Frisvad, Niels Jørgen Christensen, and Peter Falster

Technical University of Denmark

Figure 1: *A rainbow rendered in real-time using Aristotle's theory for rainbow formation.*

Abstract

Developments in the graphics discipline called realistic image synthesis are in many ways related to the historical development of theories of light. And theories of light will probably continue to inspire the ongoing search for realism in graphics. To nurture this inspiration, we present the first in-depth, source-based historical study that pinpoints events with relevance for graphics in the development of theories of light. We also show that ancient mathematical models for light scattering phenomena may still find a use in the branch of realistic image synthesis concerned with real-time rendering. As an example we use Aristotle's theory of rainbow formation to construct a method for real-time rendering of rainbows. This example serves as an invitation to use the overview and references provided in this paper, not only for understanding where many of the physical concepts used in graphics come from, but also for finding more mathematical and physical models that are useful in graphics.

CR Categories: I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism

Keywords: History, philosophy, realistic image synthesis.

1 Introduction

On immediate inspection the ancient theories of light would seem to have very little to do with the very modern phenomenon of synthesising life-like images on a computer screen. Nevertheless, we hope that you will discover in this paper how the development of various theories of light has many things in common with the development

of algorithms for producing photo-realistic computer imagery.

Realistic image synthesis is a research field which appeared at a very late point in history. At least this is true if we exclude paintings and only consider images rendered on a computer. At the time where this branch of computer graphics emerges, the quantum theory of light is able to explain every known detail of light's behaviour. And the behaviour of light is exactly what we need to simulate if we are to compute the appearance of scenery. From the outside it may then seem like a paradox that graphics research has never used the most exact theory of light. Here is the reason why: the history of theories of light embraces a long period of time in which physicists have strived to understand nature in increasingly fine detail. The finer the detail, the more complicated the complete picture. However in order to do computer graphics, we have to model the complete picture. There is no way around it, and it will probably result in a very crude model, but for every object in an image and every source of light we need a mathematical model to start from. Creating a realistic image from these artificial models quickly becomes an immensely complicated thing to do. If we want to see the result before the computer melts down, we have to start with a simple theory of light. Thus as computers grew more powerful, graphics research incorporated more and more detail and developed in a way somewhat similar to the development of theories of light.

To follow the development of theories of light, let us begin by looking at the first texts that have survived, in a more or less corrupted version, from ancient times (Sec. 2). Without doubt there have been theories of light before these, but many manuscripts have been lost [Smith 1999]. There are actually many parallels between this first groping towards an understanding of vision and the most common rendering algorithms for computer graphics. From antique theories we move on to more recent wave and radiation theories (Sec. 3). These, especially radiative transfer theories, are being used more and more often in graphics. The historical development of quantum theories is not covered because the use of these theories is too far away from the current state of graphics. Then we give a short account of the developments in realistic image synthe-

sis (Sec. 4) and so as to exemplify how we can exploit the insight that realistic rendering is related to theories of light, we conclude the paper by demonstrating that Aristotle's theory of rainbows provides an easy way to render rainbows in real-time (Sec. 5). The result of such a rendering is shown in Figure 1.

2 Ray Theories

The stories about the early Greek philosophers compiled by Diogenes Laërtius [~200 A.D., 1901], provide an opportunity to get an understanding of the philosophy leading to the first theories of light. Reading Laërtius' account of the theories of Pythagoras (c. 575 – c. 495 B.C.), we understand that the light from the sun was thought of as a source of heat and life rather than a direct cause of human vision. Laërtius writes that one of Pythagoras' theories was [Laërtius ~200 A.D., 1901, p. 349]:

> that the sun, and the moon, and the stars, were all Gods; for in them the warm principle predominates which is the cause of life. [...] Moreover, that a ray from the sun penetrated both the cold aether and the dense aether, and they call the air the cold aether, and the sea and moisture they call the dense aether. And this ray descends into the depths, and in this way vivifies everything.

Laertius explains Pythagoras' theory of the senses on this basis. Since man is alive, he contains the warmth received through rays of light from the sun. By emanating vapour of excessive warmth from the eyes, he is allowed to see through air, and through water. Laërtius tells us that Pythagoras "calls the eyes the gates of the sun".

This very early theory describes an indirect relation between light and sight (and heat). If there is no light, we do not receive heat and consequently have no excess warmth by which we can gather impressions using our eyes. Since everything which emits light also emits heat, it is easier to understand why it took several centuries before it was finally concluded that vision is not caused by rays from the eyes. This does not mean that the ancient Greek philosophers did not discuss the possibility of the eye playing only a passive role as a receptor of visual impressions. Very early on, such a theory was put forth by Democritus (c. 460 – c. 375 B.C.). According to Theophrastus (c. 371 – c. 287 B.C.), Democritus explains vision by a reflection or image in the eye as follows [Theophrastus ~300 B.C., 1917, §§50–51]:

> the air between the eye and the object of sight is compressed by the object and the visual organ, and thus becomes imprinted (*typousthai*); since there is always an effluence of some kind arising from everything. Thereupon, this imprinted air, because it is solid and of a hue contrasting [with the pupil], is reflected in the eyes, which are moist. [...] Democritus himself, in illustrating the character of the "impression", says that "it is as if one were to take a mould in wax".

What Democritus describes seems most of all akin to a mechanical process. His description does not involve light. It is more like "a sort of stamping-process, the result of which can be seen in the images reflected at the cornea's surface" [Smith 1999, p. 25].

This explanation of vision was not Democritus' own personal opinion, but rather Democritus' way of describing the supposition of a number of presocratic thinkers referred to as the natural philosophers. To counteract it, Plato (c. 427 – c. 347 B.C.) and Aristotle (384 – 322 B.C.) had to go through some trouble to explain why light plays a part in the workings of vision (see for example Plato's *The Republic*, 507c–508b). Plato was influenced by the Pythagoreans and Aristotle was a student of Plato. So in a way they developed the thoughts of the Pythagoreans to a more advanced state. The argument of Plato and Aristotle as to why light must have a role to play in vision, is really quite simple. Essentially the argument is that we can look at a colourful object and even if we do not change our way of looking, the object can still loose its colour. Conclusively there must be a third thing, outside object and eye, influencing our ability to see. This third thing is, of course, light.
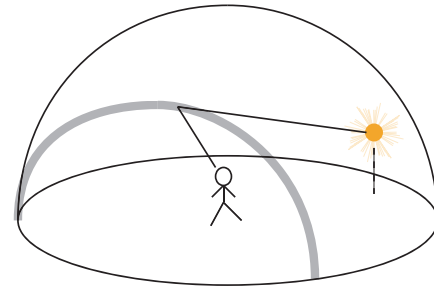

Figure 2: *An illustration of Aristotle's rainbow theory. Clouds on a hemisphere resting on the circle of the horizon reflect sunlight to the observer where the angle is equal (to some constant angle).*

In his later work, Plato presents a theory of vision which is a pleasant compromise between the previous theories. In *Timaeus* [~360 B.C., 1989, 67c] he writes that "colours [...] are a flame which emanates from every sort of body, and has particles corresponding to the sense of sight". This is quite analogous to the account of Democritus, but he also gives the following account of how vision works [Plato ~360 B.C., 1989, 45b–45d]:

> When the light of day surrounds the stream of vision, then like falls upon like, and they coalesce, and one body is formed by natural affinity in the line of vision, wherever the light that falls from within meets with an external object. [...] But when night comes on and the external and kindred fire departs, then the stream of vision is cut off; for going forth to an unlike element it is changed and extinguished, being no longer of one nature with the surrounding atmosphere which is now deprived of fire: and so the eye no longer sees, and we feel disposed to sleep.

The interesting development is that light is more directly involved in the process in Plato's account of vision. It is also interesting to note that Plato refers to a stream of vision as "the light from within". The meaning of light is changing. It is no longer only thought of as the life-giving fire emanated from the sun. Aristotle makes this change of conception more clear by saying that light is not an emanation from the sun or the eye, but rather an instantaneous thing which exists when the potentially transparent (e.g. air and water) is actually transparent (or "is excited to actuality" as he puts it) [Aristotle ~350 B.C., 1941, II:7].

What we have discussed so far are the different theories of the antiquity which try to explain the physics behind visual perception. It is evident that at this point the concept of light is in a far too fuzzy state to enable mathematical treatment of the subject. The theory of vision is, however, an entirely different matter. It is easy to follow the line of sight and realise that we can describe it in a mathematical way. This is the subject of *optics* which was already well developed at the time of Aristotle [Smith 1999]. Book III of Aristotle's *Meteorology* [~350 B.C., 1984] demonstrates quite advanced thoughts. He explains the appearance of halos and rainbows by considering reflection of the line of sight in mist around the sun (for halos) and clouds just before they condense into rain (for rainbows). After arguing that these phenomena are the result of reflection, he uses the idea that the angle between the line of sight and the direction from the cloud to the "luminous body" should be equal wherever the rainbow or halo is seen. Using this principle and placing clouds on "a hemisphere resting on the circle of the horizon", he is able to explain the appearance of halos and rainbows in a mathematical way. Figure 2 illustrates the rainbow theory.

Unfortunately no manuscripts have survived from the initial phase of optical studies. Euclid's *Optics* [~300 B.C., 1945] and *Catoptrics* [~300 B.C., 1895] are the oldest surviving works dedicated entirely to optics. The principles of *perspective* are established and the perceptual distortions resulting from our conical vi-

sion (as well as a few propositions on binocular vision) are considered in the *Optics*. The *law of reflection* is the first proposition of the *Catoptrics* [Euclid ∼300 B.C., 1895, p. 287]:

> Rays are reflected at equal angles by plane, convex, and concave mirrors. (Modern version: *The reflected ray lies in the plane of incidence; the angle of reflection equals the angle of incidence.*)

Using this proposition a number of effects resulting from reflections in concave and convex mirrors were derived by Euclid.

With optics at such an advanced state only relatively shortly after Aristotle's *Meteorology*, we might wonder whether Aristotle knew about the law of reflection or not[1]. He does use a principle of equal angles as the key to describe halos and rainbows by reflection off mist and clouds (respectively), but on first inspection it seems different from the Euclidean proposition. We do not believe it is a different principle. Suppose Aristotle's derivations are based on the fact that the orientation of the cloud surface, where the visual ray impinges, is unknown. This is a perfectly reasonable assumption because his writings very elegantly avoid having to specify this orientation. What we have to assume then, to make the proofs work with the correct law of reflection, is that the cloud (or mist) surfaces have the same orientation towards the sun across the entire hemisphere. If this is kept in mind when reading Book III of *Meteorology*, the explanations make a lot more sense (in our opinion). That the cloud surface is able to exhibit this unusual behaviour is explicable by its particulate nature, the same nature which Aristotle uses to explain why we do not see a perfect reflection of the sun in the cloud.

Although optics started out being a theory of vision rather than a theory of light, developments took an interesting turn when the mathematicians took an interest in burning mirrors. The earliest known work on this subject is Diocles' treatise *On Burning Mirrors* [∼190 B.C., 1975]. It treats the focusing properties of parabolic sections. This means that it had been realised that rays of light from the sun follow the same general rules as rays of sight from the eyes. The theory of light is getting less fuzzy. By applying the rules of optics to rays of light, it can be established that light moves in straight lines and that the law of reflection is also valid for rays of light.

The supposition that visual flux issues forth from the eyes persists. But in light of Diocles' work, we are allowed to believe that what is true for rays of sight is equally true for rays of light. A few centuries later, a new insight into the behaviour of rays is obtained by Hero (or Heron) of Alexandria. In his *Catoptrics* [∼50 A.D., 1900] Hero uses an arrow as an example and says that "because of the impelling force the object in motion strives to move over the shortest possible distance, since it does not have the time for slower motion, that is, for motion over a longer trajectory. And so, because of its speed, the object tends to move over the shortest path" [Smith 1999, §2.6]. Then he observes that "the rays emitted by us travel at an immeasurable velocity" as "it takes no perceptible time for [them] to reach the heavens". The implicit conclusion is that as rays travel at an almost infinite speed, they do not only strive to take the shortest path, they have to take it. This principle that rays take the path of minimum distance is now known as *Hero's Principle*. Hero himself uses it in his *Catoptrics* to demonstrate the law of reflection.

With respect to refraction, Hero's *Catoptrics* [∼50 A.D., 1900] only attempts an explanation of why light is partially reflected and partially transmitted into water and glass. Another century had to pass before proper treatment of refraction was to be presented by Ptolemy (c. 100 – c. 178 A.D.) in his *Optics* [∼160 A.D., 1996]. From a physical point of view, the work of Ptolemy is immensely important because he makes extensive use of carefully contrived

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[1]Some authors, e.g. Smith [1999], say that Aristotle's writings violate the law of reflection, but this is not necessarily so.

experiments to support his arguments. This signals the beginning of a new era where pure philosophical reasoning is not necessarily the main authority. Ptolemy's measurements of the angle of refraction are surprisingly exact. He found, in Book V of the *Optics*, the angle that rays make when moving "from rarer and more tenuous to denser media" (i.e. from air to water to glass) and the other way around, and he was able to describe this behaviour qualitatively, however he did not succeed in formulating the mathematical law of refraction. Ptolemy's *Optics* contains many fine results and we can certainly think of it as the culmination of ancient mathematical optics.

Ptolemy also writes qualitatively about the *shading* of objects as depending on the angle of the incident rays. In Book II of his *Optics*, he explains concepts which are surprisingly similar to diffuse and glossy reflections of light. He writes [Ptolemaeus ∼160 A.D., 1996, §§18–19 (pp. 76–77)]:

> everything that falls orthogonally strikes its subjects more intensely than whatever falls obliquely. Also, what is polished is seen more clearly than what is rough, because there is disorder in a rough object resulting from the fact that its parts are not arranged in a regular way. But the parts of a polished object have a certain regularity, and [so] brightness is inherent to it.

After Ptolemy the development in optics and theories of light is almost stagnant for several centuries. The only thing to mention is a description of *colour bleeding* by Galen (c. 130 – c. 200 A.D.). Colour bleeding is the phenomenon where light is tinted by the colour of nearby surfaces due to diffuse reflections. In the words of Galen [∼180 A.D., 1984, §7.7]: "when a person reclines under a tree [...], you can see the color of the tree enveloping him. And often when bright air touches the color of a wall, it receives the color and transmits it to another body, especially when the wall is blue or yellow or some other bright hue". By "bright air" Galen probably means air excited by light.

Perhaps the downfall of the Roman Empire was the reason for the period of stagnation after Ptolemy. The prosperity of the Abbasid Caliphate which followed moved the scientific lead to the Arab world. In the second half of the 9th century, the Arabs start contributing to optics and theories of light. Ya'qūb al-Kindī (c. 801 – c. 873) addresses a subject which also troubled Ptolemy somewhat. Ptolemy had problems with the Euclidean idea that rays are distributed discretely in the visual cone. He opted that "the nature of the visual radiation is perforce continuous rather than discrete" [Ptolemaeus ∼160 A.D., 1996, II, §50]. Still he treated rays as "virtually discrete" [Smith 1999] such that he could follow their rectilinear trajectories in reflection and refraction. What Ptolemy was groping for, was the concept of solid angles and their differentials, but the mathematics available to him were not sufficiently sophisticated. The idea that radiation is spread in a continuum over solid angles is, however, important and it is further developed by al-Kindī. In his optical treatise [Al-Kindī ∼870, 1997] called *De aspectibus* in Latin, al-Kindī analyses the spread of radiation from a point source and writes: "what lies closer to [the] point is more intensely illuminated than what lies farther from it" [Smith 1999, p. 162].

After al-Kindī this development gathered momentum and Ibn Sahl (c. 940 – c. 1000) composed an impressive work *On the Burning Instruments* [Ibn Sahl ∼984, 1993]. In this he finds the *law of refraction*. If θ_1 and θ_2 are the angles formed by the normal of a plane surface and (1) the refracted light ray in a crystal and (2) the ray in the air, then [Rashed 1990, p. 478]:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{1}{n} \ ,$$

where n is the reciprocal of what we today would refer to as the relative index of refraction. With this law Ibn Sahl is able to couple

the theory of burning mirrors as described by Diocles [∼190 B.C., 1975] with the theory of refraction as advanced by Ptolemy [∼160 A.D., 1996, V]. This led him to the first specifications of lenses.

Had the concept of lenses been in place, it might have been easier to comprehend that the eye really works as a passive sensor of light. Not long after Ibn Sahl's work on burning lenses the renowned Arab scientist Ibn al-Haytham (965–1039), known to Europeans as Alhacen, incorporated a peculiar type of lens in his model of the eye and dedicated the entire first book of his *Kitāb al-Manāẓir* ("Book of Optics") [Ibn al-Haytham ∼1016, 2001] to the discouragement of the theory that vision issues forth from the eye (especially confer the conclusive line of arguments in §§6.45–6.60 of the reference). Even though Ibn al-Haytham opposes the theory of visual rays, he also explicitly makes it clear that all the mathematical results involving rays of sight are still true, but in reality the rays consist of light travelling in the opposite direction. In the spirit of Ptolemy's *Optics*, Ibn al-Haytham's *Kitāb al-Manāẓir* comprises seven books covering all aspects of optics known at that time. He also carries Ptolemy's extensive use of experimentation further and uses it for verification of his theories throughout his treatise. Clearly al-Haytham's work is monumental in optics and upon its translation into Latin (c. 1200), it spawned renewed interest in the field.

Despite the efforts of al-Haytham, the new western scientific works on optics did not immediately discard the Greek tradition involving rays of sight. Perhaps the reason was the peculiar lens in al-Haytham's eye model, which he describes as only being sensitive to rays of light striking the surface of the lens orthogonally [Ibn al-Haytham ∼1016, 2001, I:7]. At the beginning of the 17th century, Johannes Kepler [1604, 2000] finally brought an end to the theories involving rays of sight. This was done by demonstrating that the lens of the eye is a perfectly ordinary lens merely serving the purpose of drawing an upside down image of what we are looking at point-by-point on the retinal screen behind the eye (the fact that the resulting image is upsidedown led al-Haytham to form his lens with special sensitivity). With Kepler's work the scene is set for further investigation into the nature of light.

Unfortunately the work of Ibn Sahl had not been translated into Latin, so the Europeans had to reinvent the sine-law of refraction. Kepler [1611] found an approximation of the law and discovered the existence of *total internal reflection*, which is the phenomenon that light cannot refract out of a dense transparent medium (e.g. glass) at a grazing angle, instead it will only reflect internally. According to Kwan et al. [2002], Thomas Harriot had already discovered the sine-law in 1602 and, likewise, Willebrord Snel van Royen (Latinized as Snellius) reinvented the law of refraction in 1621, but neither of the two published their results. The law was first published by René Descartes in his *Discourse on Method* containing a scientific treatise on optics [Descartes 1637, 2001]. Nevertheless the law of refraction is today called *Snell's law*.

Descartes [1637, 2001, pp. 65–83] explains refraction by thinking of light as particles on which different friction-like forces act. The forces depend on the type of media which the particles are moving from and to. For Descartes' arguments to fit the experimental behaviour of light, he must draw the rather peculiar conclusion that light is received more easily by water (and even more easily by glass) than by air. To Pierre de Fermat this explanation was not convincing. Rather he felt that there should be a minimum principle from which the law of refraction can be derived [Fermat 1891–1912, pp. 354–359, letter from Fermat to De la Chambre, 1657], just like Hero used his principle of shortest path to derive the law of reflection. After putting his mind to it, Fermat [1891–1912, pp. 457–463, letter from Fermat to De la Chambre, 1662] found that he was able to derive the law of refraction from precisely such a principle. Conclusively he writes:

> my principle, and there is nothing that is as probable and as apparent as this proposition, [is] that nature always acts by the easiest means, or, in

> other words, by the shortest paths when they do not take longer time, or, in any case, by the shortest time

In short, *Fermat's Principle* is that light follows the path of least time. This is a very powerful principle by which many things can be predicted, among them the laws of reflection and refraction. Today we know that light takes the path along which the time of travel is an extremum compared to neighbouring paths, and the wording of Fermat's Principle has been adjusted accordingly in modern books on optics. The extremum is, however, usually a minimum and consequently Fermat's original formulation is true in most cases. In a homogenous medium (where the index of refraction is everywhere the same) the speed of light is the same everywhere. The path of least time is then also the shortest path. Thus Hero's Principle is a special case of Fermat's Principle.

This concludes our discussion of ray theories of light. Many of the ideas and principles from the two millennia of history that we have now discussed are indispensable in computer graphics today. They are the backbone of most photo-realistic rendering algorithms. In particular we use: the fact that rays of light move in straight lines in homogeneous media; the laws of reflection and refraction; total internal reflection; the concepts of shading and colour bleeding; the concept that the energy carried by light is spread over solid angles; and Fermat's Principle by which we are able to find the path of light in heterogeneous media (where the speed of light may change throughout the medium).

3 Wave and Radiative Transfer Theories

The work of Aristotle was widely read and quite influential in the the seventeenth century [Shapiro 1973]. As mentioned previously, Aristotle thought of light as an excitation of potentially transparent media rendering them actually transparent. This idea caused many seventeenth century scholars to seek a theory in which light is a state propagating through a continuum (rather than particles following straight lines).

Inspired by Aristotle, Kepler [1604, 2000, Chapter 1] promotes the view that rays of light are merely a geometrical representation of what, physically, is a luminous spherical surface propagating from the centre of a light source. Following the same tradition, but carrying the concept further, Hobbes [1644, Prop. 4] writes "a ray is, in fact, a path along which a motion is projected from the luminous body, it can only be the motion of a body; it follows that a ray is the place of a body, and consequently has three dimensions". Elaborating on the concept that rays of light are three-dimensional (parallelograms), Hobbes is able to derive the law of refraction without having to make the the same counterintuitive assumption as Descartes (which was that light is easier received in glass than in air). Hobbes [1644] refers to the front of his rays as "propagated lines of light" and states that the width of the ray should be taken to be "smaller than any given magnitude" [Shapiro 1973]. This shows how remarkably similar Hobbes' concept of solid rays is to infinitesimal portions of an expanding wave.

The works of Hobbes had considerable influence on subsequent theories of light. In an attempt to explain the *interference* colours of thin films, Robert Hooke [1665] proposes a peculiar mixture of Descartes' and Hobbes' theories of light. He uses Descartes' way of explaining refraction (and assumes that light moves faster in water than air), but he uses Hobbes' concept of solid rays, only he calls them light pulses. Hooke qualitatively arrives at the right conclusion about interference, namely that the colours of thin films are caused by reflection beneath the transparent film layer resulting in a delayed (weaker) pulse being "confused" with the pulse reflected at the surface. Hooke is, in other words, able to explain interference phenomena by treating the "propagated line of light" as the surface of constant phase. This clearly speaks in favour of a wave theory of light.

About the same time as Hooke investigates interference colours,

Francesco Maria Grimaldi [1665, Book I, Prop. 1] observes that the path of light not only differs from a straight line when it is reflected or refracted, but also "when parts of light, separated by a manifold dissection, do in the same medium proceed in different directions". In other words, if you shine light at a very finely sliced object (a manifold dissection), you will observe light in the geometrical shadow. He calls this phenomenon *diffraction* and finds that it is best explained if light is thought of as a very fluid and very subtle substance.

Only a few years later Isaac Newton [1671] finds the correct explanation for the spectrum of colours seen when light is refracted through a prism. This phenomenon is called *dispersion* and it is due to the fact that, in the words of Newton [1671, p. 3079], "Light itself is a *Heterogeneous mixture of differently refrangible Rays*". In other words, Newton observes that each ray is disposed to exhibit only one particular colour and when rays of all the primary colours are mixed in a "confused aggregate of rays" light attains "whiteness" [Newton 1671, p. 3083]. Newton uses a cunning experiment to illustrate his theory. If he lets sunlight pass through a single prism, he sees a spectrum on the wall. But using a second prism after the first one, he sees light which is no different from the light coming directly from the sun.

During the same period of time, a strange phenomenon, which we today call *birefringence*, was discovered by Rasmus Bartholin [1670]. In his experiments with the crystal called Iceland spar, he observes that not only the ordinary image predicted by Snell's law, but also an "extraordinary" image is seen through the crystal. Bartholin regards this remarkable phenomenon to be a property of the crystal alone. It is, however, discovered a few years later that the experiment says quite a lot about the nature of light as well.

Yet another property of light was ascertained in this period. While Descartes (in the Aristotelian tradition) was of the opinion that light is an instantaneous thing. Others such as Hobbes and Grimaldi (like Hero of Alexandria), were of the opinion that light travels at a finite, but imperceptible, velocity. By actually giving an empirically based estimate of *the speed of light*, Ole Rømer [1676] finally discounted the hypothesis that light is an instantaneous thing.

Many of the properties that have been discovered at this point (interference, diffraction, finite speed) lead towards a wave theory of light. In 1678 Christiaan Huygens completes his *Traité de la lumière* [1690] in which he presents a wave theory of light based on the theory of sound waves as it was known at the time. Huygens assumes that every particle of a luminous body emits a spherical wave. Moreover he enunciates the principle that every element of the wave fronts also gives rise to a spherical wave, and the envelope of all these secondary waves determines the subsequent positions of the wave front. This principle is today named after him [Born and Wolf 1999] and with it he is able to explain not only the laws of reflection and refraction, but also the extraordinary refraction in Iceland spar. However, letting light pass through a sequence of two Iceland spar, Huygens discovers that the waves of light change. They "acquire a certain form or disposition" [Huygens 1690, p. 94] because when the second crystal is in a certain position the two wave fronts emerging from the first crystal are not split again. In this way Huygens discovered *polarisation* of light, but he was not able to explain it theoretically.

With all these newly found properties of light and a wave theory ready for action, things take an unexpected turn. Enthused by his explanation for dispersion, Newton publishes his *Opticks* [1704] where he advocates strongly in favour of a ray theory of light. Two theories are then available: Newton's theory of "differently refrangible rays" and the wave theory of Huygens which explains birefringence. But being strongly in favour of a ray theory, Newton attempts, in a set of queries added in the first Latin version of the *Opticks* (1706), to give a ray-based explanation of birefringence

and polarisation. His explanations are incorrect, but fact is that Newton's work became highly influential in the eighteenth century while Huygens' treatise was almost forgotten.

Concerning another aspect of light, namely how the intensity of light changes under different circumstances as observed by Ptolemy (shading) and al-Kīndi (spread of radiation), there is no major development for several centuries. What is missing, in order to develop the subject quantitatively, are the means to measure the intensity of light. Such means are discovered by Pierre Bouguer in 1725 [Middleton 1964]. He invents a *photometer* by letting light from the source that he wants to measure the intensity of, fall on a screen. He compares this light to light falling on the same screen from a number of candles. By changing the distance of the candles to the screen, he is able to adjust the intensity due to the candlelight until it fits the other light. He uses the fact that the intensity of light is proportional to the inverse of the square of the distance from the source, and thus he is able to measure the light intensity in terms of candles. Bouguer's description of the technique is available in his *Essai d'optique* [1729]. We are uncertain whether Bouguer was the first to describe the fact that light falls off with the the square of the distance to the source (the inverse square law for radiation), but he was the first to describe the formulae for finding the illumination I at a distance r from a source of intensity I_0 in a partially transparent medium [Bouguer 1729]. The modern form of the law is [Middleton 1964]:

$$ I = I_0 r^{-2} e^{-\sigma_t r} \ , $$

where σ_t is the *extinction coefficient* of the semitransparent medium. If we consider a collimated beam of light the inverse square of the distance, of course, does not appear in the formula. The law stating the exponential falloff in intensity, $I = I_0 e^{-\sigma_t r}$, for a collimated beam, is often referred to as Beer-Lambert's law. The correct name is *Bouguer-Lambert's law*.

The contribution of Johann Heinrich Lambert [1760] to Bouguer-Lambert's law is that he gives it a mathematical formulation using logarithms. His *Photometria* [Lambert 1760] is an impressive work in this new field of research founded with Bouguer's photometer. Lambert [1760] also finds the *cosine law* which says that light reflected by a perfectly diffuse surface (also called a *Lambertian surface*) decreases in intensity with the cosine between the surface normal and the direction towards the incident illumination. This is the quantitative description of Ptolemy's observations about the shading of rough surfaces.

In the middle of the eighteenth century, but without reference to Huygens, Leonhard Euler [1746] gives a wave-based description of dispersion. This is accomplished by realizing that the colour of a light pulse is determined by its frequency. The next sign of weakness in the Newtonian ray theory appears in 1788 when René-Just Haüy investigates the birefringence of Iceland spar and finds a definite disagreement with Newton's results, but better agreement with the results of Huygens [Shapiro 1973]. When Thomas Young [1802] qualitatively explains the colours of thin films using the principle of interference between waves, the wave theory gets the upper hand. Especially as Huygens' explanation of double refraction is confirmed by both William Hyde Wollaston [1802] and Étienne Louis Malus [1810]. Malus also discovers a previously unknown property of light which is that reflection causes polarisation.

With two very different theories striving for supremacy, the supporters of the Newtonian ray theory "proposed the subject of diffraction for the prize question set by the Paris Academy for 1818" [Born and Wolf 1999, p. xxvii]. To their dissatisfaction, the prize went to Augustin Jean Fresnel [1816] who, by an impressive synthesis of Huygens' envelope construction and Young's principle of interference, managed to overcome some theoretical difficulties in the previous wave theories and was also able to explain the diffraction phenomenon.

At the same time Fresnel was working on polarisation in cooperation with Dominique François Arago. They found that waves polarised at right angles to each other never interfere [Levitt 2000]. With this information Fresnel realized that the waves must be transverse rather than longitudinal and in 1821–1822 he presents three *Mémoires* [Fresnel 1827] in which he uses transverse waves to explain the birefringence and polarisation observed in crystals by Bartholin and Huygens. Shortly after these very strong arguments in favour of the wave theory of light (in 1823) Fresnel gives the ray theories the final blow: He presents formulae finding the intensities of the reflected and refracted waves and he even includes polarisation in these formulae [Fresnel 1832]. In this way he is able to explain Malus' observation that reflection causes polarisation. The *Fresnel equations*, as they are called today, are still used extensively.

At the beginning of the nineteenth century John Leslie [1804] firmly establishes that "heat and light are commonly associated". Thus when Julius Robert Mayer [1842] finds a relation between heat and mechanical energy and when James Prescott Joule [1843] subsequently discovers a similar relationship between heat and electromagnetic energy, there is "only" one step missing in the connection between heat, electromagnetism, and waves of light. This step is taken by James Clerk Maxwell [1873] who puts forth his famous theory of the electromagnetic field and gives substantial theoretical evidence to the fact that light waves are electromagnetic waves. His theory relies on the assumption that the speed of an electromagnetic wave, within experimental error, should be the same as the speed of light. Heinrich Hertz [1888] later verifies this conjecture by direct experiment.

Another theory was initiated at the beginning of the century when Young [1802] suggested that the eye most probably has three types of "fibres", each only sensitive to one of three different "principal colours". While first abandoned, this theory is revived by Hermann von Helmholtz [1867] who records three curves over wavelengths which represent the light sensitivity of each cone receptor in the eye. Each cone represents one of the three principal colours: Red, green, and blue. The colours that we see are according to this theory (which is still generally thought to be true) a mix of the three principal colours weighted according to the wavelengths in the incident illumination. Today this theory of trichromatic colour vision is sometimes referred to as *Young-Helmholtz theory*.

With respect to quantitative theories of light scattering, John William Strutt [1871], who was later the third Baron Rayleigh, is able to explain the colours of the sky using very simple arguments involving scattering of light waves. Assuming (as others before him) that the atmosphere has a suspension of particles which are very small compared to all the visible wavelengths, Rayleigh finds that for particles of this size, the ratio of the intensity of scattered to incident light varies inversely as the fourth power of the wavelength. This means that the shorter blue wavelengths are scattered more frequently in the atmosphere than the longer red wavelengths, and this is the cause of the blue sky and the red sunrises and sunsets. This type of scattering is today referred to as *Rayleigh scattering*.

A few decades later, a more general result describing the scattering of plane waves of light by spherical particles was derived by Ludvig Lorenz [1890]. Later Gustav Mie [1908] derives the same equations once again, but he uses Maxwell's electromagnetic field instead of a simpler wave equation to represent the light waves and he also provides experimental verification. This theory of light scattering which is useful for deriving the scattering properties of many different materials, is today referred to as *Lorenz-Mie theory*.

Many things have been said at this point about the nature, propagation, absorption, emission, and scattering of light, but we still lack a way to combine all these ideas. What is missing, is a theory describing the flux of radiation that would be found in some particular direction at some point in a scattering medium as the result of some incident illumination progressing through the medium. Equations for such treatment of light at a macroscopic, quantitative level were introduced by Arthur Schuster [1905] in order that he could take scattering into account when considering an atmosphere. Similar equations were given a more elegant formulation by Karl Schwarzschild [1906] in his investigations of the atmosphere of the sun. The equations are a mathematical model describing the phenomenon of scattering rather than they are based on physical foundations such as Maxwell's equations. But they have subsequently been shown to give correct results in most cases. The mathematical formulations of Schuster and Schwarzschild became the birth of the quantitative *radiative transfer theory*.

During the following years the radiative transfer theory developed to a very advanced state with analytical solutions for many special cases. With a series of papers on multiple scattering Subrahmanyan Chandrasekhar was an important influence in this development. In 1950 he published the first definitive text of the field [Chandrasekhar 1950] and it is still today an important reference in all works on the subject.

An abundance of the results that have been discussed in this section are used extensively in computer graphics. The quantitative theories are particularly useful because we have to compute the visual effects due to light scattering in complicated scenarios. It is in other words of great importance that the theories we use work at a macroscopic level. Nevertheless, we see again and again that the wave theory of light must be taken into account for the correct simulation of some visual phenomenon.

4 Realistic Image Synthesis

In the early days of computer science, an image drawn on a screen using a computer was a small miracle. The first computer graphics were created in 1950 by Ben F. Laposky [1953]. He generated artistic works using a cathode ray oscilloscope controlled by an electronic machine. Shortly after (on 20 April 1951) the MIT Whirlwind Computer was demonstrated for the first time. It had a large modified oscilloscope as its screen and was able to display text and graphics in real-time. At this time only very few people actually had access to a computer, but as the technology developed, it was used increasingly for different tasks of computation in companies. The General Motors Research Laboratories used computers for engineering and scientific analyses in 1952, and in 1959 they started implementing a system for Design Augmented by Computers (DAC-1). As part of DAC-1 they started developing hardware for graphical man-machine communication in cooperation with IBM [Krull 1994]. These commercial interests in visualization of three-dimensional design must have made computer graphics research very appealing to young computer science researchers.

At the beginning of the 1960s, research in 3D graphics gathered momentum. Ivan E. Sutherland [1963] presents a system called Sketchpad, which allows the user to draw line drawings on a computer screen interactively using a light pen. His work is extended by Timothy E. Johnson [1963] and Lawrence G. Roberts [1963] who start developing algorithms for displaying line sketches of 3D solid shapes. This way of doing computer graphics is remarkably similar to Democritus' way of explaining vision. To simulate the appearance of an object, we print its outline onto the screen by steering the electron beam of a CRT (Cathode Ray Tube) display monitor. This resembles Democritus' idea that the object would be imprinted in the air reaching the eye.

The early computer displays were modified oscilloscopes and they displayed vector graphics, but relatively quickly the raster techniques known from TV technology became the display technology of choice. This means that the CRT monitor displays an array of dots (or picture elements - pixels) of different intensities. With this development a raster technique for line drawing was needed

and in the years after it had been presented by Jack E. Bresenham [1965], the work of Johnson [1963] and Roberts [1963] was recast to suit the rasterization approach.

Based on the methods for rendering of solids as line drawings, Arthur Appel [1968] and John E. Warnock [1969] take the next logical step when they introduce shaded display of solids. Just as in the optical theories known at the time of Aristotle, Appel introduces a *ray casting* method which, in essence, corresponds to rays of sight moving in straight lines from the eye to a surface. Warnock uses a non-physical version of Lambert's cosine law to shade each polygon differently depending on its distance and orientation towards the light source. Henri Gouraud [1971] notices the "cartoon-like" appearance of these flat shading schemes and introduces continuous shading across the polygons. This is today referred to as *Gouraud shading*, but Lambert actually also investigated this subject in the 18th century when he enunciated his cosine law [Lambert 1760]. What the graphics researchers are investigating at this point is the shading of objects which was also noticed by Ptolemy. The Lambertian-like shading introduced by Warnock captures diffuse objects, but a quantitative model is missing for rendering of specular highlights. Such a model is provided by Bui Tuong Phong [1975]. It is not physically accurate, but it is very efficient and the *Phong model* is famous in graphics today.

Following the Phong model, more physically-based reflection models for rough surfaces were imported to graphics by James F. Blinn [1977]. He even includes the Fresnel equations in his models. But the real breakthrough towards realism in computer-generated images comes when Turner Whitted [1980] introduces *ray tracing*. In analogy with Euclidean optics, rays proceed from the eye (the image plane) and interact with the surfaces they arrive at. When they interact with a diffuse surface, a shading model is used. When they interact with a specular surface, the laws of reflection and refraction are employed and new rays are traced in the specular directions.

Based on ray tracing, several techniques related to the ancient mathematical optics were tested in the first half of the 1980s. Cone tracing [Amanatides 1984], for example, is closely related to the Ptolemaic concept of radiation distributed continuously in the visual cone. Many of the results discovered by both Euclid, Diocles, and Ptolemy in their works on optics and catoptrics, are useful in cone tracing. Distribution ray tracing [Cook et al. 1984] takes into account how radiation is spread at each point of intersection. This corresponds to the spread of radiation investigated by Ptolemy and al-Kindī. A few years later, graphics also find a use for the idea of letting rays issue from the light sources instead of the eyes [Arvo 1986]. At the time it is called "backward ray tracing" as the tracing direction is opposite to the usual approach. This terminology is later abandoned because it easily causes confusion. *Light ray tracing*, as we now call it, allows us to capture light phenomena known as *caustics* more easily. Caustics are the bright highlights that occur when light has been focussed through a lens or a fluid onto a surface. In a way this is similar to al-Haytham and Kepler's discovery that the eye is merely a lens focussing light on the retina. Fermat's principle was introduced for rendering of mirages [Berger and Trout 1990] at a rather late time in graphics.

Being far ahead of his time, Blinn [1982] introduces scattering effects inspired by thermal radiation and the work of Chandrasekhar [1950], and only a few years later the subject of realistic rendering is firmly connected to heat transfer [Goral et al. 1984] and radiative transfer theories [Kajiya 1984]. The methods inspired by heat transfer are referred to as *radiosity* methods in graphics. They capture the colour bleeding effects observed by Galen. The rendering methods based on radiative transfer are the most general methods used in graphics so far.

Towards the beginning of the 1990s the discipline of realistic rendering starts proceeding in two directions: One branch seek-

ing algorithms that are fast enough to allow real-time interaction with rendered scenes, and another branch seeking improved realism without worrying about the time it takes to render it. In the latter branch graphics has continued to move closer to the wave theories of light. The connection between trichromatic (Young-Helmholtz) colour theory and wavelengths has been introduced in graphics [Meyer and Greenberg 1980], and Bouguer-Lambert's law and the Fresnel equations have been incorporated as standard elements in realistic rendering [Glassner 1995]. A rendering method based on simplified wave theory is first considered for graphics by Hans P. Moravec [1981]. This approach is, however, very expensive and subsequent methods based on wave theory have mostly been used to derive local shading models [Kajiya 1985; Bahar and Chakrabarti 1987]. In a slightly different order than the development described in Section 3 (rather following the difficulties in implementation), but not completely off target, we have seen graphics simulations of dispersion [Thomas 1986], interference [Smits and Meyer 1990; Dias 1991], birefringence and polarisation [Tannenbaum et al. 1994], and diffraction effects [Stam 1999]. Even Rayleigh scattering is used for rendering of a realistic sky [Klassen 1987].

More recently the Lorenz-Mie theory has been used [Rushmeier 1995; Callet 1996] for computing the coefficients needed in the realistic rendering methods that are based on radiative transfer theory. This means that we can find macroscopic input coefficients using Maxwell's equations, but we are yet to see a complete rendering method based on the electromagnetic field theory. Of the rendering methods currently in use, the ones closest to the wave theories of light are the ones based on geometrical optics. This type of rendering was introduced by Stam and Languénou [1996]. Geometrical optics is a simplification of Maxwell's equations which assumes that the wavelength of light is so small that we can think of light as rays following trajectories which are not necessarily straight. In other words, the methods currently in use are simply the correct way to handle heterogeneous media using ray tracing. The attempt by Moravec [1981] is to our knowledge still the only attempt on a complete rendering method based on the wave theory of light.

The real-time branch is different in the sense that it more often chooses a compromise between physics and a simplified mathematical model. The Phong model is a good example. It is often the case that old theories, like the ones we have discussed throughout this paper, present a simple, but not entirely physically correct explanation of a light phenomenon. These old mathematical models often give surprisingly good visual results and their simplicity makes them well suited for real-time implementation on modern programmable hardware. This means that we can still find useful mathematical models for computer graphics by digging into the history of the theories of light. As an example we show, in the following section, how Aristotle's rainbow theory makes us able to render rainbows in real-time. Rendering rainbows more physically correctly is certainly not a real-time process. It has been done by Jackèl and Walter [1997] as follows. Lorenz-Mie theory is used to compute how small water drops scatter light. The result is used as input for the radiative transfer equation, which is used for a full volume visualisation of the air containing the water drops. This expensive computation captures the rainbow. Let us see how the Aristotelian theory works.

5 Rendering the Aristotelian Rainbow

Aristotle's [∼350 B.C., 1984] theory for the formation of rainbows is really quite simple. We have discussed it briefly in Section 2, but now we will add a few extra details. Aristotle thinks of the sky as a hemisphere (see Figure 2). This is very similar to the sky domes used in graphics. We draw a sphere with inward facing polygons and map a texture onto it to obtain a sky rendering. If we place a sun on the sky dome, it makes us able to use Aristotle's rainbow

theory.

For every point on the dome that we render, we will know the direction toward the eye $\vec{\omega}$ and the direction toward the sun $\vec{\omega}'$. Aristotle's theory states that the rainbow forms on the hemisphere of the sky where the angle between $\vec{\omega}$ and $\vec{\omega}'$ is equal. Aristotle does not say what the angle is, however from newer rainbow theories we know that $42°$ is a good choice. If the sun were a point source, the result would be an infinitely thin circular arc reflecting the intensity of the sun when

$$\vec{\omega} \cdot \vec{\omega}' = \cos 42° \approx 0.7431 \ .$$

This test is easily done in a fragment shader on modern graphics hardware. To get a rainbow instead of a bright line across the sky, we simply take into account that the sun has an extension which covers a range of directions $\vec{\omega}'$. We use the lowest and the highest point of the sun on the sky dome. This gives two cosine values

$$a = \vec{\omega} \cdot \vec{\omega}'_{\text{high}} \quad , \quad b = \vec{\omega} \cdot \vec{\omega}'_{\text{low}} \ .$$

With a `smoothstep` function (Hermite interpolation between a and b) we use a and b to find a shade value $c \in [0, 1]$ for the rainbow:

$$c = \texttt{smoothstep}(a, b, 0.7431) \ .$$

To let the value c determine the colour of the rainbow, we have to involve some more recent colour theory. Each value of c corresponds to a wavelength in the visible spectrum such that $\lambda(c = 0) = 400 \, \text{nm}$ and $\lambda(c = 1) = 780 \, \text{nm}$. To find RGB colour values for each wavelength, we use the RGB colour matching functions [Stiles and Burche 1959; Stockman and Sharpe 2000]. A look-up using c in a 1D texture holding the colours of the rainbow, i.e. the RGB colour matching functions, is one way to get the desired colours. Another option is to choose a few RGB colours at significant wavelengths (e.g. at $\lambda = 445 \, \text{nm}, 540 \, \text{nm}, 600 \, \text{nm}$) and then interpolate between them using c. Finally an alpha value is used to blend the rainbow with the background sky. For $c = 0$ and $c = 1$ the alpha value is 0 (the rainbow does not show in these regions), in-between that the alpha value should be set depending on how intensely the user wants the rainbow to appear in the sky.

The Aristotelian rainbow is very simple to render and it is easily run in real-time. It runs at 116 frames per second in a 1200×400 resolution on an NVIDIA GeForce Go 7400 graphics card. Sample renderings are shown in Figure 1 and in the colour plate. The colour plate also shows where the lowest and highest points of the sun are placed in the sky. The distance between these two points determine the width of the rainbow. In addition, it is easy to modify the position and intensity of the rainbow in the sky by moving the points and adjusting the alpha value. Originally Aristotle thought of the sun as sitting on the hemisphere (the sky dome), where the rainbow also appears. This gives rainbows which are very stretched out compared to real rainbows. If we move the points on the sun, which determine $\vec{\omega}'_{\text{high}}$ and $\vec{\omega}'_{\text{low}}$, away from the sky dome in the radial direction, the rainbow gets a more natural arc. This is another parameter we can use to modify the appearance of the Aristotelian rainbow. Using these different parameters, we have, qualitatively, tried to match the appearance of real rainbows in the colour plate.

With the Aristotelian rainbow we have given a brief example of what we can learn by taking an interest in the history of theories of light and vision. The first sections of this paper present pointers to relevant developments in the history of these theories. Hopefully these references provide an overview and a starting point for finding more mathematical and physical models that can be useful in graphics. And if we follow the development of the theories of light further than this paper does, we may get ideas and inspiration about future developments in realistic image synthesis.

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The Aristotelian Rainbow: From Philosophy to Computer Graphics

Jeppe Revall Frisvad, Niels Jørgen Christensen, and Peter Falster

Rainbows rendered in real-time using Aristotle's theory for rainbow formation. The black and green points (in the smaller images) show the lowest and the highest point of the sun. They determine the position and size of the rainbows (in the larger images). Of course the rainbow is always found when we look in the direction opposite the sun.

Comparison of rainbow pictures from the real world (top row) and rainbows rendered using Aristotle's theory (bottom row).