Heuristic Algorithms for NP-Complete Problems

Thomas V. Christensen

Supervisor: Paul Fischer

Kongens Lyngby 2007
IMM-BSc-2007-12

Technical University of Denmark
Informatics and Mathematical Modelling
Building 321, DK-2800 Kongens Lyngby, Denmark
Phone +45 45253351, Fax +45 45882673
reception@imm.dtu.dk
www.imm.dtu.dk

Summary

I have implemented a small framework of NP-complete problems in Java, where it is possible to transform instances of one NP-Complete problem into instance of another NP-Complete problem. The framework also consists of a few heuristic algorithms, which makes it possible to find a heuristic solution for instances of an NP-Complete problem.

Combining these two features it is possible to transform an instance of one NP-Complete problem into another instance. This transformed instance may then be solved by the available heuristic algorithm and the found heuristic solution may then be detransformed to the original instance. Last but not least it is possible to view this scheme through a nice user friendly GUI.

This report analyzes the design of the framework, the behavior of the heuristic algorithms to transformed instances and how one may find the best transformation automatically to a given NP-Complete problem.

Preface

This project was prepared at the institute of Informatics and Mathematical Modelling, at the Technical University of Denmark from February through June 2007. It was one of the requirements for acquiring the B.Sc. degree in engineering.

Lyngby, June 2007

Thomas Vesterløkke Christensen
s042075@student.dtu.dk

Acknowledgements

I would like to thank my supervisor, Paul Fischer, who first of all introduced me to this interesting field of computer science, but he has also advised and helped me through the five months this project has lasted.

I would also like to thank my local pizza place for always giving me carbohydrates for my continuous respiration and sufficient fat and cholesterol, which made any escape attempts from my desktop demand way more energy than I can muster. An ideal solution for maintaining work, which can only be truly appreciated in the eyes of an engineer.

Contents

Chapter 1

Introduction

In Computer Science there is a complexity class of problems known as NP-Complete (NPC), because they contain complete information about all problems in the complexity class NP. The way of proving a problem is NPC is to make a polynomial(fast) transformation from a known NPC problem to the problem considered and showing that this transformation yields the same solutions for both problems. The relationship between NPC, NP and the class of problems, where a solution can be found in polynomial time, P, can be seen in Figure 1.1.

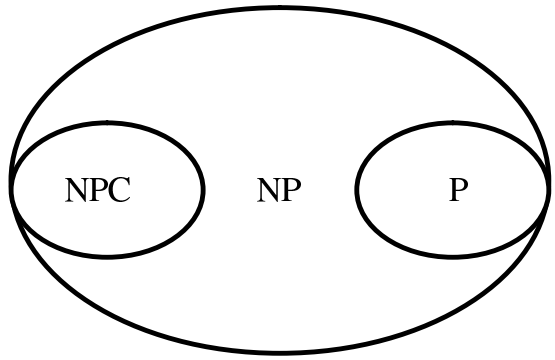


Figure 1.1: Relation between complexity classes

At the time of this project it is unknown whether or not these problems can be solved in polynomial time, but it is strongly believed that it requires super-polynomial time. However if it would be possible to solve one NPC problem in polynomial time then all NPC and NP problems can be solved in polynomial time. This is because of the polynomial transformations between the NPC problems. On the other hand if it is possible to show that one of the problems cannot be solved in polynomial time, then none of the problems can be solved in polynomial time.

Presently no valid proof for any of the two cases exists, there is however a strong belief that no polynomial time algorithm exist for the NPC problems.

In general there is two kinds of NPC problems called decision and optimization problems, where the first consists of finding if it is true or not and the other represents a problem where an optimal solution needs to be found. This project will be working with the optimization problems.

The method used so far for solving NPC optimization problems is making heuristic algorithms that gives approximate or suboptimal solutions to the considered problem. These algorithms can as well use the transformations to give approximate solutions to other NPC optimization problems like shown in Figure 1.2. The argument for doing this is that the transformation is very often easier and less time-consuming to implement and develop than creating a new heuristic algorithm from scratch.

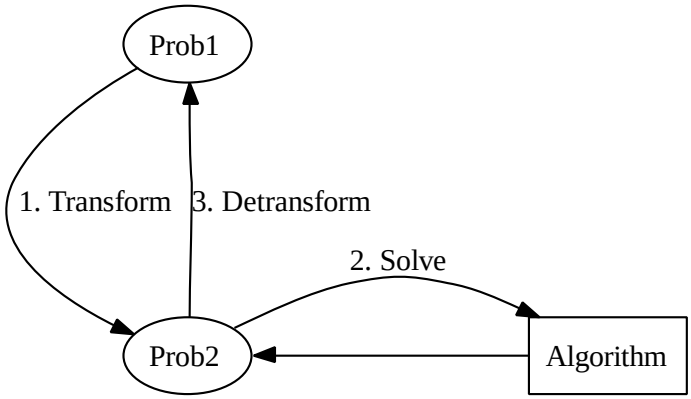


Figure 1.2: Using an algorithm to solve more than just one kind of NPC problem.

The goal of this project is to make it possible to solve instances of NPC problems by transforming them to problems where heuristic algorithms are available.

Furthermore the behavior of the heuristic algorithms will be analyzed when the transformations are used. The project should also allow the user to use a graphical user interface, such that the transformed problems and their approximate solutions can be seen by the user.

Chapter 2

Motivation

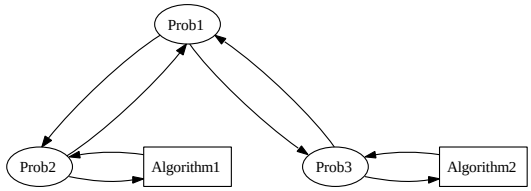
The motivation for doing this project is first of all to see if it is beneficial to reuse heuristic algorithms by using the available transformations. Should it prove not to be feasible, then it can be used to find transformations that create bottlenecks.

Another benefit of the transformation is that for a single NPC problem one can get access to a variety of different algorithms.

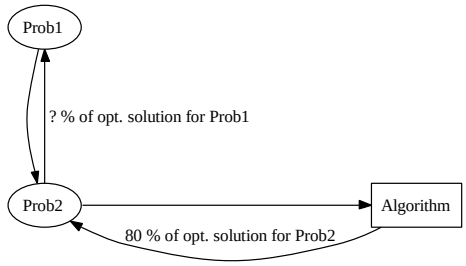
With the graphical user interface this project will be excellent for teaching purposes, such that one can get an intuitive understanding of how instances are transformed. This feature will also make it an excellent tool, when trying to create and verify new transformations.

Another interesting aspect, which can be seen in Figure 2.1a, is that the transformations allow one to compare two different algorithms from two completely different problems. With this opportunity one might be able to investigate the characteristics of good heuristic algorithms.

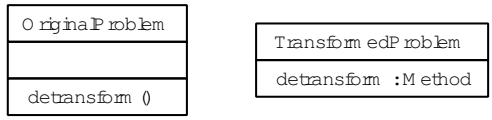
However what if a heuristic algorithm gives great results for the problem it was created for, see Figure 2.1b, does this necessarily mean that it will also give great results when detransformed to other problems? All we know is that if a transformed instance can be solved then the solution will also solve the



(a)



(b)



(c)

Figure 2.1: Diagram of (a) how to use and compare different algorithms from different NPC problems detransformation (b), compare heuristic solutions to see if the it is equally good for the transformed and the original instance (c) and detransforming of an algorithm to another NPC problem.

original instance, but when the instance cannot be solved we instead find a good estimate. Is this estimate still good when it is detransformed?

It is time-consuming to develop new heuristic algorithms, but what if one manually detransformed the entire heuristic algorithm like shown in Figure 2.1c, then the transformations of instances to other NPC problems would no longer be done and all we had to do was run the algorithm.

These are all just a handful of interesting aspects, which this project can help find answers for.

Chapter 3

Scope of Project

It is the aim of the project to implement a decent number of transformations to different kinds of NPC problems and develop various kinds of algorithms for a subset of these problems. The algorithms I implement will be my own, because I figure it might be a good idea to start from scratch, this however does not guaranty that the algorithms I end up with are original, and I will not spend any time investigating whether or not that be the case.

In the project I emphasize the following:

- This is not just a proof of concept. It is meant to be optimized for practical use,
- it should be easy for third party to extend the framework with new problems and transformations that uses the existing heuristic algorithms,
- it should be easy for third party to extend the framework with new heuristic algorithms,
- the framework should be able to find the best suited transformations automatically,
- and for the problems implemented the transformed problem input/output should be viewable in nice graphical user interface.

It should be mentioned that I do not intend the developed software to be Open Source Software. This means that certain measures need to be taken, if it should be easy for the third party to extend it without actually knowing the source code. In this sense I see it as a benefit that the source code is not Open Source, since less information is needed. However I feel that making this choice, requires me to make a manual describing how the framework should be extended.

Furthermore to optimize the framework I chose not to use proper encapsulation of data in order to avoid the overhead of get- and set-methods. This is however only carried out in the data structures, because they are the ones, which will be used the most.

This project does not in any way aim to prove or disprove the $NP = P$ problem.

Chapter 4

The NP-Complete Problems

This section shortly describes precisely what a NP-Complete problem is and the process of proving NP-Completeness for a problem. This section will not give a complete explanation regarding the theory of complexity classes or why the process is a valid proof, but instead focus on how the theory can be used in this project and what obstacles one should expect just by looking at the theory.

4.1 The necessary theory

There is one complexity class of problems called NP, which stands for non-deterministic polynomial time. Given a solution for such a problem, one can in polynomial time verify that it is indeed a solution for the problem.

However there is a set of problems in NP, which are proven to be more or at least as difficult to solve as all other problems in NP. These problems are called NP-Complete(NPC), because they are said to contain complete information about all problems in NP, the relationship between NPC, NP and P can also be seen in Figure 1.1. The immediate result of this is that if one can solve a NPC problem efficient then all problems in NP can be solved efficient.

The first problem to be proven NP-Complete was the Satisfiability problem (for

a proof see [4]). In general there are two types of NPC problems called decision problems and optimization problems. A decision problem is a NPC problem where the answer is yes or no, while the optimization problem is the best solution for a given problem. So if I have a instance of the Satisfiability problem and I ask if it can satisfy e.g three clauses then it is a decision problem, however if I instead want to know the assignment that satisfies most clauses then it is a optimization problem. This project will be dealing with optimization problems.

Now given one NPC problem, P_1, one can prove a problem, P_2, is NP-Complete if it is harder or just as hard as P_1[1]. The P_2 is proven NP-Complete by:

1. Showing it is in NP,
2. and constructing a transformation, that transforms instances, I_1, of P_1 to instances, I_2, of P_2.
    - This transformation must have polynomial running time
    - If there is a solution for I_2 then there also is a solution for I_1.
    - If there is a solution for I_1 then there also is a solution for I_2.
    - The transformation should show that P_2 is harder or just as hard as P_1, because P_1 can be solved by solving P_2.

It is easy to prove the problem is in NP, however constructing the transformation can require a great amount of creativity. To show the transformation makes P_2 harder than P_1, as a rule of thumb one just have to find an instance of P_2, which cannot be detransformed to P_1. This means all of the instances of P_1 will be transformed into special cases of P_2. Therefore P_2 is harder than P_1, because all instances of P_1 is just a subset of all instances of P_2. This rule of thumb does not apply when the problems are exactly the same or very close related, which is the case with Independent set, Clique and Vertex Cover.

If the transformation yields special cases as described above, then one might confuse a transformation with a problem reduction and maybe it is. But one should remember that all NPC problems are equally hard and this does not contradict with transformations producing special cases, because the cost of special cases are padding. In other words the transformations make the problem size increase, which is important to note, since it pose a great problem for this project.

[1] The problem can off course not be harder since a NP-Complete problem is defined by being harder than all other problems.

4.2 Theory in use

The general goal in computer science is to find the best possible solution to all problems as fast as possible, which in most cases means in polynomial time. The idea behind this project is to reuse the polynomial transformations that were created just to proof NP-Completeness.

Let us say we have a transformation going from a problem, P_1, to another problem, P_2, and for this specific problem we have a good heuristic, polynomial time algorithm. Then it it possible to transform any instance, I_1, of P_1 to an instance, I_2, of P_2, solve I_2 with the algorithm and detransform the solution back for I_1. Note that if this project had worked with decision problems instead of optimization problems then the detransformation process would be unnecessary since the answer would remain the same after a detransformation.

The process just described can all be done in polynomial time and by the definition a satisfying solution for I_2 can be detransformed into a satisfying solution for I_1. If one cannot find a solution for I_2 then it is not possible to find a solution for I_1 either. This also means that if one can find a polynomial time algorithm which finds the optimal solution for one NPC problem then all NPC problems can be solved in polynomial time.

However no such algorithm are currently known, so [6] describes three ways of solving a NPC problem:

1. using exhaustive search to find the optimal solution for small problems which gives a exponential running time,

2. to recognize special cases of a NPC problem, which can be solved optimally in polynomial running time,

3. or to use an algorithm with polynomial running time which finds a near-optimal solution.

This project will work with the last two types of algorithms mostly dealing with the last type, which is also called heuristic algorithms.

Now if one gets a good solution for I_2 using a heuristic algorithm, then the definition does not guaranty that the detransformed solution is just as good for I_1. But of course if the solution was optimal, then it will also be optimal when detransformed.

Furthermore it is interesting to see if the special cases, that the transformations produce, have a certain structure, which makes them easier to solve. If it is the case that a transformation always produce a certain kind of instances of a problem, that are easy to find the optimal solution for, then one has a polynomial time algorithm which solves a NPC problem optimally.

One of the goals of this project is to make it easy to extend the framework with new NPC problems. As explained above to show a problem, P, is NP-Complete one has to show it is in NP and construct a valid transformation, $T1$, from a known NPC problem to P. However the reason for extending the framework with new problems is to solve these problems with the existing algorithms, but this is not possible with the $T1$, since it cannot transform instances of P to any problems in the framework, which has an algorithm. So furthermore one has to create another transformation, $T2$, going from P to one of the existing NPC problems, preferably one which has access to many good or fast algorithms. This scheme can be seen in Figure 4.1.

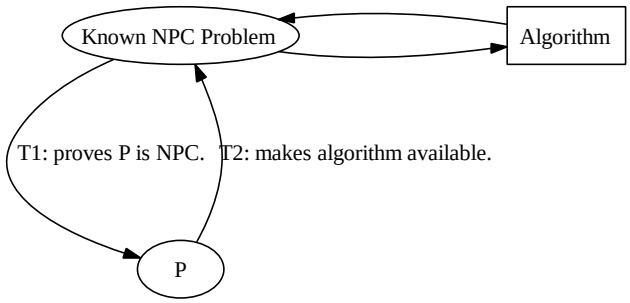


Figure 4.1: What transformation to make when extending with new NPC problems.

Chapter 5

Extending the framework

This section analyzes the design of different solutions for making the framework easy to extend for third party.

Basically there are only three kinds of extensions the framework needs to support – new problems, new transformations and new heuristic algorithms.

Thus I define three properties of the framework, that the final design should be able to handle in a proper way.

Property 1 An instance can be transformed to different problems and have multiple heuristic algorithms.

Property 2 A solution for a transformed instance can be detransformed to a solution for the original instance.

Property 3 A problem can be transformed to the same problem with different transformations.

Notice that these properties should not only be handled by the design in general, but they should also be given to the user that wishes to extend the framework.

Also note that the third property is not necessarily needed, since most problems only have one transformation. This is mainly because it only takes one transformation to prove a problem is NP-Complete, but if one locates a bottleneck transformation, then it should be possible to add a new and better transformation without removing the existing one.

5.1 Design of Problems

The most obvious choice for implementing a problem is by a class of its own. The methods of this class could then do transformations, detransformations and heuristic algorithms.

So if the framework is to be extended with a new problem, it should just inherit an abstract class called `NPCProblem` , see Figure 5.1. Then all the new class should contain is the input for the given problem, so the design would look something like this:

```
1 class P extends NPCProblem{
2 // data fields
3 ...
4 // transformation methods
5 ...
6 // detransformation methods
7 ...
8 // heuristic algorithm methods
9 ... }
```

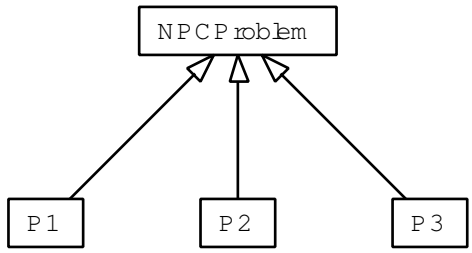


Figure 5.1: Class diagram of NPC problems inheriting `NPCProblem`.

Using this representation the user still have the opportunity of extending existing problems simply by inheriting those problems and supplying new methods(transformations, algorithms, etc.).

Overall this seems as the only right way to represent problems in an Object-Oriented Programming Language.

5.2 Design of Heuristic Algorithms

Given that problems are represented by classes the heuristic algorithms are more or less bound to be methods of the class, if it should have easy access to the data for the given problem. Thus the algorithms can be implemented as:

```
1 Solution algo1(){...}
2 Solution algo2(){...}
3 ...
```

This design also makes it possible to supply new algorithms for existing problems without overriding the old algorithms.

However using this design one can chose to simply return the solution, which would work just fine, but it may be a good idea to store the solution in the instance of the given problem. Doing this would make it necessary to store a solution for each of the algorithms, which would require a great amount of memory – possibly more memory than the input for the problem.

I chose not to save the solution, because the motivation for saving them is not to compute the solution again in case the user should think of calling the algorithm every time he[1] wanted the solution.

Just to underline that it is a bad idea to store the solution, consider the case when an instance can get a solution from a transformed instance. This means that the instance should be able to store a solution for each algorithm available in the transformed instance and so on. In the case a solution was found in a transformed instance, then the solution would also had to be stored in the transformed instance. In other words it is a waste of space to store the solution at this point.

[1]Throughout this report I will refer to third party as he or him. This should not be seen as an act of prejudice but rather a sad statistical observation.

5.3 Design of Transformations

As with the algorithms the transformations are bound to be methods in the `NPCProblem` class or its subclasses and it is also required that this method should somehow return another `NPCProblem`. However the best structure of this method is anything but easy to find. Therefore I list in the following a number of suggested solutions, which all have their advantages and disadvantages.

First suggestion The first way it could be structured is like:

```
1 NPCProblem transform()
```

It is very general, which means that it can be put in the abstract class `NPCProblem`, such that all classes inheriting `NPCProblem` will be bound to implement the transformation like shown in Figure 5.2a.

However not all NPC problems have a transformation, which makes it a bad idea furthermore only one transformation is available and the user have no idea what kind of problem the transformation creates. Now if the user were to supply a new transformation for an existing NPC problem he would have to override the existing transformation.

In other words this seems as a very poor choice for implementing transformations since none of the desired properties for the framewrok are achieved.

Second suggestion Now consider another way in which the transformation can be structured:

```
1 NPCProblem transform(Class<? extends NPCProblem> c){
2 if(c.equals(P1))
3     // return transformed prob P1
4 else if(c.equals(P2))
5     // return transformed prob P2
6 ... }
```

Using this structure one can still place the method in the abstract class `NPCProblem` as shown in Figure 5.2b and it supports the first property by supplying transformations for different problems.

If new transformations are to be added the method would have to be overwritten, however the existing transformations could still be used:

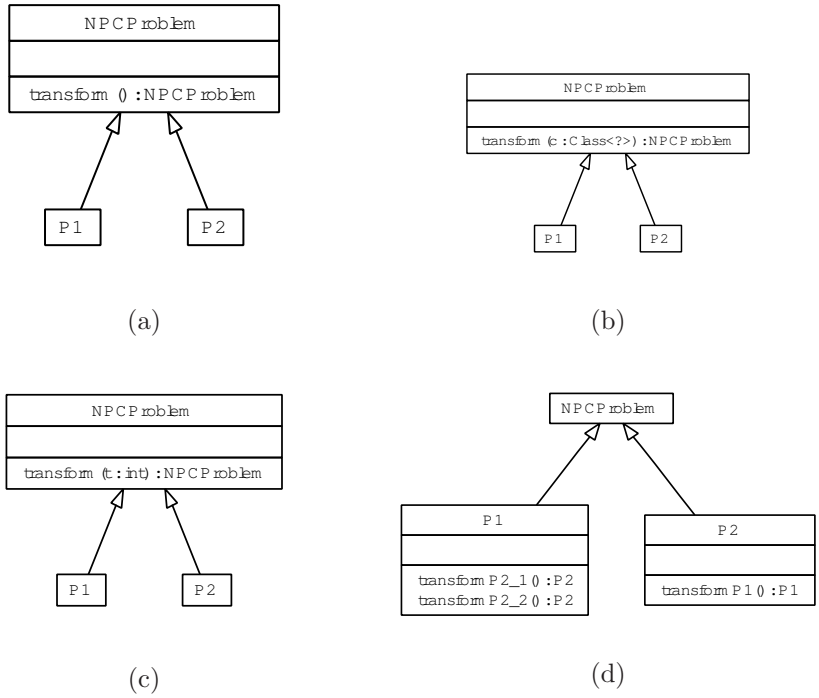


Figure 5.2: UML diagram of (a) the first suggested solution (b), the second suggested solution (c), the third suggested solution (d) and the fourth suggested solution.

```
1 //overridden method
2 NPCProblem transform(Class<? extends NPCProblem> c){
3 super(c);
4 if(c.equals(newP1))
5     // return transformed prob newP1
6 else if(c.equals(newP2))
7     // return transformed prob newP2
8 ... }
```

But this design does not allow multiple transformations to the same problem – second property of the framework is unavailable. Beside that I find that all the if-statements look terrible, but from the perspective of the user it makes it easy to just have one transformation-method.

Third suggestion By modifying the previous design one could use the one shown in Figure 5.2c and below:

```
1 NPCProblem transform(int t){
2 switch(t){
3 case TRANSFORM1_TO_P1: //return P1 using first transformation
4 case TRANSFORM2_TO_P1: //return P1 using second transformation
5 case TRANSFORM1_TO_P2: //return P2 using first transformation
6 ...}
7 }
```

This design allows transformations to different problems and multiple transformations to the same problems and it can be placed in the abstract class `NPCProblem`. Furthermore it still allows the user to extend existing problems with new transformations without overriding the existing transformations in the same way as in the lat suggestion. The downside of this design is that the user has to know the values of the existing constants(`TRANSFORM1_TO_P1`, ...) when adding his own transformation. Another undesired thing about this design is like before that one do not know for sure what type of `NPCProblem` the method returns.

Fourth suggestion My last and final suggestion for designing the transformations can be seen in Figure 5.2d and below:

```
1 SAT transformToSAT1(){ .. }
2 SAT transformToSAT2(){ .. }
```

This design gives every transformation its own unique method, which makes it easy to extend, because when one inherits the class a new transformation

method can just be added. Another benefit is that one actually knows what type of NPC problem is returned and like the previous suggestion it achieves all three properties of the framework, which explains why this is how the transformations have been implemented.

However not all is well, the above suggestions all suffer from the same problem, which seems to be difficult to solve unless the source code is open. Consider the case in Figure 5.3, where the class, `SAT`, representing the Satisfiability problem is extended by a third party with new algorithms and transformations, which is called `SAT2`. The problem arises when the other NPC problems have transformation to the `SAT` and not `SAT2` class. This can be solved my extending all the other problems and make them make transformation to `SAT2` instead or by making a constructor for `SAT2`, which takes a `SAT` object as argument.

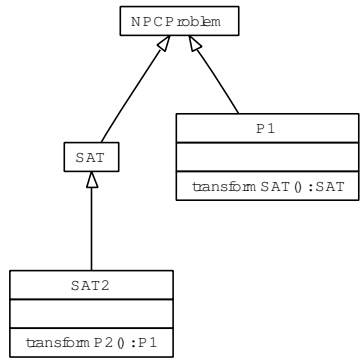


Figure 5.3: Problematic situation for third party to extend a existing `NPCProblem` class.

But still none of those two solutions are satisfying compared to how easy it would be if the source code was available. The extensions can nonetheless be done with this design.

5.4 Design of Detransformations

I find that the greatest problem in detransforming a solution from a transformed instance is that one cannot distinguish between a transformed and a user-defined instance. Furthermore if one could, then it would still be necessary to know what NPC problem to detransform the solution to.

Detransforming a solution is more or less bound to be a function in the `NPCProblem` classes. Note now that the previous choice of not storing the solution found by the algorithms, means that the solution needs to be a parameter for the detransformation method. This actually solves two problems in the detransformation process. First of all the solution is provided as a parameter by the user, so it does not need to bother about whether or not a transformed instance has found a solution. Secondly the method no longer has to decide what solution to detransform if there is more than one algorithm for a `NPCProblem`. The same goes if there are solutions from two different transformed instances.

The downside is that the user can now give a parameter which has never been a solution for the transformed instance.

There are two obvious approaches, which one can use for the detransformation:

1. The method can be called from the transformed instance and send the detransformed solution to its original instance as in Figure 5.4a,
2. or it can be called from the original instance and it will detransform the solution from its transformed instance like in Figure 5.4b.

In both cases the detransformed solution needs to be returned, so both approaches yields the same result since nothing is stored, which is a consequence of the choice described above.

With the first approach one would have to store a reference to the original instance it was transformed from, since it is necessary to know what kind of `NPCProblem` to detransform back to. When detransforming one needs to know a lot of details of the original instance, which means that every `NPCProblem` would need a lot of get-Methods in order to make the detransformation. In the case where an instance can use more than one transformation to the same `NPCProblem` , the transformed instance would have to somehow know, which of the transformations that were used, which could be done with some sort of constant variable.

Using the second approach most of the above problems are gone except for the problem of deciding what detransformation method to use if the problem contains more than one transformation. This is a problem since a solution contains little information on what transformed instance it came from. Using this solution the transformation and detransformation methods will be placed in the same class, which seems to be beneficial.

It might seem as an obvious choice, however if I had chosen that the solutions

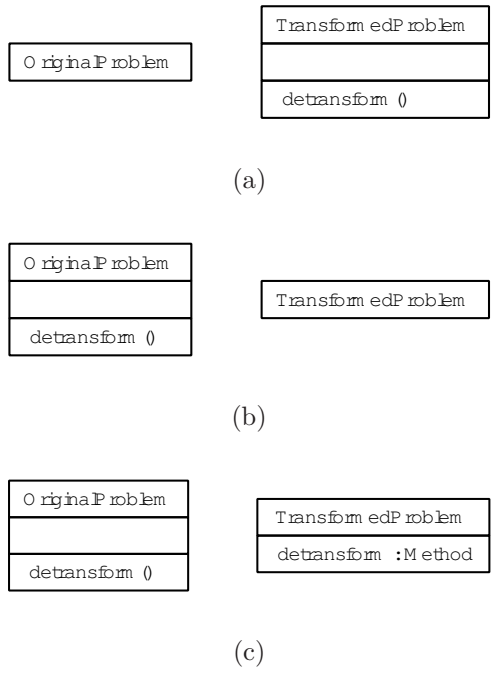


Figure 5.4: UML diagram of (a) first suggested solution for detransformation (b), second suggested solution for detransformation (c) and the third suggested solution for detransformation.

from the algorithms were stored in the problems, then it might not had been so obvious, because then a lot of work could possibly be removed from the user by choosing the first suggested solution.

There is actually also a third option where the actual detransformation method is placed at the original instance but called by the transformed instance, which is shown in Figure 5.4c. This can be done elegantly by using Javas reflection library to give the `Method` object for detransforming to the transformed instance, which will then know how to detransform a solution. I chose however to stick with the second solution because in order to use the last solution the method has to be public(or protected if the classes are in the same package), which is what has to be done in the second solution.

Now from the previous choices made the detranformation is bound to take a solution as parameter and return a detransformed solution, which gives the following design:

```
1 deTransSolution detransform3SAT(Solution s){ .. }
```

In order to support multiple detransformations for the same problem the same scheme can be used as for the transformation methods:

```
1 deTransSolution detransform3SAT1(Solution s){ .. }
2 deTransSolution detransform3SAT2(Solution s){ .. }
```

The final design for the framework now handles all the wanted properties and can be extended by a third-party. It does not waste memory on solutions that are never made and it gives a lot of control to the user. This on the other hand also makes the design vulnerable to human errors.

Chapter 6

Automatic solving

Solving a problem automatically with the best suited transformations and heuristic algorithm can be done in a number of different ways. This chapter describes the problems when solving problems automatically.

It is obvious that this problem is nothing else but to find the shortest route between two problems in terms of transformations. Because problems are connected together with the transformations, one can construct a directed graph with the problems as vertices and the transformations as directed edges as shown in Figure 6.1. Now in order to find the shortest route between two problems one just have to implement Bread-First-Search or the Dijkstra algorithm. This all seems straight ahead however there are some issues one should consider.

6.1 Constructing the graph

When constructing the graph, it is crucial not only for it to contain all NPC problems and transformations but also information on what method to call in order to use the given transformation or algorithm.

The simplest way of doing this is to manually type in all transformations and algorithms and their respective methods. However this would not only be time

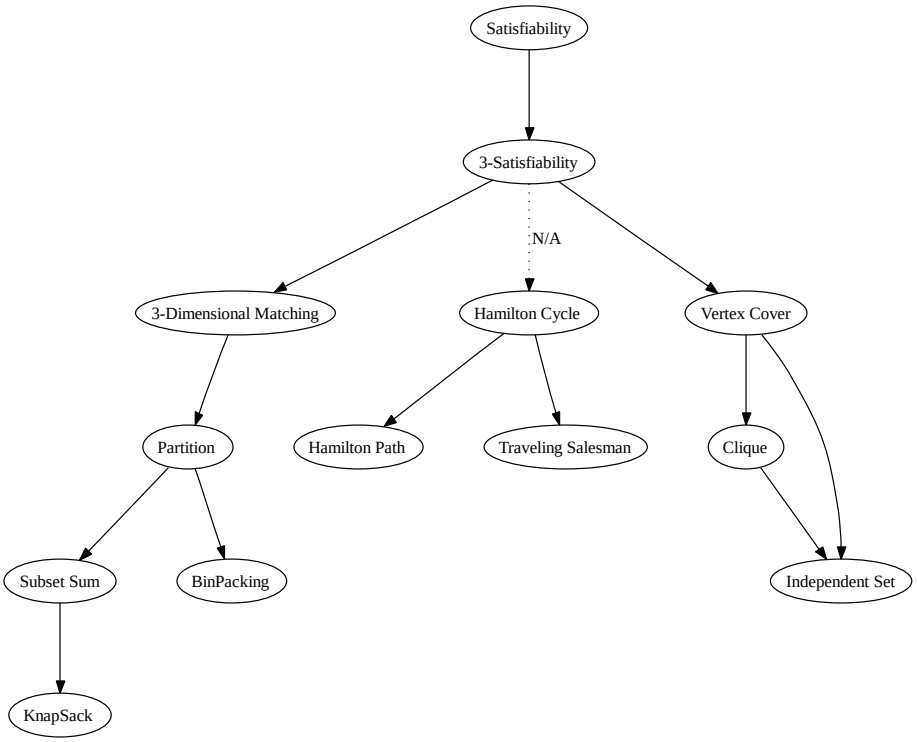


Figure 6.1: Transformation graph showing all NPC problems and transformations.

consuming but also very inflexible, because one might make a long switch-case statement, which looks at what transformation should be called and then calls it.

All these problems can nonetheless be solved very elegantly using Javas reflection library, which allows one to see a complete list of all methods within a given class. This functionality can be used to automatically retrieve all transformation methods from a class in order to create the needed graph. It does however not contain information on the subclasses of a class, which makes it necessary to give all the subclasses of `NPCProblem` to the program.

Also if one has the name of a method, then Java allows one to call the method on an object, just by passing a string containing the name of the method. This solves all the problems from above. The downside in using this is that if the program should automatically recognize transformation and algorithm methods, then they must have a certain structure or name. So if a third party should extend the framework with new transformations then it is something he should be aware of.

To be specific I have chosen that all transformation methods must start with the keyword "transformTo" and ended with some suffix. The corresponding detransformation method must start with the keyword "detransform" and be ended with the same suffix as with their transformation method. The methods for the algorithms must start with the keyword "algo". Below is shown an example:

```
1 public <V>VertexCover<V> transformToVC(){...}
2 public ThreeDimensionalMatching transformTo3DM() {...}
3 public boolean[] detransformVC(boolean[] vcsolution){...}
4 public boolean[] detransform3DM(boolean[] tdmsolution){...}
5 public boolean[] algoSomeName() {...}
6 public boolean[] algoSomeOtherName() {...}
```

Another issue using the reflection libraries is that the compiler cannot make typechecking to see if the correct parameters are used or returned, which is like playing with fire.

6.2 Finding the shortest route

Finding the shortest way to solve a instance is entirely easy to solve, but I see three different ways this can be done:

- Every transformation is given the same weight,
- or one looks at the asymptotic running time for each transformation,
- or one uses a statistical value from previous runs of a transformation.

6.2.1 Shortest route using Breadth-First-Search

The first solution is very simple to implement, since it is basically just breadth-first-search. The benefit from using this method is that it is easy to implement, it is flexible to new transformations, and it is a somewhat good estimate for the shortest transformation path. It however does not take into account that the transformations have different running time and a different degree of padding, so in this sense it is a bit too naive.

6.2.2 Shortest route using Asymptotic running time

The second solution looks from a theoretical point of view much better and should be easy to implement, however it introduces some difficulties.

Normally we write the asymptotic running time using the O-notation, which shows the most significant term as a function of the input size, this means that if one uses two transformations for a given instance of a problem, where the first have the running time $O(n^3)$ and the second $O(n^2)$, then the running time of taking both transformations would yield a running time of $O(n^3 + n^2) = O(n^3)$. This is a very simple scheme which can easily be implemented by letting the weight of a path be determined by the largest exponent from the asymptotic running times of all the transformations in a path.

This solution is clearly a bit better than the first, however it assumes that there is no padding when doing a transformation and this is rarely the case. Now consider the same two transformations from before, where the first transformation is given an input of size n and after the first transformation the size of the transformed input has increased to a size of n^2 as a result of padding. Now the running time of first transformation is still $O(n^3)$, however the running time of the second transformation is now $O((n^2)^2) = O(n^4)$, which yields a total running time for the two transformations to $O(n^3 + n^4) = O(n^4)$.

Obviously this is a more descriptive solution, however there are a couple of problems.

The first problem is that the above strategy assumes that the input size is determined by one parameter, which is rarely the case. For instance a graph transformation may take $O(V^2 + E)$, where there is not only two different parameters but also two different exponents, which makes the scheme above a bit more tricky to implement.

Another problem is that one needs to feed the transformation with some kind of function, such that it is able to calculate how much the input size will increase.

Last problem is when the Dijkstra algorithm is running, the instance to be transformed has not yet been created which means that Dijkstra does not know the input size of the instance and can therefore not do the calculations.

One could however create the instance before running Dijkstra, but if one is creating multiple instances of the same problems, then Dijkstra would have to run each time to find the shortest route. This may however be satisfying if the instances are very large and time can be saved by finding the shortest transformation path each time.

6.2.3 Shortest route using statistical data

Using the asymptotic running time for finding the shortest route seems to be a very clever choice from a theoretical point of view, but it has its limits. First of all the O-notation removes all constants and minor terms, which may be very significant even for medium sized input, however this is not really interesting since this project aims to solve large problems efficiently.

What is interesting is that the running time of computers is highly dependent on, how much time is has to wait for retrieving data from main memory. To speed up this process most computers stores recently used data close to the CPU in Cache Memory, which takes less time to access. The great problem is that the Cache Memory has different sizes and operates differently on different computers.

So bottom line is that using the asymptotic running time may not be a good idea, since it does not take into account how the computer operates. On the other hand it is way beyond this project to try and predict how an instance is transformed optimally on a specific computer. It would however seem as a good idea to find the shortest route based on empirical data for the used computer.

Doing this introduces a number of issues. The first is what data to store and how to use is for future runs, another issue is that the data should be updated

every time the transformation is used, such that it is up-to-date. It is obvious that data needs to be stored for each transformation if it is to be changed.

Clearly it would be a bad idea to store data from each time the transformation was used, since the data then would be grow tremendously in no time. Another problem of having all that data is that it has to be processed every time Dijkstra is used. The way to solve this is to some how find an average of the data gathered and then to store this average. One could choose to simply take the average time of all the transformation runs, however since the time depends very much on the input size, n,this solution would be a bad idea. Instead the average time should be weighted with respect to the input size. Since all the transformations run in polynomial time one can assume that the time depends on the input size by some polynomial:

$$time = n^k \Leftrightarrow k = \frac{\log time}{\log n}$$

So for each run of the transformation one could calculate the exponent k take the average and use this as the actual asymptotic exponent like described above.

The drawback of using this simple method, is like explained above that not all transformations depends on just one input size e.g most graph transformations depends both on the number of edges and vertices. If the function depends on more than one value then it is likely that the values are not equally significant and that they should be weighted. Another problem is that not all transformations have a running time which can be described by a polynomial e.g $n \lg n$.

Whenever a transformation has been used the stored value should be updated, however if the old value is based on the results of hundreds of transformation runs, then it should not be affected too much by a single outlier. In other words the stored data should include information on how many transformation runs, m, the average value, v_{old}, is based on. So when a new piece of data, d, is collected the old average value should be updated like this to include the new data:

$$v_{new} = \frac{v_{old}m + d}{m + 1}$$
$$m_{new} = m_{old} + 1$$

Compared to the simple way of using the asymptotic running time, this statisti-

cal data takes padding into account, but instead of taking a simple average one could instead try and find a polynomial with highest degree d, which fits the data as well as possible and then use it along with data on a specific problem. This would only require that all the coefficients for the polynomial is saved, however it is very tricky to update them. It is beyond the scope of this project to investigate on how to do it, but if one have knowledge of how many former runs the polynomial is based on and one assumes that these samples are uniformly distributed over an interval for the polynomial, then it is possible to calculate the new updated coefficients.

As previously mentioned the transformations produce instances, which have a certain structure and some of these instances are transformed further on which produce another kind of instances with a special structure. This means that the statistical data gathered may very well be based on a certain kind of input, which is a problem if this kind of input has been used for a period of time and then a new kind of data is using the transformation. The new kind of data may now be choosing a bad transformation for this kind of input, which is a problem when using the empirical data the way it is suggested above.

Furthermore if a transformation has initially proven to be bad for a number of instances then it will get a expensive weight which makes it unlikely that Dijkstra will choose to use this transformation ever again. This may prove to be wrong for other types of instances, than the ones the empirical data was based upon. This suggests that it would be a good idea to introduce some sort of randomness in Dijkstras algorithm when using empirical data.

6.3 Heuristic Algorithms

So far it has been discussed how to find the route from one problem to another which takes as little times as possible, however it also takes time to find the heuristic solution, so it may be desired to let the algorithms be part of the calculations when finding the cheapest route. On the other hand the main purpose is usually to find the optimal solution for a problem, which makes it naive to think a fast algorithm can create as good results as a slower algorithm.

If the algorithms should be included in the calculations then I see the same three ways of doing it:

- One could give all algorithms the same weight,
- or one could use their asymptotic running time and optionally take padding

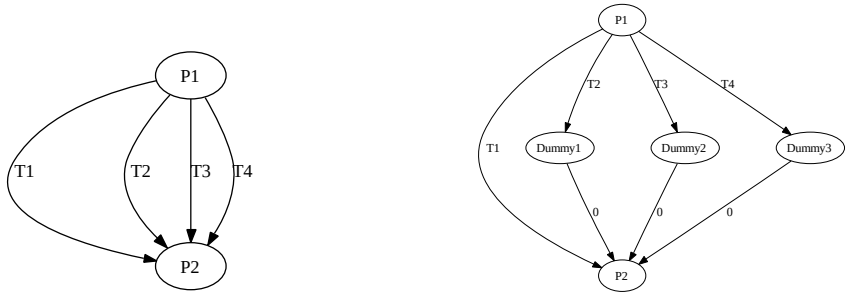
into account in the same way as with the transformations

- or use empirical data for the algorithms.

All three solutions can more or less be done in the same way as with the transformations.

6.4 Multiple transformations

When having more than one transformation, which transforms from problem, P_1, to another problem, P_2, as shown in Figure 6.2a, then the graph used with Dijkstra should be able to contain parallel edges. Parallel edges is not a problem if the graph data structure uses adjacency list, however when using adjacency matrix only one edge can be store between two vertices. This can be solved by letting the remaining edges go to its own new intermediate vertex, which has an edge to the end vertex with no weight. This solution can be seen in Figure 6.2b.



(a) (b)

Figure 6.2: Example of (a) multiple transformations between two NPC problems (b) and how this may be solved without parallel edges.

6.5 Implemented solutions

For this project I have made a general purpose graph data structure, which can contain any type of data in both vertices and edges, however in order to use

Dijkstras algorithm the edges have to be of a type, XComparable (extended version of Comparable interface), which knows how it can be added to other XComparable objects.

Demanding that Dijkstra only runs on graphs with edges implementing this interface makes it extremely simple and dynamic to define how Dijkstra should calculate the weight of a path.

Of the mentioned solution above I have implemented the following:

- It is possible to find the shortest route between two problems assuming all transformations have the same weight e.g. breadth first search.
- One can give an approximate exponent for each transformation and the weight of taking two transformations with $O(n^a)$ and $O(n^b)$ is calculated to be $c = ab$. This does not take account of padding, however one can adjust the exponent to simulate the padding of the transformation.
- The last solution implemented finds an empirical value for the exponent like described above. In order to solve the problem when an instance contains multiple input sizes, it is demanded that each instance knows how to calculate one input size based on all of its contents e.g. a graph problem can calculate its input size as $n = V + E$ and this value for n is then used as mentioned.

To store the empirical data a function have been made, which saves the entire transformation graph in a XML-document, which has to be loaded before the Dijkstra algorithm is used.

I have not implemented support of parallel transformations, since no such transformations are currently available in the transformation graph.

Furthermore the running time of the heuristic algorithms are not part of any of the calculations, because it is assumed that the user knows what heuristic algorithm he wants to use and he knows what NPC problems it is available at. This has been done because in the authors opinion it is more important, that the heuristic solution is close to the optimal rather than it is found quickly.

Finally it should be mentioned that even though it has been implemented the automatic solving does not correctly simulate the behavior of the transformations in its current form. The reason for this is that little time has been spent on finding the best approximate exponent for the asymptotic running time and the estimate of the input size used for the empirical data is in most cases a bit too

naive. However both of these issues can easily be fixed by experimenting with some different values. This has however not been prioritized in this project.

Chapter 7

Design of Graphical User Interface

There are many different NPC problems and not all of them are suitable to view as a simple text string in a console, which is one of the reasons development of a Graphical User Interface(GUI) is part of this project. This chapter describes the development of the GUI on top of the existing the model.

The computational complexity theory is far from intuitive for most people, so a lot of work have been put in making the GUI as user friendly and foolproof as possible given the limited time and resources.

The GUI for this project has been developed using the Model-View-Control paradigm. This means that the basic model, consisting of the `NPCProblem` classes, for the NPC problems was created before any development of the GUI began and it has been done so to emphasize that the basic model should be slim and as fast as possible. Another consequence of this is that the development of the GUI has been restricted from changing the basic model, because the basic model should remain able to work without any GUI. The only exception to this restriction is addition of new methods that allow functionality such as drawing an instance of a NPC problem or manipulating the problem. These added new methods are not meant to be effective, since they are only supposed to be used for the GUI and not the basic model.

The most essential features for the GUI are:

1. to create an instance of any NPC problem and to show it through the GUI,
2. to transform an instance of a NPC problem to an instance of another NPC problem and show this transformed instance using the GUI,
3. to solve an instance by one of the available heuristic algorithms and show the found solution using the GUI,
4. and to detransform the solution from a transformed instance to its original instance through the GUI.

First the problems surrounding all these features will be described in more detail and then later the final design is chosen at the end of this chapter.

Before one can use the first feature it is necessary to have an overview of the available NPC problems one can create. The preferred overview would probably be the transformation graph shown in Figure 6.1, showing all the NPC problems and their transformations. Creating an instance is very different from problem to problem and will be discussed individually further down.

The second feature requires that one selects an instance to be transformed and that one can choose between the different transformations for this given NPC problem. Having more than one instance to display introduces the need for an overview of all the constructed instances and it would probably be a good idea to show the relationship between them e.g. that I_2 was transformed from I_1. Furthermore it would also be nice to transform an instance to more than one NPC problem such that one have more than one branch of transformed instances, which would be easy to display using a JTree.

Like with the transformations third feature requires that one select a problem and choose between the available algorithms. If one can choose more than one algorithm then one might want to find a solution using both, which gives a problem when the solution should be displayed. This is however easily solved if one can just switch between the solutions, but this means that the solutions now has to be stored with the object, the issue of doing this was previously discussed and handled in Chapter 5, where I chose not to store the solutions in the basic model in order to preserve memory. However in the GUI memory should not be considered an issue since only smaller problems will be created compared to those used in the basic model without the GUI.

Another problem with storing the solutions is how to tell where they came from, such that the same solution is not found again. This can be solved by giving each solution a string stating what heuristic algorithms and detransformations it has been through.

The last property concerns detransforming and the only problem is that the basic model lets the user keep track of where the problems were transformed from and how to detransform their solutions back. This is an issue because there is actually only one right method for detransforming a solution from a transformed instance, and it would be wrong to let the user of the GUI try and find it.

7.1 Creating problems

To get an overview of all the NPC problems one can create instances of, a graph needs to be created as explained above. The easiest way of making the graph nice would probably be to make tools like Graphviz draw it and then show the image. However because the program needs to know what NPC problem, which was chosen from the graph, one has to know where each NPC problem(vertex) in the graph is located and this information is lost when using Graphviz. The alternative is to automatic placing the nodes, which is a difficult problem to solve and beyond the scope of this project. So the only choice left is to manually place the vertices, which leaves a lot of work for those who would want to extend the GUI with new problems.

7.1.1 Satisfiability problems

Satisfiability problems contain a finite number of clauses over a finite number of boolean variables. It is fairly simple to show the contents of a satisfiability problem just display one clause after the other. However some care needs to be taken when the clause cannot be displayed on one line or when the size of the screen is resized.

The solution for a Satisfiability instance is an assignment for the boolean variables, which is also easy to do, one can just display the assignment for each variable and highlight the clauses that are satisfied.

Giving the input is however a bit more difficult, since one could choose to have some sort of editor where one can write and edit the clauses. The downside in

this is that it requires a lot of work if it should work nicely. An alternative is to write a console where one can issue commands using some simple syntax.

I choose to use the last solution, because I feel it is less prone to human errors and it is easier to implement. The most necessary commands needed are:

- Create problem with n variables
- Add clause
- Remove clause

Regarding error handling the console should be able to recognize invalid input such as invalid arguments. However because of the limited time I will not check for reoccurring clauses, since it would require some kind of sorting of the clauses if it were to run fast. One might argue that this is not an issue since the instances used in the GUI are small, but the GUI uses the basic model, which should be as fast as possible. So I will not implement a simple algorithm, which slows the basic model unnecessarily.

7.1.2 Set problems

The problems containing a set of numbers can be dealt with in much the same way as the Satisfiability problems. They are fairly easy to display one just needs to handle the situation, where they cannot be displayed on one line etc. The solution for a set problem is typically a single subset of the numbers, which could just be highlighted in order to show that it represents the solution.

Regarding the input I see the same two choices as with the Satisfiability problems and again I choose to go with the console for the same reasons as above.

The console needs the following commands:

- Create problem
- Add number
- Remove number

More specialized versions of the commands will be needed depending on what set problem one is dealing with.

7.1.3 3-Dimensional Matching

3-Dimensional matching contains a set of triples, where each of the three numbers in a triple lies between 1 and a value q. A simple way of displaying it would be to draw three vertical lines next to each other with q dots and then draw the triples as two line segments going across the tree vertical lines. The drawback of this solution is when a triple goes through the same dot in the middle vertical line, because then it is impossible to see how each of the two triples continues unless they have different colors or shapes.

The solution for this kind of problems is as subset of the triples. Like with the set problems the subset can just be highlighted in order to show the solution.

Like with the previous problems I think the best way of creating and manipulating an instance of 3-Dimensional matching is by using a console, which should have the following commands:

- Create problem with the value q
- Add triple
- Remove triple

Note that it is now difficult to use the remove triple command unless the user know the identifier of each triple. This could be solved by writing the whole triple and then search through all the triples until a match was found, which of course would be inefficient. A hybrid solution is to allow both since they have different number of arguments (1 index versus 3 numbers) and this is exactly what has been done.

7.1.4 Single Graph problems

Creating a graph requires two things:

1. Vertices
2. and edges between the vertices.

I feel that the most intuitive way to create a graph is to place vertices on the screen and to draw lines between vertices to create edges. However one could

just as well use a console as done with the previous NPC problems and then initialize a graph with a specific number of vertices and then add edges to the graph. Doing this would probably be easier in term of creating and changing a graph, but another and much more difficult problem would then arise namely how to display the graph.

As mentioned earlier it is not easy to display a graph in a way such that the fewest number of edges intersect each other. So if one chooses to give input via the console one has to determine how to display the graph. An easy way of doing this would be to let Graphviz make a drawing of the graph and then show this drawing.

However all of this can be avoided if the burden is placed on the user instead, which is done with the first solution, since the user is placing the vertices by himself. On the other hand using this solution makes it a bit more difficult to create edges and to change the position of vertices fast. But because the GUI consists of small problems and creating graphs through the GUI does not affect the basic model, I choose to use a simple slow algorithm for finding the nearest vertex to the mouse cursor, which takes linear time. This could be improved to $O(\lg n)$ by using a kd-tree or similar data structures.

The problem however does not disappear, because instances of a graph problems, which are not created by the user, but by a transformation still needs to be displayed. One might still think that the best way of solving this is by letting Graphviz do it, but this is not necessarily the case. As discussed in Section 4.1 transformations yields special cases and these special cases have a certain structure, and this makes it easier to display them in a nice way. An example of such a transformation is the one going from 3-Satisfiability to Vertex Cover.

However displaying the special structure of the transformed instances requires the transformation to calculate a position for each vertex, which is waste of time and memory for the basic model. This is solved by making two transformation methods one for the GUI and one for the optimized model.

Doing all of this may work nicely when transforming from a non-graph problem to a graph-problem, but when the transformation is from a graph-problem to another graph-problem which includes padding e.g. the transformation from Hamilton Cycle to Hamilton Path, then it is any thing but easy to figure out where to place the new vertices, such that it looks nice.

I however still feel it is best to display graph problems by hand instead of leaving it to tools like Graphviz.

Because some of the transformed graphs will look bad even with my best intentions, I have made it possible to move the vertices around, so that the user have the possibility to adjust the graphs layout to his liking.

7.2 Transforming, solving and detransforming instances

To transform one instance to another requires that one knows the available transformation methods and it would be preferred if they were obtained automatically to make the program flexible for extensions or changes.

This problem has previously been solved in section 6.1 by giving all transformation methods a certain prefix, that made them distinct from other methods in a class. This problem is however a bit more difficult because some classes may now have two methods for the same transformation where one of them is used for GUI and the second for the basic model. However they still both use the same detransformation method.

Like with the transformations it is necessary to know the available heuristic algorithms, before they can be used and they are found by giving their methods a special prefix, that makes them distinct.

Regarding detransformations it should be possible to tell a transformed instance, I_2, to detransform one of its solution and then the solution would automatically be detransformed and given to its original instance, I_1.

Now if the solution from I_2 is to be given to I_1, which it came from then it must have a reference to it. This reference can also be used to see if it is a transformed instance e.g. if the reference is null then it is not a transformed instance. Secondly I_2 needs to know what method to call on I_1, because all detransformation methods lies in the instance, which created the transformed instance. This would normally seem difficult to do dynamically, however by using Javas reflection library once again the needed method can just be sent to I_2. The detransformation method needs to be invoked on I_1, but I_2 already contains a reference to I_1, which completely removes the problem.

7.3 Final Design

The prettiest way to implement the GUI would be to extend as much of the basic model as possible, however a number of things makes this impossible. First of all each problem now needs information on all available transformations, algorithms and solutions, and it now also needs a reference to the instance that transformed it and the necessary method to detransform any solution. All of this data is redundant to the basic model, which is why a new class has to be made.

To sum up all NPC problems in the GUI needs:

- List of transformations, algorithms and solutions,
- reference to the instance that transformed this instance and corresponding method for detransformation
- and a field containing the problem represented by the basic model, the `NPCProblem` class.

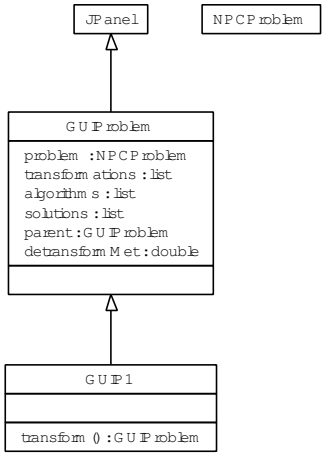


Figure 7.1: UML diagram of the design for NPC Problems in the GUI.

An abstract class called `GUIProblem` , see Figure 7.1, has been made containing all of the above data, since it is data all NPC problems for the GUI will need. This means that all NPC problems will have to inherit this class. It also means that the transformations should no longer produce instances of the basic model, the `NPCProblem` class, but instead return instances of the `GUIProblem` class.

This means that for each transformation method in the `NPCProblem` class, a corresponding method will have to be made for the `GUIProblem` class. This may seem like a lot of work, but because `GUIProblem` contains a `NPCProblem` object, *problem*, which contains all the data, it only has to call the transformation method on *problem*.

The fact that `GUIProblem` objects calls the transformation method in the corresponding `NPCProblem` object, means that the instance with two methods for the same transformation is gone, because the `GUIProblem` object can just choose to call the transformation method for the GUI.

7.4 More sophisticated features

A lot of features, which are not discussed in detail here has been implemented in the GUI, however there are still room for improvements.

Whenever an instance,I_1, of a problem has been transformed, this instance may no longer be modified and that goes as well for the transformed instance. However it would be practical to try and change the existing problem, I_1, to see how the change affects the solutions one achieves. In order to do this, one has to make a new instance, which is identical to I_1 before one can make the change. In this scenario it would be a lot easier if one could use an existing instance as a template for new instances.

It is tedious to create the same problems every time one starts the program, so it would be nice if one could save and load instances to and from a file.

Furthermore it would also be nice to remove old instance which are no longer needed, this goes both for userdefined and transformed instances and old solutions as well.

The implementation of Dijkstras algorithms is mostly meant for automatically solving a given instance, while the GUI is meant as a tool or playground, where one can get a intuitive feeling of how the transformation scheme works. However I still find it interesting to support the automatic problem solving in the GUI as well.

The tools for creating and modifying a graph do in its current form not support removal of edges or vertices, which would seem as a desired operation to support.

Even though the GUI works with small problems compared to the basic model

it should be no excuse for not making it efficient. There are a number of places where more fitted data structures may be used especially for the graph tool. Some of the display methods are also a bit naive drawing all of its contents even though only a small part of it may be visible.

I have made an effort to make the panels flexible to size change, such that scroll panes appear if the content cannot be fitted onto the given panel. I did however not put the same effort in removing or resizing the scroll pane again, when the content could be displayed on the given panel.

Because of the limited time I have not prioritized descriptive error messages, which means that even though I consider the GUI somewhat foolproof, I still feel that certain errors are not quite self explaining. For instance using the transformation from 3-Dimensional Matching to Partition may very easily fail. This is not because it does not work, but because of the overflow, which can occur if the integers for partition needs more than 63 bits to be represented. This kind of error is not fully explained. The same goes for most of the commands in the console, which will only tell whether or not it was executed.

Chapter 8

Code overview

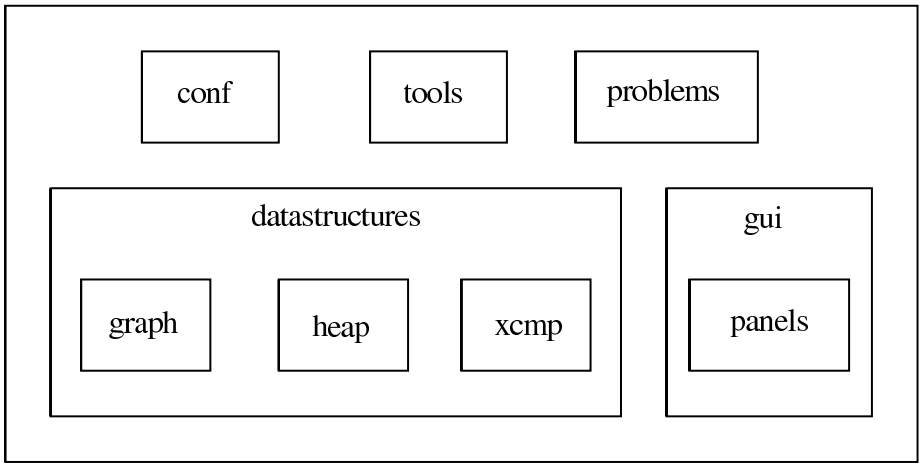


Figure 8.1: Package diagram over the source code.

conf package: The conf package contains miscellaneous global constants, such as location for Graphviz, regular expressions used for the console, prefixes used

for recognition of methods. It also contains a class for loading and saving the transformation graph to XML.

datastructures package: The datastructures package contains all the general data structures, such as clauses, triples, point class for positioning and drawing vertices etc.

datastructures.graph package: Datastructures.graph is a subpackage containing a abstract definition of a graph, and three different implementations of it.

datastructures.heap package: Dijkstra and different algorithms uses priority queues, which are implemented using either min or max heaps. Both of these heaps and their auxiliary classes are located in the subpackage datastructures.heap.

datastructures.xcmp package: The Dijkstra algorithm may not run on just any graph, it is required that the type of data on the edges can be added together. To make it a little more specific an abstract definition of the kind of data allowed has been implemented called XComparable. This class and its implementations are located in the datastructures.xcmp subpackage.

gui package: The gui package contains the main components for displaying the gui.

gui.panels package: Gui.panel is a subpackage which contains all the different JPanels for each NPC problem in the GUI.

problems package: The problems package contains the data model, consisting of all the NPC problems inheriting `NPCProblem`.

tools package: The tools contains miscellaneous tools, such as a generator of random satisfiability instances, a class that collects empirical data from random satisfiability problems and a class containing about eight different sorting algorithms.

Chapter 9

Strategy for data gathering

The goal of this project is to analyze the results from transforming and solving a problem and then detransforming the found solution. This section describes a strategy for gathering results from using this transformation scheme.

In order to compare the results achieved with as many instances as possible it would be best to create instances of a problem which can be transformed to as many different NPC problems as possible. The NPC problem that fits this description best is the Satisfiability problem. It was the first NPC problem found, which all other NPC problems was transformed from. Put in other words it should indirectly have transformations to all other known NPC problems.

It is obvious that the results obtained depends on the input, which means that one should strive to try as many different kinds of input as possible to make sure a single outlier does not mess up the final result. The best way of doing this would be to make the computer create random input and collect data automatically, since it otherwise would be too time consuming to create problems instances.

9.1 Random Satisfiability input

Creating random Satisfiability problems however contains a couple of issues. For instance let us say that we create instances with 1 to n boolean variables, should we create equally many instances with 1 boolean variable compared to those with n. This would of course be foolish, because problems with 1 variable can contain two different clauses, yielding a total of 3 different Satisfiability instances with 1 boolean variable.

To be more general when a Satisfiability instance has n boolean variables, then each variable can have three states in any of the clauses - it can be present, negated or not negated. This yields that with n boolean variables one can create $3^n - 1$ different clauses. In a Satisfiability instance each of these $3^n - 1$ clauses can be present or not present, which gives $2^{3^n-1} - 1$ different problems when having n boolean variables. Since we would want to make problems with 1 to n variables the total number of different instances will be:

$$\sum_{i=1}^{n} 2^{3^i-1} - 1$$

So with respect to the kinds of instances one can get, the probability of making an instance with j variables should be:

$$\frac{2^{3^j-1} - 1}{\sum_{i=1}^{n} 2^{3^i-1} - 1}$$

While this calculation may be correct with respect to the problem size it is not feasible to use. From working with this project I know that an average computer can solve any Satisfiability instance with 10 boolean variables with exhaustive search within 1 minutes. This means that it would be reasonable to assume $n \approx 10$, however if this be the case then the above calculations would overflow unless special data structures are used.

The point to be made is, that my intentions are to create somewhat evenly distributed instances according to the number of instances a variable can create, but it should not be done without respect to the system I work on. Instead I choose the probability with respect to the number of clauses j variables can create:

$$\frac{3^j - 1}{\sum_{i=1}^{n} 3^i - 1}$$

Now given an instance with j boolean variables, one should create k clauses, where k is randomly selected between 1 and $3^j - 1$. The problem is now, how to create k clauses.

One way of doing it would be to find k randomly selected numbers between 1 and $3^j - 1$ and to interpret each of these numbers into a clause. The problem of doing this is that there is a probability of getting the same clause twice, which would make the final result quite misleading, since the probability of getting the same clause twice increases the closer k is to $3^j - 1$.

So what we really want is to find k different numbers between 1 and $3^j - 1$ without sacrificing too much extra time.

I have two possible algorithms for this, both can be found in Appendix A.

A description of the first algorithm can be found in Appendix A.1. The main strength with this algorithm is that it runs in linear time with respect to the number of clauses, however its drawback is that there are instances which are way much more likely to get than others.

In other words there is a very low probability of getting the clauses in the lower end of the interval, so the random instances are not distributed uniformly.

Another problem is how to determine the random step length, between each clause. Depending on the chosen value one will risk getting instances with either many or few clauses.

The second algorithm is described in Appendix A.2. This algorithm runs in $O(m \lg m)$ and is therefore a bit slower than the previous. However where the previous algorithm had a very low probability of choosing the clauses in the lower part of the interval, this algorithm chooses all clauses with somewhat equal probability.

Furthermore it chooses the clauses such that, there is a high probability of them getting uniformly distributed over the interval and this is done without sacrificing the opportunity of getting instances where the clauses are clustered together. For the results to be as valid as possible it is important that all instances can be created even though the probability of getting each instance is different. This is why I have chosen to use this algorithm for generating random

input to the testing.

One drawback the algorithm however has in its present form is that it is recursive, which becomes a problem when m gets really big. This can however be solved either by making the algorithm iterative or by allocating more memory to the stack, the last solution is what I refer to as cheating. The aim of this project is to work well with the given resources and not with the resources it does not have at its disposal.

Note that because the algorithm is a divide and conquer algorithm it is very easy to make it multithreaded, which gives a running time of $O(\frac{m}{t} \lg m)$, where t is the number of threads(processors) used.

When the algorithm is not threaded the clauses in the randomly made instances of Satisfiability will be sorted e.g. only the last clauses will contain the nth variable and of the first clauses, which did not contain the nth variable only the last part contains the $n-1$th variable and so on. This also goes for the first algorithm and it may have a great influence on the final results, that the clauses are sorted this way, which should be kept in mind when evaluating the results.

Chapter 10

Results

To see whether or not the scheme of this project can be used for practical use, results will have to be gathered and analyzed. In the previous chapter is was discussed how to create instances this chapter will analyse the obtained results. The results will of course depend heavily on the heuristic algorithm and the transformations used, which is why they are discussed separately.

Before analyzing the obtained results one should first analyze the NPC problems from a theoretically point of view in order to see whether or not the results are good or bad.

10.1 Theoretical results from Satisfiability

The data gathering strategy used, detransforms all heuristic solutions found to the Satisfiability problem. This means that it suffices to make an analysis of the Satisfiability problem.

As earlier mentioned in Chapter 9, a Satisfiability instance which has n boolean variables can have 1 and up to $3^n - 1$ clauses. The question is then how many of the clauses can be expected to be satisfied.

It is trivial to see that it only takes two clauses before an instance can no longer be satisfied e.g. $\bar{x}_i$ and x_i. This is a lower bound to how many clauses one can expect to be satisfied.

To find an upper bound one should consider the case where all $3^n - 1$ clauses are present. Of all the clauses $\frac{2}{3}3^n$ of them will contain either the literal $\bar{x}_i$ or x_i, which means that no matter what truth value x_i gets one is guarantied to have $\frac{1}{3}3^n$ clauses which are satisfied. This means that there are $\frac{2}{3}3^n - 1$ clauses which are not yet satisfied. Almost like before $\frac{2}{3}\frac{2}{3}3^n$ of the remaining clauses contain either $\bar{x}_{i+1}$ or x_{i+1}. So $\frac{2}{9}3^n$ of the $\frac{2}{3}3^n - 1$ clauses are guarantied to be satisfied. Generalizing this will give the following sum, which describes how many clauses that can be satisfied at most:

$$\sum_{i=0}^{n-1} \frac{2^i}{3^{i+1}} 3^n = 3^n \sum_{i=0}^{n-1} \frac{2^i}{3^{i+1}} = \frac{3^n}{2} \sum_{i=0}^{n-1} \left(\frac{2}{3}\right)^{i+1} = 3^{n-1} \sum_{i=0}^{n-1} \left(\frac{2}{3}\right)^{i}$$

$$\sum_{i=0}^{n-1} \frac{2^i}{3^{i+1}} 3^n = 3^{n-1} \frac{\left(\frac{2}{3}\right)^n - 1}{\frac{2}{3} - 1} = 3^n - 2^n$$

So one should expect the number of clauses satisfied for each problem to be between 1 and $3^n - 2^n$. Converting this to percent one sees that as n increases the number of satisfied clauses will be between 0 and 100 percent of all the clauses.

However the way the random instances are created for the data gathering it is very unlikely that the lower bound will be reached. Actually if one assumes all clauses in an instance are chosen with equal probability and this is almost the case with the used algorithm, then one can model the number of satisfiable clauses with a hypergeometric distribution. In this distribution one has $3^n - 1$ clauses to pick between and $3^n - 2^n$ of the clauses can be satisfied. So if an instance contains m clauses then the mean value or average number of clauses satisfiable will be:

$$m \frac{3^n - 2^n}{3^n - 1} \approx m \left(1 - \left(\frac{2}{3}\right)^n\right)$$

From this result one can see that as n the higher percentage of the clauses should be expected to be satisfied. This result does however not state whether or not it gets easier to satisfy clauses.

10.2 Testing setup

All the results obtained has been achieved on the DTU Bohr-server. It is a Sun Fire E6900, which consists of 48 UltraSPARC IV CPUs (1200 MHz/dual-core/8 MB L2-cache per core) and has 98 GB of memory.

This may seem as a lot of computing power, however because the tests have been running under a student user-id, only a fraction of the resources has been available and I am sure the others user are glad this is the case.

Furthermore most of the algorithms tested runs in a single thread, which means that the running time would not be affected to much by all the CPUs available.

10.3 The Zeus algorithm

The first algorithm implemented is the algorithm I call Zeus and was developed for the Independent Set problem. The description of the Zeus algorithm can be found in appendix B.1.

To use the Zeus algorithm on instances of the Satisfiability problem, the instances were transformed to 3-Satisfiability through Vertex Cover and finally to Independent Set. One should note two things about these transformations the first is that the graph produced for Vertex Cover is a very sparse graph. Secondly, this graph is exactly the same for Independent Set, which means the transformation from Vertex Cover to Independent Set consists only of copying the Vertex Cover graph.

The first implementation, which was tested, used adjacency matrix for the graph, because it was easier to use and it solved some minor implementation issues when dealing with undirected graphs. However the punishment of doing so was that any instances with more than $1,000$ clauses lead to an outOfMemoryError by Java after about five minutes of waiting.

As mentioned above the graph, which is produced by the transformation is very sparse, so the problem was overcome by using adjacency list instead, which uses less or just as much memory as the adjacency matrix. The drawback of using adjacency list is that certain operations are slower and it is only when receiving transformed instances from 3-Satisfiability one is guarantied that the graph will be sparse.

The improvement of the graph data structure improved the running time of the transformation a great deal, but the memory limit was only shifted to around 2,000 clauses. This limit was however not caused by the transformation, but by the recursive quicksort routine used in Zeus for sorting the heap, which gave stackOverflowException, when reaching the limit.

To deal with this an iterative version of the quicksort routine was developed, which had its own stack. This seems to have completely removed the limit to how many clauses a Satisfiability instance can have[1]. A new problem however occurred, because the sorting procedure now ran very slow. As an example $51,000$ clauses took about a day to solve. When transformed $51,000$ clauses corresponds to roughly 1 million vertices for Independent Set.

The reason for the sorting procedure running so slowly seems to be, that the iterative quicksort uses up the given RAM and then uses the HDD instead for storage, which makes it extremely slow to retrieve the data when needed. The only explanation for this behavior is that the sorting routine has its own stack, which somehow seems to use much more memory than the internal stack, which Java uses for its method calls.

It should be mentioned that the iterative quicksort has no problems sorting 1 million elements within reasonable time, when there is not used memory on transformed instances and such. So it seems to be the lack of memory that causes the iterative quicksort routine to be slow.

Simply by changing the sorting routine used in Zeus from the iterative quicksort to a standard mergesort routine, the running time was improved greatly. An instance with $51,000$ clauses could now be solved within a minute opposed to 24 hours, when using the iterative quicksort. The benefit from using quicksort was the unstable sorting, which was used to randomly select vertices. This feature is lost with mergesort, which makes this implementation of the Zeus algorithm a normal deterministic one.

The reason for using an iterative quicksort was that the recursive version resulted in stack overflow at about $2,000$ clauses, so it seems a bit strange that a similar reaction is not obtained when using a simple recursive mergesort procedure. The best explanation for this absence is that quicksort in general produces a higher stack than mergesort, which always will produce a stack of size $O(\lg n)$. It still however seems a bit odd that mergesort can handle almost up to $100,000$ clauses, when quicksort cannot handle $2,000$.

The deterministic implementation of Zeus did in general get very close to the

[1] There will of course always be a memory limit set by the systems hardware.

optimal result. This is shown in Figure 10.1a, which shows in percent how close the heuristic solution was to the optimal. The precise data can be found in Appendix C.

Because of these fine results the final implementation of Zeus reintroduced some randomization to the sorted heap, just to see if the randomization would deliver even better results. The results from this implementation is shown in Figure 10.1b, where the results seems to be just as well as before. The only difference worth mentioning is that the implementation runs a bit slower because of the extra calculations for the randomization.

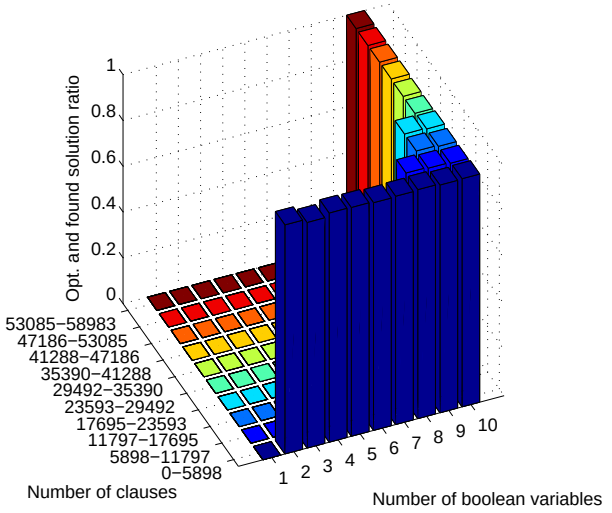
What is interesting about both implementations, is that the heuristic solution is always so close to the optimal. The greatest absolute difference measured for the deterministic version was 527 clauses in an instance where the best solution was $28,000$ clauses, which is less than 2 percent from the optimal. The greatest relative difference measured was 25 percent in a problem where 8 clauses was the best solution and only 6 clauses was satisfied with the heuristic solution. With the randomized version the greatest absolute and relative difference was 562 clauses and 25 percent respectively.

It is also very interesting that the percentage seems to be independent of the number of variables and clauses in an instance. So if this percentage remains constant even as the number of variables and clauses increases, then it is possible to make a heuristic algorithm for a NPC problem, which gets very close to the optimal solution.

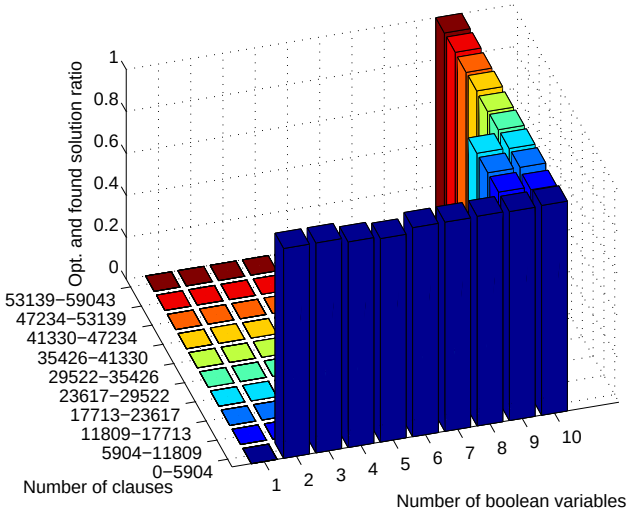
So the results are quite amazing however this is not quite the case for the running times. Figure 10.2a-b and 10.3a-b shows the running time for each transformation and the deterministic implementation of Zeus. The running time for the detransformation is not shown since it is practically zero. Again the data shown in the bar charts can be found in Appendix C.

The first thing one notes when looking at the running time is that, there are a few outliers for the last variables and they all have in common that they are close to the maximum number of clauses for the given number of variables. How this makes the running time increase significantly compared to the other samples remains unclear. It should however be mentioned that these outliers are based only on one to five samples, while all the other bars displayed are based on hundreds. So to view the other bars a bit better the outliers are replaced yielding the bar charts shown in Figure 10.4a-c.

Now what one should note is that the running time increases after each transformation, which happens because of the padding done at most transformations. This makes the input size increase for the next transformation, which needs to

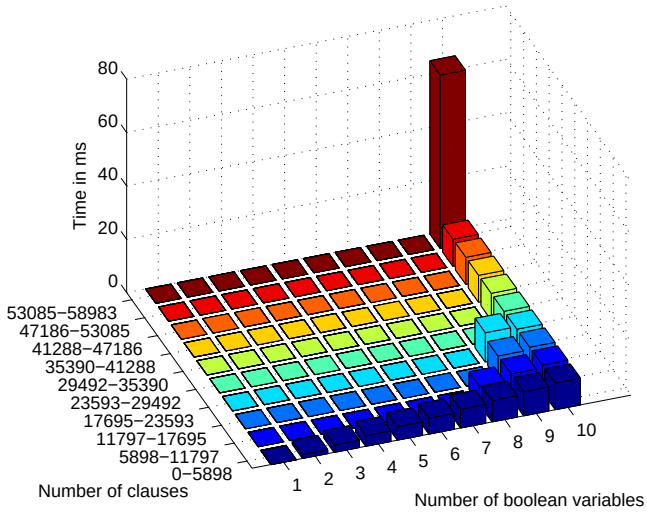


(a)

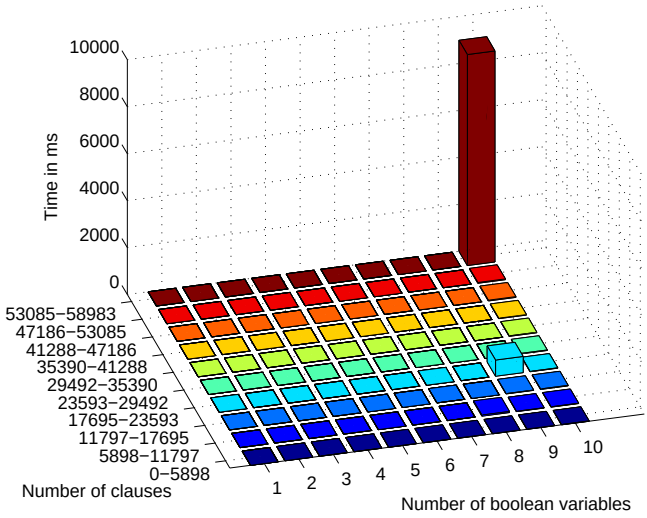


(b)

Figure 10.1: Data showing how close the heuristic solution was to the optimal solution in percent (a) for the deterministic version of Zeus using $10,000$ samples (See Table C.1 in Appendix C) (b) and the implementation of Zeus with randomization using $11,000$ samples (See Table C.2 in Appendix C).

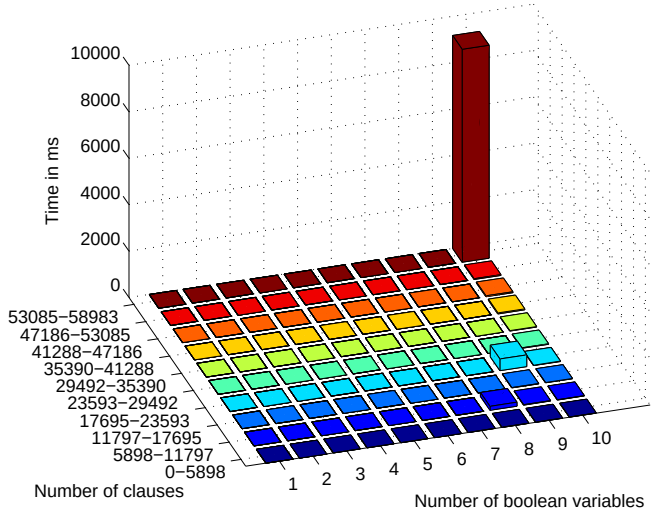


(a)

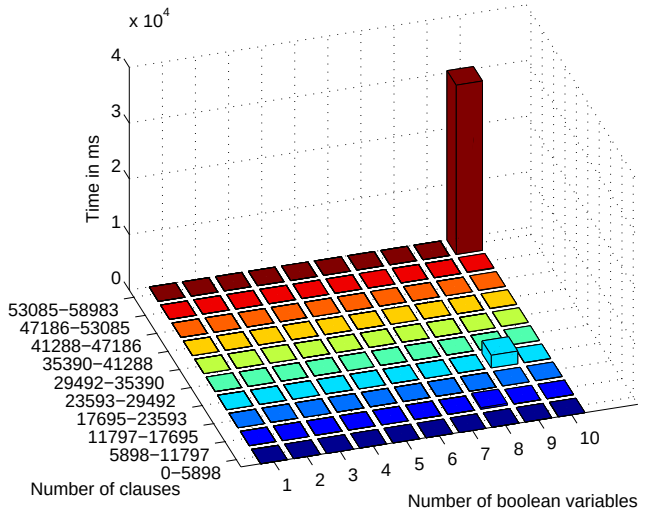


(b)

Figure 10.2: Running time for (a) the transformation from Satisfiability to 3Satisfiability (See Table C.3 in Appendix C)(b), the transformation from 3Satisfiability to Vertex Cover (See Table C.4 in Appendix C) both using 10,000 samples.

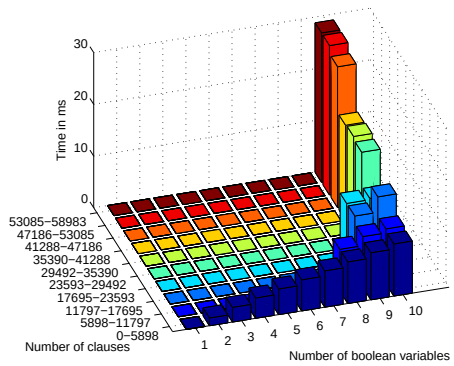


(a)

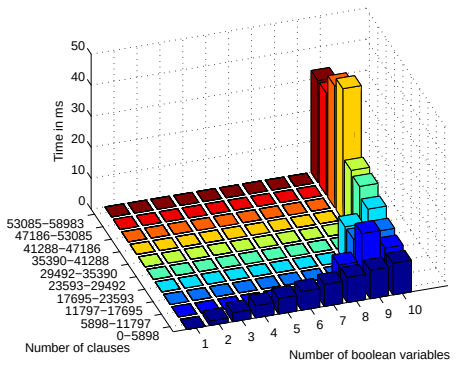


(b)

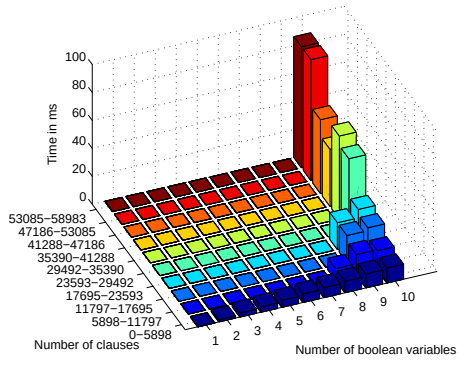
Figure 10.3: Running time for (a) the transformation from Vertex Cover to Independent Set (See Table C.5 in Appendix C)(b) and the deterministic implementation of Zeus (See Table C.6 in Appendix C) both using 10,000 samples.



(a)



(b)



(c)

Figure 10.4: Running time for (a) the transformation from 3Satisfiability to Vertex Cover (b), the transformation from Vertex Cover to Independent Set (c) and the deterministic implementation of Zeus all using 10,000 samples without outliers.

be applied, and this increase causes the running time to increase as it does.

It should also be noted that the first two transformations have the asymptotic running time $O(m+n)$ where m is the number of clauses and n is the number of variables. However the results are gathered in such a way that for a problem with n variables, there will be created instances with 1 and up to $3^n - 1$ clauses which will make the testing scheme take exponential time($O(3^n + n)$) as n increases.

This means that one should keep the number of clauses constant, because otherwise the input size is guarantied to increase exponentially, which will also give exponential running time. Normally when m is not constant the running time of exhaustive search would be $O(3^n 2^n) = O(6^n)$, because each clause is tested with each possible assignment, however when m is constant then the time is $O(m2^n) = O(2^n)$. This means that exhaustive search still takes exponential time, but the time however does not increase nearly as fast as before.

If one looks at Figure 10.4a-c again now only at rows with the same number of clauses one will see, that the running time is somewhat linear. However the problem is still that each transformation not only padds data to the instances but also itself requires memory. As an example each transformation creates a transformed instance and this instance has to be stored just as the original instance it was created from otherwise detransformation cannot be done later on.

So the input, which the heuristic algorithm needs to take, will grow for each transformation used, but even if there is no padding, which is the case for the transformation that goes from Vertex Cover to Independent Set, each transformation still takes up at least as much memory as the original problem. This makes use of the transformations unsuitable for large sized problems, which was the aim of this project.

In Figure 10.4a-c the running time may look somewhat good if the number of clauses are kept constant, but it does not show how much memory the transformation takes and pads. The fact is if the number of boolean variables where increased a bit more then Java would return an outOfMemoryError as a result of all the memory the transformation uses and pads to transformed instances.

What should also be mentioned is that this example is actually one of the cheapest ones. Remember that the Vertex Cover graph gotten from transformed 3-Satisfiability instances was sparse. If the heuristic algorithm now was for Clicque instead of Independent Set, then the transformation would require one to complement the sparse graph from Vertex Cover. This graph will not be sparse and it is very likely that one will get outOfMemoryError at about $1,000$ clauses, which was the case when adjacency matrix was used.

I am not blind to the fact that the implementation can still be improved but it would not change my conclusion that it is not feasible to solve NPC problems using the transformation. It is nonetheless possible to do, which has been proven in this project.

10.4 The Hera algorithm

Even though it is impractical to use the transformations for solving NPC problems, it is interesting that the Zeus algorithm obtained results very close to the optimal.

In order to see if the results was caused by the transformation or the heuristic algorithm, I detransformed the Zeus algorithm for Independent Set to an algorithm for Satisfiability, which I refer to as Hera. A more precise description of the algorithm can be found in appendix B.2.

The detransformation of Zeus to Hera was done by looking at the transformations one at a time. For instance in Independent Set Zeus chooses the vertex with fewest neighbors first, so to detransform it to Vertex Cover it would now have to choose the vertex with most edges first and so on.

If the good results was caused by the transformations then the results produced by Hera will decrease in quality, because the instances for Hera are not transformed and detransformed.

However surprisingly Hera seems to get solutions even closer to the optimal solution than Zeus, which is shown in Figure 10.5. Like before the data from the bar chart can be found in Appendix C. The greatest absolute difference was 420 clauses with an instance containing $116,000$ clauses and the greatest relative difference was 10 percent in an instance containing $12,500$ clauses.

This result could of course be explained by the fact that the heuristic algorithm may have changed a bit during the detransformation, which means that it cannot be directly compared with Zeus. Luckily there is a very easy way to check if the solution gets worse when being found for the transformed instance. One can simply transform the Satisfiability instance to an instance of 3-Satisfiability, because 3-Satisfiability instances can also be solved by Hera.

There however seems to be no notable difference between using Hera on Satisfiability or on transformed 3-Satisfiability instances, which means no conclusion can be made. The results can be seen in Figure 10.6 and in Appendix C.

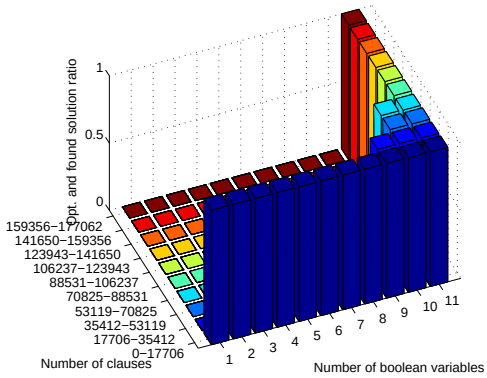


Figure 10.5: Data showing how close the heuristic solution was to the optimal solution in percent from Hera(See Table C.7 in Appendix C) using 16,000 samples

10.5 The Poseidon algorithm

Just to show how difficult padding make things, I have made a simple heuristic algorithm called Poseidon for the NPC problem, Subset Sum. Its decription can be found in Appendix B.3. The catch is, in order to reach Subset Sum from Satisfiability a very expensive path has to be taken in terms of padding. The route which will be taken goes from Satisfiability to 3-Satisfiability and then to 3-Dimensional Matching moving on to Partition and then Subset Sum. There are two transformations in this path, which are worth noting. The first is the transformation from 3-Satisfiability to 3-Dimensional Matching, where m clauses over n boolean variables will be transformed in to approximately $2(nm)^2$ triples with $q = 2nm$. So a single clause over 3 boolean variables will be transformed into approximately 20 triples with $q = 6$.

Now the transformation from 3-Dimensional Matching to Partition assumes that an instance with a given value q and k triples can be transformed into integers with $3pq$ bits, where $p = \lceil \log_2(k+1) \rceil$. So given the transformed instance from 3-Satisfiability one would need 90 bits to store the integer. At this time 32 bits are more or less a standard for variables and 64 bits are available when larger values or greater precision is needed. So the point is that just one single clause would give an overflow even when using 64 bits for the variables, which explains why I did not implement the transformation at first, it is simply too intractable.

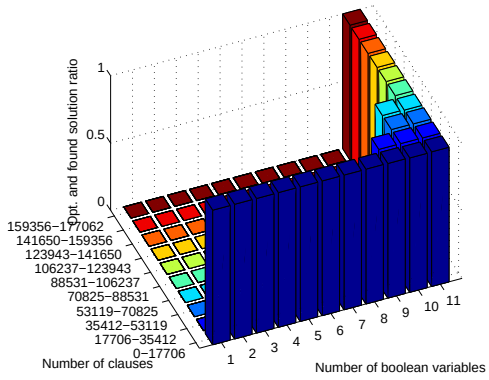


Figure 10.6: Data showing how close the heuristic solution was to the optimal solution in percent from Hera(See Table C.8 in Appendix C) on instances of 3-Satisfiability using $10,000$ samples

Now what one might be tempted to do at this point is to make a special class giving any precision wanted for an integer or floating point for that matter. But if this is done, then addition, subtraction, multiplication and division may no longer be assumed to run in constant time, which will probably make heuristic algorithms for the Set NPC problems slow.

10.6 What was not completed

As a result from the conclusions made above certain aims of this project became impossible or intractable.

One of the aims of this project was to compare different algorithms from different NPC problems. Clearly this is quite difficult to do unless both the NPC problems are very close to the NPC problem which will be made instances of.

It would also have been interesting to see if the heuristic solution found for the transformed instance is just as good here as when it is detransformed, see Figure 2.1b. A part of this has been done when analyzing how the transformation affected the solutions found with Hera. However the best way of doing this would have been to find the optimal solution for the transformed instance and compare it with the heuristic solution found. But this cannot be done in reasonable time

because of padding, it would simply take too much time to find the optimal solution with exhaustive search.

Chapter 11

Conclusion

In this project I have accomplished most of the desired goals:

- I have made a framework containing about a dozen different NPC problems ranging from set problems to graph problems and I think its design makes it flexible for extensions.
- It is possible to transform instances of one NPC problem to an instance of another NPC problem and it is possible to solve an instance of one NPC problem using a heuristic algorithm for another type of NPC problem.
- One can make the computer automatically find the shortest route to a certain algorithm using either unit weight, asymptotic running time or empirical running time as the weight for the transformations.
- I have made what I think is a user friendly GUI, where it is possible to create, transform and solve instances of different NPC problems.
- A couple of different heuristic algorithms for different NPC problems have been created.

I have however not succeeded in making it feasible to solve NPC problems by transforming the given instance to another type of NPC problem. The reason for

this is the tremendous amount of padding, which the transformations introduce and the only way to fight this is to make a new and better transformation with less padding.

The motivation for transforming instances of a NPC problem was to avoid spending time developing new heuristic algorithms for the given NPC problem. So to me it seems like a bad idea spending time on developing better transformations instead of algorithms. I therefore conclude that it is not beneficial to solve any instance of greater size by transforming the instance to another NPC problem, where a heuristic algorithm exists.

Regarding the part of this project which tries to make the transformation of NPC problems more intuitive by making a GUI, I would like to note that it still lacks some proper error messages. I also feel that it does not give the proper amount of freedom, which I wanted it to have when constructing and manipulating instances.

Concerning the framework I am satisfied with the amount of problems and transformations implemented, but I think it could have contained more heuristic algorithms. It should however be noted that the primary motivation for developing the heuristic algorithms was to experiment with their behavior on transformed instances, which came from the same NPC problem, Satisfiability. But because it was intractable to transform instances I felt my work were put to better use on other matters than developing new heuristc algorithms for the framework.

In the last phase of this project I tried to see how the heuristic solutions acted to the transformations by running the same heuristic algorithm on different NPC problems. No clear answer was however achieved, but from the result with the Hera algorithm, it would seem that the quality of the heuristic algorithm remains the same with and without its solution being detransformed.

Working with this project I find that what has taken most of my time has been implementing the many different data structures needed for the many different NPC problems and heuristic algorithms. The fact that some of the data structures are needed for different purposes made it necessary to implement them in such a way that they where flexible for those different purposes. Especially the graph data structure has taken some time to develop, because it had to be flexible for both directed and undirected graphs, graphs with vertex data such as color, graphs where edges had unit weight and graphs where the edges had other types of weights.

At this point the generics in Java has been a great help for reusing the data structures in an elegant way. At the same time it should also be said that

overuse of generics, which was sometimes necessary, made it quite difficult to grasp the code.

Another very positive experience I have made with Java during this project is with their reflection library which has been very helpful on more than one occasion.

Overall I have been very satisfied with working with Java and I find that the tools for java are very good and increases production a great deal. Java version 6, which is at the moment the most current version, however should have one last comment on the way, because it does not seem to handle outOfMemory errors too well and sometimes gives misinforming error messages, but then again who am I to talk about error messages.

11.1 Where to go from here

Even though it is intractable to transform instances of NPC problems, I still find that all the transformations made for the NPC proofs possess a lot of information, which are begging to be exploited.

One of the ideas, I find that may be worth pursuing, is to transform heuristic algorithms instead of instances. Utilizing this idea one would only have to make the transformation once for a single heuristic algorithm and then it could be used freely without any memory problems like this project suffered from. This may however proof to be very difficult to do, because it would require that the strategy used by the algorithm can be transformed automatically. The same goes for the data structures, if the data structures cannot be transformed effectively, then the transformed heuristic algorithm will probably be too slow to be of use. But under all circumstances it may be an idea worth exploring given that a transformed heuristic algorithm in fact can give just as good results as the algorithm it was transformed from.

When working with this project I found that when transforming instances of X taking two transformations, t_1 going from X to Y and t_2 going from Y to Z, one could replace the two transformations with one going from X to Z. If this can be automated then eventually all NPC problems in a strongly connected component would be able to reach each other using just one transformation, which may make the transformation scheme more feasible.

Appendix A

Creating Random Satisfiability Instances

This section presents two algorithms for creating random instances for the NPC problem called Satisfiability. Both algorithms preserves the property that no clause occurs twice in the instance and no boolean variable exists twice in the same clause.

Both algorithm assumes that it is known how many boolean variables the instance should contain. If the instance contains n boolean variables then 1 to $3^n - 1$ clauses can be created and it is assumed that one knows how to convert a number between 1 and $3^n - 1$ to a clause with at most n literals, where no boolean variable is present more than once.

A.1 First suggested algorithm

The first algorithm simply chooses a random starting point in the given interval and then randomly walks through the interval until the end is reached.

In pseudo code it can be described like this:

High Level Description A.1.1 $rsi1(n)$

Find a random number, r, between 1 and $3^n - 1$.
while $r < 3^n - 1$ **do**
  Convert the r to a clause and add it to the Satisfiability instance.
  Find a small random number,k, and let $r = r + k$.
end while
return the created Satisfiability Instance.

The running time of this algorithm is at most $O(3^n)$ or $O(m)$, where m is the number of clauses in the instance.

A.2 Second suggested algorithm

The second algorithm is a recursive divide and conquer algorithm. It assumes one knows how many clauses should be created and in what interval they should be created from.

The base case of the algorithm is when there is just one clause to be chosen in the interval. This is handled simply by choosing a random clause from this interval.

The recursive case is when there are m clauses to be created, where m is more than one. In this case the interval is broken randomly into two new intervals, where $\lfloor \frac{m}{2} \rfloor$ clauses is found in the first interval and $m - \lfloor \frac{m}{2} \rfloor$ clauses are found in the other interval.

The pseudocode for this algorithm can be seen below:

High Level Description A.2.1 $rsi2(n, m, start, end)$

if $m = 1$ **then**
  Return random clause from the interval *start* to *end* to the instance of satisfiability.
else
  $c_1 = \lfloor \frac{m}{2} \rfloor$ and $c_2 = m - c_1$
  Chose the value, *split*, between the interval $start + c_1$ to $end - c_2$
  $rsi(n, c_1, start, split)$
  $rsi(n, c_2, split + 1, end)$
end if
return the clauses for the Satisfiability instance.

If $T(m)$ is the running time of the algorithm then one can make the following recurrence equation, $T(m) = 2T(\frac{m}{2})$, because in the recursive case two new method calls are made but this time the input is half the size. In the base case one has T(1)=1. By solving the equation one gets that the running time is $O(m \lg m)$.

Appendix B

Heuristic Algorithms

This section contains description of the heuristic algorithms developed and used in this project.

B.1 Zeus

The algorithm I refer to as Zeus is a greedy algorithm for Independent Set and it uses the strategy to always pick the vertex which has fewest neighbors. This vertex is now in the independent set and all of it neighbors can now never be in the independent set.

This algorithm is described a bit more precise in the following psedocode, which will be used to analyze its running time:

High Level Description B.1.1 *zeus*()

```
Create minimum priority queue, Q, for the vertices using their number of
neighbors as key.
while Q is not empty do
  min = Q.extractMin()
  sol[min] = true
  for each neighbor v to min do
    if v is in Q then
      Q.remove(v)
      sol[v] = false
      for each neighbor w to v do
        if w is in Q then
          Q.decreaseKey(w)
        end if
      end for
    end if
  end for
end while
return sol
```

Before one can create the priority queue one has to know the key value for each vertex. With a graph represented by an adjacency matrix this would take $O(n^2)$, where n is the number of vertices. Using an adjacency list however brings it down to $O(n + m)$, where m is the number of edges in the graph.

If the priority queue is represented by a heap, then it will take $O(n \lg n)$ to initialize it as described in [6]. However at this point there are some interesting opportunities when implementing the algorithm, because basically a priority queue using a heap is just a list where the elements are arranged in a particular way, which means insertion and extraction takes $O(\lg n)$. Now if one chooses to sort this list increasingly then it has been initialized as wanted. Of course the asymptotic running time will still be $O(n \lg n)$, however if one uses a randomized quicksort implementation, then the Zeus algorithm will automatically be non deterministic, because of the unstable sorting[1] that randomized quicksort has. Using a divide and conquer algorithm like quicksort for this step, one will be able to increase the running time by making it work in parallel.

This non determinism can of course also be achieved by permuting elements with the same value after the heap has been initialized.

[1] Unstable sorting means that two elements with the same value are not guarantied to have the same place in the sorted list in two different runs of the sorting routine.

Above it was mentioned that the initialization takes $O(n \lg n)$, this is however not exactly true. If one looks at the data for the priority queue, then one will see that the keys are all integers and the values are all smaller than n, because a single vertex can not have more edges than there are vertices, assuming there are no parallel edges. This means that the sorting(initialization) can be done with countsort in $O(n)$, however countsort uses a bit more memory and implementing it will not change the overall asymptotic running time.

After the initialization comes the while-loop. In each iteration of the loop at least one vertex is removed from the queue and there are never inserted new vertices into the queue. This means that the loop will have $O(n)$ iterations.

In each iteration the smallest element in the queue is removed which takes $O(\lg n)$ and because this is done in every iteration the total time would be $O(n \lg n)$.

The for-loop, which iterates over all neighbors will be executed in $O(m)$, because each edge will only be processed once.

Inside the first for-loop is a remove operation on the heap. This operation can at most be called $n - 1$ times, since that is the maximum number of elements the heap contains at this point. This gives a total running time of $O(n \lg n)$.

Now one should note that the second for-loop takes $O(m)$ in total and not $O(m^2)$. The reason is that edges which are processed in the first loop are not processed in the second loop and vice versa.

In the innermost loop there is a decreaseKey operation, which is done in $O(\lg n)$. Because the loop iterates in total $O(m)$, the running time for all decreasekey operations is $O(m \lg n)$.

So the running time of the entire algorithm is $O(n + m + n \lg n + m \lg n) = O(n + m + (n + m) \lg n) = O((n + m) \lg n)$.

B.2 Hera

The algorithm I call Hera is a greedy algorithm for Satisfiability and it uses the same greedy strategy as Zeus. However instead of picking the vertex with the lowest degree, it picks the literal which appears in most clauses, which are not yet satisfied.

Like before the algorithm is described more precise in the following pseudocode:

High Level Description B.2.1 *hera*()

```
Create maximum priority queue, Q, for all the literals using the number of
occurrences in the clauses as key.
while Q is not empty do
  max = Q.extractMax()
  if max is a negated literal then
    sol[max] = false
  else
    sol[max] = true
  end if
  for each clause, C, max occurs in do
    if C is not already satisfied then
      C is now satisfied
      for each literal, l, in C do
        if l is in Q then
          Q.decreaseKey(l)
        end if
      end for
    end if
  end for
end while
return sol
```

The Hera algorithm is a direct translation of Zeus to Satisfiability so most of the data structures will be the same.

To find the number of appearances of a single literal in all the clauses the time would be $O(mn)$, where m is the number of clauses and n the number of variables. So the total time for all the literals would be $O(mn^2)$, which is not optimal.

To improve this I make a data structure, which can best be described as an adjacency list for literals. This data structure contains a list for each literal, stating which clauses the literal appears in. Initialing this data structure takes $O(mn)$ and creating the data for the priority queue now takes $O(n)$.

Like with Zeus this algorithm is also best suited with a heap for the priority queue, because of the many remove and decrease operations needed. Regarding the initialization of the heap, there exists the exact same opportunities as with Zeus, where one can introduce randomization by using randomized quicksort

for building the heap and so on. Unless countsort is used initialization will take $O(n \lg n)$.

Like before the while-loop is guarantied not to iterate mote than $O(n)$ times, because in each iteration at least one variable is removed.

Removing the literal(variable) with most references in the clauses takes $O(\lg n)$ and because this is done $O(n)$ times the resulting time is $O(n \lg n)$.

The first for-loop is at most done k times, where k is how many clauses the literal occurs in. Note that the data structure created in the start can give the k clauses in $O(1)$.

Because this loop is done for each literal in every clause the total time of the loop is $O(nm)$.

The second for-loop will only be executed once for each clause. One clause can have no more than $O(n)$ literals, which means the total time for this loop is $O(nm)$.

For each variable, which has no yet been removed, in each clause a decrease operation is called on the priority queue. This operation takes $O(\lg n)$, so in total the time is $O(nm \lg n)$.

So the overall running time of the algorithm with the suggested data structures is $O(nm \lg n)$.

B.3 Poseidon

This algorithm, which I call Poseidon is a simple randomized algorithm for subset sum. If M is the sum of the integers in the subset, which has been chosen so far, then the algorithm randomly selects a number, k, between 0 and $B - M$ from the remaining integers. It then adds the integer closest to the value k to the subset.

Here is the pseudocode for the algorithm:

High Level Description B.3.1 *hera*()

Create an Binary Search Tree, L, with all the integers from the set S, which are less than or equal to B.
Let the variable, *remaining*, contain the value telling how close the chosen subset is to the value B. *remaining* is initialized to B.
if L is not empty and smallest element in L is less than *remaining* **then**
  while *remaining* > 0 and L is not empty **do**
    Let v be a randomly chosen value between 0 and *remaining*.
    Find the closest value to v in L.
    Add the closest value to v to the subset and remove it from L.
    update *remaining*.
  end while
end if
return the found subset.

The first step takes $O(nh)$, where n is the number of elements in S and h is the height of the Binary Search Tree.

The while-loop will be executed at most $O(n)$ because one element is ensured to be removed in every iteration.

Finding the closest value and removing it can both be done in $O(h)$, so the total running time of the algorithm is $O(nh)$. This can however be improved by using a balanced binary search tree, because then the height of the tree will be $O(\lg n)$, which gives a total running time of $O(n \lg n)$.

Appendix C

Data Tables

Clauses\Variables	1	2	3	4	5	6	7	8	9	10
$0 - 5898$	0	1.00	0.98	1.00	1.00	0.99	1.00	1.00	0.99	1.00
$5898 - 11797$	0	0	0	0	0	0	0	1.00	1.00	0.99
$11797 - 17695$	0	0	0	0	0	0	0	0	1.00	0.99
$17695 - 23593$	0	0	0	0	0	0	0	0	1.00	0.99
$23593 - 29492$	0	0	0	0	0	0	0	0	0	1.00
$29492 - 35390$	0	0	0	0	0	0	0	0	0	0.99
$35390 - 41288$	0	0	0	0	0	0	0	0	0	0.99
$41288 - 47186$	0	0	0	0	0	0	0	0	0	1.00
$47186 - 53085$	0	0	0	0	0	0	0	0	0	1.00
$53085 - 58983$	0	0	0	0	0	0	0	0	0	1.00

Table C.1: Data containing ratio between optimal and heuristic solution found with the deterministic version of Zeus.

Clauses\Variables	1	2	3	4	5	6	7	8	9	10
$0 - 5898$	0	1.00	1.00	0.98	0.97	1.00	1.00	1.00	1.00	1.00
$5898 - 11797$	0	0	0	0	0	0	0	0	1.00	0.98
$11797 - 17695$	0	0	0	0	0	0	0	0	1.00	1.00
$17695 - 23593$	0	0	0	0	0	0	0	0	1.00	1.00
$23593 - 29492$	0	0	0	0	0	0	0	0	0	1.00
$29492 - 35390$	0	0	0	0	0	0	0	0	0	0.99
$35390 - 41288$	0	0	0	0	0	0	0	0	0	1.00
$41288 - 47186$	0	0	0	0	0	0	0	0	0	0.99
$47186 - 53085$	0	0	0	0	0	0	0	0	0	1.00
$53085 - 58983$	0	0	0	0	0	0	0	0	0	1.00

Table C.2: Data containing ratio between optimal and heuristic solution found with randomized version of Zeus.

Clauses\Variables	1	2	3	4	5	6	7	8	9	10
$0 - 5898$	0	1.5	2.8	3.9	5.0	6.0	7.0	8.0	9.0	10.0
$5898 - 11797$	0	0	0	0	0	0	0	6.7	9.0	10.0
$11797 - 17695$	0	0	0	0	0	0	0	0	9.0	10.0
$17695 - 23593$	0	0	0	0	0	0	0	0	11.0	10.0
$23593 - 29492$	0	0	0	0	0	0	0	0	0	10.0
$29492 - 35390$	0	0	0	0	0	0	0	0	0	10.0
$35390 - 41288$	0	0	0	0	0	0	0	0	0	10.0
$41288 - 47186$	0	0	0	0	0	0	0	0	0	10.0
$47186 - 53085$	0	0	0	0	0	0	0	0	0	10.0
$53085 - 58983$	0	0	0	0	0	0	0	0	0	65.00

Table C.3: Data containing running times in ms for transforming from Satisfiability to 3-Satisfiability.

Clauses\Variables	1	2	3	4	5	6	7	8	9	10
$0 - 5898$	0	1.5	2.8	3.9	5.0	6.0	7.0	8.0	9.1	10.1
$5898 - 11797$	0	0	0	0	0	0	0	42.3	9.8	10.5
$11797 - 17695$	0	0	0	0	0	0	0	0	11.3	14.3
$17695 - 23593$	0	0	0	0	0	0	0	0	685.0	11.5
$23593 - 29492$	0	0	0	0	0	0	0	0	0	18.0
$29492 - 35390$	0	0	0	0	0	0	0	0	0	19.0
$35390 - 41288$	0	0	0	0	0	0	0	0	0	18.0
$41288 - 47186$	0	0	0	0	0	0	0	0	0	27.2
$47186 - 53085$	0	0	0	0	0	0	0	0	0	28.9
$53085 - 58983$	0	0	0	0	0	0	0	0	0	9000.0

Table C.4: Data containing running times in ms for transforming from 3-Satisfiability to Vertex Cover.

Clauses\Variables	1	2	3	4	5	6	7	8	9	10
0 – 5898	0	1.5	2.8	3.9	5.0	6.0	7.0	8.1	9.2	10.1
5898 – 11797	0	0	0	0	0	0	0	168.3	17.0	11.1
11797 – 17695	0	0	0	0	0	0	0	0	11.1	12.4
17695 – 23593	0	0	0	0	0	0	0	0	503.0	15.6
23593 – 29492	0	0	0	0	0	0	0	0	0	18.9
29492 – 35390	0	0	0	0	0	0	0	0	0	20.0
35390 – 41288	0	0	0	0	0	0	0	0	0	42.3
41288 – 47186	0	0	0	0	0	0	0	0	0	39.2
47186 – 53085	0	0	0	0	0	0	0	0	0	32.8
53085 – 58983	0	0	0	0	0	0	0	0	0	9200.0

Table C.5: Data containing running times in ms for transforming from Vertex Cover to Independent Set.

Clauses\Variables	1	2	3	4	5	6	7	8	9	10
0 – 5898	0	1.5	2.9	3.9	5.0	6.0	7.0	8.2	9.9	10.7
5898 – 11797	0	0	0	0	0	0	0	275.7	14.8	14.0
11797 – 17695	0	0	0	0	0	0	0	0	19.2	21.3
17695 – 23593	0	0	0	0	0	0	0	0	2300.0	25.0
23593 – 29492	0	0	0	0	0	0	0	0	0	52.4
29492 – 35390	0	0	0	0	0	0	0	0	0	59.4
35390 – 41288	0	0	0	0	0	0	0	0	0	40.8
41288 – 47186	0	0	0	0	0	0	0	0	0	52.7
47186 – 53085	0	0	0	0	0	0	0	0	0	86.5
53085 – 58983	0	0	0	0	0	0	0	0	0	30,400

Table C.6: Data containing running times in ms for the deterministic implementation of Zeus.

Clauses\Variables	1	2	3	4	5	6	7	8	9	10	11
0 – 17706	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17706 – 35412	0	0	0	0	0	0	0	0	1.00	1.00	1.00
35412 – 53119	0	0	0	0	0	0	0	0	0	1.00	1.00
53119 – 70825	0	0	0	0	0	0	0	0	0	1.00	1.00
70825 – 88531	0	0	0	0	0	0	0	0	0	0	1.00
88531 – 106237	0	0	0	0	0	0	0	0	0	0	1.00
106237 – 123943	0	0	0	0	0	0	0	0	0	0	1.00
123943 – 141650	0	0	0	0	0	0	0	0	0	0	1.00
141650 – 159356	0	0	0	0	0	0	0	0	0	0	1.00
159356 – 177062	0	0	0	0	0	0	0	0	0	0	1.00

Table C.7: Data containing ratio between optimal and heuristic solution found with Hera for Satisfiability instances.

Clauses\Variables	1	2	3	4	5	6	7	8
$0 - 656$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$656 - 1312$	0	0	0	0	0	0	1.00	1.00
$1312 - 1968$	0	0	0	0	0	0	1.00	1.00
$1968 - 2624$	0	0	0	0	0	0	0	1.00
$2624 - 3280$	0	0	0	0	0	0	0	1.00
$3280 - 3935$	0	0	0	0	0	0	0	1.00
$3935 - 4591$	0	0	0	0	0	0	0	1.00
$4591 - 5247$	0	0	0	0	0	0	0	1.00
$5247 - 5903$	0	0	0	0	0	0	0	1.00
$5903 - 6559$	0	0	0	0	0	0	0	1.00

Table C.8: Data containing ratio between optimal and heuristic solution found with Hera for Satisfiability instances.

Appendix D

User Manual to Program

This is a guide in using the Graphical User Interface to create, transform and solve instances of a given NPC problem.

D.1 Creating Instances

Creating an instance of a given NPC problem varies depending on what problem it is. The following sections will describe how an instance of each problem can be created and modified.

However all problems requires that a skeleton is made first. This is done by going to the `Problem Overview`-tab and selecting the NPC problem, that the user wants an instance of.

D.1.1 Satisfiability and 3-Satisfiability

Creating and modifying instances of Satisfiability is all done in the Console with the following commands.

Make: The command `make` proceeded by a positive integer, n, creates a new instance with n boolean variables e.g. "make 4" creates an instance with four boolean variables. Using this command multiple times will remove the old instance and create a new with the given number of variables.

Add: The command `add` proceeded by one or more integers adds a new clause to the instance e.g. "add 1 -2 3 -4" adds the clause $x_1 \vee \bar{x}_2 \vee x_3 \vee \bar{x}_4$.

NB! The integers must not be zero or have an absolute value greater than n, otherwise the command is invalid.

Remove: The command `remove` proceeded by a positive integers removes a given clause e.g. "remove 2" removes the second clause.

NB! If the integer is greater than the numbers of clauses then the command is invalid.

Creating and modifying instances of 3-Satisfiability is done exactly the same way as Satisfiability, however clauses now has to contain three literals.

D.1.2 Graph Problems

All the graph problems(Vertex Cover, Clique, Independent Set, Hamilton Cycle, Hamilton Path and Traveling Salesman),which contains a single graph, are created by point and click.

Left Click: Will place a vertex at the given location.

Right Click: Will begin a new edge at the nearest vertex to the mouse cursor. Right clicking again will end the edge at the nearest vertex to the mouse cursor. If the start and end vertex is the same, then the edge is canceled. Left clicking while creating a new edge will place a new vertex at the given position and end the edge at that vertex.

Mode: The command `mode` proceeded by either "insert" or "move", changes the current mode to insert and move mode respectively. In insert mode one can

place new vertices and create new edges as describe above. In move mode one can move vertices around using left click dragging. Note it is possible to move vertices even if the instance is locked for modification.

In instances of Traveling Salesman the edges have weights. The initial weight of new edges is 1. To change this weight for new edges the following command should be used.

Set: The command `set` proceeded by a positive integer changes the weight of new edges e.g. "set 1337" will give new edges the weight 1337.

D.1.3 Partition

Creating and modifying instances of Partition is done in the Console with the following commands.

Add: The command `add` proceeded by a positive integer adds a new value to the instance e.g. "add 1337" adds the value 1337 to the set.

NB! Giving integers larger than $2^{63} - 1$ will yield fierce and unforgiving retaliation from the program.

Remove: The command `remove` proceeded by a positive integers removes a given value e.g. "remove 2" removes the second value in the set.

D.1.4 Subset Sum

Creating and modifying instances of Subset Sum is done in the Console with the following commands.

Make: The command `make` proceeded by a positive integer creates a new instance e.g. "make 4" creates an instance with $B = 4$. Using this command multiple times will remove the old instance and create a new with the given value for B.

Add: The command `add` proceeded by a positive integer adds a new value to the instance e.g. "add 1337" adds the value 1337 to the set.

Remove: The command `remove` proceeded by a positive integers removes a given value e.g. "remove 2" removes the second value in the set.

D.1.5 Knapsack

Creating and modifying instances of Knapsack is done in the Console with the following commands.

Make: The command `make` proceeded by two positive integer creates a new instance e.g. "make 13 37" creates an instance with $B = 13$ and $K = 37$. Using this command multiple times will remove the old instance and create a new with the given values.

Add: The command `add` proceeded by two positive integer adds a new value to the instance e.g. "add 1 337" adds the value 1 to the first set and the value 337 to the other set.

Remove: The command `remove` proceeded by a positive integers removes a given value from the two sets e.g. "remove 2" removes the second value in both sets.

D.1.6 Bin Packing

Creating and modifying instances of Bin Packing is done in the Console with the following commands.

Make: The command `make` proceeded by a positive integer creates a new instance e.g. "make 4" creates an instance with 4 Bins. Using this command multiple times will remove the old instance and create a new with the given number of bins.

Add: The command `add` proceeded by a positive floating point adds a new value to the instance e.g. "add .1337" or "add 0.1337" adds the value 0.1337 to the set.

Remove: The command `remove` proceeded by a positive integers removes a given value e.g. "remove 2" removes the second value in the set.

D.1.7 3 Dimensional Matching

Creating and modifying instances of 3 Dimensional Matching is done in the Console with the following commands.

Make: The command `make` proceeded by a positive integer creates a new instance e.g. "make 1337" creates an instance with $q = 1337$. Using this command multiple times will remove the old instance and create a new with the given value for q.

Add: The command `add` proceeded by three positive floating point adds a new triple the instance e.g. "add 1 2 3" adds the triple $(1, 2, 3)$.

NB! If any of the values are greater than q then the command is invalid.

Remove: The command `remove` proceeded by a positive integers removes a given triple e.g. "remove 2" removes the second triple from the instance. It is also possible to remove a triple with `remove` proceeded by three positive integers e.g. "remove 1 2 3" removes the triple $(1, 2, 3)$.

D.1.8 Random Instances

For the NPC problems, Satisfiability and Subset Sum, it is possible to create a randomly made instance. To do this go to the `Problem Overview`-tab and select either the `Satisfiability` or `SubsetSum` node, which will take the user to the `Instances`-tab, where an instance of the given problem is selected. Then click the button, `Random Problem`.

Clicking it multiple times, will remove the old instance and create a new random instance.

D.1.9 Exiting

It is at any time possible to exit and close the program from the console using the command `exit`.

D.2 Transforming Instances

When an instance of a given problem is selected in the `Instances`-tab the available transformations can be seen in the top-left combo box. Any of these transformations may then be used by clicking the button next to the combo box.

This will transform the instance and select this transformed instance. This new instance will also appear as a subnode to original problem in the tree to the left.

NB! When a problem is transformed it is locked and may no longer be modified. The same goes for all transformed instances.

When a transformation has been used it can not be used again for the same instance, because it will only return a transformed instance equal to the previous.

D.3 Solving Instances

When an instance of a given problem is selected in the `Instances`-tab the available heuristic algorithms appear in the combo box in the topmiddle. Any of these heuristic algorithms can then be used by clicking the button next to the combo box.

A new solution will then be added to the topright combo box and the solution will be displayed or highlighted in the instance.

Because the heuristic algorithms are random they may be called many times, which will only add more solutions to the combo box containing solutions.

D.4 Finding Best Solution

For the Satisfiability problems it is possible to find the optimal solution with exhaustive search. This can be done by clicking the button `Find Best Solution`.

This will add a new solution called "optimal", which contains the optimal solution for the problem.

NB! Clicking the button twice will give an error message, since the optimal solution has already been found.

Also note that if the problem is to big e.g. contains more than 7 boolean variables then an error will occur.

D.5 Detransforming Instances

If a transformed instance has been chosen and this instance have one or more solutions, then the solution may be detransformed to the instance it was transformed from.

This is done by choosing the solution, which the user wants to transform and then clicking the button `Detransform`. This will detransform the solution all the way to the problem, which the user created initially.

NB! Clicking the button twice for the same solution will give an error, because the solution has already been detransformed.

Appendix E

API Manual

This is a manual describing how to extend the framework with new NP Complete(NPC) problems, transformations, heuristic algorithms and how the framework should be updated to support the extensions.

For the interested developer, there is also a guide on how to incorporate the new problems in the graphical user interface

E.1 New Problems

Every NPC problem is implemented as a class and the only requirement for new NPC problem is that they inherit the abstract class called `NPCProblem` and implement the method `getSize()`. The method `getSize()` should return an estimate of the instance size. This information is used when updating empirical data for each transformation.

Here is an example of how this is done with the NPC problem WeightedHamiltonCycle, which is a problem containing a single, weighted graph.

```
1  public class WeightedHamiltonCycle<V> extends problems.NPCProblem{
2      private Graph<V,Double> g;
3
4      public WeightedHamiltonCycle<V>(Graph<V,Double> g){
5          this.g=g;
6      }
7
8      public int size() { return g.vertices + g.getNoEdges();//
           estimate of objects size
9      }
10 }
```

E.2 New Transformations

Creating a transformation requires two new methods, which are both placed in the class that should be transformed. The first method should transform an instance and the second should detransform a solution from the transformed instance.

The method which transforms an instance may not take any arguments and the name of the method should have the prefix "transformTo" and it must return the transformed instance.

The detransform method takes as argument a solution from the transformed instance and it must return a detransformed solution. Furthermore the detransform method must have the prefix "detransform" and the rest of the method name must match that of the transformation method.

Here is a simple example of a transformation, which transforms an instance of WeightedHamiltonCycle to TravellingSalesman.

```
1  public class WeightedHamiltonCycle<V> extends problems.NPCProblem{
2      ...
3      public TravelingSalesman<V> transformToTS() {
4          return new TravelingSalesman<V>(g.copy());
5      }
6
7      public int[] detransformTS(int[] sol) {
8          int[] whc = new int[sol.length];
9          for(int i=0; i< sol.length; i++)
10             whc[i]=sol[i];
11         return whc;
12     }
13     ...
14 }
```

E.3 New Heuristic Algorithms

Supplying a new heuristic algorithm for a problem is merely a new method in the class, whose instances should be solved by the algorithm. The requirements for this method is that it has the prefix "algo", that it takes no arguments and that the return type is the solution for the given instance.

Here is a very simple algorithm for the problem WeightedHamiltonCycle:

```
1  public class WeightedHamiltonCycle<V> extends problems.NPCProblem{
2      ...
3
4      public int[] algoSNAFU() {
5          int[] whc = new int[g.vertices];
6          for(int i=0; i< sol.length; i++)
7              whc[i]=i;
8          return whc;
9      }
10
11     ...
12 }
```

E.4 Updating the Framework

To use the tool for automatically finding the shortest transformation path between two algorithms, it is necessary to inform the framework of the extensions.

The relationship between NPC problems are stored in an XML file and when new problems are added this file gets obsolete and a new has to be created.

To do this one has to inform what NPC problems the file now consists of, which is done as follows:

```
1  // Problems to be added
2  ArrayList<Class<? extends NPCProblem>> p;
3  p = new ArrayList<Class<? extends NPCProblem>>();
4  p.add(HamiltonCycle.class);
5  p.add(WeightedHamiltonCycle.class); ...
6
7  XMLHandler xml = new XMLHandler();
8  xml.saveDefault(p);
```

All the empirical data and the registered asymptotic running time for the transformation is then reset in the file "default.xml".

E.5 New GUI Problems

Given that one has created a new `NPCProblem` for the model, one can incorporate it in the GUI as follows.

For each `NPCProblem` one has to make a new class which inherits `GUIProblem`. The generic parameter of this class must take the `NPCProblem` class, which we want to represent.

Secondly it must have two constructors. The first is for user defined instances and must take no arguments. The second constructor is for transformed instances and must take the instance, a reference to the `GUIProblem` it was transformed from and the Method, which should be used for detransformation.

Inheriting `GUIProblem` requires one to implement the method `consoleInput(String)`, which is for handling the commands issued from the GUI console.

To control how an instance is visualized, one has to override the method *paint*(*Graphics*) as one usually would.

Continuing with the previous example one would get the following code:

```
1  public class GUIWHCycle extends GUIProblem<WeightedHamiltonCycle<
       XPoint>>{
2
3      public GUIWHamiltonCycle() {
4          super(GUIWHamiltonCycle.class, WeightedHamiltonCycle.class)
               ;
5      }
6
7      public GUIWHamiltonCycle(WeightedHamiltonCycle<XPoint> w,
           GUIProblem parent, Method dt) {
8          super(GUIWHamiltonCycle.class, w, parent, dt);
9      }
10
11     public boolean consoleInput(String command) {
12         // parse and obey command
13         // return false if it was a invalid command
14     }
15
16     public void paint(Graphics g) {
17         super.paint(g); problem.draw(g);
18     }
19
20     public String toString() {return "WHamiltonCycle";}
21 }
```

E.6 GUI Transformations

Making the transformations available in the GUI requires a method for each transformation. The method must take no arguments and the prefix has to be "guitransform".

Before a transformation method had to return a `NPCProblem` instance, now it has to return its `GUIProblem` instance instead.

The way it is done with the previous example is shown below:

```
1  public class GUIWHamiltonCycle extends GUIProblem<
       WeightedHamiltonCycle>{
2      ...
3
4      public GUITravelingSalesman guitransformTravelingSalesman() {
5          TravelingSalesman<XPoint> ts=problem.transformToTS();
6          Method df = getDetransformation("detransformTS");
7          return new GUITravelingSalesman(ts,this,df);
8      }
9
10     ...
11 }
```

E.7 GUI Update

In order to display and make the new NPC problems available in GUI on has to give the topframe the classes as argument. Furthermore one has to inform the frame how to place the different classes. An example of this is done below.

```
1  ArrayList<ClassPoint<? extends GUIProblem>> p;
2  p = new ArrayList<ClassPoint<? extends GUIProblem>>();
3
4  // Problems to be shown and used
5  p.add(new ClassPoint(GUIWHCycle.class, 350, 150));
6  p.add(new ClassPoint(GUITravelingSalesman.class, 250, 200));
7  p.add(new ClassPoint(GUIHamiltonPath.class, 400, 200));
8  ...
9
10 new MountOlympus(p);
```

Bibliography

[1] Gilad Bracha. Generics in the java programming language. A good all around description of generics in Java. Accessed: Feb 2, 2007. http://java.sun.com/j2se/1.5/pdf/generics-tutorial.pdf.

[2] Paul Fischer. *Computationally Hard Problems*. 2006.

[3] Emden Gansner, Eleftherios Koutsofios, and Stephen North. Drawing graphs with dot, 2006. A guide to drawing directed graphs with dot. Accessed: Feb 2, 2007. http://graphviz.org/Documentation/dotguide.pdf.

[4] Rajeev Motwani John E. Hopcroft and Jeffrey D. Ullman. *Introduction to automata theory, languages and computation*. 2003.

[5] Stephen C. North. Drawing graphs with neato, 2004. A guide to drawing undirected graphs with NEATO. Accessed: Feb 2, 2007. http://graphviz.org/Documentation/neatoguide.pdf.

[6] Ronald L. Rivest Thomas H. Cormen, Charles E. Leiserson and Clifford Stein. *Introduction to Algorithms*. MIT Press, second edition, 2001.